

# ENGINEER 3PX3 – Integrated Engineering Design Project

## Complex Report Final

Group FP-37

Alex Stewart, 400323167 | Syed Moosa Askari, 400336400 |

Shyavan Sridhar, 400388748 | Sean McCafferty 400332698

March 30, 2024

## Contents

|  |    |
|--|----|
| Contribution .....   | 3  |
| Change Log .....   | 3  |
| Problem Identification .....                                   | 5  |
| Detailed NVF and Conversion Factors .....                      | 5  |
| Initial Solution Comparison .....                              | 7  |
| Alternative #1 – Resistive heating .....                       | 7  |
| Alternative #2 – Air-Blown Heating System .....                | 8  |
| Technical Analysis Overview & Detailed NVF .....               | 9  |
| Analysis of Ideal Areas for System Implementation – Sean ..... | 9  |
| Power Consumption of Pump – Alex .....                         | 9  |
| Energy Requirements per Square Foot – Shyavan .....            | 10 |
| Designing Concrete Composition – Moosa .....                   | 10 |
| Fleshing out the NVF .....                                     | 11 |
| Sensitivity Analysis .....                                     | 11 |
| Optimization .....   | 13 |
| Final Design .....   | 17 |
| Project Plan .....   | 18 |
| Work Breakdown Structure .....                                 | 19 |
| Critical Path Method .....                                     | 20 |
| Task List .....  | 21 |
| NPV .....  | 21 |
| Risk Management .....  | 24 |
| Stochastic Sensitivity Analysis .....                          | 27 |
| Final Recommendations .....                                    | 28 |
| References .....   | 29 |
| Appendices .....   | 32 |
| NVF Additional Calculations .....                              | 32 |
| Technical Analyses .....                                       | 33 |
| Analysis of Ideal Areas for System Implementation – Sean ..... | 33 |
| Power Consumption of Pump – Alex .....                         | 34 |
| Energy Requirements per Square Foot – Shyavan .....            | 35 |
| Designing Concrete Composition – Moosa .....                   | 40 |

## Contribution

| Name              | Contribution  |
|-------------------|---|
| Sean McCafferty   | Problem, NVF, Technical Analysis, Final Design, Project Plan, Risk Management, WBS                      |
| Syed Moosa Askari | Alternative Solutions, Technical Analysis, Discount Rate, CPM, WBS, Final Recommendations               |
| Shyavan Sridhar   | Optimization, NVF, Technical Analysis, IRR calculations, Risk Management, WBS, CPM, Deterministic Model |
| Alexander Stewart | Sensitivity Analysis, Technical Analysis, Detailing NVF, NPV, CPM, WBS, TVM, Stochastic Model           |

## Change Log

| Change No. | Type     | Description/ Source of change | Original                        | New   | Section                         | Page                  |
|------------|----------|-------------------------------|---------------------------------|---|---------------------------------|-----------------------|
| 1          | Revision | TA feedback                   | Figure 7 captioned as table 3   | Figure 7's caption is now corrected                                     | Optimization                    | 15                    |
| 2          | Revision | TA feedback                   | Referencing diagram by location | Referencing diagram by Figure #   | Various                         | 10,12,13,17,20,22, 23 |
| 3          | Revision | TA feedback                   | Task list appears before CPM    | Task list appears after CPM   | Project Plan                    | 19                    |
| 4          | Revision | TA feedback                   | Inconsistent number formatting  | Consistent number formatting  | Final Design, Project Plan      | 17, 18                |
| 5          | Addition | Required                      | -                               | Addition of stochastic sensitivity analysis with Monte Carlo simulation | Stochastic Sensitivity Analysis | 25                    |
| 6          | Addition | Required                      | -                               | Addition of Final recommendations                                       | Final Recommendations           | 26                    |
| 7          | Addition | TA feedback                   | -                               | Addition of regular maintenance to task                                 | Project Plan                    | 19,20, 21             |

|   |          |             |   |  |                 |              |
|---|----------|-------------|---|--|-----------------|--------------|
|   |          |             |   | list, CPM,<br>and WBS  |                 |              |
| 8 | Addition | TA feedback | - | Time value<br>of money<br>conversion<br>factors<br>provided  | NPV             | 23           |
| 9 | Addition | TA feedback | - | Risks 4-6<br>added in<br>risk matrix,<br>provided<br>explanation<br>of updated<br>NVF, more<br>detail for<br>models. | Risk Management | 25,26,<br>27 |

## Problem Identification

While the current winter maintenance system for McMaster's main campus is adequate, there are areas for improvement with regards to safety and accessibility for students, staff, and visitors. Currently, McMaster hires workers for snow removal and salting across campus, as well as cordoning off certain areas to address snowfall [1]. However, exploring a solution with greater longevity and reduced maintenance costs could improve the quality for visitors and reduce the funds McMaster diverts to winterizing. For example, a radiant heating system embedded in the pavement across the campus to prevent snow and ice from accumulating might reduce the maintenance overhead from paying personnel to winterize these areas and prevent injuries due to slips and falls. Through this project, numerous positive benefits can be attained such as improved public relations, increased accessibility for users with disabilities and reduced maintenance and insurance costs. The project also provides environmental benefits by reducing the amount of salting and other chemical applications that would otherwise run off into local watersheds. Provided that the project's net value is significantly large to acquire funding from either the federal or provincial government or McMaster itself, it could then be implemented as determined to be most beneficial in areas around campus. The project's size would be constrained by the level of funding acquired from the University and the government. In order to meet the constraints of the project, a solution must produce a reduction in the maintenance costs compared to existing policy, reduce the number of personnel to be employed, and provide increased safety, accessibility, and comfort to the students, staff, and visitors by effectively facilitating the removal of snow build-up.

## Detailed NVF and Conversion Factors

Shown below is the proposed net value function to estimate the benefits and costs of the project to aid in determining the feasibility and scale for implementation. Each parameter has a corresponding conversion factor to help estimate the value in dollars per a related dependent unit. For the purpose of providing a rough estimate of the projects net present value we are operating on the assumption that the project size will be half a million square feet. Because of the nature of the project, there would be several observed benefits for underrepresented groups such as people with disabilities. Given the increased accessibility and safety from the project this may directly translate to increased attendance during the winter months, helping McMaster in its attempts to improve equity, diversity and inclusion. We also expect to receive a grant from the federal government under their Enabling Accessibility Fund which contributes up to three million dollars for mid-sized projects, as shown in our NVF. With that in mind, the reputation of the university would see similar positive impacts which would likely result in further increases in enrollment and subsequently larger quantities of tuition to help fund the university. These incoming students would likely come from aforementioned communities requiring increased accessibility. The effects on insurance would be directly proportional to the amount of ice related accidents that take place, and as the project improves safety in areas of high pedestrian traffic we would see a decrease in the quantity of accidents thereby reducing insurance costs. Another main benefit of the project is the minimization of winter maintenance costs, as there wouldn't be as much of a need for ice and snow removal. Costs reductions would account for the hourly wages for snow removal crews, as well as fees associated with the operation of equipment such as gasoline purchases. The environmental benefits of the project are mainly related to reducing the amount of salt and other potentially harmful chemicals used in traditional winterization policy. The less chemicals that run off into the McMaster watershed, the less money the university will have to spend in the future to meet their environmental targets. Regarded regulatory concerns, the project would ensure that McMaster continues to meet snow removal

requirements outlined by the municipal government. However, there are components of the NVF that decrease the total value such as construction related costs. The conversion factors for Construction Labour and Cost of Materials were chosen based on estimates of the cost of the systems piping, the cost of our specified mix design as well as contractor salaries. There are also ethical concerns relating to the sourcing of materials for the project and disruptions to vulnerable groups. All of the conversion factors and variables used in the NVF were based on educated assumptions and information from [2], [3], [4], [5], [6], [7], [8], [9], [10], [11] as well as the technical analyses explored further in the report. Below is the equation developed from the above data:

$$\text{Land Use} = 500,000 \text{ sqft}$$

$$\text{Time} = 1 \text{ year} = 52 \text{ weeks} = 365 \text{ days}$$

$$NVF = \frac{\$750000}{\text{year}} + \frac{\$600000}{\text{year}} + \frac{\$30000}{\text{year}} + \frac{\$720000}{\text{year}} - \frac{\$1400000}{\text{year}} - \frac{\$183169.79}{\text{year}} + \frac{\$26082}{\text{year}} = \frac{\$542912.29}{\text{year}}$$

$$\begin{aligned} NVF &= \text{Branding} \uparrow + \text{Accessibility} \uparrow + \text{Reduced Insurance} \\ &\uparrow + \text{Reduced Maintenance} + \text{Environmental Impacts} \uparrow - \text{Construction Costs} \\ &\downarrow - \text{Electricity Usage} \downarrow \end{aligned}$$

$$\text{Branding} = 50 \text{ incoming students} * \$15000 = \$750000 \text{ tuition}$$

$$\text{Accessibility} = \frac{\$3000000 \text{ in grants}}{5 \text{ years}} = \$600000 \text{ annually}$$

$$\text{Insurance Reduction} = 3 \text{ serious slips per year} * \$10000 = \$30000$$

$$\text{Reduced Maintenance} = \$720,000$$

$$\text{Construction Costs} = \text{Cost of Materials} + \text{Cost of Labour} = \frac{\$7000000}{5 \text{ years}} = \frac{\$1400000}{\text{year}}$$

*Cost of Materials*

$$\begin{aligned} &= \frac{\$316\text{CAD}}{\text{m}^3 \text{ of concrete mix}} * \frac{3539.6}{\text{m}^3 \text{ of concrete mix}} + \frac{\$1.05\text{CAD}}{\text{m of piping}} * \frac{152400}{\text{m of piping}} \\ &+ \frac{\$1478.71\text{CAD}}{\text{Pump}} = 1280012.31 \end{aligned}$$

$$\text{Cost of Labour} = 5719987.69$$

$$\text{Electricity Usage} = \text{Energy to Power Pump} + \text{Energy to Heat Water}$$

$$\begin{aligned} \text{Energy to Power Pump} &= 9.936 \text{ kW} \times 4 \frac{\text{hours}}{\text{day}} \times 36 \frac{\text{day}}{\text{year}} \times 0.13033 \frac{\text{CAD}}{\text{kWh}} \\ &= 186.47 \frac{\text{CAD}}{\text{year}} \end{aligned}$$

*Cost of Energy to Heat Water*

$$= \left( \frac{0.09 \text{ kWh}}{\text{ft}^2} * 500,000 \text{ ft}^2 * 36 \text{ snowfalls} * 0.13033 \frac{\text{CAD}}{\text{kWh}} \right) - (216000 \text{ kWh} * 0.13033 \frac{\text{CAD}}{\text{kWh}}) = \$182983.32 \text{ annually}$$

$$\text{Environmental Impacts} = \left( \frac{\$1150}{\text{Tonne of Salt}} \right) * \frac{90 \text{ kg} * 7 \text{ lane km} * 36 \text{ days}}{1000} = \$26082 \text{ annually}$$

$$\text{Construction Costs} = \frac{\$14}{\text{ft}^2} * 500000 \text{ ft}^2 = \$7000000$$

## Initial Solution Comparison

### Alternative #1 – Resistive heating

One alternative to the proposed solution was the use of resistive heating coils within the concrete. Advantages to this design are simplicity and reduced equipment such as tanks and pumps. The major disadvantages are high electricity costs and higher environmental impact. The resistive heater converts electricity directly to heat and is 100% efficient [12], however the process of electricity conversion is not as efficient with high energy consumption [12]. Heat pumps can have upwards of 50% less electricity consumption in contrast [12]. In addition, heat pumps can also help in re-distributing heat from other sources and recycling it to heat the water such as taking the heat produced from the server rooms on campus. The initial NVF composed from altering the information found from the previous solution and the above [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] for this concept was:

|  |
|--|
| $NVF = \frac{\$750000}{\text{year}} + \frac{\$600000}{\text{year}} + \frac{\$30000}{\text{year}} + \frac{\$720000}{\text{year}} - \frac{\$1400000}{\text{year}} - \frac{\$219803.6}{\text{year}} + \frac{\$5000}{\text{year}} = \frac{\$485196.40}{\text{year}}$ |
| $NVF = \text{Branding} \uparrow + \text{Accessibility} \uparrow + \text{Reduced Insurance} \uparrow + \text{Reduced Maintenance} + \text{Environmental Impacts} \uparrow - \text{Construction Costs} \downarrow - \text{Electricity Usage} \downarrow$           |

Comparing the NVF of the resistive heating system with the proposed solution, the environmental aspect is hindered by to the inability to transfer heat from McMaster's server rooms and in turn offset the heat required. This means it is not as efficient as the proposed design. The value derived for accessibility will remain the same as the proposed design since they both serve the same purpose and will receive a federal accessibility grant. Similarly, the money saved from insurance reduction would also remain equal since safety would equally improve. The maintenance cost would be greater than the proposed design, however comparing the resistive heating system to the current system of snow removal, it its more efficient, ultimately resulting in a net positive value of \$485,196.40. The cost of construction would be greater than that of the radiant heating system due to the implementation of resistive heating coils which cost a larger amount per square feet. The electricity usage would be significantly higher than the radiant heating system since this heating system requires the production of electricity via the coils. Finally, there is a greater negative environmental impact of this alternative design due to the excessive energy consumption which is shown in the decreased conversion factor.

## Alternative #2 – Air-Blown Heating System

Another alternative to the design was a system that used heated air to melt the snow. This design is comprised of an air heater capable of generating hot air at high temperatures and a blower motor which would, in tandem, heat the walkways and melt the snow. One advantage of this alternative system was the ability to provide targeted heating which would be beneficial where certain areas have excess amounts of snow. However, there are definite drawbacks for this design including the heat lost to the surrounding environment, the significant amount of energy required to maintain operational temperatures, and the fact that it would require powering a motor in addition to a resistive heater. These all reduce feasibility, especially considering the project is outdoors. The initial NVF composed from altering the information found from the previous solution and the above [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] for this concept was:

|  |
|--|
| $NVF = \frac{\$500000}{year} + \frac{\$600000}{year} + \frac{\$30000}{year} + \frac{\$620000}{year} - \frac{\$1600000}{year} - \frac{\$225000}{year} + \frac{\$10000}{year} = \frac{\$-65000}{year}$   |
| $NVF = \text{Branding} \uparrow + \text{Accessibility} \uparrow + \text{Reduced Insurance} \uparrow + \text{Reduced Maintenance} + \text{Environmental Impacts} \uparrow - \text{Construction Costs} \downarrow - \text{Electricity Usage} \downarrow$ |

For the NVF of this alternative design, the branding conversion factor is worse due to the lack of heat transfer and recycled heat, which would have a negative appeal as there is no recyclability of energy. Despite having a lower NVF than the radiant heating system the overall effect is a net positive due to the university still receiving other similar positive benefits. Like the resistive heating and radiant heating systems the accessibility grants and savings on insurance would remain the same for the aforementioned reasons. Compared to the radiant and resistive heating systems the air blown heating system would have higher repair costs due to the cost of fans and heating block, reflected in the maintenance conversion factor. The cost of construction would be the highest out of the three designs, due to the placement of fans and heating units scattered throughout the campus. The cost of electricity would be significantly greater than the two systems mentioned previously due to the cost to power the fans and heating units scattered throughout the campus.



## Technical Analysis Overview & Detailed NVF

### Analysis of Ideal Areas for System Implementation – Sean

Given that funding for the project may be limited, it is important to establish key areas to prioritize within the NVF. To do this, factors for consideration were established to help decide which areas were of most importance. These factors include:

1. Quantity of foot traffic during peak hours
2. Preestablished accessibility zones
3. Areas of higher slope or grade

To maximize value in the project, McMaster should apply a conversion factor to help quantify the benefit to affected parties. This can be done by quantifying the opportunity costs of implementing the project in a function so that the value can be determined for a variety of situations.

This would quantify the value of several different costs and benefits, which impact the value of the project depending on where it is implemented. The total size of the project impacts several areas, such as the accessibility benefits, the opportunity costs of not having to remove snow or salt, the cost of power to heat the walkways annually as well as the construction costs.

Based on the findings of the technical analysis, highlighted areas include the Childrens Hospital, main arteries between large buildings, accessible parking lots and primary pedestrian entrances.

### Power Consumption of Pump – Alex

Finding the amount of electrical and mechanical power required by the system to pump water around the radiant heating loop is important because that will indicate the size of pump required, and the electrical cost of running the pump. Initial results found were about \$186.48 CAD per year to run the pump, operating at 7.5 kW mechanical power, and 9.9 kW electrical; this was for a 10 km length of 12 mm diameter tubing and assuming laminar flow of 4°C water. The general formulas used for this analysis were:

$$\dot{W}_{motor} = \frac{\dot{W}_{pump}}{\eta_{motor}}$$

The efficiency of the motor was assumed to be 75% [determining electric motor load and efficiency] and the pumps was 90%[how to calculate pump efficiency].

$$\dot{W}_{pump} = \frac{\rho \dot{V} g h_{pump}}{\eta_{pump}}$$

Density was assumed to be  $1000 \text{ kg/m}^3$ , gravity  $9.81 \text{ m/s}^2$ . Volumetric flow rate of  $2.831376 \times 10^{-3} \text{ m}^3/\text{s} = v_{avg} A$ .

$$h_{pump} = h_{L_f} + h_{L_m} = \frac{f L v_{avg}^2}{2 D g} + \frac{k_L v_{avg}^2}{2 g}$$

The length of pipe was assumed to be 10 km and an average velocity of  $0.298799834 \text{ m/s}$ . The pipe has a diameter of  $0.012065 \text{ m}$ ,  $f$  of  $0.027826087$ .

The temperature of water used was because 4°C is water in its densest form, meaning that the power required by the pump would be less at higher temperatures, since the water would be less dense.

As mentioned, it was found that this system would require 9.9 kW of electrical power for the motor. Assuming an average run time of 4 hours per day, running 36 days per year gives the final cost per year, and that the cost of electricity would be the average of the different pricing peak per kWh (\$0.13033  $\frac{CAD}{year}$ ).

$$186.47 \frac{CAD}{year} = 9.936 kW \times 4 \frac{hours}{day} \times 36 \frac{day}{year} \times 0.13033 \frac{CAD}{kWh}$$

### Energy Requirements per Square Foot – Shyavan

It is also necessary to identify the energy required to melt the snow via the radiant system. For this, the heat transfer equations for both conduction and radiation were used as those were deemed the most important methods of heat transfer for this system [13]. They are provided below:

$$\dot{q} = -k\nabla T$$

$$\dot{q} = \sigma eA(T_{snow}^4 - T_{inf}^4)$$

An FEM model was built in FlexPDE for a 3" thick slab of concrete [14] that was 1 ft<sup>2</sup> with an 8.64 cm thick slab of snow covering the same area on top from the average snowfall data for Hamilton in January [15]. The temperature was set to be -4°C from [15] and windspeed was assumed to not affect the analysis. The system's output power was also set at 37 W/ft<sup>2</sup> as per [16]. Further assumptions were made about the thermal and mechanical properties of the snow and concrete as per [17], [18], [19], [20], [21], [22] (*Refer to Appendices*). The energy required to raise the snow to 0°C was determined as 57720 J. Due to the concrete layer, the system was only about 12% efficient, and extrapolating this and calculating the energy for the latent heat of fusion for the snow, the total energy required would be 2,363,758 J/ft<sup>2</sup> for this system. Note however that this value is assuming the entire block of snow to melt as a whole, not factoring in fluid dynamics, and so, the real-world value would likely be around 15% of this.

### Designing Concrete Composition – Moosa

The concrete must be able to withstand the heat from the radiant heating system, while maintaining its strength and integrity. It was estimated that a strength of 40 MPa would prevent breaking under load [23]. The concrete must be air entrained so that there are air bubbles in the concrete so when its cooler the water may expand into the bubbles, increasing the concrete's resistance to damage caused by freeze-thaw cycles [23]. The slump must not be too runny, so it doesn't affect longevity, so the ideal slump estimation is 125 mm. Furthermore, the determined water cement ratio was 0.4. Using tables provided CIVENG 3P04, the water required is 197 kg/m<sup>3</sup> (*Refer to Appendices*). The required cement weight is calculated using the equation below:

$$Required\ Cement\ Weight = \frac{W}{\frac{w}{c} ratio} [23]$$

The bulk volume of coarse aggregate was 0.645 determined via the tables [23] and air makes up 7% [23] of the mix. With all this in mind, the design mix was created for 1 m<sup>3</sup> of concrete, this would entail 0.197 m<sup>3</sup> of water, 0.156 m<sup>3</sup> of cement, 0.07 m<sup>3</sup> of air, 0.388 m<sup>3</sup> of coarse aggregate and 0.189 m<sup>3</sup> of fine aggregate. This was calculated after adjusting the weight for 1 m<sup>3</sup> of concrete. To find the volume for fine aggregate, the following equation was used:

$$V_{FA} = 1 - (V_{water} + V_{cement} + V_{air} + V_{coarse\ aggregate}) [23]$$

## Fleshing out the NVF

The NVF determined above can now be factored into 4 main decision variables, Project Size in sqft, Electricity Rate in \$/kWh, the number of computers used to offset the heat energy required, and the heat output of those computers. The values for those were chosen based on [8], [24], [25]. From here, it is possible to now complete the optimization.

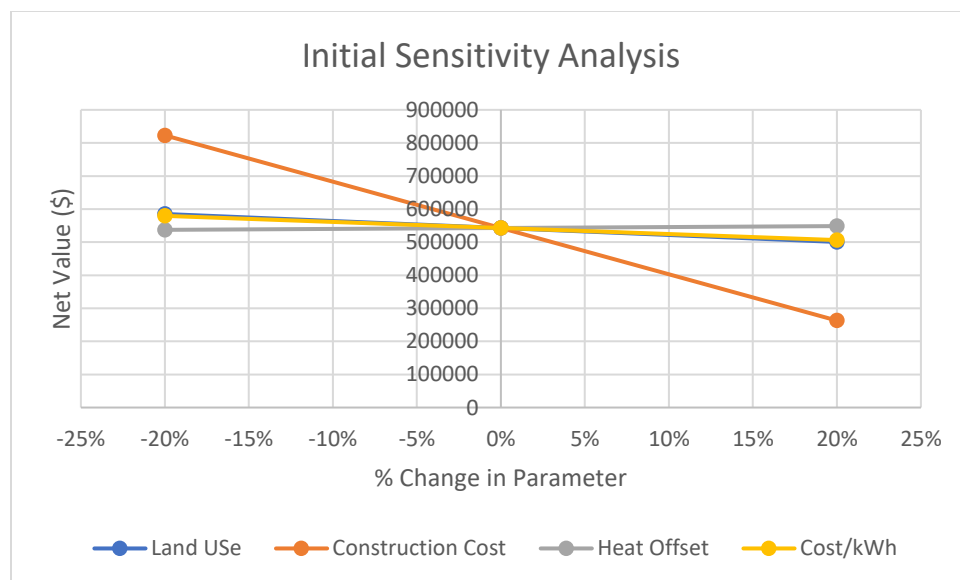
$$NVF = \left( \frac{\text{All Benefits}}{\text{sqft}} - \frac{\text{Materials Cost}}{\text{sqft}} - \frac{\text{Labour Cost}}{\text{sqft}} \right) \text{Project Size} - \frac{\text{Energy Cost}}{\text{Year}} (\# \text{ of Computers} * \text{Heat Output} * \text{Electricity Rate}) = \frac{\text{Net Value}}{\text{Year}}$$

## Sensitivity Analysis

Based on the sensitivity analysis as seen in *Table 1*, it is possible to gauge how each parameter affects the project should costs balloon, or in the event that the project proves cheaper than expected. Based on *Figure 1* it is evident that the projects most sensitive parameter is its construction costs, as they make up the bulk of the expenditures.

*Table 1 - Table for Construction of Spider Plot*

| Parameters        | 20%         | 0%          | -20%        |
|-------------------|-------------|-------------|-------------|
| Land Used         | 600000      | 500000      | 400000      |
| NV                | 500685.2859 | 542912.2059 | 585139.1259 |
| Construction Cost | 1680000     | 1400000     | 1120000     |
| NV                | 262912.2059 | 542912.2059 | 822912.2059 |
| Heat Offset       | 259200      | 216000      | 172800      |
| NV                | 548542.4619 | 542912.2059 | 537281.9499 |
| Cost/kWh          | 0.156396    | 0.13033     | 0.104264    |
| NV                | 506315.5419 | 542912.2059 | 579508.8699 |



*Figure 1 - Initial Sensitivity Analysis*

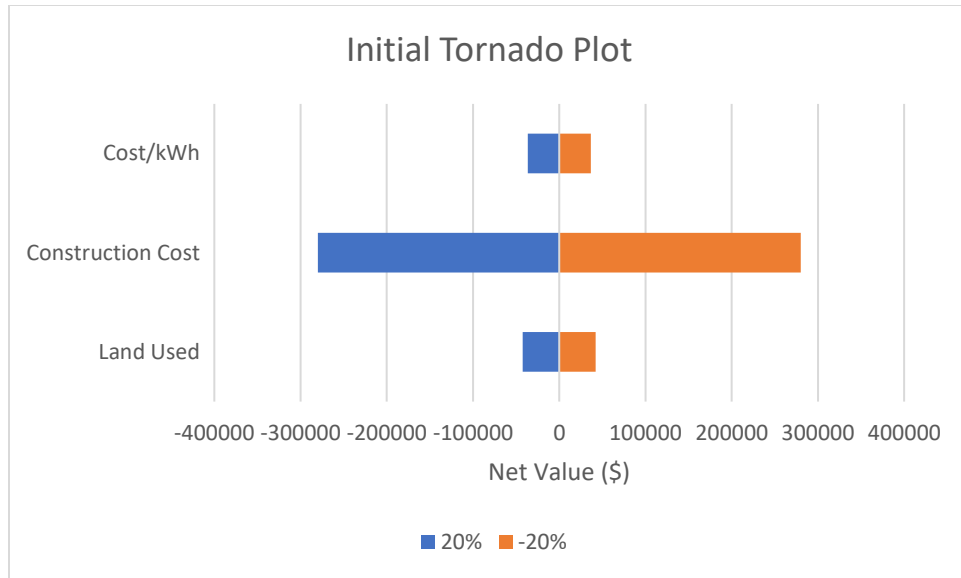


Figure 2 - Tornado Plot of Initial Values

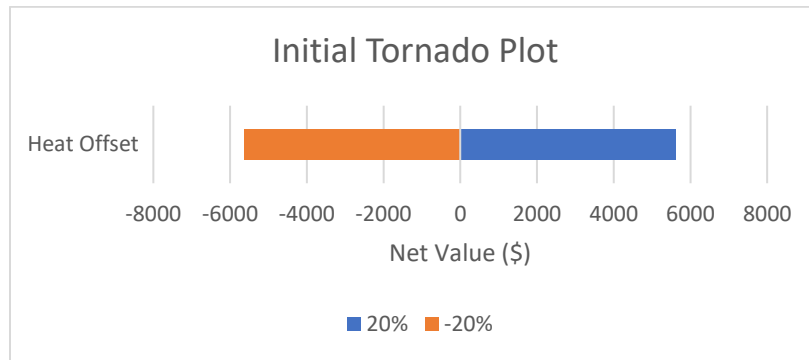


Figure 3 - Tornado Plot of Initial Values (Magnified for Small Quantities)

Overall, from the sensitivity analysis it was found that the cost of construction was the most sensitive to changes in their conversion factors, and thus going forward it would be ideal to decrease the of construction per amount of land; this would have the greatest affect on increasing the net value of the design. Apart from that factor, the remainder still have an effect, albeit not as large. From the analysis it was found that the least sensitive conversion factor was the cost per kWh; this indicates that trying to reduce the power consumed by the system is not a top priority and will also help reinforce the decision to use the radiant heat system for its power efficiency.

## Optimization

$$\max_{x,y,z} \phi = \text{NetValue}(x,y,z)$$

$$\begin{aligned} \text{NetValue}(x,y,z) &= -xy + 316 \times 0.00141584x + 1.05 \times 0.06096x + 295.742 \\ &+ 0.09x \times 36 \times 0.13033 - (3z \times 4 \times 36 \times 0.13033) + 9.936 \times 4 \times 36 \times 0.13033 \\ &+ 1.2x + (50 \times 0.03 + 3 \times 2.88/36/4 + 207.36/36/4 + 0.052164)x \\ &s. t. \end{aligned}$$

$$x = \text{Project Size (sqft)}, \quad y = \text{Labour Rate} \left( \frac{\$}{\text{sqft}} \right), \quad z = \# \text{ of Computers}$$

$$100,000 \leq x \leq 1,000,000$$

$$1.29 \leq y \leq 3.29$$

$$0 \leq z \leq 1,000 \quad (z \in \mathbb{Z})$$

Equation 1 - Objective Function

Table 2 - Optimization Setup in Excel

| Optimization              |                   |                             |                   |                |
|---------------------------|-------------------|-----------------------------|-------------------|----------------|
| NVF Components            | Cost              | Parameters                  | Conversion Factor |                |
| Labour Cost               | \$1,290,000.00    | Cost of Concrete            | \$316.00          |                |
| Energy Cost (Minimum \$0) | \$366,153.11      | Runtime (hours/day)         | 4                 |                |
| Offset Heat               | \$56,302.56       | # of Students Enabled (int) | 50                | environmental  |
| Materials Cost            | \$511,709.18      | # of Snowfalls (int)        | 36                | maintenance    |
| Grant (up to \$600000)    | \$600,000.00      |                             |                   | insurance      |
| All Other Benefits        | \$3,052,164.00    |                             |                   | tuition        |
| Decision Variables        | Conversion Factor | Min Constraint              | Type of Equality  | Max Constraint |
| Project Size (sqft)       | 1000000           | 100000                      | $\leq x \leq$     | 1000000        |
| Labour Rate (\$/sqft)     | \$1.29            | \$1.29                      | $\leq x \leq$     | \$3.29         |
| # of Computers            | 1000              | 0                           | $\leq x \leq$     | 1000           |
| Total Net Value           |                   |                             |                   |                |
| \$1,484,301.70            |                   |                             |                   |                |

As evidenced in Table 2 the optimization was done on 3 major decision variables. These all had a non-linear relationship with the NVF and so the GRG Nonlinear solver was used as it is more efficient than evolutionary. We are unable to use a simplex solver because it is only used in linear models. In the end, it was determined that maximizing the project size was advantageous as well as minimizing the electricity rate and increasing the number of computers and heat output from each computer. These results indicate that the designed solution is best implemented on as large a scale as possible. All calculations were made based on the NVF formula and solved in Excel with no extensive calculations required.

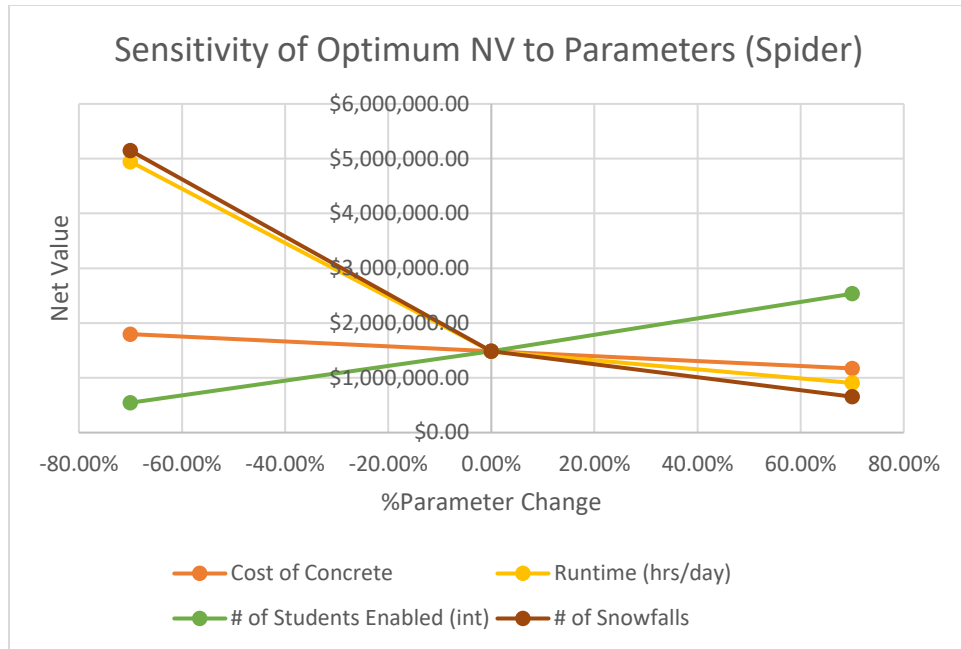


Figure 4 – Optimized Net Value Sensitivity Analysis (Spider Plot)

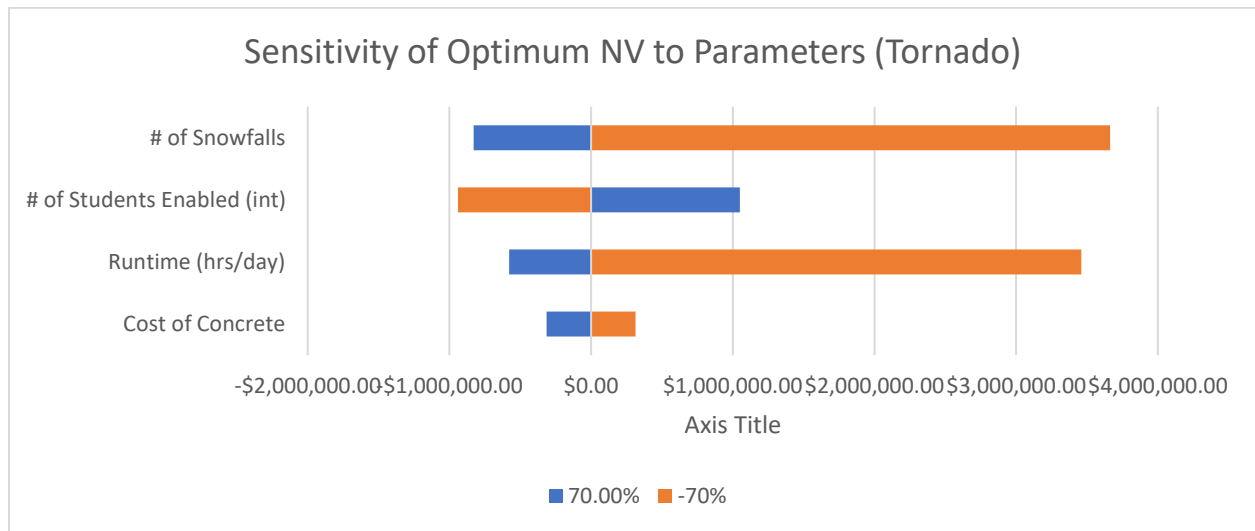


Figure 5 - Tornado Plot for Optimized Values for NV

From the optimized sensitivity analysis, the number of snowfalls were found to have the greatest effect on the net value, which is unfortunately out of our control. The effects of energy costs, heat offset, and material costs were found to be negligible. Given that the project would likely be on a lowest bid basis, this could help us to reduce the labour costs and cost of materials, further providing increases to net value. *Figure 6, Figure 7, Figure 8 and Figure 9* represent different parameters that we have no control over, but show the effect on net value up to a 70% increase or decrease. The parameters that have flat slopes in the spider plot had their corresponding tornado plots omitted as they would appear to be blank.

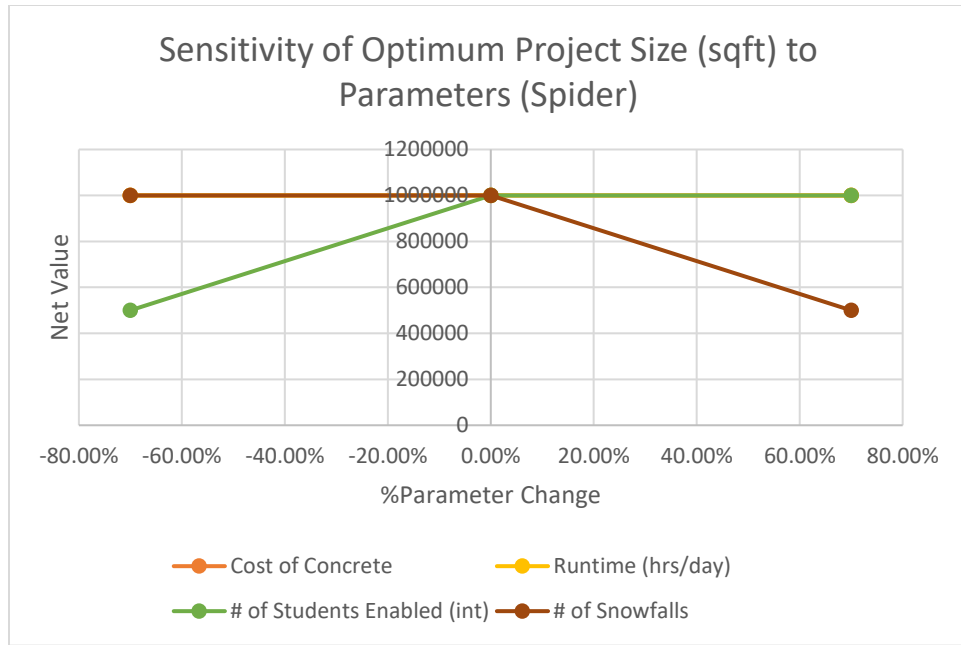


Figure 6 - Optimized Project Size Sensitivity Analysis (Spider Plot)

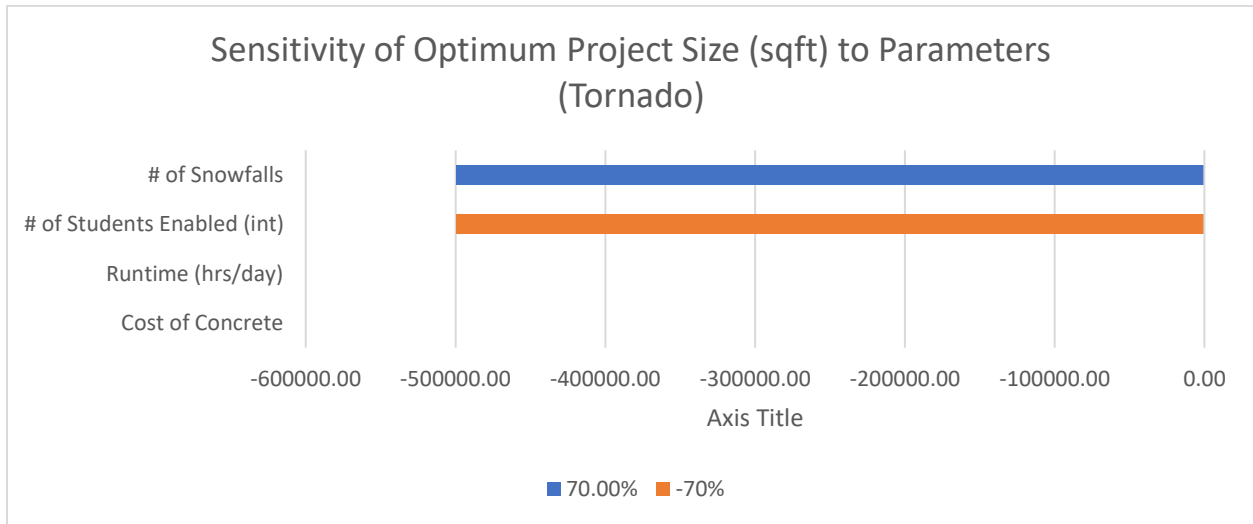


Figure 7 Tornado Plot for Optimized Project Size

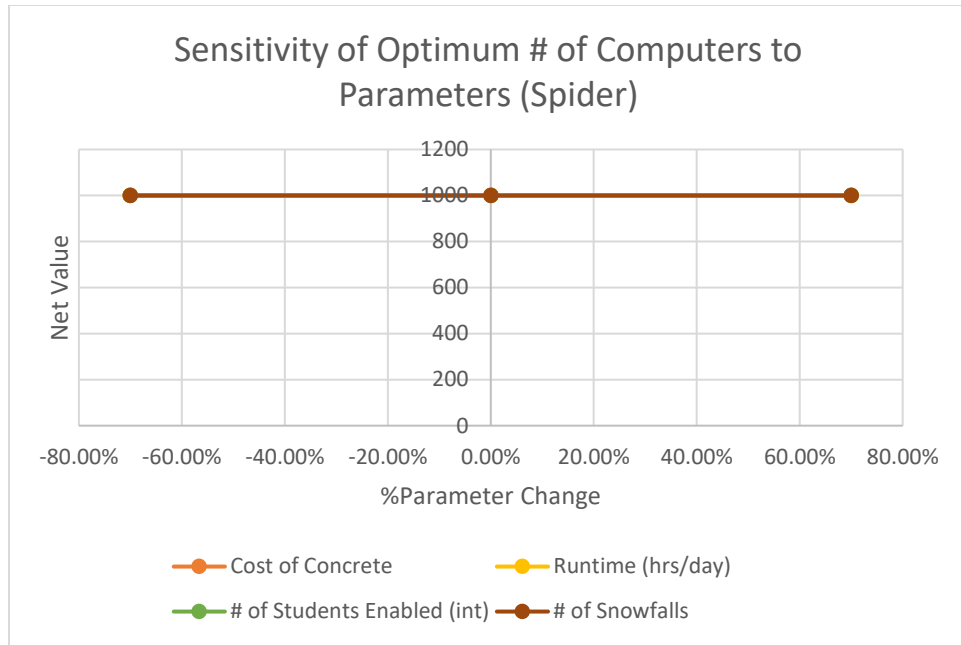


Figure 8 - Optimized # of Computers Sensitivity Analysis (Spider Plot)

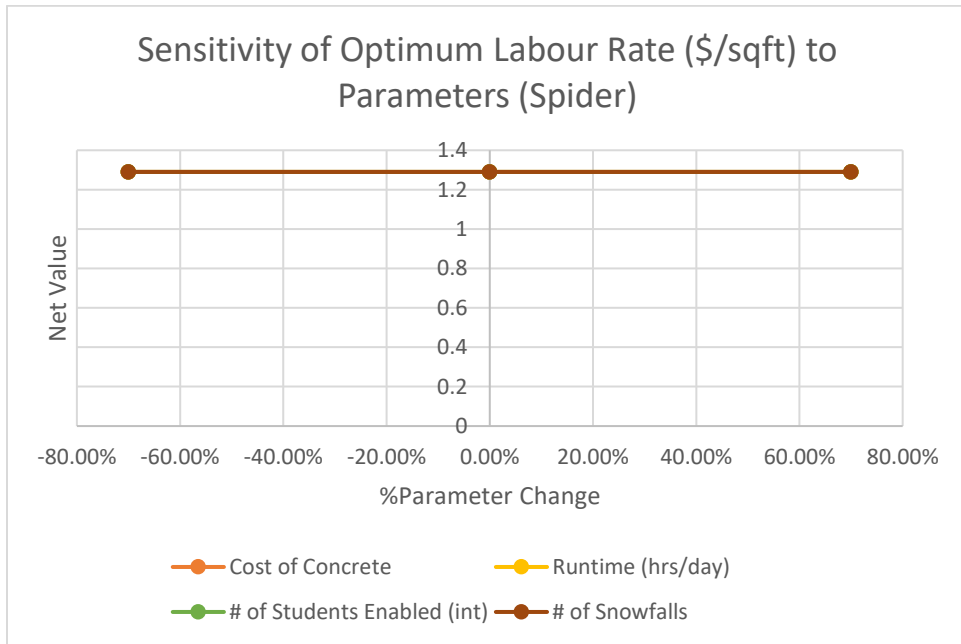


Figure 9 - Optimized Net Value Sensitivity Analysis (Spider Plot)



## Final Design

Using all of the processes above, the solution moving forward is decided to be the radiant heating system design consisting of hydronic tubing encased within the sidewalks, parking lots and roadways around McMaster's main campus. This will enable addressing all the original project objectives as mentioned in the problem identification section. Based on the findings of the technical analyses, sensitivity analysis and optimization analysis, several things have been determined. The proposal has covered all the necessary regulatory, ethical, environmental and EDI concerns, providing a net value function with a final value of \$1,193,073.27. This value was higher than the other alternative solutions due to their increased energy and maintenance costs as well as their environmental impacts. Based on the results of the sensitivity analysis and the optimization, it was determined that increasing the decision variables of project size would be highly beneficial for the project. When the project size is increased to one million square feet the NVF increases significantly from \$542,912.29. Finding ways to reduce labour costs also provide increased value, so choosing contractors based on this should help in this. Other factors that impact the net value are decreasing energy costs and the amount of heat we can recycle from other McMaster sources such as computers. The limitations of the project relate to the dependence on less tangible factors for benefits, such as the effects of branding on McMaster's enrollment. The project's value also relies on maintenance of the system remaining lower than the existing fees. The NVF's value had changed significantly as we amended the optimization section, with the optimum project size being adjusted to one million square feet and the standard labour rate for the contract bidding being lowered.

## Project Plan

In order to effectively plan the project delivery, the life cycle of the project was mapped in several areas to provide stakeholders with an idea of the stages of the project as well as their duration. The initiation stage of the project is broken up into four subtasks, with problem identification, stakeholder and cost benefit analysis taking place concurrently across the first two weeks of the project. Following this, the client pitch would occur over the next two weeks to lead us into the planning stage. This stage begins with possible solution proposals occurring from weeks four to six, with net value identification happening concurrently. Once this has occurred technical analysis' will be conducted in order to narrow the possible solutions down. Optimization and Sensitivity analysis will occur in weeks nine to twelve to allow for the final design implementation to round off the planning stage in week fifteen. This moves the project into the execution stage, wherein affected areas around campus are identified and site preparation begins for the initial construction cycle. Construction, site preparation, and safety inspection are spread through over the span of 5 years where construction would occur during the four months of summer. This would be beneficial since student population would be at a yearly low during the four months in addition to the weather allowing for construction to continue with minimal delays. Post construction, the safety inspection to ensure that all project infrastructure meets regulatory requirements and is functioning as intended. Several years later the retrospective performance analysis is conducted to observe whether the projects estimated value held up in reality. Finally, at the end of the life cycle of the project the materials involved will be recycled where applicable to help improve the environmental aspects of the project.

Using the critical path method, as seen in *Figure 11*, the sequence of the tasks over the life span of the project are further outlined to show what would occur if an activity were delayed. The critical path is displayed using the red arrows that go through each critical node. For this project the critical path begins with problem identification, followed by client pitch, solution proposal, technical analysis, optimization and sensitivity analysis, final design, identification of affected areas, site preparation, construction, safety inspection, and regular maintenance. The activities that aren't part of the critical path are those that can afford to be delayed without affecting the subsequent task in line. Despite problem identification, stakeholder analysis and cost benefit analysis having the same start and end day, it was assumed that problem identification is the most critical of the three. Note that while there does appear to be a fairly large respite with minimal being done between the end of construction and the start of the recycling (Tasks 3.3 and 4.4), this is because this is the expected operational lifetime of the system. The system requires extremely minimal regular maintenance during this time, about once every decade, with a performance analysis being done simply some point after the first five years of operation to confirm the project is working as intended.

We also determined the NV impact of each task in nominal terms, shown in *Table 3*, the task list. The sum of all tasks in the initiation stage was -\$200,000.00, with the planning stage costing \$250,000.00. The execution stage had a nominal NV impact \$3,738,289.95 across years 2-6, with the final closure stage having an NV impact of \$947,207.68.

Any values provided in this section were cited from the work breakdown structure, *Figure 10* (CPM diagram) and *Table 3* (task list), shown below.

## Work Breakdown Structure

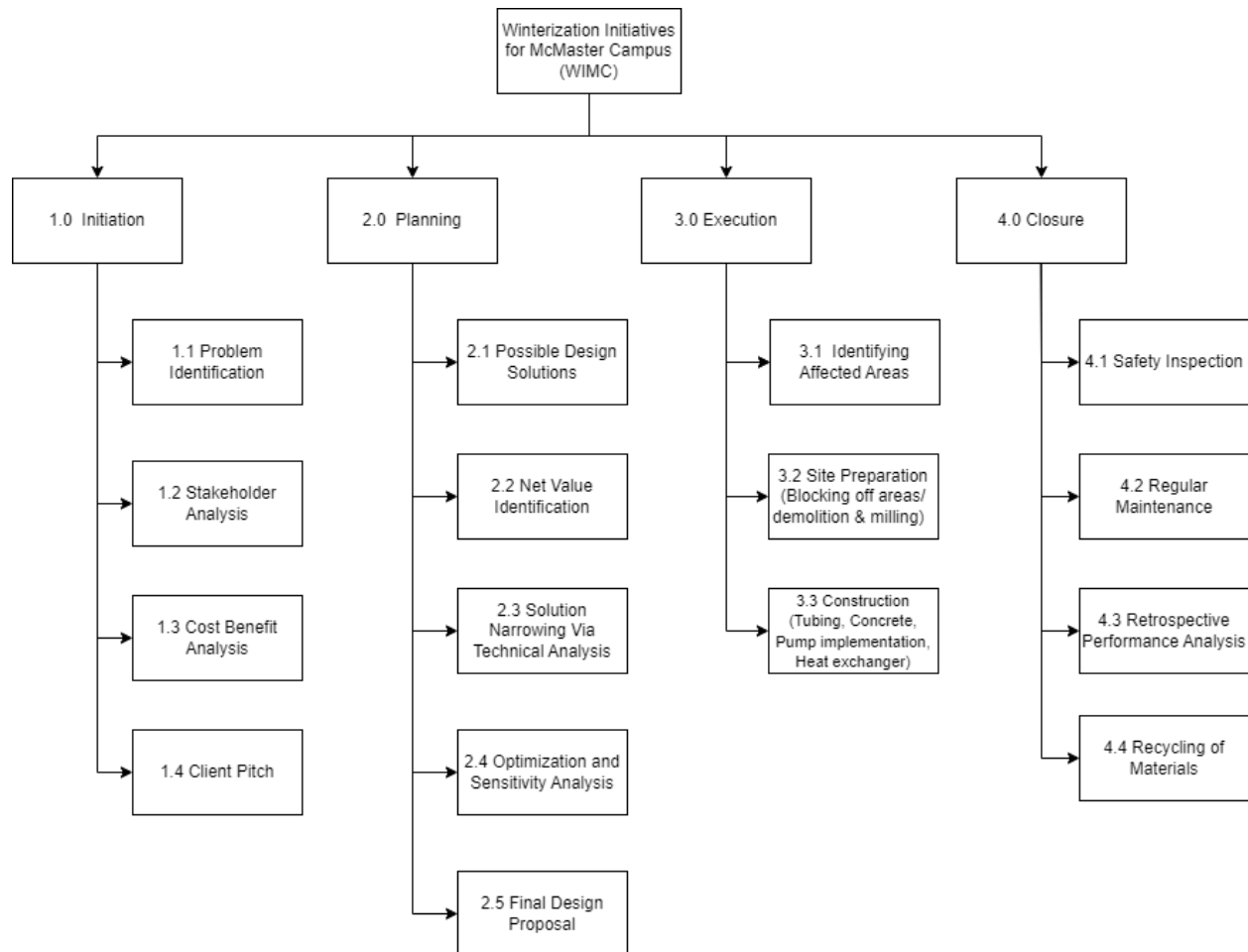


Figure 10 - Work Breakdown Structure for the Project Lifetime

## Critical Path Method

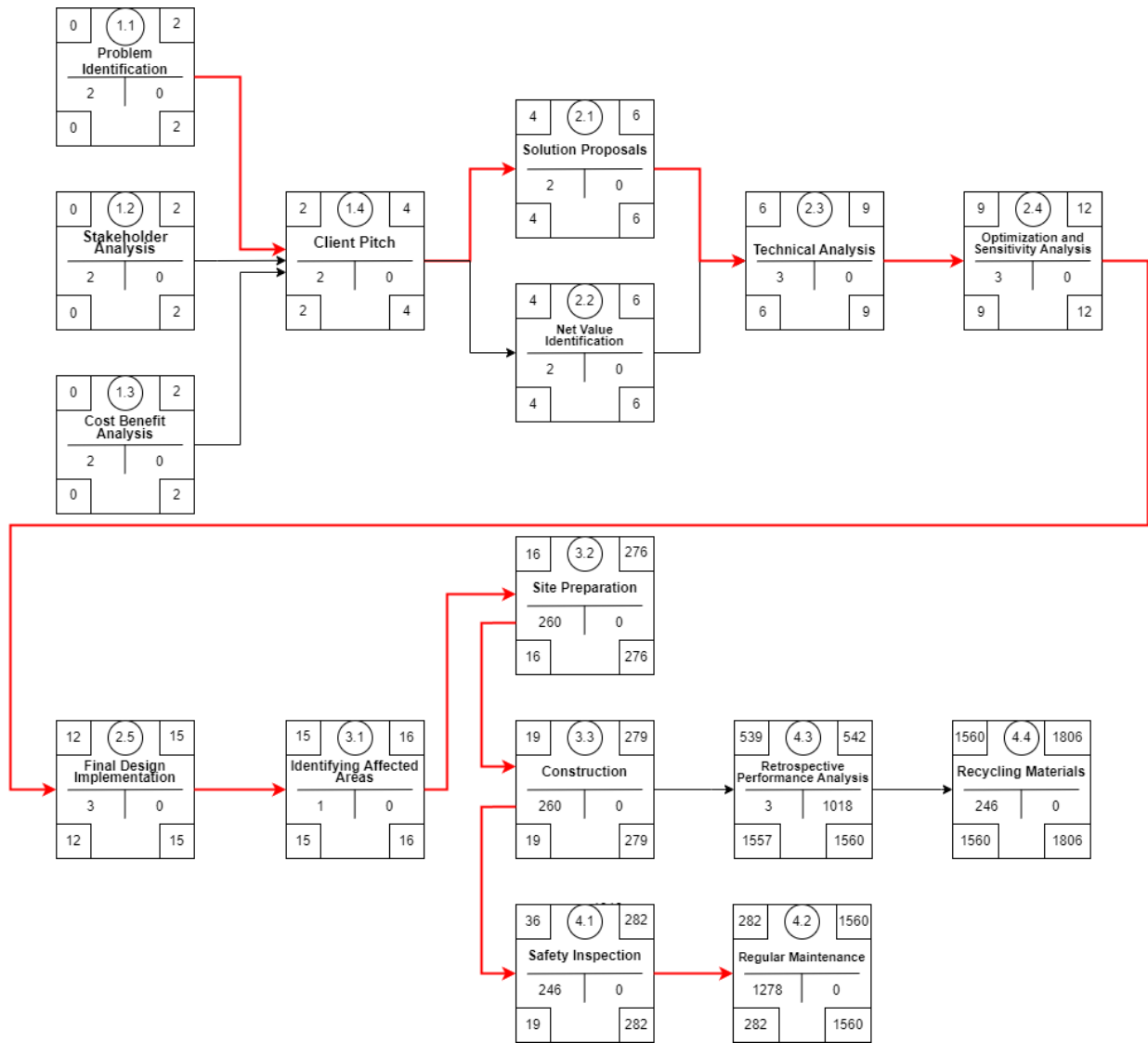


Figure 11 - Project Lifetime Critical Path Method

## Task List

Table 3 - Initial Nominal Task List

| Task ID | Task Description   | Start Date | End Date | Duration (Weeks) | Nominal Net Value |
|---------|--|------------|----------|------------------|-------------------|
| 1       | Initiation   | 0          | 4        | 4                | -\$200,000.00     |
| 1.1     | Problem Identification   | 0          | 2        | 2                | -\$50,000.00      |
| 1.2     | Stakeholder Analysis   | 0          | 2        | 2                | -\$50,000.00      |
| 1.3     | Cost Benefit Analysis  | 0          | 2        | 2                | -\$50,000.00      |
| 1.4     | Client Pitch   | 2          | 4        | 2                | -\$50,000.00      |
| 2       | Planning   | 4          | 15       | 11               | -\$250,000.00     |
| 2.1     | Possible Solution Proposals  | 4          | 6        | 2                | -\$50,000.00      |
| 2.2     | Net Value Identification   | 4          | 6        | 2                | -\$50,000.00      |
| 2.3     | Narrowing Solutions Via Technical Analysis                           | 6          | 9        | 3                | -\$50,000.00      |
| 2.4     | Optimization and Sensitivity Analysis                                | 9          | 12       | 3                | -\$50,000.00      |
| 2.5     | Final Design Implementation  | 12         | 15       | 3                | -\$50,000.00      |
| 3       | Execution  | 15         | 279      | 264              | \$3,550,499.55    |
| 3.1     | Identifying Affected Areas   | 15         | 16       | 1                | -\$523,657.92     |
| 3.2     | Site Preparation (Blocking off areas/demolition & milling)           | 16         | 276      | 260              | \$1,808,052.49    |
| 3.3     | Construction (Tubing, Concrete, Pump implementation, Heat exchanger) | 19         | 279      | 260              | \$2,266,104.98    |
| 4       | Closure  | 36         | 1806     | 1770             | \$947,207.68      |
| 4.1     | Safety Inspection  | 36         | 282      | 246              | \$226,342.08      |
| 4.2     | Regular Maintenance  | 282        | 1560     | 1278             | -\$10,000.00      |
| 4.3     | Retrospective Performance Analysis                                   | 539        | 542      | 3                | \$700,000.00      |
| 4.4     | Recycling Materials  | 1560       | 1806     | 246              | \$20,865.60       |

## NPV

Figure 12 is the value flow diagram for the project for each successive year. Each year has its associated costs and benefits, with most of the project benefits being added each year after the respective construction cycle is finished. The initial year has a low NPV because only administrative costs are associated with the initiation and planning stages. Year two would see large losses in value as there are only construction fees with no observed benefits. The following years begin to add value as the benefits of each successive completed construction zone take effect. While the construction of the project ends in year six, a seventh year was added to demonstrate the value of the project in the closure stage.

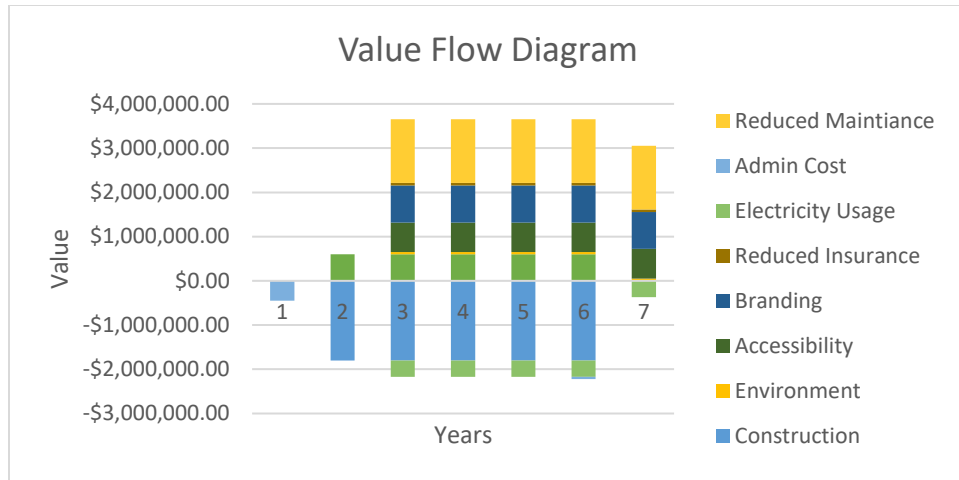


Figure 12 - Flow of the Project's Positive and Negative Value with Time

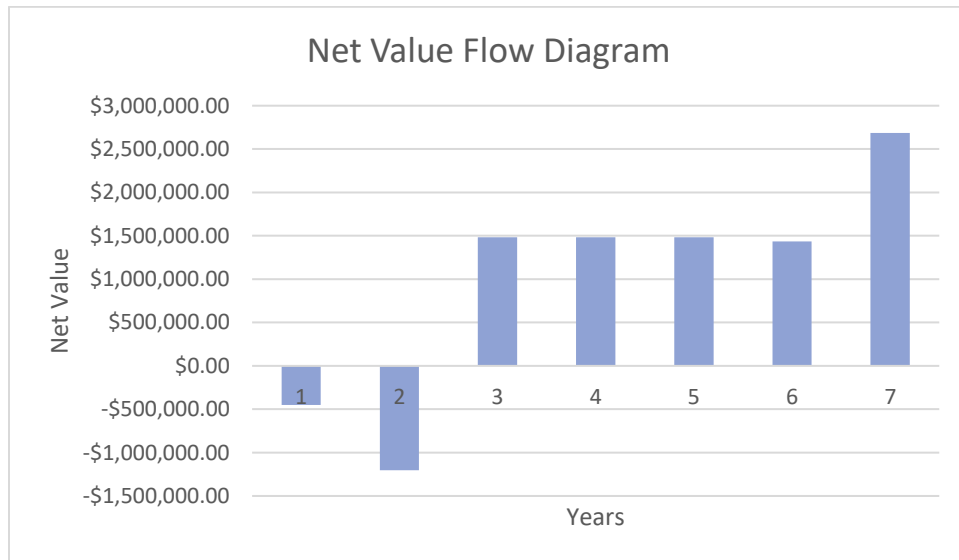


Figure 13 - Flow of the Project's Net Value with Time

Based on the Compound Annual Growth Rate formula and the aforementioned change in the cost of concrete, the inflation rate was able to be calculated for construction materials. This was done because the construction stage represented the majority of cost incurred throughout the project. This in combination with the opportunity cost, found using the risk free rate of government bonds, allowed us to select our estimated discount rate of 8.57%. The benchmark year for the project was chosen as January 1<sup>st</sup> 2025, to allow the initiation and planning stages to occur prior to the spring and summer terms of that year.

*Table 4 – Task List Accounting for Net Present Value*

| Task ID | Task Description   | Start Date | End Date | Duration (Weeks) | Nominal Net Value | TVM Conversion | Net Present Value |
|---------|--|------------|----------|------------------|-------------------|----------------|-------------------|
| 1       | Initiation   | 0          | 4        | 4                | -\$200,000.00     | 100%           | -\$200,000.00     |
| 1.1     | Problem Identification   | 0          | 2        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 1.2     | Stakeholder Analysis   | 0          | 2        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 1.3     | Cost Benefit Analysis  | 0          | 2        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 1.4     | Client Pitch   | 2          | 4        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2       | Planning   | 4          | 15       | 11               | -\$250,000.00     | 100%           | -\$250,000.00     |
| 2.1     | Possible Solution Proposals  | 4          | 6        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2.2     | Net Value Identification   | 4          | 6        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2.3     | Narrowing Solutions Via Technical Analysis                           | 6          | 9        | 3                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2.4     | Optimization and Sensitivity Analysis                                | 9          | 12       | 3                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2.5     | Final Design Implementation  | 12         | 15       | 3                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 3       | Execution  | 15         | 279      | 264              | \$3,550,499.55    | 69%            | \$2,447,267.30    |
| 3.1     | Identifying Affected Areas   | 15         | 16       | 1                | -\$523,657.92     | 78%            | -\$410,595.69     |
| 3.2     | Site Preparation (Blocking off areas/demolition & milling)           | 16         | 276      | 260              | \$1,808,052.49    | 72%            | \$1,300,898.85    |
| 3.3     | Construction (Tubing, Concrete, Pump implementation, Heat exchanger) | 19         | 279      | 260              | \$2,266,104.98    | 69%            | \$1,556,964.14    |
| 4       | Closure  | 36         | 1806     | 1770             | \$947,207.68      | 80%            | \$758,192.38      |
| 4.1     | Safety Inspection  | 36         | 282      | 246              | \$226,342.08      | 80%            | \$180,964.63      |
| 4.2     | Regular Maintenance  | 282        | 1560     | 1278             | -\$10,000.00      | 80%            | -\$8,000.00       |
| 4.3     | Retrospective Performance Analysis                                   | 539        | 542      | 3                | \$700,000.00      | 80%            | \$558,262.04      |
| 4.4     | Recycling Materials  | 1560       | 1806     | 246              | \$20,865.60       | 91%            | \$18,965.71       |

*Table 4* is our NPV model results for each section of the task list. The values outlined in our NVP equation were distributed across each task as appropriate, with the majority of benefits occurring after the first construction cycle. Using these value flows we were able to calculate the IRR of the project of 59%. Based on the predetermined MARR of 15%, we are able to justify pursuing the project due to the fact that the IRR outweighs the MRR. Even in the event that the returns were not as high as estimated, we would still choose to pursue the project if the IRR was equal to the MARR as the project provides intangible benefits to vulnerable groups. The addition of administrative costs as a decision variable also had a negative effect on our IRR as we needed to account for the cost of planning separate from construction

labour costs. Note that while nominal and present values are drastically different in some cases, this makes sense over the much larger timescale of this project alongside the discount rate chosen. Please view the provided Excel sheet for the formulas used in each cell to clarify this.

## Risk Management

In order to account for risks outside of our control, we identified several events that could affect our projects net value. From these, risks were picked out for usage in *Table 5* risk matrix. Each risk was assigned an NV Impact, as well as an expected time period before they should occur. The three most critical risks were as follows: cost of materials increasing, construction delays leading to increased labour costs and contractor implementation not to code. The impact value of the cost of materials increasing was determined by analyzing the yearly price increase of concrete and other materials, which has risen steadily over the past few years. Construction delays are also expected to occur at least once over the duration of construction, resulting in potentially large losses of up to half a million dollars in labour costs. Finally, in the rarer case that any of the contractor work is found to be subpar, entire sections of the project would have to be ripped up and replaced leading to millions in lost value. Because of this, we had to concoct mitigation strategies for these risks in the event they did occur. In order to offset the rising price of materials, we suggested that an agreement with suppliers to purchase materials at a set price in advance should be signed, with delivery occurring when each construction cycle actually occurs. By ensuring that proper communication is established between client and contractor, with adequate supervision occurring to ensure construction teams meet estimated deadlines, we are able to mitigate the chance of construction delays as much as possible. For the final critical risk, we plan to ensure inspection teams check for any deficiencies prior to construction teams leaving so that any significant errors are caught before its too late.

Using this risk assessment, we developed probabilistic models in order to revise the NVF to include the impact of risk. We converted the NVF into a stochastic model by using the RAND function in excel within an IF statement, so that if the random value generated was less than or equal to the percentage chance of that risk occurring (as a decimal), then it would output the total cost of that risk occurring in that year. Otherwise, it would output a value of zero for that year. We then converted this to our draft deterministic model by taking that same percentage and multiplying it by the cost per year determined, applied to each year. This gave us values from which we could estimate the cost of potential mitigation strategies. We then updated our task list as seen in *Table 6* with three mitigation strategies and their corresponding costs in order to offset the potential risk impacts when possible.

The final NPV of the project with all these changes was found to be \$2,181,648.77 accounting for the risk mitigation strategies. This is a summary of the NV impacts in present value terms for all of the stages and subtasks provided in our project plan section with the addition of risk included. Because this improved our NPV we know that the mitigation strategies are working as intended.



## 6 Events Affecting Net Value:

1. Cost of Materials Increasing
2. Construction Delays Leading to Increased Labour Costs
3. Contractor Implementation Not to Code
4. Cost of Materials Decreasing
5. Construction Ahead of Estimate
6. Increased Federal Grant from Estimate

| Expected Period Before Occurrence |                  | NV Impact for Each Occurrence |                 |                  |                    |            |
|-----------------------------------|------------------|-------------------------------|-----------------|------------------|--------------------|------------|
|                                   |                  | <15000\$                      | \$15000-\$40000 | \$40000-\$250000 | \$250000-\$1500000 | >\$1500000 |
|                                   | 1 every 20 years |                               |                 |                  | 5.                 | 3. 6.      |
|                                   | 1 every 5 years  |                               |                 | 4.               | 2.                 |            |
|                                   | 1 every year     |                               |                 | 1.               |                    |            |
|                                   | 1 every 2 months |                               |                 |                  |                    |            |
|                                   | 1 every week     |                               |                 |                  |                    |            |

Table 5 Risk Matrix

Critical Risk 1: \$60,000\*90%/year = \$54,000

Critical Risk 2: \$500,000\*20%/year = \$100,000

Critical Risk 3: \$2,000,000\*5%/year = \$100,000

Discount rate:

\$316 price of concrete in 2024 [26] and \$274 cost of concrete in 2021 [27]

$$\text{Compound annual growth rate} = \frac{\text{End Value}}{\text{Beginning Value}}^{\frac{1}{n}} - 1$$

$$CAGR = \left( \frac{316}{274} \right)^{\frac{1}{4}} - 1$$

$$CAGR = 0.04868$$

$$\text{Inflation rate} = CARG * 100\%$$

$$\text{Inflation rate} = 4.87\%$$

Opportunity cost:

Risk free rate (Bonds provided by government of Canada):

For a 5-year period: 3.6% [28]

$$\text{Discount rate} = 4.87\% + 3.6\% = 8.47\%$$

Table 6 - Updated Task List with Risk Mitigation Strategies

| Task ID | Task Description   | Start Date | End Date | Duration (Weeks) | Nominal Net Value | TVM Conversion | Net Present Value |
|---------|--|------------|----------|------------------|-------------------|----------------|-------------------|
| 1       | Initiation   | 0          | 4        | 4                | -\$200,000.00     | 100%           | -\$200,000.00     |
| 1.1     | Problem Identification   | 0          | 2        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 1.2     | Stakeholder Analysis   | 0          | 2        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 1.3     | Cost Benefit Analysis  | 0          | 2        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 1.4     | Client Pitch   | 2          | 4        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2       | Planning   | 4          | 15       | 11               | -\$250,000.00     | 100%           | -\$250,000.00     |
| 2.1     | Possible Solution Proposals  | 4          | 6        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2.2     | Net Value Identification   | 4          | 6        | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2.3     | Narrowing Solutions Via Technical Analysis                           | 6          | 9        | 3                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2.4     | Optimization and Sensitivity Analysis                                | 9          | 12       | 3                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 2.5     | Final Design Implementation  | 12         | 15       | 3                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 3       | Execution  | 15         | 279      | 264              | \$3,550,499.55    | 69%            | \$2,447,267.30    |
| 3.1     | Identifying Affected Areas   | 15         | 16       | 1                | -\$523,657.92     | 78%            | -\$410,595.69     |
| 3.2     | Site Preparation (Blocking off areas/demolition & milling)           | 16         | 276      | 260              | \$1,808,052.49    | 72%            | \$1,300,898.85    |
| 3.3     | Construction (Tubing, Concrete, Pump implementation, Heat exchanger) | 19         | 279      | 260              | \$2,266,104.98    | 69%            | \$1,556,964.14    |
| 4       | Closure  | 36         | 1806     | 1770             | \$947,207.68      | 80%            | \$758,192.38      |
| 4.1     | Safety Inspection  | 36         | 282      | 246              | \$226,342.08      | 80%            | \$180,964.63      |
| 4.2     | Regular Maintenance  | 282        | 1560     | 1278             | -\$10,000.00      | 80%            | -\$8,000.00       |
| 4.3     | Retrospective Performance Analysis                                   | 539        | 542      | 3                | \$700,000.00      | 80%            | \$558,262.04      |
| 4.4     | Recycling Materials  | 1560       | 1806     | 246              | \$20,865.60       | 91%            | \$18,965.71       |
| 5       | Risk Mitigations   | 0          | 282      | 282              | -\$690,000.00     | 83%            | -\$573,810.90     |
| 5.1     | Buy Futures  | 14         | 16       | 2                | -\$50,000.00      | 100%           | -\$50,000.00      |
| 5.2     | Communications Liaison   | 0          | 282      | 282              | -\$540,000.00     | 82%            | -\$444,936.19     |
| 5.3     | Frequent Inspections   | 104        | 282      | 178              | -\$100,000.00     | 79%            | -\$78,874.71      |

Based on the risk management assessment, the NVF can be amended to account for two options, the inclusion of risk impact costs or the risk mitigation fees. Both will appear as negatives in our NVF but provided the risk mitigation strategies are effective they should provide a higher total NVF then the one without risk mitigation. In the next section we will cover whether these strategies are beneficial based on the probability models.

$NVF = \text{Branding} \uparrow + \text{Accessibility} \uparrow + \text{Reduced Insurance}$   
 $\uparrow + \text{Reduced Maintenance} + \text{Environmental Impacts} \uparrow - \text{Construction Costs}$   
 $\downarrow - \text{Electricity Usage} \downarrow - \text{Risk Impacts} \downarrow$

OR

$NVF = \text{Branding} \uparrow + \text{Accessibility} \uparrow + \text{Reduced Insurance}$   
 $\uparrow + \text{Reduced Maintenance} + \text{Environmental Impacts} \uparrow - \text{Construction Costs}$   
 $\downarrow - \text{Electricity Usage} \downarrow - \text{Risk Mitigation Costs} \downarrow$

## Stochastic Sensitivity Analysis

We generated a deterministic sensitivity analysis using the prior stochastic model, by using the expected net value considering the frequency of risks occurring multiplied by the negative net value impact of said risk. This allowed us to estimate the expected net value of each impact per year and amend the stochastic model. The values for the deterministic model were unchanged as they were already optimized during risk management and could not be altered any further. Using the values we found in the deterministic model in the prior stochastic model, we generated a Monte Carlo simulation in order to create a histogram of our NPV's, as seen in *Figure 14*.

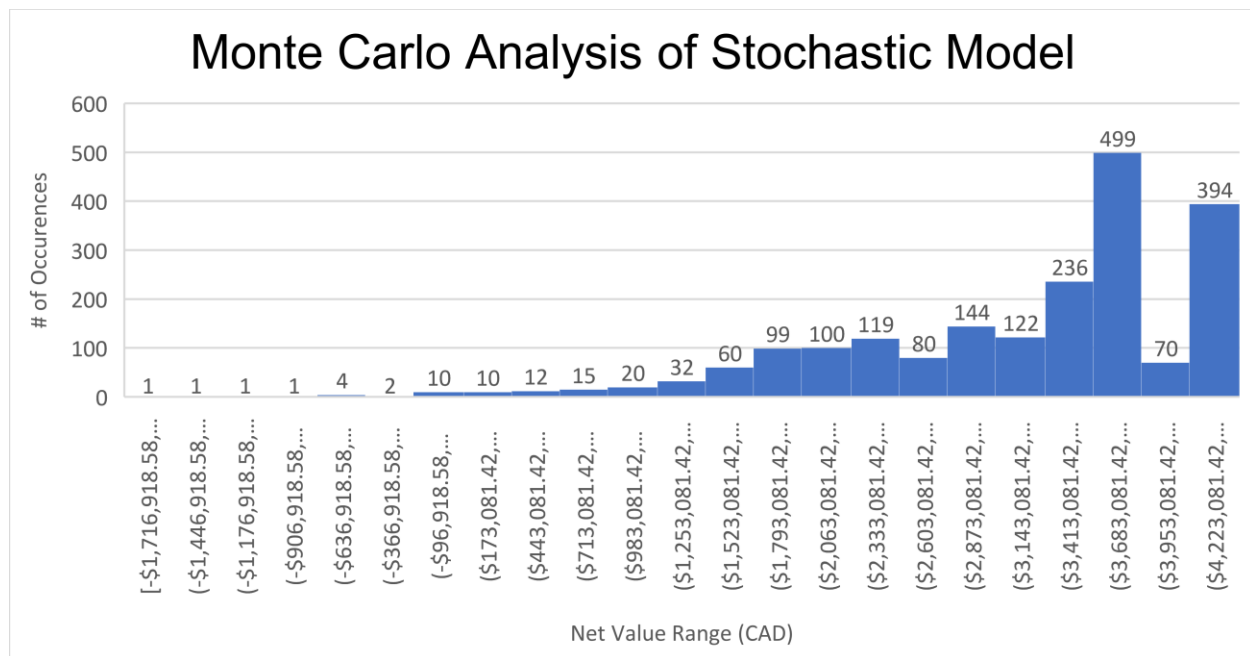


Figure 14 – Monte Carlo Simulation

From this we were able to determine the average net value as well as the standard deviation, which were \$3,275,360.02 and \$970,094.92 respectively. Because of the occurrence frequency of our risks and the disparity in impact, the simulation generated a wide range of possible net values. However, the majority of the distribution is in the positive net value range which bodes well for the project's success.

No decision variables were changed in this section, and the results suggest that we should proceed with the mitigation strategies. Because the net value with our mitigation strategies is higher than the average without, we can feel comfortable in paying the excess money to avoid the risk impacts.

Based on the Monte Carlo simulation and our associated risks, it appears that the project has a 2% chance of having negative net value using the optimized project parameters found earlier. Since mitigation was used the final NVF was amended to include those costs as our final impacting factor.

$$\begin{aligned} NVF = & \textit{Branding} \uparrow + \textit{Accessibility} \uparrow + \textit{Reduced Insurance} \\ & \uparrow + \textit{Reduced Maintenance} + \textit{Environmental Impacts} \uparrow - \textit{Construction Costs} \\ & \downarrow - \textit{Electricity Usage} \downarrow - \textit{Risk Mitigation Costs} \downarrow \end{aligned}$$

## Final Recommendations

The existing winter maintenance strategy employed by McMaster has certain limitations, in that it fails to provide accessible and economical snow removal across the campus. We have identified radiant heating systems within campus grounds and parking lots as a sustainable and cost-effective strategy to existing methods.

Using principles of economic engineering, conversion factors were implemented for all considerations affecting the project's net value. The incorporation of technical analyses to finalize decision variables alongside the use of sensitivity analysis determined that the biggest influence on the NV is cost of construction. This would make sense because the cost of construction was the highest cost present in the sensitivity analysis. After the optimization of the NV, the value was determined to be \$1,193,073.27, significantly greater than any other alternative solutions.

Additionally, we expanded our focus to encompass the entire lifecycle of the project, including time value of money, associated risks, and potential mitigation strategies. After the inclusion of stochastic and deterministic probability of risk models, further optimization of the deterministic model alongside a Monte Carlo simulation was completed for the stochastic model. The refined average NPV was calculated to be \$3,275,360.02, which is significantly greater than the previous optimized value. Due to the NPV consistently outperforming current solutions we feel that the project should be implemented.

We recognize that our research and assumptions maybe the core to the limitations for the project. Parts of the net value are based on intangible benefits and may not necessarily be represented as actual cash benefits.

In conclusion, the implementation of the radiant heating system will significantly improve safety and accessibility while aligning with the universities environmental objectives by diminishing the reliance on chemical de-icing agents. With this proposition McMaster University can set a standard for campus infrastructure, showcasing their commitment to safety, sustainability, and accessibility.

## References

- [1] "Prevent a slip and fall this winter season – Daily News." Accessed: Feb. 06, 2024. [Online]. Available: <https://dailynews.mcmaster.ca/worthmentioning/reduce-your-risk-of-injury-from-a-slip-and-fall-this-winter/>
- [2] "CER – Provincial and Territorial Energy Profiles – Ontario." Accessed: Feb. 06, 2024. [Online]. Available: <https://www.cer-rec.gc.ca/en/data-analysis/energy-markets/provincial-territorial-energy-profiles/provincial-territorial-energy-profiles-ontario.html>
- [3] A. Thomas-Murphy, "2022-23 Consolidated Budget FINAL," 2022.
- [4] "Note: The above standard strength mixes are based on mix designs including a water reducing admixture \*Includes HSF Cement and Superplasticizer. \*\*65 MPa Mixes and Higher includes HSF Cement, Superplasticizer and Temperature Control. Segmentation Pricing", Accessed: Feb. 28, 2024. [Online]. Available: [www.dufferinconcrete.ca](http://www.dufferinconcrete.ca)
- [5] "Enabling Accessibility Fund - Canada.ca." Accessed: Feb. 28, 2024. [Online]. Available: <https://www.canada.ca/en/employment-social-development/programs/enabling-accessibility-fund.html>
- [6] "How to Calculate Pex Length for Radiant Heating | Solar365." Accessed: Feb. 28, 2024. [Online]. Available: <https://www.solar365.com/green-homes/plumbing/how-calculate-pex-length-radiant-heating>
- [7] "Oxygen Barrier PEX Tubing for Radiant Heating | PexUniverse." Accessed: Feb. 28, 2024. [Online]. Available: <https://www.pexuniverse.com/barrier-pex-tubing>
- [8] "Time-of-Use Pricing | Alectra Utilities." Accessed: Feb. 28, 2024. [Online]. Available: <https://alectrautilities.com/time-use-pricing>
- [9] "Where are Hamilton's snow plows? This snow storm, you can track them | CBC News." Accessed: Feb. 28, 2024. [Online]. Available: <https://www.cbc.ca/news/canada/hamilton/plow-tracker-1.4907410>
- [10] "415V 30 250m Meters Head 80m3h 40HP 8 Inch Submersible Tube Well Water Pump for Hot Water - China Pump and Pumps." Accessed: Feb. 28, 2024. [Online]. Available: <https://dragonpump.en.made-in-china.com/product/uFytzbGMhxkL/China-415V-30-250m-Meters-Head-80m3h-40HP-8-Inch-Submersible-Tube-Well-Water-Pump-for-Hot-Water.html>
- [11] "SUBJECT: Sidewalk Snow Removal (PW19022(a)) (City Wide)-Page 2 of 9".
- [12] "Electric Resistance Heating | Department of Energy." Accessed: Feb. 06, 2024. [Online]. Available: <https://www.energy.gov/energysaver/electric-resistance-heating>
- [13] "Radiation | Physics." Accessed: Feb. 06, 2024. [Online]. Available: <https://courses.lumenlearning.com/suny-physics/chapter/14-7-radiation/>
- [14] "ISORAD V2: Hydronic radiant floor heating insulation - Under slab | Isofoam." Accessed: Feb. 06, 2024. [Online]. Available: <https://isofoam.com/en/product/isorad-v2-r10-r15/>

- [15] "Hamilton Climate, Weather By Month, Average Temperature (Canada) - Weather Spark." Accessed: Feb. 06, 2024. [Online]. Available: <https://weatherspark.com/y/19864/Average-Weather-in-Hamilton-Canada-Year-Round#Sections-Wind>
- [16] "Power Requirements and Operating Costs of a Heated Driveway | Warmzone." Accessed: Feb. 06, 2024. [Online]. Available: <https://www.warmzone.com/blog/hotnews/power-requirements-and-operating-costs-of-a-heated-driveway/>
- [17] T. S. Yun, Y. J. Jeong, and K. S. Youm, "Effect of surrogate aggregates on the thermal conductivity of concrete at ambient and elevated temperatures," *The Scientific World Journal*, vol. 2014, 2014, doi: 10.1155/2014/939632.
- [18] "Density (Unit weight) of Concrete - Civil Engineering." Accessed: Feb. 06, 2024. [Online]. Available: <https://civiltoday.com/civil-engineering-materials/concrete/361-density-of-concrete>
- [19] "Specific Heat of common Substances." Accessed: Feb. 06, 2024. [Online]. Available: [https://www.engineeringtoolbox.com/specific-heat-capacity-d\\_391.html](https://www.engineeringtoolbox.com/specific-heat-capacity-d_391.html)
- [20] S. G. Warren, "Optical properties of ice and snow," *Philos Trans A Math Phys Eng Sci*, vol. 377, no. 2146, Jun. 2019, doi: 10.1098/RSTA.2018.0161.
- [21] "7b Snow Density." Accessed: Feb. 06, 2024. [Online]. Available: [https://www.eoas.ubc.ca/courses/atasc113/snow/met\\_concepts/07-met\\_concepts/07b-newly-fallen-snow-density/](https://www.eoas.ubc.ca/courses/atasc113/snow/met_concepts/07-met_concepts/07b-newly-fallen-snow-density/)
- [22] J. W. Pomeroy and E. Brun, "Physical Properties of Snow Introduction: Snow Physics and Ecology".
- [23] O. Hisseine, "W3-Concrete," in *Civil Engineering Materials and Design*, McMaster University, 2023. Accessed: Feb. 06, 2024. [Online]. Available: <https://avenue.cllmcmaster.ca/d2l/le/content/547600/viewContent/4345760/View>.
- [24] "How to Calculate Server Room Air Conditioner Sizes | Server Room Environments." Accessed: Feb. 28, 2024. [Online]. Available: <https://www.serverroomenvironments.co.uk/blog/how-to-calculate-server-room-air-conditioner-sizes>
- [25] "Request for Proposal For Computer Acquisition".
- [26] "Note: The above standard strength mixes are based on mix designs including a water reducing admixture \*Includes HSF Cement and Superplasticizer. \*\*65 MPa Mixes and Higher includes HSF Cement, Superplasticizer and Temperature Control. Segmentation Pricing", Accessed: Mar. 14, 2024. [Online]. Available: [www.dufferinconcrete.ca](http://www.dufferinconcrete.ca)
- [27] "Enhanced Reader."
- [28] "Selected bond yields - Bank of Canada." Accessed: Mar. 14, 2024. [Online]. Available: <https://www.bankofcanada.ca/rates/interest-rates/canadian-bonds/>
- [29] "415V 30 250m Meters Head 80m3h 40HP 8 Inch Submersible Tube Well Water Pump for Hot Water - China Pump and Pumps." Accessed: Feb. 28, 2024. [Online]. Available: <https://dragonpump.en.made-in-china.com/product/uFytzbGMhxkL/China-415V-30-250m-Meters-Head-80m3h-40HP-8-Inch-Submersible-Tube-Well-Water-Pump-for-Hot-Water.html>

- [30] "Time-of-Use Pricing | Alectra Utilities." Accessed: Feb. 28, 2024. [Online]. Available: <https://alectrautilities.com/time-use-pricing>
- [31] "Section One Introduction & Overview 1," 2004.

## Appendices

### NVF Additional Calculations

#### Material Costs:

The tubing being used for the project is oxygen barrier tubing from [7]. The following are some conversion and assumptions made to find conversion factors used earlier in the report.

The tubing we were using was the half inch diameter PEX oxygen barrier tubing, which comes in a variety of lengths, the longest being 1000ft; we are using the bulk order pricing.

$$\frac{1}{2}inch(diameter) = 12.7mm \approx 13mm(diameter)$$
$$\frac{\$234.95USD}{1000ft} \times \frac{\$1.36CAD}{\$1USD} \times \frac{1ft}{0.3048m} = \frac{\$319.53CAD}{304.8m} = \frac{\$1.05CAD}{m} (of tubing)$$

The pump costed USD\$1,090.00, which converts to \$1478.71 CAD [29].

#### Power for Heat and Pump:

The average cost of electricity takes the price of electricity in Ontario at the different peak hours (on, mid, and off peak) [30].

$$\frac{18.2 + 12.2 + 8.7 \text{ cents}}{3} \frac{\text{cents}}{kWh} = 13.033 \frac{\text{cents}}{kWh} \times \frac{1 \text{ CAD}}{100 \text{ cents}} = 0.13033 \frac{CAD}{kWh}$$

#### Environmental Costs:

These calculations were based on an estimated area of the roads and sidewalks within the McMaster campus. Using a salt application rate of 90kg per lane km, and 680\$ per tonne in environmental damages, we estimated yearly damages of over fifteen thousand dollars.



## Technical Analyses

### Analysis of Ideal Areas for System Implementation – Sean

The NVF could vary quite a bit depending on where and to what extent it is implemented. The total size of the project impacts several areas, such as the accessibility benefits, the opportunity costs of not having to remove snow or salt, the cost of power to heat the walkways annually as well as the construction costs.

McMaster has already identified priority areas for investment to improve quality of life for pedestrians and cyclists, including several primary entrances and proposed bike routes seen in *Figure 15*

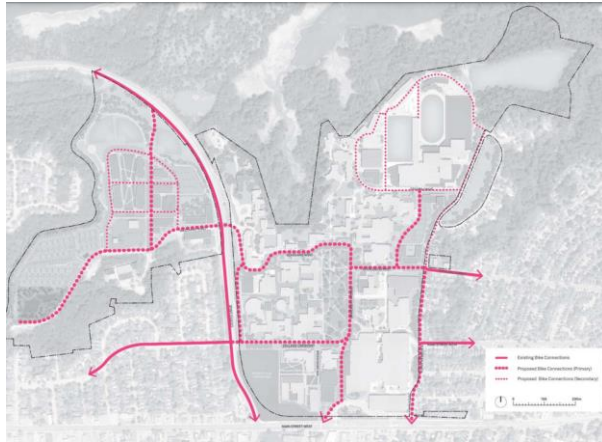


Figure 15 - Existing and Proposed Bike Paths [31]



Figure 16 – Primary Pedestrian Entrances [31]

These areas coupled with accessible parking will help decide where to implement the system, with accessibility zones having increased value.

Increases of project size will result in a greater quantity of people affected, which further increases the NPV of the project. Based on McMaster's student population of 36 000 as well as visitors, foot traffic can be estimated to be at least 5000 people per primary entrance. Increased numbers of people would pass through the accessibility and standard parking lots, which number over 4000 parking spaces. Based on the results of the sensitivity analysis it seems prudent to increase project size above all else, as the benefit from increased outreach outweighs the construction and operation costs in the net present value function.

Key areas would be the primary entrances, around the Childrens Hospital, student and faculty parking lots and key arteries between McMaster's largest buildings such as PGCLL and MDCLL.

## Power Consumption of Pump – Alex

|    |                                 |                              |   |  |                        |                                      |
|----|---------------------------------|------------------------------|---|--|------------------------|--------------------------------------|
| 2  | <b>Water</b>                    |                              |   |  |                        |                                      |
| 3  | Temperature (Celsius)           | Density (kg/m <sup>3</sup> ) | Re  | Kinematic Viscosity (mm <sup>2</sup> /s) | Velocity Avg (m/s)     | Volume Flow Rate (m <sup>3</sup> /s) |
| 4  | 4                               | 1000                         | 2300                                      | 1.5674                                   | 0.298799834            | 0.002831376                          |
| 5  |                                 |                              |   | 1.5674E-06                               |                        |                                      |
| 6  | <b>Constants</b>                |                              |   |  |                        |                                      |
| 7  | f (laminar)                     | Gravity (m/s <sup>2</sup> )  | 180 deg loss [flanged 0.2 & threaded 1.5] |  |                        |                                      |
| 8  | 0.027826087                     | 9.81                         | 1.5                                       |  |                        |                                      |
| 9  |                                 |                              |   |  |                        |                                      |
| 10 |                                 |                              |   |  |                        |                                      |
| 11 | <b>Tube Specs</b>               |                              |   |  |                        |                                      |
| 12 | Diameter (m) [inner]            | Length (m)                   | # of 180 connections                      | kL                                       | Area (m <sup>2</sup> ) |                                      |
| 13 | 0.012065                        | 10000                        | 20000                                     | 30000                                    | 0.009475829            |                                      |
| 14 |                                 |                              |   |  |                        |                                      |
| 15 | <b>Head Loss [friction] (m)</b> | <b>Head Loss [minor] (m)</b> |   |  |                        |                                      |
| 16 | 104.9509836                     | 136.5158118                  |   |  |                        |                                      |
| 17 |                                 |                              |   |  |                        |                                      |
| 18 |                                 |                              |   |  |                        |                                      |
| 19 | <b>Pump</b>                     |                              |   |  |                        |                                      |
| 20 | h_pump                          | efficiency_pump              | Power_pump (W)                            |  |                        |                                      |
| 21 | 241.4667954                     | 0.9                          | 7452.148083                               |  |                        |                                      |
| 22 |                                 |                              |   |  |                        |                                      |

Figure 17 - Specifications of Input Parameters for Model

|    |                              |                    |                               |             |
|----|------------------------------|--------------------|-------------------------------|-------------|
| 23 | <b>Motor</b>                 |                    |                               |             |
| 24 | efficiency_motor             | Power_motor (W)    | Time of Operation (hours/day) | kWh/day     |
| 25 | 0.75                         | 9936.197444        | 8                             | 79.48957955 |
| 26 |                              | 9.936197444        |                               |             |
| 27 | <b>Cost of Electricity</b>   |                    |                               |             |
| 28 | Avg Cost per kWh (cents/kWh) | Cost (dollars/day) |                               |             |
| 29 | 13.03333333                  | 10.36014187        |                               |             |
| 30 | (dollars/day)                |                    |                               |             |
| 31 | 0.130333333                  |                    |                               |             |

Figure 18 - Output of Model

The include spread sheet was used to calculate the electrical and mechanical power needed for the system along with the cost of the energy per day, it was based off the equations mentioned earlier.

## Energy Requirements per Square Foot – Shyavan

Below is the FlexPDE code used to generate the model for the heat transfer. All the material properties (thermal and mechanical) were taken from suggested values found online and referenced within the overview. Weather conditions were taken as per the average in January for Hamilton from [15] and the thickness of the concrete layer and power of the system were also taken from outside sources referenced above discussing a similar system implementation for driveways. Also provided is the output of the FlexPDE model.

### Code

```
TITLE '3PX3 Technical Analysis'      ! The problem identification

COORDINATES cartesian2              ! Coordinate system, 1D,2D,3D, etc

{
Model Assumptions:
- No convection or radiation from air
- Snow is between freshly fallen & lightly packed
- No effects on heating from windspeed
- 2D model is extrapolatable to 3D
- 1 sqft of pavement under avg snowfall & weather conditions for January in
Hamilton
- No radiative heat transfer back to the concrete from snow
}

VARIABLES
Temp(threshold=1e-6)                ! Temperature (C)

SELECT
ngrid = 19                          ! Method controls

DEFINITIONS
! Dynamic
k                                  ! Thermal Conductivity (W/cm*K^-1)
e                                  ! Emissivity (unitless)
rho                                ! Density (g/cm^3)
C_p                                ! Heat capacity at constant pressure (J/g*K^-1)
init_temp                          ! Initial temperature (C)
qvol                                ! Volumetric heat generation (W/cm^3)

! Static
sigma = 5.67e-5                    ! Stefan-Boltzmann Constant (W/cm^2*K^-1)
Q_m = 334                          ! Specific Heat of Fusion of Ice (J/g)
T_inf = -4                         ! Temperature of outdoor air (C)
L_pavement = 30.48                 ! Length of pavement (cm)
t_pavement = 7.62                  ! Thickness of pavement (cm)
t_snow = 8.64                      ! Thickness of snow block (cm)

! Sweep
P_system = 37                      ! System power (W)
```

```

! Equations
qdot = -k*grad(Temp) ! Heat flux for conduction (W/cm^2)
E_direct = (0 - T_inf)*area_integral(C_p*rho*L_pavement, "Snow") ! Energy
required for raising snow to 0 C (J)
E_latent = Q_m*area_integral(rho*L_pavement, "Snow") ! Energy required for
phase change from ice to water (J)
A_snow = 2*(L_pavement^2 + 2*t_snow*L_pavement) ! Surface are of snow block
(cm^2)

INITIAL VALUES
Temp = init_temp ! Temperature (C)

EQUATIONS ! PDE's, one for each variable
rho*C_p*dt(Temp) = qvol - div(qdot) ! Heat equation

BOUNDARIES ! The domain definition
REGION "Pavement"
k = 0.0225 ! Thermal Conductivity (W/cm*K^-1)
e = 0 ! Emissivity (unitless)
rho = 2.3 ! Density (g/cm^3)
C_p = 0.88 ! Heat capacity at constant pressure (J/g*K^-1)
init_temp = T_inf ! Initial temperature (C)

qvol = P_system/area_integral(L_pavement, "Pavement") ! Volumetric heat
generation due to hydronic system (W/cm^3)

START (-15.24, 0)
load(Temp) = 0 ! Assume insulated sides apart from contact
surface due to poor conductivity of air & no convective effects
LINE TO (15.24, 0)
LINE TO (15.24, -7.62)
LINE TO (-15.24, -7.62)
LINE TO CLOSE

REGION "Snow"
k = 0.00045 ! Thermal Conductivity (W/cm*K^-1)
e = 0.98 ! Emissivity (unitless)
rho = 0.1 ! Density (g/cm^3)
C_p = 2.090 ! Heat capacity at constant pressure
(J/g*K^-1)
init_temp = T_inf ! Initial temperature (C)

qvol = sigma*e*A_snow*((Temp - 273.15)^4 - (eval(Temp, 0, -3.81) -
273.15)^4)/area_integral(L_pavement, "Snow") ! Volumetric heat generation due
to radiant heat transfer (W/cm^3)

START (-15.24, 0)
load(Temp) = 0 ! Assume insulated sides apart from contact
surface due to poor conductivity of air & no convective effects

```

```

LINE TO (15.24, 0)
LINE TO (15.24, 8.64)
LINE TO (-15.54, 8.64)
LINE TO CLOSE

```

TIME 0 TO 3600 halt (val(Temp, 0, 8.64) > 0) ! Run is finished when snow is melted (i.e. all snow is above 0 C)

PLOTS ! Save result displays

```

for t = 0 by endtime/60 to endtime
  history(val(Temp, 0, 8.64))
  history(val(Temp, 0, -3.81))
  contour(Temp) painted
  vector(qdot) norm

```

SUMMARY ! Report energy usage

```

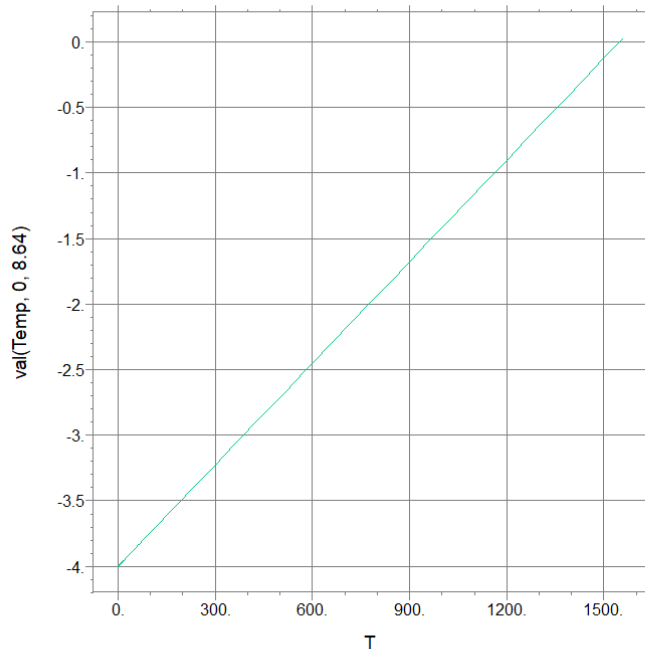
report(t*P_system) as "Energy Consumed without Phase Change "
report(E_direct) as "Energy Required without Phase Change "
report(E_direct/(t*P_system)) as "Efficiency "
report(E_latent) as "Energy Required for Phase Change "
report(t*P_system*(1 + 1/E_direct*E_latent)) as "Predicted Energy
Consumed with Phase Change "
report(t) as "Time to Finish "

```

END

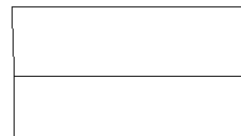
## Outputs

### 3PX3 Technical Analysis



23:42:21 2/7/24  
FlexPDE Student 6.36s/W32

HISTORY  
val(Temp, 0, 8.64)

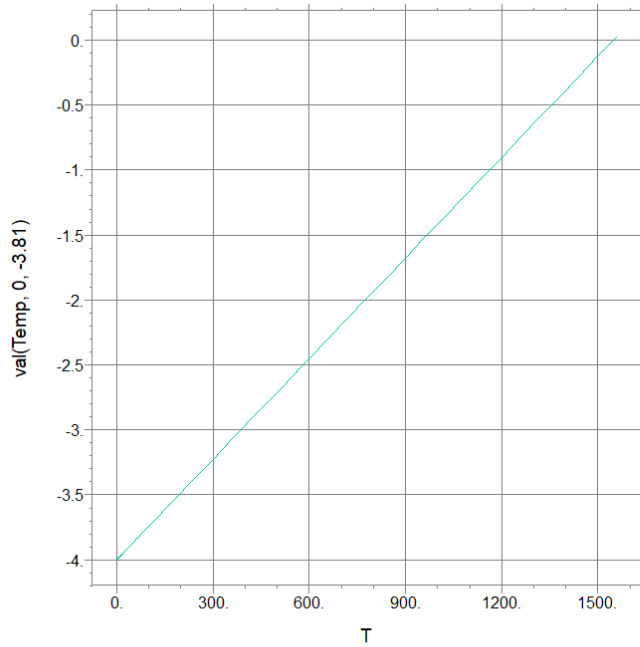


Technical Analysis: Cycle=102 Time= 1560.0 dt= 50.252 P2 Nodes=971 Cells=456 RMS Err= 1.1e-7

Figure 19 - Temperature of Snow Layer over Time

### 3PX3 Technical Analysis

23:42:21 2/7/24  
FlexPDE Student 6.36s/W32



#### HISTORY

val(Temp, 0, -3.81)

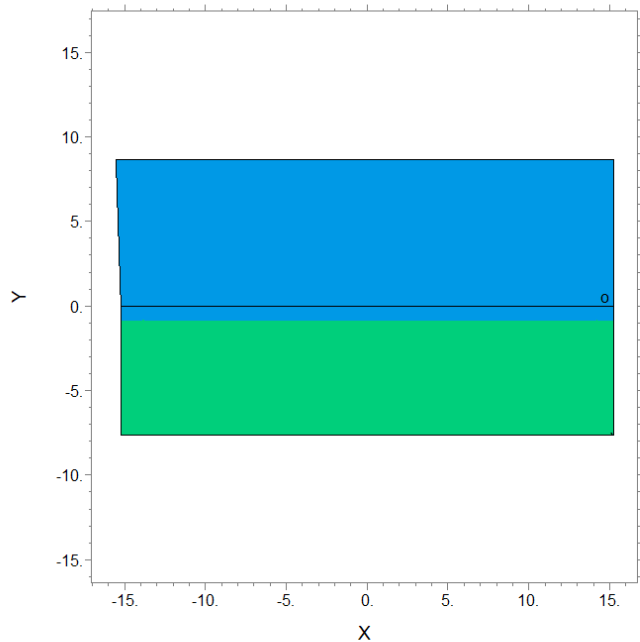


Technical Analysis: Cycle=102 Time= 1560.0 dt= 50.252 P2 Nodes=971 Cells=456 RMS Err= 1.1e-7

Figure 20 - Temperature of Concrete Layer over Time

### 3PX3 Technical Analysis

23:42:21 2/7/24  
FlexPDE Student 6.36s/W32



#### Temp

max 2.83843  
2.83860  
2.83855  
2.83850  
2.83845  
2.83840  
2.83835  
2.83830  
2.83825  
min 2.83839

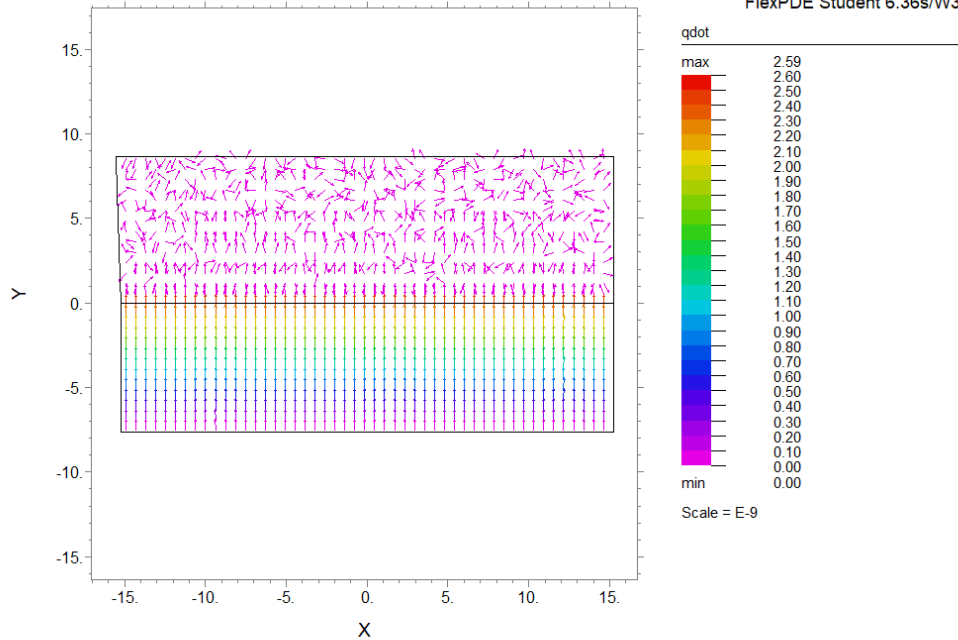
Scale = E-2

Technical Analysis: Cycle=102 Time= 1560.0 dt= 50.252 P2 Nodes=971 Cells=456 RMS Err= 1.1e-7  
Integral= 14.10405

Figure 21 - Contour Plot of Temperature over System

### 3PX3 Technical Analysis

23:42:21 2/7/24  
FlexPDE Student 6.36s/W32



Technical Analysis: Cycle=102 Time= 1560.0 dt= 50.252 P2 Nodes=971 Cells=456 RMS Err= 1.1e-7

Figure 22 - Vector Plot of Heat Flow over System

### 3PX3 Technical Analysis

23:42:21 2/7/24  
FlexPDE Student 6.36s/W32

#### SUMMARY

Energy Consumed without Phase Change = 57720.00  
Energy Required without Phase Change = 6743.447  
Efficiency = 0.116830  
Energy Required for Phase Change = 269415.2  
Predicted Energy Consumed with Phase Change = 2363758.  
Time to Finish = 1560.000

Technical Analysis: Cycle=102 Time= 1560.0 dt= 50.252 P2 Nodes=971 Cells=456 RMS Err= 1.1e-7

Figure 23 - Final Outputs of Model

## Designing Concrete Composition – Moosa

Slump = 100mm to 150mm (125mm) provides fluidity & not running  
 $f_{cr} = 40 \text{ MPa}$  thermal stress, load, longevity  
 air-entrained concrete  
 specific gravity = 3.15  
 (ideal) w/c ratio = 0.4 to 0.45 provides strength, durability, workability  
 coarse aggregate 20mm: Bulk gravity = 2.7  
 absorption = 1.5%  
 Natural moisture content = 1%  
 Dry loaded unit weight =  $1625 \text{ kg/m}^3$

fine aggregate: Bulk gravity = 2.65  
 absorption = 1%  
 Natural Moisture content = 2.5%  
 Fineness Modulus = 2.75

w/c ratio = 0.4 Required cement weight:  $197/0.4 = 492.5$   
 $W = 197$

Volume of CA/unit volume of concrete:  $0.645 \text{ m}^3 (V_{ca})$

$V_w = 197/1000 = 0.197 \text{ m}^3$   
 $V_c = 492.5/3.15(1000) = 0.156 \text{ m}^3$   
 $V_a = 7\% = 0.07$   
 $V_{ca} = 1625(0.645)/270(1000) = 0.398 \text{ m}^3$   
 $V_{ca} = 1 - (V_w + V_c + V_a + V_{ca}) = 0.199 \text{ m}^3$

Figure 24 - Technical analysis calculations for the design mix.



| Slump, mm  | Water, kilograms per cubic metre of concrete, for indicated sizes of aggregate* |        |       |        |       |         |         |          |
|--|---|--------|-------|--------|-------|---------|---------|----------|
|  | 10 mm   | 14 mm  | 20 mm | 28 mm  | 40 mm | 56 mm** | 80 mm** | 150 mm** |
| <b>Non-air-entrained concrete</b>  |   |        |       |        |       |         |         |          |
| 25 to 50   | 207   | 199    | 190   | 179    | 166   | 154     | 130     | 113      |
| 75 to 100  | 228   | 216    | 205   | 193    | 181   | 169     | 145     | 124      |
| 150 to 175   | 243   | 228    | 216   | 202    | 190   | 178     | 160     | —        |
| Approximate amount of entrapped air in non-air-entrained concrete, percent | 3   | 2.5    | 2     | 1.5    | 1     | 0.5     | 0.3     | 0.2      |
| <b>Air-entrained concrete</b>  |   |        |       |        |       |         |         |          |
| 25 to 50   | 181   | 175    | 168   | 160    | 150   | 142     | 122     | 107      |
| 75 to 100  | 202   | 193    | 184   | 175    | 165   | 157     | 133     | 119      |
| 150 to 175   | 216   | 205    | 197   | 184    | 174   | 166     | 154     | —        |
| CSA A23.1 Recommended total air content percent†                           |   |        |       |        |       |         |         |          |
| Category 1   | 6 to 9  | 5 to 8 |       | 4 to 7 |       | —       | —       | —        |
| Category 2   | 5 to 8  | 4 to 7 |       | 3 to 6 |       | —       | —       | —        |

Table 7 - Appropriate mixing water and air content requirement for different slumps and nominal maximum sizes of aggregate [23]

| Nominal maximum size of aggregate, mm | Bulk volume of dry-rodded coarse aggregate per unit volume of concrete for different fineness moduli of fine aggregate* |      |      |      |
|---------------------------------------|---|------|------|------|
|                                       | 2.40  | 2.60 | 2.80 | 3.00 |
| 10                                    | 0.50  | 0.48 | 0.46 | 0.44 |
| 14                                    | 0.59  | 0.57 | 0.55 | 0.53 |
| 20                                    | 0.66  | 0.64 | 0.62 | 0.60 |
| 28                                    | 0.71  | 0.69 | 0.67 | 0.65 |
| 40                                    | 0.75  | 0.73 | 0.71 | 0.69 |
| 56                                    | 0.78  | 0.76 | 0.74 | 0.72 |
| 80                                    | 0.82  | 0.80 | 0.78 | 0.76 |
| 150                                   | 0.87  | 0.85 | 0.83 | 0.81 |

Table 8 - Bulk volume of coarse aggregate per unit volume of concrete [23]