

Code for Figure 1

August 5, 2016

1 Create datasets for panel a)

Load the necessary libraries, source the file with the R functions:

```
library(ggplot2)

source("functions.R")
```

These results are for the $I = 2, p = 2$ case, so the parameters we vary are: $r_1, r_2, \rho_{12,1}, \rho_{12,2}$. We save the relative efficiencies for both coefficients.

We take $r_1 = r_2 = 0.5$:

```
bigMat.a <- expand.grid(r=0.5,
                      rho121=c(0, 0.25, 0.5, 0.75),
                      rho122=(-19:19)/20)
bigMat.a <- cbind(bigMat.a, RelEff1=NA, RelEff2=NA)
bigMat.a <- as.matrix(bigMat.a)
for(i in 1:nrow(bigMat.a))
{
  rho121 <- bigMat.a[i,"rho121"]
  rho122 <- bigMat.a[i,"rho122"]
  r <- bigMat.a[i, "r"]

  bigMat.a[i,c(4,5)] <- effCalc2(rho112=rho121, rho212=rho122, r1=r, r2=r)
}
##turn it back into data frame (need it as data frame for ggplot)
bigMat.a <- as.data.frame(bigMat.a)
##check that relative efficiencies are identical for the two coefficients
identical(bigMat.a$RelEff1, bigMat.a$RelEff2)

## [1] TRUE

##rename RelEff1 as RelEff
names(bigMat.a)[names(bigMat.a) == "RelEff1"] <- "RelEff"
##make rho121 into a character(required for ggplot)
bigMat.a$rho121 <- paste("rho121=", bigMat.a$rho121, sep="")##as.factor(bigMat.a$r)
```

2 Create plot for panel a)

Panel a):

```

panelA <- ggplot(bigMat.a,
                 aes(x=rho122, y=RelEff)) +
  geom_line(size=1.3, aes(linetype=rho121, color=rho121)) +
  theme_bw(base_size = 20)+
  xlab(expression(paste(rho[2]))) +
  scale_color_discrete(name = "",
                      labels =
                        c(expression(paste(rho[1], "=",
                                             0)),
                          expression(paste(rho[1], "=",
                                             0.25)),
                          expression(paste(rho[1], "=",
                                             0.5)),
                          expression(paste(rho[1], "=",
                                             0.75))))) +
  scale_linetype_discrete(name = "",
                         labels =
                           c(expression(paste(rho[1], "=",
                                                0)),
                              expression(paste(rho[1], "=",
                                                0.25)),
                              expression(paste(rho[1], "=",
                                                0.5)),
                              expression(paste(rho[1], "=",
                                                0.75))))) +
  theme(axis.line = element_line(colour = "black"),
        plot.title = element_text(size = 16, hjust = 0.5),
        panel.grid.major = element_blank(),
        panel.grid.minor = element_blank(),
        panel.border = element_blank(),
        panel.background = element_blank(),
        legend.key = element_blank(),
        legend.text.align = 0,
        axis.line.x = element_line(color="black", size = 0.5), ##this is to show axes - bug in this version
        axis.line.y = element_line(color="black", size = 0.5)) +
  labs(title=
        expression(atop("(a)",
                          paste("Fixed effects: I = 2, p = 2, ",
                                r[1], " = ", r[2], " = ", 0.5)))))

```

3 Create datasets for panel b)

These results are for the $I = 2, p \geq 2$ exchangeable case with equal within-study variances, so the parameters we vary are: r, ρ_1, ρ_2, p . We save the relative efficiencies for only one of coefficients, as they are all equal. We consider $\rho_1 = 0$.

```

##save results in data frame
bigMat <- expand.grid(rho1 = 0,
                    rho2 = 0:3/4,
                    r = c(1, 3, 5, 9)/10,
                    p = 2:20,
                    RelEff = NA)

```

```

for(i in 1:nrow(bigMat))
{
  rho1 <- bigMat[i, 1]
  rho2 <- bigMat[i, 2]
  r <- bigMat[i, 3]
  p <- bigMat[i, 4]

  ##get variance-covariance matrices
  S1 <- r*ARMAcor(phi=rho1, rho=1, n=p)
  S2 <- (1-r)*ARMAcor(phi=rho2, rho=1, n=p)
  U1 <- diag(diag(S1))
  U2 <- diag(diag(S2))

  varMVMA <- solve(solve(S1)+solve(S2))
  varUVMA <- solve(solve(U1)+solve(U2)) %*%
    (solve(U1) %*% S1 %*% solve(U1) +
     solve(U2) %*% S2 %*% solve(U2)) %*%
    solve(solve(U1)+solve(U2))

  bigMat$RelEff[i] <- varMVMA[1,1]/varUVMA[1,1]
}

##for Panel b), r = 0.5:
bigMat.b <- bigMat[bigMat[, "r"]==0.5, ]
##transform back to data frame (for ggplot)
bigMat.b <- as.data.frame(bigMat.b)
##make rho2 into factor (for ggplot)
bigMat.b$rho2 <- as.factor(bigMat.b$rho2)

```

4 Create plot for panel b)

```

panelB <- ggplot(bigMat.b,
  aes(x=p, y=RelEff))+
  geom_line(aes(color=rho2, shape=rho2)) +
  geom_point(size=3.0, aes(color=rho2, shape=rho2)) +
  theme_bw(base_size = 20)+
  scale_color_discrete(name = "",
    labels =
      c(expression(paste(rho[2], "=",
        0)),
        expression(paste(rho[2], "=",
        0.25)),
        expression(paste(rho[2], "=",
        0.5)),
        expression(paste(rho[2], "=",
        0.75))))) +
  scale_shape_discrete(name = "",
    labels =
      c(expression(paste(rho[2], "=",
        0)),
        expression(paste(rho[2], "=",

```

```

                                0.25)),
                                expression(paste(rho[2], "=",
                                0.5))),
                                expression(paste(rho[2], "=",
                                0.75)))) +
theme(axis.line = element_line(colour = "black"),
      plot.title = element_text(size = 16, hjust = 0.5),
      panel.grid.major = element_blank(),
      panel.grid.minor = element_blank(),
      panel.border = element_blank(),
      panel.background = element_blank(),
      legend.key = element_blank(),
      legend.text.align = 0,
      axis.line.x = element_line(color="black", size = 0.5), ##this is to show axes - bug in this version
      axis.line.y = element_line(color="black", size = 0.5)) +
labs(title=
      expression(atop("(b)",
                      paste("Fixed effects: I = 2, ",
                            rho[1], " = ", 0, ", ",
                            r[1], " = ", r[2], " = ", 0.5)))))

```

5 Create datasets for panel c)

The following results are for the $I = 20, p \geq 2$ exchangeable case with equal within-study variances and $S_i^2 \equiv S^2, \rho_i = \frac{\rho(i-1)}{I}$, so the parameters we vary are ρ, p . We save the relative efficiencies for only one of coefficients, as they are all equal.

```

##number of studies
I <- 20

##save results in data frame
bigMat <- expand.grid(rho = c(0, 0.25, 0.5, 0.75, 1),
                     p = 2:20,
                     RelEff = NA)

for(n in 1:nrow(bigMat))
{
  rho <- bigMat[n, 1]
  p <- bigMat[n, 2]

  ##index over the studies
  i <- 1:I

  ##calculate the two sums
  ##Sum1 is over 1/(1-rho_i)
  ##Sum2 is over 1/(1+(p-1)*rho_i)
  rho.i <- rho*(i-1)/I
  Sum1 <- sum(1/(1-rho.i))
  Sum2 <- sum(1/(1+(p-1)*rho.i))

  bigMat$RelEff[n] <- I/p * (Sum1 + (p-1)*Sum2)/(Sum1*Sum2)
}

```

```
##make rho into factor (for ggplot)
bigMat$rho <- paste("rho=", bigMat$rho, sep="")
```

6 Create plot for panel c)

```
panelC <- ggplot(bigMat,
                  aes(x=p, y=RelEff))+
  geom_line(aes(color=rho, shape=rho)) +
  geom_point(size=3.0, aes(color=rho, shape=rho)) +
  scale_y_continuous(limits = c(min(bigMat$RelEff), 1)) +
  scale_color_discrete(name = "",
                       labels =
                         c(expression(paste(rho, "=",
                                              0))),
                         expression(paste(rho, "=",
                                              0.25))),
                         expression(paste(rho, "=",
                                              0.5))),
                         expression(paste(rho, "=",
                                              0.75))),
                         expression(paste(rho, "=",
                                              1)))) +
  scale_shape_discrete(name = "",
                       labels =
                         c(expression(paste(rho, "=",
                                              0))),
                         expression(paste(rho, "=",
                                              0.25))),
                         expression(paste(rho, "=",
                                              0.5))),
                         expression(paste(rho, "=",
                                              0.75))),
                         expression(paste(rho, "=",
                                              1)))) +
  theme_bw(base_size = 20) +
  theme(axis.line = element_line(colour = "black"),
        plot.title = element_text(size = 16, hjust = 0.2),
        panel.grid.major = element_blank(),
        panel.grid.minor = element_blank(),
        panel.border = element_blank(),
        panel.background = element_blank(),
        legend.key = element_blank(),
        legend.text.align = 0,
        axis.line.x = element_line(color="black", size = 0.5), ##this is to show axes - bug in this version
        axis.line.y = element_line(color="black", size = 0.5)) +
  labs(title=
        expression(atop("(c)",
                          paste("Fixed effects: I = 20, ",
                                S[list(kk,i)], " = ", 1, ", ",
                                rho[i], " = ", rho(i-1)/I))))
```

7 Create dataset for panel d)

These results are for the $p \geq 2$ exchangeable case with equal within-study and between-study variances, with $S_i^2 \equiv S^2, \rho_i = \frac{\rho(i-1)}{I}, \Sigma = 0$ (within study). The parameter we vary is the number of studies, I . We save the relative efficiencies for only one of coefficients, as they are all equal.

```
##save results in data frame
##het is sigma^2/S^2
bigMat <- expand.grid(p = 2:20,
                     I=c(5,10,15,20),
                     RelEff = NA)
for(rr in 1:nrow(bigMat))
{
  p <- bigMat[rr, 1]
  I <- bigMat[rr, 2]

  het <- 0

  ##index over the studies
  i <- 1:I
  ##within-study correlations
  rho.iW <- (i-1)/I
  ##overall correlations
  rho.i <- rho.iW

  ##calculate the two sums
  ##Sum1 is over 1/(1-rho_i)
  ##Sum2 is over 1/(1+(p-1)*rho_i)
  Sum1 <- sum(1/(1-rho.i))
  Sum2 <- sum(1/(1+(p-1)*rho.i))

  bigMat$RelEff[rr] <- I/p * (Sum1 + (p-1)*Sum2)/(Sum1*Sum2)
}

##make these changes for ggplot
bigMat$I <- as.factor(paste("I=", bigMat$I, sep=""))
bigMat$I <- relevel(bigMat$I, ref="I=5")
```

8 Create plot for panel d)

9 Put all four panels together

```
multiplot(panelA, panelC,
           panelB, panelD, cols=2)

## Loading required package: grid
```

```

panelD <- ggplot(bigMat,
                 aes(x=p, y=RelEff))+
  geom_line(aes(color=I, shape=I)) +
  geom_point(size=3.0, aes(color=I, shape=I)) +
  ylim(c(0.45,1)) +
  theme_bw(base_size = 20) +
  theme(axis.line = element_line(colour = "black"),
        plot.title = element_text(size = 15, hjust=0.5),
        panel.grid.major = element_blank(),
        panel.grid.minor = element_blank(),
        panel.border = element_blank(),
        panel.background = element_blank(),
        legend.background = element_rect(fill="transparent"),
        legend.key = element_blank(),
        legend.text.align = 0,
        ##legend.position = c(0.15,0.28),
        axis.line.x = element_line(color="black", size = 0.5), ##this is to show axes - bug in this version
        axis.line.y = element_line(color="black", size = 0.5)) +
  scale_y_continuous(name = expression(paste(RelEff)),
                    limits=c(min(bigMat$RelEff)*0.85, 1)) +
  scale_color_manual(values=rev(gg_color_hue(4)), name="")+
  scale_shape_manual(values=c(3,15,17,16),name="")+
  labs(title=expression(atop("(d)", paste("Fixed effects: ",
                                         S[kk,i], " = ", 1, ", ",
                                         rho[i], " = ", (i-1)/I))))

## Scale for 'y' is already present. Adding another scale
## for 'y', which will replace the existing scale.

```

