Code for Figure 1

August 2, 2016

1 Create datasets for panel a)

Load the necessary libraries, source the file with the R functions:

```
library(ggplot2)

source("functions.R")
```

These results are for the $I = 2, p = 2$ case, so the parameters we vary are: $r_1, r_2, \rho_{12,1}, \rho_{12,2}$. We save the relative efficiencies for both coefficients.

We take $r_1 = r_2 = 0.5$:

```
bigMat.a <- expand.grid(r=0.5,
                        rho121=c(0, 0.25, 0.5, 0.75),
                        rho122=(-19:19)/20)
bigMat.a <- cbind(bigMat.a, RelEff1=NA, RelEff2=NA)
bigMat.a <- as.matrix(bigMat.a)
for(i in 1:nrow(bigMat.a))
{
  rho121 <- bigMat.a[i,"rho121"]
  rho122 <- bigMat.a[i,"rho122"]
  r <- bigMat.a[i, "r"]

  bigMat.a[i,c(4,5)] <- effCalc2(rho112=rho121, rho212=rho122, r1=r, r2=r)
}
##turn it back into data frame (need it as data frame for ggplot)
bigMat.a <- as.data.frame(bigMat.a)
##check that relative efficiencies are identical for the two coefficients
identical(bigMat.a$RelEff1, bigMat.a$RelEff2)

## [1] TRUE

##rename RelEff1 as RelEff
names(bigMat.a)[names(bigMat.a) == "RelEff1"] <- "RelEff"
##make rho121 into a character(required for ggplot)
bigMat.a$rho121 <- paste("rho121=", bigMat.a$rho121, sep="")##as.factor(bigMat.a$r)
```

2 Create plot for panel a)

Panel a):

```
panelA <- ggplot(bigMat.a,
                 aes(x=rho122, y=RelEff)) +
  geom_line(size=1.3, aes(linetype=rho121, color=rho121)) +
  theme_bw(base_size = 20)+
  xlab(expression(paste(rho[2]))) +
  scale_color_discrete(name = "",
                       labels =
                         c(expression(paste(rho[1], "=",
                                            0)),
                           expression(paste(rho[1], "=",
                                            0.25)),
                           expression(paste(rho[1], "=",
                                            0.5)),
                           expression(paste(rho[1], "=",
                                            0.75)))) +
  scale_linetype_discrete(name = "",
                          labels =
                            c(expression(paste(rho[1], "=",
                                               0)),
                              expression(paste(rho[1], "=",
                                               0.25)),
                              expression(paste(rho[1], "=",
                                               0.5)),
                              expression(paste(rho[1], "=",
                                               0.75)))) +
  theme(axis.line = element_line(colour = "black"),
        plot.title = element_text(size = 16, hjust = 0.5),
        panel.grid.major = element_blank(),
        panel.grid.minor = element_blank(),
        panel.border = element_blank(),
        panel.background = element_blank(),
        legend.key = element_blank(),
        legend.text.align = 0,
        axis.line.x = element_line(color="black", size = 0.5), ##this is to show axes - bug in this vers
        axis.line.y = element_line(color="black", size = 0.5)) +
  labs(title=
         expression(atop("(a)",
                         paste("Fixed effects: I = 2, p = 2, ",
                               r[1], " = ", r[2], " = ", 0.5))))
```

3 Create datasets for panel b)

These results are for the $I = 2, p \geq 2$ exchangeable case with equal within-study variances, so the parameters we vary are: r, ρ_1, ρ_2, p. We save the relative efficiencies for only one of coefficients, as they are all equal. We consider $\rho_1 = 0$.

```
##save results in data frame
bigMat <- expand.grid(rho1 = 0,
                      rho2 = 0:3/4,
                      r = c(1, 3, 5, 9)/10,
                      p = 2:20,
                      RelEff = NA)
```

```
for(i in 1:nrow(bigMat))
{
    rho1 <- bigMat[i, 1]
    rho2 <- bigMat[i, 2]
    r <- bigMat[i, 3]
    p <- bigMat[i, 4]

    ##get variance-covariance matrices
    S1 <- r*ARMAcor(phi=rho1, rho=1, n=p)
    S2 <- (1-r)*ARMAcor(phi=rho2, rho=1, n=p)
    U1 <- diag(diag(S1))
    U2 <- diag(diag(S2))

    varMVMA <- solve(solve(S1)+solve(S2))
    varUVMA <- solve(solve(U1)+solve(U2)) %*%
      (solve(U1) %*% S1 %*% solve(U1) +
         solve(U2) %*% S2 %*% solve(U2)) %*%
      solve(solve(U1)+solve(U2))

    bigMat$RelEff[i] <- varMVMA[1,1]/varUVMA[1,1]
}

##for Panel b), r = 0.5:
bigMat.b <- bigMat[bigMat[,"r"]==0.5, ]
##transform back to data frame (for ggplot)
bigMat.b <- as.data.frame(bigMat.b)
##make rho2 into factor (for ggplot)
bigMat.b$rho2 <- as.factor(bigMat.b$rho2)
```

4 Create plot for panel b)

```
panelB <- ggplot(bigMat.b,
                 aes(x=p, y=RelEff))+
  geom_line(aes(color=rho2, shape=rho2)) +
  geom_point(size=3.0, aes(color=rho2, shape=rho2)) +
  theme_bw(base_size = 20)+
  scale_color_discrete(name = "",
                       labels =
                         c(expression(paste(rho[2], "=",
                                            0)),
                           expression(paste(rho[2], "=",
                                            0.25)),
                           expression(paste(rho[2], "=",
                                            0.5)),
                           expression(paste(rho[2], "=",
                                            0.75)))) +
  scale_shape_discrete(name = "",
                       labels =
                         c(expression(paste(rho[2], "=",
                                            0)),
                           expression(paste(rho[2], "=",
```

```
                                          0.25)),
                          expression(paste(rho[2], "=",
                                          0.5)),
                          expression(paste(rho[2], "=",
                                          0.75)))) +
  theme(axis.line = element_line(colour = "black"),
        plot.title = element_text(size = 16, hjust = 0.5),
        panel.grid.major = element_blank(),
        panel.grid.minor = element_blank(),
        panel.border = element_blank(),
        panel.background = element_blank(),
        legend.key = element_blank(),
        legend.text.align = 0,
        axis.line.x = element_line(color="black", size = 0.5), ##this is to show axes - bug in this vers
        axis.line.y = element_line(color="black", size = 0.5)) +
  labs(title=
         expression(atop("(b)",
                         paste("Fixed effects: I = 2, ",
                               rho[1], " = ", 0, ", ",
                                r[1], " = ", r[2], " = ", 0.5))))
```

5 Create datasets for panel c)

The following results are for the $I = 20, p \geq 2$ exchangeable case with equal within-study variances and $S_i^2 \equiv S^2, \rho_i = \frac{\rho(i-1)}{I}$, so the parameters we vary are ρ, p. We save the relative efficiencies for only one of coefficients, as they are all equal.

```
##number of studies
I <- 20

##save results in data frame
bigMat <- expand.grid(rho = c(0, 0.25, 0.5, 0.75),
                      p = 2:1000,
                      RelEff = NA)

for(n in 1:nrow(bigMat))
{
    rho <- bigMat[n, 1]
    p <- bigMat[n, 2]

    ##index over the studies
    i <- 1:I

    ##calculate the two sums
    ##Sum1 is over 1/(1-rho_i)
    ##Sum2 is over 1/(1+(p-1)*rho_i)
    rho.i <- rho*(i-1)/I
    Sum1 <- sum(1/(1-rho.i))
    Sum2 <- sum(1/(1+(p-1)*rho.i))

    bigMat$RelEff[n] <- I/p * (Sum1 + (p-1)*Sum2)/(Sum1*Sum2)
}
```

```
##make rho into factor (for ggplot)
bigMat$rho <- paste("rho=", bigMat$rho, sep="")
```

Compare minimum values with known asymptotic limits:

```
min(bigMat$RelEff[bigMat$rho=="rho=0.25"])

## [1] 0.8901406

min(bigMat$RelEff[bigMat$rho=="rho=0.5"])

## [1] 0.7512425

min(bigMat$RelEff[bigMat$rho=="rho=0.75"])

## [1] 0.5808737

asympLimit <- function(rho, I)
{
  i <- 1:I
  rho.i <- rho*(i-1)/I

  I*(1/sum(1/rho.i)+1/sum(1/(1-rho.i)))
}

asympLimit(0.25, I)

## [1] 0.875323

asympLimit(0.5, I)

## [1] 0.7344264

asympLimit(0.75, I)

## [1] 0.5631485

##only plot the subset with p between 2 and 20
bigMat <- bigMat[bigMat$p <= 20,]
```

6 Create plot for panel c)

```
panelC <- ggplot(bigMat,
                 aes(x=p, y=RelEff))+
  geom_line(aes(color=rho, shape=rho)) +
  geom_point(size=3.0, aes(color=rho, shape=rho)) +
  scale_y_continuous(limits = c(min(bigMat$RelEff), 1)) +
  scale_color_discrete(name = "",
                       labels =
                         c(expression(paste(rho, "=",
                                            0)),
                           expression(paste(rho, "=",
                                            0.25)),
```

```
                          expression(paste(rho, "=",
                                           0.5)),
                          expression(paste(rho, "=",
                                           0.75)))) +
  scale_shape_discrete(name = "",
                       labels =
                         c(expression(paste(rho, "=",
                                           0)),
                           expression(paste(rho, "=",
                                           0.25)),
                           expression(paste(rho, "=",
                                           0.5)),
                           expression(paste(rho, "=",
                                           0.75)))) +
  theme_bw(base_size = 20) +
  theme(axis.line = element_line(colour = "black"),
        plot.title = element_text(size = 16, hjust = 0.2),
        panel.grid.major = element_blank(),
        panel.grid.minor = element_blank(),
        panel.border = element_blank(),
        panel.background = element_blank(),
        legend.key = element_blank(),
        legend.text.align = 0,
        axis.line.x = element_line(color="black", size = 0.5), ##this is to show axes - bug in this vers
        axis.line.y = element_line(color="black", size = 0.5)) +
  labs(title=
         expression(atop("(c)",
                         paste("Fixed effects: I = 20, ",
                               S[list(kk,i)], " = ", 1, ", ",
                               rho[i], " = ", rho(i-1)/I))))
```

7 Put all three panels together

```
multiplot(panelA, panelB, panelC, cols=3)

## Loading required package:  grid
```