1. Introduction

Heat Exchangers have a vital role in any industry. One of the popular types of them is Finned-Plate Heat Exchangers. Finned plate heat exchangers have several industrial applications due to their high heat transfer rates and compact design. Some of these applications include:

1- HVAC Systems: Finned plate heat exchangers are commonly used in heating, ventilation, and air conditioning (HVAC) systems to transfer heat between fluids. They are particularly effective in air conditioning systems where the temperature difference between refrigerants and air is negligible.
2- Refrigeration: Finned plate heat exchangers are used in refrigeration systems to transfer heat between the refrigerant and the surrounding environment. They are commonly used in refrigeration systems for supermarkets, food processing plants, and other industrial applications.
3- Power Generation: Finned plate heat exchangers are used in power generation systems to transfer heat between fluids, such as cooling water and steam. They are commonly used in power plants, which help improve efficiency and reduce operating costs.
4- Chemical Processing: Finned plate heat exchangers are used in chemical processing plants to transfer heat between fluids. Their usage is in applications where the fluids have different viscosities, densities, or thermal conductivities.
5- Oil and Gas: Finned plate heat exchangers are used in the oil and gas industry to transfer heat between fluids. They are commonly used in gas processing plants, refineries, and other industrial applications.

In this study, we aim to show the temperature distribution in each element of this H.Ex by 1D, 3D, Contour, Heatmap, and imagesc plot. Our other goals are calculating the lost heat and finding an optimized length for each fin to reduce the heat loss.

2. Problem Expression

There is a finned-plate H.Ex with an initial temperature of $20\ ℃$ exposed to air with a temperature of $15\ ℃$. Plate two has been rotated $45°$ clockwise and connected to plate one by four fins. Plate one generates heat at the rate of 2MW on its surface. Both plates have rectangular shapes with dimensions $50 \times 50\ cm$ for plate one and $40 \times 40\ cm$ for plate two. In addition, the top and bottom of plate one has been covered with thermal insulation. The shape of the mentioned H.Ex. is shown in the figure below:

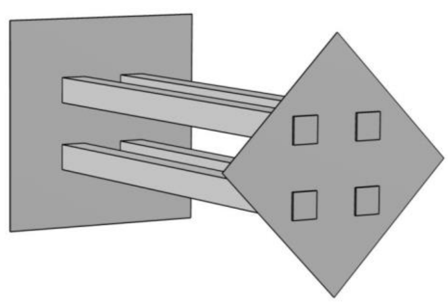

Figure 1- Schematic of H.Ex. Geometry

Each fin has a rectangular area with dimensions of $4\sqrt{2} \times 4\sqrt{2}\ cm$ and the length of $100\ cm$. The thermal condition for each plate is shown in the table below:

Plate / Side	Up	Down	Left	Right
1	Various	Various	140 ℃	160 ℃
2	130 ℃	130 ℃	100 ℃	100 ℃

The physical conditions of each plate and fin are shown in the table below:

Element	Thermal Conductivity coefficient ($\frac{W}{m.℃}$)	Density ($\frac{kg}{m^3}$)	Heat capacity ($\frac{J}{kg.℃}$)
Plate 1	302	8800	376
Plate 2	500	10510	230
Fin(s)	206	2707	896

2.1 Assumptions

To ease the calculation procedure, the several assumptions can be defined:

1) The system has reached to steady-state condition.
2) The physical properties of each element do not depend on the temperature.
3) The thickness of each plate is 1 m.
4) The mesh grid dimension is 1×1 cm.
5) The Convection Coefficient is constant.
6) The plates have convection heat transfer in their front and backward, and in plate 1, there is a convection heat transfer in z dimension.
7) For achieving more accurate results, some correlations have been applied to convection heat transfer coefficient in the cases below:

At the surface: $h' = h$ At the sides: $h' = 0.5\ h$ At the corners: $h' = 0.75\ h$

8) The temperature distribution in fins is just in z coordination.
9) Although fins have contact with air in their all areas, just 1 effective area is considered that is shown in the figure below:

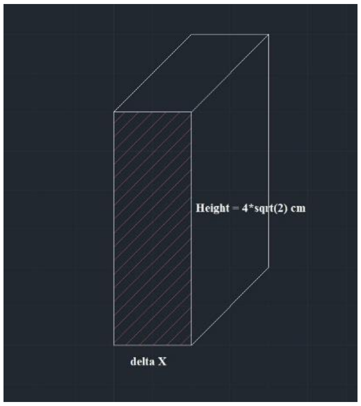

Figure 2- Effective Convection heat transfer area of fin

3. Theorem

3.1 Find the temperature distribution of each element of H.Ex.

At the beginning, the mesh and the precise location of each node must be determined. The meshes of each plate are shown in the figures below:

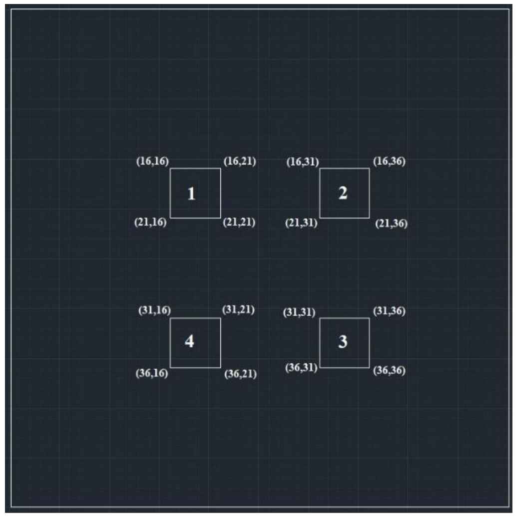

Figure 3- Mesh grid of Plate 1

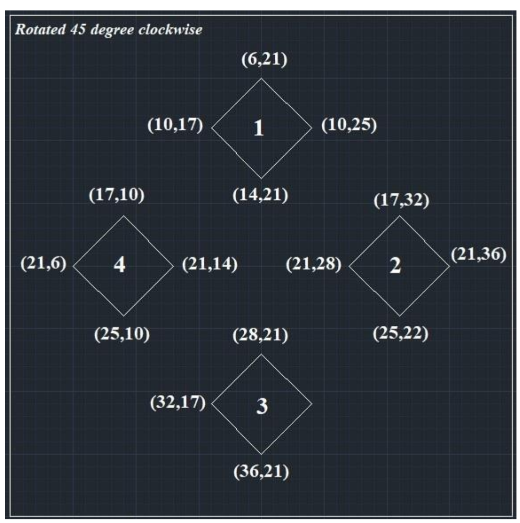

Figure 4- Mesh grid of plate 2

The used method in this study is Iterative Gauss-Seidel for the system of linear equations.

$$T_m = \frac{\left(q + \sum_n \frac{T_n}{R_{mn}}\right)}{\sum_n \frac{1}{R_{mn}}} \qquad 3\text{-}1$$

Which q represents the rate of heat generation, T represents the temperature of each node, and R_{mn} represents the thermal resistance.

3.1.1 Temperature distribution for plate 1

3.1.1.1 Temperature distribution for the top side

The schematic of each node is shown in the figure below:

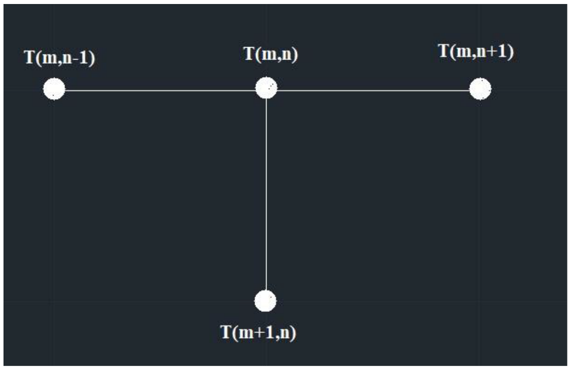

Figure 5- Schematic of each node in the top side of plate 1

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = (0.5 \times 2) \times h \times \Delta x^2$ 3-2

Conduction: $\frac{1}{R_{mn}} = \frac{k \times \Delta x}{\Delta y \times 1} = k$ 3-3

Thus, the final equation is:

$$T_{m,n} = \frac{q + k \times (T_{m,n-1} + T_{m,n+1} + 2 * T_{m+1,n}) + h \times \Delta x^2 \times T_\infty}{3k + h\Delta x^2} \quad 3\text{-}4$$

3.1.1.2 Temperature distribution for the bottom side

The schematic of each node is shown in the figure below:

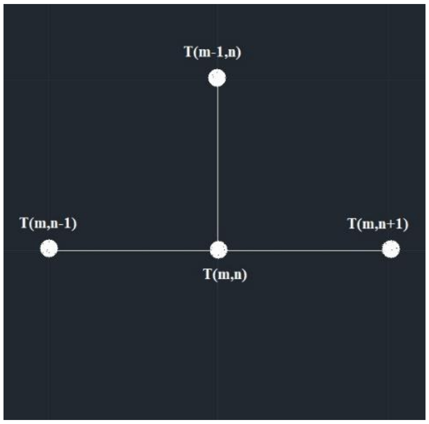

Figure 6- Schematic of each node in the bottom side of plate 1

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = (0.5 \times 2) \times h \times \Delta x^2$ 3-4

Conduction: $\frac{1}{R_{mn}} = \frac{k \times \Delta x}{\Delta y \times 1} = k$ 3-5

Thus, the final equation is:

$$T_{m,n} = \frac{q + k \times (T_{m,n-1} + T_{m,n+1} + 2 * T_{m-1,n}) + h \times \Delta x^2 \times T_\infty}{3k + h\Delta x^2} \quad 3\text{-}6$$

3.1.1.3 Temperature distribution for surface

The schematic of each node is shown in the figure below:

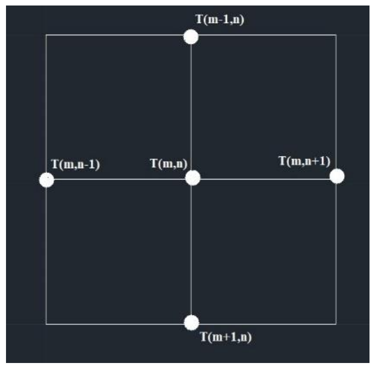

Figure 7- Schematic of each node in the surface of plate 1

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = h \times \Delta x^2$ 3-7

Conduction: $\frac{1}{R_{mn}} = \frac{k \times \Delta x}{\Delta y \times 1} = k$ 3-8

Thus, the final equation is:

$$T_{m,n} = \frac{q + k \times (T_{m,n+1} + T_{m+1,n} + T_{m,n-1} + T_{m-1,n}) + 2 \times h \times \Delta x^2 \times T_\infty}{4 * k + 2 \times h \times \Delta x^2} \quad 3\text{-}9$$

3.1.1.4 Temperature distribution for surface of fin area

The schematic of each node is shown in the figure below:

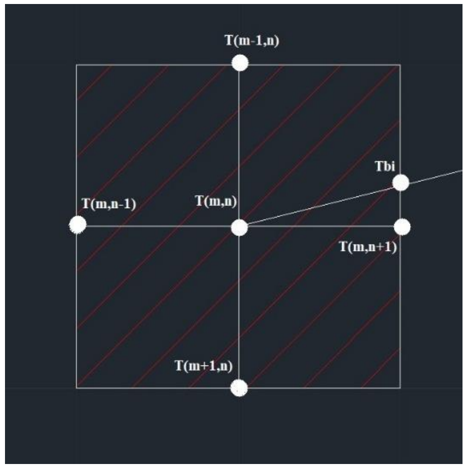

Figure 8- Schematic of each node in the surface of the fin area in plate 1

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = h \times \Delta x^2$ 3-10

Conduction: $\frac{1}{R_{mn}} = \frac{k \times \Delta x}{\Delta y \times 1} = k$ 3-11

Conduction in fin: $\frac{1}{R_{m,n}} = \frac{k \times 0.001}{\Delta y \times \Delta x} = 0.1 \times k$ 3-12

Thus, the final equation is:

$$T_{m,n} = \frac{q + (T_{m,n+1} \times k + T_{m+1,n} \times k + T_{m,n-1} \times k + T_{m-1,n} \times k + T_{bi} \times 0.1 \times k_f) + h \times \Delta x^2 \times T_\infty}{4 \times k + 0.1 \times k_f + h \times \Delta x^2} \quad 3\text{-}13$$

3.1.1.5 Temperature distribution for the corners of fin area

The schematic of each node is shown in the figure below:

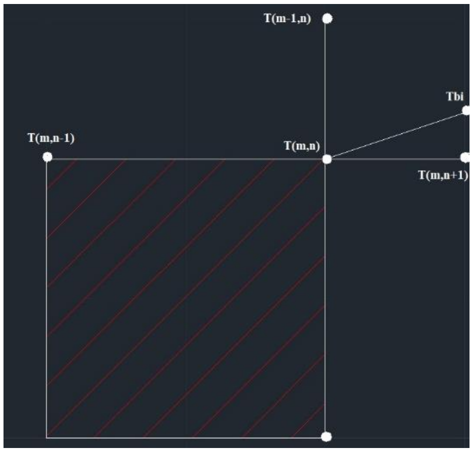

Figure 9 - Schematic of each node in the corners of the fin area in plate 1

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = (1 + 0.75) \times h \times \Delta x^2$ 3-14

Conduction: $\frac{1}{R_{mn}} = \frac{k \times \Delta x}{\Delta y \times 1} = k$ 3-15

Conduction in fin: $\frac{1}{R_{m,n}} = 0.25 \times \frac{k \times 0.001}{\Delta y \times \Delta x} = \mathrm{k} \times 0.1 \times 0.25$ 3-16

Thus, the final equation is:

$$T_{m,n} = \frac{q + (T_{m,n+1} \times k + T_{m+1,n} \times k + T_{m,n-1} \times k + T_{m-1,n} \times k + T_{bi} \times 0.1 \times 0.25 \times k_f) + (1+0.75) \times h \times \Delta x^2 \times T_\infty}{4 \times k + 0.1 \times 0.25 \times k_f + (1+0.75) \times h \times \Delta x^2}$$ 3-17

3.1.1.6 Temperature distribution for the sides of fin area

The schematic of each node is shown in the figure below:

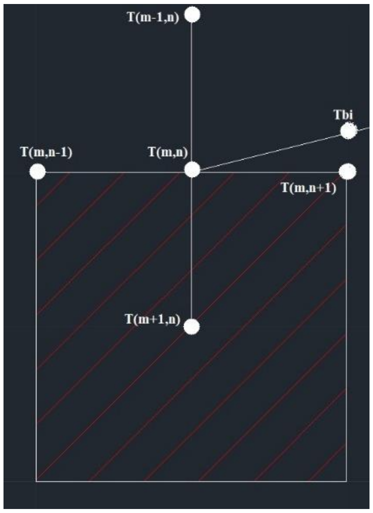

Figure 10- Schematic of each node in the sides of the fin area in plate 1

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = (1 + 0.5) \times h \times \Delta x^2$ 3-18

Conduction: $\frac{1}{R_{mn}} = \frac{k \times \Delta x}{\Delta y \times 1} = k$ 3-19

Conduction in fin: $\frac{1}{R_{m,n}} = 0.25 \times \frac{k \times 0.001}{\Delta y \times \Delta x} = \mathrm{k} \times 0.1 \times 0.5$ 3-20

Thus, the final equation is:

$$T_{m,n} = \frac{q + (T_{m,n+1} \times k + T_{m+1,n} \times k + T_{m,n-1} \times k + T_{m-1,n} \times k + T_{bi} \times 0.1 \times 0.5 \times k_f) + (1 + 0.5) \times h \times \Delta x^2 \times T_\infty}{4 \times k + 0.1 \times 0.25 \times k_f + (1 + 0.5) \times h \times \Delta x^2} \quad 3\text{-}21$$

3.1.2 Temperature distribution for plate 2

3.1.2.1 Temperature distribution for surface

The schematic of each node is shown in the figure below:

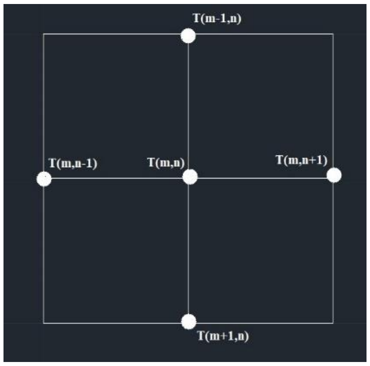

Figure 11- Schematic of each node in the surface of plate 1

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = 2 \times h \times \Delta x^2$ 3-22

Conduction: $\frac{1}{R_{mn}} = \frac{k \times \Delta x}{\Delta y \times 1} = k$ 3-23

Thus, the final equation is:

$$T_{m,n} = \frac{q + k \times (T_{m,n+1} + T_{m+1,n} + T_{m,n-1} + T_{m-1,n}) + 2 \times h \times \Delta x^2 \times T_\infty}{4 \times k + 2 \times h \times \Delta x^2} \quad 3\text{-}24$$

3.1.2.2 Temperature distribution for the surface of fin area

The schematic of each node is shown in the figure below:

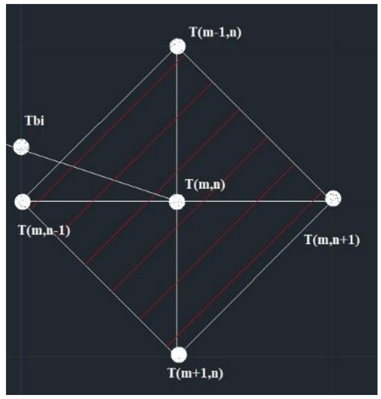

Figure 12- Schematic of each node in the surface of the fin area in plate 2

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = h \times \Delta x^2$ 3-25

Conduction: $\frac{1}{R_{mn}} = \frac{k\times\Delta x}{\Delta y\times 1} = k$ 3-26

Conduction in fin: $\frac{1}{R_{m,n}} = 0.25 \times \frac{k\times 0.001}{\Delta y\times\Delta x} = \mathrm{k} \times 0.2$ 3-27

Thus, the final equation is:

$$T_{m,n} = \frac{\left(T_{m,n+1}\times k + T_{m+1,n}\times k + T_{m,n-1}\times k + T_{m-1,n}\times k + T_{bi}\times 0.2\times k_f\right) + h\times\Delta x^2\times T_\infty}{4\times k + 0.1\times k_f + h\times\Delta x^2}$$ 3-28

3.1.2.3 Temperature distribution for the corners of fin area

The schematic of each node is shown in the figure below:

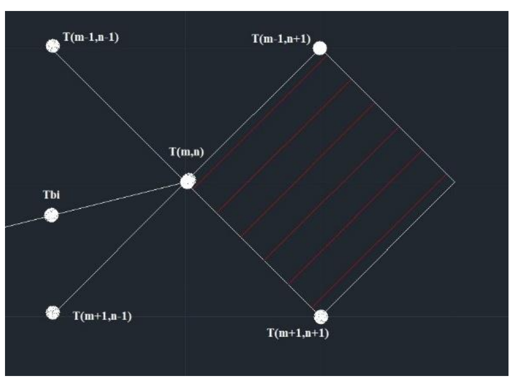

Figure 13- Schematic of each node in the corners of the fin area in plate 2

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = (1 + 0.75) \times h \times \Delta x^2$ 3-29

Conduction: $\frac{1}{R_{mn}} = \frac{k\times\Delta x}{\Delta y\times 1} = k$ 3-30

Conduction in fin: $\frac{1}{R_{m,n}} = 0.25 \times \frac{k\times 0.001}{\Delta y \times \Delta x} = \mathrm{k} \times 0.2 \times 0.25$ 3-31

Thus, the final equation is:

$$T_{m,n} = \frac{\left(T_{m-1,n+1}\times k+T_{m-1,n-1}\times k+T_{m+1,n-1}\times k+T_{m+1,n+1}\times k+T_{bi}\times 0.2\times 0.25\times k_f\right)+(1+0.75)\times h\times \Delta x'^2 * T_\infty}{4\times k+0.2\times 0.25\times k_f+(1+0.75)\times h\times \Delta x'^2} \quad 3\text{-}32$$

3.1.2.4 Temperature distribution for the sides of fin area

The schematic of each node is shown in the figure below:

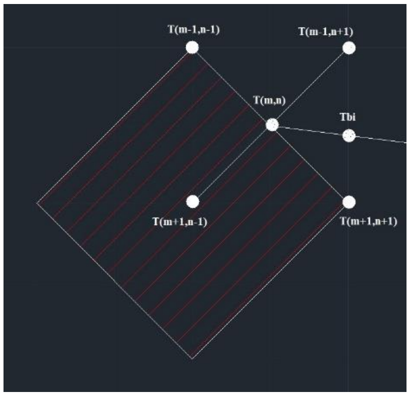

Figure 14- Schematic of each node in the sides of the fin area in plate 2

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = (1+0.5) \times h \times \Delta x^2$ 3-32

Conduction: $\frac{1}{R_{mn}} = \frac{k\times \Delta x}{\Delta y \times 1} = k$ 3-33

Conduction in fin: $\frac{1}{R_{m,n}} = 0.25 \times \frac{k\times 0.001}{\Delta y \times \Delta x} = \mathrm{k} \times 0.2 \times 0.5$ 3-34

Thus, the final equation is:

$$T_{m,n} = \frac{\left(T_{m-1,n+1}\times k+T_{m-1,n-1}\times k+T_{m+1,n-1}\times k+T_{m+1,n+1}\times k+T_{bi}\times 0.2\times 0.5\times k_f\right)+(1+0.5)\times h\times \Delta x'^2 * T_\infty}{4\times k+0.2\times 0.5\times k_f+(1+0.5)\times h\times \Delta x'^2} \quad 3\text{-}35$$

3.1.3 Temperature distribution for each fin

According to assumptions, there are 3 cases for calculating the temperature of each node:

1) Base that connected to the Plate 1 & 2:

To reach a unit temperature, calculate the average of the Base Temperature matrix elements.

$$T = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} T_{i,j}}{m\times n} \quad 3\text{-}36$$

2) Whole of the fin:

The schematic of each node is shown in the figure below:

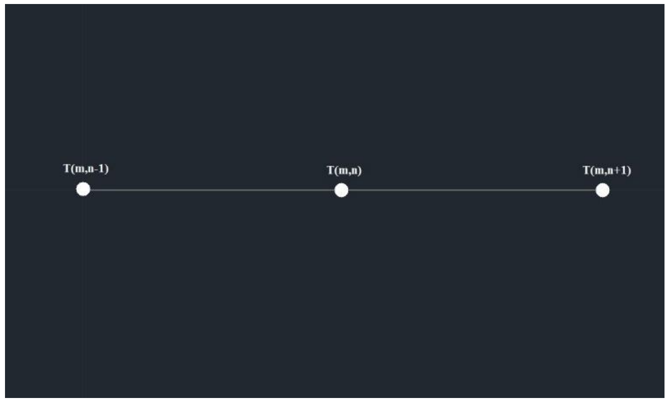

Figure 15- Schematic of each node in every fin

The values of R_{mn} have been determined for each heat transfer mechanism:

Convection: $\frac{1}{R_{mn}} = 4 \times h \times W \times \Delta x$ 3-37

Conduction: $\frac{1}{R_{mn}} = \frac{W^2 * k_f}{\Delta x} = 0.32\ k_f$ 3-38

Thus, the final equation is:

$$T_{m,n} = \frac{0.32k_f \times (T_{m,n-1} + T_{m,n+1}) + 4 \times h \times W \times \Delta x \times T_\infty}{2 \times 0.32k_f + 4 \times h \times \Delta x \times W}$$ 3-39

3.2 Find the heat loss between H.Ex. and air

In regards to assumptions, the heat transfer coefficient is considered a constant value, so the lost heat can be calculated using Newton's law of cooling:

$$Q = h.A.(T - T_\infty)$$ 3-40

Therefore, the heat loss for each node can be obtained from eq. , and the total heat loss can be resulted from summation of the heat loss vector elements.

3.3 Optimize the fin length to reduce heat loss

In this case, assume that the convection heat transfer coefficient varies with the length of fin. In another speech, the convection heat transfer coefficient of each node differs from another node in a particular fin. The convective heat transfer coefficient obeys from the following polynomial:

$$h(L)\ \left[\frac{W}{m^2.℃}\right] = -2 \times 10^{-9}L^6 + 5 \times 10^{-7}L^5 - 6 \times 10^{-5}L^4 + 0.0031L^3 - 0.0401\ L^2 - 1.0438L + 26.947$$ 3-41

The temperature distribution in each fin can be obtained from eq. . Thus, the heat loss of each node can be calculated from Newton's colling law. The optimized fin length can be calculated after the summation of the heat loss vectors.

4. Results

4.1 The temperature distribution of each H.Ex. element

The 3D plot of Temperature distribution for plate 1 is shown in the figure below:

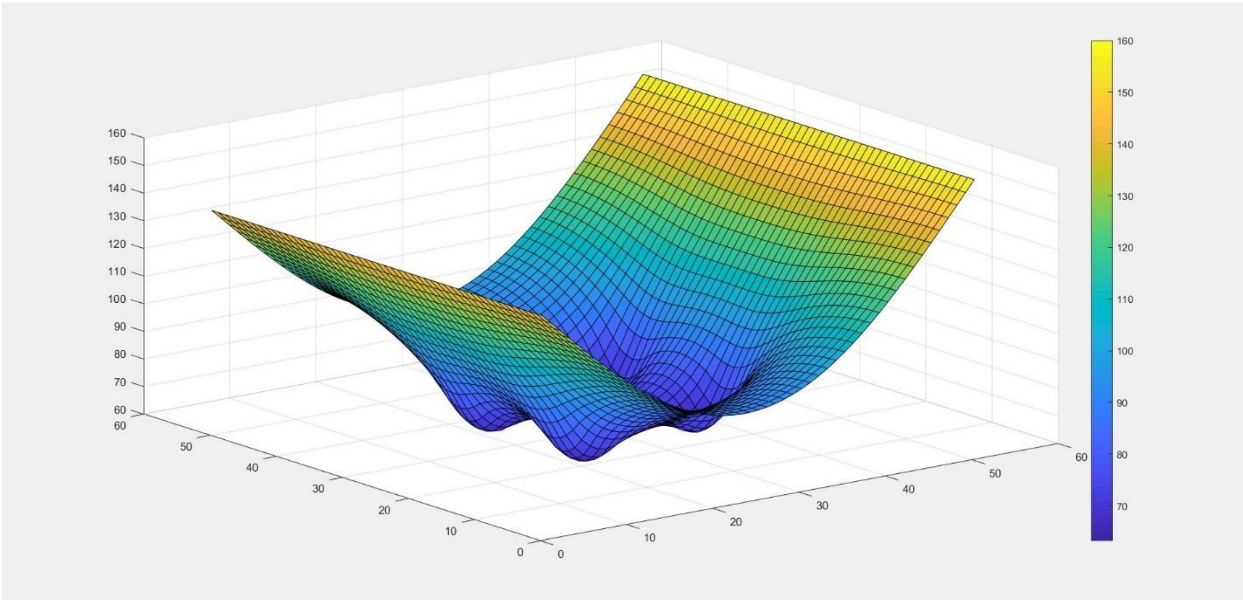

Figure 16- The temperature distribution of plate 1

The 3D plot of Temperature distribution for plate 2 is shown in the figure below:

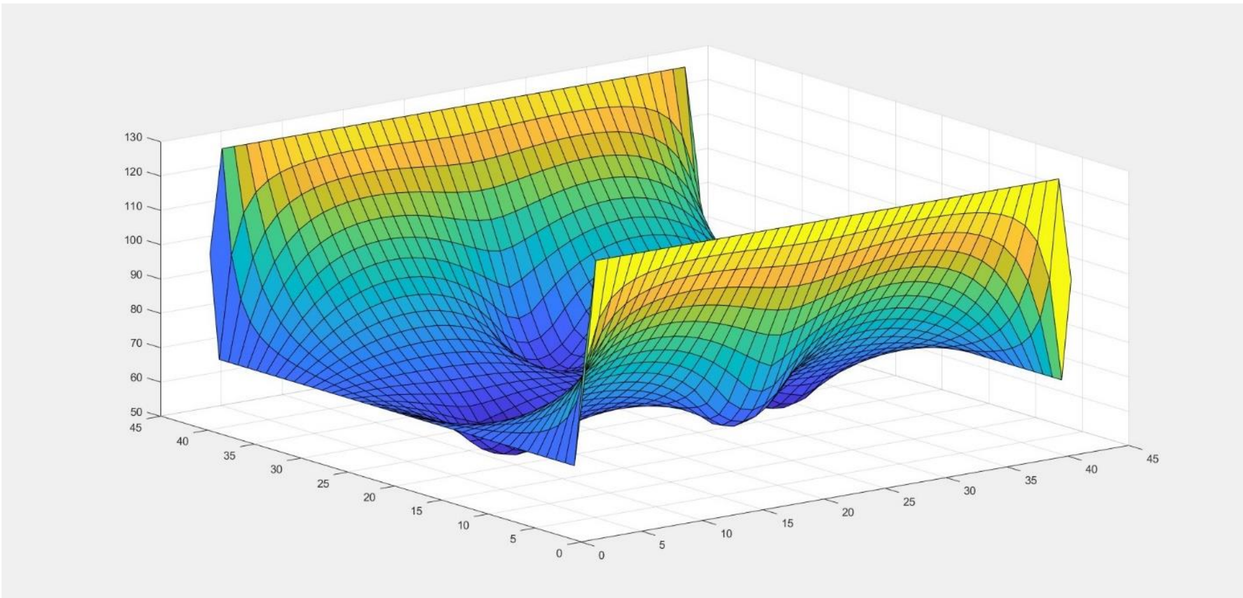

Figure 17- The temperature distribution of plate 2

The curves have a smooth shape and foveae, because of the existence of fins. Fig.17 has a sharp shape, because the temperature of all sides is bounded. The depth of the foveae of fig.17 is less than fig.16, because there is a heat generation term for plate 1 and the temperature of all sides of plate 2 is bounded.

The 1D plot of Temperature distribution for fins is shown in the figure below:

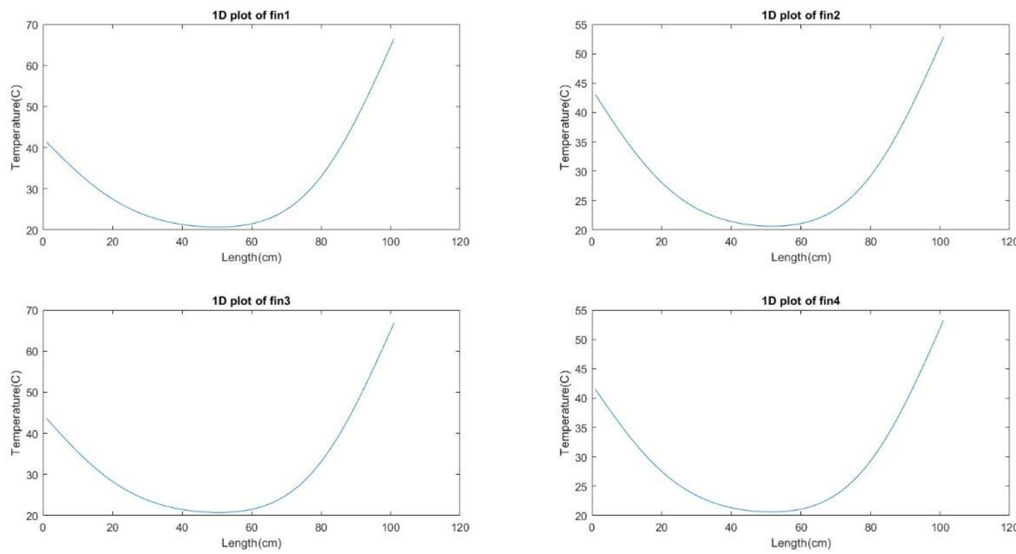

Figure 18- The temperature distribution of fins

The general form of all curves is almost the same, but there is difference in initial and final node temperature. The difference between temperature matrix of fin base caused this occurrence.

Other types of plots are available in the attached MATLAB code.

4.2 The heat loss between H.Ex. and air

Three cases have been studied. For each case, computer wants a generated heat flux value from user. The generated heat rate will be obtained from the equation below:

$$Q = q \left[\frac{W}{m^2}\right] . A = q \times 10^6 \times 0.5 \times 0.5 \times 0.001 \qquad 4.1$$

The results of study for three cases are shown in the table below:

Case	Generated heat $\left[\frac{W}{m^2}\right]$	Heat loss $[W]$	$\frac{Q}{Q_{loss}}$
No. 1	2	393.8674	1.269
No. 2	10	394.7034	6.334
No. 3	20	395.7483	12.634

The $\frac{Q}{Q_{loss}}$ versus generated heat plot is shown in the figure below:

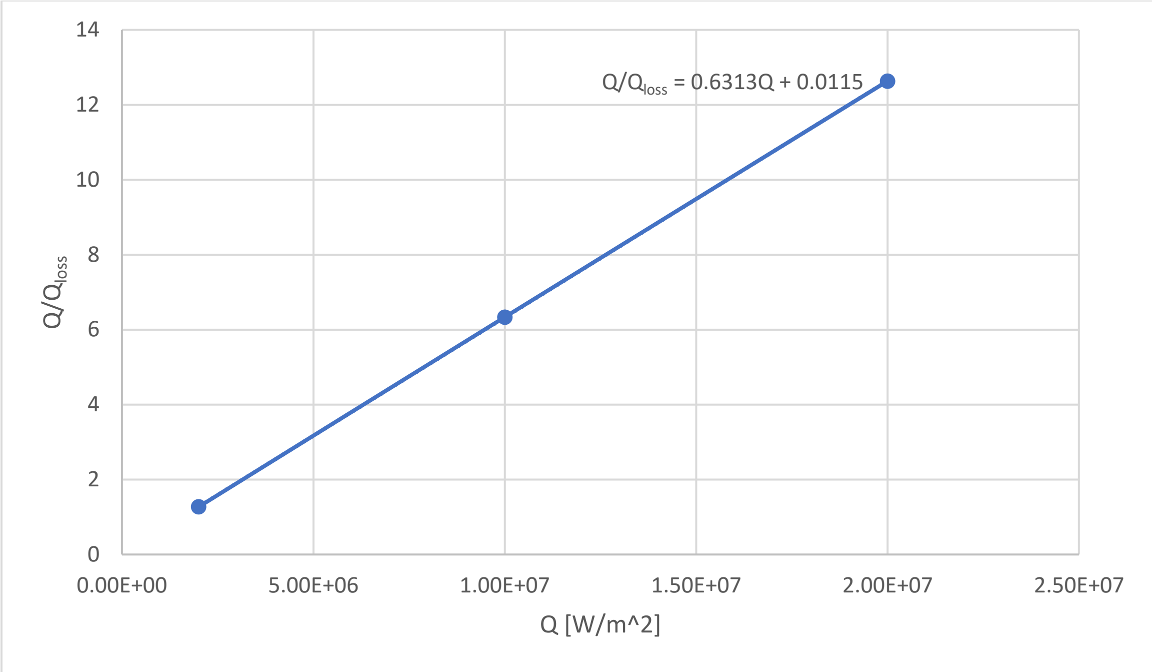

Figure 19 - Q/Q_{loss} versus Q plot

As expected, there is a linear correlation between these parameters. The assumptions and simplifications caused this occurrence.

5. References

[1]: D. Q. Kern, A. D. Kraus. (1972). *Extended surface heat transfer*.

[2]: Holman, J. P. (2010). *Heat Transfer*. Jefferson City, MO: MCGraw-Hill College.

[3]: Özışık, M. N. (1979). *Heat transfer.* McGraw-Hill College.

[4]: R. R. Harper, W. B. Brown. (1923). *Mathematical equations for heat conduction in the fins of air-cooled engines.*