ANT
Lagrange Interpolation

Q.1 Using the Lagrange Interpolation method, estimate the Interpolating function and evaluate it at x=3, for the following two Tables:

(a)

x	y
0.5	2
5	0.2
10	0.1

(b)

x	y
0.5	2
1	1
2	0.5
4	0.25
5	0.2
8	0.125
10	0.1

Also, plot the Lagrange Interpolating polynomial from x=0.5 to x=10 in steps of 0.1, for both the cases (superimposed plot) and also the dataset.

Homework - First Week:

2. For the given functions $f(x)$, let $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$, and find the absolute error.

a. $f(x) = \sin \pi x$

b. $f(x) = \sqrt[3]{x - 1}$

c. $f(x) = \log_{10}(3x - 1)$

d. $f(x) = e^{2x} - x$

11. Use Neville's method to approximate $\sqrt{3}$ with the following functions and values.

a. $f(x) = 3^x$ and the values $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 2$.

b. $f(x) = \sqrt{x}$ and the values $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 4$, and $x_4 = 5$.

c. Compare the accuracy of the approximation in parts (a) and (b).

22. Suppose $x_j = j$, for $j = 0, 1, 2, 3$ and it is known that

$$P_{0,1}(x) = x + 1, \quad P_{1,2}(x) = 3x - 1, \quad \text{and} \quad P_{1,2,3}(1.5) = 4.$$

Find $P_{0,1,2,3}(1.5)$.

Inverse Interpolation Suppose $f \in C^1[a, b]$, $f'(x) \neq 0$ on $[a, b]$ and f has one zero p in $[a, b]$. Let $x_0, \ldots, x_n$, be $n+1$ distinct numbers in $[a, b]$ with $f(x_k) = y_k$, for each $k = 0, 1, \ldots, n$. To approximate p, construct the interpolating polynomial of degree n on the nodes $y_0, \ldots, y_n$ for f^{-1}. Since $y_k = f(x_k)$ and $0 = f(p)$, it follows that $f^{-1}(y_k) = x_k$ and $p = f^{-1}(0)$. Using iterated interpolation to approximate $f^{-1}(0)$ is called *iterated inverse interpolation*.

26. Use iterated inverse interpolation to find an approximation to the solution of $x - e^{-x} = 0$, using the data

x	0.3	0.4	0.5	0.6
e^{-x}	0.740818	0.670320	0.606531	0.548812

27. Construct an algorithm that can be used for inverse interpolation.

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2nd Class - 4th Jan 2019

Q.1 For the following dataset:

x	y
1	1
4	0.25
5	0.2
10	0.1

(a) Evaluate Lagrange Interpolating polynomial $P_1(x)$ using 1st two data points
(b) Evaluate Lagrange Interpolating polynomial $P_2(x)$ using 1st three data points
(c) Evaluate Lagrange Interpolating polynomial $P_3(x)$ using all four data points
(d) Evaluate Navilles Interpolating polynomials, $f_i^{(n)}(x)$ recursively.
(e) Show that Navilles polynomials are same as Lagrange interpolating polynomials

Q.2 Show that $P_2(x) = f_0^{(2)}(x)$