Cubic Spline

Questions

Example 1 At the beginning of Chapter 3 we gave some Taylor polynomials to approximate the exponential function $f(x) = e^x$. Here we will use a natural spline and the data points $(0, 1)$, $(1, e)$, $(2, e^2)$, and $(3, e^3)$ to (a) form a new approximating function $S(x)$. Then (b) we will compare the integrals of f and S on the interval $[0, 3]$.

Q1: Use the data set given above to get the interpolating function using cubic spline method, with free or natural boundary condition.

Q2. Plot the Interpolating function in the range given above, and compare it with the Lagrange interpolating function. Also plot the exact function in the same range.

Q3. Use the functions (Cubic spline, Lagrange, exact) to get the integral in the range given above.

Q4. Solve the same problem, i.e. finding the interpolating polynomial using cubic spline method, for the above dataset using Clamped B.C., S’(x=0) = 1 and S’(x=3)=exp(3).

Q5. Use the interpolating function in Q4 to get the integral in the given range. Is the precision of this result better than that of Natural B.C.? Justify your answer.

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Q6.

A natural cubic spline S on $[0, 2]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & \text{if } 0 \le x < 1, \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if } 1 \le x \le 2. \end{cases}$$

Find b, c, and d.

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Q7

Figure 3.10 shows a ruddy duck in flight. To approximate the top profile of the duck, we have chosen points along the curve through which we want the approximating curve to pass. Table 3.17 lists the coordinates of 21 data points relative to the superimposed coordinate system shown in Figure 3.11. Notice that more points are used when the curve is changing rapidly than when it is changing more slowly.

Figure 3.10

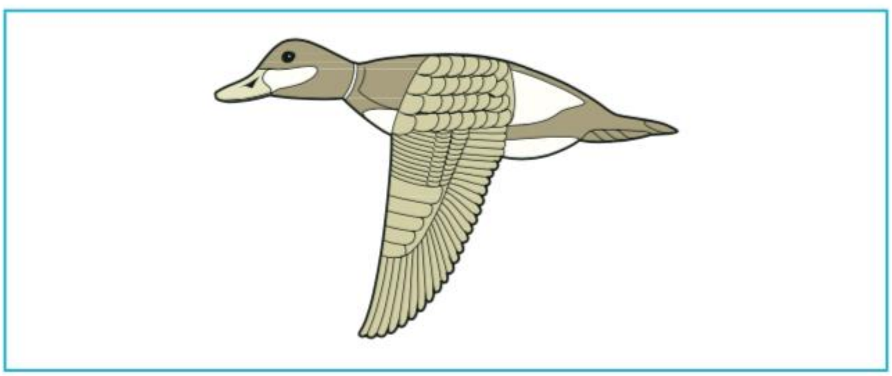

Table 3.17

x	0.9	1.3	1.9	2.1	2.6	3.0	3.9	4.4	4.7	5.0	6.0	7.0	8.0	9.2	10.5	11.3	11.6	12.0	12.6	13.0	13.3
$f(x)$	1.3	1.5	1.85	2.1	2.6	2.7	2.4	2.15	2.05	2.1	2.25	2.3	2.25	1.95	1.4	0.9	0.7	0.6	0.5	0.4	0.25

(i) Using Natural Boundary Conditions, find the cubic spline interpolating function and plot it. Superimpose the data points on the same plot.

(ii) Compare the result with the Lagrange interpolating function.

Q.7: Perform Cubic Spline Interpolation using Natural Boundary Conditions on the following dataset (x y), and plot the datapoints and the interpolating function:

-4.09091 -1

-3.28283 -0.999997

-2.67677 -0.999847

-1.9697 -0.994657

-1.56566 -0.973183

-0.959596 -0.825242

-0.757576 -0.715999

-0.151515 -0.169667

0.151515 0.169667

0.454545 0.479662

1.26263 0.92584

2.17172 0.997869

3.08081 0.999987

3.88889 1

4.59596 1

5 1