Problems on Linear Least Square Fitting

Q1. Plot the following datasets with error and fit it with straight line. Also plot all fitting lines in the same graph.

x_i	1	2	3	4	5	6	7	8	9	10
y_i	2.8	3.6	2.8	3.5	4.2	5.1	6.6	8.6	10.8	14.2
σ_1	0	0	0	0	0	0	0	0	0	0
σ_2	1	1	1	1	1	1	1	1	1	1
σ_3	0.3	0.5	0.55	0.6	0.65	0.7	0.75	0.9	1.1	1.3

Q2. Using Least Square fit method, fit the data points given below to the polynomial $P_n(x)$ with varying degrees (n = 1 to 7).

(i) For each value of n, obtain the (a) values of the coefficients and (b) corresponding minimum chi-square for the best fit.

(ii) Make a superimposed plot of the data and the best-fit polynomials for all n.

(iii) Plot the minimum chi-square vs. N

xi	yi
0.0	0.2
0.01	.231895
0.02	.264668
0.03	.289191
0.04	..332345
0.05	.368007
0.06	.403062
0.07	.43739
0.08	.472877
0.09	0.508413
0.1	0.543893
0.11	0.579221
0.12	0.614274
0.13	0.648984
0.14	0.683257
0.15	0.717008
0.16	0.717008
0.17	0.782653
0.18	.814414

Q3.

Fitting a dataset (x_i,y_i,σ_i) using a curve y=f(x)=a*exp(bx), where values of the parameters a and b are unknown, but defined within a set of range.

Dataset (x_i,y_i,σ_i) is the following:

i	1	2	3	4	5	6	7
x_i	1	2	3	4	5	6	7
y_i	4	5	8	16	30	38	70
σ_i	2	2	3	3	4	5	5

Define $\chi^2(a,b) = \sum\left[\frac{y_i - f(x_i)}{\sigma_i}\right]^2$

Prob1: Tabulate and plot $\chi^2(a = 2.101, b)$ vs b (in steps of 0.1).
(i) Find the value of b for which $\chi^2(a = 2.101, b)$ is minimum χ^2_{min}.
(ii) Find the values of b for which χ^2 value is $\chi^2_{min} + 1$
(iii) Find the values of b for which χ^2 value is $\chi^2_{min} + 4$
(iv) Find the values of b for which χ^2 value is $\chi^2_{min} + 9$

Prob2: Tabulate and plot $\chi^2(a, b = 0.498)$ vs a (in steps of 0.1).
(i) Find the value of a for which $\chi^2(a, b = 0.498)$ is minimum χ^2_{min}.
(ii) Find the values of a for which χ^2 value is $\chi^2_{min} + 1$
(iii) Find the values of a for which χ^2 value is $\chi^2_{min} + 4$
(iv) Find the values of a for which χ^2 value is $\chi^2_{min} + 9$

Prob3: Tabulate and plot $\chi^2(a, b)$ vs a, b (both in steps of 0.1).
(i) Find the values of (a, b) for which $\chi^2(a, b)$ is minimum χ^2_{min}. Plot a vs b.
(ii) Find the values of (a, b) for which χ^2 value is $\chi^2_{min} + 1$. Plot a vs b.
(iii) Find the values of (a, b) for which χ^2 value is $\chi^2_{min} + 4$. Plot a vs b.
(iv) Find the values of (a, b) for which χ^2 value is $\chi^2_{min} + 9$. Plot a vs b.

Problem 4:

Repeat question two,

(a) For the following data-sets with two different sets of errors (σ_1,σ_2).

x_i	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y_i	6.33	6.51	6.43	5.85	4.71	3.13	1.53	0.64	1.58	5.91	15.71
σ_1	0.95	0.98	0.96	0.88	0.71	0.47	0.23	0.09	0.24	0.89	2.35
σ_2	0.06	0.13	0.19	0.23	0.23	0.19	0.11	0.05	0.14	0.59	1.73

(b) fit upto 3rd degree polynomial for given data-set (n=1,2,3).

xi	1	2	3	4	5	6	7	8	9	10
yi	6.37	17.42	34.13	56.50	84.53	118.22	157.57	202.58	253.25	309.50
σ_1	3.19	8.71	17.06	28.25	42.26	59.11	78.78	101.29	126.62	154.79