DU M.Sc. Physics IV Semester
Advanced Numerical techniques (Lab)
Assignment-2

1. Integrate to an accuracy of 1 in 10^5 for given limits a and b:

$$\int_a^b \frac{\arctan x}{x^2}, \qquad a = 5, b = 10 \qquad (Answer : 0.142208)$$

Use Trapezoidal & Simpson rule.

2. The time period of a pendulum is given by the integral

$$T = 4\int_0^{\frac{\pi}{2}} \frac{1}{1 - \sin^2\left(\frac{A}{2}\right)\sin^2 x} dz$$

where $A$is the amplitude of oscillations. For small amplitudes it is possible to approximate the time period to

$$T_1 = 2\pi\left[1 + \left(\frac{A}{4}\right)^2\right]$$

Plot T, T_1 and the percentage difference between T and T_1 as functions of A for $0 < A < \pi$.

3. Let $R(\theta)$ be the polar coordinates of a particle moving under a central force. Then θ is given as a function of R by the expression:

$$\theta(R) = \int_{r_0}^{R} \frac{dr}{r^2\left[\left(\frac{2mE}{l^2}\right) - \left(\frac{2mV(r)}{l^2}\right) - \frac{1}{r^2}\right]^{\frac{1}{2}}}$$

Plot the orbit of the particle for $V(r) = -\frac{k}{r}$ (inverse square law force). Use Gauss quadrature for the evaluation of the integral. The upper limit,R is to be varied from r_0 to r_m, where r_0 and r_m are the two zeroes of the factor in the square brackets. Take $m = l = k = 1$ and

i) $E = -0.25$ (This gives $r_0 = 0.6, r_m = 3.4$ approximately)
ii)$E = 0 (r_0 \approx 0.5, r_m \approx 5)$

4. Locate the smallest positive root of the function $F(x)$, given by:

$$F(x) = \int_0^{\pi} \cos\left[x^a \cos(t)\right] \sin^{2n+1} t dt$$

to an accuracy of 4 significant figures, for $n = 1$ and $a = 1.5$.

5. Use the integral representation of the Bessel function:

$$J_n(z) = \frac{1}{2\pi} \int_0^{2\pi} \cos(z\cos(x))dx$$

to find its zeroes in the range $0 \leq z \leq 12$ by secant method.

6. The spherical Bessel function of order n is given by

$$j_n(z) = \frac{z^n}{2^{n+1}n!} \int_0^{\pi} \cos(z\cos\theta)\sin^{2n+1}\theta d\theta$$

Find all the roots of $j_2(z)$ between 0 and 10.