# STOR 565 Spring 2018 Homework 1

### Due on 01/22/2018 in Class

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*Remark.* This homework aims to introduce you to the basics in  $\mathbf{R}$ , which would be the main software we shall work on throughout this course.

Instruction. Download the whole folder for this homework. Open the **RMarkdown** document with surfix ".rmd" via **RStudio**. Click Knit to create a PDF document (remember to install the necessary packages in your **R**). By removing the results='hide' and fig.keep='none' options in the code chunks, the code outputs and the plots would display in the created file. For more information about the **RStudio**, refer to the section **Getting Started**; about the **RMarkdown**, refer to the online tutorial and the online manual of knitr by Yihui Xie from **RStudio**, Inc.

Please turn off the display of example code chunks (by specifying include=FALSE), complete the exercise code chunks (remember to turn on the eval option), fill in your name and create a PDF document, then print and submit it.

### **Getting Started**

 ${\bf R}$  is one of the most common coding language in data analysis nowadays. It's a free, open-source and powerful software environment for statistical computing and graphics. A nice description of  ${\bf R}$  can be found in the course website of German Rodriguez from Princeton. To download and install  ${\bf R}$ , click into the CRAN (Comprehensive R Archive Network) Mirrors and choose a URL that applies to you. To run with  ${\bf R}$ , click "RGui.exe" in Windows system or "R.app" in Mac OS.

The console of **R** is not the most friendly interface to work with, as compared to the more handful editor **RStudio**. You can download and install it via its website and choose the free version of **RStudio Desktop**. Note that when you run with **RStudio**, the mirror of **R** should have been installed in your system, even though you are not accessing to it directly.

The coming sections lead you to the essentials of **R**. To explore more about how **R** works, please refer to (Dalgaard 2008, W. N. Venables and R Core Team (2017)) and the tutorial of German Rodriguez.

#### Basics in R

**R** is an object-oriented language. Hence the "data" we work on are formatted as a particular object that meets some structural requirements (subjected to a particular class). Thus one should first understand which class of object he/she has on hand, and then figures out the applicable operations on it.

In a hierarchical manner, the more advanced class consists of ingredients from more fundamental classes. Vectors, matrices, lists, data frames and factors are the most commonly used fundamental classes in data analysis.

#### Vectors and Matrices

Vector is a collection of "data" that share the same type (numeric, character, logic or NULL). And matrix arranges "data" of the same type in two dimensions. Note that there doesn't exist a "scalar" object, which would be treated as a vector of length 1.

#### Create a Vector

## [1] 1 2 1 2 1 1 2 2

The combining function c() can be used to manually create a vector in  $\mathbf{R}$ . When using the c() function, numbers are entered as a list with commas between each new entry. For example, x <- c(1, 2) creates a vector and assigns it to the variable x.

To create a vector that repeats n times, we can use the replication function rep( , n). For example, a vector of five TRUE's can be obtained by  $x \leftarrow rep(TRUE, 5)$ .

Finally, we can create a consecutive sequence of numbers using the sequence generating function seq(from = , to = , by = ). Here, the from, to and by arguments specify where the sequence begins, ends, and by how much the sequence increments. For example, the vector (2,4,6,8) can be obtained using x < - seq(2,8,2). A convenient operator: similar to seq also creates the consecutive sequence stepped by 1 or -1. Try 1:4 and 4:1.

For more information, the commands ?c, ?rep and ?seq access to the online R documents for help.

Exercise 1. (5 pt) Using the c, rep or seq commands, create the following 6 vectors:

```
x1 = (2, .5, 4, 2);
x2 = (2, .5, 4, 2, 1, 1, 1, 1);
x3 = (1, 0, -1, -2);
x4 = ("Hello", "", "World, "!", "Hello World!");
Hint. Try paste function.
x5 = (TRUE, TRUE, NA, FALSE);
Hint. Check ?NA and class(NA) to learn more about the missing value object NA.
x6 = (1, 2, 1, 2, 1, 1, 2, 2).
x1 < -c(2, .5, 4, 2)
x2 <- c(x1, rep(1,4))
x3 < -1:-2
x4 <- paste("Hello"," ","World,","!","Hello World!",sep = ",")
x5 <- c(TRUE, TRUE, NA, FALSE)
x6 \leftarrow c(1, 2, 1, 2, 1, 1, 2, 2)
print(x1)
## [1] 2.0 0.5 4.0 2.0
print(x2)
## [1] 2.0 0.5 4.0 2.0 1.0 1.0 1.0 1.0
print(x3)
## [1] 1 0 -1 -2
print(x4)
## [1] "Hello, ,World,,!,Hello World!"
print(x5)
## [1] TRUE TRUE
                        NA FALSE
print(x6)
```

#### Create a Matrix

An m-by-n matrix can be created by the command  $\mathtt{matrix}(\ ,\ m,\ n)$  where the first argument admits a vector with length compatible with the matrix dimensions. For example,  $x \leftarrow \mathtt{matrix}(1:4,\ 2,\ 2)$  creates a 2-by-2 matrix that arranges the vector  $(1,\ 2,\ 3,\ 4)$  by column. To arrange the vector by row, specify the byrow option like this:  $x \leftarrow \mathtt{matrix}(1:4,\ 2,\ 2,\ \mathtt{byrow} = \mathtt{TRUE})$ .

Moreover, the command binding vectors/matrices by row  $\mathbf{rbind}$  and by column  $\mathbf{cbind}$  are also useful. Check  $\mathbf{R}$  documents for their usages.

Exercise 2. (5 pt) Using matrix, rbind and cbind, create

$$\mathbf{X} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & -1 & -2 \\ 2 & .5 & 4 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

```
X <- matrix(c(1,1,2,1,2,0,.5,1,3,-1,4,1,4,-2,2,1),4,4)
X</pre>
```

```
## [,1] [,2] [,3] [,4]
## [1,] 1 2.0 3 4
## [2,] 1 0.0 -1 -2
## [3,] 2 0.5 4 2
## [4,] 1 1.0 1 1
```

#### Indexing

There are three ways to extract specific components in a vector.

x <- 1:4

# specify the vector of indices

x[c(1,4)]

# specify a logical vector of identical length

x[c(TRUE, FALSE, FALSE, TRUE)]

# specify the indices to discard

x[-c(2,3)]

#The second approach leads to the so called "conditional selection" technique as follows.

```
x <- 1:4

x >= 3  # componentwise comparison resulting in a logical vector x[x >= 3]
```

Matrix indexing follows the same manner.

```
x < -matrix(1:12, 3, 4) x[c(1,3),-c(1,4)] x[c(TRUE,FALSE,TRUE),]
```

#### ## List

List is a more flexible container of "data" that permits inhomogeneous types. It's useful if you would

To extract the components in a list, one should use double bracket [[]] instead of a single bracket. If you've already specified the component names in a list, then the component names can be placed into the bracket directly. For example, x[["logic"]] accesses the third component of x. A more handful alternative is x\$logic.

**Punchline.** Only ONE index instead of a vector of indices can be placed into the double bracket! Explore in the following example to see the difference as compared to the single bracket indexing.

```
C11 < -\operatorname{list}(C21 = \text{``C21''}, C22 = \text{``C22''}, C23 = \text{``C23''}) C26 < -\operatorname{list}(C31 = \text{``C31''}, C32 = \text{``C32''}) C13 < -\operatorname{list}(C27 = \text{``C27''}, C28 = \text{``C28''}, C29 = \text{``C29''}) C12 < -\operatorname{list}(C24 = \text{``C24''}, C25 = \text{``C25''}, C26 = C26) P < -\operatorname{list}(C11 = C11, C12 = C12, C13 = C13)
```

#### subtree rooted at C12

P[[2]] P\$C12

# subtree (leaf) rooted at C24

P[[c(2,1)]] PC12C24

### subtree rooted at C26

P[[c(2,3)]] PC12C26

## subtree (leaf) rooted at C31

P[[c(2,3,1)]] PC12C26\$C31

#### ## Date Frame

Inheriting from matrix and list, data.frame is a container general enough for us to study a dataset. It

```
students <- data.frame(id = c("001", "002", "003"), # ids are characters score A = c(95, 97, 90), # scores
are numericss score_B = c(80, 75, 84))
students "To access the score A of student $003$, one can follow the manner in a matrix: students [3,2],
or that in a list:students[[[2]][3],students[["score_A"]][3]orstudents$score_A[3]'.
Exercise 3. (5 pt) Applying the conditional selection technique (without using subset), extract the record
of 003 in students.
students \leftarrow data.frame(id = c("001","002","003"), score_A = c(95,97,90), score_B = c(80,75,84))
students[3,2:3]
##
     score A score B
## 3
           90
One can also create a matrix or a legitimate list first and then convert it into a data.frame as follows.
scores \leftarrow matrix(c(95, 97, 90, 80, 75, 84), 3, 2)
scores <- data.frame(scores)</pre>
colnames(scores) <- c("score A", "score B")</pre>
id \leftarrow c("001", "002", "003")
students1 <- cbind(id, scores)</pre>
                                            = c("001", "002", "003"),
students2 <- data.frame( list( id</pre>
                                   score_A = c(95, 97, 90),
                                   score_B = c(80, 75, 84))
                            )
students
```

Exercise 4. (10 pt) Create a data frame object to display the calendar for Jan 2018 as follows.

**Hint.** 1) The character object ""; 2) the option row.names = FALSE in print function.

20

19

26 27

```
## Sun Mon Tue Wed Thu Fri Sat
##
       NY
                 3
             2
                     4
                         5
##
    7
        8
             9 10 11 12
                           13
       MLK 16 17 18 19 20
##
   14
##
   21
       22
           23
               24 25 26 27
   28 29 30 31
dates = matrix(c(" ",7,14,21,28,"NY",8,"MLK",22,29,2,9,16,23,30,3,10,17,24,31,4,11,18,25," ",5,12,19,26
dates = data.frame(dates)
colnames(dates) = c("Sun", "Mon", "Tue", "Wed", "Thu", "Fri", "Sat")
calendar1 = cbind(0,dates)
calendar2 = data.frame( list( Sun = c(" ",7,14,21,28), Mon = c("NY",8,"MLK",22,29), Tue = c(2,9,16,23,3
calendar2
##
     Sun Mon Tue Wed Thu Fri Sat
## 1
          NY
               2
                  3
                       4
                           5
                               6
## 2
      7
           8
               9
                 10
                      11
                         12
                             13
```

### Factors

## 3

## 5

Factor is a special data structure in  $\mathbf{R}$  in representing categorical variables and facilitating the data labels and subgroups. It's basically a character vector that keeps track of its distinct values called levels. Consider the longitudinal layout of the previous score table.

```
id \leftarrow rep(c("001","002","003"), 2)
```

16

23

30

17

24

31

18

25

14 MLK

22

29

21

28

```
\leftarrow rep(c("A","B"), each = 3)
subj
          <- c(95, 97, 90, 80, 75, 84)
score
students3 <- data.frame(id, subj, score)</pre>
students3
# students3$id and students3$subj are automatically formatted as factors
class(students3$id)
levels(students3$id)
class(students3$subj)
levels(students3$subj)
# combind student 003 with 002 via level rename
students4 <- students3
                           # work on a copy in case of direct modification of students3
levels(students4$id)[3] <- "002"</pre>
levels(students4$id)
students4
```

The factor function applied to a character vector creates a natural factor. The gl and cut functions are also useful approaches to patterned factors and that generated from numeric variables.

Exercise 5. (5 pt) Create a factor variable grade in students3, where the score variable is divided into [90,100], [80,90) and [0,80) corresponding to A, B and C in grade respectively.

Hint. Functions cut and transform.

```
id = rep(c("001","002","003"), 2)
subj = rep(c("A","B"), each = 3)
score = c(95, 97, 90, 80, 75, 84)
fscore = cut(score, breaks = c(0,80,90,100), labels = c("C", "B", "A"), right = FALSE, ordered_result = T.
grade = factor(fscore)
transform(grade)
##
     X_{data}
## 1
## 2
          Α
## 3
## 4
          В
          C
## 5
## 6
          В
students4 = data.frame(id,subj,score,grade)
students4
##
      id subj score grade
## 1 001
            Α
                 95
## 2 002
            Α
                 97
## 3 003
            Α
                 90
                        Α
## 4 001
            В
                 80
                        В
## 5 002
                 75
                         C
            В
## 6 003
                 84
                         В
```

#### **Operations and Functions**

Note that scalar operations on vectors usually apply componentwise.

• Arithmetic operations: +, -, \*, /, ^, %/% (exact division), %% (modulus), sqrt(), exp(), log()

- Logical operations
  - And &, &&; or |, ||; not!
  - Comparisons: <, <=, >, >=, == (different from =), !=
  - Summary functions: all(), any()
- Summary statistics: length, max, min, sum, prod, mean, var, sd, median, quantile
- Matrix operations
  - Matrix multiplication: %\*% (different from \*)
  - Related functions: t, solve, det, diag, eigen, svd, qr
  - Marginal operations: apply
- Factors: tapply (to apply operations/functions grouped by a factor)

Exercise 6. (5 pt) Without using the var and scale functions, compute the sample covariance X.var of the data matrix X as in Exercise 2.

```
a = c(1,1,2,1)
b = c(2,0,0.5,1)
c = c(3,-1,4,1)
d = c(4,-2,2,1)
X = cbind(a,b,c,d)
m = nrow(X)
n = ncol(X)
X_mean = matrix(data=1, nrow=m) %*% cbind(mean(a),mean(b),mean(c),mean(d))
Diff = X - X_mean
X.var = (n-1)^-1*t(Diff) %*% Diff
X.var
```

```
## a b c d
## a 0.250 -0.1250000 0.7500000 0.250
## b -0.125 0.7291667 0.9583333 1.875
## c 0.750 0.9583333 4.9166667 4.750
## d 0.250 1.8750000 4.7500000 6.250
```

Exercise 7. (10 pt) Create a variable score.mean in students3, taking value as the mean score among students who have the same subj value.

```
id     <- rep(c("001","002","003"), 2)
subj     <- rep(c("A","B"), each = 3)
score     <- c(95, 97, 90, 80, 75, 84)
students3 <- data.frame(id, subj, score)
students3</pre>
```

```
## 2 002
                 97
## 3 003
                 90
            Α
## 4 001
            В
                 80
## 5 002
            В
                 75
## 6 003
            В
score_A = c(95, 97, 90)
score_B = c(80,75,84)
score.mean = rep(c(mean(score A), mean(score B)), each = 3)
students5 = data.frame(students3, score.mean)
students5
```

```
## id subj score score.mean
## 1 001 A 95 94.00000
## 2 002 A 97 94.00000
```

id subj score

Α

##

## 1 001

```
## 3 003
            Α
                  90
                       94.00000
## 4 001
            В
                  80
                       79.66667
## 5 002
            В
                  75
                       79.66667
## 6 003
            В
                  84
                       79.66667
```

#### Writing your own functions

It's convenient to create a user-defined  $\mathbf{R}$  function, where you might encapsulate a standard, complicated or tedious procedure/algorithm in a "black box" like any built-in functions as mensioned above, so that other users might only need to care about the input and output of your function regardless the details. See the following toy example.

 $\begin{array}{l} {\rm my.fun} < -\ {\rm function}(x,\,y) \ \{ \ \# \ {\rm This} \ {\rm function} \ {\rm takes} \ x \ {\rm and} \ y \ {\rm as} \ {\rm input} \ \# \ {\rm It} \ {\rm returns} \ a \ {\rm list} \ {\rm that} \ {\rm contains} \ {\rm the} \ {\rm mean} \ {\rm of} \ x, \ y \ {\rm respectively} \ {\rm and} \ {\rm their} \ {\rm difference} \ {\rm mean}.x < -\ {\rm mean}(x) \ {\rm mean}.y < -\ {\rm mean}(y) \ {\rm mean}.{\rm diff} < -\ {\rm mean}.x - \ {\rm mean}.y \ {\rm output} < -\ {\rm list}({\rm mean\_of\_x} = {\rm mean}.x, \ {\rm mean\_of\_y} = {\rm mean}.y, \ {\rm mean\_diff} = {\rm mean.diff}) \ {\rm return}({\rm output}) \ \} \ x < -\ {\rm runif}(50,\,0,\,1) \ \# \ {\rm simulate} \ 50 \ {\rm numbers} \ {\rm from} \ {\rm U}[0,3] \ {\rm my.fun}(x,\,y) \ ``` \ {\rm contains} \ {\rm their} \ {\rm$ 

#### Flow Control

To help you write down your own  ${\bf R}$  programs, the following examples familiarize you with the conditional statements and loops.

#### **Conditional Statements**

```
mymax.if <- function(x, y)</pre>
  # The mymax function returns the larger one among x and y
  if (x > y)
  {
    message("The first argument is larger!")
    return(x)
  }
  else if (x < y)
    message("The second argument is larger!")
    return(y)
  }
  else
    message("Equal!")
    return(x)
  }
}
mymax.if(3, 4)
```

```
## [1] 4
mymax.if(4, 3)
```

## [1] 4

```
mymax.if(3, 3)
## [1] 3
Loops
factorial <- function(n)</pre>
  \# It computes the factorial of the natural number n
  if(n == 0) return(1) else if(n < 0 | n != as.integer(n)) stop("Please input a natural number!")
 n.fac <- 1
  # for loops
  for(i in 1:n)
   n.fac <- i * n.fac</pre>
  # while loops
  ## i <- 1
  ## while(i <= n)
  ## {
  ## n.fac <- i * n.fac
  ## i <- i + 1
  ## }
  # repeat loops
  ## i <- 1
  ## repeat
  ## {
  ## n.fac <- i * n.fac
  ## i <- i + 1
  ## if (i > n) break # attention to the usage of break
  ## }
 return(n.fac)
double.factorial <- function(n)</pre>
  \# It computes the double factorial of the natural number n
  if(n == 0) return(1) else if(n < 0 | n != as.integer(n)) stop("Please input a natural number!")
  n.doubfac <- 1</pre>
  for(i in 1:n)
    if(i %% 2 == n %% 2)
      n.doubfac <- i * n.doubfac</pre>
    else next # attention to the usage of next
  return(n.doubfac)
```

## [1] 720

factorial(6)

```
prod(1:6)
## [1] 720
double.factorial(6)
## [1] 48
double.factorial(5)
## [1] 15
factorial(0)
## [1] 1
## factorial.for(6.5)
## factorial.for(-6)
```

Exercise 8. (15 pt) Write a function bisect(f, lower, upper, tol = 1e-6) to find the root of the univariate function f on the interval [lower, upper] with precision tolerance  $\leq$  tol (defaulted to be  $10^{-6}$ ) via bisection, which returns a list consisting of root, f.root (f evaluated at root), iter (number of iterations) and estim.prec (estimated precision). Apply it to the function

$$f(x) = x^3 - x - 1$$

on [1,2] with precision tolerance  $10^{-6}$ . Compare it with the built-in function uniroot.

```
f = function(x) {
  return(x^3 - x - 1)
}
bisect = function(f, lower, upper, tol = 1e-6){
  if (f(lower)*f(upper) > 0){
    print("No root.")
  } else
      iter = 1
      while ((upper-lower)/2 > tol){
        m = (lower+upper)/2
        if(f(m) == 0){
          root = m
          f.root = f(root)
          estima.prec = (upper-lower)/2
          return(setNames(as.list(c(root,f.root,iter,estim.prec))),c("root","f.root","iter","estim.prec
        }else if (f(lower)*f(m) < 0){
          upper = m
        }else
          lower = m
        iter = iter + 1
      }
        root = m
        f.root = f(root)
        estim.prec = (upper-lower)/2
        return(setNames(as.list(c(root,f.root,iter,estim.prec)),c("root","f.root","iter","estim.prec"))
}
bisect(f,1,2,tol = 1e-6)
```

## \$root

```
V1 V2 V3 V4
V1 -0.29 0.68 0.2
V2 0.51 0.88
V3 0.86
```

Table 1

```
## [1] 1.324717
##
## $f.root
## [1] -5.92464e-06
##
## $iter
##
   [1] 20
## $estim.prec
## [1] 9.536743e-07
uniroot(f,c(1,2),tol=1e-6)
## $root
## [1] 1.324718
##
## $f.root
## [1] -5.634261e-07
##
## $iter
## [1] 7
##
## $init.it
## [1] NA
##
## $estim.prec
## [1] 5e-07
```

### Input/Output

scan and read.table are two main functions to read data. The main difference of them lies in that scan reads one component (also called "field") at a time but read.table reads one line at a time. Hence read.table requires the data to be well-structured as a table so as to create a data.frame in  $\mathbf R$  automatically, while scan can be flexible but might require efforts in manipulating data after reading. Their usages are quite similar. One should pay attention to the frequently used options file, header, sep, dec, skip, nmax, nlines and nrows in thier  $\mathbf R$  documents.

write.table is a converse function against read.table, while their usages are almost identical. To get familiar with thier features, explore in the next exercise.

Exercise 9. (15 pt) Without using cor, compute the sample correlation matrix X.cor from X.var in Exercise 6. Output X.cor to a text file "X\_cor.txt" which displays as in Table 1. Then input "X\_cor.txt" in R and reproduce the correlation matrix X.cor1.

Hint. 1) Functions round and lower.tri; 2) the NA trick; 3) options sep = "\t", col.names = NA.

```
V1
                ٧2
                     VЗ
## V1 1.00 -0.29 0.68 0.20
## V2 -0.29 1.00 0.51 0.88
## V3 0.68 0.51 1.00 0.86
## V4 0.20 0.88 0.86 1.00
D <- sqrt(diag(X.var))</pre>
D.inv <- matrix(OL,ncol = 4, nrow = 4)</pre>
for (i in 1:4){
  D.inv[i,i] \leftarrow 1/D[i]
}
X.cor <- D.inv %*% X.var %*% D.inv</pre>
X.cor <- round(X.cor, digits = 2)</pre>
write.table(X.cor, file = "X_cor.txt")
X.cor1 <- read.table("X_cor.txt")</pre>
X.cor1
##
               ٧2
                    VЗ
        V1
## 1 1.00 -0.29 0.68 0.20
## 2 -0.29 1.00 0.51 0.88
## 3 0.68 0.51 1.00 0.86
```

To read inline, one can specify file = stdin() (or omitted in scan function). In that case, it reads from console that the user can input line-by-line, or from the subsequent lines in a program script, until an empty line is read. However, such trick is NOT compatible in **RMarkdown**.

If you have your data stored in another format, e.g. EXCEL or SAS dataset, then you can output it as a CSV file and read in  $\mathbf R$  via read.csv function (almost identical to read.table).

## Probability and Distributions

This section explores how to create "randomness" in **R** and obtain probabilistic quantities.

#### Discrete Random Sampling

## 4 0.20 0.88 0.86 1.00

Much of the earliest work in probability theory starts with random sampling, e.g. from a well-shuffled pack of cards or a well-stirred urn. The sample function applies such procedure to a vector in  $\mathbf{R}$ . Learn more from the  $\mathbf{R}$  documents.

The following exercise means to create a five-fold cross-validating sets, which would be the starting point to assess the performance of a learned machine in, for example, classification errors.

Exercise 10. (10 pt) iris is a built-in dataset in R. Check ?iris for more information. Randomly divide iris into five subsets iris1 to iris5 stratified to iris\$Species (namely, the proportion of iris\$Species among different levels remains identical across all subsets).

```
iriss <- split(iris, sample(rep(1:5, ncol(iris), replace = FALSE)))
iris1 = iriss[1]
iris2 = iriss[2]
iris3 = iriss[3]
iris4 = iriss[4]
iris5 = iriss[5]</pre>
```

iris.5fold ## [[1]] ## [[1]]\$`1` ## Sepal.Length Sepal.Width Petal.Length Petal.Width Species ## 5 5.0 3.6 1.4 0.2 setosa ## 11 5.4 3.7 1.5 0.2 setosa ## 13 4.8 3.0 1.4 0.1 setosa ## 14 4.3 3.0 1.1 0.1 setosa ## 18 5.1 3.5 1.4 0.3 setosa ## 30 4.7 3.2 1.6 0.2 setosa ## 36 5.0 3.2 1.2 0.2 setosa ## 38 4.9 3.6 1.4 0.1 setosa ## 39 4.4 3.0 1.3 0.2 setosa ## 43 4.4 3.2 1.3 0.2 setosa ## 55 6.5 2.8 4.6 1.5 versicolor ## 61 5.0 2.0 3.5 1.0 versicolor ## 63 6.0 2.2 4.0 1.0 versicolor ## 64 6.1 2.9 4.7 1.4 versicolor ## 68 1.0 versicolor 5.8 2.7 4.1 ## 80 5.7 2.6 3.5 1.0 versicolor ## 86 6.0 3.4 4.5 1.6 versicolor ## 88 6.3 2.3 4.4 1.3 versicolor 3.0 ## 89 4.1 1.3 versicolor 5.6 ## 93 5.8 2.6 4.0 1.2 versicolor ## 105 3.0 5.8 2.2 virginica 6.5 ## 111 6.5 3.2 5.1 2.0 virginica ## 113 6.8 3.0 5.5 2.1 virginica ## 114 5.7 2.5 5.0 2.0 virginica ## 118 7.7 3.8 6.7 2.2 virginica ## 130 7.2 3.0 5.8 1.6 virginica ## 136 7.7 3.0 6.1 2.3 virginica ## 138 6.4 3.1 5.5 1.8 virginica ## 139 6.0 3.0 4.8 1.8 virginica ## 143 5.8 2.7 5.1 1.9 virginica ## ## ## [[2]] ## [[2]]\$`2` Sepal.Length Sepal.Width Petal.Length Petal.Width ## Species ## 12 4.8 1.6 3.4 0.2 setosa ## 19 5.7 1.7 3.8 0.3 setosa ## 20 5.1 3.8 1.5 0.3 setosa ## 22 5.1 3.7 1.5 0.4 setosa ## 23 4.6 3.6 1.0 0.2 setosa ## 37 5.5 3.5 1.3 0.2 setosa ## 44 5.0 3.5 1.6 0.6 setosa ## 45 5.1 3.8 1.9 0.4 setosa ## 47 5.1 3.8 1.6 0.2 setosa ## 48 4.6 3.2 1.4 0.2 setosa ## 62 5.9 3.0 4.2 1.5 versicolor 2.2 ## 69 6.2 4.5 1.5 versicolor

iris.5fold <- list(iris1, iris2, iris3, iris4, iris5)</pre>

```
## 70
                 5.6
                               2.5
                                             3.9
                                                          1.1 versicolor
## 72
                 6.1
                               2.8
                                             4.0
                                                          1.3 versicolor
## 73
                                             4.9
                 6.3
                               2.5
                                                          1.5 versicolor
                                                          1.5 versicolor
## 87
                 6.7
                               3.1
                                             4.7
## 94
                 5.0
                               2.3
                                             3.3
                                                          1.0 versicolor
## 95
                 5.6
                               2.7
                                             4.2
                                                          1.3 versicolor
## 97
                 5.7
                               2.9
                                             4.2
                                                          1.3 versicolor
## 98
                 6.2
                               2.9
                                             4.3
                                                          1.3 versicolor
## 112
                 6.4
                               2.7
                                             5.3
                                                          1.9
                                                               virginica
## 119
                 7.7
                               2.6
                                             6.9
                                                          2.3
                                                               virginica
## 120
                 6.0
                               2.2
                                             5.0
                                                          1.5
                                                                virginica
## 122
                               2.8
                                             4.9
                                                          2.0
                 5.6
                                                                virginica
## 123
                 7.7
                               2.8
                                             6.7
                                                          2.0
                                                                virginica
## 137
                 6.3
                               3.4
                                             5.6
                                                          2.4
                                                                virginica
## 144
                 6.8
                               3.2
                                             5.9
                                                          2.3
                                                                virginica
## 145
                 6.7
                               3.3
                                             5.7
                                                          2.5
                                                                virginica
## 147
                 6.3
                               2.5
                                             5.0
                                                          1.9
                                                                virginica
## 148
                 6.5
                               3.0
                                             5.2
                                                          2.0
                                                                virginica
##
##
## [[3]]
## [[3]]$`3`
       Sepal.Length Sepal.Width Petal.Length Petal.Width
##
                                                                  Species
                 4.9
                               3.0
                                             1.4
                                                                   setosa
                 4.9
## 10
                               3.1
                                             1.5
                                                          0.1
                                                                   setosa
## 15
                 5.8
                               4.0
                                             1.2
                                                          0.2
                                                                   setosa
## 16
                 5.7
                               4.4
                                             1.5
                                                          0.4
                                                                   setosa
## 24
                 5.1
                               3.3
                                             1.7
                                                          0.5
                                                                   setosa
## 27
                 5.0
                               3.4
                                             1.6
                                                          0.4
                                                                   setosa
## 35
                 4.9
                                                          0.2
                               3.1
                                             1.5
                                                                   setosa
## 40
                 5.1
                               3.4
                                             1.5
                                                          0.2
                                                                   setosa
## 41
                 5.0
                               3.5
                                             1.3
                                                          0.3
                                                                   setosa
## 49
                 5.3
                                             1.5
                               3.7
                                                          0.2
                                                                   setosa
## 52
                 6.4
                               3.2
                                             4.5
                                                          1.5 versicolor
                               2.7
## 60
                 5.2
                                             3.9
                                                          1.4 versicolor
## 65
                 5.6
                               2.9
                                             3.6
                                                          1.3 versicolor
## 66
                 6.7
                               3.1
                                             4.4
                                                          1.4 versicolor
## 74
                 6.1
                               2.8
                                             4.7
                                                          1.2 versicolor
## 77
                 6.8
                               2.8
                                             4.8
                                                          1.4 versicolor
## 85
                 5.4
                                             4.5
                                                          1.5 versicolor
                               3.0
## 90
                 5.5
                               2.5
                                             4.0
                                                          1.3 versicolor
## 91
                 5.5
                               2.6
                                             4.4
                                                          1.2 versicolor
## 99
                 5.1
                               2.5
                                             3.0
                                                          1.1 versicolor
## 102
                 5.8
                                                               virginica
                               2.7
                                             5.1
                                                          1.9
## 110
                 7.2
                               3.6
                                                               virginica
                                             6.1
                                                          2.5
## 115
                 5.8
                               2.8
                                             5.1
                                                          2.4
                                                               virginica
## 116
                 6.4
                                             5.3
                               3.2
                                                          2.3
                                                                virginica
## 124
                 6.3
                               2.7
                                             4.9
                                                          1.8
                                                                virginica
## 127
                                                          1.8
                 6.2
                               2.8
                                             4.8
                                                               virginica
## 135
                               2.6
                 6.1
                                             5.6
                                                          1.4
                                                               virginica
## 140
                 6.9
                               3.1
                                             5.4
                                                          2.1
                                                                virginica
## 141
                 6.7
                               3.1
                                             5.6
                                                          2.4 virginica
## 149
                 6.2
                               3.4
                                             5.4
                                                          2.3 virginica
##
```

```
##
## [[4]]
## [[4]]$`4`
       Sepal.Length Sepal.Width Petal.Length Petal.Width
##
                                                                  Species
## 1
                 5.1
                               3.5
                                             1.4
                                                                   setosa
## 4
                 4.6
                               3.1
                                             1.5
                                                          0.2
                                                                   setosa
## 8
                 5.0
                               3.4
                                             1.5
                                                          0.2
                                                                   setosa
## 9
                 4.4
                               2.9
                                             1.4
                                                          0.2
                                                                   setosa
## 21
                 5.4
                               3.4
                                             1.7
                                                          0.2
                                                                   setosa
## 26
                 5.0
                               3.0
                                             1.6
                                                          0.2
                                                                   setosa
## 29
                 5.2
                               3.4
                                             1.4
                                                          0.2
                                                                   setosa
## 33
                 5.2
                               4.1
                                             1.5
                                                          0.1
                                                                   setosa
## 34
                 5.5
                               4.2
                                             1.4
                                                          0.2
                                                                   setosa
## 46
                 4.8
                               3.0
                                             1.4
                                                          0.3
                                                                   setosa
## 51
                 7.0
                               3.2
                                             4.7
                                                           1.4 versicolor
## 54
                 5.5
                               2.3
                                             4.0
                                                           1.3 versicolor
## 58
                 4.9
                               2.4
                                             3.3
                                                           1.0 versicolor
## 59
                 6.6
                               2.9
                                             4.6
                                                           1.3 versicolor
## 71
                 5.9
                               3.2
                                             4.8
                                                           1.8 versicolor
## 76
                 6.6
                               3.0
                                             4.4
                                                           1.4 versicolor
## 79
                 6.0
                               2.9
                                             4.5
                                                           1.5 versicolor
## 83
                 5.8
                               2.7
                                             3.9
                                                           1.2 versicolor
## 84
                               2.7
                                                           1.6 versicolor
                 6.0
                                             5.1
## 96
                 5.7
                               3.0
                                             4.2
                                                           1.2 versicolor
## 101
                 6.3
                               3.3
                                             6.0
                                                          2.5
                                                               virginica
## 104
                 6.3
                               2.9
                                             5.6
                                                          1.8
                                                               virginica
## 108
                 7.3
                               2.9
                                             6.3
                                                           1.8
                                                               virginica
## 109
                 6.7
                               2.5
                                             5.8
                                                           1.8
                                                                virginica
## 121
                 6.9
                               3.2
                                             5.7
                                                          2.3
                                                                virginica
## 126
                 7.2
                               3.2
                                             6.0
                                                          1.8
                                                               virginica
## 129
                               2.8
                 6.4
                                             5.6
                                                          2.1
                                                                virginica
## 133
                 6.4
                               2.8
                                             5.6
                                                          2.2
                                                                virginica
## 134
                 6.3
                               2.8
                                             5.1
                                                           1.5
                                                                virginica
## 146
                 6.7
                               3.0
                                             5.2
                                                           2.3
                                                               virginica
##
##
## [[5]]
## [[5]]$`5`
##
       Sepal.Length Sepal.Width Petal.Length Petal.Width
                                                                  Species
## 3
                 4.7
                               3.2
                                             1.3
                                                          0.2
                                                                   setosa
## 6
                 5.4
                               3.9
                                             1.7
                                                           0.4
                                                                   setosa
## 7
                 4.6
                               3.4
                                             1.4
                                                          0.3
                                                                   setosa
## 17
                 5.4
                               3.9
                                             1.3
                                                          0.4
                                                                   setosa
## 25
                 4.8
                               3.4
                                             1.9
                                                          0.2
                                                                   setosa
## 28
                 5.2
                               3.5
                                             1.5
                                                          0.2
                                                                   setosa
## 31
                 4.8
                               3.1
                                                          0.2
                                             1.6
                                                                   setosa
## 32
                 5.4
                                             1.5
                               3.4
                                                          0.4
                                                                   setosa
## 42
                 4.5
                               2.3
                                             1.3
                                                          0.3
                                                                   setosa
## 50
                 5.0
                               3.3
                                             1.4
                                                          0.2
                                                                   setosa
## 53
                 6.9
                                             4.9
                               3.1
                                                           1.5 versicolor
## 56
                 5.7
                               2.8
                                             4.5
                                                           1.3 versicolor
## 57
                 6.3
                               3.3
                                             4.7
                                                           1.6 versicolor
## 67
                 5.6
                               3.0
                                             4.5
                                                          1.5 versicolor
## 75
                 6.4
                               2.9
                                             4.3
                                                          1.3 versicolor
```

##	78	6.7	3.0	5.0	1.7	versicolor
##	81	5.5	2.4	3.8	1.1	versicolor
##	82	5.5	2.4	3.7	1.0	versicolor
##	92	6.1	3.0	4.6	1.4	versicolor
##	100	5.7	2.8	4.1	1.3	versicolor
##	103	7.1	3.0	5.9	2.1	virginica
##	106	7.6	3.0	6.6	2.1	virginica
##	107	4.9	2.5	4.5	1.7	virginica
##	117	6.5	3.0	5.5	1.8	virginica
##	125	6.7	3.3	5.7	2.1	virginica
##	128	6.1	3.0	4.9	1.8	virginica
##	131	7.4	2.8	6.1	1.9	virginica
##	132	7.9	3.8	6.4	2.0	virginica
##	142	6.9	3.1	5.1	2.3	virginica
##	150	5.9	3.0	5.1	1.8	virginica

#### **Distributions**

 $\mathbf{R}$  is endowed with a set of statistical tables. To obtain the density function, cumulative distribution function (CDF), quantile (inverse CDF) and pseudo-random numbers from a specific distribution, one only needs to prefix the distribution name given below by  $\mathbf{d}$ ,  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$  respectively.

Distributions	R Names	Key Arguments	
Uniform	unif	min, max	
Normal	norm	mean, sd	
$\chi^2$	chisq	${ t df},{ t ncp}$	
Student's t	t	${ t df},{ t ncp}$	
$\mathbf{F}$	f	df1, df2, ncp	
Exponential	exp	rate	
Gamma	gamma	shape, scale	
Beta	beta	shape1, shape2, ncp	
Logistic	logis	location, scale	
Binomial	binom	size, prob	
Poisson	pois	lambda	
Geometric	geom	prob	
Hypergeometric	hyper	m, n, k	
Negative Binomial	nbinom	size, prob	

Check from their plots.

mean.x

var.x

plot(dnorm, xlim = c(-5, 5)) # bell curve of Normal density plot(plogis, xlim = c(-5, 5)) # Logistic/Sigmoid function (CDF of Logistic distribution)

```
The following two-sample t-test shows the usages of `qt`, `pt` and `rnorm`. Recall that a two-sample how $\hat{\sigma}^2 = {(n_X - 1)S_X^2 + (n_Y - 1)S_Y^2 \over n_X + n_Y -2}, \quad T = {\bar{X} - \bar{Y}} twosam <- function(x, y, alpha = 0.05) {

# It conducts a two-sample homoscedastic t-test on x and y n.x <- length(x); n.y <- length(y)
```

<- mean(x); mean.y <- mean(y)

<- var(x); var.y <- var(y)

```
mean.diff <- mean.x - mean.y</pre>
  df
             <- n.x + n.y - 2
             \leftarrow ((n.x - 1) * var.x + (n.y - 1) * var.y) / df
  var.diff <- (1/n.x + 1/n.y) * sigma
             <- mean.diff / sqrt(var.diff)</pre>
             \leftarrow qt(1 - alpha/2, df)
  t.alpha
  output
             <- list(t
                              = t.
                              = df,
                      p.value = 2 * pt(-abs(t), df),
                      confint = c(lower = mean.diff - sqrt(var.diff) * t.alpha,
                                   upper = mean.diff + sqrt(var.diff) * t.alpha),
                              = c(mu.x = mean.x, mu.y = mean.y),
                              = sigma)
                      sigma
  return(output)
}
x1 \leftarrow rnorm(40, 0, 1)
x2 <- rnorm(50, 0, 1)
x3 \leftarrow rnorm(50, 1, 1)
twosam(x1, x2)
t.test(x1, x2, var.equal = TRUE)
twosam(x1, x3)
t.test(x1, x3, var.equal = TRUE)
```

### Data Exploration and Manipulation

Data analysis in **R** starts with reading data in a data frame object via scan and read.table as discussed before. Then one would explore the profiles of data via various descriptive statistics whose usages are also introduced in the previous sections. Calling the summary function with a data frame input also provides appropriate summaries, *e.g.* means and quantiles for numeric variables (columns) and frequencies for factor variables.

This section explores more features that can be achieved through  $\mathbf{R}$ .

#### **Tables**

Statistician often works with catergorical variables via tables. Even for continuous variables, segregating them into catergorical ones in a meaningful way might provide more insights. table function generates frequency tables for factor variables. Multi-way tables, marginal and proportional displays and independence test are explored in the following example. Recall that the Pearson's  $\chi^2$  independence test statistic on an r-by-c contingency table

$$\chi^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{\left(n_{ij} - \frac{n_{i.}n_{.j}}{n_{..}}\right)^2}{n_{i.}n_{.j}/n_{..}} \stackrel{d}{\approx} \chi^2_{(r-1)(c-1)} \text{ under } H_0: p_{ij} = p_i p_j.$$

```
attach(iris0) # attach function makes columns assessible as usual variables
table(Sepal.Width)
## Sepal.Width
## (1,2] (2,3] (3,4] (4,5]
       1 82 64
iris.table0 <- table(Petal.Width)</pre>
iris.table0
## Petal.Width
## (0,1] (1,2] (2,3]
     57
         70
                  23
iris.table1 <- table(Sepal.Width, Petal.Width)</pre>
iris.table1
##
             Petal.Width
## Sepal.Width (0,1] (1,2] (2,3]
        (1,2]
                 1
                       0
         (2,3]
##
                  14
                        58
                              10
##
         (3,4]
                  39
                        12
                              13
##
         (4,5]
                  3
                         0
iris.table2 <- table(Sepal.Width, Petal.Width, Species)</pre>
iris.table2
## , , Species = setosa
##
##
              Petal.Width
## Sepal.Width (0,1] (1,2] (2,3]
##
        (1,2]
                 0
                         0
##
         (2,3]
                         0
                               0
                  8
##
         (3,4]
                  39
                         0
                               0
##
         (4,5]
                  3
## , , Species = versicolor
##
##
              Petal.Width
## Sepal.Width (0,1] (1,2] (2,3]
##
        (1,2]
                 1
                        0
                               0
         (2,3]
                        35
                               0
##
                   6
##
         (3,4]
                   0
                         8
##
         (4,5]
                  0
##
## , , Species = virginica
##
##
            Petal.Width
## Sepal.Width (0,1] (1,2] (2,3]
##
         (1,2]
               0
                        0
                        23
##
         (2,3]
                   0
                              10
##
         (3,4]
                        4
                              13
                   0
         (4,5]
                   0
                         0
# flat table in a sligtly different display
ftable(Sepal.Width, Petal.Width)
```

```
Petal.Width (0,1] (1,2] (2,3]
##
## Sepal.Width
## (1,2]
                                         0
                                               0
                                  1
## (2,3]
                                 14
                                        58
                                              10
## (3,4]
                                 39
                                        12
                                              13
## (4,5]
                                  3
                                         0
                                               0
ftable(Species, Sepal.Width, Petal.Width)
##
                            Petal.Width (0,1] (1,2] (2,3]
## Species
               Sepal.Width
## setosa
               (1,2]
                                              0
                                                     0
                                                            0
##
               (2,3]
                                              8
                                                     0
                                                            0
##
               (3,4]
                                             39
                                                     0
                                                            0
                                              3
##
               (4,5]
                                                     0
                                                            0
                                              1
                                                     0
                                                            0
## versicolor (1,2]
##
               (2,3]
                                              6
                                                    35
                                                            0
##
               (3,4]
                                              0
                                                     8
                                                            0
##
               (4,5]
                                              0
                                                     0
                                                            0
                                              0
                                                     0
                                                           0
## virginica
               (1,2]
##
               (2,3]
                                              0
                                                    23
                                                          10
##
                                              0
                                                     4
               (3,4]
                                                           13
##
               (4,5]
# data.frame display
data.frame(iris.table1)
      Sepal.Width Petal.Width Freq
##
## 1
             (1,2]
                          (0,1]
## 2
             (2,3]
                          (0,1]
                                   14
## 3
             (3,4]
                          (0,1]
                                   39
## 4
             (4,5]
                                    3
                          (0,1]
## 5
             (1,2]
                          (1,2]
                                    0
## 6
             (2,3]
                          (1,2]
                                   58
## 7
             (3,4]
                          (1,2]
                                   12
## 8
             (4,5]
                          (1,2]
                                    0
                          (2,3]
## 9
             (1,2]
                                    0
## 10
             (2,3]
                          (2,3]
                                   10
## 11
             (3,4]
                          (2,3]
                                   13
             (4,5]
                          (2,3]
                                    0
data.frame(iris.table2)
      Sepal.Width Petal.Width
                                    Species Freq
## 1
             (1,2]
                          (0,1]
                                     setosa
                                                 0
                          (0,1]
## 2
             (2,3]
                                     setosa
                                                 8
## 3
             (3,4]
                          (0,1]
                                               39
                                     setosa
             (4,5]
## 4
                          (0,1]
                                     setosa
                                                3
## 5
             (1,2]
                          (1,2]
                                     setosa
## 6
             (2,3]
                          (1,2]
                                                0
                                     setosa
## 7
             (3,4]
                          (1,2]
                                     setosa
## 8
             (4,5]
                          (1,2]
                                     setosa
                                                0
## 9
             (1,2]
                          (2,3]
                                     setosa
## 10
             (2,3]
                          (2,3]
                                                0
                                     setosa
```

setosa

setosa

## 11

## 12

(3,4]

(4,5]

(2,3]

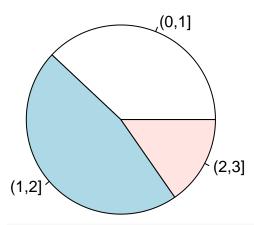
(2,3]

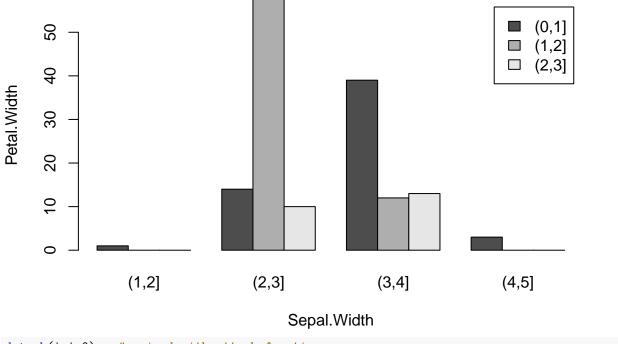
```
(1,2]
                         (0,1] versicolor
## 13
            (2,3]
## 14
                         (0,1] versicolor
                                              6
## 15
             (3,4]
                         (0,1] versicolor
## 16
            (4,5]
                         (0,1] versicolor
                                              0
## 17
            (1,2]
                         (1,2] versicolor
                                              0
## 18
            (2,3]
                         (1,2] versicolor
                                             35
## 19
            (3,4]
                         (1.2] versicolor
## 20
            (4,5]
                         (1,2] versicolor
                                              0
## 21
            (1,2]
                         (2,3] versicolor
                                              0
## 22
            (2,3]
                         (2,3] versicolor
                         (2,3] versicolor
## 23
            (3,4]
## 24
            (4,5]
                         (2,3] versicolor
                                              0
## 25
            (1,2]
                         (0,1]
                               virginica
                                              0
## 26
            (2,3]
                         (0,1]
                                virginica
## 27
            (3,4]
                         (0,1]
                                virginica
                                              0
            (4,5]
## 28
                         (0,1]
                                virginica
                                              0
## 29
            (1,2]
                         (1,2]
                                virginica
                                              0
## 30
            (2,3]
                         (1,2]
                                virginica
                                             23
## 31
            (3,4]
                         (1,2] virginica
                                              4
## 32
            (4,5]
                         (1,2] virginica
                                              0
## 33
            (1,2]
                         (2,3]
                               virginica
                                              0
## 34
            (2,3]
                         (2,3]
                                virginica
                                             10
## 35
                                virginica
            (3,4]
                         (2,3]
                                             13
## 36
            (4,5]
                         (2,3] virginica
# marginal display
margin.table(iris.table1, 1)
## Sepal.Width
## (1,2] (2,3] (3,4] (4,5]
##
       1
            82
                  64
margin.table(iris.table1, 2)
## Petal.Width
## (0,1] (1,2] (2,3]
##
      57
            70
                  23
# proportional display relative to the (margin) total
prop.table(iris.table1)
##
              Petal.Width
## Sepal.Width
                      (0,1]
                                   (1,2]
                                               (2,3]
##
         (1,2] 0.006666667 0.000000000 0.000000000
##
         (2,3] 0.093333333 0.386666667 0.066666667
         (3,4] 0.260000000 0.080000000 0.086666667
##
         (4,5] 0.020000000 0.000000000 0.000000000
##
prop.table(iris.table1, 1)
##
              Petal.Width
## Sepal.Width
                    (0,1]
                              (1,2]
##
         (1,2] 1.0000000 0.0000000 0.0000000
##
         (2,3] 0.1707317 0.7073171 0.1219512
##
         (3,4] 0.6093750 0.1875000 0.2031250
         (4,5] 1.0000000 0.0000000 0.0000000
##
```

```
prop.table(iris.table1, 2)
             Petal.Width
                                         (2,3]
## Sepal.Width (0,1]
                              (1,2]
         (1,2] 0.01754386 0.00000000 0.00000000
##
         (2,3] 0.24561404 0.82857143 0.43478261
##
         (3,4] 0.68421053 0.17142857 0.56521739
         (4,5] 0.05263158 0.00000000 0.00000000
prop.table(iris.table2, 1)
## , , Species = setosa
##
##
             Petal.Width
## Sepal.Width (0,1]
                             (1,2]
                                         (2.3]
        (1,2] 0.00000000 0.00000000 0.00000000
##
         (2,3] 0.09756098 0.00000000 0.00000000
##
         (3,4] 0.60937500 0.00000000 0.00000000
##
         (4,5] 1.00000000 0.00000000 0.00000000
##
##
  , , Species = versicolor
##
##
             Petal.Width
## Sepal.Width (0,1]
                            (1,2]
##
        (1,2] 1.00000000 0.00000000 0.00000000
         (2,3] 0.07317073 0.42682927 0.00000000
##
         (3,4] 0.00000000 0.12500000 0.00000000
##
         (4,5] 0.00000000 0.00000000 0.00000000
##
##
  , , Species = virginica
##
##
             Petal.Width
## Sepal.Width (0,1]
                              (1,2]
                                          (2,3]
         (1,2] 0.00000000 0.00000000 0.00000000
         (2,3] 0.00000000 0.28048780 0.12195122
##
         (3,4] 0.00000000 0.06250000 0.20312500
##
         (4,5] 0.00000000 0.00000000 0.00000000
prop.table(iris.table2, c(1,3))
## , , Species = setosa
##
##
             Petal.Width
                (0,1]
                            (1,2]
                                      (2,3]
## Sepal.Width
##
         (1,2]
         (2.3] 1.0000000 0.0000000 0.0000000
##
         (3,4] 1.0000000 0.0000000 0.0000000
##
         (4,5] 1.0000000 0.0000000 0.0000000
##
## , , Species = versicolor
##
             Petal.Width
## Sepal.Width (0,1]
                            (1,2]
         (1,2] 1.0000000 0.0000000 0.0000000
##
         (2,3] 0.1463415 0.8536585 0.0000000
##
```

```
(3,4] 0.0000000 1.0000000 0.0000000
##
##
         (4,5]
##
##
  , , Species = virginica
##
              Petal.Width
##
## Sepal.Width
                  (0,1]
                             (1,2]
                                        (2,3]
##
         (1,2]
         (2,3] 0.0000000 0.6969697 0.3030303
##
         (3,4] 0.0000000 0.2352941 0.7647059
##
##
         (4,5]
# conduct chi-square independence test
chisq.test(iris.table1) # warning message due to some frequency entry <= 5</pre>
   Pearson's Chi-squared test
##
## data: iris.table1
## X-squared = 47.263, df = 6, p-value = 1.658e-08
pie(iris.table0, main="Petal.Width")
```

## Petal.Width





detach(iris0) # paired with attach function

#### **Plots**

Compared to other statistical softwares in data analysis,  $\mathbf{R}$  is good at graphic generation and manipulation. The plotting functions in  $\mathbf{R}$  can be classified into the high-level ones and the low-level ones. The high-level functions create complete, new plots on the graphics device while the low-level functions only add extra information to the current plots.

plot is the most generic high-level plotting function in **R**. It will be compatible with most class of objects that the user input and produce appropriate graphics. For example, a numeric vector input results in a scatter plot with respect to its index and a factor vector results in a bar plot of its frequency table. Advanced class of object like lm (fitted result by a linear model) can also be called in plot. Methods will be discussed in specific documents like ?plot.lm.

Other plotting features include

- High-level plotting options: type, main, sub, xlab, ylab, xlim, ylim
- Low-level plotting functions
  - Symbols: points, lines, text, abline, segments, arrows, rect, polygon
  - Decorations: title, legend, axis
- Environmental graphic options (?par)
  - Symbols and texts: pch, cex, col, font
  - Lines: lty, lwd
  - Axes: tck, tcl, xaxt, yaxt
  - Windows: mfcol, mfrow, mar, new
- User interaction: location

I'll suggest the beginners learn from examples and grab when needed instead of going over such an overwhelming brochure. The following sections illustrate two basic senarios in data analysis. More high-level plotting functions will also be introduced.

#### Access the empirical distribution

**Histogram:** The hist function creates a histogram in **R**, where the breaks and freq options are frequently called. The barplot function could also realize the same result as illustrated in section **Tables**.

**Kernel Density Curve:** The density function estimates an empirical density of the data and gives a density object. One can call plot function with such an object as input and picture a density curve, which is anticipated to closely envelop its historgram. Note that the bw (bandwidth) and kernel options should be carefully considered in methodology.

Empirical CDF (ECDF): The ecdf function generates an empirical CDF ("ecdf") object that can be called by the plot function, which results in a step function graphic for the empirical cumulative distribution function. Creating a graph containing multiple CDFs or ECDFs visually displays the good-of-fitness or discrepancy among them. The statistically quantified Kolmogorov-Smirnov test with statistic

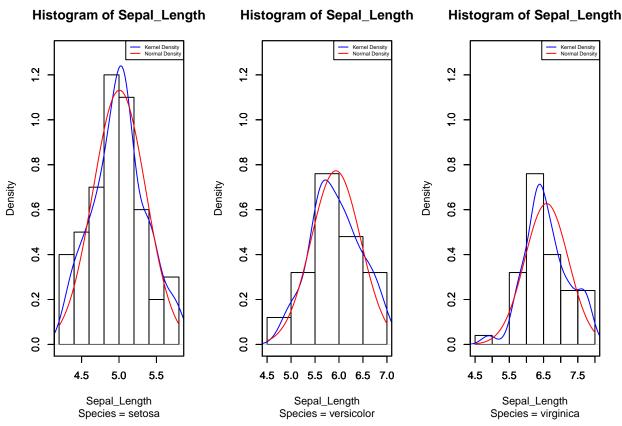
$$D_n = \sup_{x \in \mathbb{R}} |F_n(x) - F(x)|$$

realized by the ks.test function is based on it.

**Q-Q Plots:** "Q-Q" stands for the sample quantiles versus that of a given distribution or another sample. qqnorm, qqline and qqplot together is the set of functions in realizing it. **R** documents also illustrate how to create Q-Q plots against distributions other than Normal.

Box-and-Whisker Plots: With boxplot function in **R**, we can describe the data with box associated with certain quantiles and the whiskers for extremes. The box in the middle indicates "hinges" (nearly quartiles, see ?boxplot.stats) and median. The lines ("whiskers") show the largest/smallest observation that falls within a distance of 1.5 times the box size from the nearest hinge. If any observations fall farther away, the additional points are considered "extreme" values and are shown separately.

Exercise 11. (15 pt) Reproduce the code that generates the following plot about Sepal.Length in iris.



Hint. 1) Most decorations are based on defaults in hist with ylim = c(0, 1.3); 2) let h be the object resulted by hist and set xlim = range(h\$breaks); 3) set cex = 0.5 in legend; 4) use curve function in plotting the normal density with specified parameters.

```
par(mfrow=c(1,3))
for (name in c("setosa","versicolor","virginica"))
{
    species <- subset(iris,Species==name)
    sepal_length <- as.vector(species$Sepal.Length)

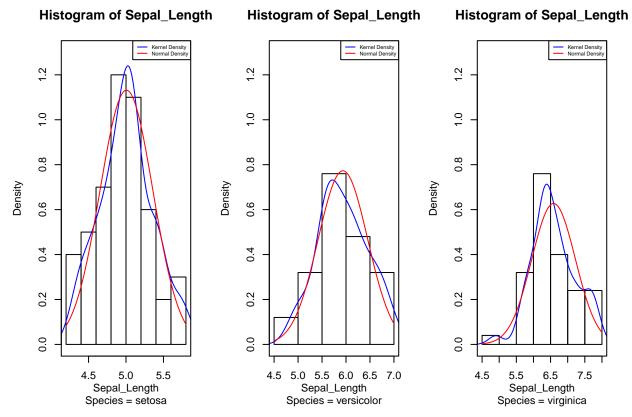
    h <- hist(sepal_length,plot=FALSE)

    plot(h,freq=FALSE,ylim=c(0,1.3),xlim=range(h$breaks),xlab=paste('Sepal_Length\nSpecies = ',name,sep=lines(density(sepal_length),col='blue')

    curve(dnorm(x,mean(sepal_length),sd(sepal_length)),add=TRUE,col='red')

    legend('topright',c('Kernel Density','Normal Density'),col=c('blue','red'),lty=c(1,1),cex=0.5)

    box(which='plot')
}</pre>
```



#### Demostrate correlation

plot(x, y) directly creates a scatter plot between the vector x and y. For a data frame input X, plot(X) would conduct pairwise scatter plots among its columns. A fitted regression line can also be added to an existing scatter plot via the abline function. And the function coplot is a power function in creating

conditioning plots given a variable segregated into different levels.

```
plot(iris[, -5])
```

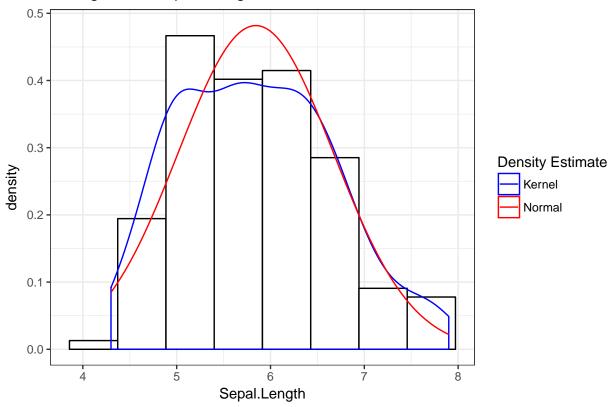
attach(iris) # attach function makes columns assessible as usual variables lm.mod <- lm(Petal.Length  $\sim$  Sepal.Length) # fit a linear model as an lm object plot(Sepal.Length, Petal.Length) ## plot(Petal.Length  $\sim$  Sepal.Length) abline(lm.mod)

coplot(Petal.Length ~ Sepal.Length | Species) detach(iris) # paired with attach function ""

### \*Create advanced plots

If you are a JAVA programmer, then you might anticipate a plotting toolbox to establish graphs layer-by-layer interactively. The ggplot2 package endows **R** with more advanced and powerful visualization techniques like this. Explore more in the online package manual. The following example solves **Exercise 11** without segregating with respect to **Species** in iris.





### \*Data Manipulation

If you are more familiar with the SQL language in manimulating data, then the dplyr package in **R** is a powerful toolkit in providing functions similar to the SQL operations (e.g. select, filter, arrange, mutate, inner\_join, group\_by, summarise and the pipe operator %>%). The following example realizes the conditional selection technique in the previous exercises in a much cleaner way. Explore more in the online package mannual.

```
# Exercise 3.
filter(students, id == '003')
      id score_A score_B
## 1 003
              90
                       84
# Exercise 7.
students6 <- group_by(students3, subj)</pre>
mutate(students6, score.mean = mean(score))
## # A tibble: 6 x 4
## # Groups: subj [2]
##
     id
           subj score score.mean
     <fct> <fct> <dbl>
                              <dbl>
##
                              94.0
## 1 001
           Α
                   95.0
## 2 002
           Α
                   97.0
                              94.0
                              94.0
## 3 003
                   90.0
           Α
## 4 001
                   80.0
                              79.7
           В
                              79.7
## 5 002
           В
                   75.0
```

```
## 6 003
           В
                  84.0
                             79.7
# Using the pipe operator makes it more compact
group_by(students3, subj) %>%
  mutate(score.mean = mean(score)) # transform function couldn't apply mean by group
## # A tibble: 6 x 4
## # Groups: subj [2]
##
     id
           subj score score.mean
     <fct> <fct> <dbl>
##
                             <dbl>
## 1 001
                  95.0
                             94.0
           Α
## 2 002
                  97.0
           Α
                             94.0
## 3 003
                  90.0
                             94.0
           Α
## 4 001
           В
                  80.0
                             79.7
## 5 002
           В
                  75.0
                             79.7
## 6 003
           В
                  84.0
                             79.7
```

## **Bibliography**

Dalgaard, Peter. 2008. Introductory Statistics with R: Statistics and Computing. Springer.

W. N. Venables, D. M. Smith, and the R Core Team. 2017. An Introduction to R: Notes on R: A Programming Environment for Data Analysis and Graphics (version 3.4.3).