

1. 3.1.2

$$S \Rightarrow aAa \Rightarrow aSSa \Rightarrow abAbaAaa \Rightarrow \\ abSSbaSSaa \Rightarrow abbabAbaa \Rightarrow abbabSSbaa \Rightarrow abbabbaa$$

2. As the hint suggests, by regularity's closure under intersection, it suffices to show  $L \cap a^*bba^* = \{a^n bba^n : n \geq 0\}$  is non-regular. Take arbitrary  $N > 0$ . Let  $w = a^N bba^N$ . Take arbitrary  $xyz = w$ , with  $|xy| \leq N$ ,  $y \neq \epsilon$ . Note that  $x = a^h, y = a^i, z = a^j bba^N$ , where  $h \geq 0, i > 0, j \geq 0$  and  $h + i + j = N$ . Let  $k = 2$ , then  $x(y^k)z = a^h a^{2i} a^j bba^N = a^{N+i} bba^N \notin L$ .

Note: we actually have a stronger conclusion in this case that  $w$  can be arbitrary and  $k > 0$  is sufficient, since any repetition has to be done both before and after  $bb$ , and repetition of  $y$  can only appear before or after  $bb$  (or containing  $bb$ ).

3.

$$L = (a \cup b)^*, S = a^n b^n$$