5.2.3

$r = \sum_{i=1}^{6} i\, I(r = i)$

$\mathbb{E}(r) = \mathbb{E}(\sum_{i=1}^{6} i\, I(r = i)) = \sum_{i=1}^{6} i \mathbb{E}(I(r = i)) = \sum_{i=1}^{6} i\, P(r = i) = 3.5$

$\mathbb{E}(\sum_{j=1}^{n} r_j) = \sum_{j=1}^{n} \mathbb{E}(r_j) = n\mathbb{E}(r) = 3.5n$

5.2.4

$X_i = 1$, if person i get back his/her hat, 0 otherwise.

$X = \sum_{i=1}^{n} X_i$, the number of people that get back hat.

$\mathbb{E}(X) = \sum_{i=1}^{n} \mathbb{E}X_i = \sum_{i=1}^{n} P(\text{person}\, i\, \text{get back hat}) = \sum_{i=1}^{n} 1/n = 1$

Note: Expectation is linear even when random variables are dependent.

5.2.5

$X_{i,j} = I((i.j)\,\text{is an inversion})$

$X = \sum_{i<j} X_{i,j}$, the number of inversions

$\mathbb{E}(X) = \sum_{i<j} \mathbb{E}X_{i,j} = \sum_{i<j} P((i.j)\,\text{is an inversion}) = \sum_{i<j} 1/2 = 1/2 \times (n^2 - n)/2$