Physics 1C: Practice Midterm 1

You have 90 minutes for this practice exam, there are 11 pages (including the cover and formula pages) and it is intended to be closed book and closed notes. The use of any form of electronics is prohibited, except for a basic scientific calculator. To receive full credit, show all your work and reasoning. No credit will be given for answers that simply "appear." If you need extra space, use the backside of the page with a note to help the grader see that the work is continued elsewhere.

Name:____________________ Signature:____________________ ID:________

Fundamental Constants

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \quad \& \quad \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$

Electric and Magnetic Force

$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad \text{or} \quad \vec{F}_B = \int_c I d\vec{\ell} \times \vec{B}$$

Kinematics with Constant Acceleration ($s = x$ or y) & Centripetal Acceleration

$$s\left(t\right) = s_0 + v_{0s}t + \frac{1}{2}a_s t^2$$

$$v_s\left(t\right) = v_{0s} + a_s t$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Magnetic Torque

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{with} \quad \vec{\mu} = \int \hat{n} A dI$$

Gauss's Law

$$\oint_{\partial\mathcal{V}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \text{with} \quad Q_{\text{encl}} = \int_{\mathcal{V}} \rho dV$$

$$\oint_{\partial\mathcal{V}} \vec{B} \cdot d\vec{A} = 0$$

Biot-Savart Law

$$\vec{B} = \frac{\mu_0}{4\pi} I \int_c \frac{d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} I \int_c \frac{d\vec{\ell} \times \vec{r}}{r^3}$$

Ampere-Maxwell Law

$$\oint_{\partial\mathcal{S}} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_{\mathcal{S}} \vec{E} \cdot d\vec{A} \equiv \mu_0 \left(I_{\text{encl}} + I_{\text{D}}\right) \quad \text{with} \quad I_{\text{encl}} = \int_{\mathcal{S}} \vec{J} \cdot d\vec{A}$$

Faraday's Law and Motional EMF

$$\mathcal{E}_{\text{ind}} = \oint_{\partial\mathcal{S}} \left(\frac{\vec{F}_{\text{EM}}}{q}\right) \cdot d\vec{\ell} = -\frac{d}{dt} \int_{\mathcal{S}} \vec{B} \cdot d\vec{A}$$

$$\mathcal{E}_{\text{mot}} = \int_c \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}$$

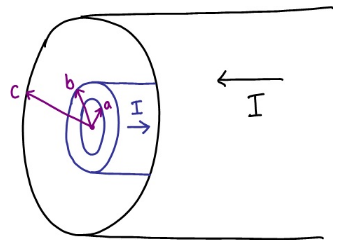
Mutual- and Self-Inductance

$$\Phi_{B1} = M_{12} I_2 \quad \& \quad \Phi_{B2} = M_{21} I_1$$
$$\Phi_B = LI$$

Energy in an Inductor and Magnetic Energy Density

$$U_B = \frac{1}{2} L I^2$$
$$u_B = \frac{B^2}{2\mu_0}$$

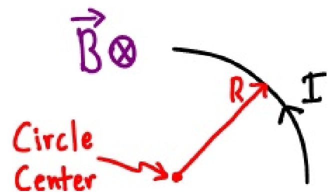
1. A long, *hollow* conducting pipe of inner radius a and outer radius b carries a current I parallel to its axis and disributed uniformly through the pipe. At radius $c > b$ is a concentric, long conducting *shell* carrying a current I in the direction opposite to the central hollow conducting pipe.



(a) Find the magnetic field everywhere. **[20]**

(b) Find the self-inductance and magnetic energy *per unit length* of this system for $r \in [b, c]$. **[15]**

2. A wire carrying a current $I = 1.50$ A passes through a region containing a magnetic field of field strength $B = 4.80 \times 10^{-2}$ T. A segment of this wire is perpendicular to the field and makes a quarter-circle turn of radius $R = 21.0$ cm as it passes through the field region, as shown below. The remaining parts of the wire (not shown), from which the current runs into the segment and to which the current goes after running through the segment, are perpendicular to the segment and are *beneath* the plane of the arc shown.



(a) Why is there no contribution to the magnetic force on this wire from the segments carrying the current to and from the circular arc? **[4]**

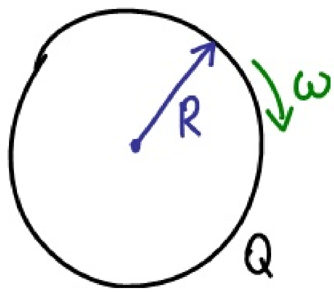
(b) Find the magnetic force (magnitude and direction) on the circular arc of wire. **[15]**

(c) Now, imagine that the external magnetic field is turned off.

i. Calculate the magnetic field (magnitude and direction) at the center of the circle (which is in the same plane as the arc) due to only the circular arc of wire. **[11]**

ii. If the segments carrying current to and from this circular arc were taken to be infinitely long beneath the plane of the arc, calculate their contribution to the field at the center of the circle described in Part (ci). **[15]**

3. Imagine an insulating *disk* of radius R carrying a total charge Q and rotating with angular frequency ω about its symmetric axis, as shown below. The rotating disk creates the effect of a bunch of concentric current loops.



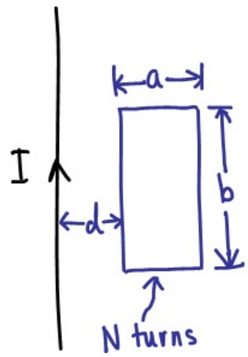
(a) If the disk has a uniform surface charge density, determine its magnetic dipole moment (magnitude and direction) in terms of the given quantities. **[12]**

(b) Now, suppose that the disk is made to be conducting and is discharged (i.e., it has NO NET CHARGE). It is then placed into a uniform magnetic field that points *into the page* with the field perpendicular to the plane of the disk with respect to the picture above.

   i. Provide a qualitative description of what happens in this system. In particular, explain what is going on when the system reaches a steady-state under these circumstances. **[7]**

   ii. Determine the emf (magnitude and direction) generated within the disk when the system reaches a steady state. **[11]**

4. In the figure below, the wire on the left is infinitely long and carries a current I. The conducting, rectangular loop on the right has N turns with width a and length b. The left end of the loop is a distance d away from the infinitely long wire. In terms of the given quantities, calculate the mutual inductance between the wire-loop system. **[25]**



5. A toroidal coil of rectangular cross-section has inner radius a, outer radius b, and height c. It consists of N total turns of wire and carries a time-varying current $I = I_0 \sin(\omega t)$. A single-turn rectangular loop encircles the toroid, outlining its cross section, as shown below. Find an expression for the peak emf induced in the rectangular loop. **[25]**