STOR 435

Homework 17

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1. $M_X^{(3)}(t) = \alpha(\alpha+1)(\alpha+2)\lambda^\alpha(\lambda-t)^{-\alpha-3}$

   $\mathbb{E}(X^3) = M_X^{(3)}(0) = \frac{\alpha(\alpha+1)(\alpha+2)}{\lambda^3} = \frac{105}{64} \approx 1.6406$

2. Yes

3. Yes

4. $\iint_A (x^2+xy)dxdy = \int_0^1\int_0^{\sqrt{y}}(x^2+xy)dxdy + \int_1^2\int_0^1(x^2+xy)dxdy$

   $= \int_0^1 \frac{y^{3/2}}{3} + \frac{y^2}{2}dy + \int_1^2 \frac{1}{3} + \frac{y}{2}dy = \frac{2}{15}y^{\frac{5}{2}} + \frac{1}{6}y^3\Big|_0^1 + \frac{1}{3}y + \frac{1}{4}y^2\Big|_1^2 = \frac{83}{60}$

   $\iint_A (x^2+xy)dxdy = \int_0^1\int_{x^2}^2(x^2+xy)dydx = \int_0^1 2x + 2x^2 - x^4 - \frac{x^5}{2}dx = x^2 + \frac{2}{3}x^3 - \frac{1}{5}x^5 - \frac{1}{12}x^6\Big|_0^1 = \frac{83}{60}$

5. $\lambda = 4$

   a) $M_X(t) = \begin{cases} \frac{4}{4-t}, \text{for } t < 4 \\ \infty, \text{for } t \geqslant 4 \end{cases}$

   b) $H(t) = \begin{cases} \left(\frac{4}{4-t}\right)^{15}, \text{for } t < 4 \\ \infty, \text{for } t \geqslant 4 \end{cases}$

   c) $\lambda = 4, \alpha = 15$

6.

   a) $(0,1)$

   b) $F_Y(y) = \Pr(Y<y) = \Pr(X^2<y) = \Pr(0<X<\sqrt{y}) + \Pr(0>X>-\sqrt{y}) = 2\sqrt{y} - y$

   c) $f_Y(y) = y^{-\frac{1}{2}} - 1$

7.

   a) $(0,\infty)$

   b) $F_Y(y) = \Pr(Y<y) = \Pr(-4\log_e(X)<y) = \Pr(X>e^{-4y}) = 1 - e^{-4y}$

   c) $f_Y(y) = 4e^{-4y}$, therefore, $Y \sim \mathrm{Exp}(4)$, is an exponential distribution.

8.

   a) $(0,\infty)$

   b) $F_Y(y) = \Pr(Y<y) = \Pr(365X<y) = \Pr\big(X<\frac{y}{365}\big) = 1 - e^{-\frac{y}{365}}$

   c) $f_Y(y) = \frac{1}{365}e^{-\frac{y}{365}}$