Problem on Nonlinear Equations

A sample of soft tissue is extracted from a pig heart muscle in order to characterize its mechanical properties. After treatments the sample is found to behave as an incompressible isotropic nonlinear elastic material characterized by the following stress tensor

$$\boldsymbol{P} = 2W_1\boldsymbol{F} - p\boldsymbol{F}^{-T}, \tag{1}$$

where $\boldsymbol{F}$ is a tensor (in this case a 3×3 matrix) that defines the deformation, p is the pressure in the tissue , $W_1 = \dfrac{a}{2}e^{b(I_1-3)}$, $a = 0.5$kPa and $b = 10$ are parameters and $I_1 = \text{trace}\left(\boldsymbol{F}^T\boldsymbol{F}\right)$.

We want to check how good the mathematical model (1) is with respect to the experiments. We consider a pure traction experiment, where the sample is assumed to have a cylindrical shape and it is pulled from one of the flat faces. In this case, the pressure field can be eliminated and the resulting force balance for model (1) can be written as

$$\frac{a}{2}e^{b\left(\lambda^2+2/\lambda-3\right)}\left(\lambda - \frac{1}{\lambda^2}\right) = f, \tag{2}$$

where f is the force (per unit area) applied on the tissue, and λ is the amount of stretch in the direction of the pull (you can think of λ as the stretch of a spring: if the resting length is L_0 and the length after pulling is L, then $\lambda = L/L_0$).

- Given the experimental results shown in Tables 1 and 2, how good is the mathematical model? Discuss your results and the numerical methods used.
  Hint: Compute the stretches given by the mathematical model (2) using the values of f shown in Table 1 and compare them with the experimental results shown in Table 2.

f [kPa]	-1.5	-0.5	-0.22	-0.08	0	0.08	0.18	0.37	0.73

Table 1: Forces applied to the tissue sample in the experiment

λ	0.8	0.85	0.9	0.95	1	1.05	1.1	1.15	1.2

Table 2: Computed values of the stretch in the experiment