We first rigorously redefine the question to avoid potential confusion:

For some $m, n \in \mathbb{N}$, find d, s.t. $\forall E_1 = \{K_1, \Sigma, \delta_1, s_1, F_1\}$, $E_2 = \{K_2, \Sigma, \delta_2, s_2, F_2\}$ with $|K_1| = m$ and $|K_2| = n$, $\exists E = \{K, \Sigma, \delta, s, F\}$ with $|K| \leq d$, s.t. $L(E_1) \cap L(E_2) = L(E)$. Denote $L(E_1) = L$, and $L(E_2) = M$.

We show that $d = mn$ satisfies such condition.

Let $E = \{K, \Sigma, \delta, s, F\}$, where $K = K_1 \times K_2$ ($\times$ denotes Cartesian product), $\delta = \delta_1 \times \delta_2$ (i.e. $\delta((p,q),a) = (\delta_1(p,a), \delta_2(q,a))$), $s = (s_1, s_2)$, and $F = F_1 \times F_2$.

Claim: $L(E) = L(E_1) \cap L(E_2)$:

Let $\{(p_i, q_i)\}_{i=1}^{i=l+1}$ denotes a sequence of states of E that recognizes string $a_1 a_2 ... a_l \in L(E)$. Note that $\forall i, \delta((p_i, q_i), a_i) = (\delta_1(p_i, a_i), \delta_2(q_i, a_i))$, so $a_1 a_2 ... a_l$ is also recoginized by the sequence $\{p_i\}_{i=0}^{i=l}$ in E_1 and $\{q_i\}_{i=0}^{i=l}$ in E_2, respectively. Therefore, $L(E) \subset L(E_1) \cap L(E_2)$. Similarly, we can show that $L(E) \supset L(E_1) \cap L(E_2)$. Therefore, $L(E) = L(E_1) \cap L(E_2)$.

$|F| = mn$, and by Claim, E recognizes $L(E_1) \cap L(E_2)$, so $d = mn$ satisfies the conditions specified by the question.