COMP 550 Assignment 1 Siyang Jing

3.1.2 Show $(n+a)^b = \Theta(n^b)$

It suffices to show $(n+a)^b = O(n^b)$, and $(n+a)^b = \Omega(n^b)$

Show $(n+a)^b = O(n^b)$:

Let $N_1 \in \mathbb{N}$ s.t. $N_1 > |a|$, let $c_1 = 2^b$, then we have

$n \geq N_1 \Rightarrow (n+a)^b > 0$, since $n > |a|$, and

$$(n+a)^b \leq (|n|+|a|)^b < 2^b n^b = c_1 n^b$$

Show $(n+a)^b = \Omega(n^b)$:

Let $N_2 \in \mathbb{N}$, s.t. $N_2 > 2|a|$, let $c_2 = 2^{-b}$, then we have

$n \geq N_2 \Rightarrow (n+a)^b \geq (n-|a|)^b \geq (\frac{n}{2})^b = c_2 n^b > 0$ □.

4.3.7.

If we use substitution: $T(n) \geq 4(c(\frac{n}{3})^{\log_3 4}) + n = cn^{\log_3 4} + n \not\leq cn^{\log_3 4}$ ↯

We first show $T(n) = \Omega(n^{\log_3 4})$ by assuming

$T(n) \geq c_1 n^{\log_3 4}$, where $c_1 = \min\{T(1), T(2)\,2^{-\log_3 4}\}$ *

Base Case: From the definition of c_1, already proved.

Inductive Step: For $n \geq 3$, suppose $\forall k < n$, * has been established, then

$$T(n) = 4T(\tfrac{n}{3}) + n \geq 4(c_1(\tfrac{n}{3})^{\log_3 4}) + n = c_1 n^{\log_3 4} + n > c_1 n^{\log_3 4}$$

We then show $T(n) = O(n^{\log_3 4})$ by assuming

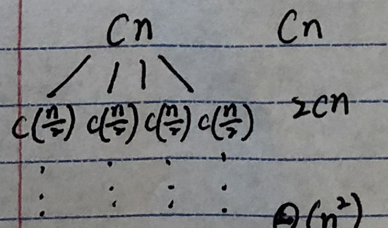
$T(n) \leq c_2 n^{\log_3 4} - 3n$, where $c_2 = \max\{T(1)+3, (T(2)+6)\,2^{-\log_3 4}\}$ **.

Base Case: Proved by definition

Inductive Step: For $n \geq 3$, suppose $\forall k < n$, ** has been established, then

$$T(n) = 4T(\tfrac{n}{3}) + n \leq 4(c_2(\tfrac{n}{3})^{\log_3 4} - 3(\tfrac{n}{3})) + n = c_2 n^{\log_3 4} - 3n$$ □.

4.4.7.



$\Omega(n^2)$: Assume $T(n) \geq d_1 n^2$ where $d_1 = \min(T(1), \frac{c}{2})$

Base Case: Proved by definition of d_1

Inductive Step: $T(n) = 4T(\lfloor n/2 \rfloor) + cn \geq$

$$4(d_1(\tfrac{n-1}{2})^2) + cn \geq d_1 n^2 + (c - 2d_1)n + d_1$$

$$\geq d_1 n^2$$

$O(n^2)$: Assume $T(n) \leq d_2 n^2 - cn$, where $d_2 = T(1) + c$.

Base Case: Proved by definition of d_2

Inductive Step: $T(n) \leq 4(d_2(\frac{n}{2})^2 - c(\frac{n}{2})) + cn = d_2 n^2 - cn$ □.