

*There are some issues that you should be able to easily address:*

*1) The matrix  $A$  is not STRICTLY diagonally dominant. Check for example row 4. Is Gauss-Seidel still convergent? Why?*

I made a miscalculation to wrongly conclude that  $A$  is strictly dominant. It's still convergent. As I wrote in the edited report, I checked the spectral radius of  $I - L^{-1}A$  is less than 1 ( $L$  is the lower triangular part of  $A$ ), which is the necessary and sufficient condition for convergence of Gauss-Seidel method.

*2) In order to say if  $A$  is not ill-conditioned you need to check the condition number of  $A$ . If you only take the lower triangular part, then you say the the lower triangular part is not ill-conditioned.*

I wrongly assumed that if  $A$  is not ill-conditioned and symmetric, then the lower triangular part of  $A$  is also not ill-conditioned. I corrected this by directly stating the condition number of  $L$  is relatively small.

*3) How can you say the matrix is not positive definite? Did you check?*

I made huge mistake on this. I made some typos in matlab code. It's actually positive definite, after checking with matlab chol function.

*4) Are you sure you could not use the conjugate gradient method? We did not cover this, but since you mention it, how can you support your claim.*

Again, I made a huge mistake on this. If  $A$  were not positive definite, then conjugate gradient and steepest descent could not be used. Conjugate gradient method and steepest descent are indeed usable. With a  $1e-4$  residual tolerance, conjugate gradient method converges in 4 iterations, and steepest descent converges in 56 iterations.

*5) You claim: "Therefore, we do not need to worry much about the speed, or convergence rate, of the method, and thus Gauss-Seidel method, though definitely not among the faster methods, satisfies our need". What is the order of convergence of Gauss-Seidel? Can other methods be used to solve this problem? Are they faster? In which sense?*

Jacobi method is not good, since the spectral radius of the iteration matrix is greater than 1. JOR is usable with proper choice of  $\omega$ , as stated by theorem 4 of chapter 4.2 of Quateroni. SOR is convergent. And conjugate gradient and steepest descent methods are also convergent.

Actually, conjugate gradient method is probably a better choice to solve the problem. Since it's guaranteed to converge in 15 (dimension of the system) iterations, which is very satisfying. The convergence of Gauss-Seidel method, as demonstrated in many exercises, is highly dependent on the initial guess and the matrix A. Typically, conjugate gradient requires less iterations than Gauss-Seidel, JOR, SOR, and steepest descent.

Nonetheless, in this question Gauss-Seidel is certainly also usable. As stated in the report, we reach  $1e-5$  residual tolerance after 15 iterations, which is a considerably small and satisfying number. And to keep consistency with my original report, I still use Gauss-Seidel in the final submission.

*6) Format your plot such that the labels, the title, and the numbers are clearly readable.*

I reformatted some of the plots.

*7) You show that at every level the pressure decreases. The different levels are separated by a distance  $L_m$ . Can you take this into account?*

I'm not sure what you mean by this. I stated that  $q_j = q_i - L_m R_m Q_m$ , demonstrating that  $q_j$  is less than  $q_i$ . And in fact, we already take  $L_m$  into account when constructing the linear system.

*8) Your arguments on the symmetry of the systems are pertinent but not enough to guarantee the same pressure for all nodes at every level. What happens if  $p_{31} = 1$ ?*

If  $p_{31}=1$ , then the binary tree is no longer balanced, and the system is not symmetric. Nonetheless, I elaborated on what I mean by the symmetry of the system.