

1. 3.5.1.(a)

By closure under concatenation, $L_1 = \{a^m b^n : m > n\} = \{a^n : n > 0\} \circ \{a^n b^n\}$ and $L_2 = \{a^m b^n : m < n\}$ are context-free languages. So by closure under union, $L_1 \cup L_2 = \{a^m b^n : m \neq n\}$ is context-free.

2. Question 2

Show $L = \{wcw : w \in \{a, b\}^*\}$ is not context-free. It's sufficient to show $L' = L \cap a^* c a^* c a^* = a^n c a^n c a^n$ is not context free.

Let $n \in \mathbb{N}$, let $w = a^n c a^n c a^n = uvxyz$, if v or y has c in it, then $uv^2xy^2z \notin L'$. If neither v nor y has c in it, then $uv^2xy^2z = a^m c a^p c a^q$, where one of m, p, q equals n , and the other two are equal but not equal to n , which means $uv^2xy^2z \notin L'$.

3. Question 3

$$\begin{aligned} & \{a^m b a^n b a^p : m = n \vee n = p \vee m = p\} \\ &= \{a^n b a^n b a^*\} \cup \{a^* b a^n b a^n\} \cup \{a^n b a^* b a^n\} \end{aligned}$$

All three languages are context-free.