STOR 435
Homework 16
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1. Yes
2. Yes
3. Yes
4. $\int_0^1 \int_0^2 \cos(x+y)dxdy = \int_0^1 \sin(x+y)|_0^2 dy = \int_0^1 \sin(2+y) - \sin(y)dy = \cos(y) - \cos(2+y)|_0^1$
$= \cos(1) - \cos(3) - \cos(0) + \cos(2) \approx 0.1141$
5. $\mathbb{E}(X) = \lambda^{-1} = 2 \rightarrow \lambda = \frac{1}{2} \rightarrow X \sim \mathrm{Exp}\left(\frac{1}{2}\right)$
$F_X(x) = 1 - e^{-\frac{1}{2}x}$
   a) $F_X(1) = 1 - e^{-\frac{1}{2}\times 1} = 0.3935$
   b) $F_X\left(\frac{1}{2}\right) = 1 - e^{-\frac{1}{2}\times\frac{1}{2}} = 0.2212$
   c) $F_X(10) = 1 - e^{-\frac{1}{2}\times 10} = 0.9933$
6.
   a) $1 - \mathrm{expcdf}(2,1) = e^{-1\times 2} = 0.1353$
   b) $\Pr(X > 10|X > 9) = \frac{\Pr(X > 10 \wedge X > 9)}{\Pr(x > 9)} = \frac{\Pr(X > 10)}{\Pr(X > 9)} = e^{-1(10-9)} = 0.3679$
7.
   a) $F_X(x) = 1 - e^{-\frac{1}{4}x}$
   b) 16
   c) $H(a) = \mathbb{E}(X-a)^2 = \mathbb{E}(X^2) - 2a\mathbb{E}(X) + a^2 = 2\lambda^{-2} - 2a\lambda^{-1} + a^2$
   $\left(\frac{\partial H(a)}{\partial a}\right)_{a=\hat{a}} = 0 \rightarrow 2\hat{a} - 2\lambda^{-1} = 0 \rightarrow \hat{a} = \lambda^{-1} = 4$
8. Let $Y \sim \mathrm{Exp}\left(\frac{1}{5}\right)$
$\Pr(X = a) = \Pr(a - 1 \leqslant Y < a) = F_Y(a) - F_Y(a-1) = e^{-\frac{1}{5}(a-1)} - e^{-\frac{1}{5}a} = \left(e^{\frac{1}{5}} - 1\right)e^{-\frac{1}{5}a}$

Notes on exponential random variable:
$F_X(a) = \int_0^a \lambda e^{-\lambda x}dx = -(e^{-\lambda x})_0^a = 1 - e^{-\lambda a}$
$\mathbb{E}(X) = \int_0^\infty \lambda e^{-\lambda x}xdx = -\int_0^\infty xde^{-\lambda x} = -(xe^{-\lambda x} + \lambda^{-1}e^{-\lambda x})_0^\infty = \lambda^{-1}$
$M_X(t) = \mathbb{E}(e^{tX}) = \int_0^\infty \lambda e^{(t-\lambda)x}dx = \left(\frac{\lambda}{t-\lambda}e^{(t-\lambda)x}\right)_0^\infty = \frac{\lambda}{\lambda - t}$, for $t < \lambda$.
$\mathbb{E}(X^2) = M_X''(0) = 2\lambda(\lambda - 0)^{-3} = 2\lambda^{-2}$
$\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}X)^2 = 2\lambda^{-2} - \lambda^{-2} = \lambda^{-2}$