STOR 435 Homework 23

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1. First we notice that X and Y are apparently independent.

   $\mathbb{E}(Z)=\mathbb{E}(X)\mathbb{E}(Y)=3.5^2=\frac{49}{4}$

   $\mathrm{Var}(Z)=\mathbb{E}(Z^2)-\mathbb{E}(Z)^2=\mathbb{E}(X^2Y^2)=\mathbb{E}(X^2)\mathbb{E}(Y^2)-\mathbb{E}(Z)^2=\frac{91}{6}\frac{91}{6}-\left(\frac{49}{4}\right)^2=\frac{11515}{144}$

2. We notice that for any independent variables X, Y, $\mathrm{Cov}(X,Y)=\mathbb{E}(XY)-\mathbb{E}(X)\mathbb{E}(Y)=0$

   $\mathrm{Var}(X-Y)=\mathrm{Var}(X)+\mathrm{Var}(Y)-2\mathrm{Cov}(X,Y)=\frac{1}{12}+\frac{1}{12}=\frac{1}{6}$

3. This question seems suitable for indicator variables.

   Reason: 1. Number of events; 2. Only want expectation.

   Let $A_i=\{\text{The } i,i+1,i+2 \text{ tossing are } HTH\}$. Let $X=\sum_{i=1}^{i=48} I_{A_i}$.

   $\mathbb{E}(X)=\sum_{i=1}^{i=48}\mathbb{E}(I_{A_i})=48\times\frac{1}{8}=6$

4. This question is suitable for indicator variables, since the linearity of $\mathbb{E}$ is regardless of dependence or independence between random variables.

   Let $A_i=\{\text{Ruoyu got the } i\text{th sort of drink at least once}\}$. We have $X=\sum_{i=1}^{i=10} I_{A_i}$.

   $\Pr(A_i)=1-\left(\frac{9}{10}\right)^{20}$

   $\mathbb{E}(X)=\sum_{i=1}^{i=10}\mathbb{E}(I_{A_i})=10\times\left(1-\left(\frac{9}{10}\right)^{20}\right)\approx 8.7842$

5. $\mathbb{E}(X)=\int_0^\infty\int_0^x x\frac{2e^{-2x}}{x}dydx=\int_0^\infty x2e^{-2x}dx=\frac{1}{2}$, we can see that X is an exponential distribution.

   $\mathbb{E}(Y)=\int_0^\infty\int_0^x y\frac{2e^{-2x}}{x}dydx=\int_0^\infty xe^{-2x}dx=\frac{1}{4}$

   $\mathbb{E}(XY)=\int_0^\infty\int_0^x xy\frac{2e^{-2x}}{x}dydx=\int_0^\infty x^2e^{-2x}dx=\frac{1}{4}$

   $\mathrm{Cov}(X,Y)=\frac{1}{8}$

6. $\mathbb{E}(3X+4Y-5)=3\mathbb{E}(X)+4\mathbb{E}(Y)-5=6$

   $\mathrm{Var}(3X+4Y-5)=3^2\mathrm{Var}(X)+4^2\mathrm{Var}(Y)+2\times 3\times 4\mathrm{Cov}(X,Y)=43$