Types of Error

$$
\begin{aligned}
\dot{\boldsymbol{x}} &= \boldsymbol{L}(\boldsymbol{x}) + \boldsymbol{\zeta} && \text{(Type I)} \\
\dot{\boldsymbol{x}} &= \boldsymbol{L}(\boldsymbol{x} + \boldsymbol{\xi}) && \text{(Type II)} \\
\dot{\boldsymbol{x}} &= \boldsymbol{L}(\boldsymbol{x} + \boldsymbol{\xi}) + \boldsymbol{\zeta} && \text{(Type III)}
\end{aligned}
$$

For simplicity, we assume that $\boldsymbol{\zeta}$ and $\boldsymbol{\xi}$ are both constant in time. when the error is small, they exhibit behaviors similar to those of the system.

$$
\begin{aligned}
\zeta_i &= A \sin(2\pi \frac{i-1}{N}) \\
\xi_i &= B \sin(2\pi \frac{i-1}{N}), \quad i = 1, \dots, N
\end{aligned}
$$

where A and B are constants.

Model 1

$$\dot{\boldsymbol{x}} = \boldsymbol{L}(\boldsymbol{x}) + \zeta \qquad \text{(Type I)}$$

$$\boldsymbol{b}_n = m(\boldsymbol{x}_{n-1}) - \hat{m}(\boldsymbol{x}_{n-1})$$

We therefore incorporate $\boldsymbol{b}_n$ to the augmented system:

$$\begin{aligned} \boldsymbol{x}_n^f &= \hat{m}(\boldsymbol{x}_{n-1}^a) + \boldsymbol{b}_n^f \\ \boldsymbol{b}_n^f &= f_b(\boldsymbol{x}_{n-1}^a, \boldsymbol{b}_n^a) \end{aligned}$$

In the case where ζ is constant, we assume $\boldsymbol{b}_n$ is constant too, and then

$$\boldsymbol{b}_n^f = f_b(\boldsymbol{x}_{n-1}^a, \boldsymbol{b}_{n-1}^a) = \boldsymbol{b}_{n-1}^a$$

Model 2

$$\dot{\boldsymbol{x}} = \boldsymbol{L}(\boldsymbol{x} + \boldsymbol{\xi}) \qquad \text{(Type II)}$$

$$\boldsymbol{c}_n = m(\boldsymbol{x}^t_{n-1}) - \hat{m}(\boldsymbol{x}^m_{n-1}) = m(\boldsymbol{x}^t_{n-1}) - \hat{m}(\boldsymbol{x}^t_{n-1} - \boldsymbol{c}_{n-1})$$

The augmented system then becomes

$$\begin{aligned} \boldsymbol{x}^f_n &= \hat{m}(\boldsymbol{x}^a_{n-1}) \\ \boldsymbol{c}^f_n &= f_c(\boldsymbol{x}^a_{n-1}, \boldsymbol{c}^a_n) \end{aligned}$$

For simplicity, we assume we know $\boldsymbol{\xi}$ is constant, which means $\boldsymbol{c}_n$ should be constant, and then

$$\boldsymbol{c}^f_n = f_c(\boldsymbol{x}^a_{n-1}, \boldsymbol{c}^a_{n-1}) = \boldsymbol{c}^a_{n-1}$$

The analysis will model the model trajectory $\hat{\boldsymbol{x}}_n$ instead of the true trajectory $\boldsymbol{x}_n$, and therefore the observation should be:

$$\boldsymbol{y}_n = H(\hat{\boldsymbol{x}}_n + \boldsymbol{c}_n)$$

Model 3

$$\dot{\boldsymbol{x}} = \boldsymbol{L}(\boldsymbol{x} + \boldsymbol{\xi}) + \boldsymbol{\zeta} \qquad \text{(Type III)}$$

Combine the previous two models together, we have the following scheme:

$$\begin{aligned}
\boldsymbol{x}_n^f &= \hat{m}(\boldsymbol{x}_{n-1}^a) + \boldsymbol{b}_n^f \\
\boldsymbol{b}_n^f &= f_b(\boldsymbol{x}_{n-1}^a, \boldsymbol{b}_{n-1}^a, \boldsymbol{c}_{n-1}^a) \\
\boldsymbol{c}_n^f &= f_c(\boldsymbol{x}_{n-1}^a, \boldsymbol{b}_{n-1}^a, \boldsymbol{c}_{n-1}^a)
\end{aligned}$$

Again, for simplicity, we assume c_n and b_n are constant, i.e.,

$$\begin{aligned}
\boldsymbol{b}_n^f &= f_b(\boldsymbol{x}_{n-1}^a, \boldsymbol{b}_{n-1}^a) = \boldsymbol{b}_{n-1}^a \\
\boldsymbol{c}_n^f &= f_c(\boldsymbol{x}_{n-1}^a, \boldsymbol{c}_{n-1}^a) = \boldsymbol{c}_{n-1}^a
\end{aligned}$$

Observation operator:

$$\boldsymbol{y}_n = H(\hat{\boldsymbol{x}}_n + \boldsymbol{c}_n)$$

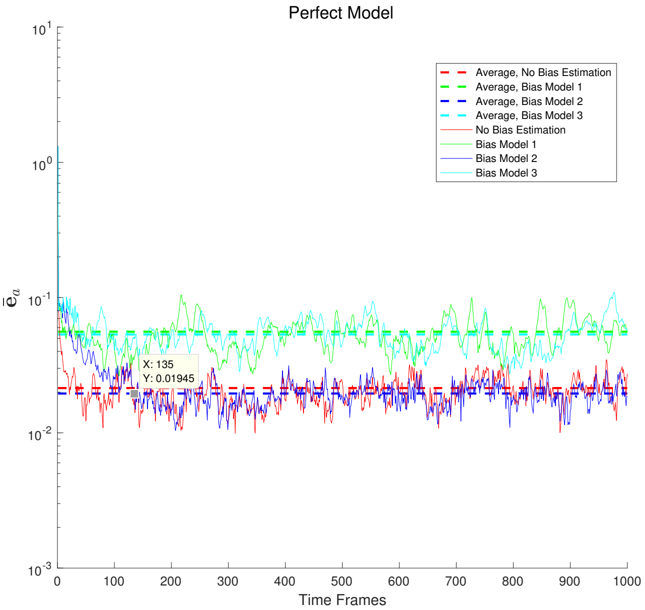
Experiment Setup

To improve the analyses in our experiments, we employ variance inflation

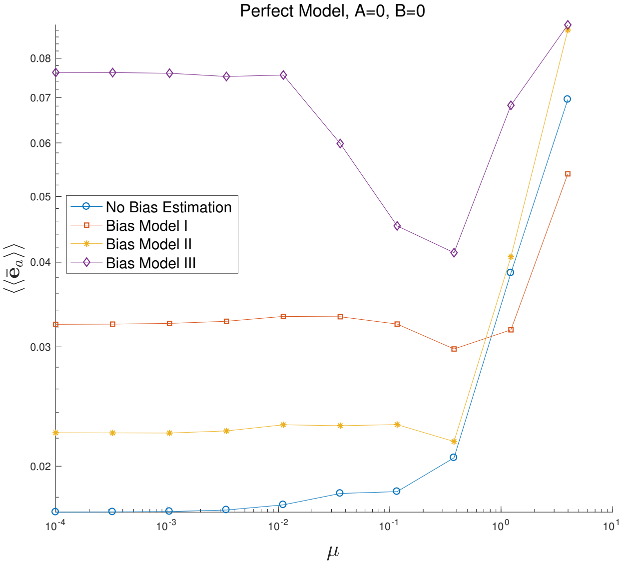
$$\boldsymbol{P}_n^a \to \boldsymbol{P}_n^a + \frac{\mu\Lambda}{K}\boldsymbol{I}_K$$

Time Step: $\Delta t = 0.05$
Initial Spread: 1.3
Ensemble Size: 40

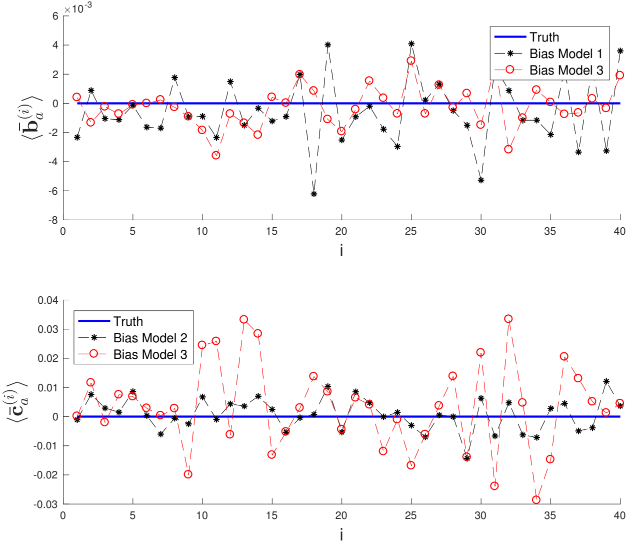
Perfect Situation: Settling Time for Error



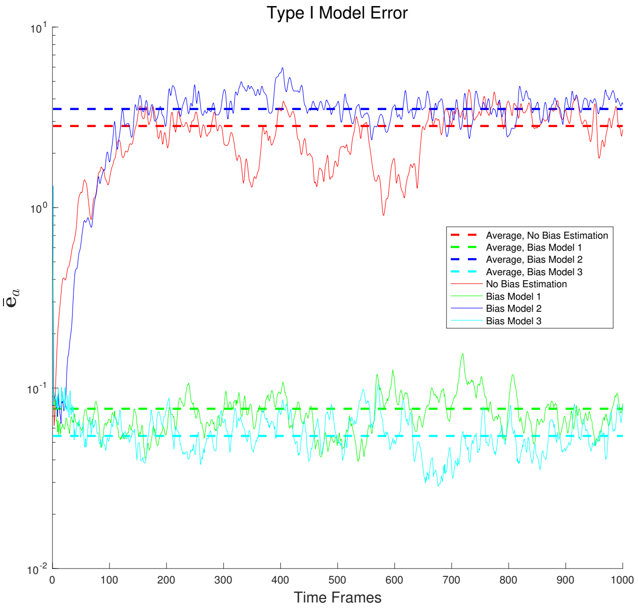
Perfect Situation: Performance and Inflation



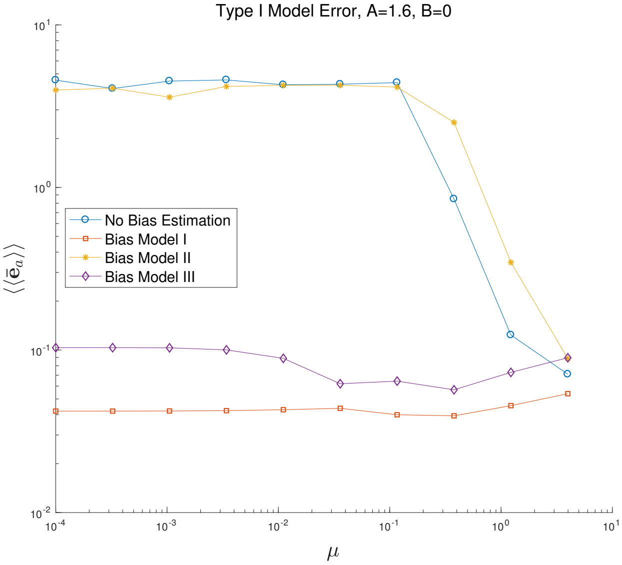
Perfect Situation: Bias Estimation



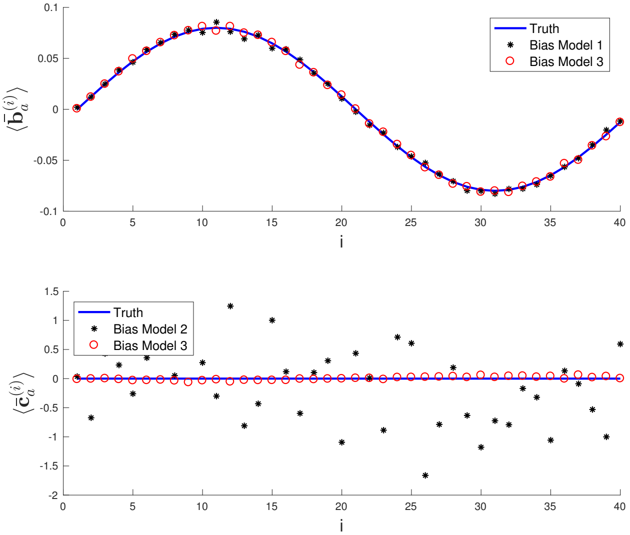
Type I Model Error: Settling Time for Error



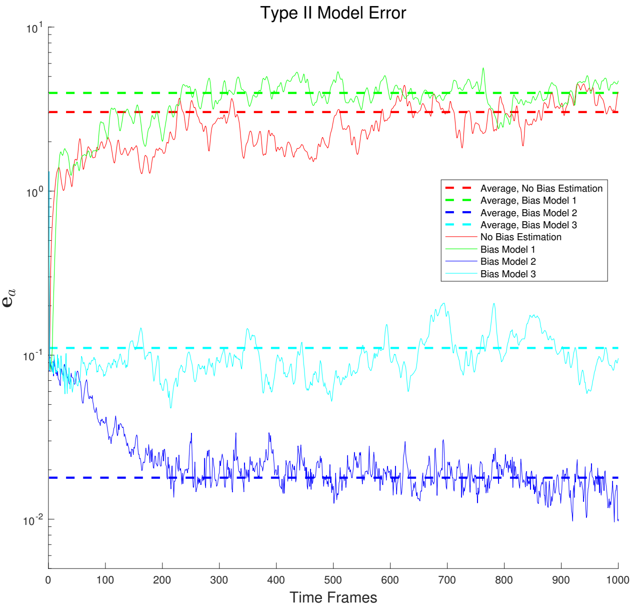
Type I Model Error: Performance and Inflation



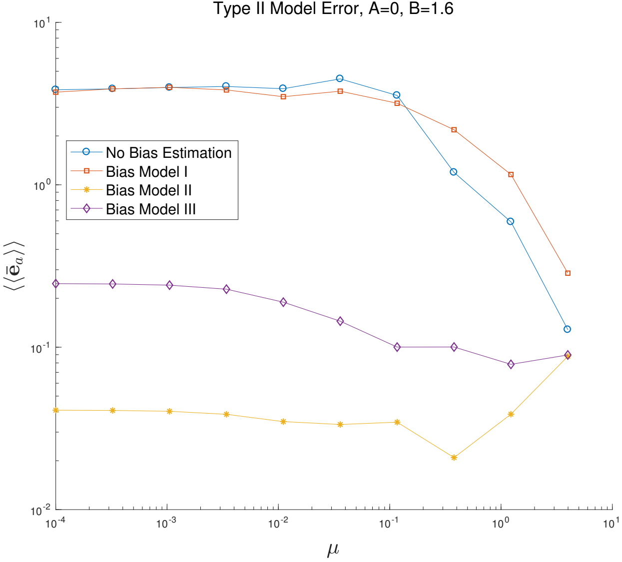
Type I Model Error: Bias Estimation



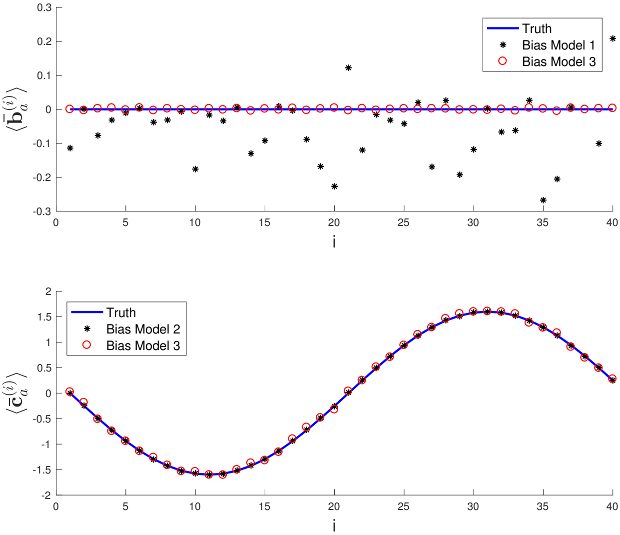
Type II Model Error: Settling Time for Error



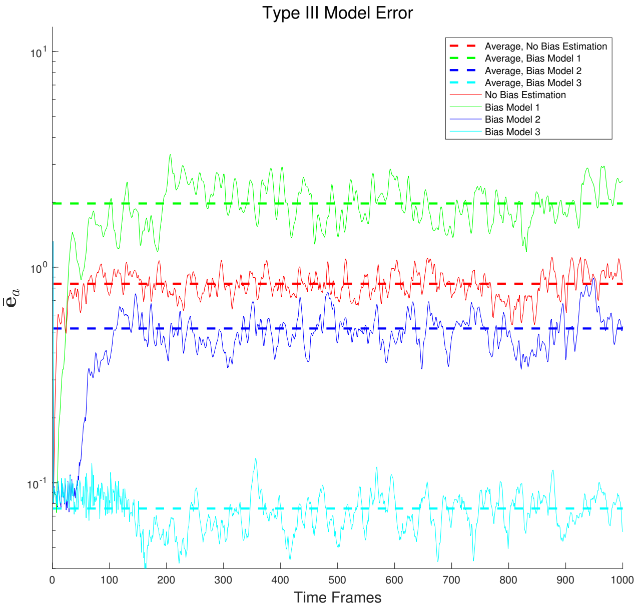
Type II Model Error: Performance and Inflation



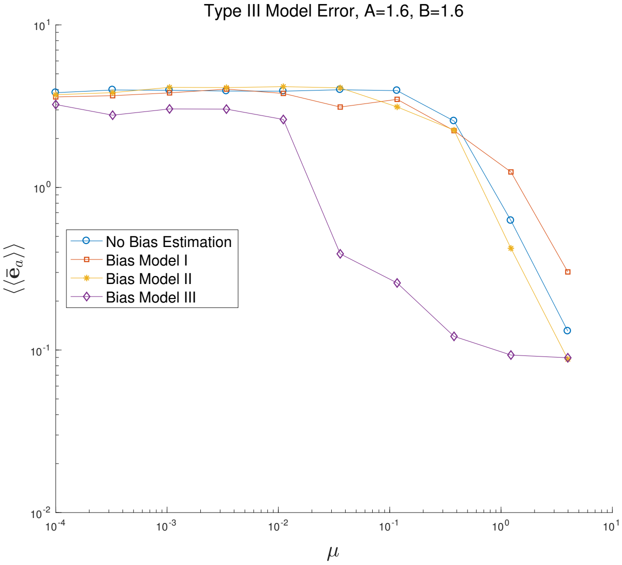
Type II Model Error: Bias Estimation



Type III Model Error: Settling Time for Error



Type III Model Error: Performance and Inflation



Type III Model Error: Bias Estimation