Data assimilation with and without a model

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Parameter estimation and UQ
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Tyrus Berry, Postdoc, GMU

Franz Hamilton, Postdoc, NC State

OUTLINE OF TALK

1. EnKF and applications
   - Parameters
   - Network edge detection
2. Problem: Don't know noise covariances
   - Adaptive filtering
3. Problem: Model error
   - Multimodel DA
4. Problem: Model unknown
   - Kalman-Takens filter

DATA ASSIMILATION

$$\begin{aligned} x_k &= f(x_{k-1}) + \eta_k \qquad & \eta_k \in N(0, Q) \\ y_k &= h(x_k) + \nu_k \qquad & \nu_k \in N(0, R) \end{aligned}$$

Main Problem. Given the model above plus observations y_k,

- **Filtering:** Estimate the current state $p(x_k \mid y_1, ..., y_k)$
- **Forecasting:** Estimate a future state $p(x_{k+\ell} \mid y_1, ..., y_k)$
- **Smoothing:** Estimate a past state $p(x_{k-\ell} \mid y_1, ..., y_k)$
- **Parameter estimation**

We'll apply the ensemble Kalman filter (EnKF) to attempt to achieve these goals.

Data assimilation

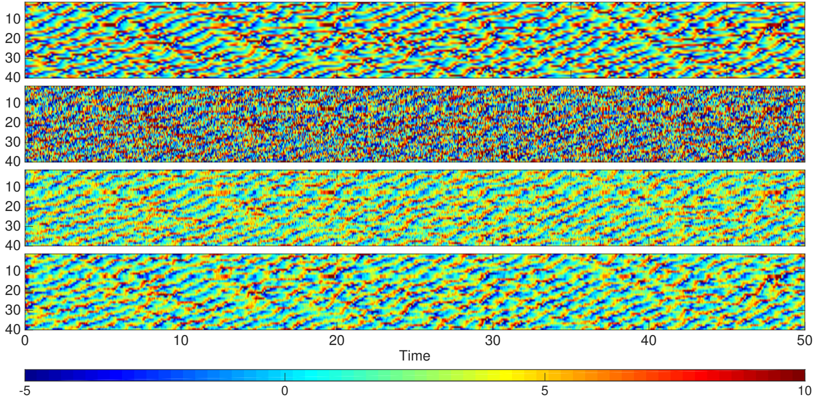
$$
\begin{aligned}
x_k &= f(x_{k-1}) + \eta_k & \eta_k \in N(0, Q) \\
y_k &= h(x_k) + \nu_k & \nu_k \in N(0, R)
\end{aligned}
$$

Possible obstructions.

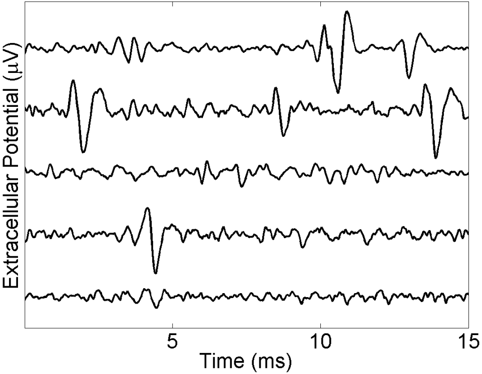
- The observations y_k mix system noise η_k with obs noise ν_k.
- Model error
  - Q and R may be unknown
  - Known model with unknown parameters
  - Wrong model, even with best fit parameters
  - Have model for some, not all of the variables

Example 1. Lorenz 96

$$\frac{dx^i}{dt} = -x^{i-2}x^{i-1} + x^{i-1}x^{i+1} - x^i + F$$



EXAMPLE 2. MEA RECORDINGS



Two Steps to Find $p(x_k \,|\, y_1, ..., y_k)$

- Assume we have $p(x_{k-1} \,|\, y_1, ..., y_{k-1})$
- **Forecast Step:** Find $p(x_k \,|\, y_1, ..., y_{k-1})$
- **Assimilation Step:** Perform a Bayesian update,

$$p(x_k \,|\, y_1, ..., y_k) \propto p(x_k \,|\, y_1, ..., y_{k-1}) p(y_k \,|\, x_k, y_1, ..., y_{k-1})$$

Posterior $\propto$ **Prior** $\times$ **Likelihood**

BEST POSSIBLE SCENARIO

$$\begin{aligned} x_k &= f(x_{k-1}) + \eta_k & \qquad \eta_k \in N(0, Q) \\ y_k &= h(x_k) + \nu_k & \qquad \nu_k \in N(0, R) \end{aligned}$$

f and h are linear, all parameters known.

$$\begin{aligned} x_k &= F_{k-1} x_{k-1} + \eta_k & \qquad \eta_k \in N(0, Q) \\ y_k &= H_k x_k + \nu_k & \qquad \nu_k \in N(0, R) \end{aligned}$$

Kalman Filter

- Assume linear dynamics/obs and additive Gaussian noise

$$
\begin{aligned}
x_k &= F_{k-1}x_{k-1} + \eta_k \qquad && \eta_k \sim \mathcal{N}(0, Q) \\
y_k &= H_k x_k + \nu_k && \nu_k \sim \mathcal{N}(0, R)
\end{aligned}
$$

- observability condition for linear systems:

$$
\tilde{H}_k^\ell = \begin{pmatrix} H_k \\ H_{k+1}F_k \\ \vdots \\ H_{k+\ell+1}F_{k+\ell}\cdots F_k \end{pmatrix}
$$

Must be full rank for some $\ell \Rightarrow$ KF guaranteed to work

Kalman Filter

- Assume linear dynamics/obs and additive Gaussian noise

$$
\begin{aligned}
x_k &= F_{k-1} x_{k-1} + \eta_k & \eta_k &\sim \mathcal{N}(0, Q) \\
y_k &= H_k x_k + \nu_k & \nu_k &\sim \mathcal{N}(0, R)
\end{aligned}
$$

- Assume current estimate is Gaussian:

$$p(x_{k-1} \mid y_1, \ldots, y_{k-1}) = \mathcal{N}(x^a_{k-1}, P^a_{k-1})$$

- **Forecast:** Linear combinations of Gaussians
  - **Prior:** $p(x_k \mid y_1, \ldots, y_{k-1}) = \mathcal{N}(x^f_k, P^f_k)$
    - $x^f_k = F_{k-1} x^a_{k-1}$
    - $P^f_k = F_{k-1} P_{k-1} F^\top_{k-1} + Q$
  - **Likelihood:** $p(y_k \mid x_k, y_1, \ldots, y_{k-1}) = \mathcal{N}(y^f_k, P^y_k)$
    - $y^f_k = H_k x^f_k$
    - $P^y_k = H_k P^f_k H^\top_k + R$

Kalman Filter

- Forecast: Linear combinations of Gaussians
  - Prior: $p(x_k \mid y_1, \ldots, y_{k-1}) = \mathcal{N}(x_k^f, P_k^f)$
    - $x_k^f = F_{k-1} x_{k-1}^a$
    - $P_k^f = F_{k-1} P_{k-1} F_{k-1}^\top + Q$
  - Likelihood: $p(y_k \mid x_k, y_1, \ldots, y_{k-1}) = \mathcal{N}(y_k^f, P_k^y)$
    - $y_k^f = H_k x_k^f$
    - $P_k^y = H_k P_k^f H_k^\top + R$
- **Assimilation:** Product of Gaussians (complete the square)

$$p(x_k \mid y_1, \ldots, y_k) = \mathcal{N}(x_k^f, P_k^f) \times \mathcal{N}(y_k^f, P_k^y) = \mathcal{N}(\color{blue}{x_k^a}, \color{red}{P_k^a})$$

- Define the **Kalman gain:** $K_k = P_k^f H_k^T (P_k^y)^{-1}$
  - $\color{blue}{x_k^a} = x_k^f + K_k(y_k - y_k^f)$
  - $\color{red}{P_k^a} = (I - K_k H_k) P_k^f$

Kalman Filter Summary

forecast

$$x_k^f = F_{k-1} x_{k-1}^a$$

covariance update

$$P_k^f = F_{k-1} P_{k-1}^a F_{k-1}^T + Q_{k-1}$$
$$P_k^y = H_k P_k^f H_k^T + R_{k-1}$$

Kalman gain

$$K_k = P_k^f H_k^T (P_k^y)^{-1}$$
$$\epsilon_k = y_k - y_k^f = y_k - H_k x_k^f$$

assimilation step

$$x_k^a = x_k^f + K_k \epsilon_k$$
$$P_k^a = (I - K_k H_k) P_k^f$$

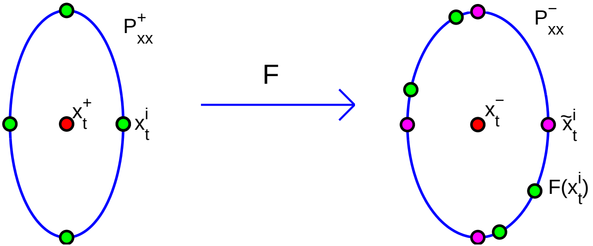
WHAT ABOUT NONLINEAR SYSTEMS?

- Consider a system of the form:

$$\begin{aligned} x_{k+1} &= f(x_k) + \eta_{k+1} & \eta_{k+1} \sim \mathcal{N}(0, Q) \\ y_{k+1} &= h(x_{k+1}) + \nu_{k+1} & \nu_{k+1} \sim \mathcal{N}(0, R) \end{aligned}$$

- More complicated observability condition (Lie derivatives)
- **Extended Kalman Filter (EKF)**:
  - Linearize $F_k = Df(\hat{x}_k^a)$ and $H_k = Dh(\hat{x}_k^f)$
- Problem: State estimate $\hat{x}_k^a$ may not be well localized
- Solution: Ensemble Kalman Filter (EnKF)

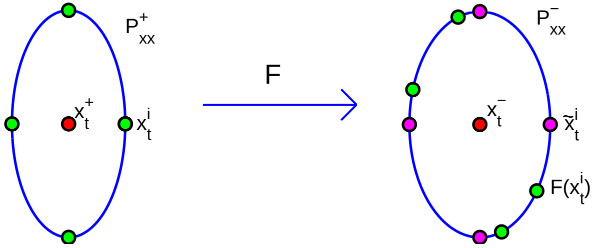
Ensemble Kalman Filter (EnKF)



Generate an ensemble with the current statistics (use matrix square root):

$$
\begin{aligned}
x_t^i &= \text{“sigma points” on semimajor axes} \\
x_t^f &= \frac{1}{2n}\sum F(x_t^i) \\
P_{xx}^f &= \frac{1}{2n-1}\sum (F(x_t^i) - x_t^f)(F(x_t^i) - x_t^f)^T + Q
\end{aligned}
$$

ENSEMBLE KALMAN FILTER (ENKF)



Calculate $y_t^i = H(F(x_t^i))$. Set $y_t^f = \frac{1}{2n}\sum_i y_t^i$.

$$
\begin{aligned}
P_{yy} &= (2n-1)^{-1}\sum(y_t^i - y_t^f)(y_t^i - y_t^f)^T + R \\
P_{xy} &= (2n-1)^{-1}\sum(F(x_t^i) - x_t^f)(y_t^i - y_t^f)^T \\
K &= P_{xy}P_{yy}^{-1} \text{ and } P_{xx}^a = P_{xx}^f - KP_{yy}K^T \\
x_{t+1}^a &= x_t^f + K(y_t - y_t^f)
\end{aligned}
$$

PARAMETER ESTIMATION

- When the model has parameters p,

$$x_{k+1} = f(x_k, p) + \eta_{k+1}$$

- Can *augment* the state $\tilde{x}_k = [x_k, p_k]$
- Introduce trivial dynamics for p

$$x_{k+1} = f(x_k, p_k) + \eta_{k+1}$$
$$p_{k+1} = p_k + \eta^p_{k+1}$$

- Need to tune the covariance of η^p_{k+1}

Example of Parameter Estimation

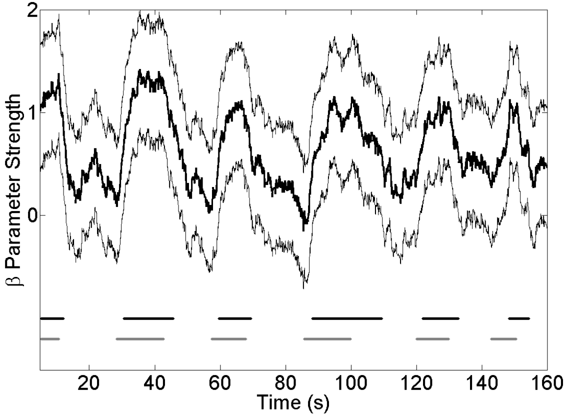
Consider the Hodgkin-Huxley neuron model, expanded to a network of n equations

$$
\begin{aligned}
\dot{V}_i &= -g_{Na} m^3 h (V_i - E_{Na}) - g_K n^4 (V_i - E_K) - g_L (V_i - E_L) \\
&\quad + I + \sum_{j \neq i}^{n} \Gamma_{\mathrm{HH}}(V_j) V_j \\
\dot{m}_i &= a_m(V_i)(1 - m_i) - b_m(V_i) m_i \\
\dot{h}_i &= a_h(V_i)(1 - h_i) - b_h(V_i) h_i \\
\dot{n}_i &= a_n(V_i)(1 - n_i) - b_n(V_i) n_i \\
\Gamma_{\mathrm{HH}}(V_j) &= \beta_{ij} / (1 + e^{-10(V_j + 40)})
\end{aligned}
$$

Only observe the voltages V_i, recover the hidden variables and the connection parameters β

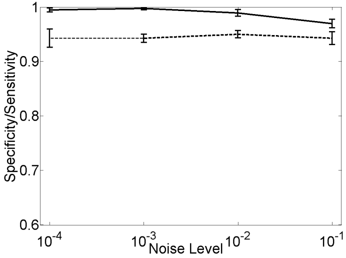
EXAMPLE OF PARAMETER ESTIMATION

Can even turn connections on and off (grey dashes)
Variance estimate ⇒ statistical test (black dashes)

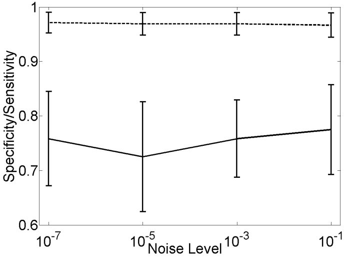


LINK DETECTION FROM NETWORKS OF MODEL NEURONS

Network of Hindmarsh-Rose neurons, modeled by Hindmarsh-Rose

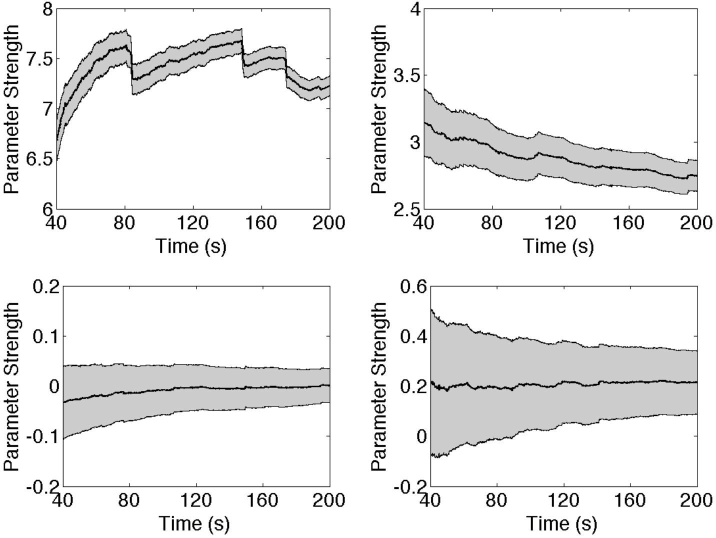


Network of Hodgkin-Huxley neurons, modeled by Hindmarsh-Rose

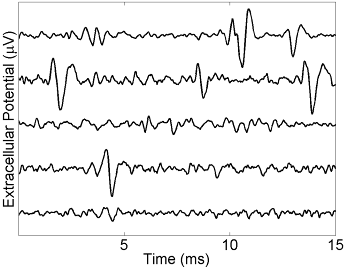


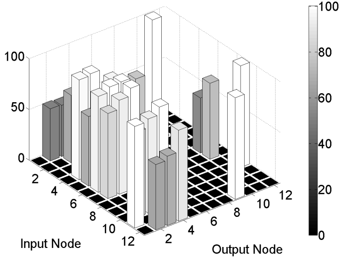
Sensitivity = true positive rate
Specificity = true negative rate

Link detection from MEA recordings



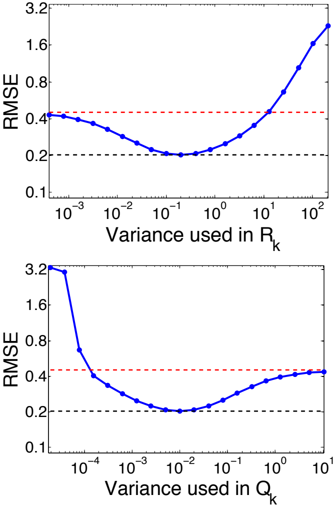
LINK DETECTION FROM MEA RECORDINGS





ENKF: WRONG Q AND R

- Simple example with full observation and diagonal noise covariances
- Red indicates RMSE of unfiltered observations
- Black is RMSE of 'optimal' filter (true covariances known)



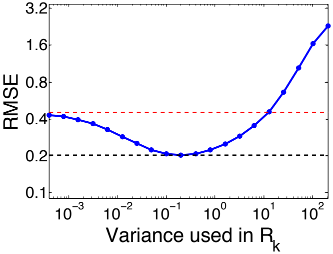
EnKF: Wrong Q and R

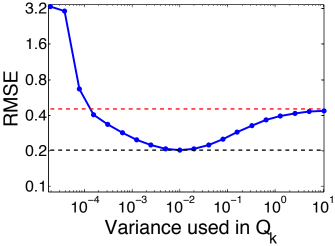
Standard Kalman Update:

$$
\begin{aligned}
P_k^f &= F_{k-1} P_{k-1}^a F_{k-1}^T + Q_{k-1} \\
P_k^y &= H_k P_k^f H_k^T + R_{k-1}
\end{aligned}
$$

$$
\begin{aligned}
K_k &= P_k^f H_k^T (P_k^y)^{-1} \\
P_k^a &= (I - K_k H_k) P_k^f
\end{aligned}
$$

$$
\begin{aligned}
\epsilon_k &= y_k - y_k^f = y_k - H_k x_k^f \\
x_k^a &= x_k^f + K_k \epsilon_k
\end{aligned}
$$





Adaptive Filter: Estimating Q and R

- Innovations contain information about Q and R

$$
\begin{aligned}
\epsilon_k &= y_k - y_k^f \\
&= h(x_k) + \nu_k - h(x_k^f) \\
&= h(f(x_{k-1}) + \eta_k) - h(f(x_{k-1}^a)) + \nu_k \\
&\approx H_k F_{k-1}(x_{k-1} - x_{k-1}^a) + H_k \eta_k + \nu_k
\end{aligned}
$$

- IDEA: Use innovations to produce samples of Q and R :

$$
\begin{aligned}
\mathbb{E}[\epsilon_k \epsilon_k^T] &\approx HP^f H^T + R \\
\mathbb{E}[\epsilon_{k+1} \epsilon_k^T] &\approx HFP^e H^T - HFK\mathbb{E}[\epsilon_k \epsilon_k^T] \\
P^e &\approx FP^a F^T + Q
\end{aligned}
$$

- In the linear case this is rigorous and was first solved by Mehra in 1970

ADAPTIVE FILTER: ESTIMATING Q AND R

- To find Q and R we estimate H_k and F_{k-1} from the ensemble and invert the equations:

$$\begin{aligned}\mathbb{E}[\epsilon_k \epsilon_k^T] &\approx HP^f H^T + R \\ \mathbb{E}[\epsilon_{k+1} \epsilon_k^T] &\approx HFP^e H^T - HFK\mathbb{E}[\epsilon_k \epsilon_k^T]\end{aligned}$$

- This gives the following *empirical* estimates of Q_k and R_k:

$$\begin{aligned}P_k^e &= (H_{k+1}F_k)^{-1}(\epsilon_{k+1}\epsilon_k^T + H_{k+1}F_k K_k \epsilon_k \epsilon_k^T)H_k^{-T} \\ Q_k^e &= P_k^e - F_{k-1}P_{k-1}^a F_{k-1}^T \\ R_k^e &= \epsilon_k \epsilon_k^T - H_k P_k^f H_k^T\end{aligned}$$

- P_k^e is an empirical estimate of the background covariance

Adaptive EnKF

We combine the estimates of Q and R with a moving average

Original Kalman Eqs.

$$
\begin{aligned}
P_k^f &= F_{k-1} P_{k-1}^a F_{k-1}^T + Q_{k-1} \\
P_k^y &= H_k P_k^f H_k^T + R_{k-1}
\end{aligned}
$$

$$
\begin{aligned}
K_k &= P_k^f H_k^T (P_k^y)^{-1} \\
P_k^a &= (I - K_k H_k) P_k^f
\end{aligned}
$$

$$
\begin{aligned}
\epsilon_k &= y_k - y_k^f \\
x_k^a &= x_k^f + K_k \epsilon_k
\end{aligned}
$$

Our Additional Update

$$
\begin{aligned}
P_{k-1}^e &= F_{k-1}^{-1} H_k^{-1} \epsilon_k \epsilon_{k-1}^T H_{k-1}^{-T} \\
&+ K_{k-1} \epsilon_{k-1} \epsilon_{k-1}^T H_{k-1}^{-T}
\end{aligned}
$$

$$
\begin{aligned}
Q_{k-1}^e &= P_{k-1}^e - F_{k-2} P_{k-2}^a F_{k-2}^T \\
R_{k-1}^e &= \epsilon_{k-1} \epsilon_{k-1}^T - H_{k-1} P_{k-1}^f H_{k-1}^T
\end{aligned}
$$

$$
\begin{aligned}
Q_k &= Q_{k-1} + (Q_{k-1}^e - Q_{k-1})/\tau \\
R_k &= R_{k-1} + (R_{k-1}^e - R_{k-1})/\tau
\end{aligned}
$$

Adaptive Filter: Application to Lorenz-96

- We will apply the adaptive EnKF to the 40-dimensional Lorenz96 model integrated over a time step $\Delta t = 0.05$

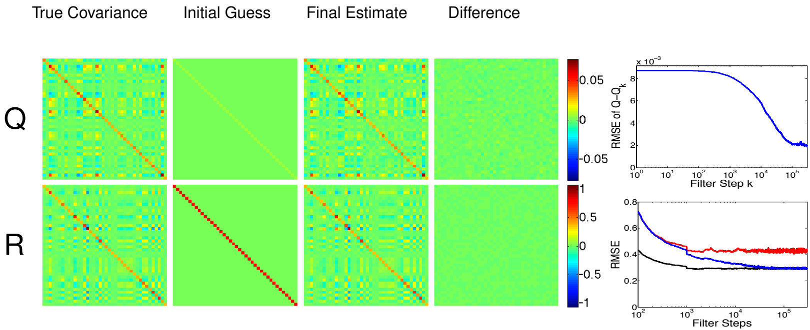
$$\frac{dx^i}{dt} = -x^{i-2}x^{i-1} + x^{i-1}x^{i+1} - x^i + F$$

- We augment the model with Gaussian white noise

$$\begin{aligned} x_k &= f(x_{k-1}) + \eta_k & \qquad \eta_k = \mathcal{N}(0, Q) \\ y_k &= h(x_k) + \nu_k & \qquad \nu_k = \mathcal{N}(0, R) \end{aligned}$$

- The Adaptive EnKF uses $F = 8$
- We will consider model error where the true $F^i = \mathcal{N}(8, 16)$

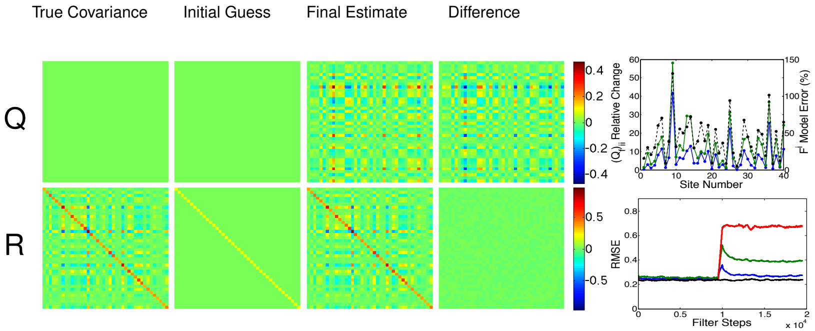
RECOVERING Q AND R, PERFECT MODEL



RMSE (bottom right) for the initial guess covariances (red) the true Q and R (black) and the adaptive filter (blue)

Compensating for Model Error

The adaptive filter compensates for errors in the forcing F^i



RMSE (bottom right) for the initial guess covariances (red) the perfect model (black) and the adaptive filter (blue)

Estimating unmodeled dynamics

Multiple models approach: Run m subfilters in parallel

$$\begin{aligned} \dot{w} &= F(w) + \eta_t \\ \begin{bmatrix} y \\ \vdots \\ y \\ S \end{bmatrix} &= H(w) + \nu_t \end{aligned}$$

where

$$w = \begin{bmatrix} x^1 \\ \vdots \\ x^m \\ c^1 \\ \vdots \\ c^m \\ d \end{bmatrix}, \; F = \begin{bmatrix} f(x, p_1) \\ \vdots \\ f(x, p_m) \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \; H(w) = \begin{bmatrix} h(x^1) \\ \vdots \\ h(x^m) \\ \sum_{i,j} c^i_j x^i_j + d \end{bmatrix}$$

Learn c^i_j and d from training data S, then can predict unmodeled S.

Selecting the Assimilation Model

- Assume a generic spiking neuron model (Hindmarsh-Rose)

$$
\begin{aligned}
\dot{\mathbf{V}} &= \mathbf{y} - a\mathbf{V}^3 + b\mathbf{V}^2 - \mathbf{z} + I \\
\dot{\mathbf{y}} &= c - d\mathbf{V}^2 - \mathbf{y} \\
\dot{\mathbf{z}} &= \tau(s(\mathbf{V}+1) - \mathbf{z})
\end{aligned}
$$

V neuron potential, **y** fast scale dynamics, **z** slow scale dynamics

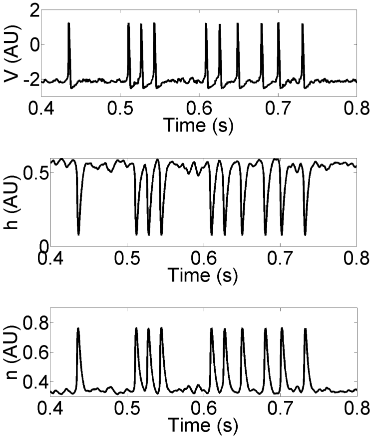
- **Goal:** Use multiple parameterized Hindmarsh-Rose models to reconstruct unmodeled neuronal quantities while assimilating recorded neuron potential

J. Hindmarsh and R. Rose, Proc. Roy. Soc., **221**, 87 (1984).

Reconstructing Unmodeled Gating Variables

Observing Hodgkin-Huxley voltage, reconstruct unmodeled gating variables (assimilation model is Hindmarsh-Rose)

A. Hodgkin and A. Huxley, J. Physiology **117**, 500 (1952).



Actual

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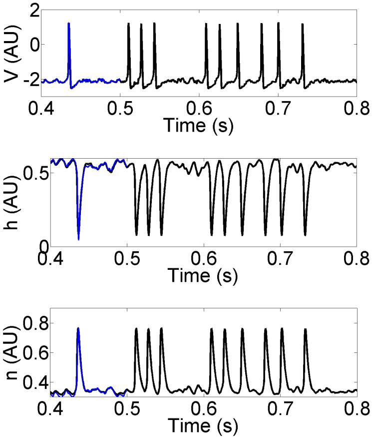

Actual **Observed**

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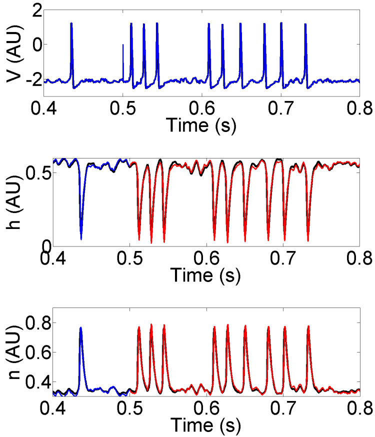
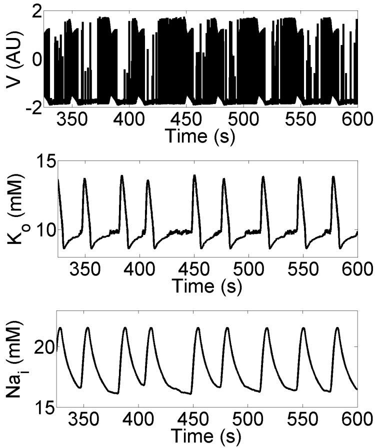


Actual **Observed** **Predicted**

Reconstructing Unmodeled Ionic Dynamics

Observing seizure voltage, reconstruct unmodeled potassium and sodium dynamics (assimilation model is Hindmarsh-Rose)

J. Cressman, G. Ullah, J. Ziburkus, S. Schiff, and E. Barreto, Journal of Comp. Neuroscience **26**, 159 (2009).



Actual

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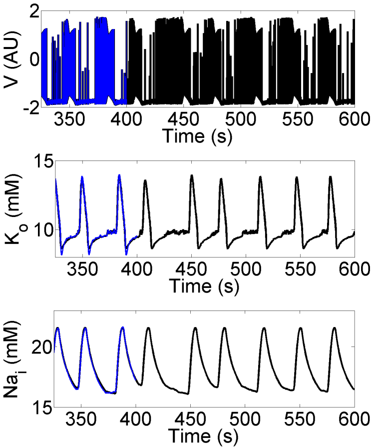
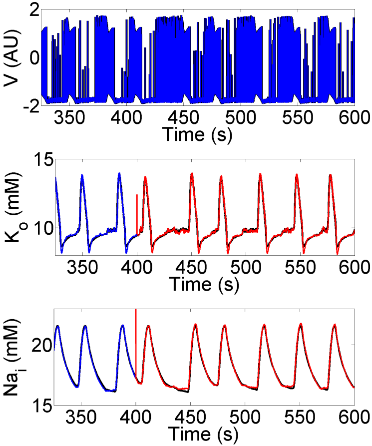


Actual **Observed**

Reconstructing Unmodeled Ionic Dynamics

Observing seizure voltage, reconstruct unmodeled potassium and sodium dynamics (assimilation model is Hindmarsh-Rose)

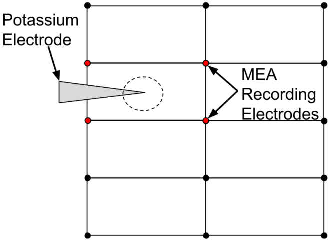
J. Cressman, G. Ullah, J. Ziburkus, S. Schiff, and E. Barreto, Journal of Comp. Neuroscience **26**, 159 (2009).



Actual **Observed** **Predicted**

RECONSTRUCTING EXTRACELLULAR POTASSIUM FROM AN *In Vitro* NETWORK

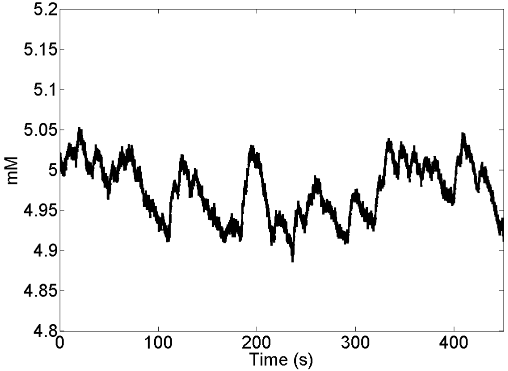
We want to track extracellular potassium dynamics in a network but measurements are difficult and spatially limited



Extracellular potassium is an **unmodeled variable**

Reconstructing Extracellular Potassium from an *In Vitro* Network

The local extracellular potassium in an MEA network can be reconstructed and predicted using our approach (assimilation model is Hindmarsh-Rose)



Actual

Reconstructing Extracellular Potassium from an *In Vitro* Network

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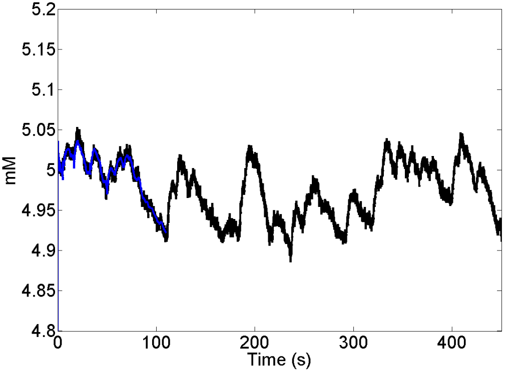
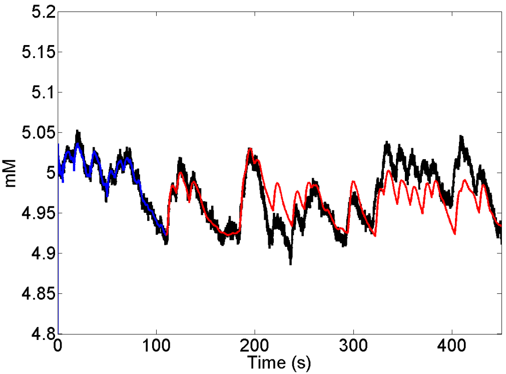


Actual **Observed**

Reconstructing Extracellular Potassium from an *In Vitro* Network

The local extracellular potassium in an MEA network can be reconstructed and predicted using our approach (assimilation model is Hindmarsh-Rose)



Actual **Observed** **Predicted**

Kalman-Takens Filter: Throwing out the model...

- Starting with historical observations $\{y_0, ..., y_n\}$
- Form Takens delay-embedding state vectors $x_i = (y_i, y_{i-1}, ..., y_{i-d})^\top$
- Build an EnKF:
  - Apply analog forecast to each ensemble member
  - Use the observation function $Hx_i = y_i$
  - Crucial to estimate Q and R

No model: Kalman-Takens filter

Reasons why this might not work:

- Nonparametric methods that depend on local reconstructions tend to fail in presence of observation noise
  - Takens' Theorem assumes no obs noise
  - Neighbors in reconstruction space are wrong
- Kalman filters depend on knowledge of model to advance dynamics
  - Filter equations need knowledge of dynamics to advance "background"

Ensemble Kalman Filtering (EnKF)

At kth step:

- Form an ensemble of state vectors centered at best guess x_{k-1}^{+}, with spread given by estimated covariance P_{k-1}^{+}
- Model f is applied to this ensemble and observed with function h. Form the *prior* state estimate x_k^- and the predicted observation y_k^-
- Construct the resulting covariance matrices P_k^{xy} and P_k^{y}
- Update the state and covariance estimates with the observation y_k

$$
\begin{aligned}
K_k &= P_k^{xy}(P_k^{y})^{-1} \\
P_k^{+} &= P_k^{-} - P_k^{xy}(P_k^{y})^{-1}P_k^{yx} \\
x_k^{+} &= x_k^{-} + K_k\left(y_k - y_k^{-}\right).
\end{aligned}
$$

Ensemble Kalman Filtering (EnKF)

At kth step:

- Form an ensemble of state vectors centered at best guess x_{k-1}^+, with spread given by estimated covariance P_{k-1}^+
- Model f is applied to this ensemble and observed with function h. Form the *prior* state estimate x_k^- and the predicted observation y_k^-
- Construct the resulting covariance matrices P_k^{xy} and P_k^y
- Update the state and covariance estimates with the observation y_k

$$
\begin{aligned}
K_k &= P_k^{xy}(P_k^y)^{-1} \\
P_k^+ &= P_k^- - P_k^{xy}(P_k^y)^{-1}P_k^{yx} \\
x_k^+ &= x_k^- + K_k\left(y_k - y_k^-\right).
\end{aligned}
$$

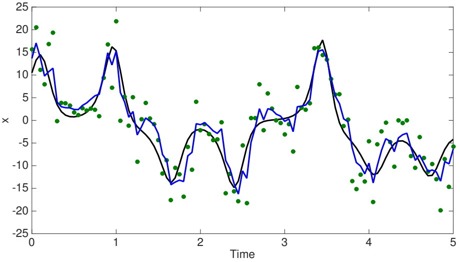
Kalman-Takens filter

At kth step:

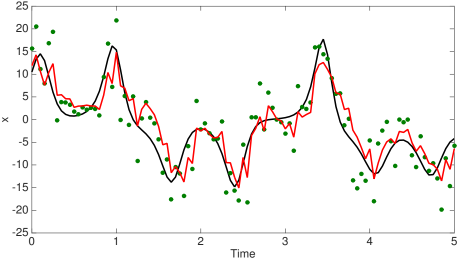
- Form ensemble of reconstructed states centered at delay-coordinate vector x_{k-1}^{+}, with spread given by estimated covariance P_{k-1}^{+}
- Apply nonparametric prediction and observe with function h. Estimate *prior* state x_k^{-} and observation y_k^{-}
- Construct the resulting covariance matrices P_k^{xy} and P_k^{y}
- Update the state and covariance estimates with the observation y_k

$$\begin{aligned}
K_k &= P_k^{xy}(P_k^{y})^{-1} \\
P_k^{+} &= P_k^{-} - P_k^{xy}(P_k^{y})^{-1}P_k^{yx} \\
x_k^{+} &= x_k^{-} + K_k\left(y_k - y_k^{-}\right).
\end{aligned}$$

Kalman-Takens filter: Lorenz 63



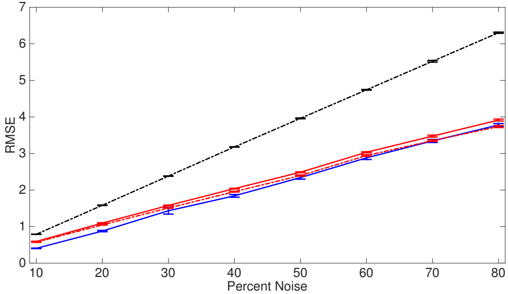
full model



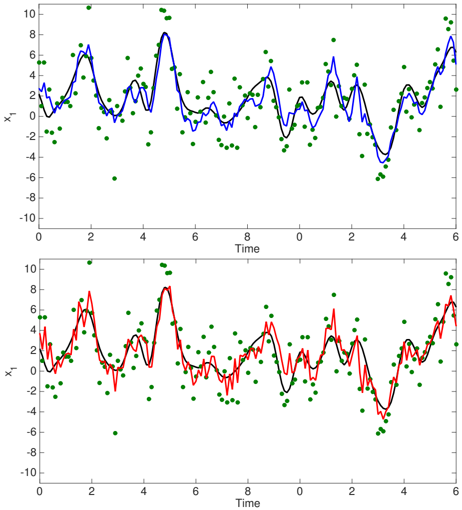
Kalman-Takens

KALMAN-TAKENS FILTER

Comparison of Kalman-Takens (red) with full model (blue)



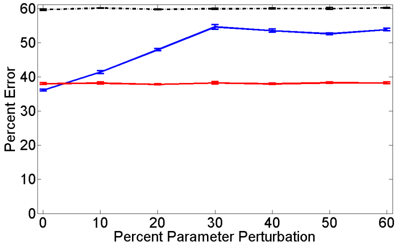
KALMAN-TAKENS FILTER: LORENZ 96



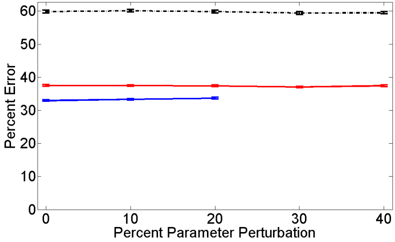
full model

Kalman-Takens

Kalman-Takens filter: Model error

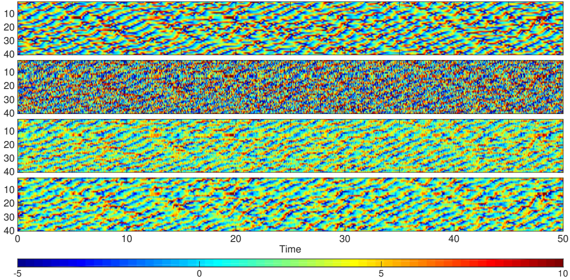


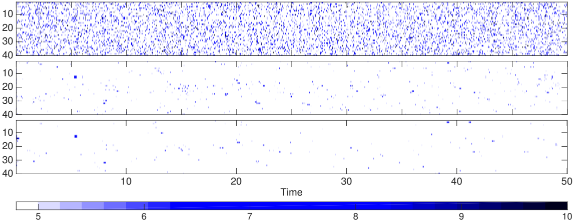
Lorenz 63



Lorenz 96

Kalman-Takens filter: Lorenz 96



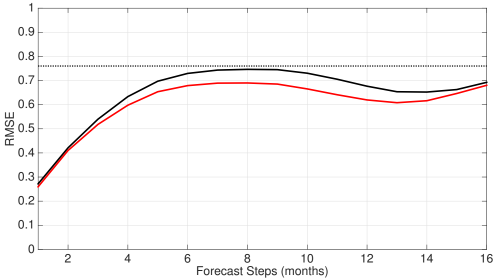


Kalman-Takens filter: Forecast error

El Nino index

Training data: Jan. 1950 - Dec. 1999.

Forecasts after 2000



SUMMARY

- EnKF is a useful data assimilation technique for neurodynamics and other types of data
- Parameter estimation
- Adaptive QR is helpful when Q and R are unknown
- Difficulties
  - Model error
  - Unmodeled variables
  - No model
- Multimodel data assimilation
- Kalman-Takens filter

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- F. Hamilton, T. Berry, T. Sauer, Ensemble Kalman filtering without a model. Phys. Rev. X 6, 011021 (2016).
- F. Hamilton, T. Berry, T. Sauer, Kalman-Takens filtering in the presence of dynamical noise. To appear, Eur. Phys. J.