Homework for Lecture 6

1. For the Coppel example, derive the fundamental matrix solution. Hint: Consider the **change of variables** $x = U^T(t)y$. Generalize the example and find fundamental matrix solutions for parameters $\alpha$ and $\omega$ where

$$A_0 = \begin{pmatrix} -1 & -\alpha \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad U(t) = \begin{pmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{pmatrix}.$$

   and determine the values of $\alpha$ and $\omega$ for which the zero solution is unstable.

2. Prove the Lyapunov Stability Theorem for the periodic Hill equation. For a proof, see

   `http://www.ams.org/journals/proc/1963-014-03/`
   `S0002-9939-1963-0149019-6/S0002-9939-1963-0149019-6.pdf`

3. Use your modification of `predcorr` with pseudo arc length continuation for equilbrium solutions of the Lorenz '63 model. Fix the parameters $a$ and $c$ and following paths of equilibria as you vary the parameter $b$ and detect potential bifurcation points.

4. In the paper

   `https://arxiv.org/abs/1712.05730v1`

   the bifurcation behavior of equilibrium solutions is investigated for the Lorenz '96 primarily for $F < 0$. Starting from the equilibrium solution $(F, ..., F)$ use your modification of `predcorr` with pseudo arc length continuation for equilbrium solutions to follow this path of equilibrium solutions starting from $F_0 = -1/4$ by decreasing $F$ in dimensions $N = 4$ and $N = 16$. Detect the first pitchfork bifurcation and continue on one or more of the three branches.