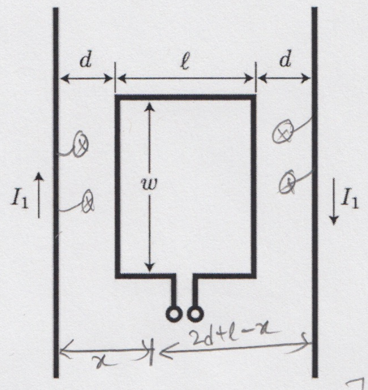
1. Two long parallel wires are carry current as shown below. A rectangular loop is placed symmetrically in between the wires in the plane of the wires. The inner loop is at a distance d = 3 mm from the wires. The loop has a length = 10 mm and a width of w = 15 mm

(a) Calculate the magnetic field in the space between the wires as a function of the current in the parallel lines.

(b) Calculate the magnetic flux crossing the inner loop.

(c) Draw the voltage pickup (EMF) in the inner loop if the wires carry the current waveforms shown below.



(a) $B(x) = \frac{\mu I_1}{2\pi x} + \frac{\mu I_1}{2\pi(2d+\ell-x)}$

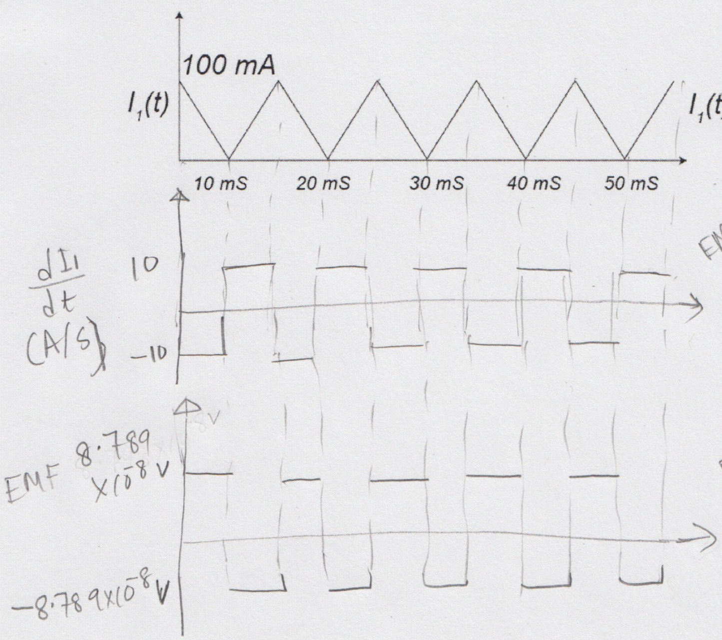
(b) $\phi = \int B\,dA = \int B\,dx\,dy = \int B\,\omega\,dx$

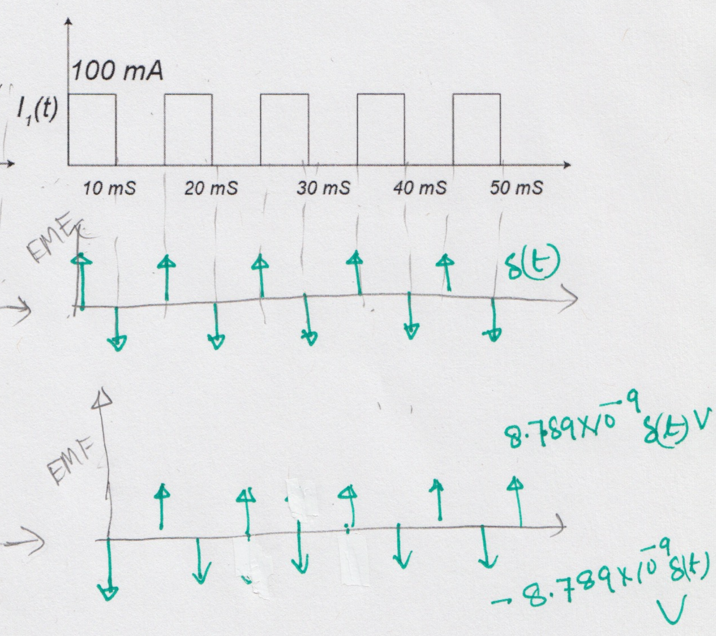
$= \frac{\mu I_1 \omega}{2\pi}\int_{x=d}^{d+\ell}\left(\frac{1}{x}+\frac{1}{2d+\ell-x}\right)dx = \frac{\mu I_1 \omega}{2\pi}\left[\ln x - \ln(2d+\ell-x)\right]_d^{d+\ell}$

$= \frac{\mu I_1 \omega}{2\pi}\left[\ln\frac{x}{2d+\ell-x}\right]_d^{d+\ell} = \frac{\mu I_1 \omega}{2\pi}\left(\ln\left(\frac{d+\ell}{\ell}\right) - \ln\left(\frac{d}{d+\ell}\right)\right)$

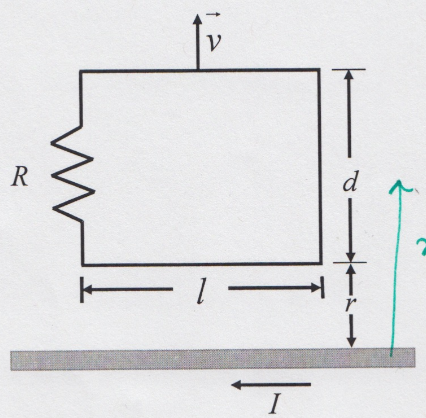
$= \frac{\mu I_1 \omega}{2\pi}\ln\left(\frac{d+\ell}{d}\right)^2 = 8.798\times10^{-9}\, I_1(t)$ Wb/A

(c) $\text{EMF} = -\frac{d\phi}{dt} = -8.798\times10^{-9}\,\frac{dI_1}{dt}$





2. A rectangular loop of dimensions l and d moves with a constant velocity $\vec{v}$ away from an infinitely long straight wire that lies in the plane of the loop and is carrying a current I in the direction shown in figure . Let the total resistance of the loop be R . What is the current in the loop at the instant the near side is a distance r from the wire?



$$B = \frac{\mu_0 I}{2\pi x}$$

$$d\phi = \vec{B} \cdot d\vec{A} = \frac{\mu_0 I}{2\pi x} \cdot l\, dx$$

$$\phi = \int d\phi = \frac{\mu_0 I l}{2\pi} \int_r^{r+d} \frac{dx}{x} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{r+d}{r}\right)$$

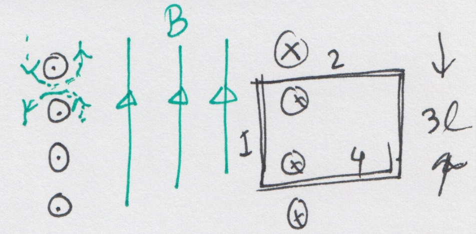
$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{\mu_0 I l}{2\pi} \frac{d}{dt}\left(\ln \frac{r+d}{r}\right) = -\frac{\mu_0 I l}{2\pi}\left(\frac{1}{r+d} - \frac{1}{r}\right)\frac{dr}{dt}$$

$$= \frac{\mu_0 I l}{2\pi} \frac{d}{r(r+d)} v = \frac{\mu_0 I l d v}{2\pi (r+d) r}$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{\mu_0 I l d v}{2\pi r (r+d)}$$

3. (a) Calculate the magnetic field inside a long solenoid with radius a, turns per unit length N_a, and current I.

(b) Inside the long solenoid and parallel to it, there is a short solenoid with radius b, turns per unit length N_b, and length l. What is the mutual inductance of the two solenoids?



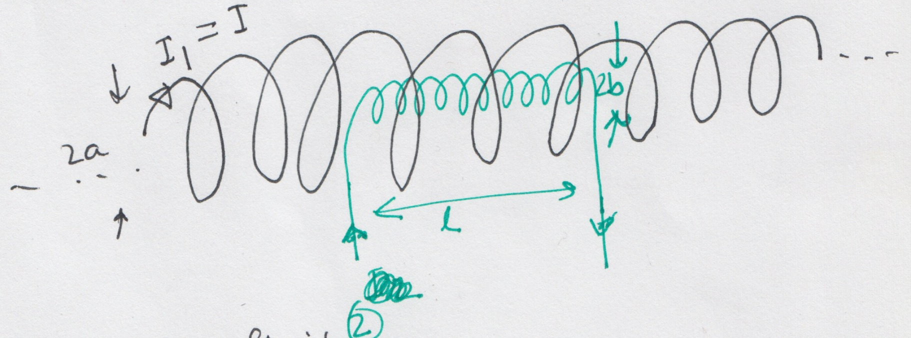
$$\int \bar{B}\cdot\bar{ds} = \int_{1,2,3,4} B\cdot ds = \int_1 B\cdot ds + \int_{2,3,4} B\cdot ds$$

$$= Bl + 0$$

now $\int \bar{B}\cdot\bar{ds} = Bl = \mu_0 I_{enc}$

$$\Rightarrow \int \bar{B}\cdot\bar{ds} = Bl = \mu_0 N_a l I$$

$$\Rightarrow \boxed{B = \mu_0 N_a I}$$



flux per unit turn of short solenoid (2)

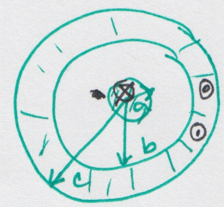
$$\phi_{21} = B_1 A_2 = \mu_0 N_a I \cdot \pi b^2$$

Mutual inductance, $M_{21} = \frac{N_2 \phi_{21}}{I_1} = \frac{N_b l \, \phi_{21}}{I}$

$$\Rightarrow \boxed{M_{21} = \mu_0 N_a N_b l \pi b^2}$$

4. Conductors of a coaxial cable are made from copper, and its dielectric is air. The radius of the inner conductor is a, whereas the inner and outer radii of the outer conductor are b and c, respectively ($a<b<c$). A steady current of intensity I established in the cable. Find the magnetic flux density vector everywhere.

$$B(r) = \frac{\mu_0}{2\pi r} \cdot \frac{I \pi r^2}{\pi a^2} = \frac{\mu_0 I r}{2\pi a^2} \quad (0<r<a)$$



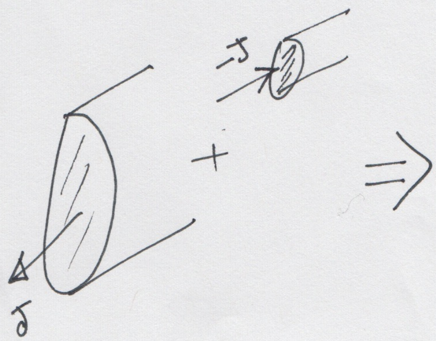
$$B(r) = \frac{\mu_0 I}{2\pi r} \quad ; \quad (a<r<b)$$

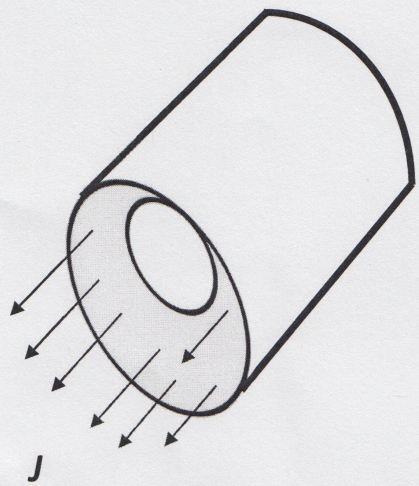
$$B(r) = -\frac{\mu_0 I (r^2 - b^2)}{2\pi (c^2 - b^2) r} + \frac{\mu_0 I}{2\pi r} \quad ; \quad (b<r<c)$$

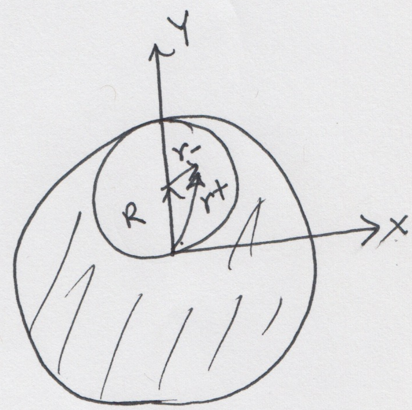
$$B(r) = 0 \quad ; \quad (r>c)$$

5. An infinite cylindrical wire with radius 2R carries a uniform current density **J**, except inside an infinite cylindrical hole parallel to the wire's axis. The hole has radius R and is tangent to the exterior of the wire. A short chunk of the wire is shown in the accompanying figure.

Calculate the magnetic field <u>everywhere inside the hole</u>, and sketch the lines of **B** on the figure.





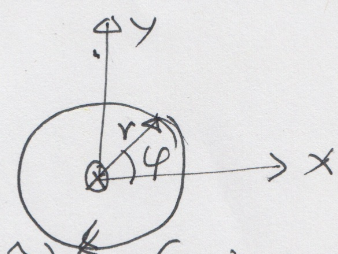


$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$\Rightarrow B \cdot 2\pi r = \mu_0 J \pi r^2$$

$$\Rightarrow \vec{B} = \frac{\mu_0 J r}{2} \hat{a}_\varphi = \frac{\mu_0 J r}{2}(-\sin\varphi\, \hat{a}_x + \cos\varphi\, \hat{a}_y)$$

$$\Rightarrow \vec{B} = \frac{\mu_0 J}{2}(-y\,\hat{a}_x + x\,\hat{a}_y)$$



$$\begin{bmatrix} r\sin\varphi = y \\ r\cos\varphi = x \end{bmatrix}$$

now, $\vec{B} = \vec{B}_+ + \vec{B}_-$

$$= \frac{\mu_0 J}{2}(-y_+\hat{a}_x + x_+\hat{a}_y) - \frac{\mu_0 J}{2}(-y_-\hat{a}_x + x_-\hat{a}_y)$$

$$= \frac{\mu_0 J}{2}(-y_+\hat{a}_x + y_-\hat{a}_x + x_+\hat{a}_y - x_-\hat{a}_y)$$

$$= \frac{\mu_0 J}{2}\left[-y\,\hat{a}_x + (y-R)\hat{a}_x + x\hat{a}_y - x\hat{a}_y\right]$$

$$x_+ = x_- = x$$
$$y_+ = y_- + R = y$$
origin transf

$$\Rightarrow \vec{B} = -\frac{\mu_0 J R}{2}\hat{a}_x$$