NP-completeness of set-partition problem

i) It is NP because we can guess two partitions & verify in polynomial time if they are equal.

ii) Reduction of SS to SP:

- Suppose given set in SS is S_1
- " " target in SS is t
- Now consider the set in SP,

$$S_2 = S_1 \cup \{\text{sum of elements of } S_1 - 2t\}$$

iii) Sum calculation works in polynomial time. So, this reduction works in " time.

iv) SP has sol^n if and only if SS has sol^n

1. $$S_1 \to \underbrace{\{\underbrace{\_,\ \_,}_{t}\ \underbrace{\dots,\ \_}_{sum-t}\}}_{sum}$$

$$S_2 \to \{\underbrace{\_,\ \_,}_{t}\ \underbrace{\dots,\ \_}_{\cancel{S-t}\ \ sum-t},\ sum-2t\}$$

$$t + (sum-2t) = sum-t$$

2. $$S_2 \to \{\underbrace{\_,\ \_,\ \dots,}_{\Sigma \text{ elements of } S_1 = sum}\ sum-2t\}$$

$$2 \cdot sum - 2t$$

If sol^n exists, each partitioned set's sum $= \frac{1}{2}(2 \cdot sum - 2t) = sum - t$

$$\underbrace{\{\_,\ \_,\ \_,\ \dots\}}_{sum-t} \qquad \underbrace{\{\_,\ \_,\ \dots,\ sum-2t\}}_{sum-t}$$

$$\Sigma \to sum - t - (sum-2t) = t$$

$\therefore S_1$ has some elements whose sum is t.