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~~5A~~.

Let SP be the given problem and H be the hamiltonian path checking problem.

$\boxed{1}$ SP is in NP.

~~For example~~ When SP returns yes, as a certifier it returns a path of length $\geq k$ ~~using~~ which we can easily check using input Graph G in polynomial time.

$\boxed{2}$ we will reduce H to SP.

$\boxed{3}$ Given input G for H we convert it to input $(G', n+1)$ for SP. Here n= number of vertices in G and G' is a extended graph of G. To make G' from G, we add two vertices s and t and join all old vertices to both s and t. and then ask SP, whether there is a n+1 length path in G' from s to t. [length = number of edges.

[4] if SP returns yes then such a path uses all nodes [both old and new s,t] and thus the path $- \{s, t\}$ is a hamiltonian path.

If SP returns no, then no n+1 length path exists and so, a hamiltonian path doesn't exist. As such a hamiltonian path could be extended to add s and t to get n+1 length path in G' so result of SP = result of H.

Therefore SP is NP-complete.