1. Definition of grid score for spatial coverage

A spatial grid i may be covered by multiple trips $N_i(t)$ within a given time step t (e.g. 1 hour). The score $r\big(N_i(t)\big)$ for such sensing coverage should satisfy the following conditions:

C1. $r_i(\cdot)$, as a function of number of trips $N_i(t)$, should be defined for the set of non-negative real numbers $\mathbb{R}_+$[1].

C2. The grid score should be monotonically increasing but with decreasing marginal benefit as $N_i(t)$ increases; i.e. $r' > 0$, $r'' < 0$.

To fix the idea, we begin with integer numbers $n \in \mathbb{Z}_+$, and define

$$r_i\big(N_i(t)\big) = \sum_{n=1}^{N_i(t)} \frac{1}{n^\alpha} \qquad \alpha \in (0,1), \tag{1}$$

which clearly satisfies C2. The summation in Eqn. (1) can be interpreted as the green curve in Figure 1. The blue one corresponds to (REFERENCE) where only a score of 0 or 1 can be awarded for a grid at a time, and the red one represents linear award. The blue one does not reasonably reflect air quality sensing requirements as more samples help to improve not only spatial inference results (REFERENCE), but also identify potential pollution hotspots. The linear reward (red), on the other hand, may encourage concentrations on one or several grids instead of coverage to a larger spatial extent.

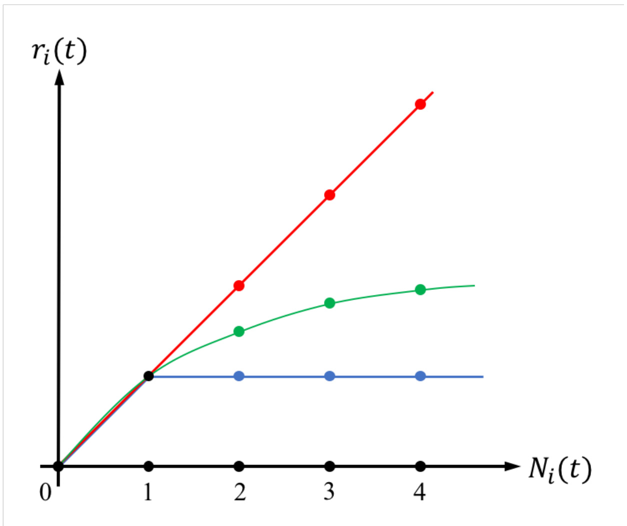

Figure 1: Illustration of the reward function.

To extend Eqn. (1) to the set of real numbers, we invoke the finite discretization $0 = n_1 < n_2 < \ldots < n_m = N_i(t)$ with step size δn:

$$r_i\big(N_i(t)\big) = \sum_{m=1}^{N_i(t)/\delta n} \frac{\delta n}{n_m^\alpha} \approx \int_0^{N_i(t)} \frac{1}{x^\alpha} dx = \frac{1}{1-\alpha} N_i(t)^{1-\alpha}$$

Therefore, we formally define the grid coverage score to be $r_i\big(N_i(t)\big) = N_i(t)^\beta$, $\beta \in (0,1)$, which satisfies both C1 and C2.

[1] The number of trips $N_i(t)$ is extended beyond integers because statistical average of $N_i(t)$ over multiple days will be later used in the algorithm.