5+5+3
=> 13/20

Exercise7

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1 Homework 7:Stability Analysis (20 points)

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[5]:
```
#Load standard libraries
import numpy as np
from scipy.sparse import diags
import matplotlib.pyplot as plt
import scipy.linalg as la
%matplotlib inline
```

In this exercise we study the evolution of 6 populations according to the following equations for population dynamics: 3 predator- (Pi) and 3 prey-species (Ni), all parameters positive, always i, j = 1, . . . , 3:

$$\frac{dN_i}{dt} = N_i \left(a_i - N_i - \sum_j b_{ij} P_j \right) \tag{1}$$

$$\frac{dP_i}{dt} = P_i \left(\sum_j c_{ij} N_j - d_i \right) \tag{2}$$

The parameters chosen are $a_1 = 56, a_2 = 12, a_3 = 35; d_1 = 85, d_2 = 9, d_3 = 35$; the parameters b_{ij} and c_{ij} are given in matrix form here:

$$b_{ij} = \begin{pmatrix} 20 & 30 & 5 \\ 1 & 3 & 7 \\ 4 & 10 & 20 \end{pmatrix}$$

$$c_{ij} = \begin{pmatrix} 20 & 30 & 35 \\ 3 & 3 & 3 \\ 7 & 8 & 20 \end{pmatrix}$$

Notice: the unusual feature here in the equations is that the prey populations Ni have a Verhulst style growth limiting factor in their equations, which limits their growth even if there is no predator (model for limited resources even in absence of predators). Another Notice: please do not try to make the equations dimensionless, just use the numbers given here.

1.0.1 1. What are the fixed points for the system of equations given above?

1. FP: $N_1 = 0; N_2 = 0; N_3 = 0; P_1 = 0; P_2 = 0; P_3 = 0$
2. FP: $N_1 = 1; N_2 = 1; N_3 = 1; P_1 = 1; P_2 = 1; P_3 = 1$
3. FP: $N_1 = a_1; N_2 = a_2; N_3 = a_3; P_1 = 0; P_2 = 0; P_3 = 0$

✓ 1. 5/5

1.0.2 2. What is the Jacobi matrix A at the non-trivial fixed point?

For the non-trivial FP :

$N_1 = 1; N_2 = 1; N_3 = 1; P_1 = 1; P_2 = 1; P_3 = 1$

The Jacobi matrix A is given as:

2. 5/5

✓

$$A = \begin{pmatrix} -1 & 0 & 0 & -20 & -30 & -5 \\ 0 & -1 & 0 & -1 & -3 & -7 \\ 0 & 0 & -1 & -4 & -10 & -20 \\ 20 & 30 & 35 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 7 & 8 & 20 & 0 & 0 & 0 \end{pmatrix}$$

1.0.3 3. Determine the eigenvalues and eigenvectors λ_i and $v_i, i = 1, ...6$ of A for this fixed point.

[6]:
```
A = np.
 ↪array([[-1,0,0,-20,-30,-5],[0,-1,0,-1,-3,-7],[0,0,-1,-4,-10,-20],[20,30,35,0,0,0],[3,3,3,0,
print(A)

eigvals, eigvecs = la.eig(A)
```

```
[[ -1   0   0 -20 -30  -5]
 [  0  -1   0  -1  -3  -7]
 [  0   0  -1  -4 -10 -20]
 [ 20  30  35   0   0   0]
 [  3   3   3   0   0   0]
 [  7   8  20   0   0   0]]
```

The eigenvalues λ are:

[7]:
```
print(eigvals)
```

```
[-0.5        +33.62558388j -0.5        -33.62558388j
 -0.5         +7.67949446j -0.5         -7.67949446j
 -1.13602378 +0.j          0.13602378 +0.j        ]
```

The eigenvectors v are:

```
[8]: print(eigvecs)
```

```
[[-0.00808048+5.43421815e-01j -0.00808048-5.43421815e-01j
  -0.86913661+0.00000000e+00j -0.86913661-0.00000000e+00j
  -0.53762066+0.00000000e+00j  0.08149945+0.00000000e+00j]
 [-0.00138394+9.30713447e-02j -0.00138394-9.30713447e-02j
   0.13809399-1.36932149e-16j  0.13809399+1.36932149e-16j
   0.29472269+0.00000000e+00j -0.04467786+0.00000000e+00j]
 [-0.00436622+2.93633078e-01j -0.00436622-2.93633078e-01j
   0.34158855+1.49181629e-16j  0.34158855-1.49181629e-16j
   0.07549029+0.00000000e+00j -0.01144379+0.00000000e+00j]
 [ 0.71189052+0.00000000e+00j  0.71189052-0.00000000e+00j
   0.01084273+1.66533346e-01j  0.01084273-1.66533346e-01j
  -0.64384894+0.00000000e+00j -0.81514582+0.00000000e+00j]
 [ 0.0829838 +6.53568437e-18j  0.0829838 -6.53568437e-18j
   0.00986382+1.51498294e-01j  0.00986382-1.51498294e-01j
   0.44208849+0.00000000e+00j  0.55970673+0.00000000e+00j]
 [ 0.30991834+2.42444559e-17j  0.30991834-2.42444559e-17j
  -0.01564017-2.40217184e-01j -0.01564017+2.40217184e-01j
  -0.09176103+0.00000000e+00j -0.11617418+0.00000000e+00j]]
```

3.
Eigenvalues/vector: 3/3

Why didn't you do evolution part? If you did but just didn't print it, please let me know (as soon as possible)