

# Z Reference Card

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## Specifications

### Schema box

<i>Name</i> [ <i>Params</i> ] —
<i>Declarations</i>
<i>Predicates</i>

```
\begin{schema}{Name} [Params]
  Declarations
\where
  Predicates
\end{schema}
```

### Axiomatic description

<i>Declarations</i>
<i>Predicates</i>

```
\begin{axdef}
  Declarations
\where
  Predicates
\end{axdef}
```

### Generic definition

[ <i>Params</i> ] =
<i>Declarations</i>
<i>Predicates</i>

```
\begin{gendef} [Params]
  Declarations
\where
  Predicates
\end{gendef}
```

`\begin{zed} ...`

### Basic type definition

`[NAME, DATE]` `[NAME, DATE]`

### Abbreviation definition

`DOC == seq CHAR` `DOC == \seq CHAR`

### Constraint

`n_disks < 5` `n\_disks < 5`

### Schema definition

`Point ≐ [x, y : ℤ]` `Point \defs [~x, y: \num~]`

### Free type definition

`Ans ::= ok⟨⟨ℤ⟩⟩ | error` `Ans ::= ok \ldata\num\rdata | error`  
`... \end{zed}`

## Logic and schema calculus

<i>true, false</i>	<code>true, false</code>	Logical constants
$\neg P$	<code>\not P</code>	Negation
$P \wedge Q$	<code>P \land Q</code>	Conjunction
$P \vee Q$	<code>P \lor Q</code>	Disjunction
$P \Rightarrow Q$	<code>P \implies Q</code>	Implication
$P \Leftrightarrow Q$	<code>P \iff Q</code>	Equivalence
$\forall x : T \mid P \bullet Q$	<code>\forall x : T \mid P \bullet Q</code>	Universal quantifier
$\exists x : T \mid P \bullet Q$	<code>\exists x : T \mid P \bullet Q</code>	Existential quantifier
$\exists_1 x : T \mid P \bullet Q$	<code>\exists_1 x : T \mid P \bullet Q</code>	Unique quantifier

## Special schema operators

$S[y_1/x_1, y_2/x_2]$	<code>S[y_1/x_1, y_2/x_2]</code>	Renaming
$S \setminus (x_1, x_2)$	<code>S \hide (x_1, x_2)</code>	Hiding
$S1 \upharpoonright S2$	<code>S1 \project S2</code>	Projection
$\text{pre } Op$	<code>\pre Op</code>	Pre-condition
$Op1 \mathbin{\text{\textcircled{;}}} Op2$	<code>Op1 \semi Op2</code>	Sequential composition
$Op1 \gg Op2$	<code>Op1 \pipe Op2</code>	Piping

## Basic expressions

$x = y$	<code>x = y</code>	Equality
$x \neq y$	<code>x \neq y</code>	Inequality
<b>if</b> $P$ <b>then</b> $E_1$ <b>else</b> $E_2$	<code>\IF P \THEN E_1</code> <code>\ELSE E_2</code>	Conditional Expression
$\theta S$	<code>\theta S</code>	Theta-expression
$E.x$	<code>E.x</code>	Selection
$(\mu x : T \mid P \bullet E)$	<code>(\mu x: T \mid P @ E)</code>	Mu-expression
$(\text{let } x == E1 \bullet E2)$	<code>(\LET x == E1 @ E2)</code>	Let-expression

## Sets

$x \in S$	<code>x \in S</code>	Membership
$x \notin S$	<code>x \notin S</code>	Non-membership
$\{x_1, \dots, x_n\}$	<code>\{x_1, ..., x_n\}</code>	Set display
$\{x : T \mid P \bullet E\}$	<code>\{~x: T \mid P @ E~\}</code>	Set comprehension
$\emptyset$	<code>\emptyset</code>	Empty set
$S \subseteq T$	<code>S \subseteq T</code>	Subset relation
$S \subset T$	<code>S \subset T</code>	Proper subset relation
$\mathbb{P} S$	<code>\power S</code>	Power set
$\mathbb{P}_1 S$	<code>\power_1 S</code>	Non-empty subsets
$S \times T$	<code>S \cross T</code>	Cartesian product
$(x, y, z)$	<code>(x, y, z)</code>	Tuple
<i>first</i> $p$	<code>first~p</code>	First of pair
<i>second</i> $p$	<code>second~p</code>	Second of pair
$S \cup T$	<code>S \cup T</code>	Set union
$S \cap T$	<code>S \cap T</code>	Set intersection
$S \setminus T$	<code>S \setminus T</code>	Set difference
$\bigcup A$	<code>\bigcup A</code>	Generalized union
$\bigcap A$	<code>\bigcap A</code>	Generalized intersection
$\mathbb{F} X$	<code>\finset X</code>	Finite sets
$\mathbb{F}_1 X$	<code>\finset_1 X</code>	Non-empty finite sets

## Relations

$X \longleftrightarrow Y$	$X \backslash \text{rel } Y$	Binary relations
$x \mapsto y$	$x \backslash \text{mapsto } y$	Maplet
$\text{dom } R$	$\backslash \text{dom } R$	Domain
$\text{ran } R$	$\backslash \text{ran } R$	Range
$\text{id } X$	$\backslash \text{id } X$	Identity relation
$Q \circledcirc R$	$Q \backslash \text{comp } R$	Composition
$Q \circ R$	$Q \backslash \text{circ } R$	Backwards composition
$S \triangleleft R$	$S \backslash \text{dres } R$	Domain restriction
$R \triangleright S$	$R \backslash \text{rres } S$	Range restriction
$S \triangleleft R$	$S \backslash \text{ndres } R$	Domain anti-restriction
$R \triangleright S$	$R \backslash \text{nrres } S$	Range anti-restriction
$R^\sim$	$R \backslash \text{inv}$	Relational inverse
$R \Downarrow S$	$R \backslash \text{limg } S \backslash \text{rimg}$	Relational image
$Q \oplus R$	$Q \backslash \text{oplus } R$	Overriding
$R^k$	$R^\sim\{k\}$	Iteration
$R^+$	$R \backslash \text{plus}$	Transitive closure
$R^*$	$R \backslash \text{star}$	Reflexive-trans. closure

## Functions

$f(x)$	$f(x)$	Function application
$(\lambda x : T \mid P \bullet E)$	$(\backslash \text{lambda } \dots)$	Lambda-expression
$X \twoheadrightarrow Y$	$X \backslash \text{pfun } Y$	Partial functions
$X \longrightarrow Y$	$X \backslash \text{fun } Y$	Total functions
$X \twoheadrightarrowtail Y$	$X \backslash \text{pinj } Y$	Partial injections
$X \twoheadrightarrowtail Y$	$X \backslash \text{inj } Y$	Total injections
$X \twoheadrightarrowtail Y$	$X \backslash \text{psurj } Y$	Partial surjections
$X \twoheadrightarrowtail Y$	$X \backslash \text{surj } Y$	Total surjections
$X \twoheadrightarrowtail Y$	$X \backslash \text{bij } Y$	Bijections
$X \twoheadrightarrowtail Y$	$X \backslash \text{ffun } Y$	Finite partial functions
$X \twoheadrightarrowtail Y$	$X \backslash \text{finj } Y$	Finite partial injections

## Numbers and arithmetic

$\mathbb{N}$	<code>\nat</code>	Natural numbers
$\mathbb{Z}$	<code>\num</code>	Integers
$+ - * \text{div mod}$	<code>+ - * \div \mod</code>	Arithmetic operations
$< \leq \geq >$	<code>&lt; \leq \geq &gt;</code>	Arithmetic comparisons
$\mathbb{N}_1$	<code>\nat_1</code>	Strictly positive integers
<i>succ</i>	<code>succ</code>	Successor function
$m .. n$	<code>m \upto n</code>	Number range
$\#S$	<code>\# S</code>	Size of a set
$\min S$	<code>min~S</code>	Minimum of a set
$\max S$	<code>max~S</code>	Maximum of a set

## Sequences

$\text{seq } X$	<code>\seq X</code>	Finite sequences
$\text{seq}_1 X$	<code>\seq_1 X</code>	Non-empty sequences
$\text{iseq } X$	<code>\iseq X</code>	Injective sequences
$\langle x_1, \dots, x_n \rangle$	<code>\langle ... \rangle</code>	Sequence display
$s \frown t$	<code>s \cat t</code>	Concatenation
<i>rev s</i>	<code>rev~s</code>	Reverse
<i>head s</i>	<code>head~s</code>	Head of sequence
<i>last s</i>	<code>last~s</code>	Last element of sequence
<i>tail s</i>	<code>tail~s</code>	Tail of sequence
<i>front s</i>	<code>front~s</code>	All but last element
$U \upharpoonright s$	<code>U \extract S</code>	Extraction
$s \upharpoonright V$	<code>s \filter V</code>	Filtering
<i>squash f</i>	<code>squash~f</code>	Compaction
$s \text{ prefix } t$	<code>s \prefix t</code>	Prefix relation
$s \text{ suffix } t$	<code>s \suffix t</code>	Suffix relation
$s \text{ in } t$	<code>s \inseq t</code>	Segment relation
$\frown/ss$	<code>\dcat ss</code>	Distributed concat.
disjoint $SS$	<code>\disjoint SS</code>	Disjointness
$SS$ partition $T$	<code>SS \partition T</code>	Partition relation

## Bags

$\text{bag } X$	<code>\bag X</code>	Bags
$\llbracket x_1, \dots, x_n \rrbracket$	<code>\lbag ... \rbag</code>	Bag display
$\text{count } B \ x$	<code>count~B~x</code>	Count of an element
$B \# x$	<code>B \bcount x</code>	Infix count operator
$n \otimes B$	<code>n \otimes B</code>	Bag scaling
$x \in B$	<code>x \inbag B</code>	Bag membership
$B \sqsubseteq C$	<code>B \subbageq C</code>	Sub-bag relation
$B \uplus C$	<code>B \uplus C</code>	Bag union
$B \ominus C$	<code>B \uminus C</code>	Bag difference
$\text{items } s$	<code>items~s</code>	Items in a sequence

## *f*UZZ flags

Usage: `fuzz [-aqrstv] [-p prelude] [file ...]`

<code>-a</code>	Don't use type abbreviations
<code>-p prelude</code>	Use <i>prelude</i> in place of the standard one
<code>-q</code>	Assume implicit quantifiers for undeclared variables
<code>-d</code>	Dependency analysis
<code>-s</code>	Syntax check only
<code>-t</code>	Report types of global definitions
<code>-v</code>	Echo formal text as it is parsed