Tools for secondary analyses for individualized effects

Raaz Dwivedi, Tuesday afternoon session

To think about

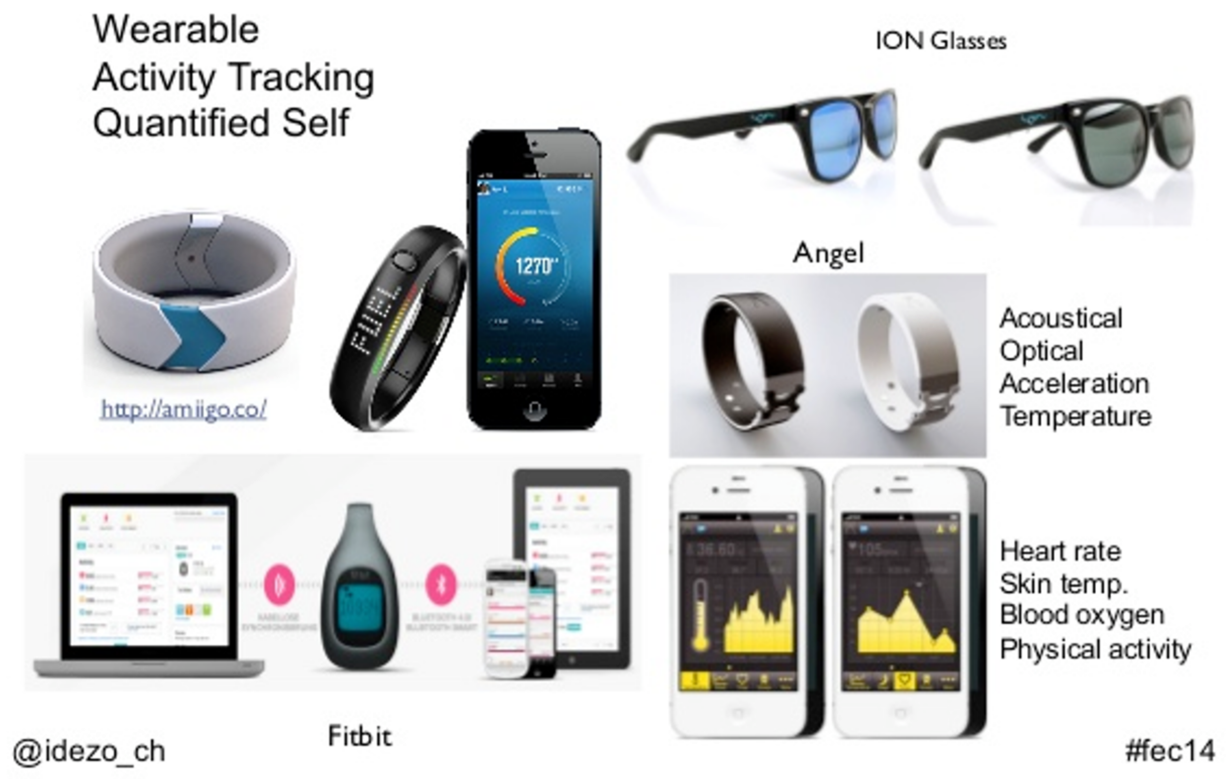
- Once the trial is over and the data is collected, what would we like to do with the data?
    - As scientists who ran the trial
    - As other scientists who did not run, but are interested in the trial

(Personalized) Sequential decision making

- Mobile health: Personalized app notifications to promote healthy behavior
- Online education: Personalized teaching strategies for better learning
- Online advertising: Personalized ads / placements to increase revenue

Physical activity

Wearable/trackers



Personalized decision making in medicine

- Precision medicine with RCTs
    - Subgroup analysis – typically very limiting
    - ITE, counterfactual, conditional average treatment effect – Helps with better decisions **in hindsight,** but becomes useful step for hypothesis generation for the future
    - Precision medicine is a doctor's goal – https://nam.edu/wp-content/uploads/2019/08/Caring-for-the-Individual-Patient-prepub.pdf

Primary analyses

- Trials are designed to help answer primary questions of interest, aka, primary analyses
    - **On average**: Does the mobile app with the RL algorithm help people become more active?
    - **Subgroup analysis**: Same question can be asked for pre-specified subgroups of population
- These analyses are done by the scientists involved in the trials
- These analyses are pre-specified and in best cases, pre-registered at clinicaltrials.gov or openscience – "***Part of conservative traditions in clinical trials***"

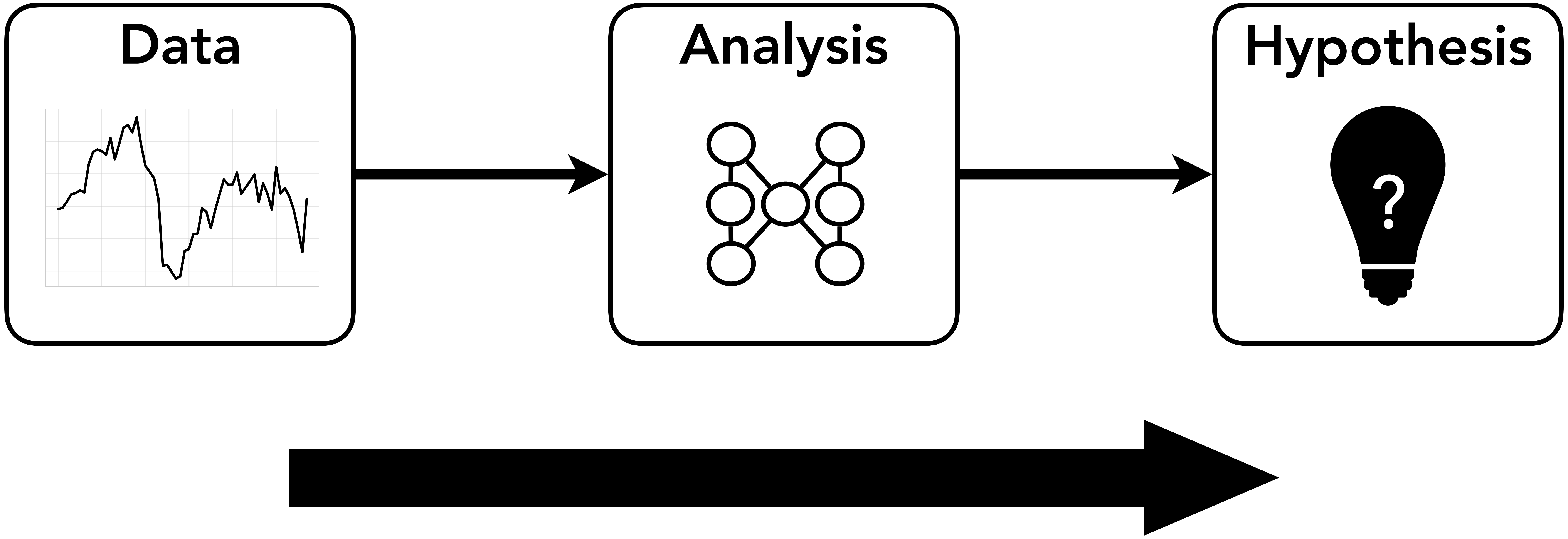
Examples in mHealth trials

- **HeartSteps V1**: Delivering (vs. not delivering) a contextually-tailored activity suggestion increases average step count in the 30 min following a decision point
- **SARA**: Offering (vs. not offering) an inspirational quote increases the likelihood to fully complete current day's survey and/or active tasks
- **ORALYTICS**: Delivering an engagement prompt increases proximal oral self-care behaviors (OSCB) in the subsequent brushing window as compared to not delivering an engagement prompt
- Typically these analyses are marginal in nature across users, (sometimes even across time), i.e., about treatment effect on average

Secondary analyses

- Given the data from the trials, what else can we learn from it? What new theories can be conjectured from it?
    - What group of people benefit the most from the app?
    - How does the benefit vary over time?
- Can we assess these conjectures on the same data?
- Essentially, exploratory data analyses with the goal to provide insights beyond the primary analyses – highly desirable, scientists invest a lot of time and money to run the trials and get this data

Secondary analyses



Similar to Induction in logical reasoning

Examples for HeartSteps

- Secondary:
    - Which states are more useful for personalizing the intervention?
    - Is there heterogeneity in how the people respond? Which group of people respond the best to the app?

Are the two related?

- The questions of interest are generally formulated as hypothesis
    - And the goal of primary analyses is to illustrate whether the data from the trial accepts or refutes the hypothesis with quantitative support (often in terms of statistical significance)
    - But where do these questions come from?

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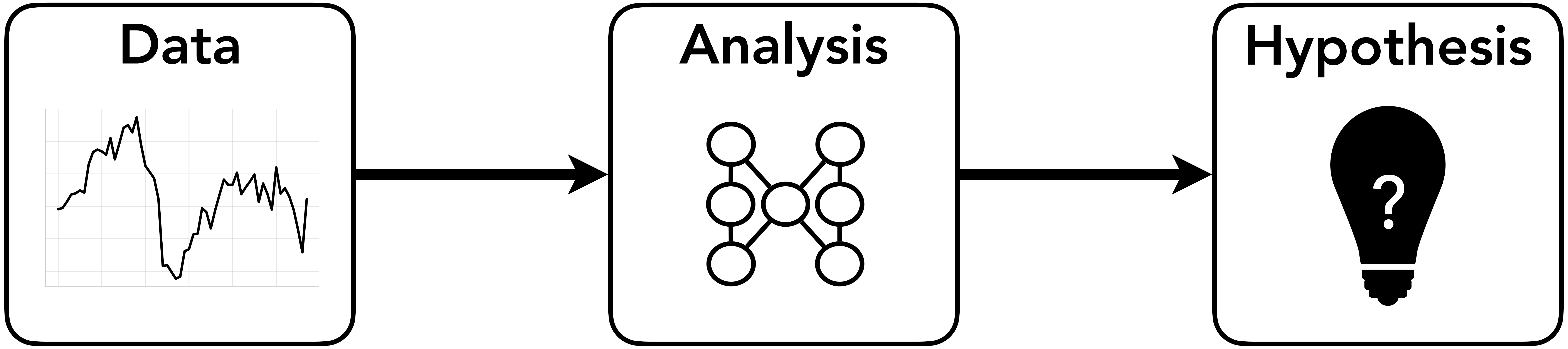
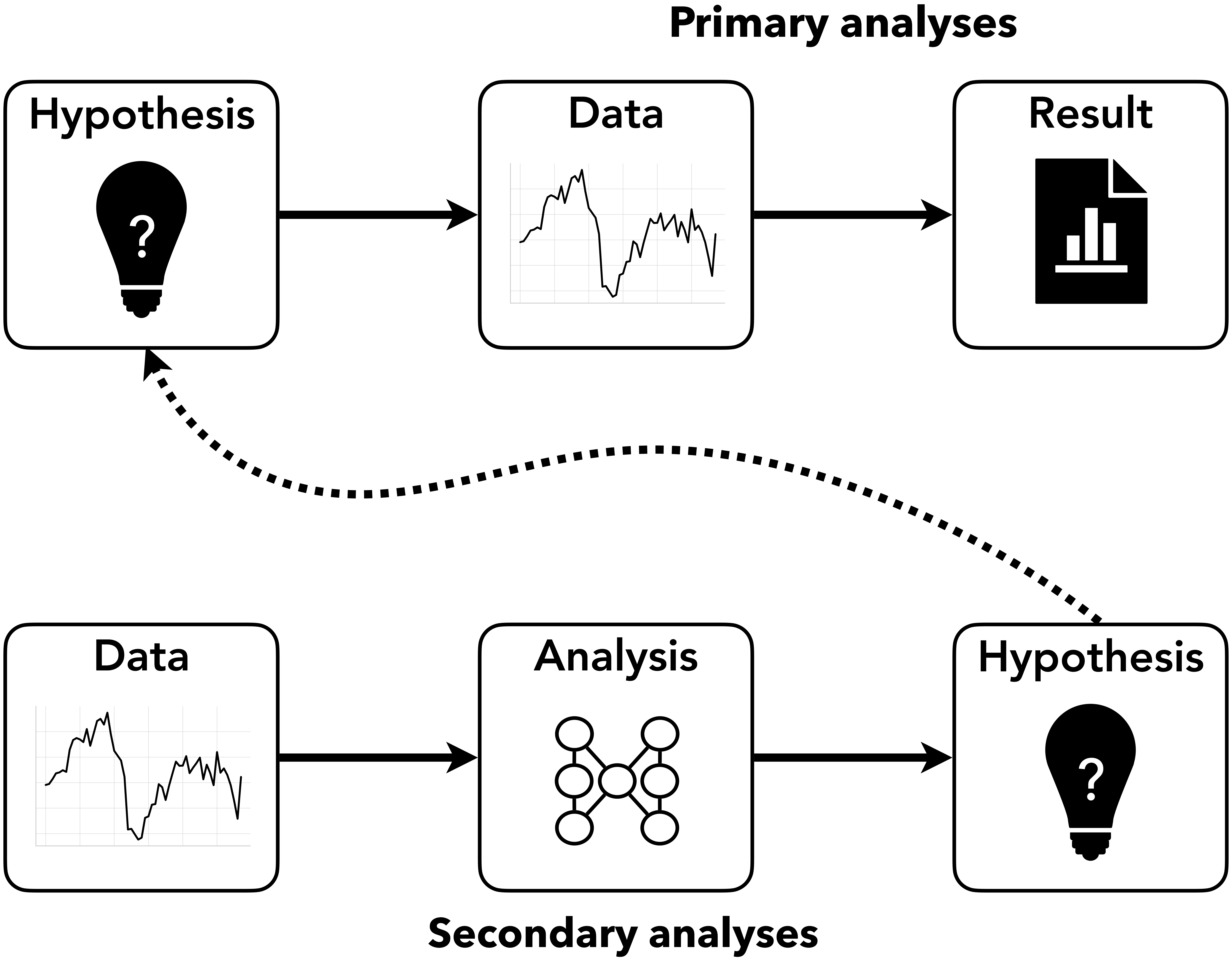
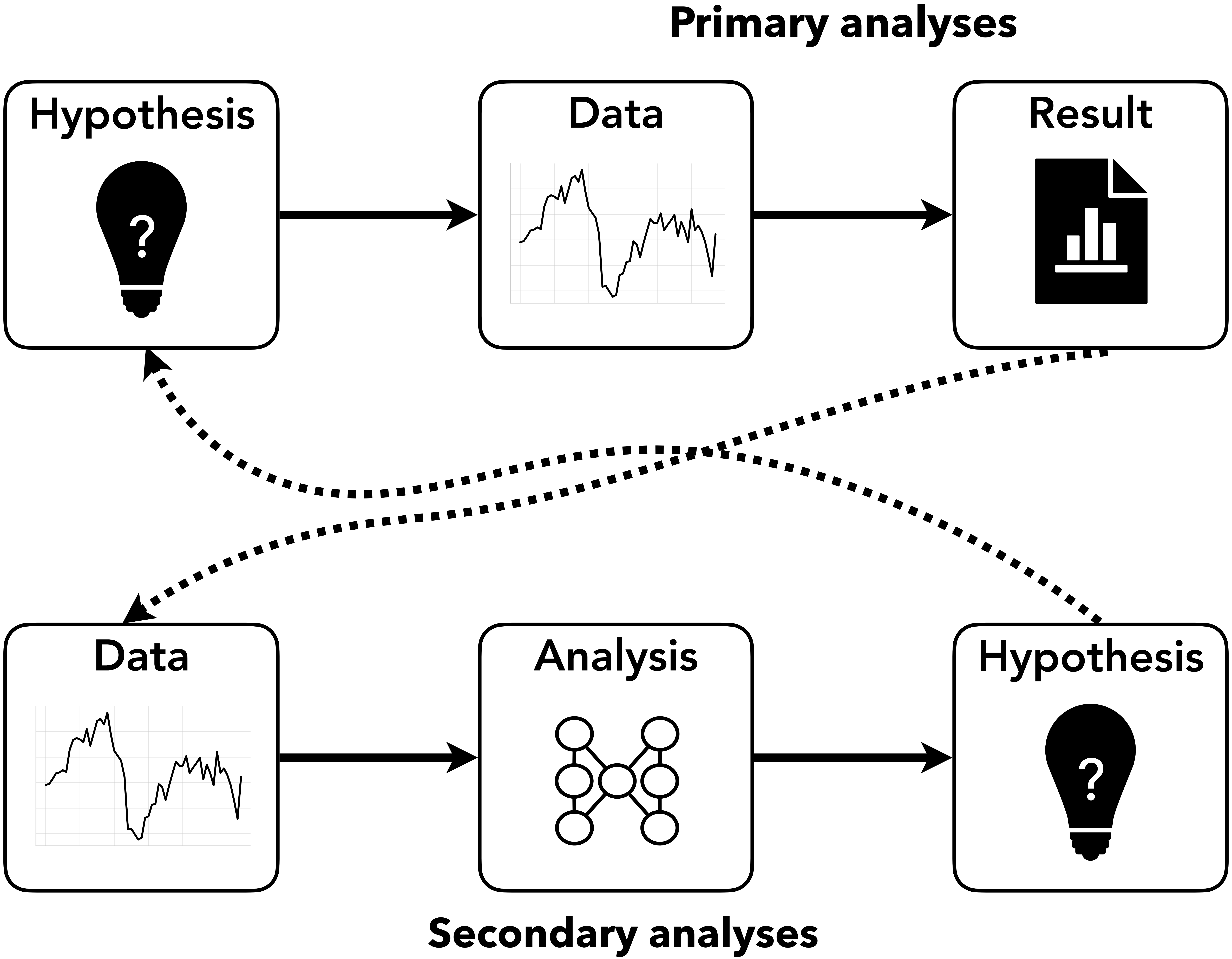
    - And the goal of primary analyses is to illustrate whether the data from the trial accepts or refutes the hypothesis with quantitative support (often in terms of statistical significance)
    - But where do these questions come from?
    - Typically the output of secondary analyses from some other data

Primary analyses



Secondary analyses


Primary analyses
Hypothesis
Data
Result
Data
Analysis
Hypothesis
Secondary analyses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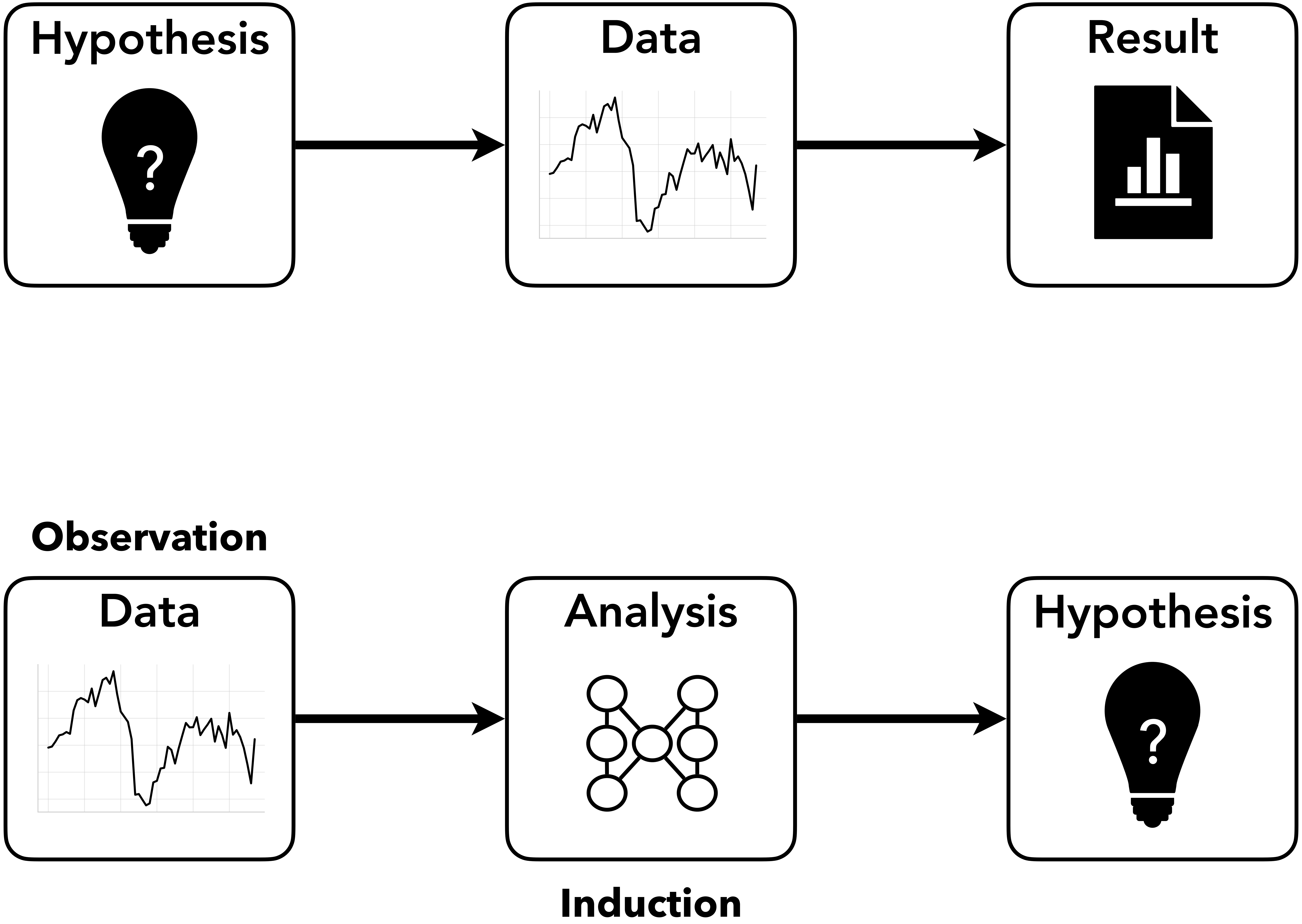
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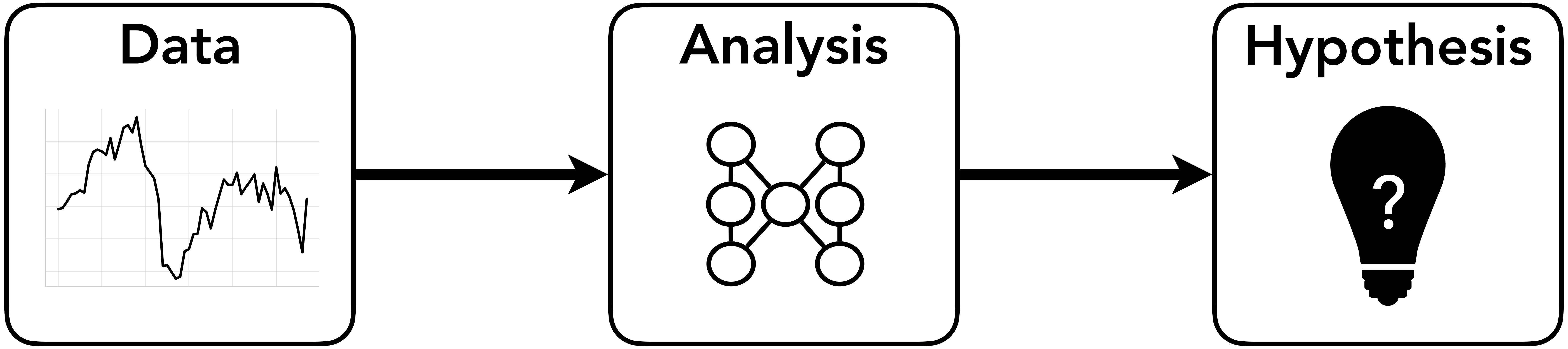
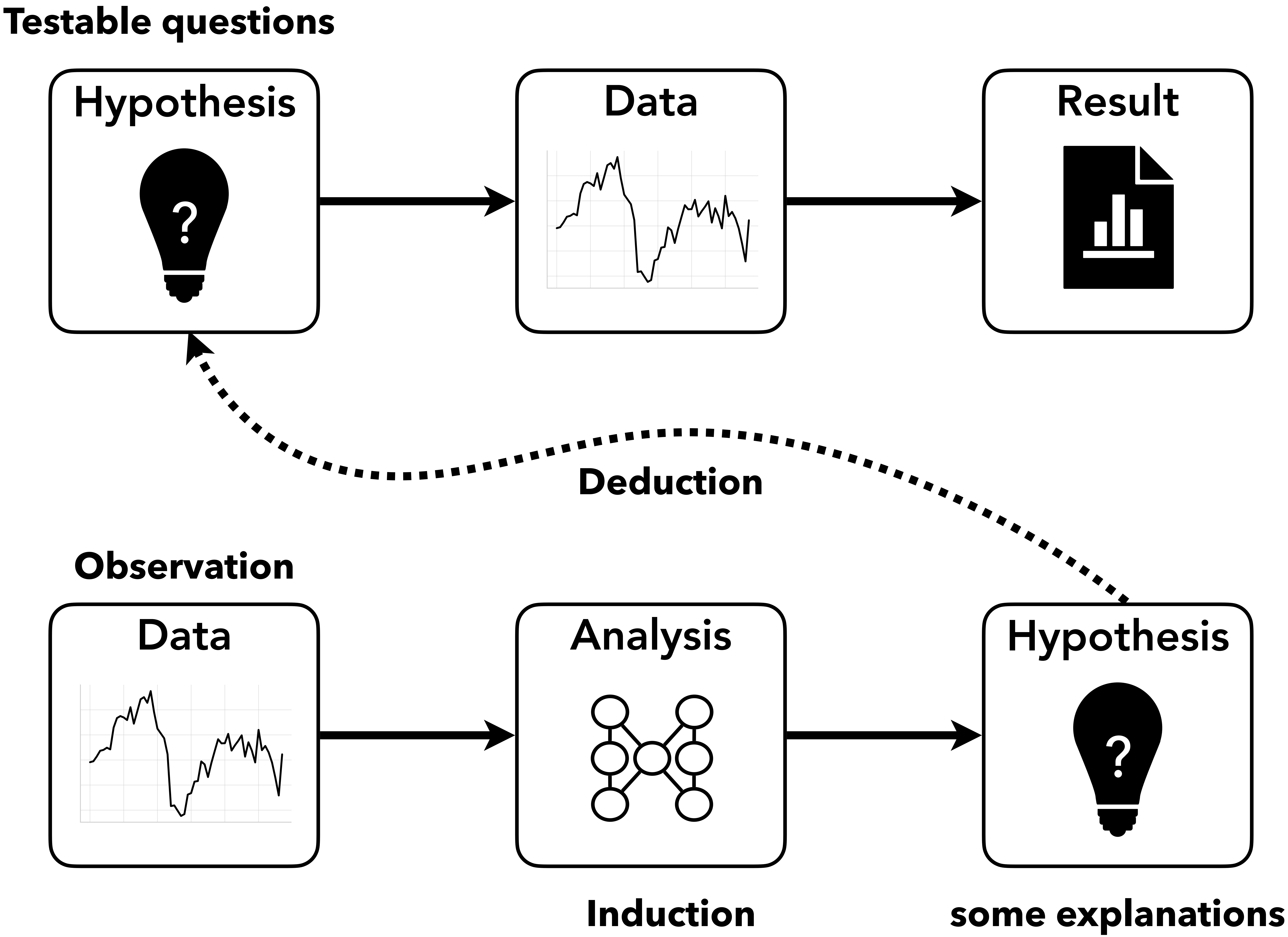
Observation



Induction

some explanations



Testable questions
Hypothesis
?
Data
Result
Deduction
Observation
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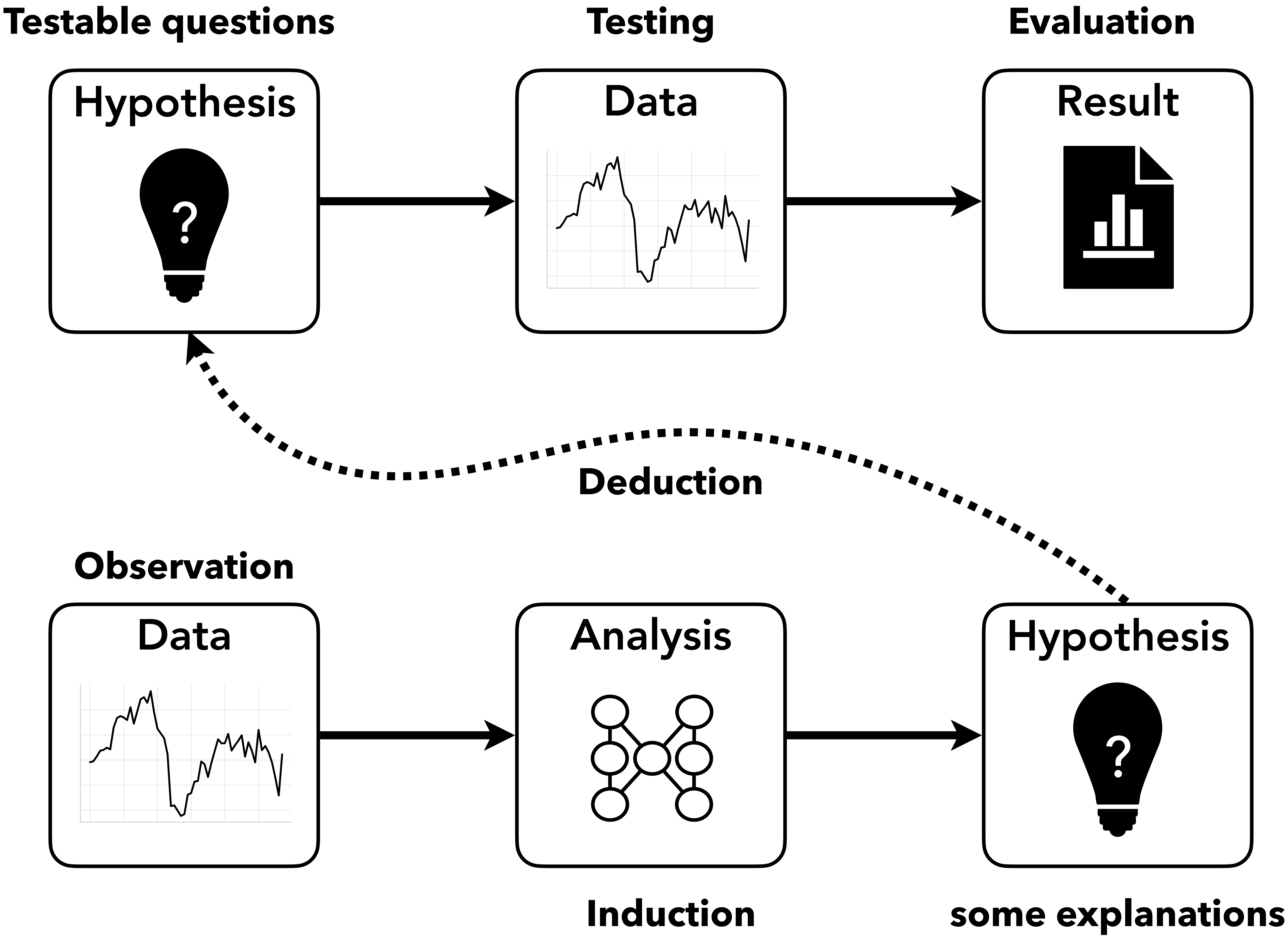
Induction
some explanations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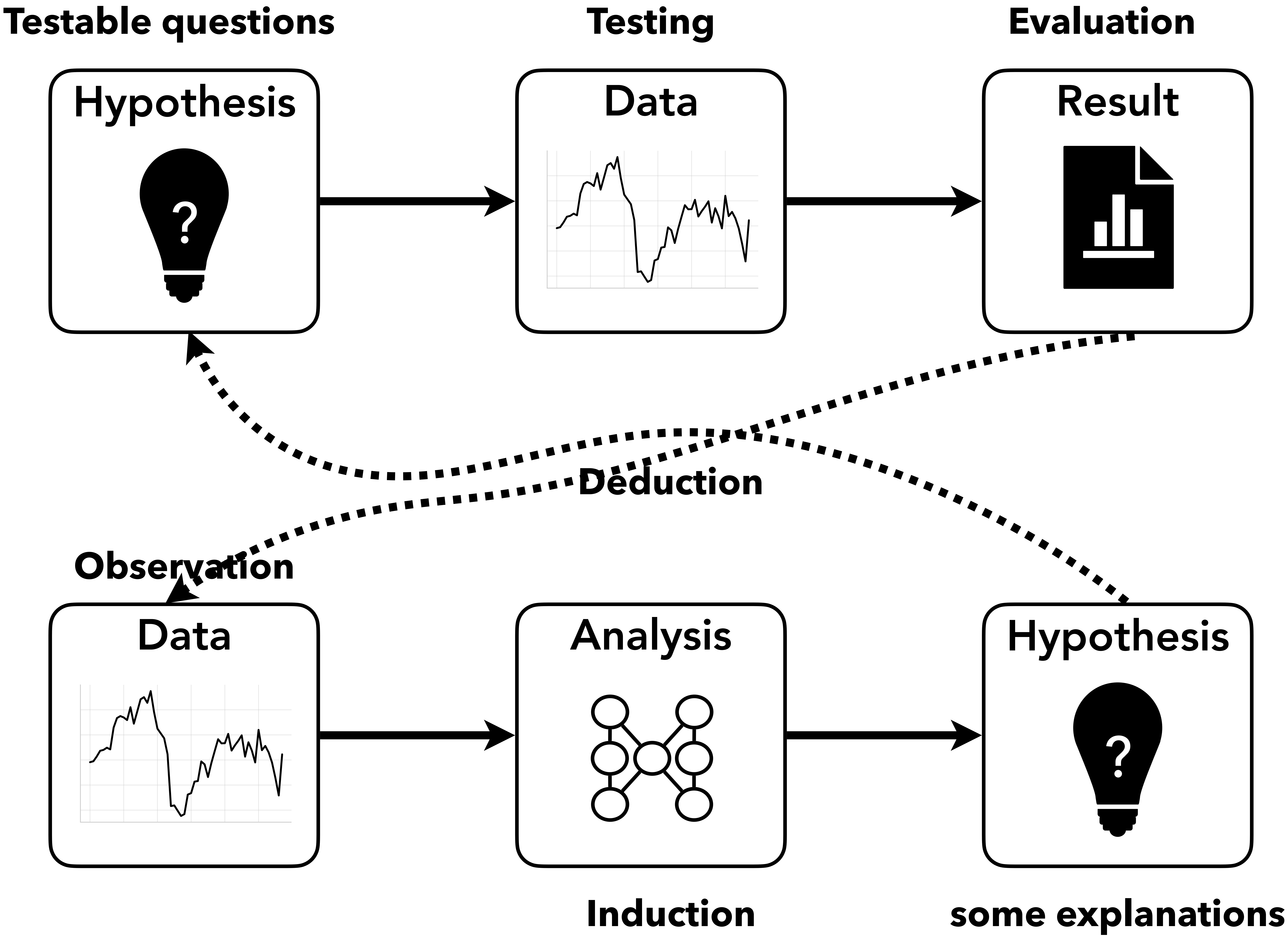
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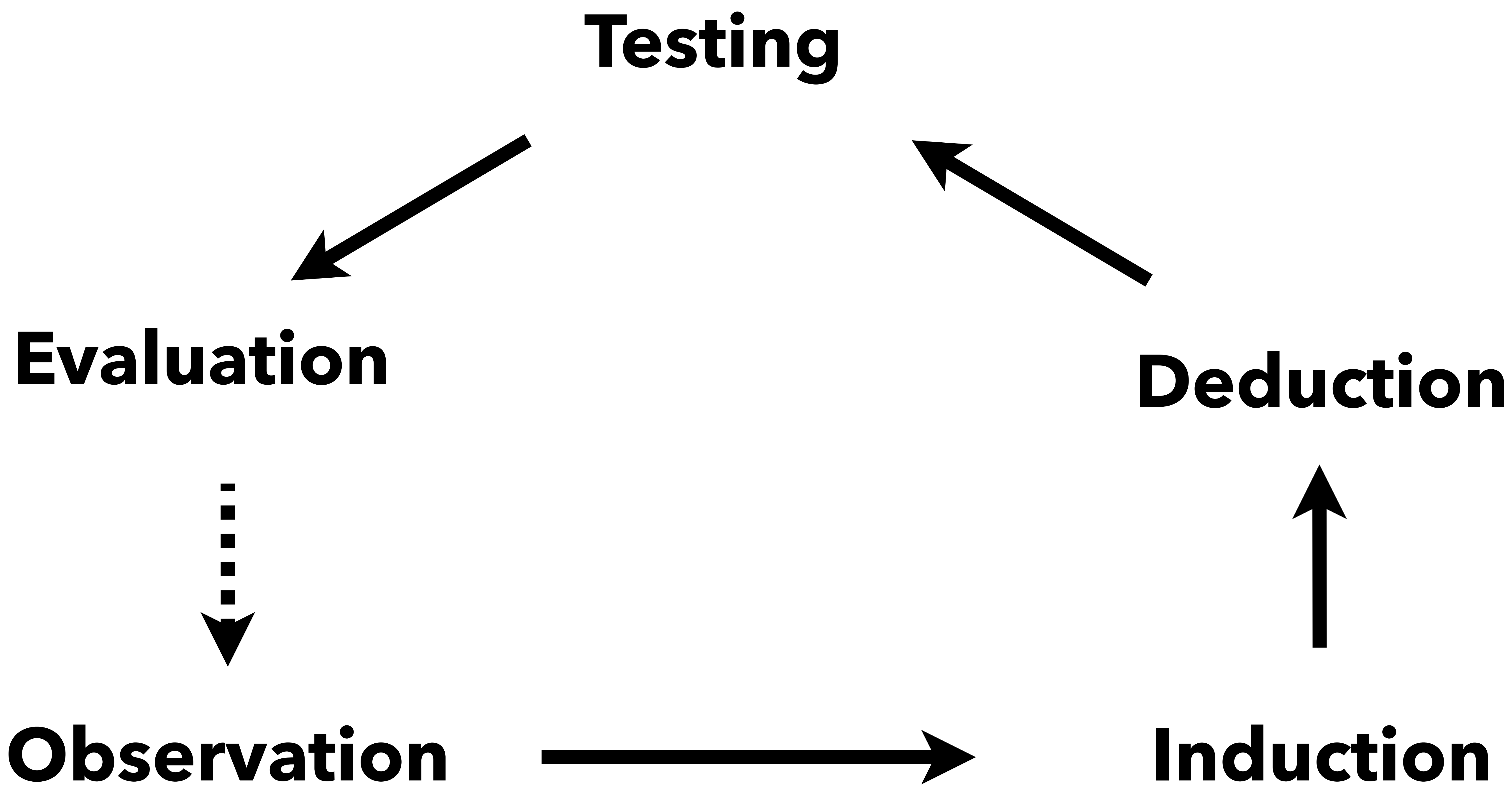
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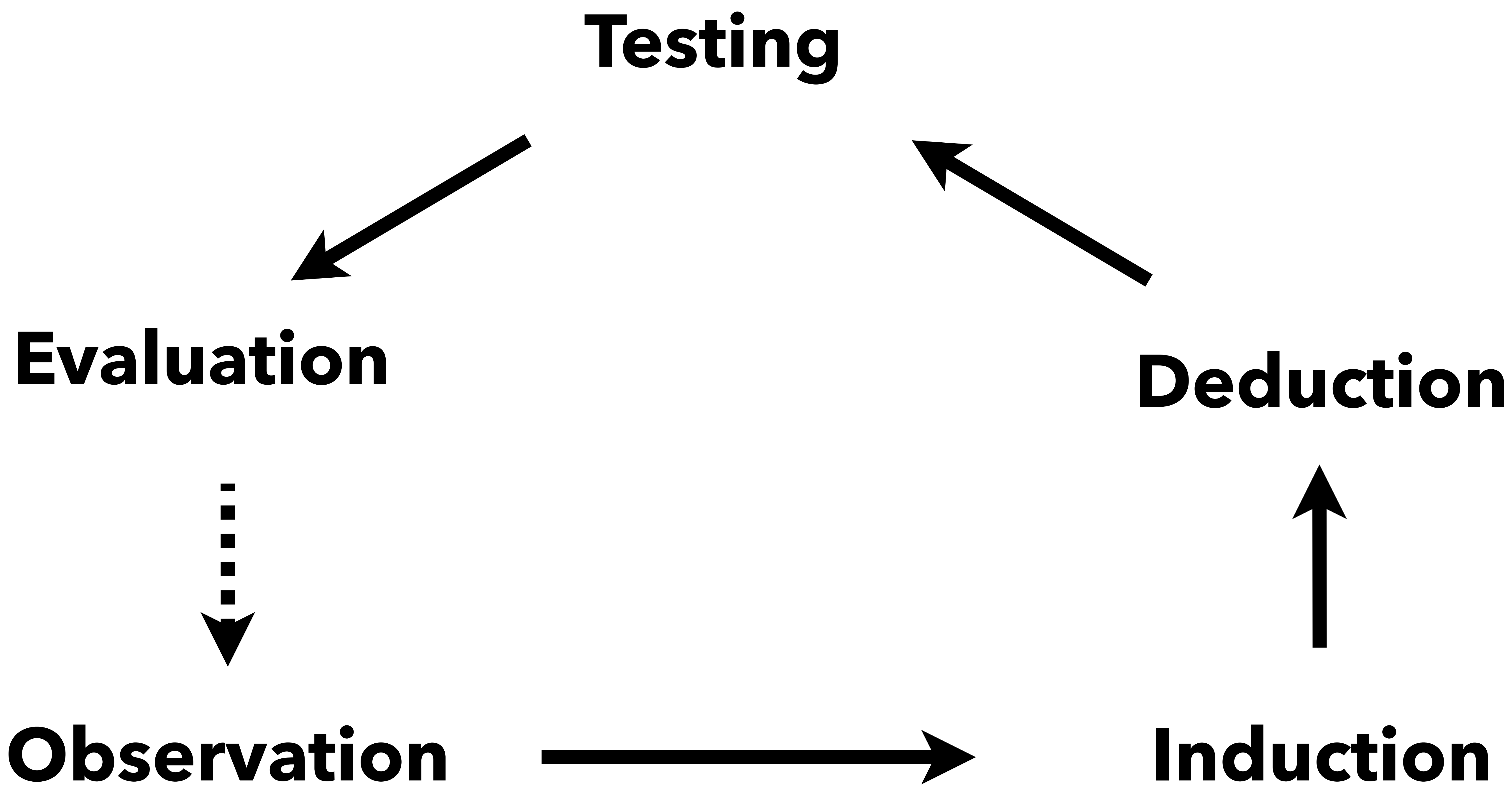
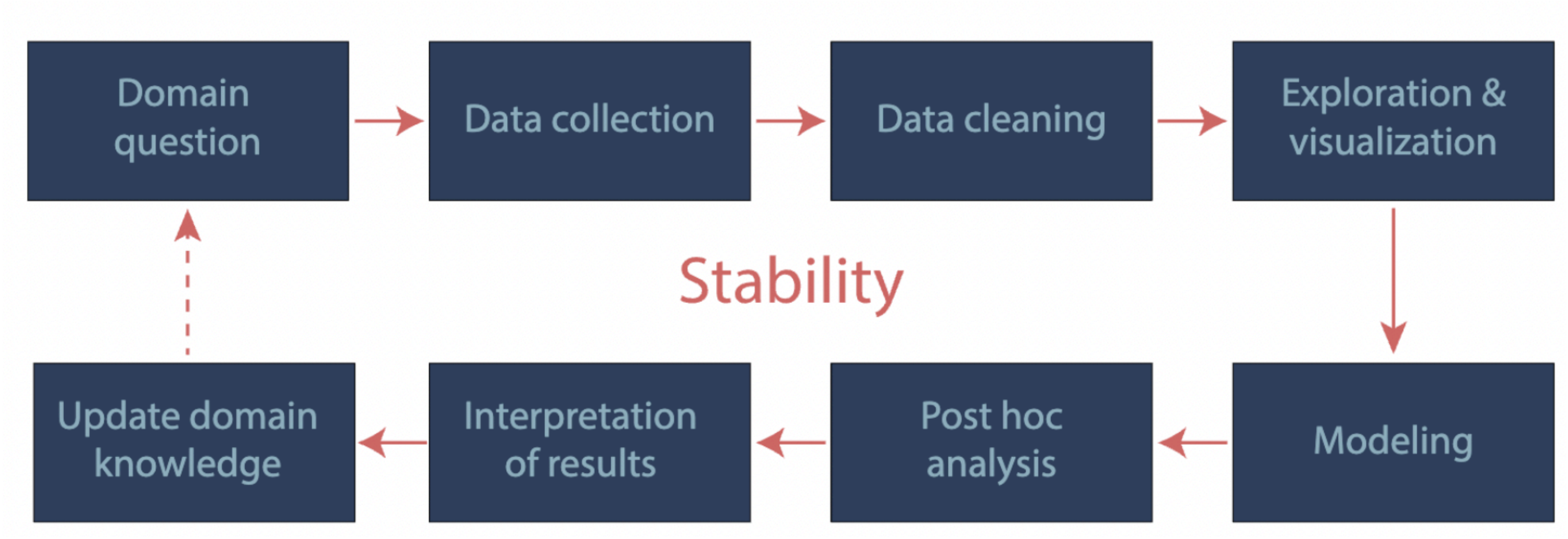
Empirical research cycle / Scientific method

[De Groot 1961]



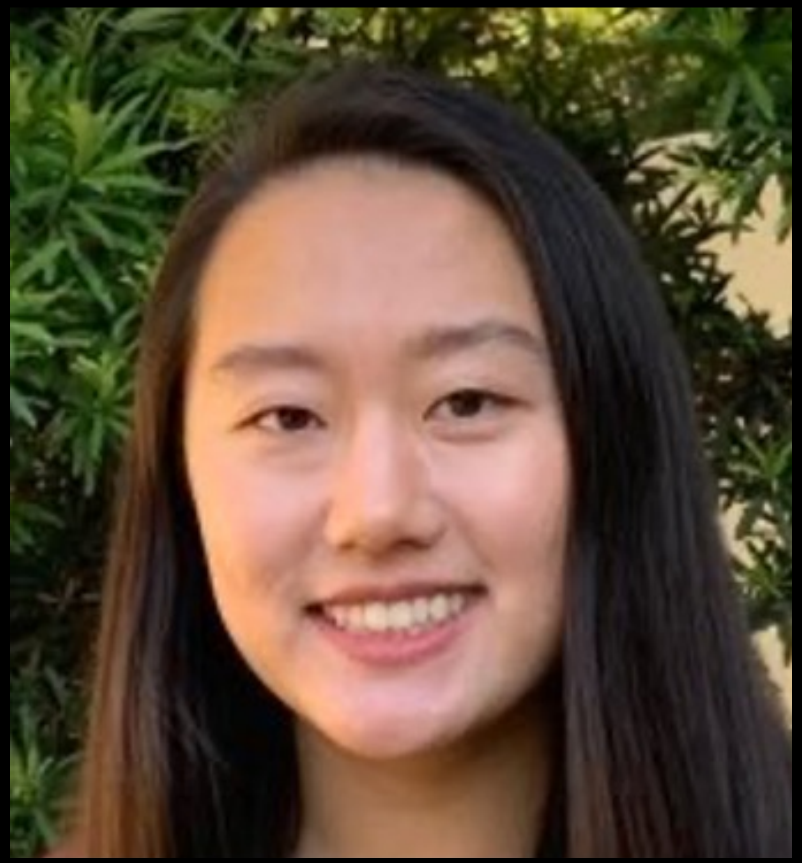
Data science life cycle [Yu & Kumbier 2021]



Next:
Two approaches for secondary analyses with adaptively collected data

Today + Tomorrow morning: Using factor models

Tomorrow second session: Using simulators



Katherine Tian

Sabina Tomkins

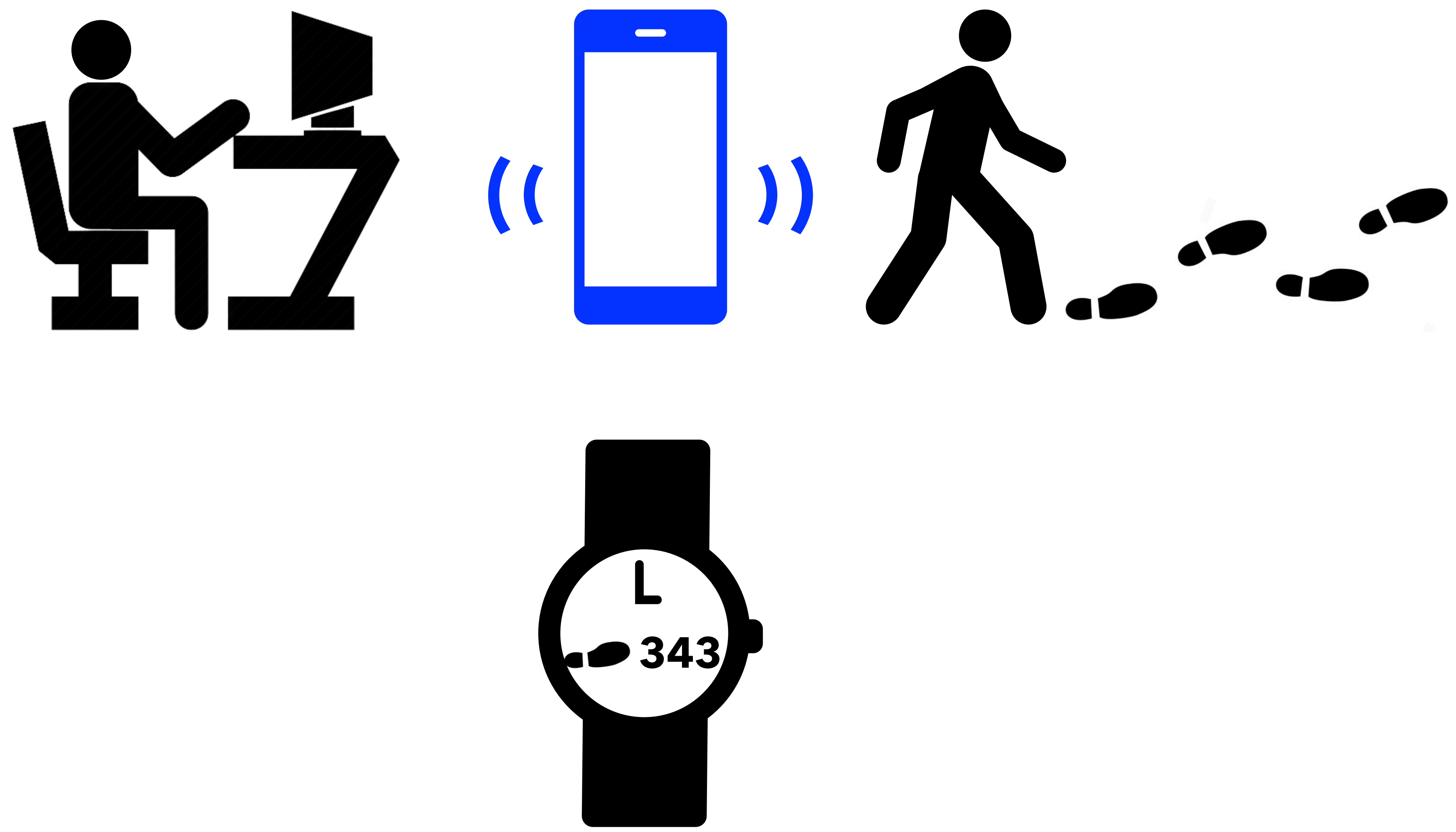
Predrag Klasnja

Susan Murphy

Devavrat Shah

Building AI agents for personalized treatments

How to assign personalized digital treatments to help you?



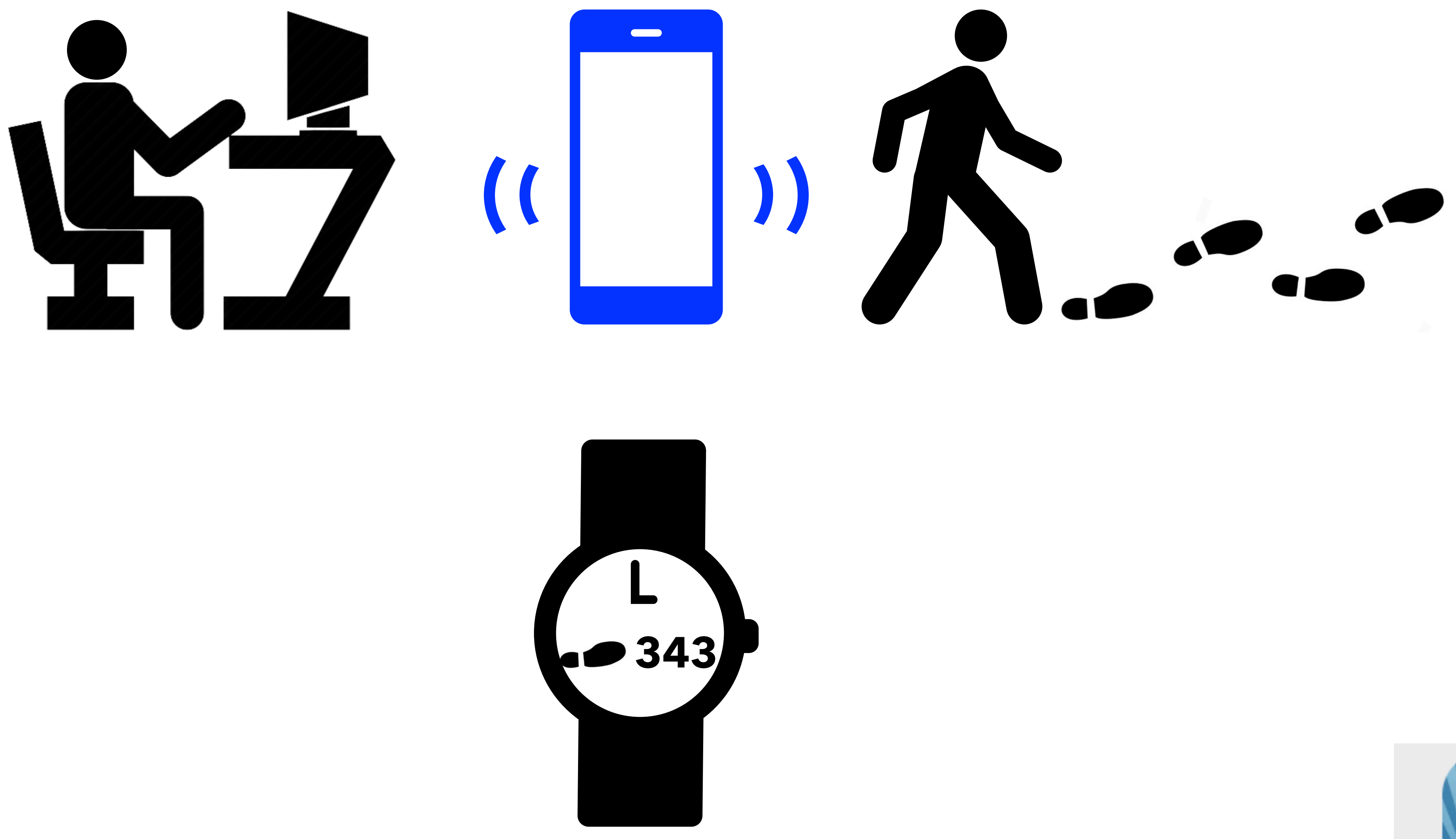
Mobile health study:
Personalized HeartSteps

[Liao+ '20]

- **Goal**: Promote physical activity via mobile app
- **Population:** 91 hypertension patients, 90 days
- **Treatment:** Mobile notifications upto 5 times/ day assigned by a bandit algorithm
- **Outcome**: 30-min step count after decision time

Building AI agents for personalized treatments

How to assign personalized digital treatments to help you?



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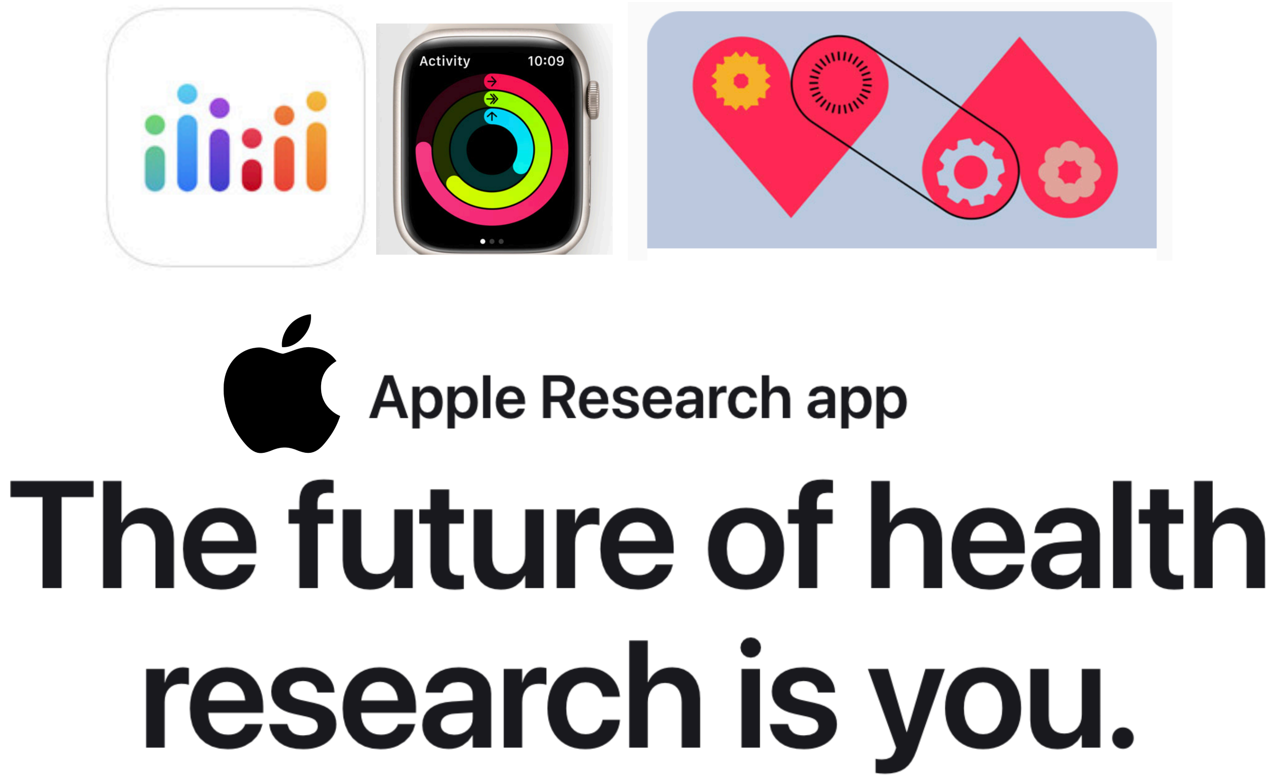

image credits
va.gov
apple.com

Motivating questions for our secondary analyses

- Was sending the notification effective for different users? (E.g., was there a treatment effect for a given user?)
- Was the RL algorithm effective in personalizing the timing of these notifications? (E.g., did the RL algorithm learn these effects and send notifications accordingly?)

The two approaches

- Today: When we **do not** have access to the RL algorithm

  –Proceed by modeling and some estimation procedure, provide theoretical guarantees under strong assumptions

- Tomorrow: When we **have** access to the RL algorithm

  –Proceed by modeling and simulations using the RL algorithm, provide empirical evidence under weak assumptions

An ambitious question

- For a given user i at a given time t, should we have sent the notification?
    - **Mathematatizing this question is non-trivial. Why?**

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    - **Mathematizing this question is non-trivial. Why?**
    - When we have delayed effects, to answer this question, we have to ask what we did before time t and what we are going to do after time t?

An ambitious question: Simplified setting

- For a given user i at a given time t, should we have sent the notification?
    - Let's simplify the problem: Suppose we are in a bandit-like setting and have no delayed effects. Suppose we don't even have states. **Can we easily answer the question without any assumptions?**

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  - **No.** Because answering this question requires us to estimate counterfactual quantities.
  - **What if the potential outcomes $R_t(1), R_t(0)$ are iid at each time? Still no.** We need to have some notion of repeated measurements for the ``quantity of interest''.

A factor model for potential outcomes with "no delayed effects"

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  - The potential outcomes for unit i at time t satisfy

$$\mathbb{E}[R_{i,t}(a) \mid u_i^{(a)}, v_t^{(a)}] = \langle u_i^{(a)}, v_t^{(a)} \rangle \triangleq \theta_{i,t}^{(a)}$$

$$\text{and } R_{i,t} = R_{i,t}(A_{i,t}) \triangleq \theta_{i,t}^{(A_{i,t})} + \varepsilon_{i,t}^{(A_{i,t})}$$

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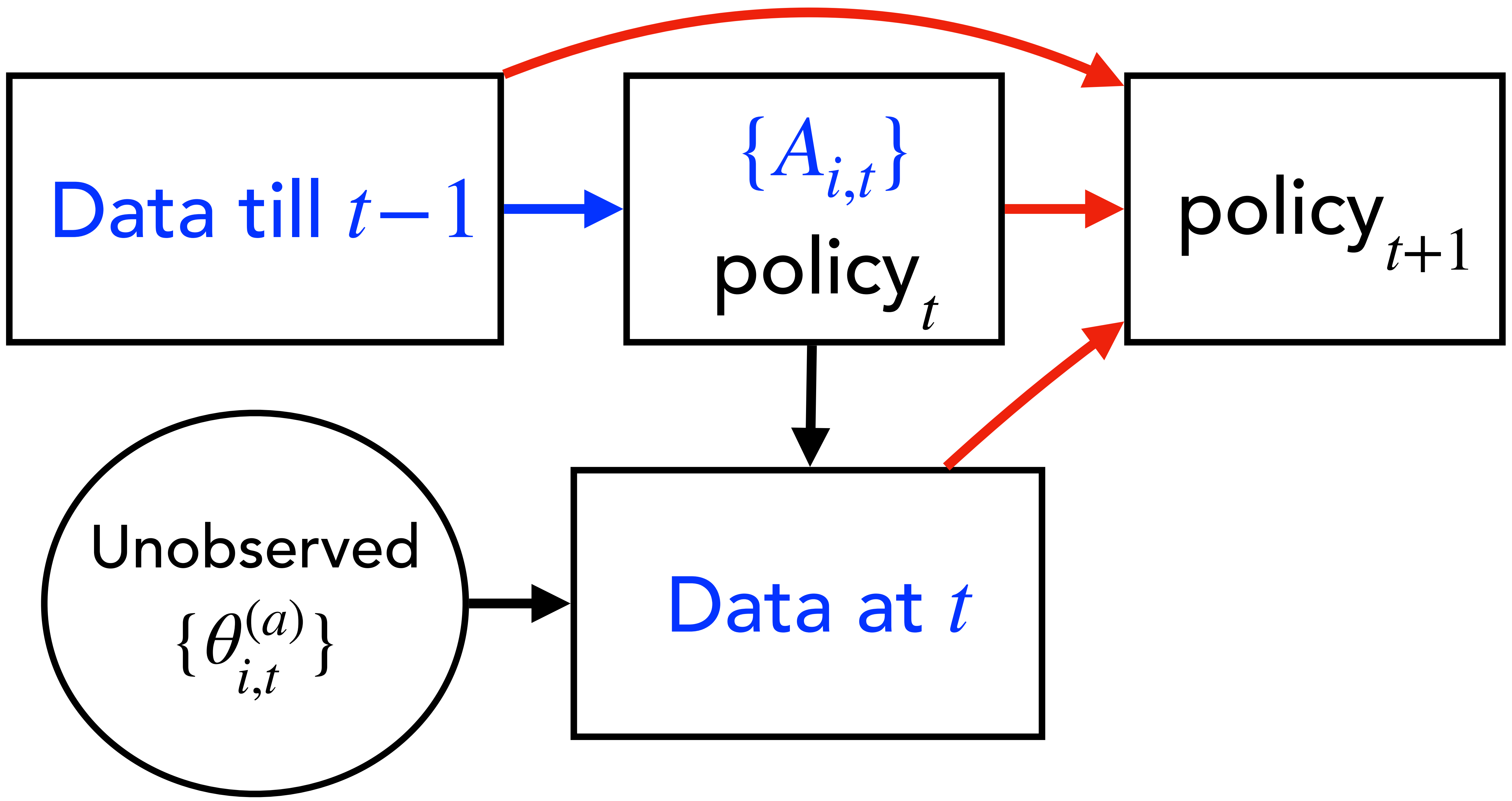
Where is the no delayed effects assumption kicking in?

A factor model for potential outcomes



We allow $A_{i,t}$ to be assigned by a bandit algorithm that may be **pooling** data across users

Is the ambitious question now tractable?

- $\mathbb{E}[R_{i,t}(a) \,|\, u_i^{(a)}, v_t^{(a)}] = \langle u_i^{(a)}, v_t^{(a)} \rangle \triangleq \theta_{i,t}^{(a)}$

  and $R_{i,t} = R_{i,t}(A_{i,t}) \triangleq \theta_{i,t}^{(A_{i,t})} + \varepsilon_{i,t}^{(A_{i,t})}$

- **Our goal:**
  - Can we now hope to learn $\theta_{i,t}^{(1)}$ – the conditional mean parameter for a unit with factor $u_i^{(1)}$ at decision time with factor $v_t^{(1)}$?
  - If yes, we can then also estimate $\theta_{i,t}^{(1)} - \theta_{i,t}^{(0)}$: the "treatment effect" for unit i at time t.

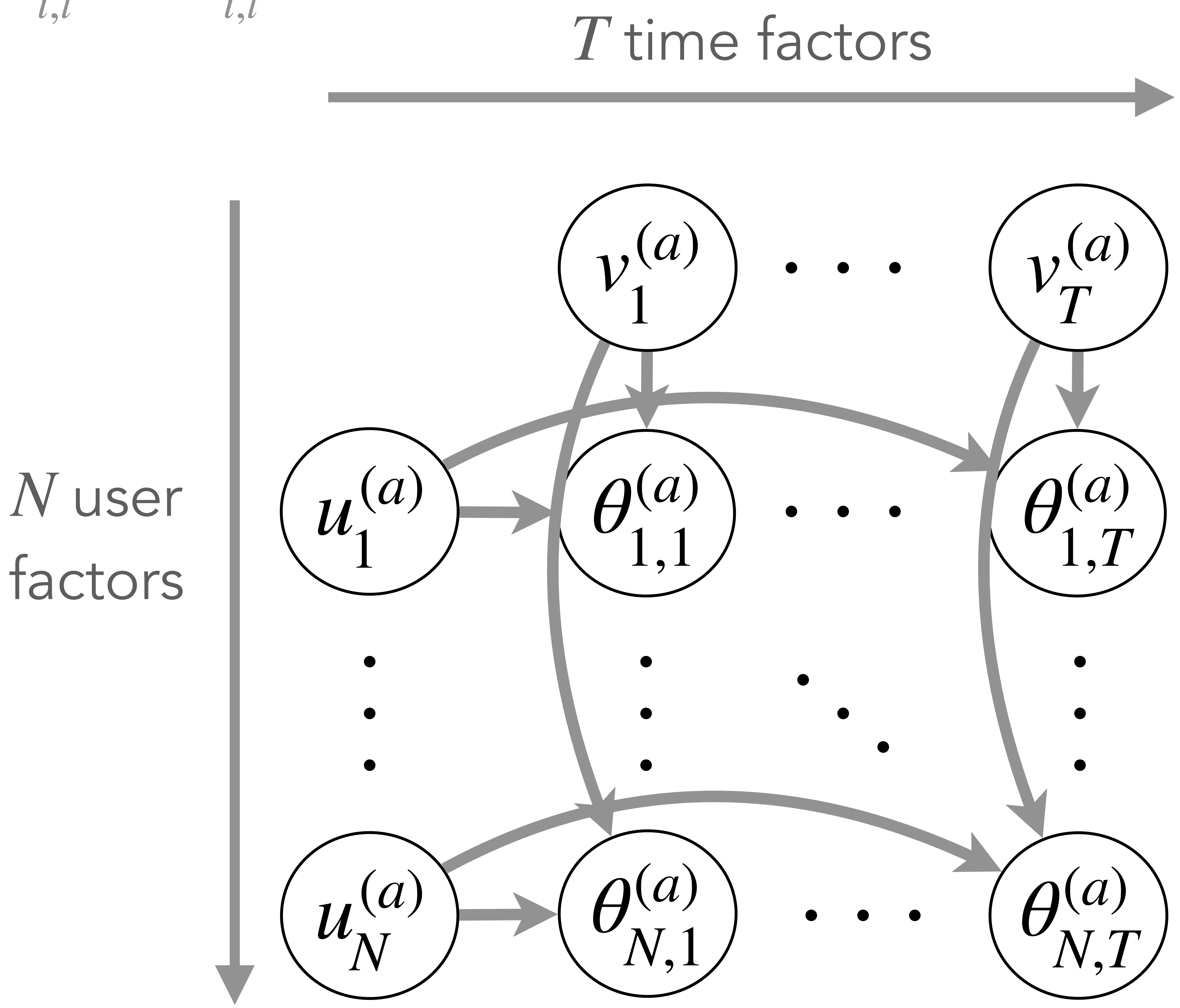
Another look at the factor model

$$R_{i,t} = R_{i,t}(A_{i,t}) \triangleq \theta_{i,t}^{(A_{i,t})} + \varepsilon_{i,t}^{(A_{i,t})}$$

$$\theta_{i,t}^{(a)} \triangleq \langle u_i^{(a)}, v_t^{(a)} \rangle$$

user factor (e.g., personal traits)

time factor (e.g., societal, weather changes)



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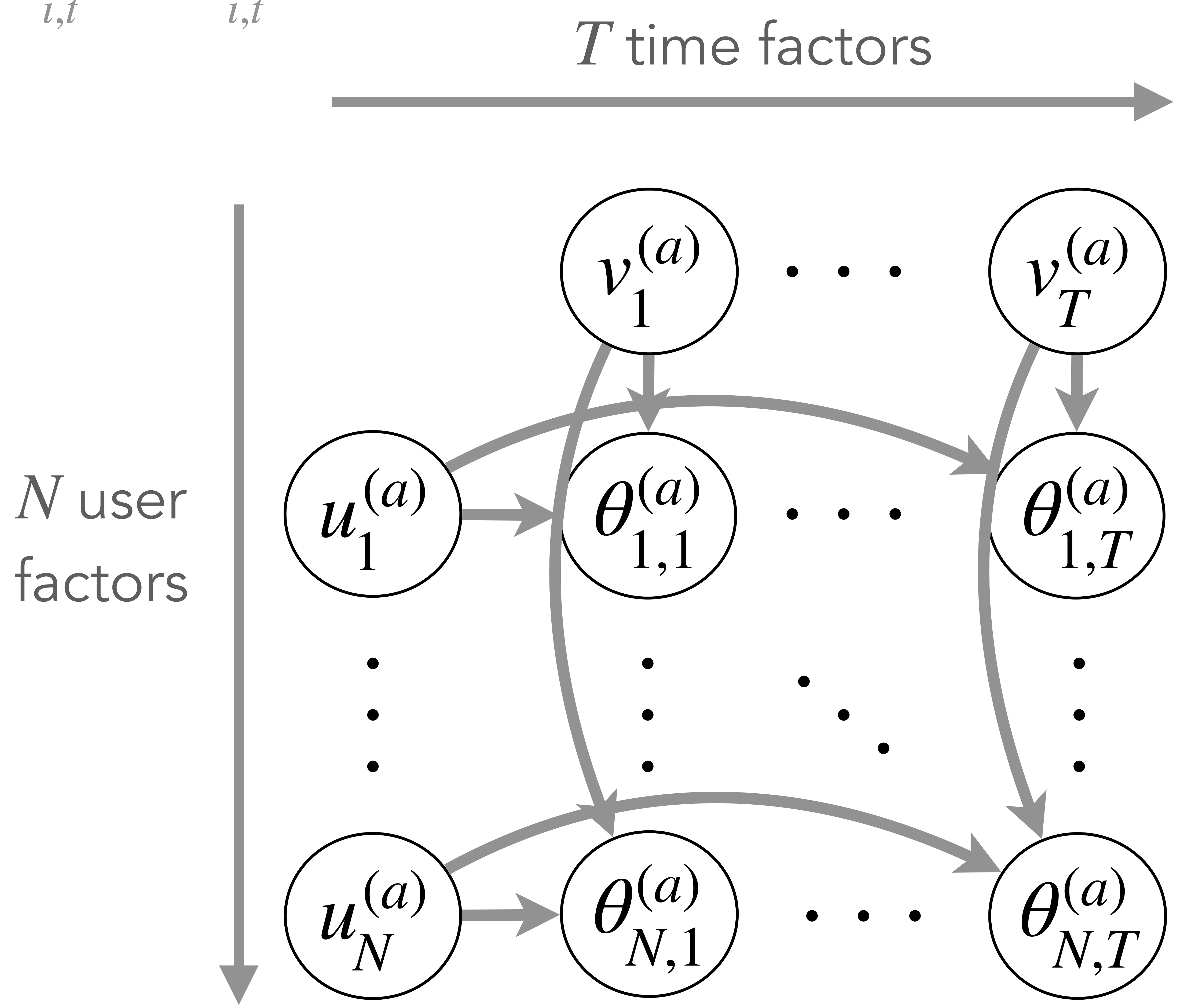
Factor model also a form of dimensionality reduction for number of unknowns



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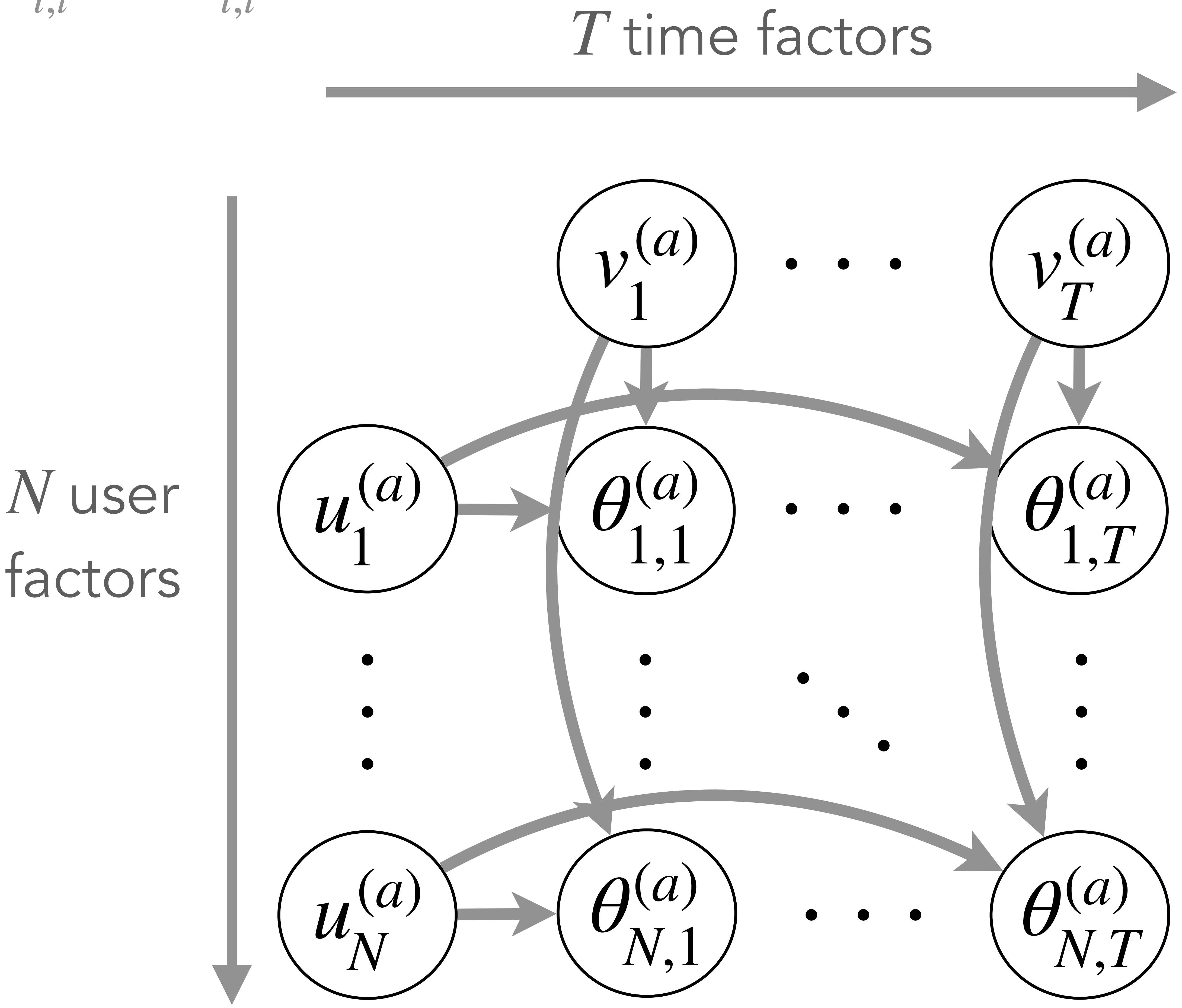
Food for thought:
Is our goal similar to estimating individualized margins in factored pooled bandits?



Next: User nearest neighbors estimator for treatment a

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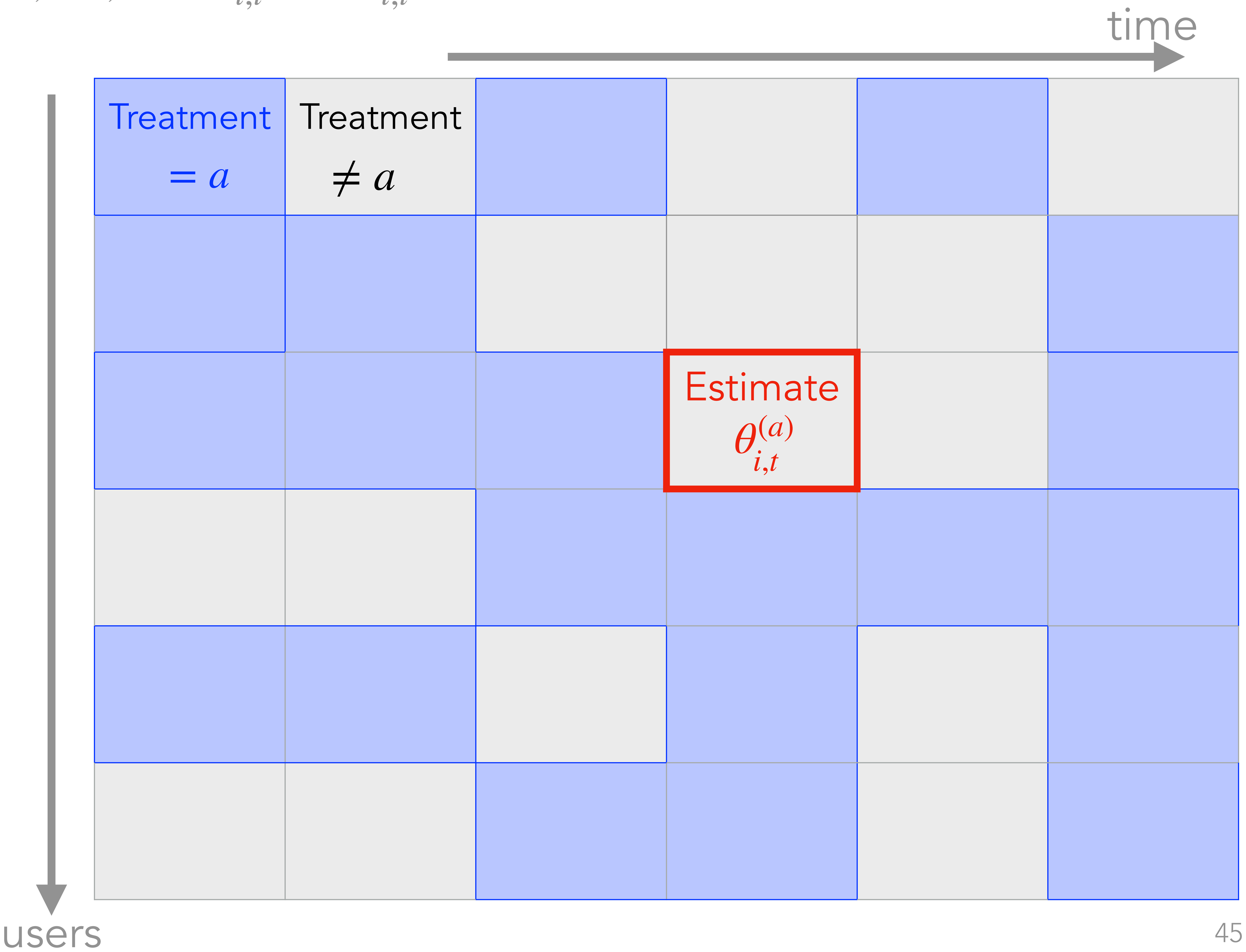


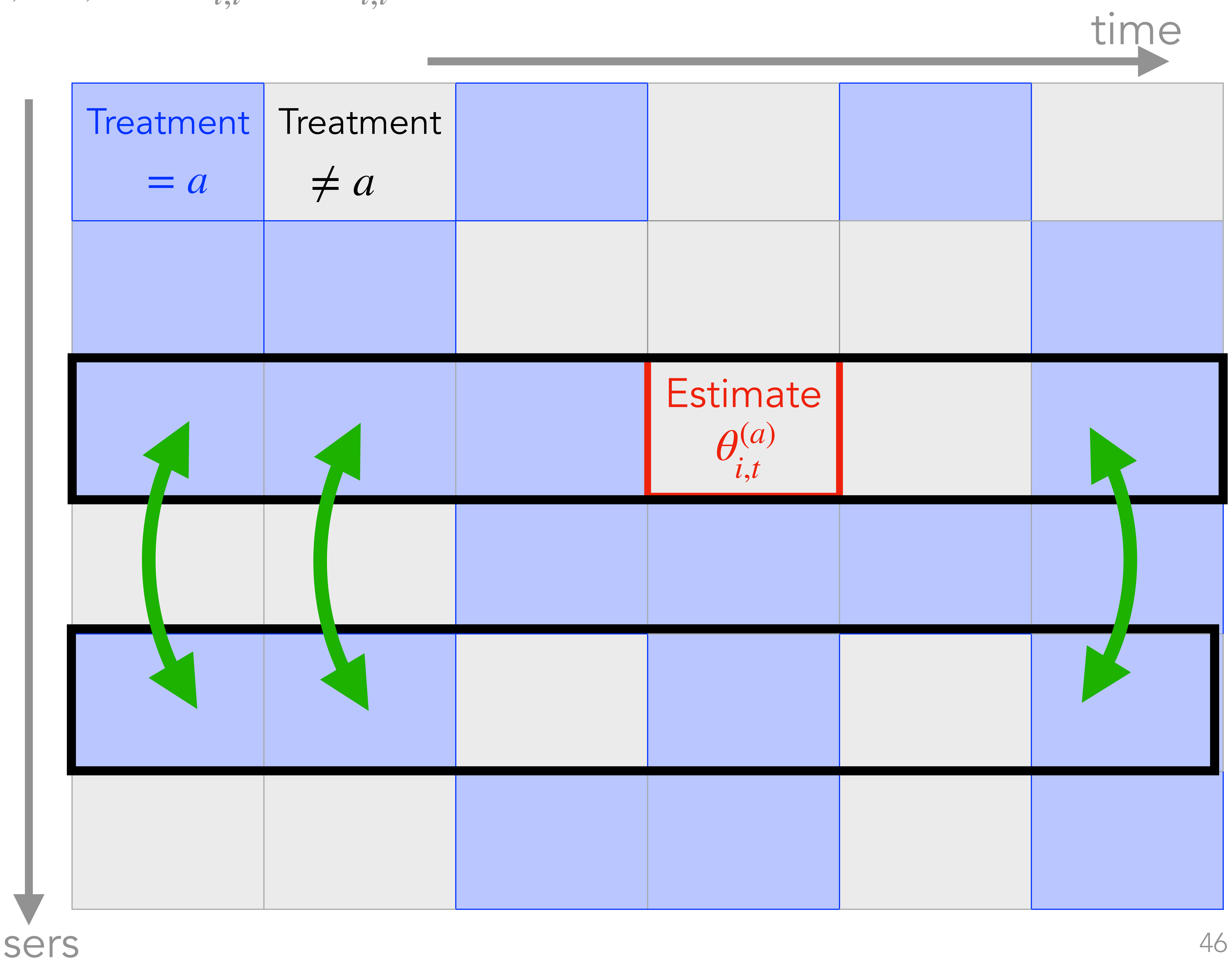
User nearest neighbors estimator for treatment a

$$R_{i,t} = R_{i,t}(A_{i,t}) \triangleq \theta_{i,t}^{(A_{i,t})} + \varepsilon_{i,t}^{(A_{i,t})}$$

1. Compute distance between user pairs i, j under treatment a **using all data**

$$\rho_{i,j}^{(a)} = \frac{\sum_{t'=1}^{T} (R_{i,t'} - R_{j,t'})^2 \cdot \mathbf{1}(A_{i,t'} = A_{j,t'} = a)}{\sum_{t'=1}^{T} \mathbf{1}(A_{i,t'} = A_{j,t'} = a)} + \frac{\hat{\sigma}_\rho}{\sqrt{\sum_{t'=1}^{T} \mathbf{1}(A_{i,t'} = A_{j,t'} = a)}}$$

$\hat{\sigma}_\rho^2$ = Variance estimate for $(\varepsilon_{i,t'} - \varepsilon_{j,t'})^2$



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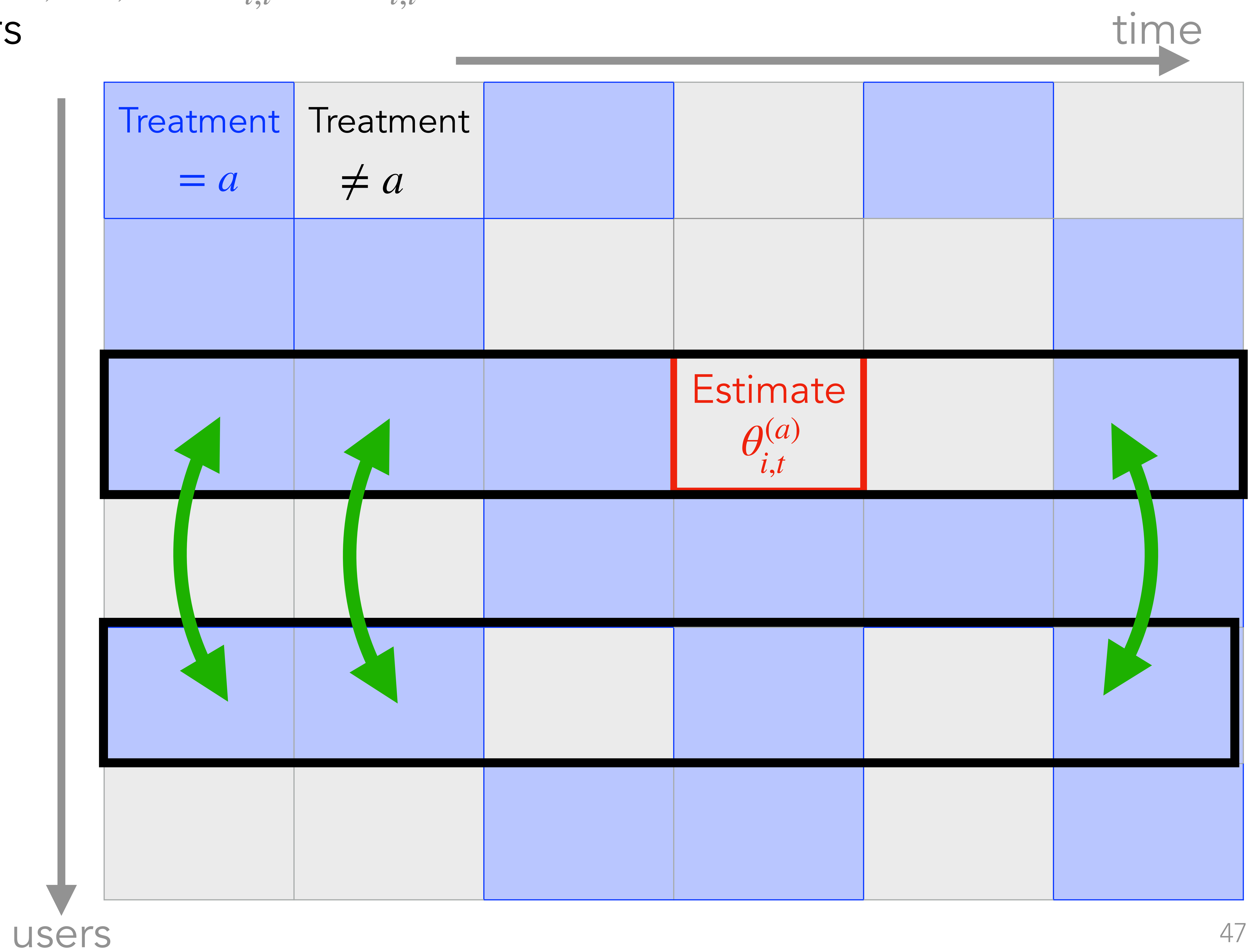
Why do we have a second term? Is there another way to operationalize it?



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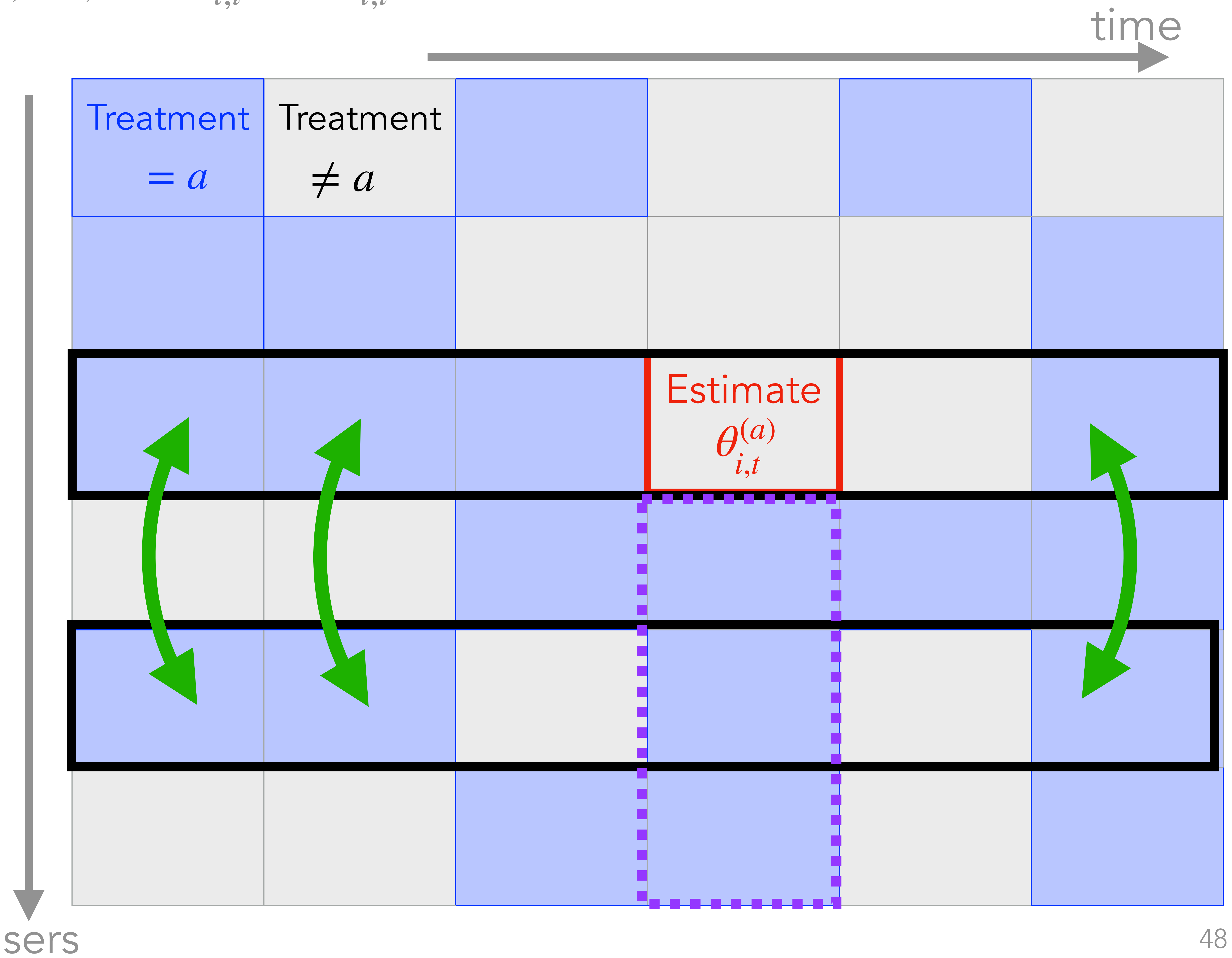
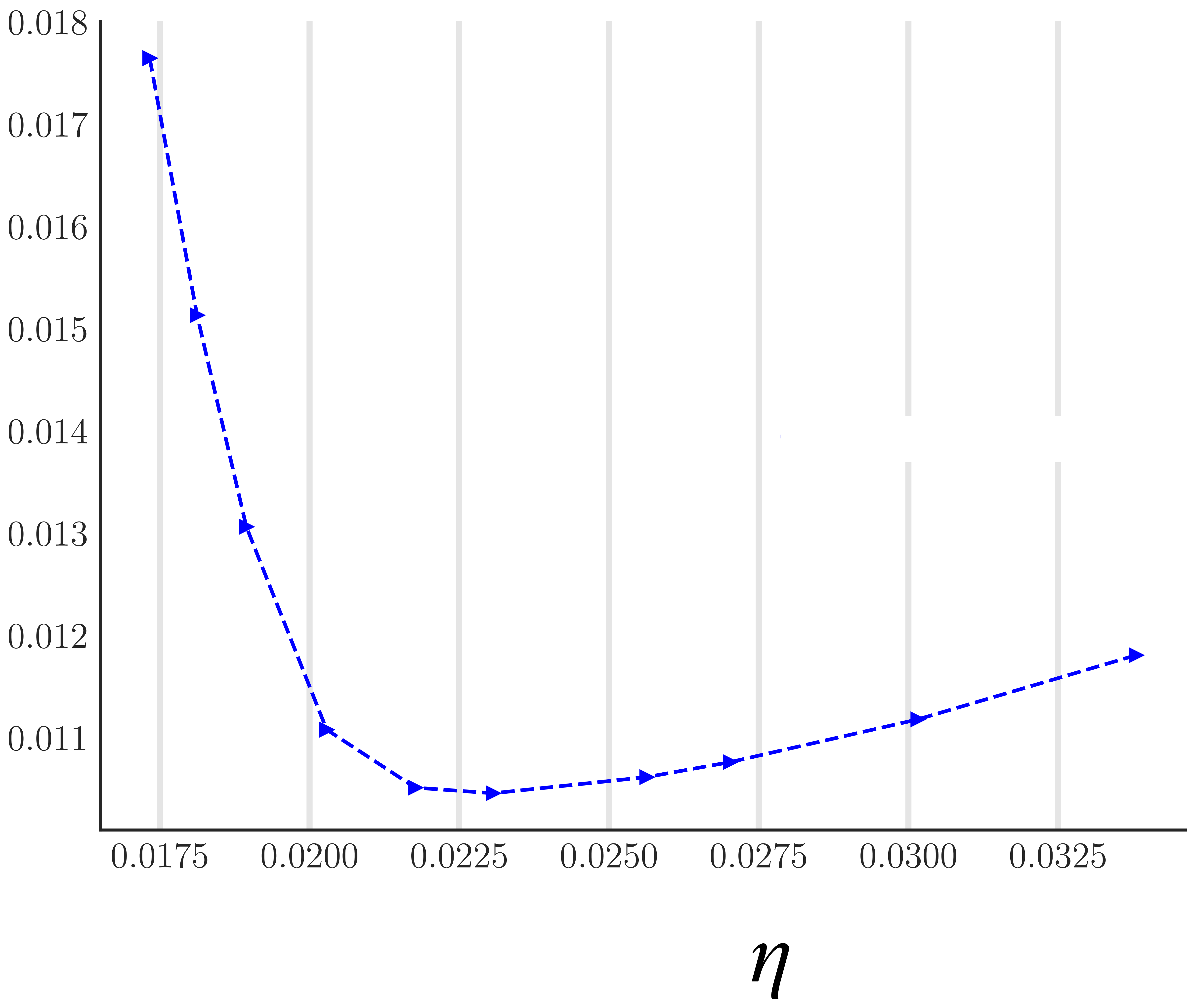
2. Average over user neighbors treated with a at time t

$$\hat{\theta}_{i,t,\text{user-NN}}^{(a)} = \frac{\sum_{j=1}^{N} R_{j,t} \mathbf{1}(\rho_{i,j}^{(a)} \leq \eta, A_{j,t} = a)}{\sum_{j=1}^{N} \mathbf{1}(\rho_{i,j}^{(a)} \leq \eta, A_{j,t} = a)}$$



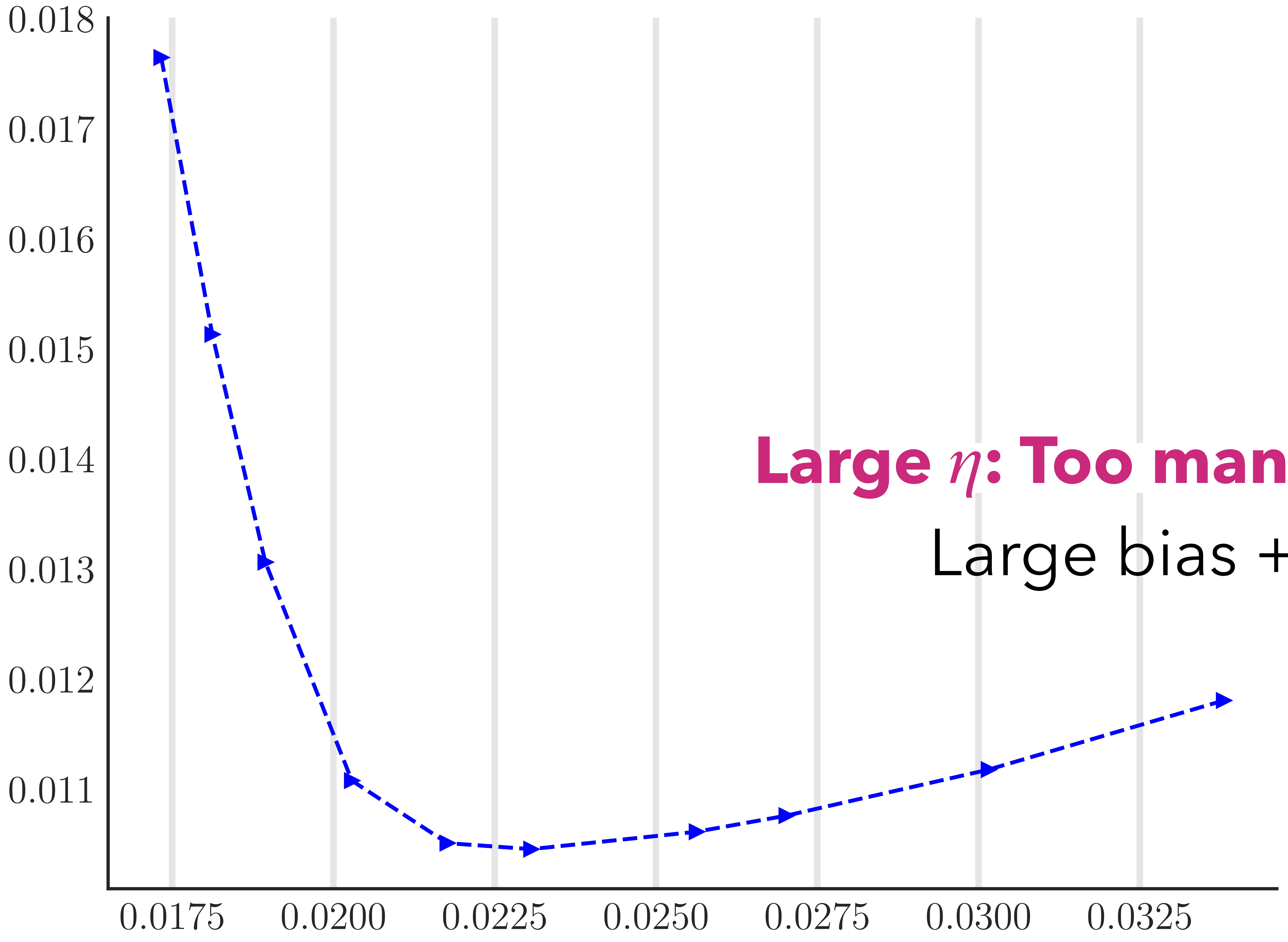
MSE for estimates on observed entries



Question: Why do we expect this U-shaped curve?

Bias-variance tradeoff for the nearest neighbors with η

MSE for estimates on observed entries



Large η: Too many "noisy" neighbors

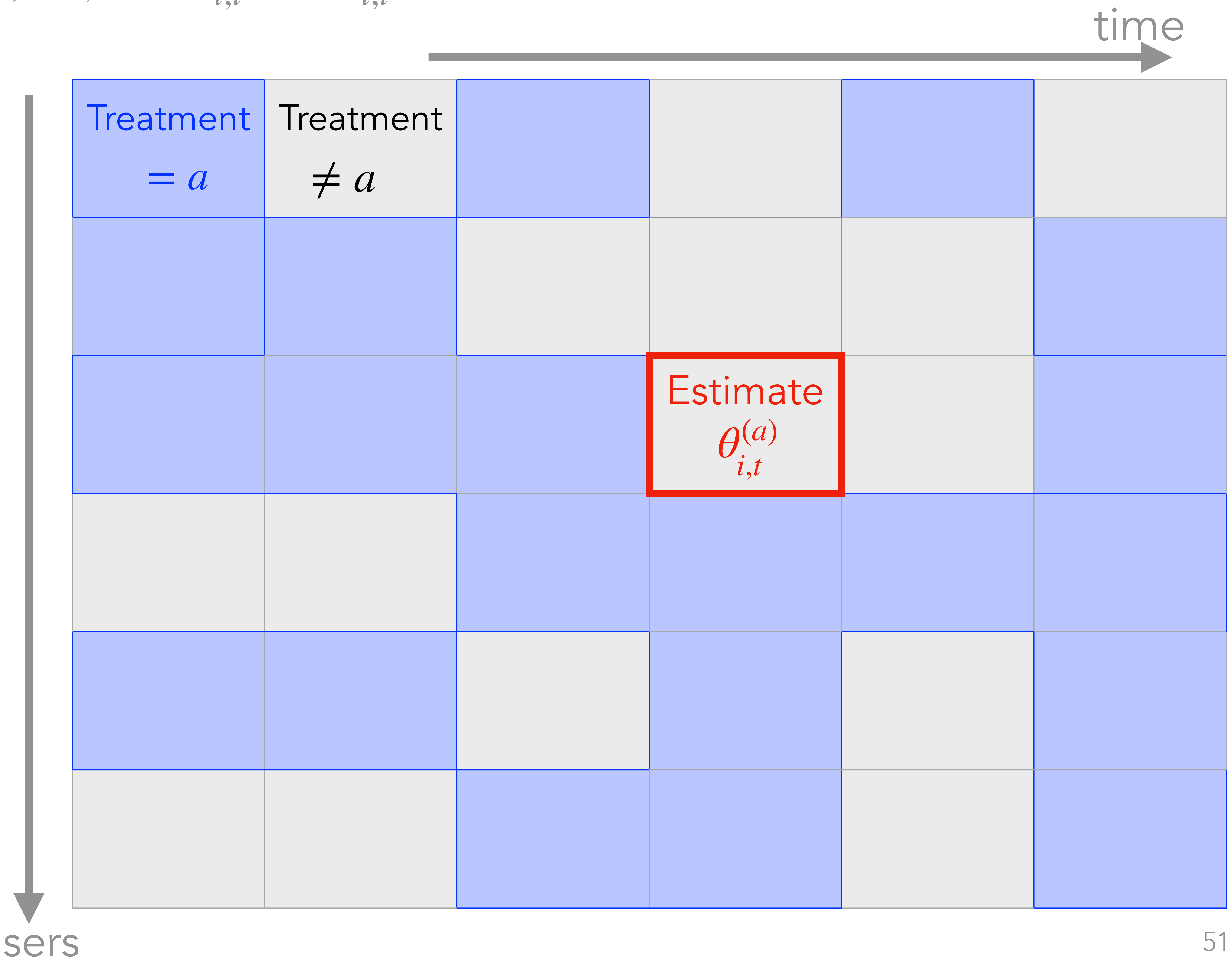
Large bias + Small variance

Small η: Few "good" neighbors η

Small bias + Large variance

Can you think of another variant of nearest neighbors?

$$R_{i,t} = R_{i,t}(A_{i,t}) \triangleq \theta_{i,t}^{(A_{i,t})} + \varepsilon_{i,t}^{(A_{i,t})}$$



Time nearest-neighbor estimators

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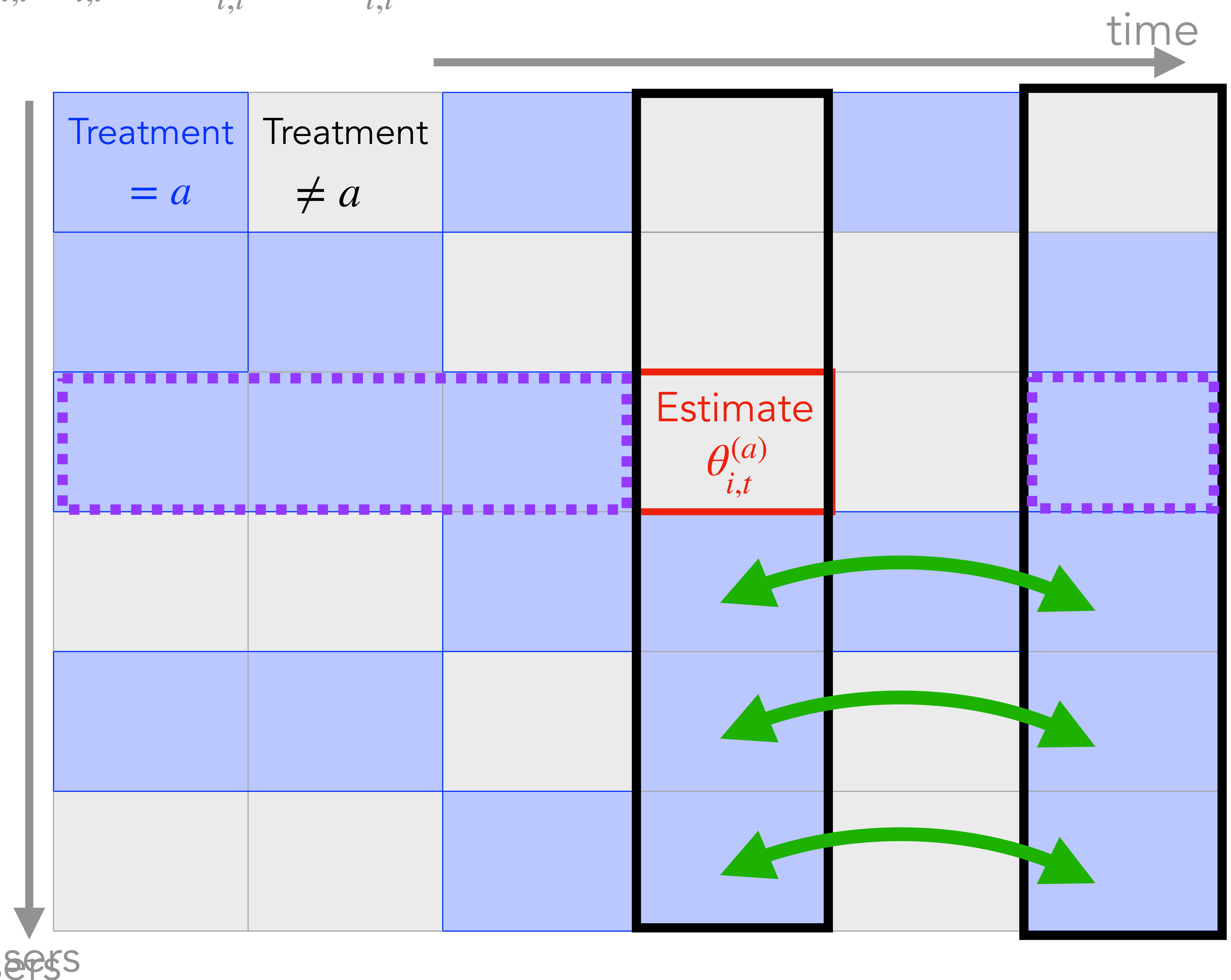
2. Average outcomes of user i at time neighbors when treated with a

$$\hat{\theta}_{i,t,\text{user-NN}}^{(a)} = \frac{\sum_{t'=1}^{T} R_{i,t'} \mathbf{1}(\rho_{t,t'}^{(a)} \leq \eta, A_{i,t'} = a)}{\sum_{t'=1}^{T} \mathbf{1}(\rho_{t,t'}^{(a)} \leq \eta, A_{i,t'} = a)}$$



users

Discussion questions

- Which of the two estimators, time NN or user NN, might you prefer?
    - Do you think these estimators are interpretable? Is there an easy way to diagnose for which (i, t) pairs, the NN estimates, are likely to be reliable?
      Hint: Think about a unique user on a unique day.
- Given all these counterfactual estimates, what kind of quantities could you investigate? How would you use them for between study analyses or to help the design of next study?

Tomorrow first lecture: Deep dive into the NN analysis

- **Theory:** When would these estimators do well? Can you design a best of the both estimators?

- **Discussion:**
  Pros and cons of the factor model, and how to generalize it to include states / delayed effects – you can begin to think tonight