Causal Excursion Effects (Individual RL Algorithms)

Monday Afternoon Session

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Just-In-Time Adaptive Interventions (JITAIs)

- Interventions that are delivered **whenever and wherever needed**
- Examples
    - Heartsteps
    - Oralytics
    - Sense2Stop
- **Goal of Micro-Randomized Trial:** To inform the design of JITAIs

Questions to Inform the Design of JITAI (from Susan's Heartsteps Slides)

- Do tailored activity suggestions have an effect at all?
- Do less and more burdensome activity suggestions work equally well?
- Does the effect of suggestions change over time? (e.g., do people get tired of them after a while?)
- When should we send suggestions for optimal effect?
  - Do they work better during certain parts of the day?
  - Do they work better when weather is good vs. bad?
- Is the suggestion effectiveness, including states in which they work, different for different types of people?

What is a Micro-Randomized Trial?

(1) Intervening on people repeatedly over an extended period of time

(2) Treatments or Actions $A_{i,t}$ are randomized.

States $S_{i,t}$ is a subset of $O_{i,t}$

Time-Varying Covariates $X_{i,t}$ is a subset of O_t

For each user $i \in [1 : n]$,

$$\underbrace{(O_{i,1}, A_{i,1}, Y_{i,2})}_{D_{i,1}} \qquad \underbrace{(O_{i,2}, A_{i,2}, Y_{i,3})}_{D_{i,2}} \qquad \dots \qquad \underbrace{(O_{i,T}, A_{i,T}, Y_{i,T+1})}_{D_{i,T}}$$

How to probabilistically choose actions?

(1) Randomize with a constant probability

- $A_{i,t} \sim \text{Bernoulli}(p)$

(2) Randomize with a constant probability when person is available

- HeartSteps V1: $p = 0.6$ when not driving, walking, etc. otherwise, $p = 0$
  - Cannot answer any causal questions about "unavailable" times
- **Drawback:** User burden and habituation due to interrupting people in states for which they are not responsive.

How to probabilistically choose actions?

(1) Ra

- A

Data Collection Policy –>

What questions you can assess after study is over

(2) Randomize with a constant probability when person is available

- HeartSteps V1: $p = 0.6$ when not driving, walking, etc. otherwise, $p = 0$
  - Cannot answer any causal questions about "unavailable" times
- **Drawback:** User burden and habituation due to interrupting people in states for which they are not responsive.

How to probabilistically choose actions?

(3) Randomization Depends on Person's State

- Interventions may only be useful in certain states
- **Example:** Mornings randomize with $p = 0.8$ and evenings randomize with $p = 0.2$
- Can be informed by prior data
- **Drawbacks:**
    - Prior data not always available, or prior data from target population may not be available
    - Distribution shift and non-stationarity between prior data and MRT

How to probabilistically choose actions?

(4) Stochastic Online Algorithm

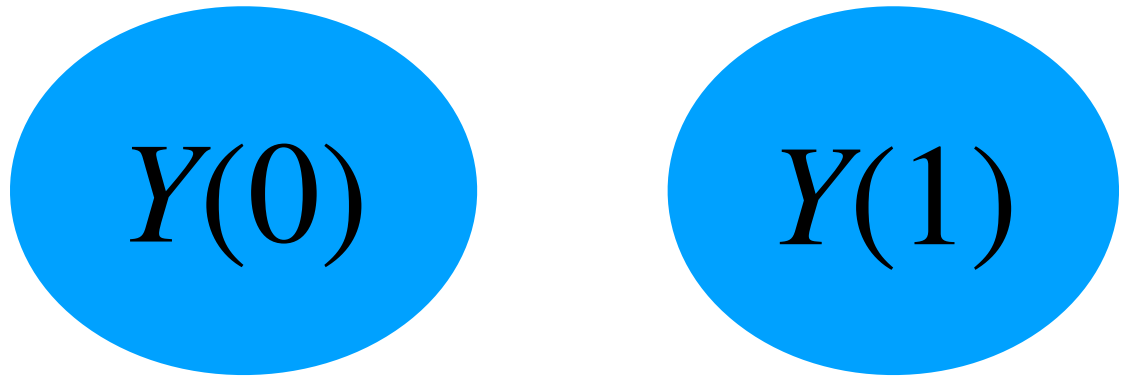
- Rather than choosing a policy apriori, specify some objective that the decision making algorithm optimizes online
- **Example:** Use posterior sampling RL algorithm to bias the randomization in favor of actions that should maximize rewards (Heartsteps V2/V3)
- **Example:** Heartsteps anti-sedentary messages - spread treatments uniformly across anti-sedentary times

Suppose we ran an MRT…

Do the treatments differentially impact the proximal outcome?

- In certain states?
- For people with certain traits?
- On average?

Standard Treatment Effect



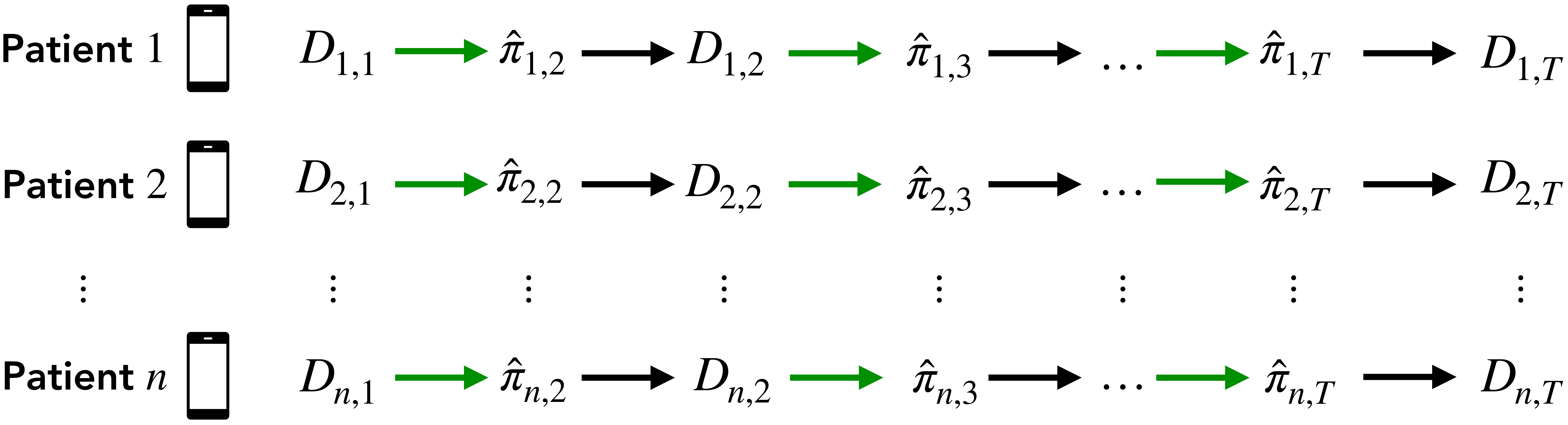
$$\mathbb{E}\left[Y(1) - Y(0) \,|\, X = x\right]$$

$D_{i,t} \triangleq \left(O_{i,t}, A_{i,t}, R_{i,t+1}\right)$

Longitudinal Patient Data

Algorithm Update

Data Collection



Outcomes are dependent over time within for each patient

Issues in the MRT setting…

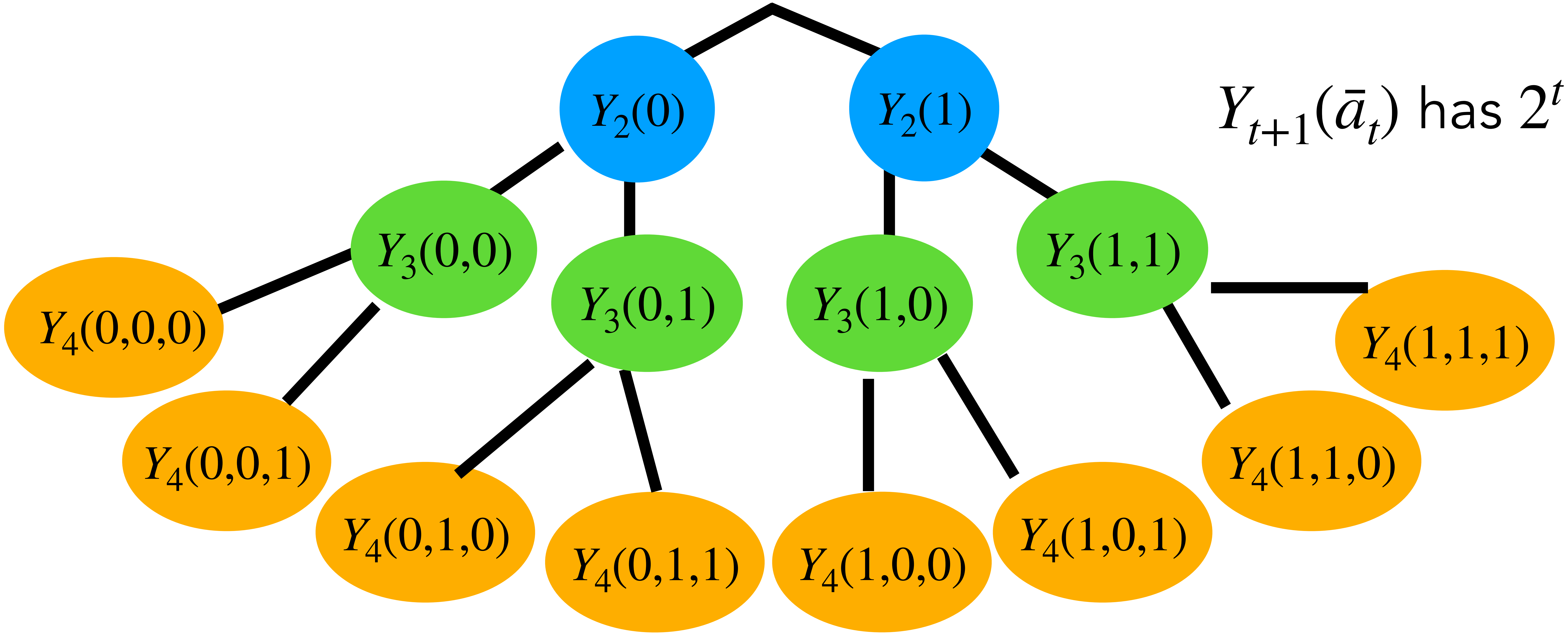
- Delayed effects of treatments
  - Responsiveness to treatment today depends on previous treatments
- Non-Stationarity
  - Responsiveness to treatment changes over time

Not Captured by "Standard Treatment Effect"

$Y(0)$ $Y(1)$

$$\mathbb{E}\left[Y(1) - Y(0) \mid X = x\right]$$

Longitudinal Data Setting: Potential Outcomes



$Y_{t+1}(\bar{a}_t)$ has 2^t possibilities

(1) Patients (potential outcome tree) drawn i.i.d. from a population

(2) Patients "tree" of potential outcomes:

$$\left\{ O_{i,t}(\bar{a}_{t-1}), Y_{i,t+1}(\bar{a}_t) : \bar{a}_t \in \{0,1\}^t \right\}_{t=1}^{T}$$

Longitudinal Data Setting: Treatment Effects

Do the treatments differentially impact the proximal outcome on average given time-varying covariates x?

$$\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, 1) - Y_{t+1}(\bar{A}_{t-1}, 0) \,|\, X_t = x\right]$$

where $\bar{A}_{t-1} = \{A_1, A_2, \ldots, A_{t-1}\}$

Averages over randomness in

(1) Draw of patient from population (potential outcomes)

(2) Randomness in previous action selection $\bar{A}_{t-1}$, aka "behavior policy"

Interpretation as Causal "Excursion" Effects

$$\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1},1) - Y_{t+1}(\bar{A}_{t-1},0)\,|\,X_t = x\right]$$

where $\bar{A}_{t-1} = \{A_1, A_2, \ldots, A_{t-1}\}$

- Above represents the effect of treating vs not treating given time-varying covariates x
    - At time t, when the behavior policy is used to select previous actions $\bar{A}_{t-1}$
- Represents the effect of taking an "one time-step excursion" from the behavior policy

Inferential Goal: Causal Excursion Effect

$$\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1},1) - Y_{t+1}(\bar{A}_{t-1},0) \,|\, X_t = x\right] = f(x)^\top \theta$$

where $\bar{A}_{t-1} = \{A_1, A_2, \ldots, A_{t-1}\}$ and $f(x)$ is a feature mapping

We are interested in the best fitting linear model, i.e., some $\theta^\star$

Small Group Discussion Questions

(1) Recall the model $\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1},1) - Y_{t+1}(\bar{A}_{t-1},0) \,|\, X_t = x\right] = f(x)^\top \theta$. The HeartSteps RL algorithm had a different model of the rewards / outcomes. How is this coherent? **What is the purpose of having an after study analyses model that is separate RL algorithm model?**

(2) Are there other causal excursion effects that one might be interested in besides $\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1},1) - Y_{t+1}(\bar{A}_{t-1},0) \,|\, X_t = x\right]$?

We are given the following MRT dataset…

Simple behavior policy

- If participant has been recently physically active, randomize with $p = 0.3$
- Otherwise, randomize with $p = 0.5$

Task: We suppose the following model for the excursion effect

$$\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, 1) - Y_{t+1}(\bar{A}_{t-1}, 0) \,|\, X_t = x\right] = f(x)^\top \theta$$

How can we estimate some best fitting θ?

Form a model for $\mathbb{E}\left[Y_{t+1} \,|\, H_{t-1}, X_t\right]$

We can form a model for Y_{t+1}

$$\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, A_t) \,|\, H_{t-1}, X_t, A_t\right] = g(H_{t-1}, X_t)^\top \eta + A_t f(X_t)^\top \theta$$

- Model for excursion effect

$$\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, 1) - Y_{t+1}(\bar{A}_{t-1}, 0) \,|\, H_{t-1}, X_t\right] = f(X_t)^\top \theta$$

- Model for baseline outcome

$$\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, 0) \,|\, H_{t-1}, X_t\right] = g(H_{t-1}, X_t)^\top \eta$$

Fit model for $\mathbb{E}\left[Y_{t+1} \,|\, H_{t-1}, X_t\right]$

Least Squares Loss:

$$\ell(H_{i,t}; \eta, \theta) \triangleq \sum_{t=1}^{T} \left(Y_{i,t+1} - g(H_{i,t-1}, X_{i,t})^\top \eta - A_{i,t} f(X_{i,t})^\top \theta\right)^2$$

Loss Minimizer:

$$(\hat{\eta}, \hat{\theta}) = \text{argmin}_{\eta,\theta} \, \frac{1}{n} \sum_{i=1}^{n} \ell(H_{i,t}; \eta, \theta)$$

Equivalently $(\eta, \theta) = (\hat{\eta}, \hat{\theta})$ **solves:**

$$0 = \frac{1}{n} \sum_{i=1}^{n} \frac{\partial}{\partial(\eta, \theta)} \ell(H_{i,t}; \eta, \theta) \Big|_{(\eta,\theta)}$$

Fit model for $\mathbb{E}\left[Y_{t+1} \mid H_{t-1}, X_t\right]$

Least Squares Loss:

$$\ell(H_{i,t}; \eta, \theta) \triangleq \sum_{t=1}^{T} \left(Y_{i,t+1} - g(H_{i,t-1}, X_{i,t})^\top \eta - A_{i,t} f(X_{i,t})^\top \theta\right)^2$$

When do we expect the above approach to work well vs. not?

- **Concern:** When the model for outcome under $A_t = 0$ is poorly specified $\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, 0) \mid H_{t-1}, X_t\right] = g(H_{t-1}, X_t)^\top \eta$
  - η is a nuisance parameter
- **Concern:** Excursion effect model $\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, 1) - Y_{t+1}(\bar{A}_{t-1}, 0) \mid H_{t-1}, X_t\right]$ depends other entries in H_{t-1} (besides X_t)

Estimation with Nuisance Parameter

Ideal Scenario: Estimate θ consistently even if our model for the baseline outcome $\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1},0)\,|\,H_{t-1},X_t\right] = g(H_{t-1},X_t)^\top \eta$ is wrong

Turns out this is possible!

High-Level Idea: Rewrite the estimation criteria so that misspecification of the baseline model does not affect the estimation of θ in the limit (Neyman orthogonalization)

Action Centering

Least Squares Loss:

$$\ell(H_{i,t}; \eta, \theta) \triangleq \sum_{t=1}^{T} \left(Y_{i,t+1} - g(H_{i,t-1}, X_{i,t})^\top \eta - (A_{i,t} - \pi_{i,t}) f(X_{i,t})^\top \theta \right)^2$$

Why is this loss reasonable for estimating θ?

- $\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, A_t) \,|\, H_{t-1}, X_t \right] = g(H_{t-1}, X_t)^\top \eta$ now model of **average** reward
- $\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1}, 1) - Y_{t+1}(\bar{A}_{t-1}, 0) \,|\, X_t \right] = f(X_t)^\top \theta$

Critical Assumption: $\pi_{i,t}$ depends only on $X_{i,t}$

Why is adding action centering helpful?

$$\frac{\partial}{\partial\theta}\ell(H_{i,t};\eta,\theta) \triangleq \sum_{t=1}^{T}\left\{Y_{i,t+1} - g(H_{i,t-1},X_{i,t})^\top\eta - (A_{i,t}-\pi_{i,t})f(X_{i,t})^\top\theta\right\}(A_{i,t}-\pi_{i,t})f(X_{i,t})$$

- $\theta = \theta^\star$ solves $0 = \mathbb{E}\left[\frac{\partial}{\partial\theta}\ell(H_{i,t};\eta,\theta)\right]$
  - The solution $\theta^\star$ is not affected by η at all!!!
- Key observation:

$$\mathbb{E}\left[g(H_{i,t-1},X_{i,t})^\top\eta\left(A_{i,t}-\pi_{i,t}\right)f(X_{i,t})\middle|H_{i,t-1},X_{i,t}\right]$$
$$= g(H_{i,t-1},X_{i,t})^\top\eta\,\mathbb{E}\left[A_{i,t}-\pi_{i,t}\middle|H_{i,t-1},X_{i,t}\right]f(X_{i,t}) = 0$$

Why is adding action centering helpful?

Why include $g(H_{i,t-1}, X_{i,t})^\top \eta$ in the model at all?

- Action centering ensures the limiting $\theta^\star$ is not affected by the model $g(H_{i,t-1}, X_{i,t})^\top \eta$
- In small samples $\hat{\theta}$ could have lower variance if $g(H_{i,t-1}, X_{i,t})^\top \eta$ helps reduce noise
- Can replace $g(H_{i,t-1}, X_{i,t})^\top \eta$ with neural network / random forest using double machine learning (large data setting) [Chernozhukov, 2018]

What if the MRT data was collected with an RL algorithm?

- Complication is that $\pi_{i,t}$ no longer depends on just $X_{i,t}$, but also $H_{i,t-1}$
- Need to incorporate weights to ensure that action centering trick "works"
    - i.e., only need to assume that the excursion effect model is correct $\mathbb{E}\left[Y_{t+1}(\bar{A}_{t-1},1) - Y_{t+1}(\bar{A}_{t-1},0) \,|\, X_t\right] = f(X_t)^\top \theta$
- Weights $W_{i,t} = \left(\frac{p(X_{i,t})}{\pi_{i,t}}\right)^{A_{i,t}} \left(\frac{1 - p(X_{i,t})}{1 - \pi_{i,t}}\right)^{1-A_{i,t}}$

See "*Note on Excursion Effects and Action Centering*" for formal justification

$$(\hat{\eta}, \hat{\theta}) = \operatorname{argmin}_{\eta,\theta} \frac{1}{n} \sum_{i=1}^{n} \sum_{t=1}^{T} W_{i,t} \left\{ Y_{i,t+1} - g(H_{i,t-1}, X_{i,t})^\top \eta - (A_{i,t} - p(X_{i,t})) f(X_{i,t})^\top \theta \right\}^2$$