– Variable Neighborhood Search for the Weighted Total Domination Problem and Its Application in Social Network Information Spreading: supplementary material –

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Contents

1 Partial fitness function technical details

Algorithm 1 and Algorithm 2 show the partial fitness function calculation after node addition and removal, respectively.

Algorithm 1 Function RecalcNodeAdded

Input: S: solution; $G = (V, Ef_1, f_2)$: problem instance; v: node to add; $viol$: number of nodes that have zero neighbors in S; $objValue$: objective function value of the solution S; $external$: list of sorted (by edge weights) sets of external edges w.r.t. S.
$viol_{new} \leftarrow viol$
$objValue_{new} \leftarrow objValue + f_1(v)$ // add node weight
if $external[v] \neq \emptyset$ **then** // v had a neighbor in S
    $objValue_{new} \leftarrow objValue_{new} - f_2(external[v][0])$
end if
for $e \in external[v]$ **do** // add internal weights
    $objValue_{new} \leftarrow objValue_{new} + f_2(e)$
end for
for $e = (v, v') \in external[v]$ **do**
    $weight \leftarrow f_2(e)$
    if $external[v'] \neq \emptyset$ **then**
        $viol_{new} \leftarrow viol_{new} - 1$
        if $v' \notin S$ **then** // internal already added, add only external
            $objValue_{new} \leftarrow objValue_{new} + weight$
        end if
    else
        $prev_{min\_weight} \leftarrow f_2(external[v'][0])$
        if $v' \notin S$ and $prev_{min\_weight} < weight$ **then** // v is (v')'s new nearest neighbor in S
            $objValue_{new} \leftarrow objValue_{new} - prev_{min\_weight} + weight$
        end if
    end if
end for
Output: $viol_{new} + \frac{objValue_{new}}{W_{tot}+1}$

Algorithm 2 Function RecalcNodeRemoved

Input: S: solution; $G = (V, E, f_1, f_2)$: problem instance; v: node to remove; $viol$: number of nodes that have zero neighbors in S; $objValue$: objective function value of S; $external$: list of sorted (by edge weights) sets of external edges.
$viol_{new} \leftarrow viol$
$objValue_{new} \leftarrow objValue - f_1(v)$ // subtract node weight
if $external[v] \neq \emptyset$ **then** // if node had a neighbor in S, add its external edge weight
    $objValue_{new} \leftarrow objValue_{new} + f_2(external[v][0])$
end if
for $e \in external[v]$ **do** // subtract internal edge weights
    $objValue_{new} \leftarrow objValue_{new} - f_2(edge)$
end for
for $e = (v, v') \in external[v]$ **do**
    $weight \leftarrow f_2(e)$
    if $|external[v']| = 1$ **then** // set cardinality is 1
        $viol_{new} \leftarrow viol_{new} + 1$
        if $v' \notin s$ **then** // internal edge weights are already subtracted, now subtract only external
            $objValue_{new} \leftarrow objValue_{new} - weight$
        end if
    else
        $prev_{min\_edge} = (v', u) \leftarrow external[v'][0]$
        if $v' \notin S$ and $u = v$ **then** // v is not (v')'s nearest neighbor in S anymore
            $objValue_{new} \leftarrow objValue_{new} - f_2(prev_{min\_edge}) + f_2(external[v'][1])$
        end if
    end if
end for
Output: $viol_{new} + \frac{objValue_{new}}{W_{tot}+1}$

2 Additional Results

Table 1: Detailed VNS comparison to ILP for MA-20 instances.

		ILP		VNS			
instance	*best*	*obj*	*ind.*	*t*	*obj*	*pg%*	*ind.*
MA-20-0.2-5-5-1	63	63	opt	2.4	63	0	opt
MA-20-0.2-5-5-2	58	58	opt	1.5	58	0	opt
MA-20-0.2-5-5-3	58	58	opt	1.4	58	0	opt
MA-20-0.2-5-5-4	51	51	opt	1.5	51	0	opt
MA-20-0.2-5-5-5	55	55	opt	1.4	55	0	opt
MA-20-0.5-5-5-1	44	44	opt	1.7	44	0	opt
MA-20-0.5-5-5-2	47	47	opt	1.6	47	0	opt
MA-20-0.5-5-5-3	46	46	opt	1.6	46	0	opt
MA-20-0.5-5-5-4	40	40	opt	1.5	40	0	opt
MA-20-0.5-5-5-5	41	41	opt	1.5	41	0	opt
MA-20-0.8-5-5-1	37	37	opt	1.4	37	0	opt
MA-20-0.8-5-5-2	35	35	opt	1.7	35	0	opt
MA-20-0.8-5-5-3	40	40	opt	1.6	40	0	opt
MA-20-0.8-5-5-4	34	34	opt	1.4	34	0	opt
MA-20-0.8-5-5-5	34	34	opt	1.8	34	0	opt

Table 2: Detailed VNS comparison to ILP for MA-50 instances.

		ILP		VNS			
instance	*best*	*obj*	*ind.*	*t*	*obj*	*pg%*	*ind.*
MA-50-0.2-5-5-1	111	111	opt	5.8	111	0	opt
MA-50-0.2-5-5-2	106	106	opt	4.5	106	0	opt
MA-50-0.2-5-5-3	111	111	opt	4.5	111	0	opt
MA-50-0.2-5-5-4	101	101	opt	4.6	101	0	opt
MA-50-0.2-5-5-5	108	108	opt	4.7	108	0	opt
MA-50-0.5-5-5-1	82	82	opt	5.1	82	0	opt
MA-50-0.5-5-5-2	85	85	opt	4.8	85	0	opt
MA-50-0.5-5-5-3	84	84	opt	5.5	84	0	opt
MA-50-0.5-5-5-4	82	82	opt	5.4	82	0	opt
MA-50-0.5-5-5-5	82	82	opt	5.4	82	0	opt
MA-50-0.8-5-5-1	77	77	opt	5.7	77	0	opt
MA-50-0.8-5-5-2	72	72	opt	5.7	72	0	opt
MA-50-0.8-5-5-3	74	74	opt	5.7	74	0	opt
MA-50-0.8-5-5-4	76	76	opt	5.5	76	0	opt
MA-50-0.8-5-5-5	79	79	opt	5.7	79	0	opt

Table 3: Detailed VNS comparison to ILP for MA-100 instances.

		ILP		VNS			
instance	*best*	*obj*	*ind.*	*t*	*obj*	*pg%*	*ind.*
MA-100-0.2-5-5-1	175	175	opt	16.3	175	0	opt
MA-100-0.2-5-5-2	174	174	opt	15.4	174	0	opt
MA-100-0.2-5-5-3	177	177	opt	15.2	177	0	opt
MA-100-0.2-5-5-4	169	169	opt	15.6	169	0	opt
MA-100-0.2-5-5-5	167	167	opt	15.5	167	0	opt
MA-100-0.5-5-5-1	147	147	opt	18.9	147	0	opt
MA-100-0.5-5-5-2	144	144	opt	19.8	144	0	opt
MA-100-0.5-5-5-3	147	147	opt	19.5	147	0	opt
MA-100-0.5-5-5-4	146	146	opt	20.5	146	0	opt
MA-100-0.5-5-5-5	139	139	opt	20.9	139	0	opt
MA-100-0.8-5-5-1	136	136	opt	23.5	136	0	opt
MA-100-0.8-5-5-2	140	140	opt	21	140	0	opt
MA-100-0.8-5-5-3	141	141	opt	22.5	141	0	opt
MA-100-0.8-5-5-4	141	141	opt	22.9	141	0	opt
MA-100-0.8-5-5-5	134	134	opt	22.2	134	0	opt

Table 4: Detailed VNS comparison to GRASP and Grasp+Ga for AMS-75 instances.

		ILP		VNS				GRASP				Grasp+Ga			
instance	*best*	*obj*	*ind.*	*t*	*obj*	*pg%*	*ind.*	*t*	*obj*	*pg%*	*ind.*	*t*	*obj*	*pg%*	*ind.*
AMS-75-0.2-10-50-1	686	686	opt	10.9	686	0	opt	1	769	12.1		5	686	0	opt
AMS-75-0.2-10-50-2	770	770	opt	10	770	0	opt	1	871	13.12		6	794	3.12	
AMS-75-0.2-10-50-3	661	661	opt	10.1	661	0	opt	1	765	15.73		6	661	0	opt
AMS-75-0.2-10-50-4	703	703	opt	10.7	703	0	opt	1	762	8.39		7	740	5.26	
AMS-75-0.2-10-50-5	758	758	opt	9.8	758	0	opt	1	857	13.06		6	779	2.77	
AMS-75-0.2-25-25-1	498	498	opt	10.2	498	0	opt	1	556	11.65		6	504	1.2	
AMS-75-0.2-25-25-2	546	546	opt	9.6	546	0	opt	1	607	11.17		6	546	0	opt
AMS-75-0.2-25-25-3	518	518	opt	9.5	518	0	opt	1	603	16.41		5	518	0	opt
AMS-75-0.2-25-25-4	498	498	opt	9.7	498	0	opt	1	521	4.62		6	498	0	opt
AMS-75-0.2-25-25-5	513	513	opt	9.9	513	0	opt	1	526	2.53		6	513	0	opt
AMS-75-0.2-50-10-1	339	339	opt	9.1	339	0	opt	1	340	0.29		6	339	0	opt
AMS-75-0.2-50-10-2	382	382	opt	8.3	382	0	opt	1	414	8.38		5	382	0	opt
AMS-75-0.2-50-10-3	335	335	opt	9.1	335	0	opt	1	341	1.79		5	341	1.79	
AMS-75-0.2-50-10-4	333	333	opt	8.8	333	0	opt	1	338	1.5		6	333	0	opt
AMS-75-0.2-50-10-5	347	347	opt	9	347	0	opt	1	353	1.73		6	347	0	opt
AMS-75-0.5-10-50-1	581	581	opt	13.1	581	0	opt	1	590	1.55		13	581	0	opt
AMS-75-0.5-10-50-2	602	602	opt	12.1	602	0	opt	1	641	6.48		11	602	0	opt
AMS-75-0.5-10-50-3	545	545	opt	13.3	545	0	opt	1	545	0	opt	10	545	0	opt
AMS-75-0.5-10-50-4	540	540	opt	12.7	540	0	opt	1	580	7.41		10	540	0	opt
AMS-75-0.5-10-50-5	519	519	opt	12.7	519	0	opt	1	551	6.17		10	519	0	opt
AMS-75-0.5-25-25-1	387	387	opt	12.2	387	0	opt	1	402	3.88		10	387	0	opt
AMS-75-0.5-25-25-2	384	384	opt	11.5	384	0	opt	1	413	7.55		10	384	0	opt
AMS-75-0.5-25-25-3	362	362	opt	11.6	362	0	opt	1	380	4.97		10	362	0	opt
AMS-75-0.5-25-25-4	366	366	opt	11.3	366	0	opt	1	371	1.37		9	371	1.37	
AMS-75-0.5-25-25-5	331	331	opt	12.2	331	0	opt	1	331	0	opt	10	331	0	opt
AMS-75-0.5-50-10-1	240	240	opt	10.5	240	0	opt	1	244	1.67		9	240	0	opt
AMS-75-0.5-50-10-2	238	238	opt	10.5	238	0	opt	1	245	2.94		9	238	0	opt
AMS-75-0.5-50-10-3	215	215	opt	10.5	215	0	opt	1	215	0	opt	9	215	0	opt
AMS-75-0.5-50-10-4	235	235	opt	10.6	235	0	opt	1	235	0	opt	9	235	0	opt
AMS-75-0.5-50-10-5	206	206	opt	11	206	0	opt	1	206	0	opt	8	206	0	opt
AMS-75-0.8-10-50-1	571	571	opt	13.8	571	0	opt	2	613	7.36		16	571	0	opt
AMS-75-0.8-10-50-2	520	520	opt	14.2	520	0	opt	2	520	0	opt	15	520	0	opt
AMS-75-0.8-10-50-3	543	543	opt	14.4	543	0	opt	2	543	0	opt	15	543	0	opt
AMS-75-0.8-10-50-4	571	571	opt	13.9	571	0	opt	2	571	0	opt	15	571	0	opt
AMS-75-0.8-10-50-5	509	509	opt	14.4	509	0	opt	2	509	0	opt	17	509	0	opt
AMS-75-0.8-25-25-1	357	357	opt	13.3	357	0	opt	2	360	0.84		15	357	0	opt
AMS-75-0.8-25-25-2	338	338	opt	13.2	338	0	opt	2	356	5.33		15	338	0	opt
AMS-75-0.8-25-25-3	323	323	opt	13	323	0	opt	2	323	0	opt	13	323	0	opt
AMS-75-0.8-25-25-4	345	345	opt	13.7	345	0	opt	2	345	0	opt	13	345	0	opt
AMS-75-0.8-25-25-5	311	311	opt	13.5	311	0	opt	2	311	0	opt	15	311	0	opt
AMS-75-0.8-50-10-1	182	182	opt	12.5	182	0	opt	2	182	0	opt	14	182	0	opt
AMS-75-0.8-50-10-2	188	188	opt	12.3	188	0	opt	2	188	0	opt	11	188	0	opt
AMS-75-0.8-50-10-3	191	191	opt	12	191	0	opt	2	191	0	opt	11	191	0	opt
AMS-75-0.8-50-10-4	196	196	opt	12.1	196	0	opt	2	196	0	opt	12	196	0	opt
AMS-75-0.8-50-10-5	192	192	opt	12.2	192	0	opt	2	192	0	opt	15	192	0	opt

Table 5: Detailed VNS comparison to GRASP and Grasp+Ga for AMS-100 instances.

		ILP		VNS				GRASP				Grasp+Ga			
instance	*best*	*obj*	*ind.*	*t*	*obj*	*pg%*	*ind.*	*t*	*obj*	*pg%*	*ind.*	*t*	*obj*	*pg%*	*ind.*
AMS-100-0.2-10-50-1	873	873	opt	19.3	873	0	opt	1	930	6.53		12	873	0	opt
AMS-100-0.2-10-50-2	944	944	opt	17.7	944	0	opt	1	983	4.13		13	944	0	opt
AMS-100-0.2-10-50-3	878	878	opt	18	878	0	opt	1	905	3.08		11	878	0	opt
AMS-100-0.2-10-50-4	837	837	opt	18.3	837	0	opt	1	879	5.02		11	837	0	opt
AMS-100-0.2-10-50-5	840	840	opt	17.8	840	0	opt	1	907	7.98		12	870	3.57	
AMS-100-0.2-25-25-1	591	591	opt	18.2	591	0	opt	1	591	0	opt	12	591	0	opt
AMS-100-0.2-25-25-2	653	653	opt	15.8	653	0	opt	1	687	5.21		11	655	0.31	
AMS-100-0.2-25-25-3	612	612	opt	16.5	615	0.49		1	648	5.88		12	616	0.65	
AMS-100-0.2-25-25-4	552	552	opt	15.9	552	0	opt	1	602	9.06		11	552	0	opt
AMS-100-0.2-25-25-5	606	606	opt	16.8	606	0	opt	1	646	6.6		12	607	0.17	
AMS-100-0.2-50-10-1	418	418	opt	15.2	418	0	opt	1	422	0.96		12	420	0.48	
AMS-100-0.2-50-10-2	447	447	opt	14.3	447	0	opt	1	472	5.59		11	456	2.01	
AMS-100-0.2-50-10-3	419	419	opt	15.2	419	0	opt	1	427	1.91		11	419	0	opt
AMS-100-0.2-50-10-4	403	403	opt	15	403	0	opt	1	418	3.72		12	410	1.74	
AMS-100-0.2-50-10-5	375	375	opt	15.7	375	0	opt	1	379	1.07		13	379	1.07	
AMS-100-0.5-10-50-1	743	743	opt	22.4	743	0	opt	2	749	0.81		26	749	0.81	
AMS-100-0.5-10-50-2	698	698	opt	21.5	698	0	opt	3	705	1		25	700	0.29	
AMS-100-0.5-10-50-3	699	699	opt	22.1	699	0	opt	3	730	4.43		24	718	2.72	
AMS-100-0.5-10-50-4	726	726	opt	22.2	726	0	opt	2	775	6.75		26	726	0	opt
AMS-100-0.5-10-50-5	702	702	opt	22.4	702	0	opt	2	743	5.84		25	702	0	opt
AMS-100-0.5-25-25-1	461	461	opt	20.3	461	0	opt	3	461	0	opt	25	461	0	opt
AMS-100-0.5-25-25-2	437	437	opt	21.3	437	0	opt	2	448	2.52		19	437	0	opt
AMS-100-0.5-25-25-3	434	434	opt	22.2	434	0	opt	3	443	2.07		22	434	0	opt
AMS-100-0.5-25-25-4	482	482	opt	20.5	482	0	opt	2	489	1.45		25	482	0	opt
AMS-100-0.5-25-25-5	456	456	opt	20.4	456	0	opt	3	470	3.07		23	457	0.22	
AMS-100-0.5-50-10-1	260	260	opt	18.2	260	0	opt	2	260	0	opt	22	260	0	opt
AMS-100-0.5-50-10-2	271	271	opt	18.9	271	0	opt	2	271	0	opt	21	271	0	opt
AMS-100-0.5-50-10-3	283	283	opt	20.6	283	0	opt	3	283	0	opt	21	283	0	opt
AMS-100-0.5-50-10-4	291	291	opt	18.3	291	0	opt	2	296	1.72		22	291	0	opt
AMS-100-0.5-50-10-5	269	269	opt	18.5	269	0	opt	2	269	0	opt	21	269	0	opt
AMS-100-0.8-10-50-1	730	730	TL	25.2	730	0	best	4	730	0	best	39	730	0	best
AMS-100-0.8-10-50-2	683	683	opt	23.5	683	0	opt	4	688	0.73		37	683	0	opt
AMS-100-0.8-10-50-3	718	718	opt	24.8	718	0	opt	4	718	0	opt	37	718	0	opt
AMS-100-0.8-10-50-4	709	709	opt	28.1	709	0	opt	4	709	0	opt	41	709	0	opt
AMS-100-0.8-10-50-5	700	700	opt	26.2	700	0	opt	4	710	1.43		39	704	0.57	
AMS-100-0.8-25-25-1	442	442	opt	22.5	442	0	opt	5	452	2.26		40	442	0	opt
AMS-100-0.8-25-25-2	430	430	opt	23.4	430	0	opt	4	430	0	opt	32	430	0	opt
AMS-100-0.8-25-25-3	426	426	opt	23.3	426	0	opt	4	426	0	opt	36	426	0	opt
AMS-100-0.8-25-25-4	428	428	opt	22.7	428	0	opt	4	428	0	opt	35	428	0	opt
AMS-100-0.8-25-25-5	432	432	opt	23	432	0	opt	4	432	0	opt	42	432	0	opt
AMS-100-0.8-50-10-1	259	259	opt	21	259	0	opt	4	259	0	opt	32	259	0	opt
AMS-100-0.8-50-10-2	246	246	opt	20.6	246	0	opt	4	246	0	opt	9	246	0	opt
AMS-100-0.8-50-10-3	238	238	opt	21.6	238	0	opt	4	238	0	opt	34	238	0	opt
AMS-100-0.8-50-10-4	253	253	opt	22.2	253	0	opt	4	258	1.98		34	253	0	opt
AMS-100-0.8-50-10-5	248	248	opt	22.8	248	0	opt	5	250	0.81		31	248	0	opt

3 Additional statistical analysis and discussion

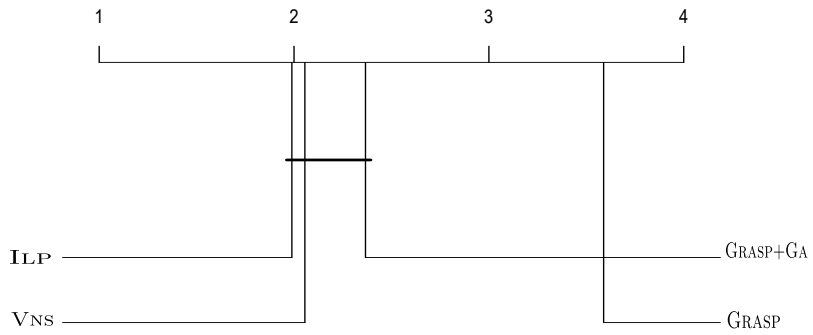

Figure 1: CD plot for the results of the instances from AMS-125.

By Figure 1 a statistical analysis for the four approaches on benchmark set AMS-125 is given by means of a CD plot. One could see that ILP and VNS are best according to the ranking and much better than VNS. However, the statistical difference between these three approaches is not statistical in terms of solution quality. Significant difference exists between them and the GRASP approach.

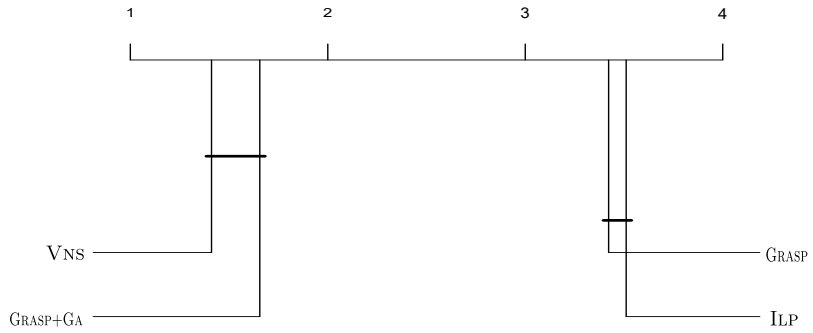

Figure 2: CD plot for the results of the instances from New-250.

Statistical comparisons w.r.t. solution quality of the four algorithms is shown in Figure 2. We conclude that the results of VNS and GRASP+VNS are significantly better than the results of the other two competitors. Average rankings of the results of VNS are better than that of the GRASP+VNS. However, difference is not significant.

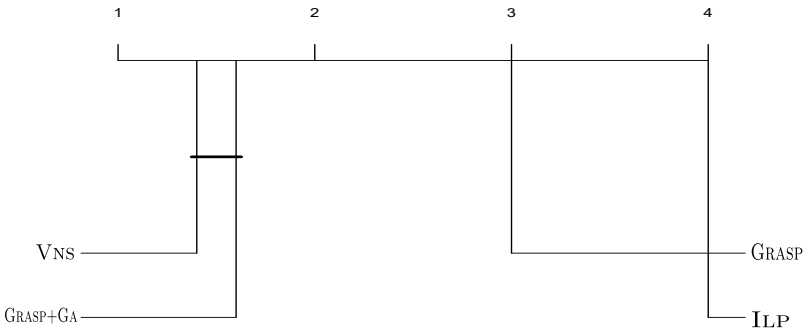

Figure 3: CD plot for the results of the instances from New-500.

Statistical comparison of the four approaches by means of a CD plot is shown in Figure 3. One can see that VNS and GRASP+GA delivers statistically better results than the other two competitors. Although the average ranking is in favour of VNS, there is no statistical difference between the results of this approach and GRASP+GA.

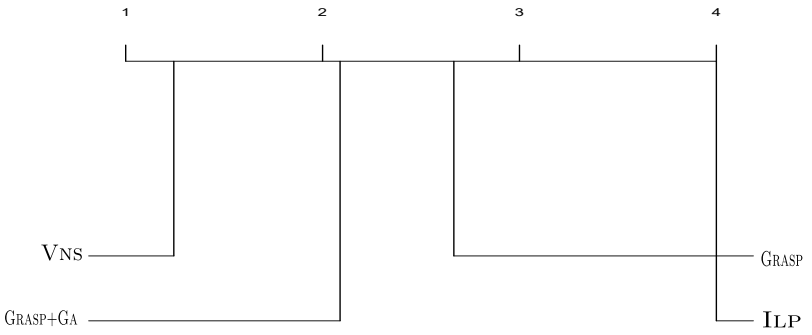

Figure 4: CD plot for the results of the instances from New-1000.

Statistical comparison of the four approaches on the 45 instances is presented by means of a CD plot given by Figure 4. One can see that the differences between the results delivered by VNS and GRASP+GA are statistically significant in favour of VNS approach.