Role of poles and zeros

- For any $s_0 \in \mathbb{C}$

$$G(s_0) = \frac{(s_0 - z_1)\cdots(s_0 - z_k)}{(s_0 - p_1)\cdots(s_0 - p_n)} = \frac{|s_0 - z_1| e^{j\angle(s_0 - z_1)} \cdots |s_0 - z_k| e^{j\angle(s_0 - z_k)}}{|s_0 - p_1| e^{j\angle(s_0 - p_1)} \cdots |s_0 - p_n| e^{j\angle(s_0 - p_n)}}$$

$$= \underbrace{\frac{|s_0 - z_1|\cdots|s_0 - z_k|}{|s_0 - p_1|\cdots|s_0 - p_n|}} e^{j\overbrace{[\angle(s_0 - z_1) + \cdots + \angle(s_0 - z_k) - \angle(s_0 - p_1) - \cdots - \angle(s_0 - p_n)]}}$$

$$= |G(s_0)| \quad e^{i\angle G(s_0)}$$

Principle of the argument

- As $s_0 \in \mathbb{C}$ moves around, $G(s_0) \in \mathbb{C}$ also moves
- If travels around a closed curve $C \subseteq \mathbb{C}$, $G(s_0) \in \mathbb{C}$ will travel around another closed curve $L \subseteq \mathbb{C}$

Principle of the argument:
Assume that the curve C does not pass through any poles or zeros of $G(s)$. Let:

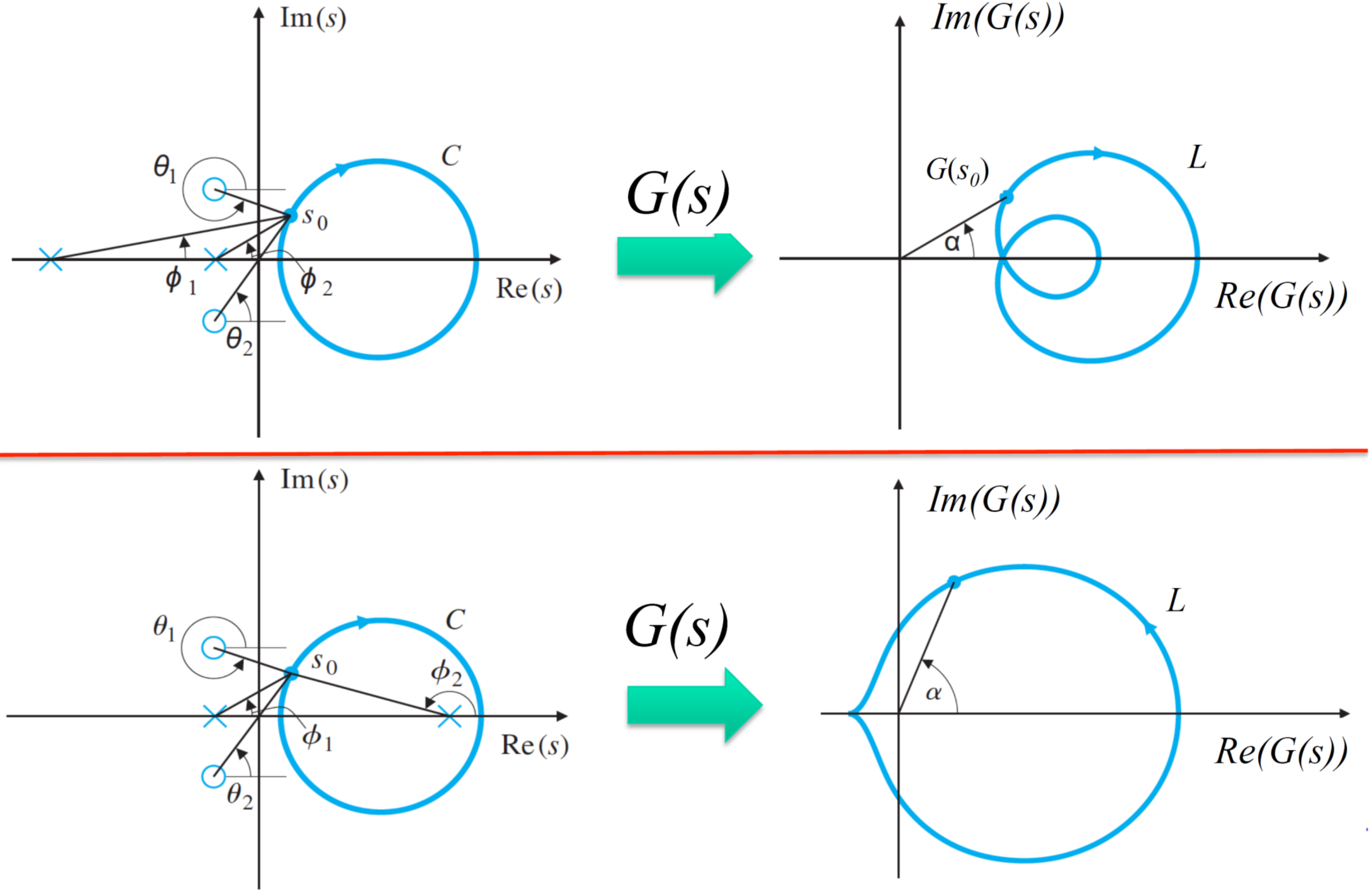
1. N = Number of times L encircles $(0,0)$ in the clockwise direction (clockwise $+1$, anticlockwise -1).
2. Z = Number of zeros of $G(s)$ encircled by C.
3. P = Number of poles of $G(s)$ encircled by C.

Then

$$N = Z - P$$

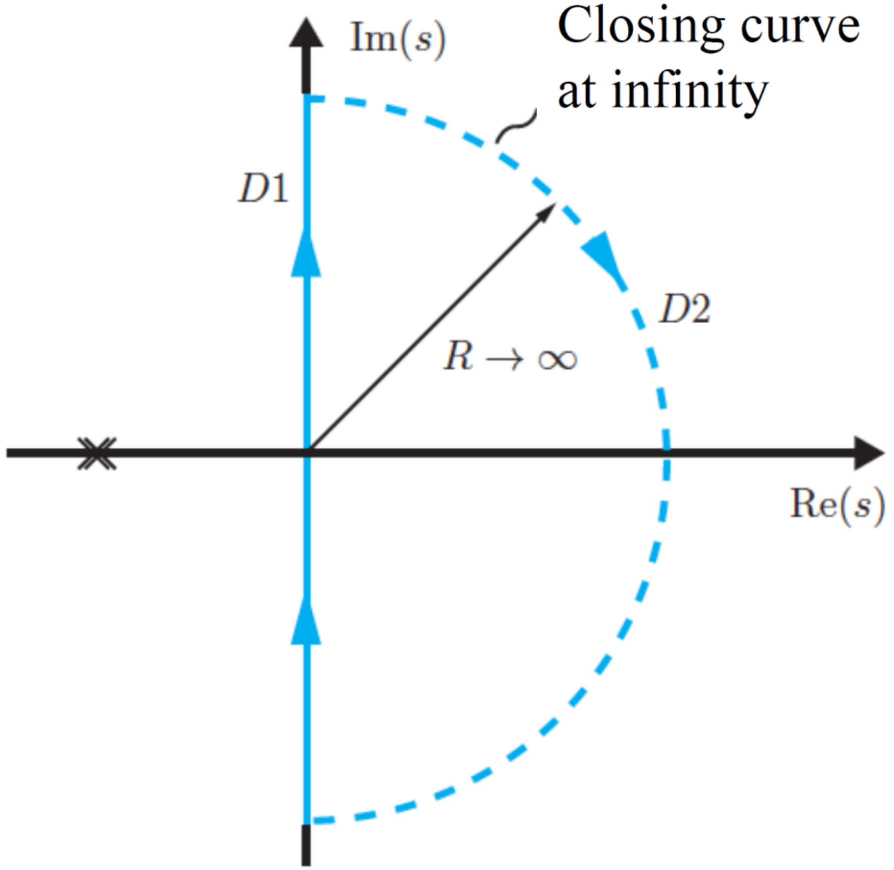
Figures from Franklin, Powell, Emami-Naeini,
“Feedback control of Dynamic Systems”

Example

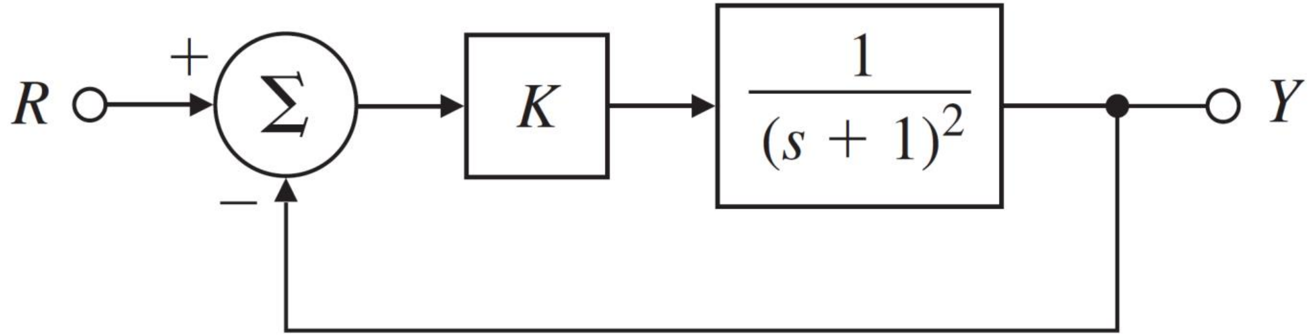


Nyquist diagram

- Stability related to real part of the poles
  - All negative → asymptotic stability
  - Some positive → instability
- Curve K enclosing positive half plane → "D curve"
- Only top half necessary bottom half complex conjugate
- Resulting curve L known as Nyquist diagram



Example



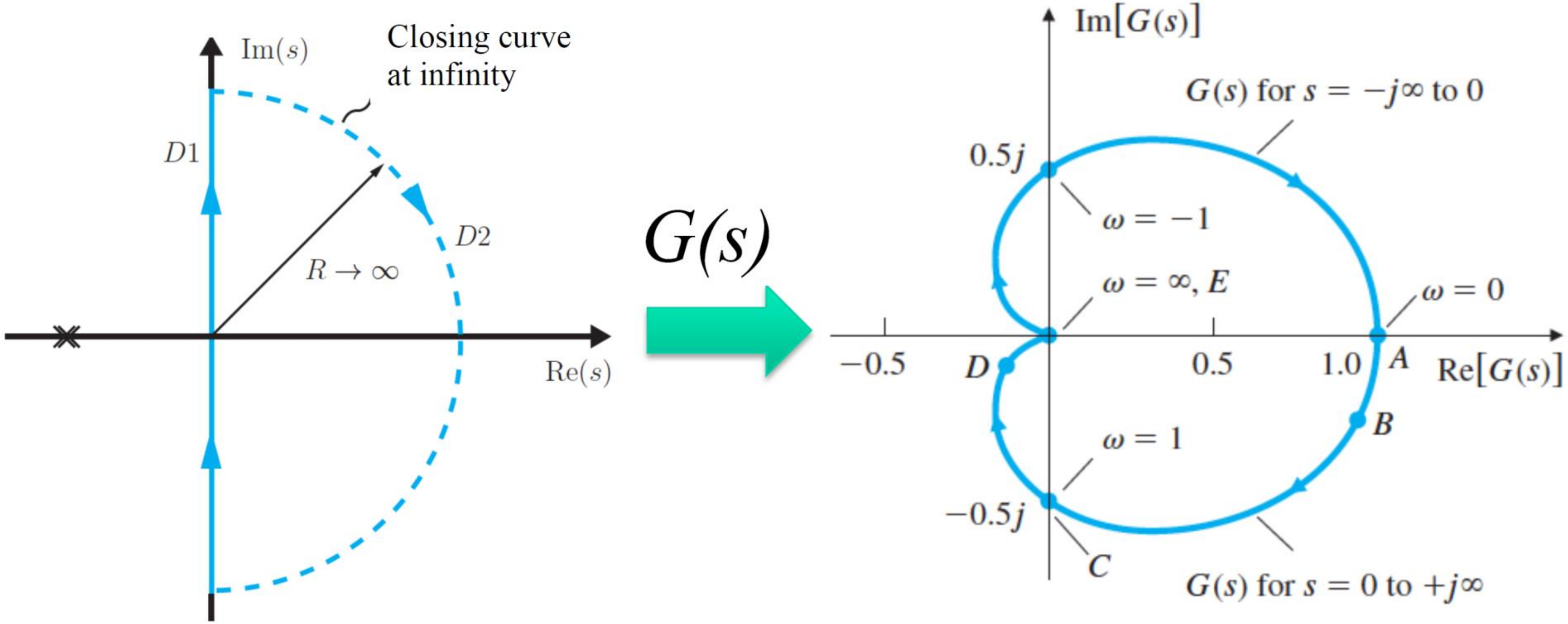


Figure from Franklin, Powell, Emami-Naeini, "Feedback control of Dynamic Systems"

Non-monotone Bode plots

Example: $G(s) = \dfrac{10(s+1)(2s+1)}{(100s+1)(20s+1)(10s+1)(0.5s+1)}$