AE353: Equivalent Representations of a System

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The following three representations of a system with input u and output y are equivalent:

- an nth-order ordinary differential equation

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_{n-1} \frac{dy}{dt} + a_n y = b_0 \frac{d^n u}{dt^n} + b_1 \frac{d^{n-1} u}{dt^{n-1}} + \cdots + b_{n-1} \frac{du}{dt} + b_n u$$

- a proper transfer function, the denominator of which is an nth-order polynomial

$$H(s) = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

- a state-space system, in controllable canonical form, in which the state has length n

$$\dot{x} = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} b_1 - a_1 b_0 & b_2 - a_2 b_0 & \cdots & b_{n-1} - a_{n-1} b_0 & b_n - a_n b_0 \end{bmatrix} x + \begin{bmatrix} b_0 \end{bmatrix} u$$