

ORIGINAL PROBLEM

$$\begin{array}{ll}\text{minimize} & h(x(t_1)) + \int_{t_0}^{t_1} g(x(t), u(t)) dt \\ & u_{[t_0, t_1]} \\ \text{subject to} & \frac{dx(t)}{dt} = f(x(t), u(t)), \quad x(t_0) = x_0\end{array}$$

VALUE FUNCTION

$$v(t, x) = \min_{\bar{u}_{[t, t_1]}} \left\{ h(\bar{x}(t_1)) + \int_t^{t_1} g(\bar{x}(s), \bar{u}(s)) ds : \right. \\ \left. \frac{d\bar{x}(s)}{ds} = f(\bar{x}(s), \bar{u}(s)), \quad \bar{x}(t) = x \right\}$$

HAMILTON-JACOBI-BELLMAN EQUATION

$$-\frac{\partial v(t, x)}{\partial t} = \min_u \left\{ \frac{\partial v(t, x)}{\partial x} f(x, u) + g(x, u) \right\}$$