AE403 Homework #5: Control

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(due at the beginning of class on Tuesday, April 26)

1. (25 points) *Be able to design and simulate a 1-axis linear control law for a spacecraft with reaction wheels in deep space.* In class we saw that, for a spacecraft with three orthogonal reaction wheels in deep space, the dynamics about each axis are approximately decoupled and have the form

$$\tau + u = J\ddot{\theta} \tag{1}$$

$$-u = J_w \dot{v}, \tag{2}$$

where J is a principal moment of inertia, u is the input from the reaction wheel, τ is an external disturbance torque, θ is an Euler angle, and v is the spin rate of the wheel relative to the spacecraft. Throughout this problem, we will assume that $J = 10$, $J_w = 1$, $\theta(0) = 0.1$, and $\dot{\theta}(0) = v(0) = 0$. We will also assume that $\tau = 0$, so that it is not possible to control both the orientation of the spacecraft and the spin rate of the reaction wheel simultaneously. We will design a feedback control law that drives the system from arbitrary initial conditions to $(\theta, \dot{\theta}) = 0$ as follows:

(a) Write (1) in state-space form

$$\dot{x} = Ax + Bu,$$

where we define

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}.$$

(b) For state feedback of the form

$$u = -Kx = -\begin{bmatrix} k_p & k_d \end{bmatrix} x,$$

find the poles of the closed-loop system. What are the conditions on k_p and k_d in order for this system to be stable?

(c) Say we want to match the closed-loop characteristic polynomial

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0,$$

where $\zeta \geq 0$ is the *damping ratio* and $\omega_n > 0$ is the *natural frequency*. Assuming that $\zeta = 1/\sqrt{2}$, express k_d and k_p in terms of ω_n.

(d) On the same figure, plot $\theta(t)$ for $\omega_n = 10^{-2}, 10^{-1}, 10^0, 10^1$, and 10^2, using the controller you derived in part (c). On separate figures (but with a consistent time scale), do the same thing for $u(t)$ and for $v(t)$.

Hint: Notice that the solution of the differential equation

$$\dot{x} = (A - BK)x$$

is

$$x(t) = \exp\left[(A - BK)t\right] x(0)$$

In MATLAB, you can use the function `expm` to express the matrix exponential function.

(e) Consider a quadratic cost function of the form

$$J_{\text{LQR}} = \frac{1}{2} \int_0^\infty \left(x^T Q x + u^T R u \right) dt,$$

where $Q = I$ and $R = r > 0$. Recall that the matrix Q penalizes non-zero states, and that the matrix R penalizes non-zero inputs. The optimal gain matrix has the form

$$K = R^{-1} B^T P,$$

where P is the solution to the algebraic Riccati equation

$$0 = A^T P + PA - PBR^{-1} B^T P + Q.$$

You can find K by using the function `K = lqr(A,B,Q,R)` in MATLAB.

On three separate figures, plot $\theta(t)$, $u(t)$, and $v(t)$ for $r = 10^{-2}, 10^{-1}, 10^0, 10^1$, and 10^2. Discuss your results, as compared to part (d). Again, you may need the MATLAB function `expm` to find $\theta(t)$, $u(t)$, and $v(t)$.

2. (25 points) *Be able to design and simulate a pitch linear control law for a spacecraft with reaction wheels in LEO subject to gravity-gradient torque.* In HW4, we derived equations of motion for the pitch dynamics of a spacecraft in LEO subject to a gravity-gradient torque. We can extend these equations to include a pitch reaction wheel. In this case, the gravity-gradient torque makes it possible to control both the pitch angle and the wheel spin rate simultaneously. In this problem, you will design a control law to do this as follows:

(a) Express the linearized equations of motion in the form

$$\dot{x} = Ax + Bu,$$

where

$$x = \begin{bmatrix} \theta_2 \\ \dot{\theta}_2 \\ v_2 \end{bmatrix}$$

and $u = u_2$. These equations should be functions of the principal moments of inertia J_1, J_2, and J_3, the wheel moment of inertia J_w, and the orbital angular velocity n.

Hint: Note that ω is not part of the x vector in this problem; ω can be expressed in terms of θ and $\dot{\theta}$ using HW4 Problem 4.

(b) Assume that $J_w = 1$, $J_1 = 12$, $J_2 = 14$, and $J_3 = 8$. Also assume that $n = 0.0011$. Given the weights $Q = I$ and $R = 10^0$, design an LQR control law that drives arbitrary initial conditions to zero using the function `[K,S,E] = lqr(A,B,Q,R)` in MATLAB. Find the closed-loop poles. Is the system stable with active control? Discuss.

(c) Simulate the (linear) closed-loop system for the initial conditions $\theta_2(0) = 0.5$, $\dot{\theta}_2(0) = 0$, and $v_2(0) = 0$. Plot $x(t)$. Discuss your results.

3. (25 points) *Be able to design and simulate a roll/yaw linear control law for a spacecraft with reaction wheels in LEO subject to gravity-gradient torque.* In HW4, we derived equations of motion for the roll/yaw dynamics of a spacecraft in LEO subject to a gravity-gradient torque. We can extend these equations to include roll/yaw reaction wheels. In this case, the gravity-gradient torque makes it possible to control both the roll/yaw angle and the wheel spin rates simultaneously. In this problem, you will design a control law to do this as follows:

(a) Express the linearized equations of motion in the form

$$\dot{x} = Ax + Bu,$$

where

$$x = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \dot{\theta}_1 \\ \dot{\theta}_3 \\ v_1 \\ v_3 \end{bmatrix} \qquad \text{and} \qquad u = \begin{bmatrix} u_1 \\ u_3 \end{bmatrix}.$$

These equations should be functions of the principal moments of inertia J_1, J_2, and J_3, the wheel moment of inertia J_w, and the orbital angular velocity n. Refer to the Hint for Problem 2 (a).

(b) Assume that $J_w = 1$, $J_1 = 12$, $J_2 = 14$, and $J_3 = 8$. Also assume that $n = 0.0011$. Given the weights $Q = I$ and $R = 10^0 \cdot I$, design an LQR control law that drives arbitrary initial conditions to zero using the function `[K,S,E] = lqr(A,B,Q,R)` in MATLAB. Find the closed-loop poles. Is the system stable with active control? Discuss.

(c) Simulate the (linear) closed-loop system for the initial conditions $\theta_1(0) = 0.1$, $\theta_3(0) = 0.5$, $\dot{\theta}_1(0) = \dot{\theta}_3(0) = 0$, and $v_1(0) = v_3(0) = 0$. Plot $x(t)$. Discuss your results.

4. Create animated simulations of the resulting spacecraft motion for
   (a) (10 points) any one of Problems 1-3.
   (b) (Extra Credit: 10 points) the remaining two of Problems 1-3.

5. (Extra Credit: 10 points) In all of Problems 1-3, the cost functions have penalized the input torque u_i for simplicity, but this is not strictly correct; we should penalize the internal torque u_i' instead of the input torque u_i for the actual reaction wheel design. Redo Problems 1-3 by penalizing u_i'.

6. (15 points) *Be able to design a spacecraft attitude control system.* Given the following scientific requirement, what types of sensors and actuators would you use for this spacecraft? (You can use multiple sensors and actuators.) Consider various disturbances at LEO and explain how you would cancel them to achieve the requirements.

   Explain your reasons in one page. High-level discussion is sufficient; no need to use equations of motion for detailed analysis. If you find the requirements insufficient, make appropriate assumptions.

   Orbit: Low-Earth-orbit (LEO)

   Spacecraft Size: 50cm*50cm*50cm

   Science Objective: Deep-space astronomical observation (The target is inertially fixed.)

   Payload: Near-Infrared Telescope

   Attitude Determination Requirement: 10 arcsec

   Attitude Control Requirement: 50 arcsec over 10-min observation