Chapter 12

1. Find the cosine of the angle between the vectors **u** and **v**. (12.3)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
   $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
   $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
   $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
   $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
   $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
   $\mathbf{v} = \langle 6, 4, 2 \rangle$

2. Find the projection of **u** onto **v**. (12.3)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
   $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
   $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
   $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
   $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
   $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
   $\mathbf{v} = \langle 6, 4, 2 \rangle$

3. Use the cross product to find a vector normal to both **u** and **v**. (12.4)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
   $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
   $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
   $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
   $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
   $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
   $\mathbf{v} = \langle 6, 4, 2 \rangle$

4. Give a vector equation and parametric equations for the line. (12.5)

   (a) The line passing through $(1, 3, -2)$ and parallel to $\langle 3, 0, 1 \rangle$.

   (b) The line passing through $(-2, 0, 4)$ and $(1, 3, 3)$.

   (c) The line parallel to $\mathbf{r}(t) = \langle t, 2 - t, 2 + t \rangle$ and passing through $(2, 4, 5)$.

   (d) The line with equation $x = -3z + 1$ in the xz plane.

   (e) The line normal to the plane with equation $x + y + 2z = 4$ and passing through $(1, 1, 1)$.

5. Give the distance from the point to the line: (12.5)

   (a) $(4, 5, 3)$ to $\mathbf{r}(t) = \langle 1 + t, 2 + 2t, 2t \rangle$

   (b) $(-1, -2, 2)$ to $\mathbf{r}(t) = \langle -1 + 3t, -4, 2 + 4t \rangle$

   (c) $(3, 0)$ to $\mathbf{r}(t) = \langle 4 - 4t, 7 - 3t \rangle$

   (d) $(2, 6)$ to $\mathbf{r}(t) = \langle -3 + 3t, 1 + t \rangle$

6. Give an equation for the plane. (12.5)

   (a) The plane passing through $(1, 3, -2)$ and normal to $\langle 3, 0, 1\rangle$.

   (b) The plane passing through $(1, -2, 0)$ and parallel to $2x - y + 3z = 3$.

   (c) The plane passing through $(1, 1, 1)$ and normal to the line with equation $\mathbf{r}(t) = \langle 4 - 3t, t, 2 + 2t\rangle$.

   (d) The plane passing through $(-2, 0, 4)$, $(1, 3, 3)$, and $(0, 0, 2)$.

7. Give the distance from the point to the plane: (12.5)

   (a) $(5, 1, 1)$ to $x - 2y + 2z = 2$

   (b) $(4, -1, 3)$ to $3x + 4z = 4$

   (c) $(0, 1, 1)$ to $-2x - 3y - 6z = 5$

   (d) $(-1, 5, 2)$ to $x + y + z = 3$

8. Sketch the curve given by the equation in the appropriate coordinate plane, and then sketch the cylinder in xyz space given by the equation. (12.6)

   (a) $y = x^2$

   (b) $x = z^3$

   (c) $y = \sin z$

   (d) $z = e^x$

   (e) $z = \ln y$

   (f) $xy = 1$

9. Sketch the three coordinate plane cross-sections for the quadric surface given by the equation, sketch the surface itself, and give the name of the surface. (12.6)

   (a) $x^2 - y = -z^2$

   (b) $y^2 + z^2 = 4 - 4x^2$

   (c) $z^2 - 9y^2 = x^2$

   (d) $y^2 - z^2 = 4 - 4x^2$

   (e) $4x^2 - y^2 - 4z^2 = 16$

   (f) $z = y^2 - 4x^2$

Chapter 13

10. Give a parametrization of the curve described as a vector function. (13.1)

    (a) The parabola $y = x^2$ in the xy plane.

    (b) The directed line segment beginning at $(1, 2, -3)$ and ending at $(0, 3, 0)$.

    (c) The circle $x^2 + y^2 = 9$.

    (d) The ellipse $x^2 + 9y^2 = 9$.

11. Find the limit of the vector function. (13.1)

    (a) $\lim_{t\to -1} \left\langle \text{Arctan } t, \frac{e^{1+t}}{1-t} \right\rangle$

    (b) $\lim_{t\to 2} \left\langle t^2 - 4, \frac{t^2-4}{t-2} \right\rangle$

    (c) $\lim_{t\to 0} \left(\frac{\sin 3t}{4t}\mathbf{i} + \frac{1-\cos t}{t}\mathbf{j} \right)$

    (d) $\lim_{t\to \pi/2} \langle \sin t, \cos t, \cot t \rangle$

    (e) $\lim_{t\to 0} \left(\frac{e^t}{t+1}\mathbf{i} + \frac{e^t - 1}{t}\mathbf{j} + \frac{2^{2t}-1}{t}\mathbf{k} \right)$

    (f) $\lim_{t\to 1} \left\langle \frac{3t^2-3}{t+1}, \frac{\sin(2t-2)}{2t-2}, \frac{3t^2-3}{t-1} \right\rangle$

12. Find the derivative $\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t)$ of the vector function. (13.2)

    (a) $\mathbf{r}(t) = \langle x^2, 3 + t \rangle$

    (b) $\mathbf{r}(t) = \langle 3\sin 4t, -3\cos 4t \rangle$

    (c) $\mathbf{r}(t) = \left\langle \frac{1}{t^2}, \frac{t}{t^2+1} \right\rangle$

    (d) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$

    (e) $\mathbf{r}(t) = (\ln 2t)\mathbf{i} + (e^{2t} - 2)\mathbf{j} + \frac{1}{e^t}\mathbf{k}$

    (f) $\mathbf{r}(t) = \langle \text{Arcsin } t, \text{Arccsc } t, \text{Arctan } t \rangle$

13. Find the indefinite integral $\int \mathbf{r}(t)\,dt$ of the vector function (13.2)

    (a) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t+1\rangle$
    (b) $\mathbf{r}(t) = \langle 2\sin 2t, -2\cos 2t\rangle$
    (c) $\mathbf{r}(t) = \langle e^t, 2e^{-t}, e^{3t}\rangle$
    (d) $\mathbf{r}(t) = \left\langle \frac{1}{t+2}, \frac{1}{(t+2)^2}, \frac{2t}{t^2+2}\right\rangle$
    (e) $\mathbf{r}(t) = (e^{2t}e^t + 2t)\mathbf{i} + \frac{\ln t}{t}\mathbf{k}$

14. Solve the differential vector equation to find $\mathbf{r}(t)$. (13.2)

    (a) $\mathbf{r}'(t) = \langle 3t^2, 2t^3\rangle$, $\mathbf{r}(0) = \langle 3, 4\rangle$
    (b) $\mathbf{r}'(t) = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $\mathbf{r}(1) = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$
    (c) $\mathbf{r}'(t) = \left\langle 2e^t, 4, \frac{1}{t}\right\rangle$, $\mathbf{r}(\ln 3) = \langle 5, 0, \ln(\ln 3)\rangle$
    (d) $\mathbf{r}'(t) = \left\langle \frac{3}{2}\sqrt{t}, 8t, 3t^2+3\right\rangle$, $\mathbf{r}(1) = \langle 1, -3, 6\rangle$
    (e) $\mathbf{r}'(t) = \left\langle \frac{1}{1+t^2}, \frac{2t}{1+t^2}\right\rangle$, $\mathbf{r}(0) = \langle 0, 1\rangle$

15. Find the arclength parameter $s(t)$ where $s(0) = 0$ and $\frac{ds}{dt} \geq 0$ for the given curve, and use it to find the arclength of the given portion of the curve. (13.3)

    (a) $\mathbf{r}(t) = \langle 1+2t, 2-t, 3-2t\rangle$, $1 \leq t \leq 3$
    (b) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t\rangle$, $0 \leq t \leq 1$
    (c) $\mathbf{r}(t) = \langle 3e^t, -4e^t\rangle$, $0 \leq t \leq \ln 2$
    (d) $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + \mathbf{k}$, $0 \leq t \leq \frac{\sqrt{5}}{3}$
    (e) $\mathbf{r}(t) = \langle 6t, t^3, 3t^2\rangle$, $-3 \leq t \leq -2$

16. Find the unit vectors $\mathbf{T}, \mathbf{N}$ to the curve in terms of the parameter t. (13.3)

    (a) $\mathbf{r}(t) = \langle 3\cos 2t, 3\sin 2t\rangle$
    (b) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t\rangle$
    (c) $\mathbf{r}(t) = \left\langle \sqrt{2}\sin t, 2\cos t, \sqrt{2}\sin t\right\rangle$
    (d) $\mathbf{r}(t) = \langle e^t, e^t\sin t, e^t\cos t\rangle$

17. Given the information about $\mathbf{r}(t)$ at a point, evaluate the binormal vector $\mathbf{B}$ and curvature κ at that same point. (13.3)

    (a) $\frac{d\mathbf{r}}{dt} = \langle 3, 0, -4 \rangle$, $\frac{d\mathbf{T}}{dt} = \langle 0, 10, 0 \rangle$
    $\mathbf{T} = \left\langle \frac{3}{5}, 0, -\frac{4}{5} \right\rangle$, $\mathbf{N} = \langle 0, 1, 0 \rangle$

    (b) $\frac{d\mathbf{r}}{dt} = \left\langle -3, 0, 3\sqrt{3} \right\rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle -\sqrt{3}, 0, -1 \right\rangle$
    $\mathbf{T} = \left\langle -\frac{1}{2}, 0, \frac{\sqrt{3}}{2} \right\rangle$, $\mathbf{N} = \left\langle -\frac{\sqrt{3}}{2}, 0, -\frac{1}{2} \right\rangle$

    (c) $\frac{d\mathbf{r}}{dt} = \langle 1, 1, 1 \rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle \frac{1}{\sqrt{3}}, 0, -\frac{1}{\sqrt{3}} \right\rangle$
    $\mathbf{T} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$, $\mathbf{N} = \left\langle \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right\rangle$

    (d) $\frac{d\mathbf{r}}{dt} = \left\langle \frac{3\sqrt{2}}{2}, -4, -\frac{3\sqrt{2}}{2} \right\rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle -\frac{3\sqrt{2}}{10}, 0, -\frac{3\sqrt{2}}{10} \right\rangle$
    $\mathbf{T} = \left\langle \frac{3\sqrt{2}}{10}, -\frac{4}{5}, -\frac{3\sqrt{2}}{10} \right\rangle$, $\mathbf{N} = \left\langle -\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right\rangle$

18. Sketch $\mathbf{r}(t)$ in the plane and plot the point where $t = a$, and then find and sketch $\mathbf{v}$, $\mathbf{a}$ at $t = a$ on the curve. (13.4)

    (a) $\mathbf{r}(t) = \langle t, t^2 \rangle$, $t = 2$
    (b) $\mathbf{r}(t) = \langle 2\sin t, -2\cos t \rangle$, $t = \pi/2$
    (c) $\mathbf{r}(t) = \langle e^{2t}, 2t \rangle$, $t = 0$
    (d) $\mathbf{r}(t) = \langle \sin(\ln t), \cos(\ln t) \rangle$, $t = 1$

19. Assuming ideal projectile motion and $g = 10\frac{m}{s^2}$, find the following.

    (a) Height of a projectile shot from the ground at an angle of $\pi/4$ with initial speed $16\sqrt{2}\frac{m}{s}$ after 2 seconds.
    (b) Flight time of a projectile shot from the ground at an angle of $\pi/6$ with initial speed $100\frac{m}{s}$.
    (c) Maximum height of a projectile shot from the ground at an angle of $\pi/3$ with initial speed $50\sqrt{3}\frac{m}{s}$.
    (d) Total horizontal distance traveled by a projectile shot from the ground at an angle of $\pi/4$ with initial speed $10\sqrt{2}\frac{m}{s}$.
    (e) Initial speed of a projectile shot from the ground at an angle of $\pi/3$ which has traveled 60 meters horizontally after 4 seconds.

20. Find the tangential and normal components of acceleration for the given position function at the given value of t.

    (a) $\mathbf{r}(t) = \langle 3t, t^2 \rangle$, $t = 2$
    (b) $\mathbf{r}(t) = \langle \sin t, \cos t \rangle$, $t = \pi/4$
    (c) $\mathbf{r}(t) = \left\langle \frac{1}{3}t^3, 2t, t^2 \right\rangle$, $t = 1$
    (d) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$, $t = \pi/2$

Chapter 14

21. Sketch the domain of the function f in the xy plane, sketch and label the three level curves of f within its domain for each given k value, and then sketch the graph of f. (14.1)

    (a) $f(x,y) = 2x - y + 1$, $k = -3, 0, 3$
    (b) $f(x,y) = 4x^2 + y^2$, $k = 0, 4, 16$
    (c) $f(x,y) = \sqrt{x^2 + 9y^2}$, $k = 0, 3, 6$
    (d) $f(x,y) = \sqrt{1 - x^2 - y^2}$, $k = 0, \frac{1}{\sqrt{2}}, 1$
    (e) $f(x,y) = \sqrt{1 - x^2 + y^2}$, $k = 0, 1, \sqrt{2}$
    (f) $f(x,y) = \ln(4 - x^2 - y^2)$, $k = \ln 1, \ln 2, \ln 3$

22. Sketch the level surface of f for the given value of k. (14.1)

    (a) $f(x,y,z) = x + y + z$, $k = 2$
    (b) $f(x,y,z) = x^2 + y^2 + z^2$, $k = 9$
    (c) $f(x,y,z) = \sqrt{x^2 + 4y^2 + z^2}$, $k = 2$
    (d) $f(x,y,z) = z - x^2$, $k = 3$

23. Prove the limit does not exist by comparing two paths of approach. (14.2)

    (a) $\displaystyle\lim_{P \to (0,0)} \frac{x^2 + y^2}{xy}$
    (b) $\displaystyle\lim_{P \to (0,0)} \frac{|xy|}{xy}$
    (c) $\displaystyle\lim_{P \to (0,0)} \frac{y^6 + x^2}{y^3 x + y^6}$
    (d) $\displaystyle\lim_{P \to (3,4)} \frac{25 - x^2 - y^2}{7 - x - y}$

24. Compute the value of the limit. (14.2)

    (a) $\displaystyle\lim_{P\to(1,-3)} \frac{6-xy}{3x+y}$

    (b) $\displaystyle\lim_{P\to(0,0)} \frac{2x^2+4y^2}{\sqrt{x^2+2y^2+1}-1}$

    (c) $\displaystyle\lim_{P\to(0,0)} \frac{x\sin 2y-\sin 2y}{y-xy}$

    (d) $\displaystyle\lim_{P\to(1,2)} \frac{x^2+2xy+y^2-3x-3y}{x+y}$

    (e) $\displaystyle\lim_{P\to(1,2)} \frac{x^2+2xy+y^2-3x-3y}{x+y-3}$

25. Find all the first-order and second-order partial derivatives of f. (14.3)

    (a) $f(x,y) = 4x^2 - 5y^3 + x - 1$
    (b) $f(x,y) = 3x^2y^2 - x^3 + y^4 - 7$
    (c) $f(x,y) = \sin(x+3y)$
    (d) $f(x,y) = e^{xy^2}$
    (e) $f(r,\theta) = r\cos(\theta)$
    (f) $f(u,v) = \text{Arctan}\,(uv)$

26. Find the linearization $L(x,y)$ of $f(x,y)$ at (a,b), and use it to approximate the value of f at (c,d). (14.4)

    (a) $f(x,y) = 3x^2 - 2y^3$, $(a,b) = (1,2)$, $(c,d) = (0.9, 2.2)$
    (b) $f(x,y) = 7y + 3xy - 1$, $(a,b) = (5,1)$, $(c,d) = (5.1, 0.9)$
    (c) $f(x,y) = 2xy - x^2 - y^2$, $(a,b) = (3,-1)$, $(c,d) = (2.9, -1.05)$
    (d) $f(x,y) = \sqrt{25-x^2-y^2}$, $(a,b) = (-3,0)$, $(c,d) = (-3.04, 0.09)$

27. Find the given derivative for the given nested functions at the given point. (14.5)

    (a) Find $\frac{df}{dt}$ at $t = 1$:
    $f(x, y, z) = xyz^2$, $x(t) = 2t + 1$, $y(t) = t^2 + 1$, $z(t) = 1 - t^3$

    (b) Find $\frac{\partial g}{\partial u}$ at $(u, v) = (2, 0)$:
    $g(x, y) = 2x + 3x^2y$, $x(u, v) = 1 - u$, $y(u, v) = 1 - uv$

    (c) Find $\frac{df}{dt}$ at $t = \pi/3$:
    $f(x, y) = 4x^2 + 2y$, $x(t) = \cos t$, $y(t) = 2\sin^2 t$

    (d) Find $\frac{\partial f}{\partial t}$ at $(t, u) = (0, 1)$:
    $f(x, y, z) = ye^x + 2z$, $x(t, u) = t^2$, $y(t, u) = t + u$, $z(t, u) = u + 1$

    (e) Find $\frac{dh}{dt}$ at $t = 1$:
    $h(x, y) = x + 2y$, $x(u, v) = uv$, $y(u, v) = v^2$, $u(t) = t^2$, $v(t) = t + 1$

28. Use partial derivatives to find the rate of change $\frac{dy}{dx}$ for the equation at the given point. (14.5)

    (a) $3x^2 + 5y = 8$ at $(1, 1)$
    (b) $4x^3y = 3xy^3 + 16$ at $(-1, 2)$
    (c) $-xy^2 + y^3 = -5x + 5$ at $(-3, 2)$
    (d) $x^3y^4 = x^4y^3$ at $(2, 2)$
    (e) $e^{xy} = \ln(xy + e)$ at $(1, 0)$
    (f) $\sin(2x + y) = \cos(2x + y) + 1$ at $(\pi/8, \pi/4)$

29. Find the derivative of f in the direction of the given vector at the given point. (14.6)

    (a) $f(x, y) = x + 2y$, $\mathbf{A} = \langle -4, 3\rangle$, $P_0 = (1, 3)$
    (b) $f(x, y) = xy^2 + 3y$, $\mathbf{A} = \langle 2, 2\rangle$, $P_0 = (2, 0)$
    (c) $f(x, y) = e^{x+xy}$, $\mathbf{A} = 5\mathbf{i} - 12\mathbf{j}$, $P_0 = (\ln 2, 0)$
    (d) $f(x, y, z) = x^2 + 4y^2 + z^2$, $\mathbf{A} = \langle 3, -2, -6\rangle$, $P_0 = (1, 1, 2)$
    (e) $f(x, y, z) = xz^3 + 3yz$, $\mathbf{A} = \langle 1, -2, 2\rangle$, $P_0 = (-2, 0, 1)$
    (f) $f(x, y, z) = \ln(y^2) + 4xz$, $\mathbf{A} = 6\mathbf{i} - 8\mathbf{k}$, $P_0 = (3, 1, 2)$

30. Find and label all the points yielding local maximum values, local minimum values, and saddle points for f. (14.7)

    (a) $f(x,y) = x^2 + 9y^2 + 3$
    (b) $f(x,y) = x^2 - 2xy + y^2 - 3$
    (c) $f(x,y) = x^3 + 3xy + y^3 + 2$
    (d) $f(x,y) = x^3 - 3xy^2 + 6y^2 + 6x^2 + 1$
    (e) $f(x,y) = (x^2 + y^2)e^{x+y+2}$
    (f) $f(x,y) = x^2y - xy^2 + 12x - 12y$