Ch 11 Study Guide **Your Name:** _____ **Class: 9am / 1pm**

Calculus III - Math 2630 - Spring 2013 **Instructor: Steven Clontz**

Draw a box around your answer. Show your work. Calculators not allowed.

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1. Find the velocity and acceleration functions associated with the position function

$$\mathbf{r}(t) = \langle 2\sin t, -2\cos t \rangle$$

which corresponds to the circle

$$x^2 + y^2 = 4$$

(a) Compute correct velocity function (5 points)

(b) Compute correct acceleration function (5 points)

2. Sketch the curve given by $\mathbf{r}(t) = \langle 2\sin t, -2\cos t \rangle$ from #1 along with its velocity and acceleration vectors at $t = \pi$.

   (a) Sketch correct curve (2 points)

   (b) Mark $t = \pi$ accurately (2 points)

   (c) Sketch $\mathbf{v}$ correctly (3 points)

   (d) Sketch $\mathbf{a}$ correctly (3 points)

3. Evaluate $\int_0^{\pi/4} (\sec^2 t)\mathbf{i} + (6)\mathbf{j} + \left(e^t\right)\mathbf{k}\, dt$.

(a) (3, 6, or 10 points for 1, 2, or 3 correctly evaulated components)

4. Find $\mathbf{r}(t)$ given $\mathbf{r}'(t) = \langle 2t, 2t - t^2, 2e^{2t} \rangle$ and $\mathbf{r}(0) = \langle 0, 7, -1 \rangle$.

   (a) (3, 6, or 10 points for 1, 2, or 3 correctly evaulated components)

5. Assume $g = 10$ m/s^2. What is the flight time of a projectile launched from the ground with an initial speed of 100 m/s and launch angle of $\frac{\pi}{6}$?

- If using formula:
  (a) Write correct formula (2 points)
  (b) Compute corect answer (8 points)
- If using calculus:
  (a) Write correct position function (2 points)
  (b) Attempt to use correct method to find answer (6 points)
  (c) Compute correct answer (2 points)

6. Give the length of the arc on the curve $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t\rangle$ between $t = 0$ and $t = 1$.

   (a) Use arclength formula $\int_a^b |\mathbf{v}(t)|\, dt$ (2 points)

   (b) Compute $\mathbf{v}(t)$ correctly (2 points)

   (c) Simplify $|\mathbf{v}(t)|$ correctly (3 points)

   (d) Compute arclength correctly (3 points)

7. Write the equation of the circle of curvature to a curve at a point $(4, 0)$ with curvature $\frac{1}{10}$ and normal vector $\mathbf{N} = \left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle$.

   (a) Calculate the radius $a = \frac{1}{\kappa}$ correctly (3 points)

   (b) Calculate the center $\langle x_0, y_0 \rangle = \mathbf{r}(t_0) + a\mathbf{N}$ correctly (3 points)

   (c) Write a correct equation for the circle (4 points)

8. Find $\mathbf{T}$, $\mathbf{N}$ for $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$.

   (a) Compute $\mathbf{v}$ correctly (2 points)

   (b) Compute $\mathbf{T}$ correctly (3 points)

   (c) Compute $\frac{d\mathbf{T}}{dt}$ correctly (2 points)

   (d) Compute $\mathbf{N}$ correctly (3 points)

9. Given a point $(1, 1, 0)$ on a curve where $\mathbf{v} = \left\langle \frac{3\sqrt{2}}{2}, -4, -\frac{3\sqrt{2}}{2} \right\rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle -\frac{3\sqrt{2}}{10}, 0, -\frac{3\sqrt{2}}{10} \right\rangle$, $\mathbf{N} = \left\langle -\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right\rangle$, $\frac{d\mathbf{B}}{dt} = \left\langle -\frac{4\sqrt{2}}{5}, 0, -\frac{4\sqrt{2}}{5} \right\rangle$, compute κ, τ, and $\mathbf{B}$ at that point.

   (a) Write a correct formula for each of κ, τ, and $\mathbf{B}$ (1, 2, or 4 points)

   (b) Compute κ, τ, and $\mathbf{B}$ correctly (2, 4, or 6 points)

10. Given the polar parametric equations $r(t) = 2 - \cos t$ and $\theta(t) = 2t$, find $\mathbf{v}$ in terms of $\mathbf{u}_r$ and $\mathbf{u}_\theta$ at $t = 0$.

    (a) Write correct formula $\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta$ (2 points)

    (b) Compute $\dot{r}$ correctly (3 points)

    (c) Compute $\dot{\theta}$ correctly (3 points)

    (d) Compute correct expression for $\mathbf{v}$ (2 points)

Include extra scratch work below:

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