Ch 12 Study Guide **Your Name:** ____________________________ **Class: 9am / 1pm**

Calculus III - MATH 2630 - Spring 2013 **Instructor: Steven Clontz**

Draw a $\boxed{\text{box}}$ around your answer. Show your work. Pens/calculators not allowed.

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1. Give the domain and range of the function $f(x, y) = \frac{3}{x-y}$ in set notation or interval notation.

   - Write the domain in set notation (3 pts)
   - Write the range in set notation or interval notation (1 pt)

   Now, sketch the function's domain, tell if it is open/closed/both/neither, and tell if it is bounded/unbounded.

   - Sketch the domain correctly (2 pts)
   - Identify the domain as open/closed/both/neither (2 pts)
   - Identify the domain as bounded/unbounded (2 pts)

2. Sketch three typical level curves for the function $f(x, y) = \frac{3}{x-y}$.

- Use the format $f(x, y) = c$ for level curves (2 pts)
- Simplify $f(x, y) = c$ into an identifiable curve equation (3 pts)
- Correctly express $f(x, y) = c$ for three values of c (2 pts)
- Sketch each $f(x, y) = c$ in the xy-plane accurately (3 pts)

3. Compute $\lim_{(x,y)\to(0,0)} \dfrac{x\sin 2y - \sin 2y}{y - xy}$ without restricting to a path of approach.

- Correctly identify method to simplify fraction (4 pts)
- Execute method of simplification correctly (4 pts)
- Compute the correct value of the limit (2 pts)

4. Prove that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2y}{y^2+x^4}$ doesn't exist.

- Restrict the limit to two paths of approach, or a path of approach depending on k (3 pts)
- Conclude the limit DNE since the limit equals different real numbers along two different paths of approach (3 pts)
- Correctly compute the limits along two paths of approach (4 pts)

5. Find all of the first-order and second-order partial derivatives for $f(x, y) = x^3 + 3xy^2 + e^y$.

- Compute each of f_x, f_y, f_{xx}, $f_{xy} = f_{yx}$, and f_{yy} correctly (2 pts each)

6. Find $\frac{df}{dt}$ at $t = \frac{\pi}{4}$ given $f(x, y, z) = \ln(xyz)$, $x = \cos t$, $y = \sec t$, and $z = 4t^2$.

- If using Chain Rule:
  - Use a correct Chain Rule formula $\frac{df}{dt} = \nabla f \cdot \frac{d\vec{r}}{dt}$ (2 pts)
  - Compute each of $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$ correctly (1 pt each)
  - Plug in value of t and simplify (2 pts)
- If using substitution:
  - Plug x, y, z into $f(x, y, z)$ to get f as a function of t (3 pts)
  - Differentiate correctly (5 pts)
  - Plug in value of t and simplify (2 pts)

7. Find the directional derivative of $f(x, y, z) = 3xy^2+6yz^2$ at the point $(1, 2, -1)$ in the direction of $\langle 2, -1, 2\rangle$.

   - Use the formula $\frac{df}{ds_{\vec{u}}} = \nabla f \cdot \vec{u}$ (3 pts)
   - Compute the direction $\vec{u}$ (2 pts)
   - Compute the gradient ∇f (2 pts)
   - Compute the directional derivative (3 pts)

8. Find the plane tangent to the surface $xy + 2yz - xz = 24$ at the point $(1, 2, -1)$.

- Compute the gradient ∇f (3 pts)
- Use the plane equation $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ (4 pts)
- Plug into the plane equation correctly (3 pts)

9. Find and label all the points yielding local maximum values, local minimum values, and saddle points for $f(x, y) = x^3 + 3xy + y^3 + 2$.

- Compute the gradient ∇f (2 pts)
- Set $\nabla f = \vec{0}$ to find its critical points (1 pt)
- Find all the critical points (2 pts)
- Compute f_{xx}, f_{yy}, f_{xy} correctly (1 pt)
- Compute f_D correctly (1 pt)
- Use the 2nd derivative test to correctly label each critical point (1 pt for one, 3 pts for all)

10. Find the radius and height of a cylinder with volume 2π such that the surface area is as small as possible. (HINT: Use the Method of Lagrange Multipliers along with the formulas $V = \pi r^2 h$ and $SA = 2\pi rh + 2\pi r^2$.)

- Identify and label the function f to optimize (2 pt)
- Identify and label the restriction function $g = c$ (2 pt)
- Use the method of Lagrange Multipliers $\nabla f = \lambda \nabla g$ (3 pts)
- Find the optimizing input (3 pts)

Include extra scratch work below:

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