Chapter 12

1. Find the cosine of the angle between the vectors **u** and **v**. (12.3)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
       $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
       $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
       $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
       $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
       $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
       $\mathbf{v} = \langle 6, 4, 2 \rangle$

2. Find the projection of **u** onto **v**. (12.3)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
       $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
       $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
       $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
       $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
       $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
       $\mathbf{v} = \langle 6, 4, 2 \rangle$

3. Use the cross product to find a vector normal to both $\mathbf{u}$ and $\mathbf{v}$. (12.4)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
       $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
       $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
       $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
       $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
       $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
       $\mathbf{v} = \langle 6, 4, 2 \rangle$

4. Give a vector equation and parametric equations for the line. (12.5)

   (a) The line passing through $(1, 3, -2)$ and parallel to $\langle 3, 0, 1 \rangle$.

   (b) The line passing through $(-2, 0, 4)$ and $(1, 3, 3)$.

   (c) The line parallel to $\mathbf{r}(t) = \langle t, 2 - t, 2 + t \rangle$ and passing through $(2, 4, 5)$.

   (d) The line with equation $x = -3z + 1$ in the xz plane.

   (e) The line normal to the plane with equation $x + y + 2z = 4$ and passing through $(1, 1, 1)$.

5. Give the distance from the point to the line: (12.5)

   (a) $(4, 5, 3)$ to $\mathbf{r}(t) = \langle 1 + t, 2 + 2t, 2t \rangle$

   (b) $(-1, -2, 2)$ to $\mathbf{r}(t) = \langle -1 + 3t, -4, 2 + 4t \rangle$

   (c) $(3, 0)$ to $\mathbf{r}(t) = \langle 4 - 4t, 7 - 3t \rangle$

   (d) $(2, 6)$ to $\mathbf{r}(t) = \langle -3 + 3t, 1 + t \rangle$

6. Give an equation for the plane. (12.5)

    (a) The plane passing through $(1, 3, -2)$ and normal to $\langle 3, 0, 1 \rangle$.

    (b) The plane passing through $(1, -2, 0)$ and parallel to $2x - y + 3z = 3$.

    (c) The plane passing through $(1, 1, 1)$ and normal to the line with equation $\mathbf{r}(t) = \langle 4 - 3t, t, 2 + 2t \rangle$.

    (d) The plane passing through $(-2, 0, 4)$, $(1, 3, 3)$, and $(0, 0, 2)$.

7. Give the distance from the point to the plane: (12.5)

    (a) $(5, 1, 1)$ to $x - 2y + 2z = 2$

    (b) $(4, -1, 3)$ to $3x + 4z = 4$

    (c) $(0, 1, 1)$ to $-2x - 3y - 6z = 5$

    (d) $(-1, 5, 2)$ to $x + y + z = 3$

8. Sketch the curve given by the equation in the appropriate coordinate plane, and then sketch the cylinder in xyz space given by the equation. (12.6)

    (a) $y = x^2$

    (b) $x = z^3$

    (c) $y = \sin z$

    (d) $z = e^x$

    (e) $z = \ln y$

    (f) $xy = 1$

9. Sketch the three coordinate plane cross-sections for the quadric surface given by the equation, sketch the surface itself, and give the name of the surface. (12.6)

    (a) $x^2 - y = -z^2$

    (b) $y^2 + z^2 = 4 - 4x^2$

    (c) $z^2 - 9y^2 = x^2$

    (d) $y^2 - z^2 = 4 - 4x^2$

    (e) $4x^2 - y^2 - 4z^2 = 16$

    (f) $z = y^2 - 4x^2$

Chapter 13

10. Give a vector function which parametrizes the given curve without nontrivial overlaps. (13.1)

   (a) The parabola $y = x^2$ in the xy plane.

   (b) The directed line segment beginning at $(1, 2, -3)$ and ending at $(0, 3, 0)$.

   (c) The circle $x^2 + y^2 = 9$.

   (d) The ellipse $x^2 + 9y^2 = 9$.

11. Find the limit of the vector function. (13.1)

   (a) $\displaystyle\lim_{t \to -1} \left\langle \text{Arctan } t, \frac{e^{1+t}}{1-t} \right\rangle$

   (b) $\displaystyle\lim_{t \to 2} \left\langle t^2 - 4, \frac{t^2 - 4}{t - 2} \right\rangle$

   (c) $\displaystyle\lim_{t \to 0} \left(\frac{\sin 3t}{4t}\mathbf{i} + \frac{1 - \cos t}{t}\mathbf{j} \right)$

   (d) $\displaystyle\lim_{t \to \pi/2} \left\langle \sin t, \cos t, \cot t \right\rangle$

   (e) $\displaystyle\lim_{t \to 0} \left(\frac{e^t}{t+1}\mathbf{i} + \frac{e^t - 1}{t}\mathbf{j} + \frac{2^{2t} - 1}{t}\mathbf{k} \right)$

   (f) $\displaystyle\lim_{t \to 1} \left\langle \frac{3t^2 - 3}{t + 1}, \frac{\sin(2t - 2)}{2t - 2}, \frac{3t^2 - 3}{t - 1} \right\rangle$

12. Find the derivative $\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t)$ of the vector function. (13.2)

   (a) $\mathbf{r}(t) = \langle t^2, 3 + t \rangle$

   (b) $\mathbf{r}(t) = \langle 3 \sin 4t, -3 \cos 4t \rangle$

   (c) $\mathbf{r}(t) = \left\langle \frac{1}{t^2}, \frac{t}{t^2 + 1} \right\rangle$

   (d) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$

   (e) $\mathbf{r}(t) = (\ln 2t)\mathbf{i} + (e^{2t} - 2)\mathbf{j} + \frac{1}{e^t}\mathbf{k}$

   (f) $\mathbf{r}(t) = \langle \text{Arcsin } t, \text{Arccsc } t, \text{Arctan } t \rangle$

13. Find the indefinite integral $\int \mathbf{r}(t)\,dt$ of the vector function (13.2)

   (a) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$

   (b) $\mathbf{r}(t) = \langle 2\sin 2t, -2\cos 2t \rangle$

   (c) $\mathbf{r}(t) = \langle e^t, 2e^{-t}, e^{3t} \rangle$

   (d) $\mathbf{r}(t) = \left\langle \frac{1}{t+2}, \frac{1}{(t+2)^2}, \frac{2t}{t^2+2} \right\rangle$

   (e) $\mathbf{r}(t) = (e^{2t}e^t + 2t)\mathbf{i} + \frac{\ln t}{t}\mathbf{k}$

14. Solve the differential vector equation to find $\mathbf{r}(t)$. (13.2)

   (a) $\mathbf{r}'(t) = \langle 3t^2, 2t^3 \rangle$, $\mathbf{r}(0) = \langle 3, 4 \rangle$

   (b) $\mathbf{r}'(t) = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $\mathbf{r}(1) = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$

   (c) $\mathbf{r}'(t) = \langle 2e^t, 4, \frac{1}{t} \rangle$, $\mathbf{r}(\ln 3) = \langle 5, 0, \ln(\ln 3) \rangle$

   (d) $\mathbf{r}'(t) = \langle \frac{3}{2}\sqrt{t}, 8t, 3t^2 + 3 \rangle$, $\mathbf{r}(1) = \langle 1, -3, 6 \rangle$

   (e) $\mathbf{r}'(t) = \left\langle \frac{1}{1+t^2}, \frac{2t}{1+t^2} \right\rangle$, $\mathbf{r}(0) = \langle 0, 1 \rangle$

15. Find the arclength parameter $s(t)$ where $s(0) = 0$ and $\frac{ds}{dt} \geq 0$ for the given curve, and use it to find the arclength of the given portion of the curve. (13.3)

   (a) $\mathbf{r}(t) = \langle 1 + 2t, 2 - t, 3 - 2t \rangle$, $1 \leq t \leq 3$

   (b) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$, $0 \leq t \leq 1$

   (c) $\mathbf{r}(t) = \langle 3e^t, -4e^t \rangle$, $0 \leq t \leq \ln 2$

   (d) $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + \mathbf{k}$, $0 \leq t \leq \frac{\sqrt{5}}{3}$

   (e) $\mathbf{r}(t) = \langle 6t, t^3, 3t^2 \rangle$, $-3 \leq t \leq -2$

16. Find the unit vectors $\mathbf{T}, \mathbf{N}$ to the curve in terms of the parameter t. (13.3)

   (a) $\mathbf{r}(t) = \langle 3\cos 2t, 3\sin 2t \rangle$

   (b) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$

   (c) $\mathbf{r}(t) = \langle \sqrt{2}\sin t, 2\cos t, \sqrt{2}\sin t \rangle$

   (d) $\mathbf{r}(t) = \langle e^t, e^t \sin t, e^t \cos t \rangle$

17. Given the information about $\mathbf{r}(t)$ at a point, evaluate the binormal vector $\mathbf{B}$ and curvature κ at that same point. (13.3)

   (a) $\frac{d\mathbf{r}}{dt} = \langle 3, 0, -4 \rangle$, $\frac{d\mathbf{T}}{dt} = \langle 0, 10, 0 \rangle$
       $\mathbf{T} = \langle \frac{3}{5}, 0, -\frac{4}{5} \rangle$, $\mathbf{N} = \langle 0, 1, 0 \rangle$

   (b) $\frac{d\mathbf{r}}{dt} = \langle -3, 0, 3\sqrt{3} \rangle$, $\frac{d\mathbf{T}}{dt} = \langle -\sqrt{3}, 0, -1 \rangle$
       $\mathbf{T} = \langle -\frac{1}{2}, 0, \frac{\sqrt{3}}{2} \rangle$, $\mathbf{N} = \langle -\frac{\sqrt{3}}{2}, 0, -\frac{1}{2} \rangle$

   (c) $\frac{d\mathbf{r}}{dt} = \langle 1, 1, 1 \rangle$, $\frac{d\mathbf{T}}{dt} = \langle \frac{1}{\sqrt{3}}, 0, -\frac{1}{\sqrt{3}} \rangle$
       $\mathbf{T} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$, $\mathbf{N} = \langle \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \rangle$

   (d) $\frac{d\mathbf{r}}{dt} = \langle \frac{3\sqrt{2}}{2}, -4, -\frac{3\sqrt{2}}{2} \rangle$, $\frac{d\mathbf{T}}{dt} = \langle -\frac{3\sqrt{2}}{10}, 0, -\frac{3\sqrt{2}}{10} \rangle$
       $\mathbf{T} = \langle \frac{3\sqrt{2}}{10}, -\frac{4}{5}, -\frac{3\sqrt{2}}{10} \rangle$, $\mathbf{N} = \langle -\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \rangle$

18. Sketch $\mathbf{r}(t)$ in the plane and plot the point where $t = a$, and then find and sketch $\mathbf{v}$, $\mathbf{a}$ at $t = a$ on the curve. (13.4)

   (a) $\mathbf{r}(t) = \langle t, t^2 \rangle$, $t = 2$

   (b) $\mathbf{r}(t) = \langle 2 \sin t, -2 \cos t \rangle$, $t = \pi/2$

   (c) $\mathbf{r}(t) = \langle e^{2t}, 2t \rangle$, $t = 0$

   (d) $\mathbf{r}(t) = \langle \sin(\ln t), \cos(\ln t) \rangle$, $t = 1$

19. Assuming ideal projectile motion and $g = 10\frac{m}{s^2}$, find the following.

   (a) Height of a projectile shot from the ground at an angle of $\pi/4$ with initial speed $16\sqrt{2}\frac{m}{s}$ after 2 seconds.

   (b) Flight time of a projectile shot from the ground at an angle of $\pi/6$ with initial speed $100\frac{m}{s}$.

   (c) Maximum height of a projectile shot from the ground at an angle of $\pi/3$ with initial speed $50\sqrt{3}\frac{m}{s}$.

   (d) Total horizontal distance traveled by a projectile shot from the ground at an angle of $\pi/4$ with initial speed $10\sqrt{2}\frac{m}{s}$.

   (e) Initial speed of a projectile shot from the ground at an angle of $\pi/3$ which has traveled 60 meters horizontally after 4 seconds.

20. Find the tangential and normal components of acceleration for the given position function at the given value of t.

   (a) $\mathbf{r}(t) = \langle 3t, t^2 \rangle$, $t = 2$

   (b) $\mathbf{r}(t) = \langle \sin t, \cos t \rangle$, $t = \pi/4$

   (c) $\mathbf{r}(t) = \langle \frac{1}{3}t^3, 2t, t^2 \rangle$, $t = 1$

   (d) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$, $t = \pi/2$

Chapter 14

21. Sketch the domain of the function f in the xy plane, sketch and label the three level curves of f within its domain for each given k value, and then sketch the graph of f. (14.1)

   (a) $f(x, y) = 2x - y + 1$, $k = -3, 0, 3$

   (b) $f(x, y) = 4x^2 + y^2$, $k = 0, 4, 16$

   (c) $f(x, y) = \sqrt{x^2 + 9y^2}$, $k = 0, 3, 6$

   (d) $f(x, y) = \sqrt{4 - x^2 - y^2}$, $k = 0, \frac{1}{\sqrt{2}}, 1$

   (e) $f(x, y) = \sqrt{4 - x^2 + y^2}$, $k = 0, 1, \sqrt{2}$

   (f) $f(x, y) = \ln(4 - x^2 - y^2)$, $k = \ln 1, \ln 2, \ln 3$

22. Sketch the level surface of f for the given value of k. (14.1)

   (a) $f(x, y, z) = x + y + z$, $k = 2$

   (b) $f(x, y, z) = x^2 + y^2 + z^2$, $k = 9$

   (c) $f(x, y, z) = \sqrt{x^2 + 4y^2 + z^2}$, $k = 2$

   (d) $f(x, y, z) = z - x^2$, $k = 3$

23. Prove the limit does not exist by comparing two paths of approach. (14.2)

   (a) $\displaystyle\lim_{P \to (0,0)} \frac{x^2 + y^2}{xy}$

   (b) $\displaystyle\lim_{P \to (0,0)} \frac{|xy|}{xy}$

   (c) $\displaystyle\lim_{P \to (0,0)} \frac{y^6 + x^2}{y^3 x + y^6}$

   (d) $\displaystyle\lim_{P \to (3,4)} \frac{25 - x^2 - y^2}{7 - x - y}$

24. Compute the value of the limit. (14.2)

    (a) $\displaystyle \lim_{P \to (1,-3)} \frac{6 - xy}{3x + y + 1}$

    (b) $\displaystyle \lim_{P \to (0,0)} \frac{2x^2 + 4y^2}{\sqrt{x^2 + 2y^2 + 1} - 1}$

    (c) $\displaystyle \lim_{P \to (0,0)} \frac{x \sin 2y - \sin 2y}{y - xy}$

    (d) $\displaystyle \lim_{P \to (1,2)} \frac{x^2 + 2xy + y^2 - 3x - 3y}{x + y}$

    (e) $\displaystyle \lim_{P \to (1,2)} \frac{x^2 + 2xy + y^2 - 3x - 3y}{x + y - 3}$

25. Find all the first-order and second-order partial derivatives of f. (14.3)

    (a) $f(x, y) = 4x^2 - 5y^3 + x - 1$

    (b) $f(x, y) = 3x^2y^2 - x^3 + y^4 - 7$

    (c) $f(x, y) = \sin(x + 3y)$

    (d) $f(x, y) = e^{xy^2}$

    (e) $f(r, \theta) = r \cos(\theta)$

    (f) $f(u, v) = \text{Arctan}\,(uv)$

26. Find the linearization $L(x, y)$ of $f(x, y)$ at (a, b), and use it to approximate the value of f at (c, d). (14.4)

    (a) $f(x, y) = 3x^2 - 2y^3$, $(a, b) = (1, 2)$, $(c, d) = (0.9, 2.2)$

    (b) $f(x, y) = 7y + 3xy - 1$, $(a, b) = (5, 1)$, $(c, d) = (5.1, 0.9)$

    (c) $f(x, y) = 2xy - x^2 - y^2$, $(a, b) = (3, -1)$, $(c, d) = (2.9, -1.05)$

    (d) $f(x, y) = \sqrt{25 - x^2 - y^2}$, $(a, b) = (-3, 0)$, $(c, d) = (-3.04, 0.09)$

27. Find the given derivative for the given nested functions at the given point. (14.5)

   (a) Find $\frac{df}{dt}$ at $t = 1$:
      $f(x, y, z) = xyz^2$, $x(t) = 2t + 1$, $y(t) = t^2 + 1$, $z(t) = 1 - t^3$

   (b) Find $\frac{\partial g}{\partial u}$ at $(u, v) = (2, 0)$:
      $g(x, y) = 2x + 3x^2y$, $x(u, v) = 1 - u$, $y(u, v) = 1 - uv$

   (c) Find $\frac{df}{dt}$ at $t = \pi/3$:
      $f(x, y) = 4x^2 + 2y$, $x(t) = \cos t$, $y(t) = 2\sin^2 t$

   (d) Find $\frac{\partial f}{\partial t}$ at $(t, u) = (0, 1)$:
      $f(x, y, z) = ye^x + 2z$, $x(t, u) = t^2$, $y(t, u) = t + u$, $z(t, u) = u + 1$

   (e) Find $\frac{dh}{dt}$ at $t = 1$:
      $h(x, y) = x + 2y$, $x(u, v) = uv$, $y(u, v) = v^2$, $u(t) = t^2$, $v(t) = t + 1$

28. Use partial derivatives to find the rate of change $\frac{dy}{dx}$ for the equation at the given point. (14.5)

   (a) $3x^2 + 5y = 8$ at $(1, 1)$

   (b) $4x^3y = 3xy^3 + 16$ at $(-1, 2)$

   (c) $-xy^2 + y^3 = -5x + 5$ at $(-3, 2)$

   (d) $x^3y^4 = x^4y^3$ at $(2, 2)$

   (e) $e^{xy} = \ln(xy + e)$ at $(1, 0)$

   (f) $\sin(2x + y) = \cos(2x + y) + 1$ at $(\pi/8, \pi/4)$

29. Find the gradient vector ∇f. (14.6)

   (a) $f(x, y) = x^3 + 3xy$

   (b) $f(x, y) = \sqrt{2xy + y^2}$

   (c) $f(x, y) = \frac{x+1}{y+1}$

   (d) $f(x, y, z) = \ln(x + y + z) + z^2$

   (e) $f(x, y, z) = yz\sin(\frac{1}{x})$

   (f) $f(x, y, z) = xye^{yz}$

30. Find the derivative of f in the direction of the given vector at the given point. (14.6)

    (a) $f(x, y) = x + 2y$, $\mathbf{A} = \langle -4, 3 \rangle$, $P_0 = (1, 3)$

    (b) $f(x, y) = xy^2 + 3y$, $\mathbf{A} = \langle 2, 2 \rangle$, $P_0 = (2, 0)$

    (c) $f(x, y) = e^{x+xy}$, $\mathbf{A} = 5\mathbf{i} - 12\mathbf{j}$, $P_0 = (\ln 2, 0)$

    (d) $f(x, y, z) = x^2 + 4y^2 + z^2$, $\mathbf{A} = \langle 3, -2, -6 \rangle$, $P_0 = (1, 1, 2)$

    (e) $f(x, y, z) = xz^3 + 3yz$, $\mathbf{A} = \langle 1, -2, 2 \rangle$, $P_0 = (-2, 0, 1)$

    (f) $f(x, y, z) = \ln(y^2) + 4xz$, $\mathbf{A} = 6\mathbf{i} - 8\mathbf{k}$, $P_0 = (3, 1, 2)$

31. Find and label all the points yielding local maximum values, local minimum values, and saddle points for f. (14.7)

    (a) $f(x, y) = x^2 + 9y^2 + 3$

    (b) $f(x, y) = x^2 - 2xy + 2y^2 + 4y - 3$

    (c) $f(x, y) = x^3 + 3xy + y^3 + 2$

    (d) $f(x, y) = x^3 - 6xy + \frac{3}{2}y^2 - 1$

    (e) $f(x, y) = x^2y - xy^2 + 12x - 12y$

    (f) $f(x, y) = (x^2 + y^2)e^{x+y+2}$

32. Find the absolute maximum and absolute mininum value of f within the closed bounded region R. (14.7)

    (a) $f(x, y) = x^2 + y^2$, R : square with vertices $(-1, 2)$, $(2, 2)$, $(2, 5)$, $(-1, 5)$

    (b) $f(x, y) = x^2 + y^2 - 2x - 2y$, R : triangle with vertices $(0, 0)$, $(2, 4)$, $(2, 0)$

    (c) $f(x, y) = x^2 + 2y^2 + 2xy + 4x$, $R = \{(x, y) : |x| \le 4, |y| \le 4\}$

    (d) $f(x, y) = 2xy$, $R = \{(x, y) : x^2 + y^2 \le 4\}$

33. Use Lagrange Multipliers to find the solution to the word problem. (14.8)

    (a) Find the maximum volume of a rectangular box without a lid which uses 108 square units of material.

    (b) Find the minimum surface area of a right circular cylinder with volume equal to 54π cubic units. ($V = \pi r^2 h$, $SA = 2\pi r(r + h)$)

    (c) Find the area of the largest rectangle which has its base on the x-axis and fits in the triangle with vertices $(-4, 0)$, $(0, 8)$, $(4, 0)$.

    (d) Find the highest and lowest points which lay on the curve of intersection for the cylinder $x^2 + y^2 = 8$ and the plane $2x + 2y + z = 16$.

Chapter 15

34. Divide R into four 2-by-2 equal pieces and use the midpoint rule to approximate the double integral. (15.1)

    (a) $\iint_R 2x + 2y + 4\,dA$, $R = \{(x,y) : 0 \le x \le 4, 0 \le y \le 2\}$
    (b) $\iint_R 3y^2 - 4xy\,dA$, $R = \{(x,y) : -1 \le x \le 3, -3 \le y \le 1\}$
    (c) $\iint_R 12x^2y\,dA$, $R = \{(x,y) : -2 \le x \le 2, 0 \le y \le 2\}$
    (d) $\iint_R \cos(x + y)\,dA$, $R = \{(x,y) : 0 \le x \le \pi/2, 0 \le y \le \pi/2\}$

35. Evaluate the double integral. (15.2)

    (a) $\iint_R 2x + 2y + 4\,dA$, $R = \{(x,y) : 0 \le x \le 4, 0 \le y \le 2\}$
    (b) $\iint_R 3y^2 - 4xy\,dA$, $R = \{(x,y) : -1 \le x \le 3, -3 \le y \le 1\}$
    (c) $\iint_R 12x^2y\,dA$, $R = \{(x,y) : -2 \le x \le 2, 0 \le y \le 2\}$
    (d) $\iint_R \cos(x + y)\,dA$, $R = \{(x,y) : 0 \le x \le \pi/2, 0 \le y \le \pi/2\}$
    (e) $\iint_R 3x(1 + xy)^2\,dA$, $R = \{(x,y) : 1 \le x \le 3, 0 \le y \le 1\}$
    (f) $\iint_R 2xy\sqrt{16 + x^2}\,dA$, $R = \{(x,y) : 0 \le x \le 3, 0 \le y \le 3\}$

36. Evaluate the iterated integral or double integral of two variables. (15.3)

    (a) $\displaystyle\int_0^2 \int_0^x 11x^2 + 3y^2\,dy\,dx$

    (b) $\displaystyle\int_{-1}^2 \int_{-1}^{y^2} 20xy\,dx\,dy$

    (c) $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 6x + 30y\,dx\,dy$

    (d) $\displaystyle\int_1^2 \int_{1/x}^{2/x} xe^x\,dx\,dy$

    (e) $\iint_R 8xy\,dA$, $R = \{(x,y) : 0 \le x \le y, 0 \le y \le 1\}$
    (f) $\iint_R \frac{6}{5}y\,dA$, $R :$ triangle with vertices $(-2, 0)$, $(0, 1)$, $(3, 0)$

37. Evaluate the iterated integral of two variables. (15.3)

    (a) $\displaystyle\int_0^1 \int_x^1 \frac{2}{\sqrt{4+y^2}} \, dy \, dx$

    (b) $\displaystyle\int_0^2 \int_y^2 y(8-x^3)^{1/3} \, dx \, dy$

    (c) $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 3\pi \sin(\pi x^3) \, dx \, dy$

    (d) $\displaystyle\int_0^1 \int_{e^x}^e \frac{y}{\ln y} \, dy \, dx$

38. Find an expression involving iterated integrals for the given area or average value. (15.3)

    (a) Area of the rectangle with vertices $(-1,0)$, $(2,0)$, $(2,4)$, $(-1,4)$

    (b) Area of the parallelogram with vertices $(-1,2)$, $(3,2)$, $(4,1)$, $(0,1)$

    (c) Area of the triangle with vertices $(1,3)$, $(1,1)$, and $(2,2)$

    (d) Area between $x = 4 - y^2$ and $x = y^2 - 4$

    (e) Average value of $f(x,y) = e^{x^2 y}$ over the square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$

    (f) Average value of $f(x,y) = \sin\left(\frac{x}{2y}\right)$ over the triangle with vertices $(0,1)$, $(1,1)$, $(0,2)$

39. Evaulate the iterated integral of three variables. (15.7)

    (a) $\displaystyle\int_0^1 \int_0^1 \int_0^1 8xz - y^2 \, dy \, dx \, dz$

    (b) $\displaystyle\int_1^2 \int_0^x \int_x^{2z} 24y \, dy \, dz \, dx$

    (c) $\displaystyle\int_{-1}^1 \int_{1+y}^{2+y} \int_0^2 z \, dx \, dz \, dy$

    (d) $\displaystyle\int_{-\pi}^0 \int_0^{\pi/2} \int_0^x - \sin(z) \, dz \, dy \, dx$

    (e) $\displaystyle\int_0^1 \int_0^1 \int_0^1 \frac{2xy^2}{(1+xyz)^3} \, dz \, dx \, dy$

40. Find an expression involving iterated integrals for the volume of the given solid. (15.7)

    (a) The pyramid with vertices $(0,0,0)$, $(3,0,0)$, $(0,2,0)$, and $(0,0,1)$

    (b) The solid in the first octant bounded by the coordinate planes, $z = 1 - y^2$, and $x = 4$

    (c) The sphere $x^2 + y^2 + z^2 \leq 4$

    (d) The solid bounded by the surfaces $z = 4 - x^2 - y^2$ and $z = 4x^2 + 4y^2 - 16$

41. Find a transformation from either the unit square or triangle in the uv plane into the given region R in the xy plane. (15.10)

    (a) R : parallelogram bounded by $y = 3x + 1$, $y = 3x - 3$, $y = x - 3$ $y = x + 1$

    (b) R : triangle bounded by $y = x$, $y = 2x$, $y = 6 - x$

    (c) R : square with vertices $(2, 1)$, $(-2, 3)$, $(0, 7)$, $(4, 5)$

    (d) R : triangle with vertices $(0, -2)$ $(-1, 1)$, $(1, 3)$

42. Evaulate the double integral of variables x, y using the given transformation from the uv plane. (15.10)

    (a) $\iint_R 2x - y \, dA$, $\mathbf{r}(u, v) = \langle u + v, 2u - v + 3 \rangle$ from unit square into the parallelogram R with vertices $(0, 3)$, $(1, 5)$, $(2, 4)$, $(1, 2)$

    (b) $\iint_R (x + y)(x - y - 2) \, dA$, $\mathbf{r}(u, v) = \langle 4 - u - v, v - u + 2 \rangle$ from unit triangle into the triangle R with vertices $(4, 2)$, $(3, 1)$, $(2, 2)$

    (c) $\iint_R (x + y)e^{x^2 - y^2} \, dA$, $\mathbf{r}(u, v) = \langle u + 2v, u - 2v \rangle$ from unit square into the rectangle R bounded by $y = x$, $y = x - 4$, $y = -x$, $y = 2 - x$

    (d) $\iint_R e^x \cos(\pi e^x) \, dA$, $\mathbf{r}(u, v) = \langle \ln(u + v + 1), v \rangle$ from unit triangle into the region R bounded by $y = 0$, $y = e^x - 2$, $y = \frac{e^x - 1}{2}$

43. Use polar coordinates to evaluate the double integral or iterated integral. (15.4)

    (a) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 2y\,dx\,dy$

    (b) $\displaystyle\int_{0}^{1}\int_{0}^{x} 3xy\,dy\,dx$

    (c) $\displaystyle\iint_{R} e^{x^2+y^2}\,dA,\ R:$ disk with boundary $x^2+y^2=9$

    (d) $\displaystyle\int_{0}^{4}\int_{0}^{\sqrt{4x-x^2}} dy\,dx$

44. Use cylindrical coordinates to give an equivalent iterated integral which can be directly evaluated. (15.8)

    (a) $\displaystyle\int_{0}^{3}\int_{0}^{\sqrt{9-y^2}}\int_{0}^{1} 2z\,dz\,dx\,dy$

    (b) $\displaystyle\iiint_{D} \sqrt{x^2+y^2}\,dV,\ D:$ right circular cylinder bounded by $|z|\le 2$ and $x^2+y^2=1$

    (c) $\displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} dz\,dy\,dx$

    (d) The volume of the solid bounded by the xy plane and $z = 1-x^2-y^2$

45. Use spherical coordinates to give an equivalent iterated integral which can be directly evaluated. (15.9)

    (a) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{0}^{\sqrt{1-x^2-y^2}} dz\,dx\,dy$

    (b) $\displaystyle\iiint_{D} x\,dV,\ D:$ hemisphere bounded by $x = \sqrt{4-y^2-z^2}$ and the yz plane

    (c) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} 3xz\,dz\,dx\,dy$

    (d) The volume of the "ice cream cone" shaped solid
    $$D = \{(x,y,z) : \sqrt{x^2+y^2} \le z \le \sqrt{1-x^2-y^2}+1\}$$

Chapter 16

46. Evaluate the line integral with respect to arclength. (16.2)

   (a) $\int_C 2x + y \, ds$, C : line segment given by $\mathbf{r}(t) = \langle 4t + 1, 4 - 3t \rangle$ for $0 \leq t \leq 2$
   (b) $\int_C z + 2xy \, ds$, C : line segment from $(0, -1, 3)$ to $(2, 2, -3)$
   (c) $\int_C xy^3 \, ds$, C : arc on the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(1, \sqrt{3})$
   (d) $\int_C 2x \, ds$, C : parabolic arc on $y = x^2$ from $(0, 0)$ to $(1, 1)$

47. Evaluate the line integral with respect to a variable. (16.2)

   (a) $\int_C 2x + y \, dx$, C : line segment given by $\mathbf{r}(t) = \langle 4t + 1, 4 - 3t \rangle$ for $0 \leq t \leq 2$
   (b) $\int_C z + 2xy \, dz$, C : line segment from $(0, -1, 3)$ to $(2, 2, -3)$
   (c) $\int_C xy^3 \, dy$, C : arc on the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(1, \sqrt{3})$
   (d) $\int_C 2x \, dy$, C : parabolic arc on $y = x^2$ from $(0, 0)$ to $(1, 1)$

48. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ directly. (16.2)

   (a) $\mathbf{F} = \langle y, x + y \rangle$, C : line segment from $(1, 3)$ to $(-4, -9)$
   (b) $\mathbf{F} = \langle z, xy, z \rangle$, C : line segment from $(0, -1, 3)$ to $(2, 2, -3)$
   (c) $\mathbf{F} = \langle y^2, x^2 \rangle$, C : one counter-clockwise revolution of the circle $x^2 + y^2 = 9$
   (d) $\mathbf{F} = \langle y, 2y \rangle$, C : trigonometric arc on $y = \sin x$ from $(0, 0)$ to $(\pi, 0)$

49. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ using the Fundamental Theorem of Line Integrals. (16.3)

   (a) $\mathbf{F} = \langle x, y \rangle$, C : line segment from $(1, 1)$ to $(3, -2)$
   (b) $\mathbf{F} = \langle yz, xz, xy \rangle$, C : line segment from $(0, -3, 2)$ to $(4, -1, 3)$
   (c) $\mathbf{F} = \langle 4, z^2, 2yz \rangle$, C : curve given by $\mathbf{r}(t) = \langle 2^t, \sin(\pi t), 4t^2 \rangle$ for $0 \leq t \leq 1$
   (d) $\mathbf{F} = \langle 2x, 1 \rangle$, C : counter-clockwise oriented boundary of the unit square
   (e) $\mathbf{F} = \langle 12x^2 y^2 + 3y, 8x^3 y + 3x \rangle$, C : one clockwise revolution of the ellipse $x^2 + 4y^2 = 4$
   (f) $\mathbf{F} = \langle ye^{xy+z}, xe^{xy+z}, e^{xy+z} \rangle$, C : curve given by $\mathbf{r}(t) = \left\langle \frac{1}{1+t^2}, \cos t, e^{1-t^2} \right\rangle$ for $-1 \leq t \leq 1$

50. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Green's Theorem. (16.4)

   (a) $\mathbf{F} = \langle x^2 + y, x + y \rangle$, C : boundary of the unit square oriented counter-clockwise

   (b) $\mathbf{F} = \langle x, x^2 + xy^3 \rangle$, C : boundary of the rectangle
   $R = \{(x, y) : 1 \le x \le 2, 1 \le y \le 3\}$ oriented clockwise

   (c) $\mathbf{F} = \langle y, 2x \rangle$, C : boundary of the triangle with vertices $(1, 2)$, $(3, -2)$, $(-1, -2)$ oriented counter-clockwise

   (d) $\mathbf{F} = \langle x + y, x - y \rangle$, C : boundary of the upper semicircle $0 \le y \le \sqrt{4 - x^2}$ oriented counter-clockwise

51. Find the divergence and curl for each vector field. (16.5)

   (a) $\mathbf{F} = \langle x^2, y^2, z^2 \rangle$

   (b) $\mathbf{F} = \left\langle \frac{1}{x}, \frac{1}{y^2}, \frac{1}{x+y+z} \right\rangle$

   (c) $\mathbf{F} = \langle xyz, 2xyz, 3xyz \rangle$

   (d) $\mathbf{F} = \langle e^z \cos x, e^y \cos z, e^x \cos y \rangle$

   (e) $\mathbf{F} = \left\langle \frac{2}{x+yz}, 2, 2x + 2yz \right\rangle$

   (f) $\mathbf{F} = \left\langle \ln x, \frac{1}{x}, yz^3 \right\rangle$

52. Find a parametrization from the region G to the surface S. (16.6)

   (a) S : the portion of the elliptical paraboloid $z = x^2 + y^2$ above G
   $G : 0 \le x \le 2$ and $0 \le y \le 1$

   (b) S : the triangle with vertices $(0, 0, 8)$, $(2, 0, 6)$, $(2, 4, 2)$
   G : triangle with vertices $(0, 0)$, $(2, 0)$, $(2, 4)$

   (c) S : the lateral surface on the cylinder $x^2 + y^2 = 9$ between $z = -1$ and $z = 4$
   $G : 0 \le \theta \le 2\pi$ and $-1 \le z \le 4$

   (d) S : the sphere $x^2 + y^2 + z^2 = 4$
   $G : 0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$

   (e) S : the portion of the conical surface $z = \sqrt{x^2 + y^2}$ within the first octant and between the planes $z = 1$ and $z = 2$
   $G : 0 \le \theta \le \frac{\pi}{2}$ and $1 \le r \le 2$

   (f) S : the portion of the conical surface $z = \sqrt{x^2 + y^2}$ inside the sphere $x^2 + y^2 + z^2 = 9$
   $G : 0 \le \theta \le 2\pi$ and $0 \le \rho \le 3$

53. Given the parametrization $\mathbf{r}$ from the region G to the surface S, express the surface area of the S as a double iterated integral. (16.6)

   (a) S: portion of the plane $x + 2y + 3z = 6$ in the first octant
   G: triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$
   $\mathbf{r}(u,v) = \langle 6u, 3v, 2 - 2u - 2v \rangle$

   (b) S: elliptical region given by the portion of the plane $4x + y + z = 8$ inside the cylinder $x^2 + y^2 = 4$
   G : circlular region given by $0 \le r \le 2$ and $0 \le \theta \le 2\pi$
   $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 8 - 4r\cos\theta - r\sin\theta \rangle$

   (c) S : surface $z = \sqrt{x^3} + \sqrt{y^3}$ above G
   G : square $0 \le x \le 9$, $0 \le y \le 9$
   $\mathbf{r}(x,y) = \left\langle x, y, \sqrt{x^3} + \sqrt{y^3} \right\rangle$

   (d) S : hemisphere $x^2 + y^2 + z^2 = 4$ above the xy plane
   G: rectangle $0 \le \phi \le \frac{\pi}{2}$, $0 \le \theta \le 2\pi$
   $\mathbf{r}(\phi,\theta) = \langle 2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi \rangle$

54. Given the parametrization $\mathbf{r}$ from the region G to the surface S, rewrite the surface integral as a double iterated integral. (16.7)

   (a) $\iint_S x + y + z \, d\sigma$
   S: portion of the plane $x + z = 2$ above G
   G: square $0 \le x \le 1$, $0 \le y \le 1$
   $\mathbf{r}(x,y) = \langle x, y, 2 - x \rangle$

   (b) $\iint_S x^2 + y^2 \, d\sigma$
   S: lateral surface of the cylinder $x^2 + y^2 = 4$ where $0 \le z \le 1$
   G: rectangle $0 \le \theta \le 2\pi$, $0 \le z \le 1$
   $\mathbf{r}(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z \rangle$

   (c) $\iint_S 3z \, d\sigma$
   S: cone $z = \sqrt{x^2 + y^2}$ below $z = 2$
   G: rectangle $0 \le \rho \le 2\sqrt{2}$, $0 \le \theta \le 2\pi$
   $\mathbf{r}(\rho, \theta) = \left\langle \frac{\sqrt{2}}{2}\rho\cos\theta, \frac{\sqrt{2}}{2}\rho\sin\theta, \frac{\sqrt{2}}{2}\rho \right\rangle$

55. Given the positively oriented parametrization **r** from the region G to the surface S, rewrite the vector field surface integral as a scalar double iterated integral. (16.7)

    (a) $\iint_S \langle x + y, y + z, z + x \rangle \cdot d\vec{\sigma}$
        S: parallelogram with vertices $(4, 0, 3)$, $(5, -2, 2)$, $(4, -1, -1)$, $(3, 1, 0)$
        G: square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$ in the uv plane
        $\mathbf{r}(u, v) = \langle 4 + u - v, -2u + v, 3 - u - 3v \rangle$

    (b) $\iint_S \langle y, -x, 1 - z \rangle \cdot d\vec{\sigma}$
        S: portion of the elliptical paraboloid $z = x^2 + y^2$ above G with concave orientation
        G: triangle $0 \leq x \leq 2$, $0 \leq y \leq 2x$ in the xy plane
        $\mathbf{r}(x, y) = \langle x, y, x^2 + y^2 \rangle$

    (c) $\iint_S \langle x, y, z \rangle \cdot d\vec{\sigma}$
        S: surface of the unit sphere oriented outwards
        G: rectangle $0 \leq \phi \leq \pi$, $0 \leq \theta \leq 2\pi$
        $\mathbf{r}(\phi, \theta) = \langle \sin\phi \cos\theta, \sin\phi \sin\theta, \cos\phi \rangle$

56. Use Stokes' Theorem to rewrite the given surface integral as a definite integral, where S is the upper hemisphere $z = \sqrt{1 - x^2 - y^2}$ with convex orientation.

    (a) $\iint_S \langle 1, 1, 1 \rangle \cdot d\vec{\sigma}$

    (b) $\iint_S \langle -2z, 0, -2y \rangle \cdot d\vec{\sigma}$

    (c) $\iint_S \langle 0, 1 - 2x, 2x - 1 \rangle \cdot d\vec{\sigma}$

    (d) $\iint_S \langle 1, 1, 3 - 2y \rangle \cdot d\vec{\sigma}$

57. Use the Divergence Theorem to rewrite the given surface integral as a triple iterated integral, where S is the surface of the unit cube oriented outwards.

    (a) $\iint_S \langle x, y, z \rangle \cdot d\vec{\sigma}$

    (b) $\iint_S \langle x + y, y^2 + z^2, z^3 + x^3 \rangle \cdot d\vec{\sigma}$

    (c) $\iint_S \langle xyz, xyz, xyz \rangle \cdot d\vec{\sigma}$

    (d) $\iint_S \langle xy + yz, yz + zx, zx + xy \rangle \cdot d\vec{\sigma}$