## Chapter 12

- 1. Find the cosine of the angle between the vectors  $\mathbf{u}$  and  $\mathbf{v}$ . (12.3)
  - (a)  $\mathbf{u} = \langle 4, -3, 0 \rangle$

$$\mathbf{v} = \langle 2, 6, -3 \rangle$$

(b) 
$$\mathbf{u} = \langle 1, 2, 2 \rangle$$

$$\mathbf{v} = \langle 0, 0, -3 \rangle$$

(c) 
$$\mathbf{u} = \langle 1, 4, 2 \rangle$$

$$\mathbf{v} = \langle 4, 1, -2 \rangle$$

(d) 
$$\mathbf{u} = \langle -4, -4, -6 \rangle$$

$$\mathbf{v} = \langle 2, 2, 3 \rangle$$

(e) 
$$\mathbf{u} = \langle 0, 5, -11 \rangle$$

$$\mathbf{v} = \langle 2, 0, 0 \rangle$$

(f) 
$$\mathbf{u} = \langle 3, 2, 1 \rangle$$

$$\mathbf{v} = \langle 6, 4, 2 \rangle$$

2. Find the projection of  $\mathbf{u}$  onto  $\mathbf{v}$ . (12.3)

(a) 
$$\mathbf{u} = \langle 4, -3, 0 \rangle$$

$$\mathbf{v} = \langle 2, 6, -3 \rangle$$

(b) 
$$\mathbf{u} = \langle 1, 2, 2 \rangle$$

$$\mathbf{v} = \langle 0, 0, -3 \rangle$$

(c) 
$$\mathbf{u} = \langle 1, 4, 2 \rangle$$

$$\mathbf{v} = \langle 4, 1, -2 \rangle$$

(d) 
$$\mathbf{u} = \langle -4, -4, -6 \rangle$$

$$\mathbf{v} = \langle 2, 2, 3 \rangle$$

(e) 
$$\mathbf{u} = \langle 0, 5, -11 \rangle$$

$$\mathbf{v} = \langle 2, 0, 0 \rangle$$

(f) 
$$\mathbf{u} = \langle 3, 2, 1 \rangle$$

$$\mathbf{v} = \langle 6, 4, 2 \rangle$$

- 3. Use the cross product to find a vector normal to both  ${\bf u}$  and  ${\bf v}$ . (12.4)
  - (a)  $\mathbf{u} = \langle 4, -3, 0 \rangle$

$$\mathbf{v} = \langle 2, 6, -3 \rangle$$

(b) 
$$\mathbf{u} = \langle 1, 2, 2 \rangle$$

$$\mathbf{v} = \langle 0, 0, -3 \rangle$$

(c) 
$$\mathbf{u} = \langle 1, 4, 2 \rangle$$

$$\mathbf{v} = \langle 4, 1, -2 \rangle$$

(d) 
$$\mathbf{u} = \langle -4, -4, -6 \rangle$$

$$\mathbf{v} = \langle 2, 2, 3 \rangle$$

(e) 
$$\mathbf{u} = \langle 0, 5, -11 \rangle$$

$$\mathbf{v} = \langle 2, 0, 0 \rangle$$

(f) 
$$\mathbf{u} = (3, 2, 1)$$

$$\mathbf{v} = \langle 6, 4, 2 \rangle$$

- 4. Give a vector equation and parametric equations for the line. (12.5)
  - (a) The line passing through (1, 3, -2) and parallel to (3, 0, 1).
  - (b) The line passing through (-2,0,4) and (1,3,3).
  - (c) The line parallel to  $\mathbf{r}(t) = \langle t, 2-t, 2+t \rangle$  and passing through (2,4,5).
  - (d) The line with equation x = -3z + 1 in the xz plane.
  - (e) The line normal to the plane with equation x + y + 2z = 4 and passing through (1, 1, 1).
- 5. Give the distance from the point to the line: (12.5)

(a) 
$$(4,5,3)$$
 to  $\mathbf{r}(t) = \langle 1+t, 2+2t, 2t \rangle$ 

(b) 
$$(-1, -2, 2)$$
 to  $\mathbf{r}(t) = \langle -1 + 3t, -4, 2 + 4t \rangle$ 

(c) 
$$(3,0)$$
 to  $\mathbf{r}(t) = \langle 4-4t, 7-3t \rangle$ 

(d) (2,6) to 
$$\mathbf{r}(t) = \langle -3 + 3t, 1 + t \rangle$$

- 6. Give an equation for the plane. (12.5)
  - (a) The plane passing through (1, 3, -2) and normal to (3, 0, 1).
  - (b) The plane passing through (1, -2, 0) and parallel to 2x y + 3z = 3.
  - (c) The plane passing through (1,1,1) and normal to the line with equation  $\mathbf{r}(t) = \langle 4-3t, t, 2+2t \rangle$ .
  - (d) The plane passing through (-2,0,4), (1,3,3), and (0,0,2).
- 7. Give the distance from the point to the plane: (12.5)
  - (a) (5,1,1) to x-2y+2z=2
  - (b) (4, -1, 3) to 3x + 4z = 4
  - (c) (0,1,1) to -2x-3y-6z=5
  - (d) (-1,5,2) to x+y+z=3
- 8. Sketch the curve given by the equation in the appropriate coordinate plane, and then sketch the cylinder in xyz space given by the equation. (12.6)
  - (a)  $y = x^2$
  - (b)  $x = z^3$
  - (c)  $y = \sin z$
  - (d)  $z = e^x$
  - (e)  $z = \ln y$
  - (f) xy = 1
- 9. Sketch the three coordinate plane cross-sections for the quadric surface given by the equation, sketch the surface itself, and give the name of the surface. (12.6)
  - (a)  $x^2 y = -z^2$
  - (b)  $y^2 + z^2 = 4 4x^2$
  - (c)  $z^2 9y^2 = x^2$
  - (d)  $y^2 z^2 = 4 4x^2$
  - (e)  $4x^2 y^2 4z^2 = 16$
  - (f)  $z = y^2 4x^2$

## Chapter 13

- 10. Give a parametrization of the curve described as a vector function. (13.1)
  - (a) The parabola  $y = x^2$  in the xy plane.
  - (b) The directed line segment beginning at (1, 2, -3) and ending at (0, 3, 0).
  - (c) The circle  $x^2 + y^2 = 9$ .
  - (d) The ellipse  $x^2 + 9y^2 = 9$ .
- 11. Find the limit of the vector function. (13.1)

(a) 
$$\lim_{t \to -1} \left\langle \text{Arctan } t, \frac{e^{1+t}}{1-t} \right\rangle$$

(b) 
$$\lim_{t \to 2} \left\langle t^2 - 4, \frac{t^2 - 4}{t - 2} \right\rangle$$

(c) 
$$\lim_{t\to 0} \left( \frac{\sin 3t}{4t} \mathbf{i} + \frac{1-\cos t}{t} \mathbf{j} \right)$$

(d) 
$$\lim_{t \to \pi/2} \langle \sin t, \cos t, \cot t \rangle$$

(e) 
$$\lim_{t\to 0} \left( \frac{e^t}{t+1} \mathbf{i} + \frac{e^t - 1}{t} \mathbf{j} + \frac{2^{2t} - 1}{t} \mathbf{k} \right)$$

(f) 
$$\lim_{t \to 1} \left\langle \frac{3t^2 - 3}{t + 1}, \frac{\sin(2t - 2)}{2t - 2}, \frac{3t^2 - 3}{t - 1} \right\rangle$$

12. Find the derivative  $\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t)$  of the vector function. (13.2)

(a) 
$$\mathbf{r}(t) = \langle x^2, 3+t \rangle$$

(b) 
$$\mathbf{r}(t) = \langle 3\sin 4t, -3\cos 4t \rangle$$

(c) 
$$\mathbf{r}(t) = \left\langle \frac{1}{t^2}, \frac{t}{t^2+1} \right\rangle$$

(d) 
$$\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$$

(e) 
$$\mathbf{r}(t) = (\ln 2t)\mathbf{i} + (e^{2t} - 2)\mathbf{j} + \frac{1}{e^t}\mathbf{k}$$

(f) 
$$\mathbf{r}(t) = \langle \text{Arcsin } t, \text{Arccsc } t, \text{Arctan } t \rangle$$

13. Find the indefinite integral  $\int \mathbf{r}(t) dt$  of the vector function (13.2)

(a) 
$$\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$$

(b) 
$$\mathbf{r}(t) = \langle 2\sin 2t, -2\cos 2t \rangle$$

(c) 
$$\mathbf{r}(t) = \langle e^t, 2e^{-t}, e^{3t} \rangle$$

(d) 
$$\mathbf{r}(t) = \left\langle \frac{1}{t+2}, \frac{1}{(t+2)^2}, \frac{2t}{t^2+2} \right\rangle$$

(e) 
$$\mathbf{r}(t) = (e^{2t}e^t + 2t)\mathbf{i} + \frac{\ln t}{t}\mathbf{k}$$

14. Solve the differential vector equation to find  $\mathbf{r}(t)$ . (13.2)

(a) 
$$\mathbf{r}'(t) = \langle 3t^2, 2t^3 \rangle, \mathbf{r}(0) = \langle 3, 4 \rangle$$

(b) 
$$\mathbf{r}'(t) = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \ \mathbf{r}(1) = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

(c) 
$$\mathbf{r}'(t) = \langle 2e^t, 4, \frac{1}{t} \rangle, \mathbf{r}(\ln 3) = \langle 5, 0, \ln(\ln 3) \rangle$$

(d) 
$$\mathbf{r}'(t) = \langle \frac{3}{2}\sqrt{t}, 8t, 3t^2 + 3 \rangle, \mathbf{r}(1) = \langle 1, -3, 6 \rangle$$

(e) 
$$\mathbf{r}'(t) = \left\langle \frac{1}{1+t^2}, \frac{2t}{1+t^2} \right\rangle, \mathbf{r}(0) = \left\langle 0, 1 \right\rangle$$

15. Find the arclength parameter s(t) where s(0) = 0 and  $\frac{ds}{dt} \ge 0$  for the given curve, and use it to find the arclength of the given portion of the curve. (13.3)

(a) 
$$\mathbf{r}(t) = \langle 1 + 2t, 2 - t, 3 - 2t \rangle, 1 \le t \le 3$$

(b) 
$$\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle, \ 0 \le t \le 1$$

(c) 
$$\mathbf{r}(t) = \langle 3e^t, -4e^t \rangle, \ 0 \le t \le \ln 2$$

(d) 
$$\mathbf{r}(t) = t^3 \mathbf{i} + t^2 \mathbf{j} + \mathbf{k}, \ 0 \le t \le 1$$

(e) 
$$\mathbf{r}(t) = \langle 6t, t^3, 3t^2 \rangle, -3 < t < -2$$

16. Find the unit vectors  $\mathbf{T}, \mathbf{N}$  to the curve in terms of the parameter t. (13.3)

(a) 
$$\mathbf{r}(t) = \langle 3\cos 2t, 3\sin 2t \rangle$$

(b) 
$$\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$$

(c) 
$$\mathbf{r}(t) = \langle \sqrt{2} \sin t, 2 \cos t, \sqrt{2} \sin t \rangle$$

(d) 
$$\mathbf{r}(t) = \langle e^t, e^t \sin t, e^t \cos t \rangle$$

17. Given the information about  $\mathbf{r}(t)$  at a point, evaluate the binormal vector  $\mathbf{B}$  and curvature  $\kappa$  at that same point. (13.3)

(a) 
$$\frac{d\mathbf{r}}{dt} = \langle 3, 0, -4 \rangle, \frac{d\mathbf{T}}{dt} = \langle 0, 10, 0 \rangle$$

(b) 
$$\frac{d\mathbf{r}}{dt} = \langle -3, 0, 3\sqrt{3} \rangle, \frac{d\mathbf{T}}{dt} = \langle -\sqrt{3}, 0, -1 \rangle$$

(c) 
$$\frac{d\mathbf{r}}{dt} = \langle 1, 1, 1 \rangle, \frac{d\mathbf{T}}{dt} = \left\langle \frac{1}{\sqrt{3}}, 0, -\frac{1}{\sqrt{3}} \right\rangle$$

(d) 
$$\frac{d\mathbf{r}}{dt} = \left\langle \frac{3\sqrt{2}}{2}, -4, -\frac{3\sqrt{2}}{2} \right\rangle, \frac{d\mathbf{T}}{dt} = \left\langle -\frac{3\sqrt{2}}{10}, 0, -\frac{3\sqrt{2}}{10} \right\rangle$$

18. Sketch  $\mathbf{r}(t)$  in the plane and plot the point where t=a, and then find and sketch  $\mathbf{v}$ ,  $\mathbf{a}$  at t=a on the curve. (13.4)

(a) 
$$\mathbf{r}(t) = \langle t, t^2 \rangle, t = 2$$

(b) 
$$\mathbf{r}(t) = \langle 2\sin t, -2\cos t \rangle, t = \pi/2$$

(c) 
$$\mathbf{r}(t) = \langle e^{2t}, 2t \rangle, t = 0$$

(d) 
$$\mathbf{r}(t) = \langle \sin(\ln t), \cos(\ln t) \rangle, t = 1$$

- 19. Assuming ideal projectile motion and  $g = 10 \frac{m}{s^2}$ , find the following.
  - (a) Height of a projectile shot from the ground at an angle of  $\pi/4$  with initial speed  $16\sqrt{2}\frac{m}{s}$  after 2 seconds.
  - (b) Flight time of a projectile shot from the ground at an angle of  $\pi/6$  with initial speed  $100\frac{m}{s}$ .
  - (c) Maximum height of a projectile shot from the ground at an angle of  $\pi/3$  with initial speed  $50\sqrt{3}\frac{m}{s}$ .
  - (d) Total horizontal distance traveled by a projectile shot from the ground at an angle of  $\pi/4$  with initial speed  $10\sqrt{2}\frac{m}{s}$ .
  - (e) Initial speed of a projectile shot from the ground at an angle of  $\pi/3$  which has traveled 60 meters hoizontally after 4 seconds.

- 20. Find the tangential and normal components of acceleration for the given position function at the given value of t.
  - (a)  $\mathbf{r}(t) = \langle 3t, t^2 \rangle, t = 2$
  - (b)  $\mathbf{r}(t) = \langle \sin t, \cos t \rangle, t = \pi/4$
  - (c)  $\mathbf{r}(t) = \langle \frac{1}{3}t^3, 2t, t^2 \rangle, t = 1$
  - (d)  $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle, t = \pi/2$