

Chapter 12

1. Find the cosine of the angle between the vectors \mathbf{u} and \mathbf{v} . (12.3)

(a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
 $\mathbf{v} = \langle 2, 6, -3 \rangle$

(b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
 $\mathbf{v} = \langle 0, 0, -3 \rangle$

(c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
 $\mathbf{v} = \langle 4, 1, -2 \rangle$

(d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
 $\mathbf{v} = \langle 2, 2, 3 \rangle$

(e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
 $\mathbf{v} = \langle 2, 0, 0 \rangle$

(f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
 $\mathbf{v} = \langle 6, 4, 2 \rangle$

2. Find the projection of \mathbf{u} onto \mathbf{v} . (12.3)

(a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
 $\mathbf{v} = \langle 2, 6, -3 \rangle$

(b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
 $\mathbf{v} = \langle 0, 0, -3 \rangle$

(c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
 $\mathbf{v} = \langle 4, 1, -2 \rangle$

(d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
 $\mathbf{v} = \langle 2, 2, 3 \rangle$

(e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
 $\mathbf{v} = \langle 2, 0, 0 \rangle$

(f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
 $\mathbf{v} = \langle 6, 4, 2 \rangle$

3. Use the cross product to find a vector normal to both \mathbf{u} and \mathbf{v} . (12.4)

(a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
 $\mathbf{v} = \langle 2, 6, -3 \rangle$

(b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
 $\mathbf{v} = \langle 0, 0, -3 \rangle$

(c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
 $\mathbf{v} = \langle 4, 1, -2 \rangle$

(d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
 $\mathbf{v} = \langle 2, 2, 3 \rangle$

(e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
 $\mathbf{v} = \langle 2, 0, 0 \rangle$

(f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
 $\mathbf{v} = \langle 6, 4, 2 \rangle$

4. Give a vector equation and parametric equations for the line. (12.5)

(a) The line passing through $(1, 3, -2)$ and parallel to $\langle 3, 0, 1 \rangle$.

(b) The line passing through $(-2, 0, 4)$ and $(1, 3, 3)$.

(c) The line parallel to $\mathbf{r}(t) = \langle t, 2 - t, 2 + t \rangle$ and passing through $(2, 4, 5)$.

(d) The line with equation $x = -3z + 1$ in the xz plane.

(e) The line normal to the plane with equation $x + y + 2z = 4$ and passing through $(1, 1, 1)$.

5. Give the distance from the point to the line: (12.5)

(a) $(4, 5, 3)$ to $\mathbf{r}(t) = \langle 1 + t, 2 + 2t, 2t \rangle$

(b) $(-1, -2, 2)$ to $\mathbf{r}(t) = \langle -1 + 3t, -4, 2 + 4t \rangle$

(c) $(3, 0)$ to $\mathbf{r}(t) = \langle 4 - 4t, 7 - 3t \rangle$

(d) $(2, 6)$ to $\mathbf{r}(t) = \langle -3 + 3t, 1 + t \rangle$

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6. Give an equation for the plane. (12.5)
- (a) The plane passing through $(1, 3, -2)$ and normal to $\langle 3, 0, 1 \rangle$.
 - (b) The plane passing through $(1, -2, 0)$ and parallel to $2x - y + 3z = 3$.
 - (c) The plane passing through $(1, 1, 1)$ and normal to the line with equation $\mathbf{r}(t) = \langle 4 - 3t, t, 2 + 2t \rangle$.
 - (d) The plane passing through $(-2, 0, 4)$, $(1, 3, 3)$, and $(0, 0, 2)$.
7. Give the distance from the point to the plane: (12.5)
- (a) $(5, 1, 1)$ to $x - 2y + 2z = 2$
 - (b) $(4, -1, 3)$ to $3x + 4z = 4$
 - (c) $(0, 1, 1)$ to $-2x - 3y - 6z = 5$
 - (d) $(-1, 5, 2)$ to $x + y + z = 3$
8. Sketch the curve given by the equation in the appropriate coordinate plane, and then sketch the cylinder in xyz space given by the equation. (12.6)
- (a) $y = x^2$
 - (b) $x = z^3$
 - (c) $y = \sin z$
 - (d) $z = e^x$
 - (e) $z = \ln y$
 - (f) $xy = 1$
9. Sketch the three coordinate plane cross-sections for the quadric surface given by the equation, sketch the surface itself, and give the name of the surface. (12.6)
- (a) $x^2 - y = -z^2$
 - (b) $y^2 + z^2 = 4 - 4x^2$
 - (c) $z^2 - 9y^2 = x^2$
 - (d) $y^2 - z^2 = 4 - 4x^2$
 - (e) $4x^2 - y^2 - 4z^2 = 16$
 - (f) $z = y^2 - 4x^2$

Chapter 13

10. Give a parametrization of the curve described as a vector function. (13.1)

- (a) The parabola $y = x^2$ in the xy plane.
- (b) The directed line segment beginning at $(1, 2, -3)$ and ending at $(0, 3, 0)$.
- (c) The circle $x^2 + y^2 = 9$.
- (d) The ellipse $x^2 + 9y^2 = 9$.

11. Find the limit of the vector function. (13.1)

- (a) $\lim_{t \rightarrow -1} \left\langle \text{Arctan } t, \frac{e^{1+t}}{1-t} \right\rangle$
- (b) $\lim_{t \rightarrow 2} \left\langle t^2 - 4, \frac{t^2 - 4}{t - 2} \right\rangle$
- (c) $\lim_{t \rightarrow 0} \left(\frac{\sin 3t}{4t} \mathbf{i} + \frac{1 - \cos t}{t} \mathbf{j} \right)$
- (d) $\lim_{t \rightarrow \pi/2} \langle \sin t, \cos t, \cot t \rangle$
- (e) $\lim_{t \rightarrow 0} \left(\frac{e^t}{t+1} \mathbf{i} + \frac{e^t - 1}{t} \mathbf{j} + \frac{2^{2t} - 1}{t} \mathbf{k} \right)$
- (f) $\lim_{t \rightarrow 1} \left\langle \frac{3t^2 - 3}{t + 1}, \frac{\sin(2t - 2)}{2t - 2}, \frac{3t^2 - 3}{t - 1} \right\rangle$

12. Find the derivative $\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t)$ of the vector function. (13.2)

- (a) $\mathbf{r}(t) = \langle x^2, 3 + t \rangle$
- (b) $\mathbf{r}(t) = \langle 3 \sin 4t, -3 \cos 4t \rangle$
- (c) $\mathbf{r}(t) = \left\langle \frac{1}{t^2}, \frac{t}{t^2 + 1} \right\rangle$
- (d) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$
- (e) $\mathbf{r}(t) = (\ln 2t) \mathbf{i} + (e^{2t} - 2) \mathbf{j} + \frac{1}{e^t} \mathbf{k}$
- (f) $\mathbf{r}(t) = \langle \text{Arcsin } t, \text{Arccsc } t, \text{Arctan } t \rangle$

13. Find the indefinite integral $\int \mathbf{r}(t) dt$ of the vector function (13.2)

(a) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$

(b) $\mathbf{r}(t) = \langle 2 \sin 2t, -2 \cos 2t \rangle$

(c) $\mathbf{r}(t) = \langle e^t, 2e^{-t}, e^{3t} \rangle$

(d) $\mathbf{r}(t) = \left\langle \frac{1}{t+2}, \frac{1}{(t+2)^2}, \frac{2t}{t^2+2} \right\rangle$

(e) $\mathbf{r}(t) = (e^{2t}e^t + 2t)\mathbf{i} + \frac{\ln t}{t}\mathbf{k}$

14. Solve the differential vector equation to find $\mathbf{r}(t)$. (13.2)

(a) $\mathbf{r}'(t) = \langle 3t^2, 2t^3 \rangle, \mathbf{r}(0) = \langle 3, 4 \rangle$

(b) $\mathbf{r}'(t) = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \mathbf{r}(1) = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$

(c) $\mathbf{r}'(t) = \langle 2e^t, 4, \frac{1}{t} \rangle, \mathbf{r}(\ln 3) = \langle 5, 0, \ln(\ln 3) \rangle$

(d) $\mathbf{r}'(t) = \langle \frac{3}{2}\sqrt{t}, 8t, 3t^2 + 3 \rangle, \mathbf{r}(1) = \langle 1, -3, 6 \rangle$

(e) $\mathbf{r}'(t) = \langle \frac{1}{1+t^2}, \frac{2t}{1+t^2} \rangle, \mathbf{r}(0) = \langle 0, 1 \rangle$

15. Find the arclength parameter $s(t)$ where $s(0) = 0$ and $\frac{ds}{dt} \geq 0$ for the given curve, and use it to find the arclength of the given portion of the curve. (13.3)

(a) $\mathbf{r}(t) = \langle 1 + 2t, 2 - t, 3 - 2t \rangle, 1 \leq t \leq 3$

(b) $\mathbf{r}(t) = \langle 3 \sin t, -4t, 3 \cos t \rangle, 0 \leq t \leq 1$

(c) $\mathbf{r}(t) = \langle 3e^t, -4e^t \rangle, 0 \leq t \leq \ln 2$

(d) $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + \mathbf{k}, 0 \leq t \leq \frac{\sqrt{5}}{3}$

(e) $\mathbf{r}(t) = \langle 6t, t^3, 3t^2 \rangle, -3 \leq t \leq -2$

16. Find the unit vectors \mathbf{T}, \mathbf{N} to the curve in terms of the parameter t . (13.3)

(a) $\mathbf{r}(t) = \langle 3 \cos 2t, 3 \sin 2t \rangle$

(b) $\mathbf{r}(t) = \langle 3 \sin t, -4t, 3 \cos t \rangle$

(c) $\mathbf{r}(t) = \langle \sqrt{2} \sin t, 2 \cos t, \sqrt{2} \sin t \rangle$

(d) $\mathbf{r}(t) = \langle e^t, e^t \sin t, e^t \cos t \rangle$

17. Given the information about $\mathbf{r}(t)$ at a point, evaluate the binormal vector \mathbf{B} and curvature κ at that same point. (13.3)

(a) $\frac{d\mathbf{r}}{dt} = \langle 3, 0, -4 \rangle$, $\frac{d\mathbf{T}}{dt} = \langle 0, 10, 0 \rangle$
 $\mathbf{T} = \langle \frac{3}{5}, 0, -\frac{4}{5} \rangle$, $\mathbf{N} = \langle 0, 1, 0 \rangle$

(b) $\frac{d\mathbf{r}}{dt} = \langle -3, 0, 3\sqrt{3} \rangle$, $\frac{d\mathbf{T}}{dt} = \langle -\sqrt{3}, 0, -1 \rangle$
 $\mathbf{T} = \langle -\frac{1}{2}, 0, \frac{\sqrt{3}}{2} \rangle$, $\mathbf{N} = \langle -\frac{\sqrt{3}}{2}, 0, -\frac{1}{2} \rangle$

(c) $\frac{d\mathbf{r}}{dt} = \langle 1, 1, 1 \rangle$, $\frac{d\mathbf{T}}{dt} = \langle \frac{1}{\sqrt{3}}, 0, -\frac{1}{\sqrt{3}} \rangle$
 $\mathbf{T} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$, $\mathbf{N} = \langle \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \rangle$

(d) $\frac{d\mathbf{r}}{dt} = \langle \frac{3\sqrt{2}}{2}, -4, -\frac{3\sqrt{2}}{2} \rangle$, $\frac{d\mathbf{T}}{dt} = \langle -\frac{3\sqrt{2}}{10}, 0, -\frac{3\sqrt{2}}{10} \rangle$
 $\mathbf{T} = \langle \frac{3\sqrt{2}}{10}, -\frac{4}{5}, -\frac{3\sqrt{2}}{10} \rangle$, $\mathbf{N} = \langle -\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \rangle$

18. Sketch $\mathbf{r}(t)$ in the plane and plot the point where $t = a$, and then find and sketch \mathbf{v} , \mathbf{a} at $t = a$ on the curve. (13.4)

(a) $\mathbf{r}(t) = \langle t, t^2 \rangle$, $t = 2$

(b) $\mathbf{r}(t) = \langle 2 \sin t, -2 \cos t \rangle$, $t = \pi/2$

(c) $\mathbf{r}(t) = \langle e^{2t}, 2t \rangle$, $t = 0$

(d) $\mathbf{r}(t) = \langle \sin(\ln t), \cos(\ln t) \rangle$, $t = 1$

19. Assuming ideal projectile motion and $g = 10 \frac{m}{s^2}$, find the following.

(a) Height of a projectile shot from the ground at an angle of $\pi/4$ with initial speed $16\sqrt{2} \frac{m}{s}$ after 2 seconds.

(b) Flight time of a projectile shot from the ground at an angle of $\pi/6$ with initial speed $100 \frac{m}{s}$.

(c) Maximum height of a projectile shot from the ground at an angle of $\pi/3$ with initial speed $50\sqrt{3} \frac{m}{s}$.

(d) Total horizontal distance traveled by a projectile shot from the ground at an angle of $\pi/4$ with initial speed $10\sqrt{2} \frac{m}{s}$.

(e) Initial speed of a projectile shot from the ground at an angle of $\pi/3$ which has traveled 60 meters horizontally after 4 seconds.

20. Find the tangential and normal components of acceleration for the given position function at the given value of t .

(a) $\mathbf{r}(t) = \langle 3t, t^2 \rangle, t = 2$

(b) $\mathbf{r}(t) = \langle \sin t, \cos t \rangle, t = \pi/4$

(c) $\mathbf{r}(t) = \langle \frac{1}{3}t^3, 2t, t^2 \rangle, t = 1$

(d) $\mathbf{r}(t) = \langle 3 \sin t, -4t, 3 \cos t \rangle, t = \pi/2$

Chapter 14

21. Sketch the domain of the function f in the xy plane, sketch and label the three level curves of f within its domain for each given k value, and then sketch the graph of f . (14.1)

(a) $f(x, y) = 2x - y + 1$, $k = -3, 0, 3$

(b) $f(x, y) = 4x^2 + y^2$, $k = 0, 4, 16$

(c) $f(x, y) = \sqrt{x^2 + 9y^2}$, $k = 0, 3, 6$

(d) $f(x, y) = \sqrt{1 - x^2 - y^2}$, $k = 0, \frac{1}{\sqrt{2}}, 1$

(e) $f(x, y) = \sqrt{1 - x^2 + y^2}$, $k = 0, 1, \sqrt{2}$

(f) $f(x, y) = \ln(4 - x^2 - y^2)$, $k = \ln 1, \ln 2, \ln 3$

22. Sketch the level surface of f for the given value of k . (14.1)

(a) $f(x, y, z) = x + y + z$, $k = 2$

(b) $f(x, y, z) = x^2 + y^2 + z^2$, $k = 9$

(c) $f(x, y, z) = \sqrt{x^2 + 4y^2 + z^2}$, $k = 2$

(d) $f(x, y, z) = z - x^2$, $k = 3$

23. Prove the limit does not exist by comparing two paths of approach. (14.2)

(a) $\lim_{P \rightarrow (0,0)} \frac{x^2 + y^2}{xy}$

(b) $\lim_{P \rightarrow (0,0)} \frac{|xy|}{xy}$

(c) $\lim_{P \rightarrow (0,0)} \frac{y^6 + x^2}{y^3x + y^6}$

(d) $\lim_{P \rightarrow (3,4)} \frac{25 - x^2 - y^2}{7 - x - y}$

24. Compute the value of the limit. (14.2)

- (a) $\lim_{P \rightarrow (1, -3)} \frac{6 - xy}{3x + y}$
- (b) $\lim_{P \rightarrow (0, 0)} \frac{2x^2 + 4y^2}{\sqrt{x^2 + 2y^2 + 1} - 1}$
- (c) $\lim_{P \rightarrow (0, 0)} \frac{x \sin 2y - \sin 2y}{y - xy}$
- (d) $\lim_{P \rightarrow (1, 2)} \frac{x^2 + 2xy + y^2 - 3x - 3y}{x + y}$
- (e) $\lim_{P \rightarrow (1, 2)} \frac{x^2 + 2xy + y^2 - 3x - 3y}{x + y - 3}$

25. Find all the first-order and second-order partial derivatives of f . (14.3)

- (a) $f(x, y) = 4x^2 - 5y^3 + x - 1$
- (b) $f(x, y) = 3x^2y^2 - x^3 + y^4 - 7$
- (c) $f(x, y) = \sin(x + 3y)$
- (d) $f(x, y) = e^{xy^2}$
- (e) $f(r, \theta) = r \cos(\theta)$
- (f) $f(u, v) = \text{Arctan}(uv)$

26. Find the linearization $L(x, y)$ of $f(x, y)$ at (a, b) , and use it to approximate the value of f at (c, d) . (14.4)

- (a) $f(x, y) = 3x^2 - 2y^3$, $(a, b) = (1, 2)$, $(c, d) = (0.9, 2.2)$
- (b) $f(x, y) = 7y + 3xy - 1$, $(a, b) = (5, 1)$, $(c, d) = (5.1, 0.9)$
- (c) $f(x, y) = 2xy - x^2 - y^2$, $(a, b) = (3, -1)$, $(c, d) = (2.9, -1.05)$
- (d) $f(x, y) = \sqrt{25 - x^2 - y^2}$, $(a, b) = (-3, 0)$, $(c, d) = (-3.04, 0.09)$

27. Find the given derivative for the given nested functions at the given point.
(14.5)

(a) Find $\frac{df}{dt}$ at $t = 1$:

$$f(x, y, z) = xyz^2, x(t) = 2t + 1, y(t) = t^2 + 1, z(t) = 1 - t^3$$

(b) Find $\frac{\partial g}{\partial u}$ at $(u, v) = (2, 0)$:

$$g(x, y) = 2x + 3x^2y, x(u, v) = 1 - u, y(u, v) = 1 - uv$$

(c) Find $\frac{df}{dt}$ at $t = \pi/3$:

$$f(x, y) = 4x^2 + 2y, x(t) = \cos t, y(t) = 2\sin^2 t$$

(d) Find $\frac{\partial f}{\partial t}$ at $(t, u) = (0, 1)$:

$$f(x, y, z) = ye^x + 2z, x(t, u) = t^2, y(t, u) = t + u, z(t, u) = u + 1$$

(e) Find $\frac{dh}{dt}$ at $t = 1$:

$$h(x, y) = x + 2y, x(u, v) = uv, y(u, v) = v^2, u(t) = t^2, v(t) = t + 1$$