Chapter 12

1. Find the cosine of the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$. (12.3)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
   $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
   $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
   $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
   $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
   $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
   $\mathbf{v} = \langle 6, 4, 2 \rangle$

2. Find the projection of $\mathbf{u}$ onto $\mathbf{v}$. (12.3)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
   $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
   $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
   $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
   $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
   $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
   $\mathbf{v} = \langle 6, 4, 2 \rangle$

3. Use the cross product to find a vector normal to both $\mathbf{u}$ and $\mathbf{v}$. (12.4)

   (a) $\mathbf{u} = \langle 4, -3, 0\rangle$
   $\mathbf{v} = \langle 2, 6, -3\rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2\rangle$
   $\mathbf{v} = \langle 0, 0, -3\rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2\rangle$
   $\mathbf{v} = \langle 4, 1, -2\rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6\rangle$
   $\mathbf{v} = \langle 2, 2, 3\rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11\rangle$
   $\mathbf{v} = \langle 2, 0, 0\rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1\rangle$
   $\mathbf{v} = \langle 6, 4, 2\rangle$

4. Give a vector equation and parametric equations for the line. (12.5)

   (a) The line passing through $(1, 3, -2)$ and parallel to $\langle 3, 0, 1\rangle$.

   (b) The line passing through $(-2, 0, 4)$ and $(1, 3, 3)$.

   (c) The line parallel to $\mathbf{r}(t) = \langle t, 2 - t, 2 + t\rangle$ and passing through $(2, 4, 5)$.

   (d) The line with equation $x = -3z + 1$ in the xz plane.

   (e) The line normal to the plane with equation $x + y + 2z = 4$ and passing through $(1, 1, 1)$.

5. Give the distance from the point to the line: (12.5)

   (a) $(4, 5, 3)$ to $\mathbf{r}(t) = \langle 1 + t, 2 + 2t, 2t\rangle$

   (b) $(-1, -2, 2)$ to $\mathbf{r}(t) = \langle -1 + 3t, -4, 2 + 4t\rangle$

   (c) $(3, 0)$ to $\mathbf{r}(t) = \langle 4 - 4t, 7 - 3t\rangle$

   (d) $(2, 6)$ to $\mathbf{r}(t) = \langle -3 + 3t, 1 + t\rangle$

6. Give an equation for the plane. (12.5)

   (a) The plane passing through $(1, 3, -2)$ and normal to $\langle 3, 0, 1\rangle$.

   (b) The plane passing through $(1, -2, 0)$ and parallel to $2x - y + 3z = 3$.

   (c) The plane passing through $(1, 1, 1)$ and normal to the line with equation $\mathbf{r}(t) = \langle 4 - 3t, t, 2 + 2t\rangle$.

   (d) The plane passing through $(-2, 0, 4)$, $(1, 3, 3)$, and $(0, 0, 2)$.

7. Give the distance from the point to the plane: (12.5)

   (a) $(5, 1, 1)$ to $x - 2y + 2z = 2$

   (b) $(4, -1, 3)$ to $3x + 4z = 4$

   (c) $(0, 1, 1)$ to $-2x - 3y - 6z = 5$

   (d) $(-1, 5, 2)$ to $x + y + z = 3$

8. Sketch the curve given by the equation in the appropriate coordinate plane, and then sketch the cylinder in xyz space given by the equation. (12.6)

   (a) $y = x^2$

   (b) $x = z^3$

   (c) $y = \sin z$

   (d) $z = e^x$

   (e) $z = \ln y$

   (f) $xy = 1$

9. Sketch the three coordinate plane cross-sections for the quadric surface given by the equation, sketch the surface itself, and give the name of the surface. (12.6)

   (a) $x^2 - y = -z^2$

   (b) $y^2 + z^2 = 4 - 4x^2$

   (c) $z^2 - 9y^2 = x^2$

   (d) $y^2 - z^2 = 4 - 4x^2$

   (e) $4x^2 - y^2 - 4z^2 = 16$

   (f) $z = y^2 - 4x^2$

Chapter 13

10. Give a parametrization of the curve described as a vector function. (13.1)

   (a) The parabola $y = x^2$ in the xy plane.
   (b) The directed line segment beginning at $(1, 2, -3)$ and ending at $(0, 3, 0)$.
   (c) The circle $x^2 + y^2 = 9$.
   (d) The ellipse $x^2 + 9y^2 = 9$.

11. Find the limit of the vector function. (13.1)

   (a) $\lim\limits_{t \to -1} \left\langle \text{Arctan } t, \dfrac{e^{1+t}}{1-t} \right\rangle$
   (b) $\lim\limits_{t \to 2} \left\langle t^2 - 4, \dfrac{t^2-4}{t-2} \right\rangle$
   (c) $\lim\limits_{t \to 0} \left(\dfrac{\sin 3t}{4t}\mathbf{i} + \dfrac{1-\cos t}{t}\mathbf{j} \right)$
   (d) $\lim\limits_{t \to \pi/2} \langle \sin t, \cos t, \cot t \rangle$
   (e) $\lim\limits_{t \to 0} \left(\dfrac{e^t}{t+1}\mathbf{i} + \dfrac{e^t - 1}{t}\mathbf{j} + \dfrac{2^{2t}-1}{t}\mathbf{k} \right)$
   (f) $\lim\limits_{t \to 1} \left\langle \dfrac{3t^2-3}{t+1}, \dfrac{\sin(2t-2)}{2t-2}, \dfrac{3t^2-3}{t-1} \right\rangle$

12. Find the derivative $\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t)$ of the vector function. (13.2)

   (a) $\mathbf{r}(t) = \langle t^2, 3 + t \rangle$
   (b) $\mathbf{r}(t) = \langle 3\sin 4t, -3\cos 4t \rangle$
   (c) $\mathbf{r}(t) = \left\langle \frac{1}{t^2}, \frac{t}{t^2+1} \right\rangle$
   (d) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$
   (e) $\mathbf{r}(t) = (\ln 2t)\mathbf{i} + (e^{2t} - 2)\mathbf{j} + \frac{1}{e^t}\mathbf{k}$
   (f) $\mathbf{r}(t) = \langle \text{Arcsin } t, \text{Arccsc } t, \text{Arctan } t \rangle$

13. Find the indefinite integral $\int \mathbf{r}(t)\,dt$ of the vector function (13.2)

    (a) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t+1 \rangle$
    (b) $\mathbf{r}(t) = \langle 2\sin 2t, -2\cos 2t \rangle$
    (c) $\mathbf{r}(t) = \langle e^t, 2e^{-t}, e^{3t} \rangle$
    (d) $\mathbf{r}(t) = \left\langle \frac{1}{t+2}, \frac{1}{(t+2)^2}, \frac{2t}{t^2+2} \right\rangle$
    (e) $\mathbf{r}(t) = (e^{2t}e^t + 2t)\mathbf{i} + \frac{\ln t}{t}\mathbf{k}$

14. Solve the differential vector equation to find $\mathbf{r}(t)$. (13.2)

    (a) $\mathbf{r}'(t) = \langle 3t^2, 2t^3 \rangle$, $\mathbf{r}(0) = \langle 3, 4 \rangle$
    (b) $\mathbf{r}'(t) = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $\mathbf{r}(1) = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$
    (c) $\mathbf{r}'(t) = \left\langle 2e^t, 4, \frac{1}{t} \right\rangle$, $\mathbf{r}(\ln 3) = \langle 5, 0, \ln(\ln 3) \rangle$
    (d) $\mathbf{r}'(t) = \left\langle \frac{3}{2}\sqrt{t}, 8t, 3t^2+3 \right\rangle$, $\mathbf{r}(1) = \langle 1, -3, 6 \rangle$
    (e) $\mathbf{r}'(t) = \left\langle \frac{1}{1+t^2}, \frac{2t}{1+t^2} \right\rangle$, $\mathbf{r}(0) = \langle 0, 1 \rangle$

15. Find the arclength parameter $s(t)$ where $s(0) = 0$ and $\frac{ds}{dt} \geq 0$ for the given curve, and use it to find the arclength of the given portion of the curve. (13.3)

    (a) $\mathbf{r}(t) = \langle 1+2t, 2-t, 3-2t \rangle$, $1 \leq t \leq 3$
    (b) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$, $0 \leq t \leq 1$
    (c) $\mathbf{r}(t) = \langle 3e^t, -4e^t \rangle$, $0 \leq t \leq \ln 2$
    (d) $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + \mathbf{k}$, $0 \leq t \leq \frac{\sqrt{5}}{3}$
    (e) $\mathbf{r}(t) = \langle 6t, t^3, 3t^2 \rangle$, $-3 \leq t \leq -2$

16. Find the unit vectors $\mathbf{T}, \mathbf{N}$ to the curve in terms of the parameter t. (13.3)

    (a) $\mathbf{r}(t) = \langle 3\cos 2t, 3\sin 2t \rangle$
    (b) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$
    (c) $\mathbf{r}(t) = \left\langle \sqrt{2}\sin t, 2\cos t, \sqrt{2}\sin t \right\rangle$
    (d) $\mathbf{r}(t) = \langle e^t, e^t\sin t, e^t\cos t \rangle$

17. Given the information about $\mathbf{r}(t)$ at a point, evaluate the binormal vector $\mathbf{B}$ and curvature κ at that same point. (13.3)

    (a) $\frac{d\mathbf{r}}{dt} = \langle 3, 0, -4 \rangle$, $\frac{d\mathbf{T}}{dt} = \langle 0, 10, 0 \rangle$
    $\mathbf{T} = \left\langle \frac{3}{5}, 0, -\frac{4}{5} \right\rangle$, $\mathbf{N} = \langle 0, 1, 0 \rangle$

    (b) $\frac{d\mathbf{r}}{dt} = \left\langle -3, 0, 3\sqrt{3} \right\rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle -\sqrt{3}, 0, -1 \right\rangle$
    $\mathbf{T} = \left\langle -\frac{1}{2}, 0, \frac{\sqrt{3}}{2} \right\rangle$, $\mathbf{N} = \left\langle -\frac{\sqrt{3}}{2}, 0, -\frac{1}{2} \right\rangle$

    (c) $\frac{d\mathbf{r}}{dt} = \langle 1, 1, 1 \rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle \frac{1}{\sqrt{3}}, 0, -\frac{1}{\sqrt{3}} \right\rangle$
    $\mathbf{T} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$, $\mathbf{N} = \left\langle \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right\rangle$

    (d) $\frac{d\mathbf{r}}{dt} = \left\langle \frac{3\sqrt{2}}{2}, -4, -\frac{3\sqrt{2}}{2} \right\rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle -\frac{3\sqrt{2}}{10}, 0, -\frac{3\sqrt{2}}{10} \right\rangle$
    $\mathbf{T} = \left\langle \frac{3\sqrt{2}}{10}, -\frac{4}{5}, -\frac{3\sqrt{2}}{10} \right\rangle$, $\mathbf{N} = \left\langle -\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right\rangle$

18. Sketch $\mathbf{r}(t)$ in the plane and plot the point where $t = a$, and then find and sketch $\mathbf{v}$, $\mathbf{a}$ at $t = a$ on the curve. (13.4)

    (a) $\mathbf{r}(t) = \langle t, t^2 \rangle$, $t = 2$

    (b) $\mathbf{r}(t) = \langle 2\sin t, -2\cos t \rangle$, $t = \pi/2$

    (c) $\mathbf{r}(t) = \langle e^{2t}, 2t \rangle$, $t = 0$

    (d) $\mathbf{r}(t) = \langle \sin(\ln t), \cos(\ln t) \rangle$, $t = 1$

19. Assuming ideal projectile motion and $g = 10\frac{m}{s^2}$, find the following.

    (a) Height of a projectile shot from the ground at an angle of $\pi/4$ with initial speed $16\sqrt{2}\frac{m}{s}$ after 2 seconds.

    (b) Flight time of a projectile shot from the ground at an angle of $\pi/6$ with initial speed $100\frac{m}{s}$.

    (c) Maximum height of a projectile shot from the ground at an angle of $\pi/3$ with initial speed $50\sqrt{3}\frac{m}{s}$.

    (d) Total horizontal distance traveled by a projectile shot from the ground at an angle of $\pi/4$ with initial speed $10\sqrt{2}\frac{m}{s}$.

    (e) Initial speed of a projectile shot from the ground at an angle of $\pi/3$ which has traveled 60 meters horizontally after 4 seconds.

20. Find the tangential and normal components of acceleration for the given position function at the given value of t.

    (a) $\mathbf{r}(t) = \langle 3t, t^2 \rangle$, $t = 2$
    (b) $\mathbf{r}(t) = \langle \sin t, \cos t \rangle$, $t = \pi/4$
    (c) $\mathbf{r}(t) = \left\langle \frac{1}{3}t^3, 2t, t^2 \right\rangle$, $t = 1$
    (d) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$, $t = \pi/2$

Chapter 14

21. Sketch the domain of the function f in the xy plane, sketch and label the three level curves of f within its domain for each given k value, and then sketch the graph of f. (14.1)

    (a) $f(x, y) = 2x - y + 1$, $k = -3, 0, 3$
    (b) $f(x, y) = 4x^2 + y^2$, $k = 0, 4, 16$
    (c) $f(x, y) = \sqrt{x^2 + 9y^2}$, $k = 0, 3, 6$
    (d) $f(x, y) = \sqrt{1 - x^2 - y^2}$, $k = 0, \frac{1}{\sqrt{2}}, 1$
    (e) $f(x, y) = \sqrt{1 - x^2 + y^2}$, $k = 0, 1, \sqrt{2}$
    (f) $f(x, y) = \ln(4 - x^2 - y^2)$, $k = \ln 1, \ln 2, \ln 3$

22. Sketch the level surface of f for the given value of k. (14.1)

    (a) $f(x, y, z) = x + y + z$, $k = 2$
    (b) $f(x, y, z) = x^2 + y^2 + z^2$, $k = 9$
    (c) $f(x, y, z) = \sqrt{x^2 + 4y^2 + z^2}$, $k = 2$
    (d) $f(x, y, z) = z - x^2$, $k = 3$

23. Prove the limit does not exist by comparing two paths of approach. (14.2)

    (a) $\displaystyle\lim_{P\to(0,0)} \frac{x^2 + y^2}{xy}$
    (b) $\displaystyle\lim_{P\to(0,0)} \frac{|xy|}{xy}$
    (c) $\displaystyle\lim_{P\to(0,0)} \frac{y^6 + x^2}{y^3 x + y^6}$
    (d) $\displaystyle\lim_{P\to(3,4)} \frac{25 - x^2 - y^2}{7 - x - y}$

24. Compute the value of the limit. (14.2)

(a) $\displaystyle\lim_{P\to(1,-3)} \frac{6-xy}{3x+y+1}$

(b) $\displaystyle\lim_{P\to(0,0)} \frac{2x^2+4y^2}{\sqrt{x^2+2y^2+1}-1}$

(c) $\displaystyle\lim_{P\to(0,0)} \frac{x\sin 2y-\sin 2y}{y-xy}$

(d) $\displaystyle\lim_{P\to(1,2)} \frac{x^2+2xy+y^2-3x-3y}{x+y}$

(e) $\displaystyle\lim_{P\to(1,2)} \frac{x^2+2xy+y^2-3x-3y}{x+y-3}$

25. Find all the first-order and second-order partial derivatives of f. (14.3)

(a) $f(x,y)=4x^2-5y^3+x-1$

(b) $f(x,y)=3x^2y^2-x^3+y^4-7$

(c) $f(x,y)=\sin(x+3y)$

(d) $f(x,y)=e^{xy^2}$

(e) $f(r,\theta)=r\cos(\theta)$

(f) $f(u,v)=\text{Arctan}\,(uv)$

26. Find the linearization $L(x,y)$ of $f(x,y)$ at (a,b), and use it to approximate the value of f at (c,d). (14.4)

(a) $f(x,y)=3x^2-2y^3$, $(a,b)=(1,2)$, $(c,d)=(0.9,2.2)$

(b) $f(x,y)=7y+3xy-1$, $(a,b)=(5,1)$, $(c,d)=(5.1,0.9)$

(c) $f(x,y)=2xy-x^2-y^2$, $(a,b)=(3,-1)$, $(c,d)=(2.9,-1.05)$

(d) $f(x,y)=\sqrt{25-x^2-y^2}$, $(a,b)=(-3,0)$, $(c,d)=(-3.04,0.09)$

27. Find the given derivative for the given nested functions at the given point. (14.5)

    (a) Find $\frac{df}{dt}$ at $t = 1$:
    $f(x, y, z) = xyz^2$, $x(t) = 2t + 1$, $y(t) = t^2 + 1$, $z(t) = 1 - t^3$
    (b) Find $\frac{\partial g}{\partial u}$ at $(u, v) = (2, 0)$:
    $g(x, y) = 2x + 3x^2y$, $x(u, v) = 1 - u$, $y(u, v) = 1 - uv$
    (c) Find $\frac{df}{dt}$ at $t = \pi/3$:
    $f(x, y) = 4x^2 + 2y$, $x(t) = \cos t$, $y(t) = 2\sin^2 t$
    (d) Find $\frac{\partial f}{\partial t}$ at $(t, u) = (0, 1)$:
    $f(x, y, z) = ye^x + 2z$, $x(t, u) = t^2$, $y(t, u) = t + u$, $z(t, u) = u + 1$
    (e) Find $\frac{dh}{dt}$ at $t = 1$:
    $h(x, y) = x + 2y$, $x(u, v) = uv$, $y(u, v) = v^2$, $u(t) = t^2$, $v(t) = t + 1$

28. Use partial derivatives to find the rate of change $\frac{dy}{dx}$ for the equation at the given point. (14.5)

    (a) $3x^2 + 5y = 8$ at $(1, 1)$
    (b) $4x^3y = 3xy^3 + 16$ at $(-1, 2)$
    (c) $-xy^2 + y^3 = -5x + 5$ at $(-3, 2)$
    (d) $x^3y^4 = x^4y^3$ at $(2, 2)$
    (e) $e^{xy} = \ln(xy + e)$ at $(1, 0)$
    (f) $\sin(2x + y) = \cos(2x + y) + 1$ at $(\pi/8, \pi/4)$

29. Find the derivative of f in the direction of the given vector at the given point. (14.6)

    (a) $f(x, y) = x + 2y$, $\mathbf{A} = \langle -4, 3\rangle$, $P_0 = (1, 3)$
    (b) $f(x, y) = xy^2 + 3y$, $\mathbf{A} = \langle 2, 2\rangle$, $P_0 = (2, 0)$
    (c) $f(x, y) = e^{x+xy}$, $\mathbf{A} = 5\mathbf{i} - 12\mathbf{j}$, $P_0 = (\ln 2, 0)$
    (d) $f(x, y, z) = x^2 + 4y^2 + z^2$, $\mathbf{A} = \langle 3, -2, -6\rangle$, $P_0 = (1, 1, 2)$
    (e) $f(x, y, z) = xz^3 + 3yz$, $\mathbf{A} = \langle 1, -2, 2\rangle$, $P_0 = (-2, 0, 1)$
    (f) $f(x, y, z) = \ln(y^2) + 4xz$, $\mathbf{A} = 6\mathbf{i} - 8\mathbf{k}$, $P_0 = (3, 1, 2)$

30. Find and label all the points yielding local maximum values, local minimum values, and saddle points for f. (14.7)

    (a) $f(x, y) = x^2 + 9y^2 + 3$
    (b) $f(x, y) = x^2 - 2xy + 2y^2 + 4y - 3$
    (c) $f(x, y) = x^3 + 3xy + y^3 + 2$
    (d) $f(x, y) = x^3 - 3xy^2 + 6y^2 + 18x^2 + 1$
    (e) $f(x, y) = (x^2 + y^2)e^{x+y+2}$
    (f) $f(x, y) = x^2y - xy^2 + 12x - 12y$

31. Find the absolute maximum and absolute mininum value of f within the closed bounded region R. (14.7)

    (a) $f(x, y) = x^2 + y^2$, R : square with vertices $(-1, 2)$, $(2, 2)$, $(2, 5)$, $(-1, 5)$
    (b) $f(x, y) = x^2 + y^2 - 2x - 2y$, R : triangle with vertices $(0, 0)$, $(2, 4)$, $(2, 0)$
    (c) $f(x, y) = x^2 + 2y^2 + 2xy + 4x$, $R = \{(x, y) : |x| \leq 4, |y| \leq 4\}$
    (d) $f(x, y) = 2xy$, $R = \{(x, y) : x^2 + 4y^2 \leq 4\}$

32. Use Lagrange Multipliers to find the solution to the word problem. (14.8)

    (a) Find the maximum volume of a rectangular box without a lid which uses 108 square units of material.
    (b) Find the minimum surface area of a right circular cylinder with volume equal to 54π cubic units. ($V = \pi r^2 h$, $SA = 2\pi r(r + h)$)
    (c) Find the volume of the largest rectangle which has its base on the x-axis and fits in the triangle with vertices $(-4, 0)$, $(0, 8)$, $(4, 0)$.
    (d) Find the highest and lowest points which lay on the curve of intersection for the cylinder $x^2 + y^2 = 8$ and the plane $2x + 2y + z = 16$.

Chapter 14

33. Divide R into 2×2 equal pieces and use the midpoint rule to approximate the double integral. (15.1)

    (a) $\iint_R 2x + 2y + 4\, dA,\ R : 0 \leq x \leq 4, 0 \leq y \leq 2$
    (b) $\iint_R 3y^2 - 4xy\, dA,\ R : -1 \leq x \leq 3, -3 \leq y \leq 1$
    (c) $\iint_R 3x^2 - 2y + 4\, dA,\ R : 0 \leq x \leq 4, -2 \leq y \leq 6$
    (d) $\iint_R \cos(x + y)\, dA,\ R : 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2$

34. Evaluate the double integral. (15.2)

    (a) $\iint_R 2x + 2y + 4\, dA,\ R : 0 \leq x \leq 4, 0 \leq y \leq 2$
    (b) $\iint_R 3y^2 - 4xy\, dA,\ R : -1 \leq x \leq 3, -3 \leq y \leq 1$
    (c) $\iint_R 3x^2 - 2y + 4\, dA,\ R : 0 \leq x \leq 4, -2 \leq y \leq 6$
    (d) $\iint_R \cos(x + y)\, dA,\ R : 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2$
    (e) $\iint_R 12x^2 y\, dA,\ R : 0 \leq x \leq 3, 1 \leq y \leq 2$
    (f) $\iint_R \frac{y}{1+xy}\, dA,\ 0 \leq x \leq 2,\ 1 \leq y \leq 3$

35. Evaluate the iterated integral or double integral of two variables. (15.3)

    (a) $\displaystyle\int_0^2 \int_0^x 11x^2 + 3y^2\, dy\, dx$
    (b) $\displaystyle\int_{-1}^2 \int_{-1}^{y^2} 20xy\, dx\, dy$
    (c) $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 6x + 30y\, dx\, dy$
    (d) $\displaystyle\int_1^2 \int_{1/x}^{2/x} xe^x\, dx\, dy$
    (e) $\iint_R 8xy\, dA,\ R : 0 \leq x \leq y, 0 \leq y \leq 1$
    (f) $\iint_R \frac{6}{5} y\, dA,\ R :$ triangle with vertices $(-2, 0)$, $(0, 1)$, $(3, 0)$

36. Evaluate the iterated integral of two variables. (15.3)

    (a) $\int_0^1 \int_x^1 \frac{2}{\sqrt{4+y^2}}\, dy\, dx$

    (b) $\int_0^2 \int_y^2 y(8-x^3)^{1/3}\, dx\, dy$

    (c) $\int_0^1 \int_{y^2}^1 3\pi \sin(\pi x^3)\, dx\, dy$

    (d) $\int_0^1 \int_{e^x}^e \frac{y}{\ln y}\, dy\, dx$

37. Find an expression involving iterated integrals for the given area or average value. (15.3)

    (a) Area of the rectangle with vertices $(-1,0)$, $(2,0)$, $(2,4)$, $(-1,4)$
    (b) Area of the parallelogram with vertices $(-1,2)$, $(3,2)$, $(4,1)$, $(0,1)$
    (c) Area of the triangle with vertices $(1,3)$, $(1,1)$, and $(2,2)$
    (d) Area between $x = 4 - y^2$ and $x = y^2 - 4$
    (e) Average value of $f(x,y) = e^{x^2 y}$ over the square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$
    (f) Average value of $f(x,y) = \sin(\frac{x}{2y})$ over the triangle with vertices $(0,1)$, $(1,1)$, $(0,2)$

38. Evaulate the iterated integral of three variables. (15.7)

39. Find an expression involving iterated integrals for the given volume. (15.7)

40. Find a substitution from the given region in the uv plane into the given region in the xy plane. (15.10)

41. Use the given substitution to evaulate the iterated integral of variables x, y. (15.10)

42. (15.4)

43. (15.8)

44. (15.9)