Chapter 12

1. Find the cosine of the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$. (12.3)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
   $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
   $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
   $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
   $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
   $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
   $\mathbf{v} = \langle 6, 4, 2 \rangle$

2. Find the projection of $\mathbf{u}$ onto $\mathbf{v}$. (12.3)

   (a) $\mathbf{u} = \langle 4, -3, 0 \rangle$
   $\mathbf{v} = \langle 2, 6, -3 \rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2 \rangle$
   $\mathbf{v} = \langle 0, 0, -3 \rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2 \rangle$
   $\mathbf{v} = \langle 4, 1, -2 \rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6 \rangle$
   $\mathbf{v} = \langle 2, 2, 3 \rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11 \rangle$
   $\mathbf{v} = \langle 2, 0, 0 \rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1 \rangle$
   $\mathbf{v} = \langle 6, 4, 2 \rangle$

3. Use the cross product to find a vector normal to both $\mathbf{u}$ and $\mathbf{v}$. (12.4)

   (a) $\mathbf{u} = \langle 4, -3, 0\rangle$
   $\mathbf{v} = \langle 2, 6, -3\rangle$

   (b) $\mathbf{u} = \langle 1, 2, 2\rangle$
   $\mathbf{v} = \langle 0, 0, -3\rangle$

   (c) $\mathbf{u} = \langle 1, 4, 2\rangle$
   $\mathbf{v} = \langle 4, 1, -2\rangle$

   (d) $\mathbf{u} = \langle -4, -4, -6\rangle$
   $\mathbf{v} = \langle 2, 2, 3\rangle$

   (e) $\mathbf{u} = \langle 0, 5, -11\rangle$
   $\mathbf{v} = \langle 2, 0, 0\rangle$

   (f) $\mathbf{u} = \langle 3, 2, 1\rangle$
   $\mathbf{v} = \langle 6, 4, 2\rangle$

4. Give a vector equation and parametric equations for the line. (12.5)

   (a) The line passing through $(1, 3, -2)$ and parallel to $\langle 3, 0, 1\rangle$.

   (b) The line passing through $(-2, 0, 4)$ and $(1, 3, 3)$.

   (c) The line parallel to $\mathbf{r}(t) = \langle t, 2-t, 2+t\rangle$ and passing through $(2, 4, 5)$.

   (d) The line with equation $x = -3z + 1$ in the xz plane.

   (e) The line normal to the plane with equation $x + y + 2z = 4$ and passing through $(1, 1, 1)$.

5. Give the distance from the point to the line: (12.5)

   (a) $(4, 5, 3)$ to $\mathbf{r}(t) = \langle 1+t, 2+2t, 2t\rangle$

   (b) $(-1, -2, 2)$ to $\mathbf{r}(t) = \langle -1+3t, -4, 2+4t\rangle$

   (c) $(3, 0)$ to $\mathbf{r}(t) = \langle 4-4t, 7-3t\rangle$

   (d) $(2, 6)$ to $\mathbf{r}(t) = \langle -3+3t, 1+t\rangle$

6. Give an equation for the plane. (12.5)
   (a) The plane passing through $(1, 3, -2)$ and normal to $\langle 3, 0, 1\rangle$.
   (b) The plane passing through $(1, -2, 0)$ and parallel to $2x - y + 3z = 3$.
   (c) The plane passing through $(1, 1, 1)$ and normal to the line with equation $\mathbf{r}(t) = \langle 4 - 3t, t, 2 + 2t\rangle$.
   (d) The plane passing through $(-2, 0, 4)$, $(1, 3, 3)$, and $(0, 0, 2)$.
7. Give the distance from the point to the plane: (12.5)
   (a) $(5, 1, 1)$ to $x - 2y + 2z = 2$
   (b) $(4, -1, 3)$ to $3x + 4z = 4$
   (c) $(0, 1, 1)$ to $-2x - 3y - 6z = 5$
   (d) $(-1, 5, 2)$ to $x + y + z = 3$
8. Sketch the curve given by the equation in the appropriate coordinate plane, and then sketch the cylinder in xyz space given by the equation. (12.6)
   (a) $y = x^2$
   (b) $x = z^3$
   (c) $y = \sin z$
   (d) $z = e^x$
   (e) $z = \ln y$
   (f) $xy = 1$
9. Sketch the three coordinate plane cross-sections for the quadric surface given by the equation, sketch the surface itself, and give the name of the surface. (12.6)
   (a) $x^2 - y = -z^2$
   (b) $y^2 + z^2 = 4 - 4x^2$
   (c) $z^2 - 9y^2 = x^2$
   (d) $y^2 - z^2 = 4 - 4x^2$
   (e) $4x^2 - y^2 - 4z^2 = 16$
   (f) $z = y^2 - 4x^2$

Chapter 13

10. Give a vector function which parametrizes the given curve without nontrivial overlaps. (13.1)

    (a) The parabola $y = x^2$ in the xy plane.

    (b) The directed line segment beginning at $(1, 2, -3)$ and ending at $(0, 3, 0)$.

    (c) The circle $x^2 + y^2 = 9$.

    (d) The ellipse $x^2 + 9y^2 = 9$.

11. Find the limit of the vector function. (13.1)

    (a) $\lim_{t\to -1} \left\langle \text{Arctan } t, \frac{e^{1+t}}{1-t} \right\rangle$

    (b) $\lim_{t\to 2} \left\langle t^2 - 4, \frac{t^2-4}{t-2} \right\rangle$

    (c) $\lim_{t\to 0} \left(\frac{\sin 3t}{4t}\mathbf{i} + \frac{1-\cos t}{t}\mathbf{j} \right)$

    (d) $\lim_{t\to\pi/2} \langle \sin t, \cos t, \cot t \rangle$

    (e) $\lim_{t\to 0} \left(\frac{e^t}{t+1}\mathbf{i} + \frac{e^t - 1}{t}\mathbf{j} + \frac{2^{2t}-1}{t}\mathbf{k} \right)$

    (f) $\lim_{t\to 1} \left\langle \frac{3t^2-3}{t+1}, \frac{\sin(2t-2)}{2t-2}, \frac{3t^2-3}{t-1} \right\rangle$

12. Find the derivative $\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t)$ of the vector function. (13.2)

    (a) $\mathbf{r}(t) = \langle t^2, 3 + t \rangle$

    (b) $\mathbf{r}(t) = \langle 3\sin 4t, -3\cos 4t \rangle$

    (c) $\mathbf{r}(t) = \left\langle \frac{1}{t^2}, \frac{t}{t^2+1} \right\rangle$

    (d) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t + 1 \rangle$

    (e) $\mathbf{r}(t) = (\ln 2t)\mathbf{i} + (e^{2t} - 2)\mathbf{j} + \frac{1}{e^t}\mathbf{k}$

    (f) $\mathbf{r}(t) = \langle \text{Arcsin } t, \text{Arccsc } t, \text{Arctan } t \rangle$

13. Find the indefinite integral $\int \mathbf{r}(t)\,dt$ of the vector function (13.2)

    (a) $\mathbf{r}(t) = \langle 3t^2, 4t^3, 2t+1\rangle$
    (b) $\mathbf{r}(t) = \langle 2\sin 2t, -2\cos 2t\rangle$
    (c) $\mathbf{r}(t) = \langle e^t, 2e^{-t}, e^{3t}\rangle$
    (d) $\mathbf{r}(t) = \left\langle \frac{1}{t+2}, \frac{1}{(t+2)^2}, \frac{2t}{t^2+2}\right\rangle$
    (e) $\mathbf{r}(t) = (e^{2t}e^t + 2t)\mathbf{i} + \frac{\ln t}{t}\mathbf{k}$

14. Solve the differential vector equation to find $\mathbf{r}(t)$. (13.2)

    (a) $\mathbf{r}'(t) = \langle 3t^2, 2t^3\rangle$, $\mathbf{r}(0) = \langle 3, 4\rangle$
    (b) $\mathbf{r}'(t) = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $\mathbf{r}(1) = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$
    (c) $\mathbf{r}'(t) = \left\langle 2e^t, 4, \frac{1}{t}\right\rangle$, $\mathbf{r}(\ln 3) = \langle 5, 0, \ln(\ln 3)\rangle$
    (d) $\mathbf{r}'(t) = \left\langle \frac{3}{2}\sqrt{t}, 8t, 3t^2+3\right\rangle$, $\mathbf{r}(1) = \langle 1, -3, 6\rangle$
    (e) $\mathbf{r}'(t) = \left\langle \frac{1}{1+t^2}, \frac{2t}{1+t^2}\right\rangle$, $\mathbf{r}(0) = \langle 0, 1\rangle$

15. Find the arclength parameter $s(t)$ where $s(0) = 0$ and $\frac{ds}{dt} \geq 0$ for the given curve, and use it to find the arclength of the given portion of the curve. (13.3)

    (a) $\mathbf{r}(t) = \langle 1+2t, 2-t, 3-2t\rangle$, $1 \leq t \leq 3$
    (b) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t\rangle$, $0 \leq t \leq 1$
    (c) $\mathbf{r}(t) = \langle 3e^t, -4e^t\rangle$, $0 \leq t \leq \ln 2$
    (d) $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + \mathbf{k}$, $0 \leq t \leq \frac{\sqrt{5}}{3}$
    (e) $\mathbf{r}(t) = \langle 6t, t^3, 3t^2\rangle$, $-3 \leq t \leq -2$

16. Find the unit vectors $\mathbf{T}, \mathbf{N}$ for the given curve in terms of the parameter t. (13.3)

    (a) $\mathbf{r}(t) = \langle 3\cos 2t, 3\sin 2t\rangle$
    (b) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t\rangle$
    (c) $\mathbf{r}(t) = \left\langle \sqrt{2}\sin t, 2\cos t, \sqrt{2}\sin t\right\rangle$
    (d) $\mathbf{r}(t) = \langle e^t, e^t\sin t, e^t\cos t\rangle$

17. Given the information about $\mathbf{r}(t)$ at a point, evaluate the binormal vector $\mathbf{B}$ and curvature κ at that same point. (13.3)

    (a) $\frac{d\mathbf{r}}{dt} = \langle 3, 0, -4 \rangle$, $\frac{d\mathbf{T}}{dt} = \langle 0, 10, 0 \rangle$
    $\mathbf{T} = \left\langle \frac{3}{5}, 0, -\frac{4}{5} \right\rangle$, $\mathbf{N} = \langle 0, 1, 0 \rangle$

    (b) $\frac{d\mathbf{r}}{dt} = \left\langle -3, 0, 3\sqrt{3} \right\rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle -\sqrt{3}, 0, -1 \right\rangle$
    $\mathbf{T} = \left\langle -\frac{1}{2}, 0, \frac{\sqrt{3}}{2} \right\rangle$, $\mathbf{N} = \left\langle -\frac{\sqrt{3}}{2}, 0, -\frac{1}{2} \right\rangle$

    (c) $\frac{d\mathbf{r}}{dt} = \langle 1, 1, 1 \rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle \frac{1}{\sqrt{3}}, 0, -\frac{1}{\sqrt{3}} \right\rangle$
    $\mathbf{T} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$, $\mathbf{N} = \left\langle \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right\rangle$

    (d) $\frac{d\mathbf{r}}{dt} = \left\langle \frac{3\sqrt{2}}{2}, -4, -\frac{3\sqrt{2}}{2} \right\rangle$, $\frac{d\mathbf{T}}{dt} = \left\langle -\frac{3\sqrt{2}}{10}, 0, -\frac{3\sqrt{2}}{10} \right\rangle$
    $\mathbf{T} = \left\langle \frac{3\sqrt{2}}{10}, -\frac{4}{5}, -\frac{3\sqrt{2}}{10} \right\rangle$, $\mathbf{N} = \left\langle -\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right\rangle$

18. Sketch $\mathbf{r}(t)$ in the plane and plot the point where $t = a$, and then find and sketch $\mathbf{v}$, $\mathbf{a}$ at $t = a$ on the curve. (13.4)

    (a) $\mathbf{r}(t) = \langle t, t^2 \rangle$, $t = 2$

    (b) $\mathbf{r}(t) = \langle 2\sin t, -2\cos t \rangle$, $t = \pi/2$

    (c) $\mathbf{r}(t) = \langle e^{2t}, 2t \rangle$, $t = 0$

    (d) $\mathbf{r}(t) = \langle \sin(\ln t), \cos(\ln t) \rangle$, $t = 1$

19. Assuming ideal projectile motion and $g = 10\frac{m}{s^2}$, find the following.

    (a) Height of a projectile shot from the ground at an angle of $\pi/4$ with initial speed $16\sqrt{2}\frac{m}{s}$ after 2 seconds.

    (b) Flight time of a projectile shot from the ground at an angle of $\pi/6$ with initial speed $100\frac{m}{s}$.

    (c) Maximum height of a projectile shot from the ground at an angle of $\pi/3$ with initial speed $50\sqrt{3}\frac{m}{s}$.

    (d) Total horizontal distance traveled by a projectile shot from the ground at an angle of $\pi/4$ with initial speed $10\sqrt{2}\frac{m}{s}$.

    (e) Initial speed of a projectile shot from the ground at an angle of $\pi/3$ which has traveled 60 meters horizontally after 4 seconds.

20. Find the tangential and normal components of acceleration for the given position function at the given value of t.

    (a) $\mathbf{r}(t) = \langle 3t, t^2 \rangle$, $t = 2$

    (b) $\mathbf{r}(t) = \langle \sin t, \cos t \rangle$, $t = \pi/4$

    (c) $\mathbf{r}(t) = \left\langle \frac{1}{3}t^3, 2t, t^2 \right\rangle$, $t = 1$

    (d) $\mathbf{r}(t) = \langle 3\sin t, -4t, 3\cos t \rangle$, $t = \pi/2$

Chapter 14

21. Sketch the domain of the function f in the xy plane, sketch and label the three level curves of f within its domain for each given k value, and then sketch the graph of f. (14.1)

    (a) $f(x,y) = 2x - y + 1$, $k = -3, 0, 3$
    (b) $f(x,y) = 4x^2 + y^2$, $k = 0, 4, 16$
    (c) $f(x,y) = \sqrt{x^2 + 9y^2}$, $k = 0, 3, 6$
    (d) $f(x,y) = \sqrt{4 - x^2 - y^2}$, $k = 0, \frac{1}{\sqrt{2}}, 1$
    (e) $f(x,y) = \sqrt{4 - x^2 + y^2}$, $k = 0, 1, \sqrt{2}$
    (f) $f(x,y) = \ln(4 - x^2 - y^2)$, $k = \ln 1, \ln 2, \ln 3$

22. Sketch the level surface of f for the given value of k. (14.1)

    (a) $f(x,y,z) = x + y + z$, $k = 2$
    (b) $f(x,y,z) = x^2 + y^2 + z^2$, $k = 9$
    (c) $f(x,y,z) = \sqrt{x^2 + 4y^2 + z^2}$, $k = 2$
    (d) $f(x,y,z) = z - x^2$, $k = 3$

23. Prove the limit does not exist by comparing two paths of approach. (14.2)

    (a) $\displaystyle\lim_{P \to (0,0)} \frac{x^2 + y^2}{xy}$
    (b) $\displaystyle\lim_{P \to (0,0)} \frac{|xy|}{xy}$
    (c) $\displaystyle\lim_{P \to (0,0)} \frac{y^6 + x^2}{y^3 x + y^6}$
    (d) $\displaystyle\lim_{P \to (3,4)} \frac{25 - x^2 - y^2}{7 - x - y}$

24. Compute the value of the limit. (14.2)

   (a) $\displaystyle\lim_{P\to(1,-3)} \frac{6-xy}{3x+y+1}$

   (b) $\displaystyle\lim_{P\to(0,0)} \frac{2x^2+4y^2}{\sqrt{x^2+2y^2+1}-1}$

   (c) $\displaystyle\lim_{P\to(0,0)} \frac{x\sin 2y-\sin 2y}{y-xy}$

   (d) $\displaystyle\lim_{P\to(1,2)} \frac{x^2+2xy+y^2-3x-3y}{x+y}$

   (e) $\displaystyle\lim_{P\to(1,2)} \frac{x^2+2xy+y^2-3x-3y}{x+y-3}$

25. Find all the first-order and second-order partial derivatives of f. (14.3)

   (a) $f(x,y)=4x^2-5y^3+x-1$

   (b) $f(x,y)=3x^2y^2-x^3+y^4-7$

   (c) $f(x,y)=\sin(x+3y)$

   (d) $f(x,y)=e^{xy^2}$

   (e) $f(r,\theta)=r\cos(\theta)$

   (f) $f(u,v)=\text{Arctan }(uv)$

26. Find the linearization $L(x,y)$ of $f(x,y)$ at (a,b), and use it to approximate the value of f at (c,d). (14.4)

   (a) $f(x,y)=3x^2-2y^3$, $(a,b)=(1,2)$, $(c,d)=(0.9,2.2)$

   (b) $f(x,y)=7y+3xy-1$, $(a,b)=(5,1)$, $(c,d)=(5.1,0.9)$

   (c) $f(x,y)=2xy-x^2-y^2$, $(a,b)=(3,-1)$, $(c,d)=(2.9,-1.05)$

   (d) $f(x,y)=\sqrt{25-x^2-y^2}$, $(a,b)=(-3,0)$, $(c,d)=(-3.04,0.09)$

27. Find the given derivative for the given nested functions at the given point. (14.5)

    (a) Find $\frac{df}{dt}$ at $t = 1$:
    $f(x, y, z) = xyz^2$, $x(t) = 2t + 1$, $y(t) = t^2 + 1$, $z(t) = 1 - t^3$

    (b) Find $\frac{\partial g}{\partial u}$ at $(u, v) = (2, 0)$:
    $g(x, y) = 2x + 3x^2 y$, $x(u, v) = 1 - u$, $y(u, v) = 1 - uv$

    (c) Find $\frac{df}{dt}$ at $t = \pi/3$:
    $f(x, y) = 4x^2 + 2y$, $x(t) = \cos t$, $y(t) = 2\sin^2 t$

    (d) Find $\frac{\partial f}{\partial t}$ at $(t, u) = (0, 1)$:
    $f(x, y, z) = ye^x + 2z$, $x(t, u) = t^2$, $y(t, u) = t + u$, $z(t, u) = u + 1$

    (e) Find $\frac{dh}{dt}$ at $t = 1$:
    $h(x, y) = x + 2y$, $x(u, v) = uv$, $y(u, v) = v^2$, $u(t) = t^2$, $v(t) = t + 1$

28. Use partial derivatives to find the rate of change $\frac{dy}{dx}$ for the equation at the given point. (14.5)

    (a) $3x^2 + 5y = 8$ at $(1, 1)$

    (b) $4x^3 y = 3xy^3 + 16$ at $(-1, 2)$

    (c) $-xy^2 + y^3 = -5x + 5$ at $(-3, 2)$

    (d) $x^3 y^4 = x^4 y^3$ at $(2, 2)$

    (e) $e^{xy} = \ln(xy + e)$ at $(1, 0)$

    (f) $\sin(2x + y) = \cos(2x + y) + 1$ at $(\pi/8, \pi/4)$

29. Find the gradient vector ∇f. (14.6)

    (a) $f(x, y) = x^3 + 3xy$

    (b) $f(x, y) = \sqrt{2xy + y^2}$

    (c) $f(x, y) = \frac{x+1}{y+1}$

    (d) $f(x, y, z) = \ln(x + y + z) + z^2$

    (e) $f(x, y, z) = yz \sin(\frac{1}{x})$

    (f) $f(x, y, z) = xye^{yz}$

30. Find the derivative of f in the direction of the given vector at the given point. (14.6)

    (a) $f(x,y) = x + 2y$, $\mathbf{A} = \langle -4, 3\rangle$, $P_0 = (1,3)$
    (b) $f(x,y) = xy^2 + 3y$, $\mathbf{A} = \langle 2, 2\rangle$, $P_0 = (2,0)$
    (c) $f(x,y) = e^{x+xy}$, $\mathbf{A} = 5\mathbf{i} - 12\mathbf{j}$, $P_0 = (\ln 2, 0)$
    (d) $f(x,y,z) = x^2 + 4y^2 + z^2$, $\mathbf{A} = \langle 3, -2, -6\rangle$, $P_0 = (1,1,2)$
    (e) $f(x,y,z) = xz^3 + 3yz$, $\mathbf{A} = \langle 1, -2, 2\rangle$, $P_0 = (-2,0,1)$
    (f) $f(x,y,z) = \ln(y^2) + 4xz$, $\mathbf{A} = 6\mathbf{i} - 8\mathbf{k}$, $P_0 = (3,1,2)$

31. Find and label all the points yielding local maximum values, local minimum values, and saddle points for f. (14.7)

    (a) $f(x,y) = x^2 + 9y^2 + 3$
    (b) $f(x,y) = x^2 - 2xy + 2y^2 + 4y - 3$
    (c) $f(x,y) = x^3 + 3xy + y^3 + 2$
    (d) $f(x,y) = x^3 - 6xy + \frac{3}{2}y^2 - 1$
    (e) $f(x,y) = x^2y - xy^2 + 12x - 12y$
    (f) $f(x,y) = (x^2 + y^2)e^{x+y+2}$

32. Find the absolute maximum and absolute mininum value of f within the closed bounded region R. (14.7)

    (a) $f(x,y) = x^2 + y^2$, R : square with vertices $(-1,2)$, $(2,2)$, $(2,5)$, $(-1,5)$
    (b) $f(x,y) = x^2 + y^2 - 2x - 2y$, R : triangle with vertices $(0,0)$, $(2,4)$, $(2,0)$
    (c) $f(x,y) = x^2 + 2y^2 + 2xy + 4x$, $R = \{(x,y) : |x| \le 4, |y| \le 4\}$
    (d) $f(x,y) = 2xy$, $R = \{(x,y) : x^2 + y^2 \le 4\}$

33. Use Lagrange Multipliers to find the solution to the word problem. (14.8)

    (a) Find the maximum volume of a rectangular box without a lid which uses 108 square units of material.
    (b) Find the minimum surface area of a right circular cylinder with volume equal to 54π cubic units. ($V = \pi r^2 h$, $SA = 2\pi r(r + h)$)
    (c) Find the area of the largest rectangle which has its base on the x-axis and fits in the triangle with vertices $(-4,0)$, $(0,8)$, $(4,0)$.
    (d) Find the highest and lowest points which lay on the curve of intersection for the cylinder $x^2 + y^2 = 8$ and the plane $2x + 2y + z = 16$.

Chapter 15

34. Divide R into four 2-by-2 equal pieces and use the midpoint rule to approximate the double integral. (15.1)

    (a) $\iint_R 2x+2y+4\,dA$, $R=\{(x,y):0\leq x\leq 4, 0\leq y\leq 2\}$
    (b) $\iint_R 3y^2-4xy\,dA$, $R=\{(x,y):-1\leq x\leq 3, -3\leq y\leq 1\}$
    (c) $\iint_R 12x^2y\,dA$, $R=\{(x,y):-2\leq x\leq 2, 0\leq y\leq 2\}$
    (d) $\iint_R \cos(x+y)\,dA$, $R=\{(x,y):0\leq x\leq \pi/2, 0\leq y\leq \pi/2\}$

35. Evaluate the double integral. (15.2)

    (a) $\iint_R 2x+2y+4\,dA$, $R=\{(x,y):0\leq x\leq 4, 0\leq y\leq 2\}$
    (b) $\iint_R 3y^2-4xy\,dA$, $R=\{(x,y):-1\leq x\leq 3, -3\leq y\leq 1\}$
    (c) $\iint_R 12x^2y\,dA$, $R=\{(x,y):-2\leq x\leq 2, 0\leq y\leq 2\}$
    (d) $\iint_R \cos(x+y)\,dA$, $R=\{(x,y):0\leq x\leq \pi/2, 0\leq y\leq \pi/2\}$
    (e) $\iint_R 3x(1+xy)^2\,dA$, $R=\{(x,y):1\leq x\leq 3, 0\leq y\leq 1\}$
    (f) $\iint_R 2xy\sqrt{16+x^2}\,dA$, $R=\{(x,y):0\leq x\leq 3, 0\leq y\leq 3\}$

36. Evaluate the iterated integral or double integral of two variables. (15.3)

    (a) $\displaystyle\int_0^2\int_0^x 11x^2+3y^2\,dy\,dx$
    (b) $\displaystyle\int_{-1}^2\int_{-1}^{y^2} 20xy\,dx\,dy$
    (c) $\displaystyle\int_0^4\int_{\sqrt{y}}^2 6x+30y\,dx\,dy$
    (d) $\displaystyle\int_1^2\int_{1/x}^{2/x} xe^x\,dy\,dx$
    (e) $\iint_R 8xy\,dA$, $R=\{(x,y):0\leq x\leq y, 0\leq y\leq 1\}$
    (f) $\iint_R \frac{6}{5}y\,dA$, R : triangle with vertices $(-2,0)$, $(0,1)$, $(3,0)$

37. Evaluate the iterated integral of two variables. (15.3)

(a) $\displaystyle\int_0^1 \int_x^1 \frac{2}{\sqrt{4+y^2}}\,dy\,dx$

(b) $\displaystyle\int_0^2 \int_y^2 y(8-x^3)^{1/3}\,dx\,dy$

(c) $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 3\pi \sin(\pi x^3)\,dx\,dy$

(d) $\displaystyle\int_0^1 \int_{e^x}^e \frac{y}{\ln y}\,dy\,dx$

38. Find an expression involving iterated integrals for the given area or average value. (15.3)

(a) Area of the rectangle with vertices $(-1,0)$, $(2,0)$, $(2,4)$, $(-1,4)$

(b) Area of the parallelogram with vertices $(-1,2)$, $(3,2)$, $(4,1)$, $(0,1)$

(c) Area of the triangle with vertices $(1,3)$, $(1,1)$, and $(2,2)$

(d) Area between $x=4-y^2$ and $x=y^2-4$

(e) Average value of $f(x,y)=e^{x^2y}$ over the square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$

(f) Average value of $f(x,y)=\sin(\frac{x}{2y})$ over the triangle with vertices $(0,1)$, $(1,1)$, $(0,2)$

39. Evaulate the iterated integral of three variables. (15.7)

(a) $\displaystyle\int_0^1 \int_0^1 \int_0^1 8xz - y^2\,dy\,dx\,dz$

(b) $\displaystyle\int_1^2 \int_0^x \int_x^{2z} 24y\,dy\,dz\,dx$

(c) $\displaystyle\int_{-1}^1 \int_{1+y}^{2+y} \int_0^2 z\,dx\,dz\,dy$

(d) $\displaystyle\int_{-\pi}^0 \int_0^{\pi/2} \int_0^x -\sin(z)\,dz\,dy\,dx$

(e) $\displaystyle\int_0^1 \int_0^1 \int_0^1 \frac{2xy^2}{(1+xyz)^3}\,dz\,dx\,dy$

40. Find an expression involving iterated integrals for the volume of the given solid. (15.7)

    (a) The pyramid with vertices $(0, 0, 0)$, $(3, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 1)$

    (b) The solid in the first octant bounded by the coordinate planes, $z = 1 - y^2$, and $x = 4$

    (c) The sphere $x^2 + y^2 + z^2 \leq 4$

    (d) The solid bounded by the surfaces $z = 4 - x^2 - y^2$ and $z = 4x^2 + 4y^2 - 16$

41. Find a transformation from either the unit square or triangle in the uv plane into the given region R in the xy plane. (15.10)

    (a) R : parallelogram bounded by $y = 3x + 1$, $y = 3x - 3$, $y = x - 3$ $y = x + 1$

    (b) R : triangle bounded by $y = x$, $y = 2x$, $y = 6 - x$

    (c) R : square with vertices $(2, 1)$, $(-2, 3)$, $(0, 7)$, $(4, 5)$

    (d) R : triangle with vertices $(0, -2)$ $(-1, 1)$, $(1, 3)$

42. Evaulate the double integral of variables x, y using the given transformation from the uv plane. (15.10)

    (a) $\iint_R 2x - y\,dA$, $\mathbf{r}(u, v) = \langle u + v, 2u - v + 3\rangle$ from unit square into the parallelogram R with vertices $(0, 3)$, $(1, 5)$, $(2, 4)$, $(1, 2)$

    (b) $\iint_R (x+y)(x-y-2)\,dA$, $\mathbf{r}(u, v) = \langle 4 - u - v, v - u + 2\rangle$ from unit triangle into the triangle R with vertices $(4, 2)$, $(3, 1)$, $(2, 2)$

    (c) $\iint_R (x + y)e^{x^2-y^2}\,dA$, $\mathbf{r}(u, v) = \langle u + 2v, u - 2v\rangle$ from unit square into the rectangle R bounded by $y = x$, $y = x - 4$, $y = -x$, $y = 2 - x$

    (d) $\iint_R e^x \cos(\pi e^x)\,dA$, $\mathbf{r}(u, v) = \langle \ln(u + v + 1), v\rangle$ from unit triangle into the region R bounded by $y = 0$, $y = e^x - 2$, $y = \frac{e^x-1}{2}$

43. Use polar coordinates to evaluate the double integral or iterated integral. (15.4)

(a) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 2y\,dx\,dy$

(b) $\displaystyle\int_{0}^{1}\int_{0}^{x} 3xy\,dy\,dx$

(c) $\displaystyle\iint_R e^{x^2+y^2}\,dA$, R : disk with boundary $x^2+y^2=9$

(d) $\displaystyle\int_{0}^{4}\int_{0}^{\sqrt{4x-x^2}} dy\,dx$

44. Use cylindrical coordinates to give an equivalent iterated integral which can be directly evaluated. (15.8)

(a) $\displaystyle\int_{0}^{3}\int_{0}^{\sqrt{9-y^2}}\int_{0}^{1} 2z\,dz\,dx\,dy$

(b) $\displaystyle\iiint_D \sqrt{x^2+y^2}\,dV$, D : right circular cylinder bounded by $|z|\le 2$ and $x^2+y^2=1$

(c) $\displaystyle\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} dz\,dy\,dx$

(d) The volume of the solid bounded by the xy plane and $z=1-x^2-y^2$

45. Use spherical coordinates to give an equivalent iterated integral which can be directly evaluated. (15.9)

(a) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{0}^{\sqrt{1-x^2-y^2}} dz\,dx\,dy$

(b) $\displaystyle\iiint_D x\,dV$, D : hemisphere bounded by $x=\sqrt{4-y^2-z^2}$ and the yz plane

(c) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} 3xz\,dz\,dx\,dy$

(d) The volume of the "ice cream cone" shaped solid

$$D=\{(x,y,z):\sqrt{x^2+y^2}\le z\le\sqrt{1-x^2-y^2}+1\}$$

Chapter 16

46. Evaluate the line integral with respect to arclength. (16.2)

    (a) $\int_C 2x + y\,ds$, C : line segment given by $\mathbf{r}(t) = \langle 4t+1, 4-3t\rangle$ for $0 \le t \le 2$
    (b) $\int_C z + 2xy\,ds$, C : line segment from $(0,-1,3)$ to $(2,2,-3)$
    (c) $\int_C xy^3\,ds$, C : arc on the circle $x^2+y^2=4$ from $(2,0)$ to $(1,\sqrt{3})$
    (d) $\int_C 2x\,ds$, C : parabolic arc on $y = x^2$ from $(0,0)$ to $(1,1)$

47. Evaluate the line integral with respect to a variable. (16.2)

    (a) $\int_C 2x + y\,dx$, C : line segment given by $\mathbf{r}(t) = \langle 4t+1, 4-3t\rangle$ for $0 \le t \le 2$
    (b) $\int_C z + 2xy\,dz$, C : line segment from $(0,-1,3)$ to $(2,2,-3)$
    (c) $\int_C xy^3\,dy$, C : arc on the circle $x^2+y^2=4$ from $(2,0)$ to $(1,\sqrt{3})$
    (d) $\int_C 2x\,dy$, C : parabolic arc on $y = x^2$ from $(0,0)$ to $(1,1)$

48. Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ directly. (16.2)

    (a) $\mathbf{F} = \langle y, x+y\rangle$, C : line segment from $(1,3)$ to $(-4,-9)$
    (b) $\mathbf{F} = \langle z, xy, z\rangle$, C : line segment from $(0,-1,3)$ to $(2,2,-3)$
    (c) $\mathbf{F} = \langle y^2, x^2\rangle$, C : one counter-clockwise revolution of the circle $x^2+y^2=9$
    (d) $\mathbf{F} = \langle y, 2y\rangle$, C : trigonometric arc on $y = \sin x$ from $(0,0)$ to $(\pi,0)$

49. Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ using the Fundamental Theorem of Line Integrals. (16.3)

    (a) $\mathbf{F} = \langle x, y\rangle$, C : line segment from $(1,1)$ to $(3,-2)$
    (b) $\mathbf{F} = \langle yz, xz, xy\rangle$, C : line segment from $(0,-3,2)$ to $(4,-1,3)$
    (c) $\mathbf{F} = \langle 4, z^2, 2yz\rangle$, C : curve given by $\mathbf{r}(t) = \langle 2^t, \sin(\pi t), 4t^2\rangle$ for $0 \le t \le 1$
    (d) $\mathbf{F} = \langle 2x, 1\rangle$, C : counter-clockwise oriented boundary of the unit square
    (e) $\mathbf{F} = \langle 12x^2y^2 + 3y, 8x^3y + 3x\rangle$, C : one clockwise revolution of the ellipse $x^2 + 4y^2 = 4$
    (f) $\mathbf{F} = \langle ye^{xy+z}, xe^{xy+z}, e^{xy+z}\rangle$, C : curve given by $\mathbf{r}(t) = \left\langle \frac{1}{1+t^2}, \cos t, e^{1-t^2}\right\rangle$ for $-1 \le t \le 1$

50. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Green's Theorem. (16.4)

    (a) $\mathbf{F} = \langle x^2 + y, x + y \rangle$, C : boundary of the unit square oriented counter-clockwise

    (b) $\mathbf{F} = \langle x, x^2 + xy^3 \rangle$, C : boundary of the rectangle $R = \{(x, y) : 1 \leq x \leq 2, 1 \leq y \leq 3\}$ oriented clockwise

    (c) $\mathbf{F} = \langle y, 2x \rangle$, C : boundary of the triangle with vertices $(1, 2)$, $(3, -2)$, $(-1, -2)$ oriented counter-clockwise

    (d) $\mathbf{F} = \langle x + y, x - y \rangle$, C : boundary of the upper semicircle $0 \leq y \leq \sqrt{4 - x^2}$ oriented counter-clockwise

51. Find the divergence and curl for each vector field. (16.5)

    (a) $\mathbf{F} = \langle x^2, y^2, z^2 \rangle$

    (b) $\mathbf{F} = \left\langle \frac{1}{x}, \frac{1}{y^2}, \frac{1}{x+y+z} \right\rangle$

    (c) $\mathbf{F} = \langle xyz, 2xyz, 3xyz \rangle$

    (d) $\mathbf{F} = \langle e^z \cos x, e^y \cos z, e^x \cos y \rangle$

    (e) $\mathbf{F} = \left\langle \frac{2}{x+yz}, 2, 2x + 2yz \right\rangle$

    (f) $\mathbf{F} = \left\langle \ln x, \frac{1}{x}, yz^3 \right\rangle$

52. Find a parametrization from the region G to the surface S. (16.6)

    (a) S : the portion of the elliptical paraboloid $z = x^2 + y^2$ above G
    $G : 0 \leq x \leq 2$ and $0 \leq y \leq 1$

    (b) S : the triangle with vertices $(0, 0, 8)$, $(2, 0, 6)$, $(2, 4, 2)$
    G : triangle with vertices $(0, 0)$, $(2, 0)$, $(2, 4)$

    (c) S : the lateral surface on the cylinder $x^2 + y^2 = 9$ between $z = -1$ and $z = 4$
    $G : 0 \leq \theta \leq 2\pi$ and $-1 \leq z \leq 4$

    (d) S : the sphere $x^2 + y^2 + z^2 = 4$
    $G : 0 \leq \theta \leq 2\pi$ and $0 \leq \phi \leq \pi$

    (e) S : the portion of the conical surface $z = \sqrt{x^2 + y^2}$ within the first octant and between the planes $z = 1$ and $z = 2$
    $G : 0 \leq \theta \leq \frac{\pi}{2}$ and $1 \leq r \leq 2$

    (f) S : the portion of the conical surface $z = \sqrt{x^2 + y^2}$ inside the sphere $x^2 + y^2 + z^2 = 9$
    $G : 0 \leq \theta \leq 2\pi$ and $0 \leq \rho \leq 3$

53. Given the parametrization $\mathbf{r}$ from the region G to the surface S, express the surface area of the S as a double iterated integral. (16.6)

    (a) S: portion of the plane $x + 2y + 3z = 6$ in the first octant
    G: triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$
    $\mathbf{r}(u,v) = \langle 6u, 3v, 2 - 2u - 2v \rangle$

    (b) S: elliptical region given by the portion of the plane $4x + y + z = 8$ inside the cylinder $x^2 + y^2 = 4$
    G : circlular region given by $0 \leq r \leq 2$ and $0 \leq \theta \leq 2\pi$
    $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 8 - 4r\cos\theta - r\sin\theta \rangle$

    (c) S : surface $z = \sqrt{x^3} + \sqrt{y^3}$ above G
    G : square $0 \leq x \leq 9$, $0 \leq y \leq 9$
    $\mathbf{r}(x,y) = \left\langle x, y, \sqrt{x^3} + \sqrt{y^3} \right\rangle$

    (d) S : hemisphere $x^2 + y^2 + z^2 = 4$ above the xy plane
    G: rectangle $0 \leq \phi \leq \frac{\pi}{2}$, $0 \leq \theta \leq 2\pi$
    $\mathbf{r}(\phi,\theta) = \langle 2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi \rangle$

54. Given the parametrization $\mathbf{r}$ from the region G to the surface S, rewrite the surface integral as a double iterated integral. (16.7)

    (a) $\iint_S x + y + z \, d\sigma$
    S: portion of the plane $x + z = 2$ above G
    G: square $0 \leq x \leq 1$, $0 \leq y \leq 1$
    $\mathbf{r}(x,y) = \langle x, y, 2 - x \rangle$

    (b) $\iint_S x^2 + y^2 \, d\sigma$
    S: lateral surface of the cylinder $x^2 + y^2 = 4$ where $0 \leq z \leq 1$
    G: rectangle $0 \leq \theta \leq 2\pi$, $0 \leq z \leq 1$
    $\mathbf{r}(\theta,z) = \langle 2\cos\theta, 2\sin\theta, z \rangle$

    (c) $\iint_S 3z \, d\sigma$
    S: cone $z = \sqrt{x^2 + y^2}$ below $z = 2$
    G: rectangle $0 \leq \rho \leq 2\sqrt{2}$, $0 \leq \theta \leq 2\pi$
    $\mathbf{r}(\rho,\theta) = \left\langle \frac{\sqrt{2}}{2}\rho\cos\theta, \frac{\sqrt{2}}{2}\rho\sin\theta, \frac{\sqrt{2}}{2}\rho \right\rangle$

55. Given the positively oriented parametrization $\mathbf{r}$ from the region G to the surface S, rewrite the vector field surface integral as a scalar double iterated integral. (16.7)

   (a) $\iint_S \langle x+y, y+z, z+x\rangle \cdot d\vec{\sigma}$
   S: parallelogram with vertices $(4,0,3)$, $(5,-2,2)$, $(4,-1,-1)$, $(3,1,0)$
   G: square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$ in the uv plane
   $\mathbf{r}(u,v) = \langle 4+u-v, -2u+v, 3-u-3v\rangle$

   (b) $\iint_S \langle y, -x, 1-z\rangle \cdot d\vec{\sigma}$
   S: portion of the elliptical paraboloid $z = x^2+y^2$ above G with concave orientation
   G: triangle $0 \leq x \leq 2$, $0 \leq y \leq 2x$ in the xy plane
   $\mathbf{r}(x,y) = \langle x, y, x^2+y^2\rangle$

   (c) $\iint_S \langle x, y, z\rangle \cdot d\vec{\sigma}$
   S: surface of the unit sphere oriented outwards
   G: rectangle $0 \leq \phi \leq \pi$, $0 \leq \theta \leq 2\pi$
   $\mathbf{r}(\phi,\theta) = \langle \sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi\rangle$

56. Use Stokes' Theorem to rewrite the given surface integral as a definite integral, where S is the upper hemisphere $z = \sqrt{1-x^2-y^2}$ with convex orientation.

   (a) $\iint_S \langle 1, 1, 1\rangle \cdot d\vec{\sigma}$
   (b) $\iint_S \langle -2z, 0, -2y\rangle \cdot d\vec{\sigma}$
   (c) $\iint_S \langle 0, 1-2x, 2x-1\rangle \cdot d\vec{\sigma}$
   (d) $\iint_S \langle 1, 1, 3-2y\rangle \cdot d\vec{\sigma}$

57. Use the Divergence Theorem to rewrite the given surface integral as a triple iterated integral, where S is the surface of the unit cube oriented outwards.

   (a) $\iint_S \langle x, y, z\rangle \cdot d\vec{\sigma}$
   (b) $\iint_S \langle x+y, y^2+z^2, z^3+x^3\rangle \cdot d\vec{\sigma}$
   (c) $\iint_S \langle xyz, xyz, xyz\rangle \cdot d\vec{\sigma}$
   (d) $\iint_S \langle xy+yz, yz+zx, zx+xy\rangle \cdot d\vec{\sigma}$