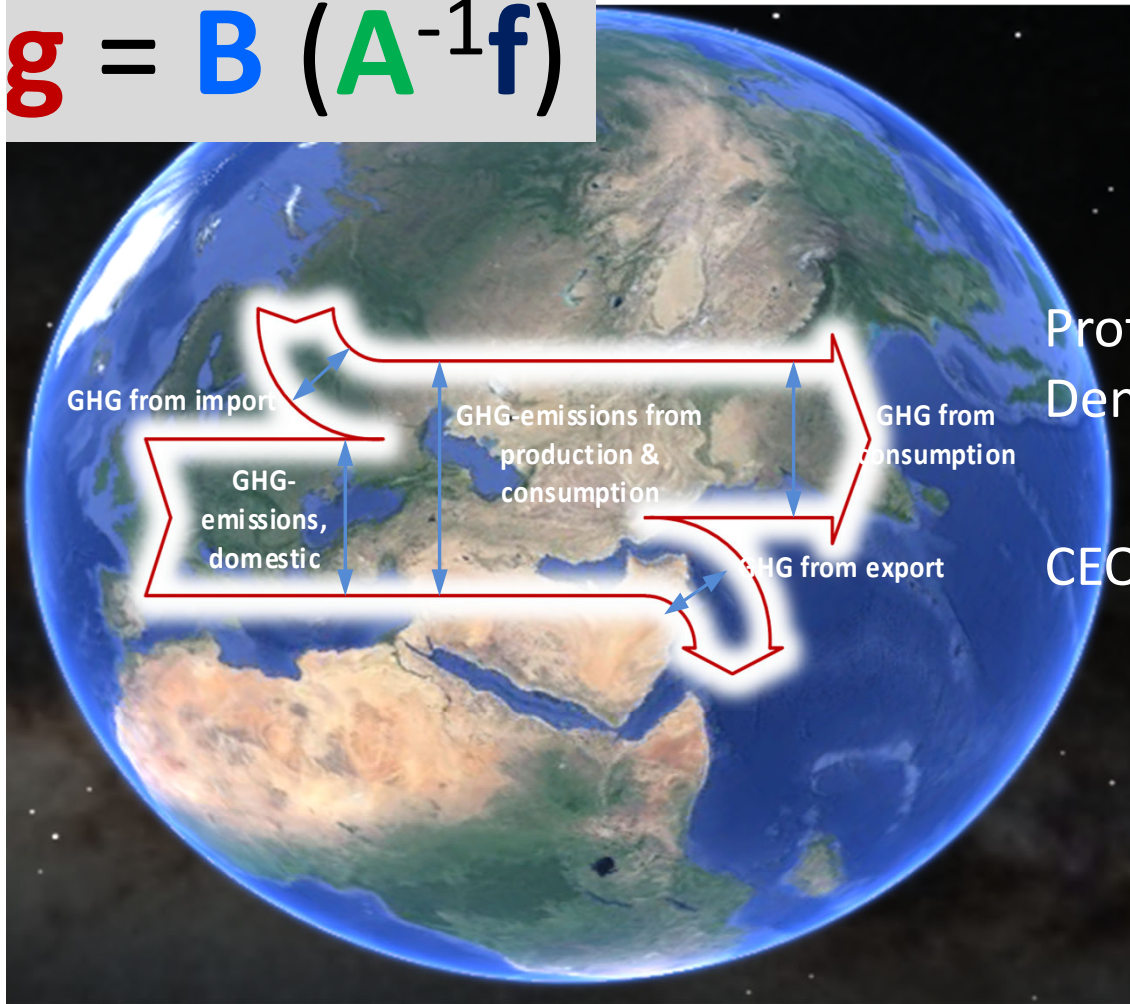


# Input-output modelling

Monetary supply-use tables and how they are constructed from raw data. Constructing consequential and attributional IO models

$$\mathbf{g} = \mathbf{B} (\mathbf{A}^{-1} \mathbf{f})$$



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Updated: 7<sup>th</sup> April 2022



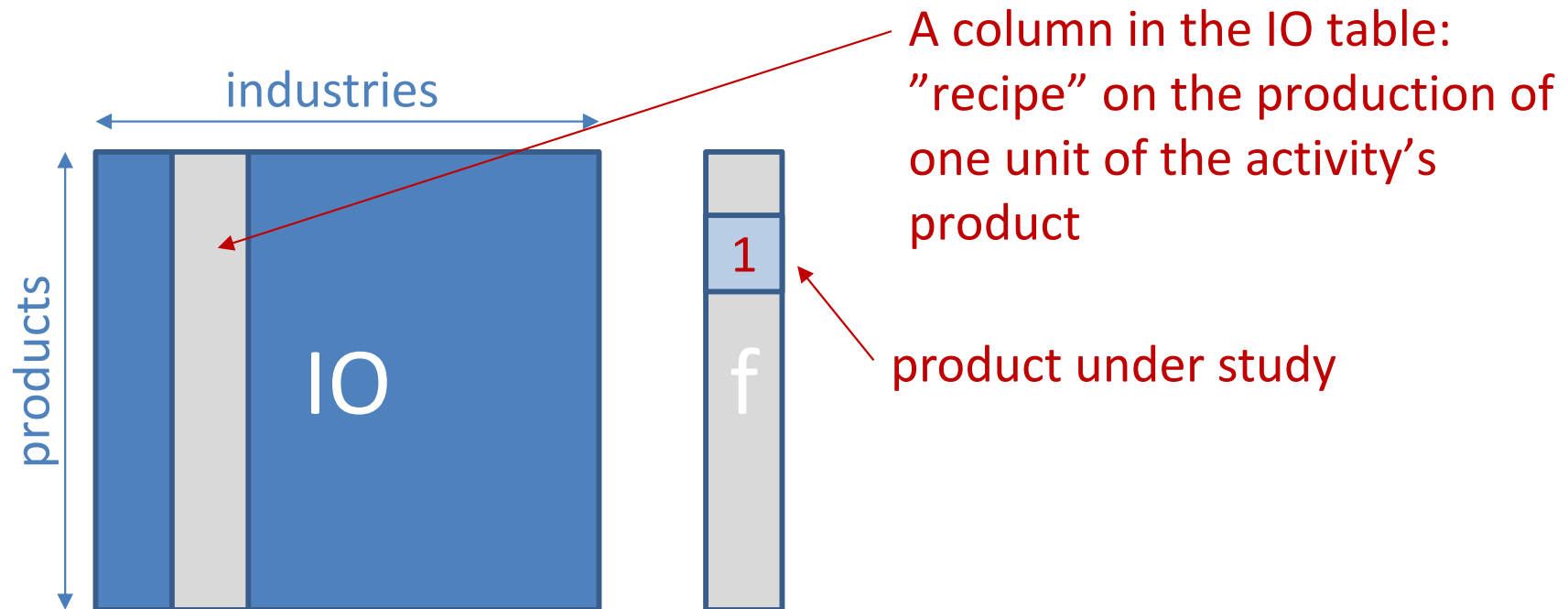
AALBORG UNIVERSITY  
DENMARK

# Agenda

- 
- What is an IO-model?
  - Supply-use tables
  - Constructing IO models - consequential and attributional models

# What is an IO model?

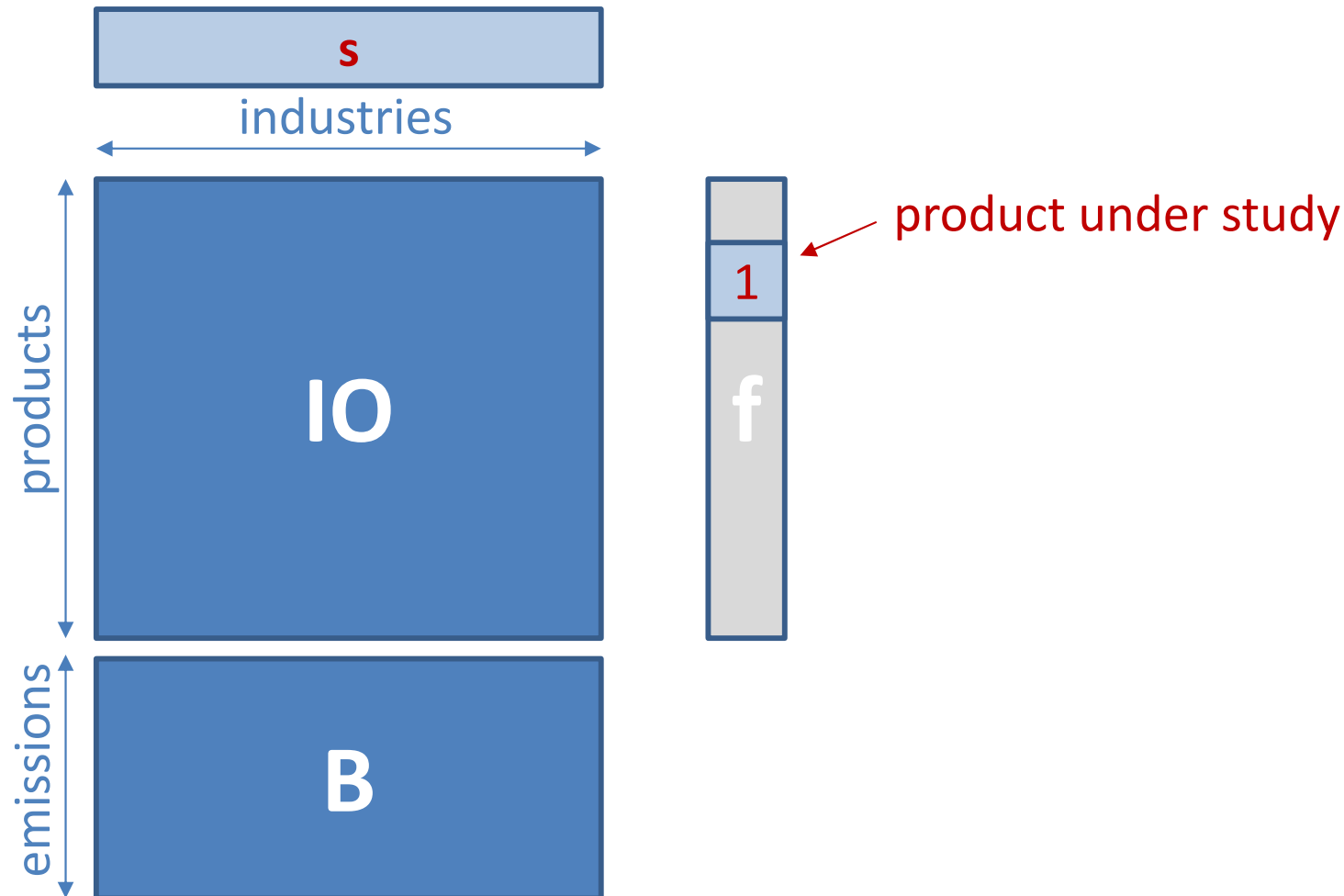
- **Model** that expresses the interdependencies of industries in economy via their flows (inputs and outputs)
- Model calculates the **effect** on all industries from a defined demand for a product (or several products)



- **"effect"**? What is calculated?

# What is an IO model?

- Effect = scaling factors



- Resulting emissions =  $Bs'$

# Agenda

- 
- What is an IO-model?
  - Supply-use tables
  - Constructing IO models - consequential and attributional models

# Framework: supply-use tables (SUT)

- Defined in System of Environmental-Economic Accounts (SEEA2012)

## System of Environmental-Economic Accounting

SUPPLY					
	Production and generation of residuals	Accumulation	Flows from the Rest of the World	Flows from the Environment	Total
	Production and generation of residuals by industries (incl. household production on own account) - classified by ISIC	Generation of residuals by households	Industries - classified by ISIC		
Natural inputs	Supply			A. Flows from environment (incl. natural resource residuals)	Total Supply of Natural Inputs (TSNI)
Products					Total Supply of Products (TSP)
Residuals	11. Residuals generated by industry (incl. natural resource residuals) 12. Residuals generated following treatment	J. Residuals generated by household final consumption	K1. Residuals from scrapping and demolition of produced assets K2. Emissions from controlled landfill sites	L. Residuals received from rest of the world M. Residuals recovered from the environment	Total Supply of Residuals (TSR)
TOTAL SUPPLY					

USE					
	Intermediate consumption of products, use of natural inputs and collection of residuals	Final consumption *	Accumulation	Flows to the Rest of the World	Flows to the Environment
	Industries - classified by ISIC	Households	Industries - classified by ISIC		
Natural inputs	B. Extraction of natural inputs				Total Use of Natural Inputs (TUNI)
	B1. Extraction used in production B2. Natural resource residuals				
Products	E. Intermediate consumption (incl. purchase of recycled and reused products)				Total Use of Products (TUP)
Residuals	N. Residuals received by waste mgmt and other industries (incl. residuals from scrapping and demolition of produced assets; excl. accumulation in controlled landfill sites)	of recycled and reused products	O. Accumulation in controlled landfill sites	P. Residuals sent to the rest of the world	Q. Residual flows direct to environment Q1. Direct from industry and households (incl. natural resource residuals & landfill emissions) Q2. Following treatment
TOTAL USE					Total Use of Residuals (TUR)

White cover publication, pre-edited text subject to official editing

European Commission • Food and Agriculture Organization • International Monetary Fund  
Organisation for Economic Cooperation and Development • United Nations • World Bank

## Supply-use tables:

- Same framework for IOA, MFA, energy accounts and LCA
- Same concepts and classifications
- Facilitates balance and completeness checks
- Stores data unallocated (co-products)

# Supply-use tables

## Final demand

- Household consumption
- Government consumption

## Investments

Monetary supply table	Activities (a)	Import	Total
Products (c)	$V'_o$	$N_c$	$q$
Total	$g'$		

Import

Monetary use table	Activities (a)	Final uses			Total
Products (c)	$U_o$	$y_o$	Capital formation	$E_c$	$q$
Primary inputs	Labour				
	Taxes				
	Use of fixed capital				
	Profit				
Total	$g'$				

Export

GDP

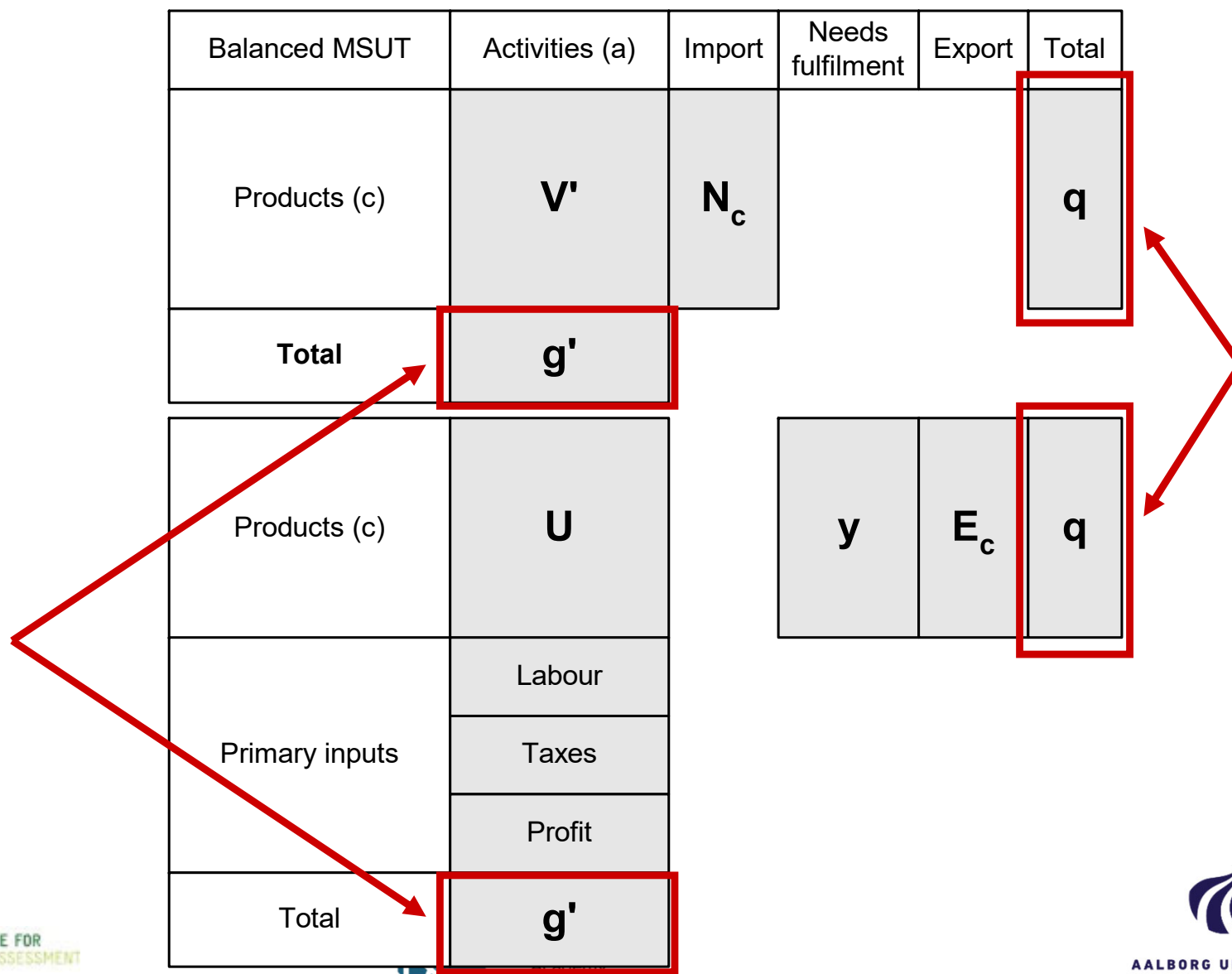
Final uses		
$y_o$	Capital formation	$E_c$

minus

$N_c$
-------

# Supply-use tables

- Supply ( $\mathbf{V'}$ ) and use ( $\mathbf{U}$ ) tables are balanced





# Agenda

- What is an IO-model?
- Supply-use tables
- ➔ ▪ Constructing IO models - consequential and attributional models

# Constructing IO models

- IO-tables must be square for IO analysis.
- Product-by-product versus industry-by-industry tables?
  - **PxP**: describes the technological relations between products and homogeneous units of production (branches). The intermediate part describes, for each product, the amounts of products that were used to produce this product, irrespective of the producing industry.
  - **IxI**: describes inter-industry relations. The intermediate part of the table describes for each industry the use of products in production.
  - **PxP is recommended**: theoretically more homogeneous in their description of the transactions than industry-by-industry tables, since a single element of IxI can refer to products that are characteristic in other industries.
- Focus only on PxP in the following. Corresponds to LCA.

Eurostat (2008, ch 11), Eurostat Manual of Supply, Use and Input-Output Tables. Eurostat.  
<http://ec.europa.eu/eurostat/ramon/statmanuals/files/KS-RA-07-013-EN.pdf>

# Constructing IO models

## - The problem of negatives in IO table

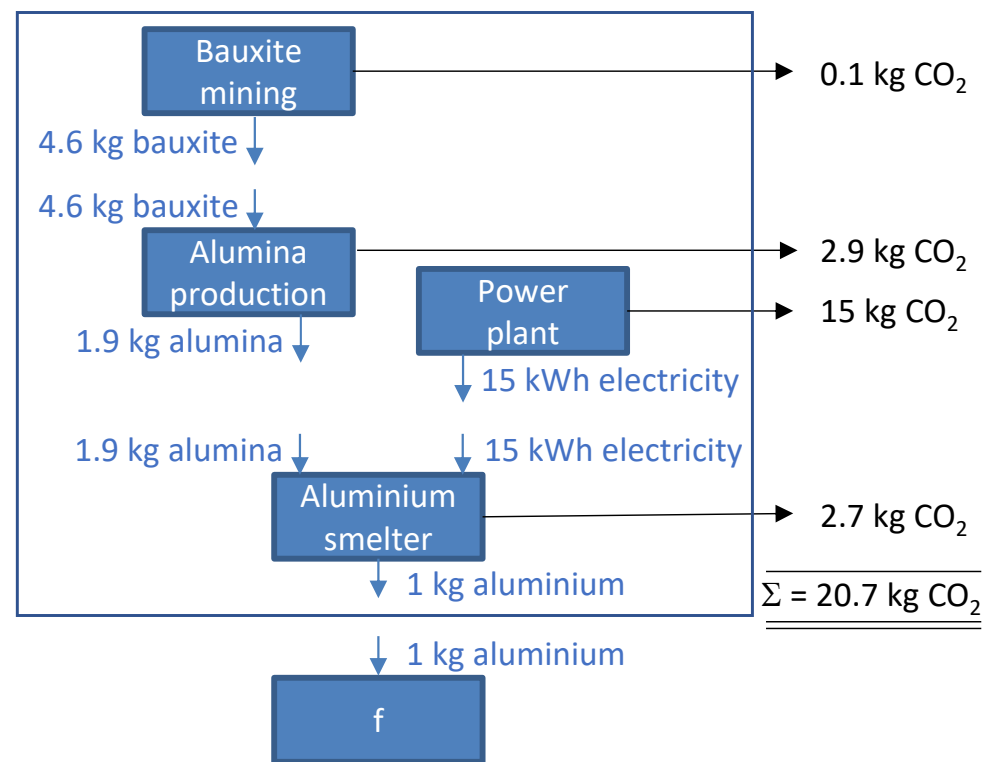
- Negative values in IO table often seen as problematic in IO literature & by practitioners.
- However, in LCA this is common, e.g.:
  - Waste incineration substitutes energy
  - Dairy cow milk production substitutes beef
  - Soybean meal substitutes vegetable oil
- Problem of negatives only relevant if it is caused by errors
  - Extreme case when scenarios cause larger negative change than current production volume.

Eurostat (2008, ch 11), Eurostat Manual of Supply, Use and Input-Output Tables. Eurostat.

<http://ec.europa.eu/eurostat/ramon/statmanuals/files/KS-RA-07-013-EN.pdf>

# From SUT to IO

## - no co-products



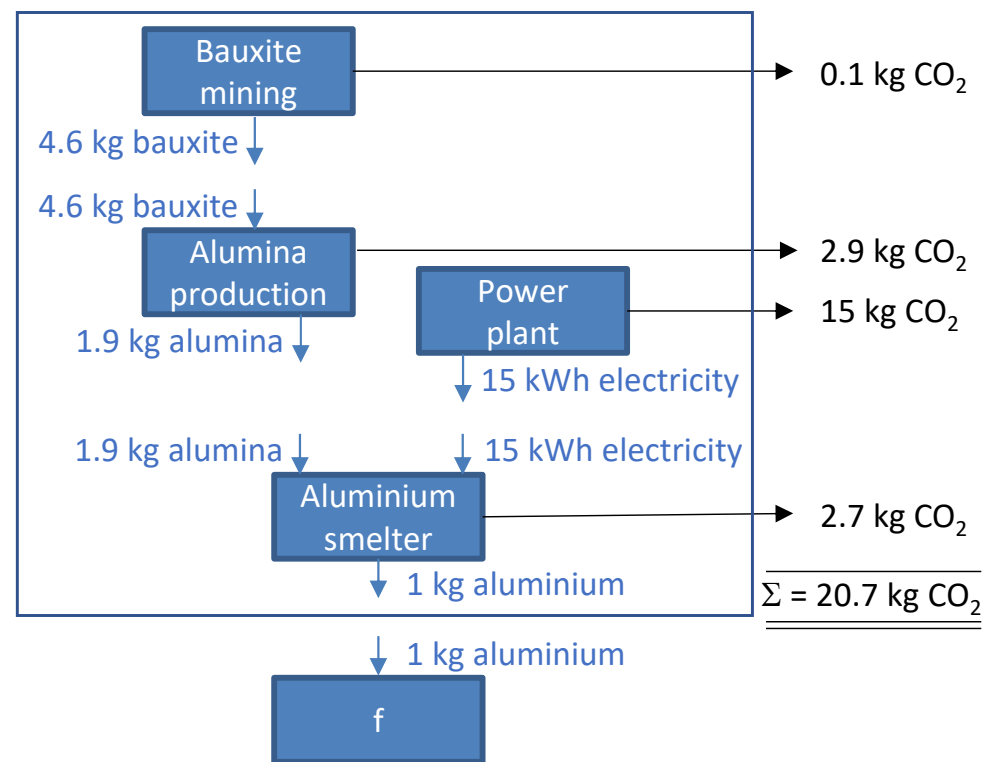
## Supply-use table

Products		Industry					
Supply	Unit	Bauxite mining	Alumina production	Power plant	Aluminium smelter		Total
Bauxite	kg						
Alumina	kg						
Electricity	kWh						
Aluminium	kg						
Use		Bauxite mining	Alumina production	Power plant	Aluminium smelter	Final use	Total
Bauxite	kg						
Alumina	kg						
Electricity	kWh						
Aluminium	kg						
Emissions		Bauxite mining	Alumina production	Power plant	Aluminium smelter		Total
CO <sub>2</sub>	kg						



# From SUT to IO

- no co-products



## Supply-use table

Products		Industry					
Supply	Unit	Bauxite mining	Alumina production	Power plant	Aluminium smelter		Total
Bauxite	kg	4.6					4.6
Alumina	kg		1.9				1.9
Electricity	kWh			15			15
Aluminium	kg				1		1
Use		Bauxite mining	Alumina production	Power plant	Aluminium smelter	Final use	Total
Bauxite	kg		4.6				4.6
Alumina	kg				1.9		1.9
Electricity	kWh				15		15
Aluminium	kg					1	1
Emissions		Bauxite mining	Alumina production	Power plant	Aluminium smelter		Total
CO <sub>2</sub>	kg	0.1	2.9	15	2.7		20.7

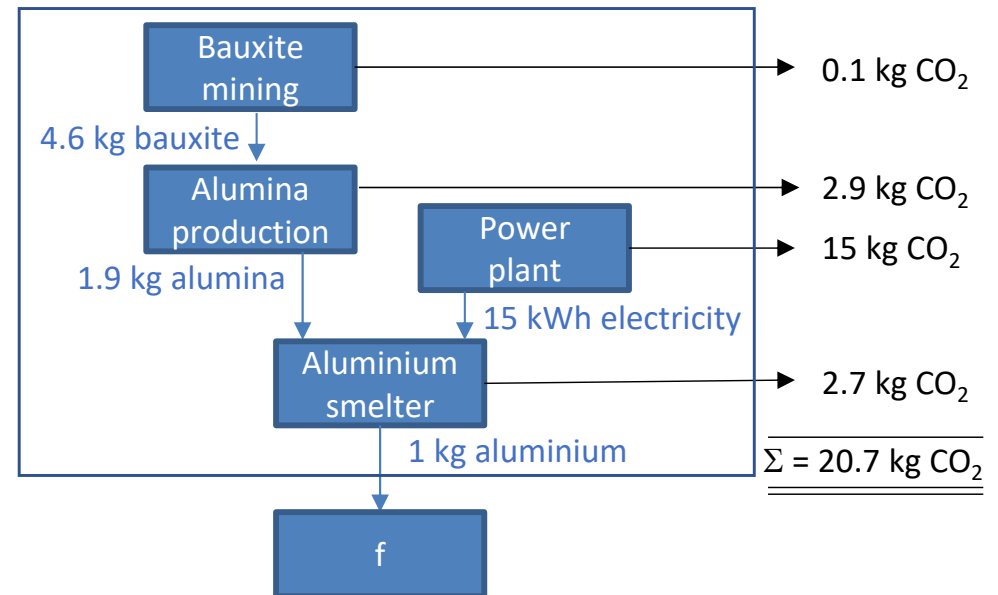


# From SUT to IO

- no co-products

Normalise by the output of industries:

Direct requirement coefficient matrix  $\tilde{Z}$



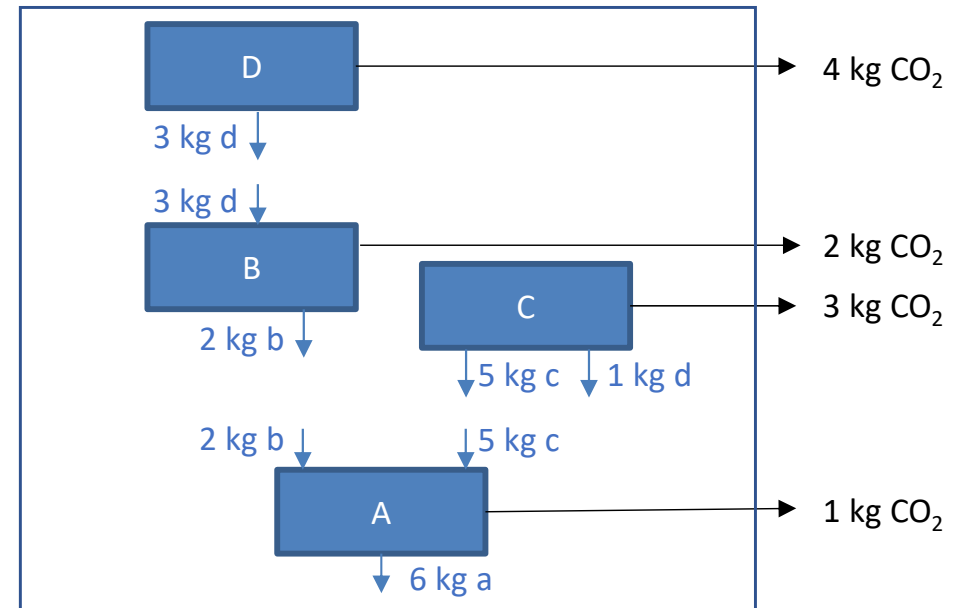
## Input-Output table

Products		Industry			
Supply		Bauxite mining	Alumina production	Power plant	Aluminium smelter
Unit		kg	kg	kWh	kg
Reference product		1	1	1	1
Use					
Bauxite	kg		2.4		
Alumina	kg				1.9
Electricity	kWh				15.0
Aluminium	kg				
Emissions					
CO <sub>2</sub>	kg	0.022	1.5	1.0	2.7

# From SUT to IO

- with co-products

Process	A	B	C	D
<b>Outputs</b>				
A	6			
B		2		
C			5	
D			1	3
<b>Inputs</b>				
A				
B	2			
C	5			
D		3		
<b>Emissions</b>				
CO <sub>2</sub>	1	2	3	4



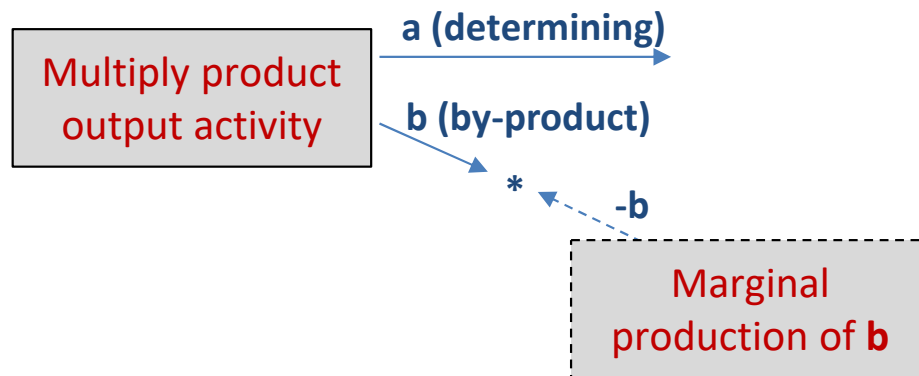
Direct requirement coefficient matrix depends on allocation/substitution

# By-products in the IO-framework

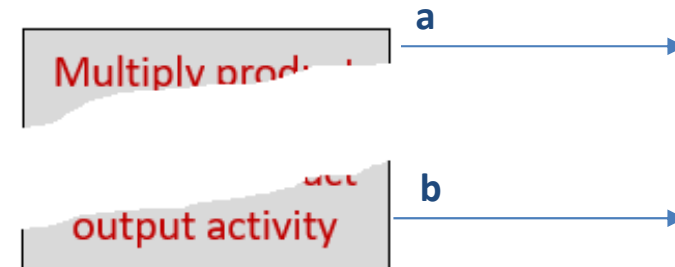
The issue is important in LCA and IO!

⇒ discussion on allocation vs. substitution  
(or attributional vs. consequential)

## Substitution



## Allocation



Substitution approach is most often referred to as the best...

- Kop Jansen, P. and ten Raa, T. (1990) "The Choice of Model in the Construction of Input-Output Coefficients Matrices", International Economic Review, 31, pp. 213-227
- United Nations (1993) Revised System of National Accounts, Studies in Methods, Series F, no. 2, rev.4
- Eurostat (2008, p 310)
- Suh S, Weidema B P, Schmidt J, Heijungs R (2010)



# Substitution

- How is it done?

- **Balances:** Substitution is only algorithm that consistently maintains mass, elementary, energy and monetary balances of the resulting single-product systems (Weidema & Schmidt 2010).
- **Simplicity:** Suh et al. (2010) clarifies the simplicity of the algorithm: By-product outputs are modelled as negative inputs.



Supply table	Activities
Products	$A$ $V'$
Use table	Activities
Products	$-B$ $U$

Weidema B P, Schmidt J (2010). Avoiding allocation in life cycle assessment revisited. *Journal of Industrial Ecology* 14(2):192-195

Suh S, Weidema B P, Schmidt J, Heijungs R (2010). Generalized make and use framework for allocation in life cycle assessment. *Journal of Industrial Ecology* 14(2):335-353

# Constructing IO models

## - Technology models

- Commodity technology model
  - $\mathbf{Z} = \mathbf{UV}'^{-1}$
  - $\mathbf{E} = \mathbf{BV}'^{-1}$
- By-product technology model
  - $\mathbf{Z} = (\mathbf{U} - \mathbf{V}'_{\text{off-diag}}) \mathbf{V}'_{\text{diag}}^{-1}$
  - $\mathbf{E} = \mathbf{BV}'_{\text{diag}}^{-1}$
- Industry technology model
  - $\mathbf{Z} = (\mathbf{U} \hat{\mathbf{g}}^{-1}) (\mathbf{V} \hat{\mathbf{q}}^{-1})$
  - $\mathbf{E} = (\mathbf{B} \hat{\mathbf{g}}^{-1}) (\mathbf{V} \hat{\mathbf{q}}^{-1})$
- Same results for life cycle emissions/extensions
- By-product technology model is more transparent and simple
- By-product technology model = substitution in LCA

# Constructing IO models

## - with by-products

- How to deal with co-products?

<b>V'</b>	Crop	Animal	Vegetable oil	Animal feed	Other food industry	Total
Crop	485	0	0	0	0	485
Animal	0	51	0	0	0	51
Vegetable oil	0	0	260	0	0	260
Animal feed	0	0	40	560	0	600
Other food industry	0	0	0	0	241	241
<b>Total</b>	<b>485</b>	<b>51</b>	<b>300</b>	<b>560</b>	<b>241</b>	<b>1637</b>

<b>U</b>	Crop	Animal	Vegetable oil	Animal feed	Other food industry	Final demand	Total
Crop	10	0	180	200	22	73	485
Animal	0	1	0	0	40	10	51
Vegetable oil	0	0	0	0	129	131	260
Animal feed	0	21	0	50	0	529	600
Other food industry	0	0	0	0	0	241	241
Primary inputs	475	29	120	310	50		
<b>Total</b>	<b>485</b>	<b>51</b>	<b>300</b>	<b>560</b>	<b>241</b>	<b>984</b>	<b>1637</b>

- Two options:

### Extensions: B

- Substitution
- Co-product allocation

# Constructing IO models

## - Substitution / by-product technology model

Extensions  
coefficient  
matrix

$$\mathbf{E} = \mathbf{B}\hat{\mathbf{g}}^{-1}$$

$$\mathbf{Z} = \mathbf{U}\hat{\mathbf{g}}^{-1}$$

$\mathbf{U}$  normalised by  
total supply from  
activities ( $\mathbf{g}$ )

$\mathbf{V}'$	Crop	Animal	Vegetable oil	Animal feed	Other food industry	Total
Crop	485	0	0	0	0	485
Animal	0	51	0	0	0	51
Vegetable oil	0	0	260	0	0	260
Animal feed	0	0	40	560	0	600
Other food industry	0	0	0	0	241	241
Total	485	51	300	560	241	1637

$\mathbf{U}$	Crop	Animal	Vegetable oil	Animal feed	Other food industry	Final demand	Total
Crop	10	0	180	200	22	73	485
Animal	0	1	0	0	40	10	51
Vegetable oil	0	0	0	0	129	131	260
Animal feed	0	21	0	50	0	529	600
Other food industry	0	0	0	0	0	241	241
Primary inputs	475	29	120	310	50		
Total	485	51	300	560	241	984	1637

$\mathbf{V}'_{\text{diag}}$	Crop	Animal	Vegetable oil	Animal feed	Other food industry	Total
Crop	485	0	0	0	0	485
Animal	0	51	0	0	0	51
Vegetable oil	0	0	260	0	0	260
Animal feed	0	0	0	560	0	560
Other food industry	0	0	0	0	241	241
Total	485	51	260	560	241	1597

$\mathbf{U}-\mathbf{V}'_{\text{off-diag}}$	Crop	Animal	Vegetable oil	Animal feed	Other food industry	Final demand	Total
Crop	10	0	180	200	22	73	485
Animal	0	1	0	0	40	10	51
Vegetable oil	0	0	0	0	129	131	260
Animal feed	0	21	-40	50	0	529	560
Other food industry	0	0	0	0	0	241	241
Primary inputs	475	29	120	310	50		
Total	485	51	260	560	241	984	1597

# Constructing IO models

## - Substitution / by-product technology model

- Substitution (by-product technology assumption)

$$\mathbf{Z} = \left( \mathbf{U} - \mathbf{V}'_{\text{off-diag}} \right) \left( \mathbf{V}'_{\text{diag}} \right)^{-1}$$

- Where  $\mathbf{V}'$  is split into  $\mathbf{V}'_{\text{diag}}$  (diagonal entries in  $\mathbf{V}'$ ) and  $\mathbf{V}'_{\text{off-diag}}$  (off-diagonal entries in  $\mathbf{V}'$ )

# Constructing IO models

- Revenue allocation / industry technology model

<b>V'</b>	Crop	Animal	Vegetable oil	Animal feed	Other food industry	Total
Crop	485	0	0	0	0	485
Animal	0	51	0	0	0	51
Vegetable oil	0	0	260	0	0	260
Animal feed	0	0	40	560	0	600
Other food industry	0	0	0	0	241	241
<b>Total</b>	<b>485</b>	<b>51</b>	<b>300</b>	<b>560</b>	<b>241</b>	<b>1637</b>

<b>U</b>	Crop	Animal	Vegetable oil	Animal feed	Other food industry	Final demand	Total
Crop	10	0	180	200	22	73	485
Animal	0	1	0	0	40	10	51
Vegetable oil	0	0	0	0	129	131	260
Animal feed	0	21	0	50	0	529	600
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Primary inputs	475	29	120	310	50		
<b>Total</b>	<b>485</b>	<b>51</b>	<b>300</b>	<b>560</b>	<b>241</b>	<b>984</b>	<b>1637</b>

$$Z = (U\hat{g}^{-1})(V\hat{q}^{-1})$$

**Extensions coefficient matrix:**  $E = (B\hat{g}^{-1})(V\hat{q}^{-1})$

**V** normalised by total supply of products (**q**)  
 => average market supply, similar to attributional modelling

**U** Normalised by total supply from activities (**g**)  
 => Economic allocation similar to attributional modelling

# Allocation problems in the SUT framework

- Years of discussions on difficulties in substitution and allocation in LCI is solved!
- Allocation versus substitution: Two different simple matrix formulas.

Suh S, Weidema B, Schmidt J H and Reinout H (2010), Generalized Make and Use Framework for Allocation in Life Cycle Assessment. *Journal of Industrial Ecology* 14(2): 335-353

# ... if you want to know more

- **Aalborg University:** Annual Advanced PhD course:  
<https://ilca.es/advanced-lca-consequential-and-io-based-life-cycle-assessment/>
- **ILCA:** The International Life Cycle Academy (<https://ilca.es/>)
- Consequential LCA (<https://consequential-lca.org/>)