Computer Science 2300: Homework 1

Due: February 7, 2011

Note: Please use rigorous, formal arguments. You will not receive full credit otherwise. Homework is due **at the beginning of lecture.**

1. (10 points) [Based on a question from Cormen *et al*'s *Introduction to Algorithms*] Consider the following procedure for generating a permutation of the numbers $1 \ldots n$. First, choose an integer r uniformly at random between 1 and n. Now consider the array A with indices from 0 to $n-1$. Fill $A[i]$ with the number $i+r+1$ if $i+r+1 \leq n$ and with the number $i+r+1-n$ otherwise. First show that each number between 1 and n has a $1/n$ probability of winding up in any particular position in A. Then show that the resulting permutation in A is not uniformly random.

2. (10 points) Let F_k be the Fibonacci numbers, with $F_0 = 0, F_1 = 1$, and $F_i = F_{i-1} + F_{i-2}$ if $i > 1$. Prove by induction that $F_{n-1}F_{n+1} = F_n^2 + (-1)^n$.

3. (10 points) Problem 1.4 in DPV.

4. (20 points) Problem 2.5 in DPV, parts a through j (2 points each) only (pages 71-72). For 5 points of extra credit if you get it right, you can also do part k.

5. (10 points) Problem 2.16 in DPV.