Problem 1 (100 points)

An object is observed is space and is determined to have position and velocity vectors expressed in terms of a geocentric despun equatorial frame (standard earth frame) of
$\vec{r} = -35144\hat{i} - 59954\hat{j} - 6057\hat{k}$ [km] and $\vec{v} = -0.8025\hat{i} - 3.133\hat{j} - 0.4078\hat{k}$ km/s]
where $\hat{i}\hat{j}\hat{k}$ are the basis vectors associated with the standard earth reference frame.

From this information determine:

A) orbit eccentricity e *(25 pts)*
B) orbit semimajor axis a [km] (20 pts)
C) true anomaly θ [deg] (25 pts)
D) Orbit inclination i [deg]. (10 pts)
E) Right ascension of the ascending node Ω [deg]. (10 pts)
F) Argument of perigee ω [deg] (10 pts)

$r = \sqrt{(-35144)^2 + (-59954)^2 + (-6057)^2} = 69{,}759\text{ km}$

$\vec{h} = \vec{r} \times \vec{v} = 5468\hat{i} - 9471\hat{j} + 62{,}020\hat{k}$ $h = \sqrt{(5468)^2 + (-9{,}471)^2 + (62020)^2} = 62{,}977\ \text{km}^2/\text{s}$

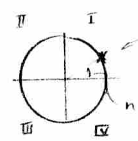
$\vec{e} = \dfrac{\vec{v} \times \vec{h}}{\mu} - \dfrac{\vec{r}}{r} = 0.00651\hat{i} + 0.9787\hat{j} + 0.1489$

A) $\Rightarrow e = \sqrt{(0.00651)^2 + (0.9787)^2 + (0.1489)^2} = \underline{0.990}$

$\text{Energy} = \frac{1}{2}\vec{v}\cdot\vec{v} - \dfrac{\mu}{r} = -0.3986\,(\text{km}/\text{s})^2$

B) $a = \dfrac{-\mu}{2\,\text{Energy}} = \underline{500{,}000\text{ km}}$

$\hat{n} = \dfrac{\hat{k} \times \vec{h}}{|\hat{k} \times \vec{h}|} = 0.8660\hat{i} + 0.5000\hat{j} + 0.000\hat{k}$



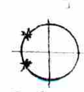
FIRST quadrant so no problem w/ $\tan^{-1}$ will return the proper value w/o adjustment

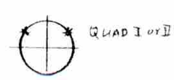
E) $\underline{\Omega = \tan^{-1}\left(\dfrac{0.5}{0.866}\right) = 30°}$

D) $i = \cos^{-1}\left\{\dfrac{\hat{k}\cdot\vec{h}}{h}\right\} = \underline{10°}$

$\hat{p} = \dfrac{\vec{e}}{e} = 0.006578\hat{i} + 0.9886\hat{j} + 0.1504\hat{k}$, $\hat{w} = \dfrac{\vec{h}}{h} = 0.08682\hat{i} - 0.1504\hat{j} + 0.9848\hat{k}$

$\hat{q} = \hat{w} \times \hat{p} = -0.9962\hat{i} - 0.00658\hat{j} + 0.08682\hat{k}$





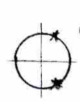
C) $r_1 = \vec{r}\cdot\hat{p} = -60412$, $r_2 = \vec{r}\cdot\hat{q} = 34879$

$\theta_1 = \cos^{-1}\left\{\dfrac{\vec{r}\cdot\hat{p}}{r}\right\} = 150°$ QUADRANT II or III

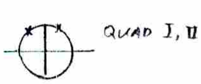
$\theta_2 = \sin^{-1}\left\{\dfrac{\vec{r}\cdot\hat{q}}{r}\right\} = 30°$

ALSO $\vec{r}\cdot\vec{v} = 218600 > 0 \Rightarrow$ OBJECT IS IN QUADRANT I OR II so $\cos^{-1}$ works $\boxed{\theta = 150°}$

F) $\omega = \cos^{-1}\{\hat{n}\cdot\hat{p}\} = 60°$



AND $\vec{\omega} = \sin^{-1}\{(\hat{n}\times\hat{p})\cdot\hat{w}\} = 60°$



$\rightarrow \boxed{\omega = 60°}$

Problem 2 (100 points)

Assume that for the problem above the following orbital parameters are given: $e=0.99$, $\theta=-150°$, $i=10°$, $\Omega=30°$, $\omega=60°$, $a = 500000$ [km]:

NOTE THAT THIS IS NEGATIVE! INBOUND!!!

a) Show that the object will (or will not) impact the earth. If this object does not impact the earth, what is its distance from the earth at closest approach? (treat the earth as a perfect sphere) (25 pts)

b) How long (in hours) until impact or closest approach? (40 pts)

c) What will its actual anomaly be when the object's velocity is 10 km/sec? (25 pts)

d) What is the objects velocity at impact ignoring the atmosphere (or closest approach)? (10 pts)

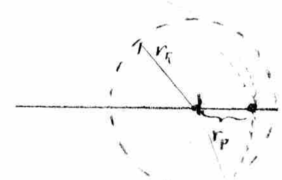
A) $r_p = \frac{h^2}{\mu(1+e)} = \frac{(62,977)^2}{398,600(1+0.99)} = 5000$ Km ≤ 6378 (radius of Earth) ⇒ collision

B) $r_E = r_{impact} = 6378 = \frac{h^2}{\mu(1+e\cos\theta)}$ ⇒ $\theta_{impact} = \cos^{-1}\left\{\frac{h^2 - r_E\mu}{r_E\mu e}\right\} = \cos^{-1}\left\{\frac{(62977)^2 - 6378(398600)}{6378(398,600)\,0.99}\right\}$

$= -55.55°$ = 0.9695 rad

$E_0 = 2\tan^{-1}\left\{\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta_0}{2}\right)\right\} = -0.5173$ rad -

$E_{impact} = 2\tan^{-1}\left\{\sqrt{\frac{1-0.99}{1+0.99}}\tan\left(\frac{0.9695\text{ rad}}{2}\right)\right\} = -0.07463$ rad



$t_0 - T_0 = \{E_0 - e\sin E_0\}\sqrt{\frac{a^3}{\mu}} = -15515$ SEC

$t_{impact} - T_0 = \{E_{impact} - 0.99\sin E_{impact}\}\sqrt{\frac{(500,000)^3}{398600}} = -456.4$ SEC

$\Delta t = 15,059$ sec = 4.183 hours

NOT ALL θ'S SHOULD BE NEGATIVE VALUED

C) $v_r = \frac{\mu}{h} e\sin\theta$ $\quad v_\theta = \left(\frac{\mu}{h}\right)[1+e\cos\theta]$ ⇒ $V = \sqrt{v_r^2+v_\theta^2} = \left(\frac{\mu}{h}\right)\left[\overbrace{e^2\sin^2\theta}^{v_r^2} + (1+2e\cos\theta+e^2\cos\theta)\right]$

→ $V = 10 = \left(\frac{\mu}{h}\right)[e^2+1+2e\cos\theta]$

⇒ $\theta_{V=10} = \cos^{-1}\left\{\frac{(Vh/\mu)^2-(1+e^2)}{2e}\right\} = -1.3071$ rad $= -74.89°$

$E_0 = \frac{1}{2}v_0^2 - \frac{\mu}{r_0} = -0.3986$ ⇒ $v_{impact} = \left\{2E_0 + \frac{\mu}{r_{Earth}}\right\}^{1/2} = 11.144$ Km/s