HW6: Introduction to Numerical Methods for Differential Equations

Due 3pm, April 29, 2011

1. Find the exact solution for the following advection equations (you can use characteristic curves of these equations)

   (a)

$$\begin{cases} u_t - 2u_x &= 0, & x \in (-\infty, \infty), t > 0 \\ u(x,0) &= \sin(x-1)e^x, & x \in (-\infty, \infty) \end{cases}$$

   (b)

$$\begin{cases} u_t - 2u_x &= 0, & x \in (0,1), t > 0 \\ u(x,0) &= \sin(x-1)e^x, & x \in (0,1) \\ u(1,t) &= 1-t, & t > 0 \end{cases}$$

2. The Fromm method for solving the advection equation $u_t + au_x = 0$ is

$$u_{i,j+1} = \frac{1}{4}\lambda(\lambda-1)u_{i-2,j} + \frac{1}{4}\lambda(5-\lambda)u_{i-1,j} - \frac{1}{4}(\lambda-1)(\lambda+4)u_{i,j} + \frac{1}{4}\lambda(\lambda-1)u_{i+1,j},$$

   with $\lambda = a\frac{k}{h}$. What is the stencil of the method, and what is the CFL condition?

3. 4.22 without question (a).

   Note:

   - In (d) it is " Redo (c)" instead of " Redo (b)".
   - Please work with the upwind method and Lax-Wendroff method. The Lax-Friedrichs scheme is optional.

4. 5.1 (a) (c)