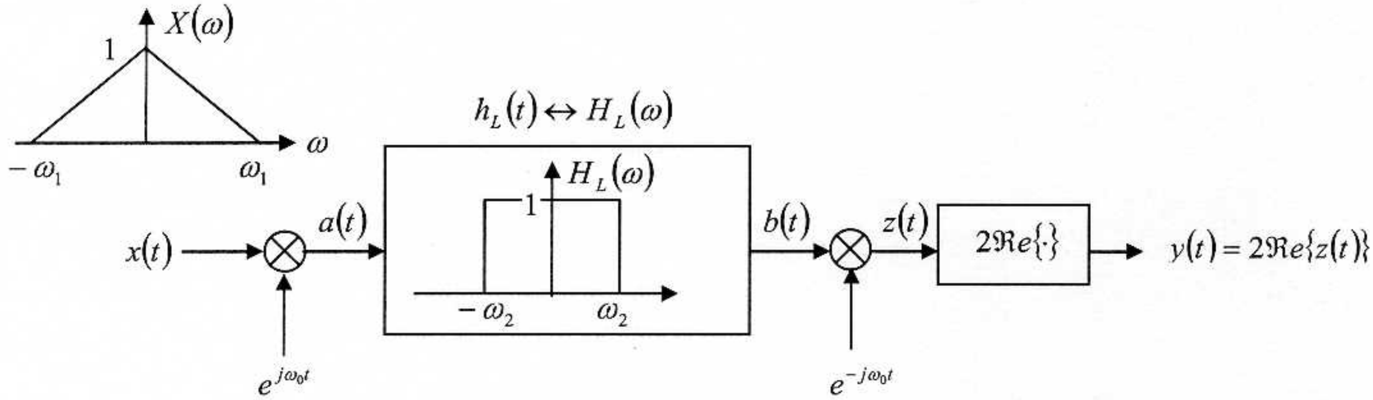
ECSE-2410 Signals & Systems - Fall 2010 Due Fri 10/29/10

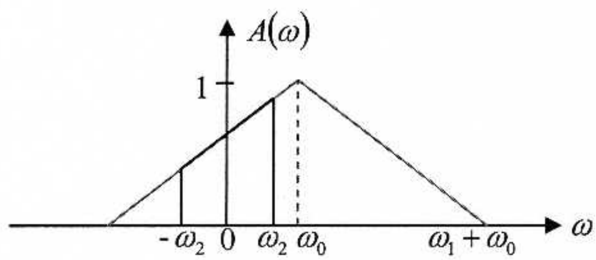
1(50). Show that the system below is basically the bandpass filter:

$$H_{BP}(\omega) = \frac{Y(\omega)}{X(\omega)} = H_L(\omega - \omega_0) + H_L(\omega + \omega_0).$$

Assume that $x(t)$ and $h(t)$ are real. Also, $y(t) = z(t) + z^*(t) \leftrightarrow Y(\omega) = Z(\omega) + Z^*(-\omega)$.



For example,



$$a(t) = x(t)e^{j\omega_0 t} \leftrightarrow A(\omega) = \frac{1}{\cancel{2\pi}} X(\omega) * \cancel{2\pi}\,\delta(\omega-\omega_0) = X(\omega-\omega_0)$$

$$B(\omega) = A(\omega) H_L(\omega)$$

$$z(t) = b(t)e^{-j\omega_0 t} \leftrightarrow Z(\omega) = \frac{1}{\cancel{2\pi}} B(\omega) * \cancel{2\pi}\,\delta(\omega+\omega_0) = B(\omega+\omega_0)$$

$$y(t) = 2\,\Re e\{z(t)\} = \cancel{2}\left(\frac{z(t)+z^*(t)}{\cancel{2}}\right) \leftrightarrow Y(\omega) = Z(\omega) + Z^*(-\omega)$$

Now work backwards

1 cont.

$$\mathcal{Y}(\omega) = Z(\omega) + Z^*(-\omega) = B(\omega+\omega_0) + B^*(-\omega+\omega_0)$$

$$A(\omega+\omega_0)\,H_L(\omega+\omega_0) \qquad A^*(-\omega+\omega_0)\,H_L^*(-\omega+\omega_0)$$

But $h_L(t)$ is real

$$\therefore h_L(t) = h_L^*(t) \Rightarrow H_L(\omega) = H_L^*(-\omega)$$

becomes

$$H_L^*\big(-(\omega-\omega_0)\big) = H_L(\omega-\omega_0)$$

Thus

$$\mathcal{Y}(\omega) = A(\omega+\omega_0)\,H_L(\omega+\omega_0) + A^*(-\omega+\omega_0)\,H_L(\omega-\omega_0)$$

subs. $A(\omega) = \mathcal{X}(\omega-\omega_0)$

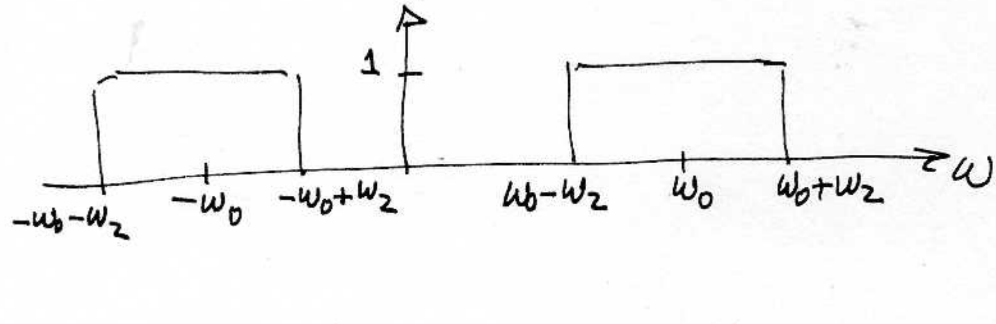
$$\therefore \mathcal{Y}(\omega) = \cancel{A(\omega+\omega_0)}\;\mathcal{X}(\omega-\cancel{\omega_0}+\cancel{\omega_0})\,H_L(\omega+\omega_0) + \mathcal{X}^*(-\omega+\cancel{\omega_0}-\cancel{\omega_0})\,H_L(\omega-\omega_0)$$

But $x(t)$ is real

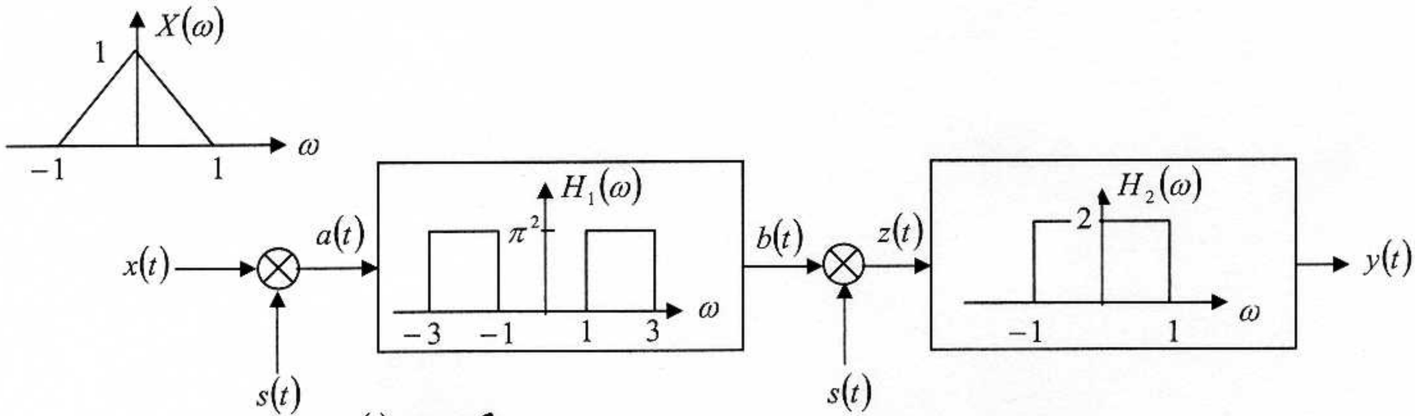
$$\therefore x(t) = x^*(t) \Rightarrow \mathcal{X}(\omega) = \mathcal{X}^*(-\omega)$$

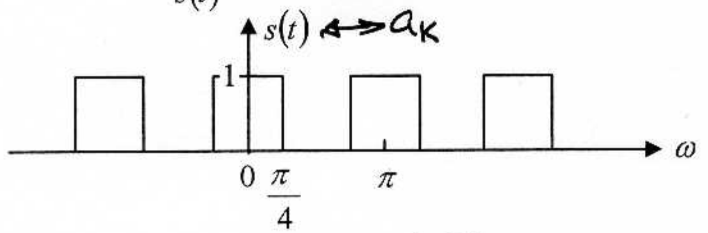
becomes $\mathcal{X}(\omega)$.

$$\therefore \mathcal{Y}(\omega) = \mathcal{X}(\omega)\Big[\underbrace{H_L(\omega+\omega_0) + H_L(\omega-\omega_0)}_{H_{BP}(\omega)}\Big]$$



2(50). The block diagram below represents a chopper stabilized amplifier used to amplify low frequency signals such as those found in transducer outputs. Assume the frequency range of $x(t)$ is given by the Fourier transform $X(\omega)$ shown. Find the overall gain of the amplifier over the frequency range of interest.





$$\omega_0 = \frac{2\pi}{\pi} = 2$$

$$a_k = \frac{2\left(\frac{\pi}{4}\right)}{\pi} \operatorname{sinc}\left(k \cdot 2\frac{\pi}{4}\right) = \frac{1}{2} \operatorname{sinc}\left(k\frac{\pi}{2}\right)$$

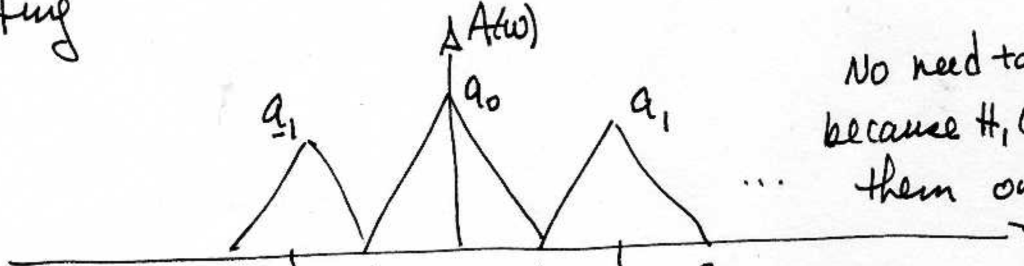
Now $s(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2t} \leftrightarrow S(\omega) = \sum_{k=-\infty}^{\infty} a_k\, 2\pi\, \delta(\omega - 2k)$

Then $a(t) = x(t)\, s(t) \leftrightarrow A(\omega) = \frac{1}{2\pi} X(\omega) * S(\omega)$

$$= \cancel{\frac{1}{2\pi}} X(\omega) * \sum_{k=-\infty}^{\infty} a_k \cancel{2\pi} \delta(\omega - 2k)$$

$$= \sum_{k=-\infty}^{\infty} a_k X(\omega - 2k)$$

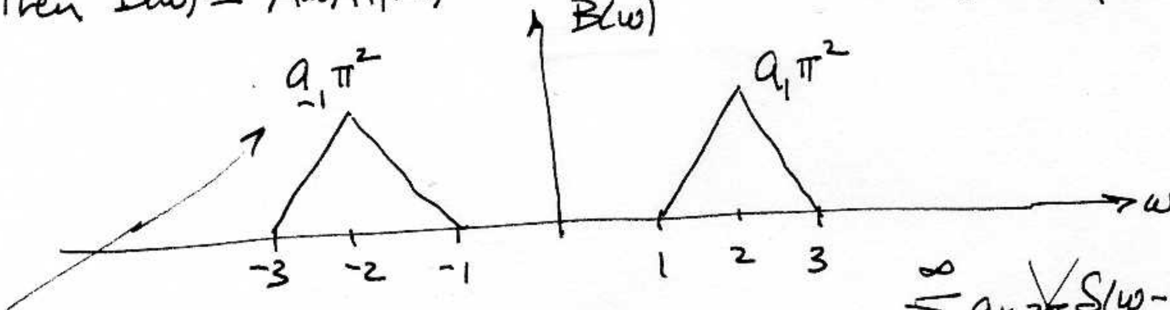
Plotting



No need to plot other terms because $H_1(\omega)$ filters them out, along with $X(\omega)$.

2. Continued.

Then $B(\omega) = A(\omega)H_1(\omega)$ (π^2 is filter gain)

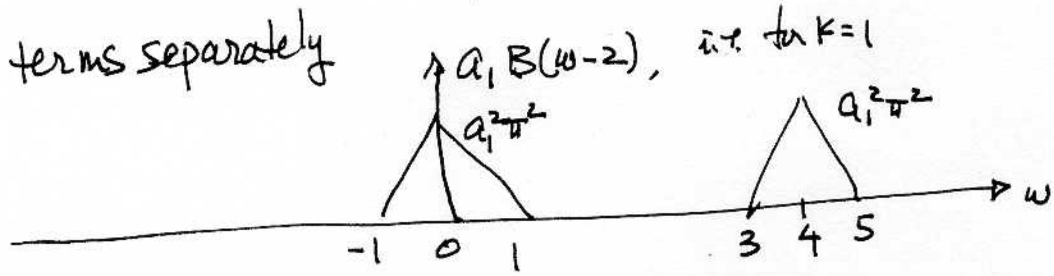


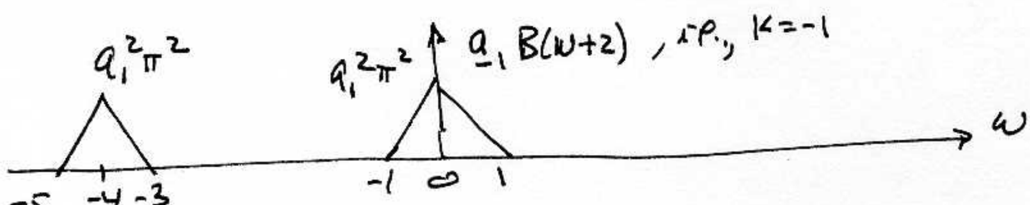
note $x(t)$ real & even $\therefore a_{-1} = a_1$

Now $z(t) = b(t)s(t) \leftrightarrow Z(\omega) = \cancel{\frac{1}{2\pi}} B(\omega) * S(\omega)$, $S(\omega) = \sum_{k=-\infty}^{\infty} a_k \cancel{2\pi}\, \delta(\omega - 2k)$

$$= \sum_{k=-\infty}^{\infty} a_k B(\omega - 2k)$$

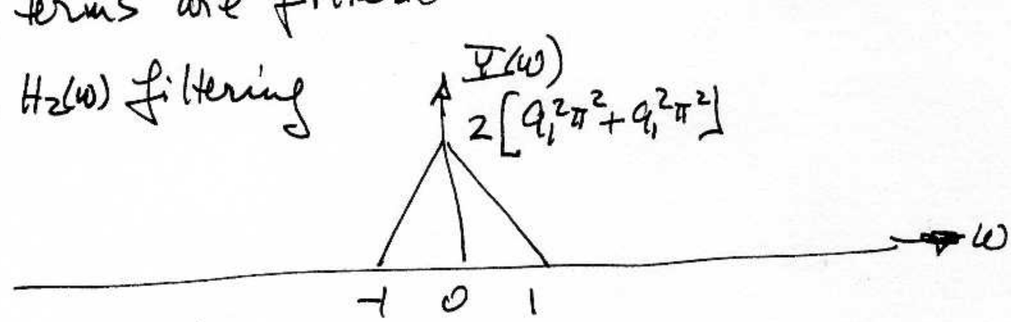
Plot terms separately





other terms are filtered out

After $H_2(\omega)$ filtering



But $a_1 = \frac{1}{2}\,\text{sinc}(\pi/2) = \frac{1}{2}\,\frac{\sin(\pi/2)}{\pi/2} = \frac{1}{\pi}$

$$\therefore \text{Gain} = 2\Big[\underbrace{\cancel{\tfrac{1}{\pi^2}}\cdot\cancel{\pi^2}}_{1} + \underbrace{\cancel{\tfrac{1}{\pi^2}}\cdot\cancel{\pi^2}}_{1}\Big] = \boxed{4}$$