SOLUTION

MANE-4100: Space Flight Mechanics

Exam #2, April 15, 2015

Topics covered:

i) All prior topics +
ii) Kepler's Equations
iii) Orbit Determination
iv) Classical Orbit Elements
v) Coordinate/basis Transformations

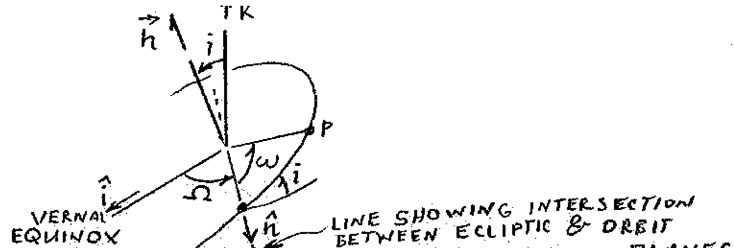
Examination Rules:

- Exam is Closed Book/Closed Note with the exception of a single sheet of 8.5"x11" handwritten crib sheet. This crib sheet must be prepared by you and be turned in with your exam.
- Pay careful attention to notation! You will only be graded on the work you show!
- Work alone, no communication with any other students of any type at any time during the exam period. ALL mobile devices must be turned off during the exam. No cell phones, Blackberry's, etc. are permitted
- Laptop computers may not be used on the exam in any capacity, but conventional pocket calculators are permitted.
- Exam will be turned in to the instructor no later that 4:00 PM 4/15/11
- Put your name and ID number and signature in the spaces provided below only. The remaining pages of this book should be void of any indication of your identity.

Agreement of Student Conduct:
In signing this statement, I agree to follow a examination rules as given above and in accord with the general principles of academic integrity as described in the Rensselaer Handbook.

Name:______________________________. ID #:____________________________

Signature:__________________________________



Problem 1 (120 points)

An object is observed in space and is determined to have the initial position and velocity vectors $\vec{r}_0 = 2.1681\hat{i} - 2.1681\hat{j} - 1.7703\hat{k}$ [AU] and $\vec{v}_0 = -1.7884\hat{i} + 3.5719\hat{j} + 2.1883\hat{k}$ [AU/yr] expressed in terms of despun solar system reference frame (centered at the sun) where $\hat{i}, \hat{j}, \hat{k}$, are the unit basis vectors associated with the standard solar system reference frame (unit vectors $\hat{i}$ and $\hat{j}$ lie in the ecliptic plane with direction $\hat{k}$ being in the direction of the Earth's orbit angular momentum (not spin angular momentum) vector). $\mu_{sun} = 4\pi^2\ [AU^3/yr^2]$

From this information determine:

A) orbit eccentricity e (~~25~~ pts)
B) orbit semimajor axis a [~~km~~ AU] (~~10~~ 6 pts)
C) true anomaly θ [deg] (25 pts)
D) Orbit inclination i [deg]. (10 pts)
E) Right ascension of the ascending node Ω [deg]. (10 pts)
F) Argument of perigee ω [deg] (10 pts)
G) What are the actual anomaly values associated with each of the two locations at which this object crosses the ecliptic plane? (10 Pts) [Remember the definitions of the classical orbit parameters]
H) What is the object's orbit radius r at each of the two locations at which it crosses the ecliptic plane? Should we be worried? Why? (20 Pts)

$r_0 = |\vec{r}_0| = \sqrt{(2.168)^2 + (-2.168)^2 + (-1.770)^2} = \underline{3.541\ AU}$ 2

$v_0 = |\vec{v}_0| = \sqrt{(-1.788)^2 + (3.572)^2 + (2.188)^2} = \underline{4.554\ AU/yr}$ 2

$\vec{h} = \vec{r}_0 \times \vec{v}_0 = 1.579\hat{i} - 1.579\hat{j} + 3.867\hat{k}$ 4

$h = \sqrt{\vec{h}\cdot\vec{h}} = \underline{4.465}$

A) $\vec{e} = \dfrac{\vec{v}_0 \times \vec{h}}{\mu} - \dfrac{\vec{r}_0}{r_0} = -0.175\hat{i} + 0.8750\hat{j} + 0.429\hat{k}$ (10) $\Rightarrow \boxed{e = \sqrt{\vec{e}\cdot\vec{e}} = 0.99}$ 2

$\varepsilon = \text{Energy} = \frac{1}{2}v_0^2 - \frac{\mu}{r_0} = -0.778\ (AU/yr)^2$ (2, 2, 2)

B) $\boxed{a = \dfrac{-\mu}{2\varepsilon} = 25.377\ AU}$ (6, 3)

C) $\hat{p} = \dfrac{\vec{e}}{e} = -0.177\hat{i} + 0.884\hat{j} + 0.433\hat{k}$ (4), $\quad \hat{w} = \dfrac{\vec{h}}{h} = 0.354\hat{i} - 0.354\hat{j} + 0.866\hat{k}$ (4)

$\hat{q} \triangleq \hat{w} \times \hat{p} = -0.919\hat{i} - 0.306\hat{j} + 0.250\hat{k}$ (4)

$\vec{r}_0 = r_1\hat{p} + r_2\hat{q}$ WHERE $r_1 = \vec{r}_0\cdot\hat{p} = -3.066\ AU$ (4), $\quad r_2 = \vec{r}_0\cdot\hat{q} = -1.770\ AU$ (4) $\left\{\begin{array}{l}r_1\ \&\ r_2\ \text{are both negative so object is in Quadrant III}\end{array}\right.$

So $\Rightarrow \boxed{\theta = 180° - \tan^{-1}\left\{\frac{r_2}{r_1}\right\} = \underline{-150°}}$ 5

D) $\boxed{i = \cos^{-1}\left\{\dfrac{\hat{k}\cdot\vec{h}}{h}\right\} = 30°}$ (2, 8), $\quad$ E) $\hat{n} = \dfrac{\hat{k}\times\vec{h}}{|\hat{k}\times\vec{h}|} = 0.707\hat{i} + 0.707\hat{j}$ (QUAD I) (2, 2)

$\Rightarrow \boxed{\Omega = \tan^{-1}\left\{\frac{0.707}{0.707}\right\} = 45°}$ 2 2

F) $\omega = \cos^{-1}\{\hat{n}\cdot\hat{p}\} = 60°$ (or $-60°$) (4) AND $\omega = \sin^{-1}\{(\hat{n}\times\hat{p})\cdot\hat{w}\} = 60°$ (or $120°$) (4, 1) $\Rightarrow \boxed{\omega = 60°}$

G) $\boxed{\underline{\theta_1} = -\omega = -60°, \quad \underline{\theta_2} = \theta_1 + 180° = \underline{120°}}$ (4, 6)

H) $R_1 = \dfrac{h^2}{\mu(1+e\cos\theta_1)} = \underline{0.338\ AU}$ (8)

$R_2 = r(\theta_2) = \dfrac{h^2}{\mu(1+e\cos\theta_2)} = \underline{\underline{1.000}}$ (4) (8)

Big problems!!

OBJECT CAN IMPACT EARTH ON ONE OF ITS ORBIT CYCLES 2

Problem 2 (80 points)

Assume that for the problem 1) the following orbital parameters are given: $e=0.95$, $\theta_0=-135°$, $i=10°$, $\Omega=30°$, $\omega=60°$, $a = 24$ [AU]:

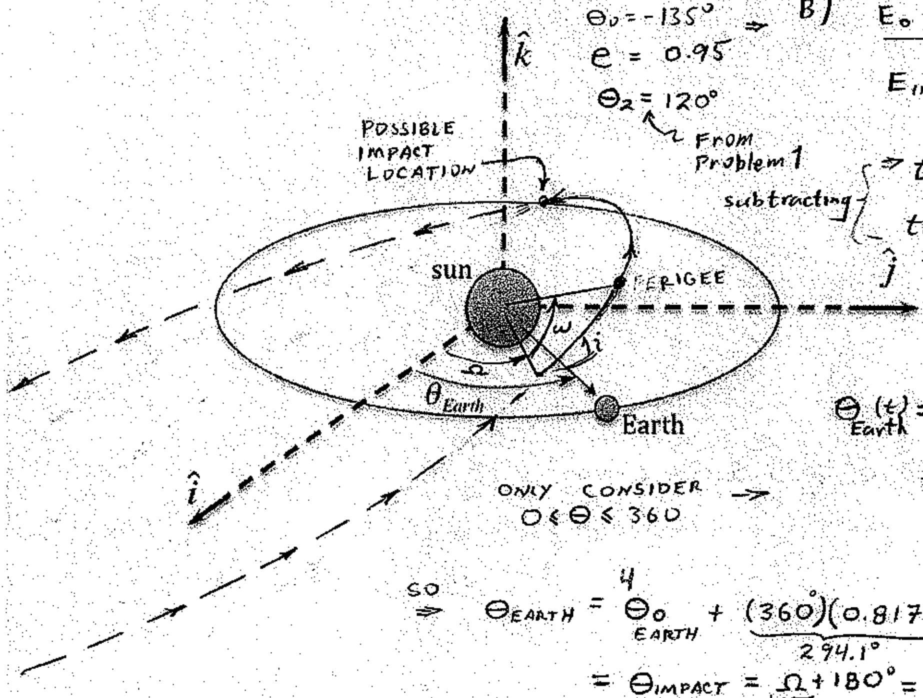
A) What is the period [in years] of this object's orbit. (5 pts)

B) How long will it take [in years] for the object to get from its initial θ_0 position to the location where it will cross the ecliptic the second time [beginning at θ_0]) {The object should come uncomfortably close to the Earth's orbit at this second ecliptic crossing}(60 pts)

C) Assuming the Earth is in an effectively circular orbit, and that its position in its orbit is described by its actual anomaly θ_{Earth} which is measured from the $\hat{i}$ axis as indicated in the figure below. At the time this object is observed, what does the actual anomaly of the Earth need to be [in degrees] such that earth and this object would/might collide during the current orbit of the object? (15 pts)

A) $\boxed{\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{24^3}{\mu}} = 117.58\ yr}$

$\Theta_0 = -135°$
$e = 0.95$
$\Theta_2 = 120°$ From Problem 1



B) $E_0 = 2\tan^{-1}\left\{\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\Theta_0}{2}\right)\right\} = -0.738\ rad$

$E_{impact} = 2\tan^{-1}\left\{\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\Theta_2}{2}\right)\right\} = 0.5411\ rad$

subtracting

$t_{impact} - T_0 = \{E_{impact} - e\sin E_{impact}\}\sqrt{\frac{a^3}{\mu}} = 0.969\ yr$

$t_0 - T_0 = \{E_0 - e\sin E_0\}\sqrt{\frac{a^3}{\mu}} = -1.848\ yr$

$\Delta t = t_{impact} - t_0 = 0.969 + 1.848 = 2.817\ yr$ B)

WHAT WE WANT

$\Theta_{Earth}(t) = \Theta_{0_{EARTH}} + \left(\frac{360°}{yr}\right)\Delta t = \Theta_{0_{EARTH}} + \left(360°/yr\right)(2 + 0.817)yr$

$= \Theta_{0_{EARTH}} + \underbrace{2(360°)}_{\text{BRINGS EARTH BACK TO SAME LOCATION IN ITS ORBIT}} + (360)(0.817)$

ONLY CONSIDER $0 \leq \Theta \leq 360$

SO $\Rightarrow \Theta_{EARTH} = \Theta_{0_{EARTH}} + \underbrace{(360°)(0.817)}_{294.1°}$

$= \Theta_{IMPACT} = \underbrace{\Omega}_{30°} + 180° = 210°$

$\Rightarrow \boxed{\Theta_{0_{EARTH}} = 210° - 294.1° = -84.1°}$