February 1, 2011

MANE 6550-01 Theory of Compressible Flow
Fall Semester 2011
Problem Set #1

Due: February 15, 2011

1) a) Prove that a gas behaving according to the perfect gas equation of state does not have a thermodynamic critical point.
b) Derive the formulas for the parameters *a* and *b* in the Van der Waals equation of state as a function of the critical pressure and temperature of the gas, and the specific gas constant. Determine the value of the compressibility factor of the Van Der Waals gas at the critical point.
c) The Redlich-Kwong equation of state is given by

$$p = RT / (v-b) - a\, T^{-1/2} / [v(v+b)]$$

Determine the values of *a* and *b* as a function of the critical pressure and temperature of the gas, and the specific gas constant. Determine the value of the compressibility factor of the Redlich-Kwong gas at the critical point.

2) Given the flow velocity field: $u(x,y,z,t) = 2x\,(1+t^2)$, $v(x,y,z,t) = 3y$, and $w(x,y,z,t)=0$.
a) Find the particle's path line and velocity vector as function of time t only for a particle which passed through the point $x=1, y=1, z=1$ at $t=0$. Construct a relationship $x_p = f_1(y_p)$ for the particle's path line.
b) Find the equation $x=f_2(y)$ which describes the streamline that passes through the point $x=1, y=1, z=1$ at $t=5$.
c) Find the equation $x=f_3(y)$ for the streak line at time $t=5$ constructed by all particles that have passed through the point $x=1, y=1, z=1$.
d) Plot all the above lines in a diagram *x vs. y* and compare between the lines.

3) Show that the double-dot product of the deformation tensor and the spin tenso: $\underline{D} : \underline{\Omega} = \Sigma_i \Sigma_k D_{ik}\,\Omega_{ik} = 0$ (here *i* and $k = x,y,z$).

4) Derive the integral and differential equations for the balance of angular momentum $\boldsymbol{f} = \boldsymbol{r} \times \boldsymbol{V}$ of a compressible and viscous fluid flowing in a field with pure external body moments $\int_{Vol} \rho \boldsymbol{M_B}\,(dVol)$. Here $\boldsymbol{r}$ is the distance vector from the origin and $\boldsymbol{V}$ is the flow velocity vector). Include in this analysis the effects of the pressure, viscous stresses, and body forces. Show that when $\boldsymbol{M}_B = 0$, the angular momentum balance can be derived from the linear momentum balance equation.