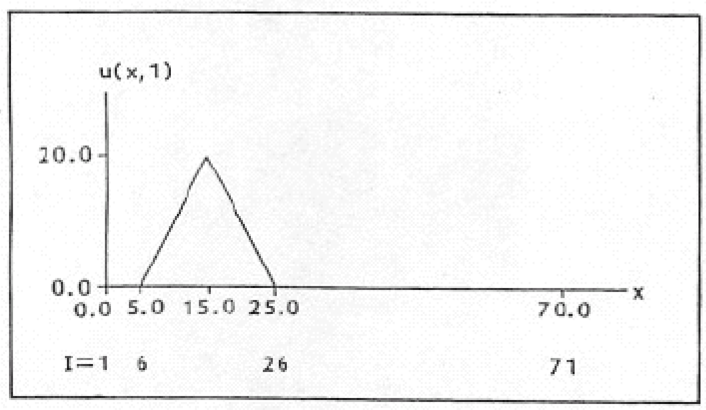
March 29, 2011

MANE 6550-01 Theory of Compressible Flow
Spring Semester 2011
HW #4
Due: April 12, 2011

Problem 1.

A 1-D acoustic wave u(x,t) is propagating, starting from t=0. The wave has an initial triangular shape (see Figure) at t = 0. Compute the wave propagation up to t = 0.2 sec by solving the first-order linear acoustic wave equation, $\partial u/\partial t + a\, \partial u/\partial x = 0$. Assume the speed of sound to be a = 250 m/sec. Use the domain 0 < x < 70.

Solve the problem by the following numerical integration methods:



(a) Lax method
(b) Lax-Wendroff method
(c) Euler's BTCS implicit method

Use three sets of step sizes that are specified as follows:

(I)	$\Delta x = 1.0$	$\Delta t = 0.004$
(II)	$\Delta x = 1.0$	$\Delta t = 0.08$
(II)	$\Delta x = 1.0$	$\Delta t = 0.002$
(III)	$\Delta x = 1.0$	$\Delta t = 0.001$

Print the solution at intervals of 0.025 sec up to t = 0.2 sec.
Discuss the results and the differences between the various cases and the three numerical methods used.

Problem 2.

A 1-D acoustic wave $u(x,t)$ is propagating, starting from $t = 0$. The wave has an initial shape at $t = 0$ that is a discontinuous jump at $x=1$, i.e. $u(x,0) = 1$ m/s for $0 < x < 2$ m, and $u(x,0) = 0$ for $2 \text{ m} < x < 4$ m. Use the domain $0 < x < 4$. Also, use $u(0,t)=1$ m/s for all t. Compute the wave propagation up to $t = 2$ sec by solving the non-linear canonic Burgers wave equation, $\partial u/\partial t + u\, \partial u/\partial x = 0$.

Solve the problem by the following numerical integration methods:

1. Lax method.

2. MacCormack predictor-corrector method.

Use three sets of step sizes that are specified as follows:

(I)	$\Delta t = 0.1$	$\Delta x = 0.1$
(II)	$\Delta t = 0.05$	$\Delta x = 0.1$
(III)	$\Delta t = 0.15$	$\Delta x = 0.1$

Print the solution at intervals of 0.4 sec up to $t = 2$ sec.
Discuss the results and the differences between the various cases and the two numerical methods.