# Probability and Statistics

Assignment -6 [1 - Nov - 2018] You may use $R$-software to determine critical region and power etc.

1. Let $X_1, X_2, \ldots$ be independent random variables with Bernouli($p$) distribution. It is desired to test the hypothesis $H_0 : p = 0.6$ against the alternative $H_1 : p = 0.3$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on $X_1, X_2, \ldots X_n$. For $n = 49$, find the power of the test.

2. Let $X_1, X_2, \ldots$ be independent random variables with normal distribution, mean 0 and variance $\theta$, $0 < \theta < \infty$. It is desired to test the hypothesis $H_0 : \theta = 4$ against the alternative $H_1 : \theta = 1$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on $X_1, X_2, \ldots X_n$. For $n = 36$, find the power of the test.

3. Let $X_1, X_2, \ldots$ be independent random variables with common density, for $\theta > 0$
$$f(x, \theta) = \frac{1}{\theta} \exp\{-\frac{x}{\theta}\}, \quad 0 < x < \infty.$$
It is desired to test the hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 4$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on $X_1, X_2, \ldots X_n$. For $n = 25$, find the power of the test.

4. Let $X_1, X_2, \ldots$ be independent random variables with Poisson distribution, mean $\theta$, $\theta > 0$. It is desired to test the hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 4$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on $X_1, X_2, \ldots X_n$. For $n = 25$, find the power of the test.

5. Let $X_1, X_2, \ldots$ be independent random variables with normal distribution, mean $\mu$ and variance $\theta$, $-\infty < \mu < \infty$, $0 < \theta < \infty$. It is desired to test the hypothesis $H_0 : \mu = 1$ against the alternative $H_1 : \mu = 4$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on $X_1, X_2, \ldots X_n$.

6. Let $X_1, X_2, \ldots$ be independent random variables with normal distribution, mean $\mu$ and variance $8$, $-\infty < \mu < \infty$. It is desired to test the hypothesis $H_0 : \mu = 1$ against the alternative $H_1 : \mu > 1$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on $X_1, X_2, \ldots X_n$. For $n = 32$, determine the power function : for $a > 1$, power($a$) is the power of the test when $\mu = a$.

7. Let $X_1, X_2, \ldots$ be independent random variables with Bernouli($p$) distribution. It is desired to test the hypothesis $H_0 : p = 0.5$ against the alternative $H_1 : p < 0.5$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on $X_1, X_2, \ldots X_n$. For $n = 49$, find the power function of the test : for $0 < a < 0.5$, power($a$) is the power of the test when $p = a$.