

## STATEMENT OF PURPOSE

VED DATAR, DATE OF BIRTH NOVEMBER 13, 1987, APPLYING TO THE DEPARTMENT OF MATHEMATICS

I am applying for a PhD in Mathematics at Princeton. My current interests are Analysis, Analytical Number Theory and Topology. I desire to make significant contributions to my field.

I was attracted to Mathematics at an early age. In high school I was introduced to the challenging problems from the Olympiads. In 2005, I was amongst the six students from the 12th grade to be shortlisted for the International Mathematics Olympiad Training Camp held at Mumbai. At the camp I encountered a plethora of beautiful problems and results. Some expository lectures in advanced topics in Mathematics in the camp encouraged me to pursue the subject. Chennai Mathematical Institute (CMI), with its excellent faculty and flexibility, was an obvious choice for a bachelors degree in mathematics.

I have benefited from the well rounded curriculum at CMI including courses in Algebra (covering most of Artin's *Algebra* except representation theory), Real Analysis (covering most of Rudin's *Principles of Mathematical Analysis*), Complex Analysis, Topology (covering the first six chapters of Munkres' book along with the one on Fundamental Groups) and some graduate courses.

On coming to CMI, I was drawn to analysis. In my second semester I read a few chapters from Tom Apostol's book, *Mathematical analysis*. While reading about Riemann integral and its various shortcomings, I had my first brush with Measure Theory. Interested in learning more, I credited a course offered by Prof. S.Kesavan at the Institute of Mathematical Sciences (IMSc). Topics like signed measures, Radon-Nikodym theorem and  $L^p$  spaces made for an interesting course. I enjoyed reading through the books by Bauer and Roydon. Solving problems from these books was educational too. I was particularly fascinated by the parallels between some of the concepts in Topology and Measure Theory (for example the similarity in the definitions of continuous functions and measurable functions). As a natural follow up to this course I also credited a Functional Analysis course in which we covered some basic topics like the Banach spaces, Hilbert spaces, locally convex spaces and Spectral Theory of bounded operators on a Hilbert space. A course on Vector Calculus has implanted in me a desire to learn about integration theories on surfaces.

In my fourth semester I credited a graduate level course in Complex Analysis offered by Prof. R.Shridharan. The topics covered included Dirichlet's theorem for a disc and some geometrical topics like calculating the automorphism groups of the plane, the punctured plane and the unit disc. The course ended with a proof of the Riemann Mapping Theorem. I found the result quite startling and the proof quite mystifying. This June, I attended a talk by Prof. M.S.Ragunathan on Harmonic Analysis at the Tata Institute of Fundamental Research (TIFR), Mumbai. Having read about classical Fourier Analysis from Apostol's book, this talk was a revelation. I have also heard of connections of Harmonic Analysis to fields like Number Theory and Differential Equations. Next semester, I plan to do a reading course in the subject to equip myself with some of its basic techniques.

Last summer, I was selected and participated in the Visiting Students Research Program (VSRP) held at TIFR, Mumbai. As a part of the program I studied Analytic Number Theory under Prof. A. Sankaranarayanan. The first part of the project was spent in understanding the analytic proof of the prime number theorem. In due course the connection of the zeroes of the zeta function to the estimation of the error term, and hence the importance of the Riemann hypothesis, became clear. The universality of the method of attack appealed to me and I tried applying the method to the Dirichlet divisor problem by considering the function  $\zeta^2$ . Prof Sankaranarayanan also talked about Lindelof's hypothesis, its application to similar problems and its connection with the Riemann hypothesis. All in all it was a great learning experience.

The second part of the program was spent trying to understand the proof of the Goldbach Schnirelmann theorem

*"There exists a constant  $c$  such that every natural numbers is the sum of at most  $c$  primes"*

En route to the main result I studied the basic Selberg sieve. I was quite enamored by the elementary yet ingenious method of minimization used. I am looking forward to reading about other sieving techniques from Richert's book, *Sieve Methods* (TIFR lecture notes), in the coming semester. As a sequel to the month at TIFR I am currently working through the problems of Ram Murthy's book *Problems in analytic number theory*. I plan to complete the book soon. Alongside this, I am reading Ingham's book on distribution of primes.

I am also interested in Topology. I credited a reading course in Differential Topology under Prof. V. Balaji this semester. The material covered included some basic theorems on Manifolds (like Sard's and Whitney's), Vector Bundles and some basic theorems on Lie groups. A few prototype manifolds like the projective space and the Grassmannians were discussed. As a part of the course, I gave talks on the proof of Sard's theorem and *Computation of global sections of some holomorphic line bundles over the complex projective space*. An effort is made in the course to get an understanding first on the topological level and then on the differentiable level. For example a discussion of Vector Bundles was preceded by a discussion of Covering Spaces. I am currently reading Frank Warner's book, *Foundations of Differentiable Manifolds and Lie Groups*. The course is to be continued next semester and the plan is to complete this book. Additional issues like Transversality and Tubular Neighborhoods will also be addressed. Being amongst the three students selected from CMI to visit the École Normale Supérieure in the coming summer, I plan to use the opportunity to study further into Differential Topology.

In December 2006, I gave a series of three talks on problem solving in elementary Number Theory at the Indian National Mathematics Olympiad Training Camp, Mumbai. I have given a few lectures on Analysis and Number Theory as a part of the student presentations initiative at CMI. Based on these experiences, I have gained confidence on my abilities as a lecturer

My education at CMI has taught me to appreciate the interplay between various areas of Mathematics and I hope to explore possible connections between different branches of the subject in graduate school. With my ambition and passion for research and teaching and my areas of interest, Princeton, with its esteemed faculty in diverse areas of Mathematics, is an ideal choice for graduate studies. I am sure, if given the opportunity to work with the excellent faculty of Princeton, I will contribute significantly to Mathematics.