Probability and Statistics

Assignment -4 [20th Sep 2018]

1. Let X_1, X_2 be independent random variables with geometric distribution with parameters p_1, p_2, $0 < p_1 < 1$, $0 < p_2 < 1$.

$$P(X_i = k) = (1 - p_i)^{k-1} p_i, \quad k \geq 0, \ i = 1, 2.$$

   Let $W = min(X_1, X_2)$. Show that W also has geometric distribution and obtain expression for $P(W = k)$ in terms of p_1, p_2.

2. Let X_1, X_2 be independent random variables with means μ_1, μ_2 and variances σ_1^2, σ_2^2. Let $Y_1 = (X_1 + X_2)$ and $Y_2 = (X_1 - X_2)$. Find means and variances of Y_1, Y_2 and correlation between Y_1, Y_2.

3. Let X have a exponential distribution with density $f(x) = \exp\{-x\}$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$. Let $U = \exp\{-X\}$. Find the distribution of U.

4. Let Y have a normal distribution with mean 0 and variance 1. Suppose $W = Y^2$ and $\xi = 1$ if $Y \geq 0$ and $\xi = -1$ if $Y < 0$. Show that W and ξ are independent random variables.

5. For $n \geq 1$, let Z_n be random variable with Poisson distribution with parameter n. Let $W_n = \frac{(Z_n - n)}{\sqrt{n}}$. Using central limit theorem, show that

$$P(W_n \leq a) \to P(X \leq a)$$

   for all a, where X has normal distribution with mean 0 and variance 1.

6. Let X_1, X_2, X_3 be iid with normal distribution, mean 0 and variance 1. For real numbers a_1, a_2, a_3, b_1, b_2, b_3 let

$$Y = a_1 X_1 + a_2 X_2 + a_3 X_3, \text{ and } Z = b_1 X_1 + b_2 X_2 + b_3 X_3.$$

   Find the characteristic function of the joint distribution of (Y, Z).

7. Let $X_1, X_2, \ldots, X_n, \ldots$ be iid random variables with $P(X_1 = 1) = p$ and $P(X_1 = 0) = 1 - p$. Let Y be a random variable with Poisson distribution with parameter $\lambda > 0$, Y being independent of $X_1, X_2, \ldots, X_n, \ldots$.
   Let

$$W = X_1 + X_2 + \ldots + X_Y$$

   (sum of random number of terms). Show that distribution of W is also Poisson. Find it's mean.

8. (Correction to Assignment 1) Suppose (X, Y) has bivariate normal distribution with each X and Y having mean 0 and variance 1. Suppose the correlation between X, Y is α. Let
   $W = \frac{1}{\sqrt{(1-\alpha^2)}}(Y - \alpha X)$.
   Show that X and W are independent random variables with Normal distribution, with mean 0 and variance 1.