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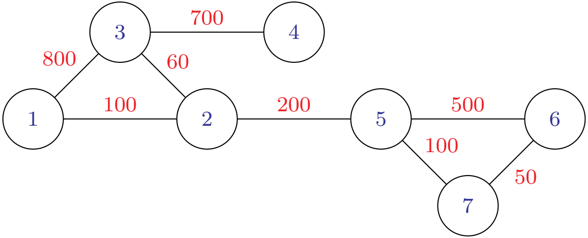
- The rules permit CmI members to fly only by the state airline which uses a rather obscure pricing policy.
- It is not always the case that the cost of a ticket is proportional to the distance.
- The travel desk has decided that they would like to find the cheapest way to reach each destination from CmI.



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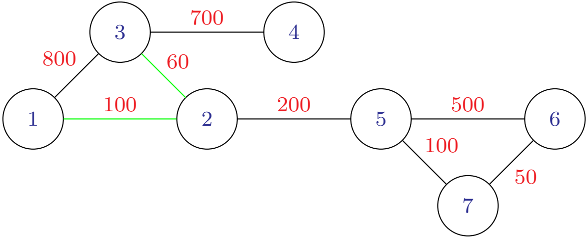

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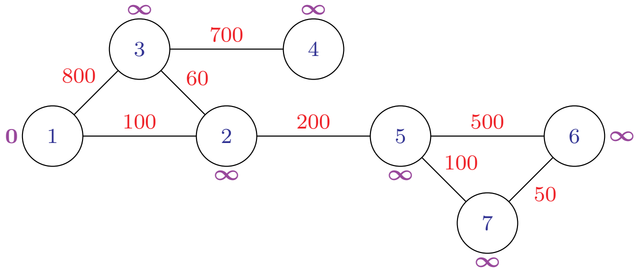
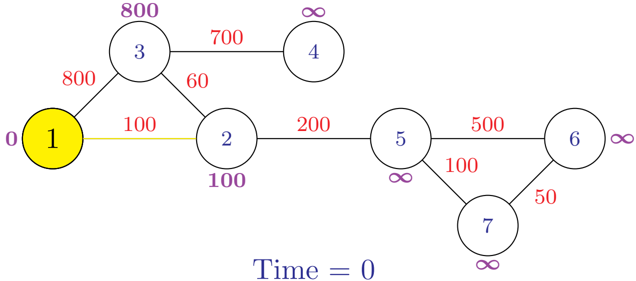
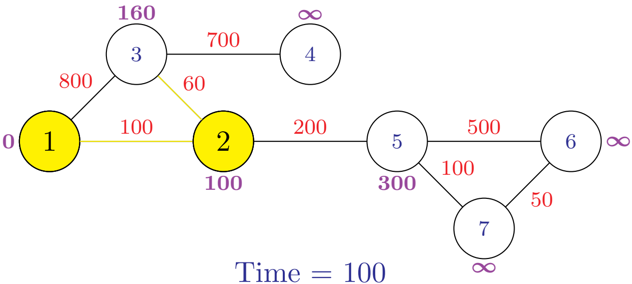
- The length of the shortest path to a vertex $\boldsymbol{v}$ is the time at which $\boldsymbol{v}$ starts burning.



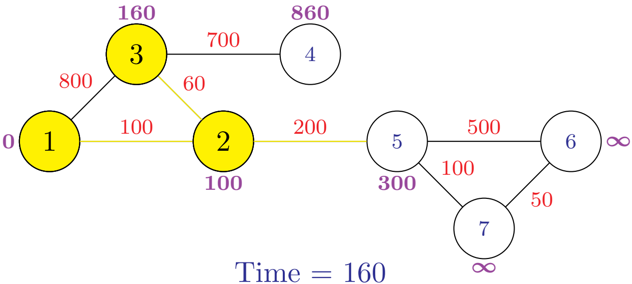
1
2
3
4
5
6
7
0
∞
∞
∞
∞
∞
∞
800
60
700
100
200
500
100
50



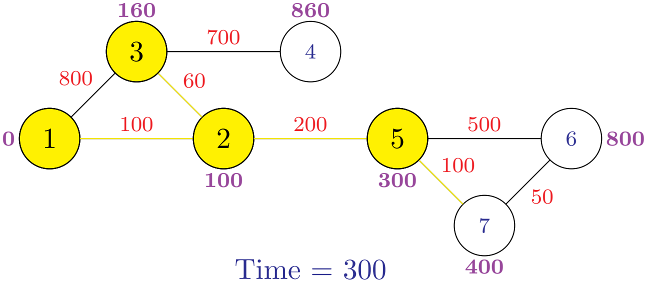
Time = 0



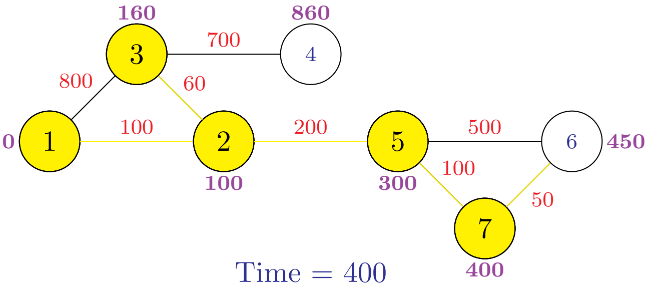
Time = 100



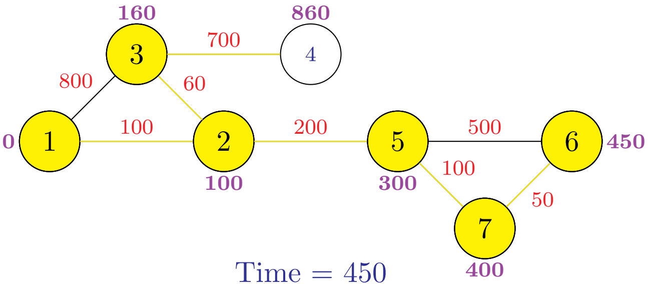
Time = 160



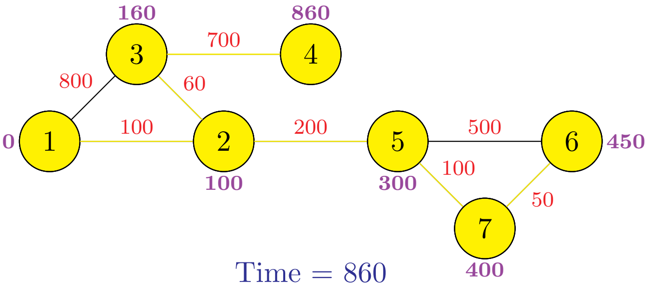
Time = 300



Time = 400



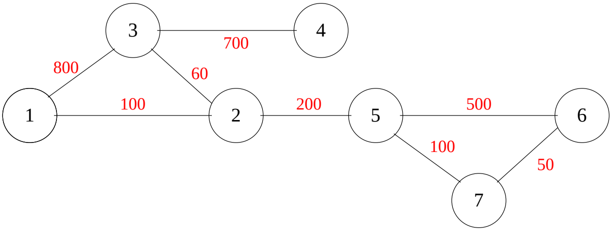
Time = 450



Time = 860

How do we translate this into an algorithm?

- For each vertex maintain information on when it is expected to burn.
- This value depends on when its neighbours start burning and the length of the edges connecting it to its neighbours.
- Initially, vertex 1 is the only one expected to burn.



The Details (Dijkstra's Algorithm)

Use two arrays $\boldsymbol{Burnt}[]$ and $\boldsymbol{ExpBurnTime}[]$.

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Repeat the following steps till all the vertices are burnt:

1. The unburnt vertex with minimum expected burning time will burn next.
2. The burning time of this vertex is used to update the expected burning time of its neighbours.

How long does it take?

- In each round we examine all the vertices twice.

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About $\boldsymbol{N^2}$ Steps

Will the Adjacency List representation help?

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About $\boldsymbol{N^2}$ Steps

Will the Adjacency List representation help? NO.

You have to at least look at the entire graph once and there could up $\boldsymbol{N^2}$ edges.

In real life many graphs have very few edges. Often the number of edges is not more than the number of vertices.

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Can we improve the algorithm for sparse graphs?

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Can we improve the algorithm for sparse graphs?

Yes, we can use Heaps to implement Dijkstra's algorithm in O(nloge).

Keep the vertices and their expected burn time in a Heap.

In each step

Dijkstra's Algorithm with Heaps

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In each step

- Pull out the vertex with minimum expected burn time. Mark it as burnt.
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Note, each edge may modify the heap only once and hence the complexity.