Personal Statement

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When I was in eighth grade in my high school in Pune (India), I studied Euclidean geometry starting from the postulates. I began to appreciate the strictly logical and axiomatic yet beautiful nature of mathematics. I continued to learn more advanced mathematics in college and explored areas that I was interested in through summer research programs and other undergraduate research opportunities. Having studied some exciting modern mathematics and seen a glimpse of current mathematical research, I am extremely keen to learn and contribute to mathematics in my future career. My interests include algebraic geometry, algebraic topology and analysis.

As I realized in high school that I enjoyed mathematics, I sought opportunities to learn mathematics outside of the regular school curriculum. In ninth grade, I came across the mathematical olympiad program in India, and started preparing for the regional level exams organized to select the Indian team for the International Mathematical Olympiad (IMO). With a group of friends, I started attending lectures and problem solving sessions at Bhaskaracharya Pratishthan, a mathematical research institute in Pune. At Bhaskaracharya Pratishthan, not only did I get to learn about topics in number theory, algebra, combinatorics and geometry, but I got a chance to struggle with challenging mathematical problems. I remember how muchI enjoyed working on tough IMO problems and then explaining my solutions to my friends. I was selected in the Indian team of 6 students for IMO 2003 and IMO 2004 and I won a bronze and a silver medal.

I did not confine my study of mathematics to the preparation of the olympiad. I started reading books like Walter Rudin's *Principles of Mathematical Analysis* and Michael Artin's *Algebra* on my own. In addition, I started discussing what I had read with scholars at Bhaskaracharya Pratishthan. I remember explaining to one of the professors my characterization of subsets of $\mathbb{R}$ that are the set of points at which some function $f : \mathbb{R} \to \mathbb{R}$ is continuous. I had immensely enjoyed the few restless days I had spent trying to arrive at a satisfactory answer.

In addition to learning interesting mathematics during these years, I realized that mathematics was an area of active research. I started considering working in the field of pure mathematics as a serious career choice, which was unusual for high school students in India. Having studied at a school for gifted students, won national scholarships like the National Talent Search scholarship and scored exceptionally at the 10th grade national board examinations, I was expected to appear for the entrance examination to the Indian Institute of Technology (IIT). However, in spite of the overwhelming trend to join the IIT or other engineering colleges, I started looking for options to study mathematics. After talking to several mathematics professors and students who had chosen to pursue mathematics, my father, who is a professor himself, and my mother, who has a degree in statistics, supported me in my decision to apply to universities with a strong mathematics program, including several US universities.

As an undergraduate at MIT, I took courses in a variety of areas of mathematics. Through graduate courses in fields like commutative algebra, algebraic geometry, algebraic topology, analysis and differential geometry, I gained knowledge in a broad range of areas in contemporary mathematics. Moreover, I developed my skills in mathematical exposition through seminars. Additionally, I studied areas of computer science including theory of computation, algorithms, artificial intelligence, and software engineering, finishing roughly the computer science section of MIT's undergraduate program in Electrical Engineering and Computer Science.

As I was exposed to contemporary mathematics in the courses I took at MIT, I became fascinated by the interplay between algebra and geometry/topology that pervades much of modern mathematics. I saw the advantage of mapping topological structures to algebraic structures through functors like homology, cohomology and homotopy. I realized the utility of converting seemingly intractable topological situations to more computationally accessible algebraic problems.

In MIT's Summer Program in Undergraduate Research (SPUR), I worked on a problem that exemplified the technique of using algebra to solve topological problems. During this six week long camp, I worked with Xiaoguang Ma, a graduate student of Prof. Pavel Etingof. Given a compact Lie group G, we sought to find the characteristic classes of principal G bundles over smooth manifolds using Chern-Weil theory. Denote by $\mathfrak{g}$ the Lie algebra of G, and let $P \to M$ a principal G bundle. Let Ω a curvature form on P arising from a connection. Denote by $S(\mathfrak{g})$ the polynomial ring over $\mathfrak{g}$ and by $S(\mathfrak{g})^G$ the ring of polynomials invariant under the adjoint action of G on $\mathfrak{g}$. We began by studying the Chern-Weil homomorphism, which naturally maps $S(\mathfrak{g})^G$ to the cohomology ring $H^*(M)$ using the curvature form Ω. It turns out that the homomorphism is independent of Ω. Denoting this homomorphism by W_P, we see that an element $q \in S(\mathfrak{g})^G$ gives a characteristic class c_q sending $P \mapsto W_P(q)$. Surprisingly, all the characteristic classes of principal G bundles arise in this way. Furthermore, if $T \subset G$ is a maximal torus with Lie algebra $\mathfrak{t}$, then the ring $S(\mathfrak{g})^G$ is isomorphic to the ring $S(\mathfrak{t})^{W(G)}$ of the polynomials on $\mathfrak{t}$ invariant under the action of the Weyl group $W(G)$ of G. Thus, the seemingly daunting problem of finding all characteristic classes of principal G-bundles is reduced to the algebraic problem of computing the ring $S(\mathfrak{t})^{W(G)}$, which is the ring of polynomials invariant under the action of a finite reflection group! We computed this ring for the exceptional Lie groups G_2 and F_4, and looked at some of the implications of the results to reduction problems involving G_2. Not only did I enjoy working on this problem, I also won an award for the best paper in the SPUR program.

As I took courses in algebraic geometry, I came across the other side of the interplay between algebra and geometry. I learned about the beautiful geometry arising from the algebra of rings and modules. I was fascinated to see how algebraic operations correspond to geometric transformations. For example, it was interesting to learn how taking the integral closure (normalization) helps in the geometric problem of resolving singularities and how algebraic constructions like primary decomposition correspond to dividing varieties into irreducible components.

I had the opportunity to work with Prof. Steven Kleiman on a problem in algebraic geometry of current interest. The problem was about the principle of specialization of integral dependence, which originated from the goal of describing sets of singularities in analytic families. This problem can be phrased in algebraic terms as a question about integral dependence. Consider a family of complex analytical spaces $X \to Y$ such that for each $y \in Y$, the fiber $X(y)$ is equidimensional. Let $E = O_X^p$ be a free module on X and $M \subset E$ a submodule. The goal is to associate numbers $e(y)$ to the pair $(E(y), M(y))$, whose constancy along Y would ensure the following: if a section h of E is such that its image in $E(y)$ is integrally dependent on $M(y)$ for all y in a Zariski open subset of Y, then h is integrally dependent on M. Observe that if $\mathrm{supp}(E/M) \to Y$ is finite then $M(y) \subset E(y)$ is of finite colength. In this case, Kleiman and Gaffney proved the principle using the Bucshbaum-Rim multiplicity. For the general case, we needed to find a suitable multiplicity for the pair (N, F) of a free module F and a submodule $N \subset F$. The proof of Kleiman and Gaffney indicated that such a multiplicity $\mu(N, F)$ would satisfy certain desirable conditions:

1. $y \mapsto \mu(M(y), E(y))$ is upper semicontinuous on Y;
2. if $N_1 \subset N_2 \subset F$ then $\mu(N_1, F) = \mu(N_2, F)$ if and only if N_2 is integrally dependent over N_1.

A possible candidate for such a multiplicity is the j^* multiplicity of Ulrich and Validashti. Although j^* satisfies (2), it doesn't give an upper semicontinuous map as in (1). To have something that satisfied (1) and (2), we needed a more general version of j^*, which I aimed to find. Constructing something that was general enough for our purposes, but still satisfied (2) was tricky, but enjoyable. Although I didn't succeed in proving (1) for the multiplicity I constructed, and could prove only a part of (2), I learned a lot about mathematical research while working with Prof. Kleiman.

Having experienced and enjoyed the challenge in doing mathematical research, I look forward to doing more of it in my future career. I hope to gain new insights in mathematics in graduate school through interactions with the faculty. I look forward to sharing my knowledge with my fellow students and learning from them. Finally, in the long run, I hope to make a valuable contribution to the mathematical area I work in.