STATEMENT OF PURPOSE

VIPUL NAIK, DATE OF BIRTH APRIL 23, 1986, APPLYING TO DEPARTMENT OF MATHEMATICS

I am applying for Ph.D. in mathematics. I wish to pursue research in algebraic geometry and group representation theory. My other interests include group theory, Galois theory, differential geometry, and automorphic function theory. I also seek to share my mathematical knowledge and insights through teaching.

Since childhood, I enjoyed doing reading mathematical topics outside the school syllabus, including number theory and combinatorics. Later, I became interested in Olympiad mathematics and won silver medals for India at the **International Mathematical Olympiad** (2003 and 2004), being India's top scorer both times. Determined to pursue mathematics, I enrolled for B.Sc. (Hons) Mathematics at **Chennai Mathematical Institute** (CMI) at IMO faculty's recommendation.

Through five semesters of coursework, seminars, summer camps, and personal sessions with professors, I built a strong foundation in group theory, representation theory, commutative algebra, analysis, number theory, differential geometry, and other areas. I extended this by attending advanced graduate-level courses, including *Global Calculus*, *Game Theory*, *Abelian varieties*, *Elliptic curves and modular forms*, and *Computational Complexity*.

Beautiful Olympiad-level number theory problems attracted me to number theory. Thus, I opted for a number theory lecture series and later an *Elliptic Curves and Modular Forms* course under Professor Balasubramanian. Here, I perceived links between number theory, complex analysis and group cohomology and this led me to credit Ramanan's *Abelian varieties* course, where I grasped the deeper meaning of cohomology and automorphic functions. I am exploring the Riemann-Roch theorem in private classes under Ramanan.

In the first semester, I studied Nielsen-Schreier theorem and began reading the Classification of Finite Simple Groups. I got a flavour of the machinery needed to compute representations from a summer camp on *Groups, Representations and Algebras* at **Institute of Mathematical Sciences**. In a representation theory elective, I learnt Brauer's and Artin's induction theorems. Attending representation theory seminars motivated me to explore Lie group, Lie algebra and algebraic group representations.

I grasped the scheme-theoretic approach to algebraic geometry in private classes under Ramanan. I applied to *Visiting Students' Research Programme* at **TIFR** listing algebraic geometry and representation theory as interests. There, I read Kostant's paper on invariant theory titled *Lie group representations of polynomial rings* under Dipendra Prasad. Working on this eighty-page research paper and supplying missing details was educative. I condensed the first section (twenty pages) into a 30-minute presentation that was highly appreciated. I plan to read Nagata's invariant theory papers under Professor Seshadri.

As part of an exchange programme, I will work with *Groupes et geometries* faculty at **Ecole Normale Superieure** this summer.

In Ramanan's *Global Calculus* course, I understood how "local patching" defines global structures on manifolds. In *Elementary Differential Geometry*, I studied curves and the classification of surfaces. I plan to study Ricci flows and Hamilton's theorem under Dr. Aravinda, to appreciate developments like the Poincare conjecture's proof.

While studying computer science, I perceived interesting connections with number theory, algebra and analysis. In the *Microsoft Research Summer School on Algorithms, Complexity and Cryptography*, I grasped how strands from number theory are combined in constructing algorithms, such as the Elkies-Atkin-Schoof algorithm, and the MOV attack. I particularly enjoyed the insight that Riemann-Roch and Weil reciprocity provided for cryptographical algorithms.

Exploring problems often leads me to exciting research literature. I began my undergraduate life trying to develop a theory of group and subgroup properties. I constantly raised new questions and tried to solve them, taking help from journals, books, and the Internet. I also corresponded with professors including Foguel (AUM), Isaacs (Wisconsin-Madison), and Robinson (UIUC). While exploring normalizers, I formulated this problem:

> Define the **hypernormalizer** of a subgroup in a group as the limit of repeatedly taking normalizers. *Is the hypernormalizer of a subnormal subgroup necessarily the whole group?*

Having read Foguel's articles on related topics, I sent him the question. He provided a counterexample and references for further study.

In my first semester, I formulated the **Extensible Automorphisms problem**:

> Call an automorphism σ of a group G **extensible** if for any group H containing G there is an automorphism ϕ of H such that $\phi|_G = \sigma$. *Is every extensible automorphism inner?*

Using group actions and representation theory, and with inputs from Ramanan, Isaacs and Alperin (Chicago), I obtained a partial solution, recorded at:

`http://www.cmi.ac.in/~vipul/unsolvedproblems/extensibleautomorphisms/`

Growing interest in Lie and algebraic groups has led me towards analogous problems for groups with extra structure (algebraic, analytic, or topological).

I formulated another problem:

> Call a subgroup $H \leq G$ **potentially characteristic** if there is a group K containing G such that H is characteristic in K. *Is every normal subgroup potentially characteristic?*

I am recording progress on this and other unsolved problems at:

`http://www.cmi.ac.in/~vipul/unsolvedproblems/`

I am passionate about teaching and sharing mathematical knowledge and insights. Given my Olympiad background, and my liking for teaching, I offered my services to national

and regional Olympiad coordinators to train students for Olympiad-related problem solving. I used my training sessions to convey love for mathematics by designing simple and interesting lessons. Study material I prepared is available at:

`http://www.cmi.ac.in/~vipul/olymp_resources`

Samasya, a mathematics problems journal, published my article on combinatorial identities.

At CMI, I initiated a series of student presentations on mathematical topics, in which I lectured on group theory, number theory, and algebraic geometry.

Given my objective to do mathematical research and my primary interest areas (number theory, automorphic functions and algebraic geometry), I feel Harvard is an ideal choice for my graduate studies. I have encountered theorems and algorithms due to Mazur, Tate, and Elkies in number theory, and look forward to learning from them. I also hope to benefit from Harvard's faculty in automorphic function theory (like Taylor), algebraic geometry(like Harris) and differential geometry (like Yau). If selected, I will use my academic background and passion for research to contribute significantly to mathematics under the guidance of Harvard's eminent faculty.

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