Regression

11th November

Dynamic Regression

$$
\begin{array}{c c}
 & \begin{array}{cc} y & X \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ T \end{array} & \left.\begin{bmatrix} y_1 & x_1 \\ \vdots & \vdots \\ y_T & x_T \end{bmatrix}\right\} \quad (\beta_T, \sigma_T^2) = \theta_T \\
T+1 & \begin{array}{cc} y_{T+1} & x_{T+1} \end{array} \quad (\beta_{T+1}, \sigma^2_{T+1}) = \theta_{T+1}
\end{array}
$$

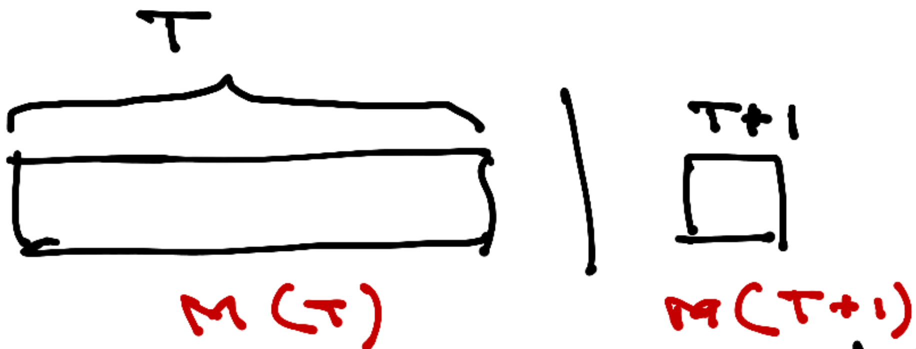
If y_{T+1} & x_{T+1} arrives together, then its just about the updation

But if it's not

then it's dynamic regression type problem
(Kalman filter problem)

Right now we care about the updating problem.

Suppose you have information upto T.



Now T+1 comes, can model has to get updated.

The optimal way to do so is using Bayes rule.

[* Bayes rule is the most optimal way to update information.]

Fixing us to linear model framework.

$$y_t = X_t \beta_t + \varepsilon. \qquad \varepsilon_t \sim N(0, \sigma)$$

$$\beta_t \sim N_p(\beta_0, C_0)$$

$\hookrightarrow$ the prior

and from the data model, find the likelihood. $Y \sim N_n(X\beta, V)$.

Apply Bayes computation.

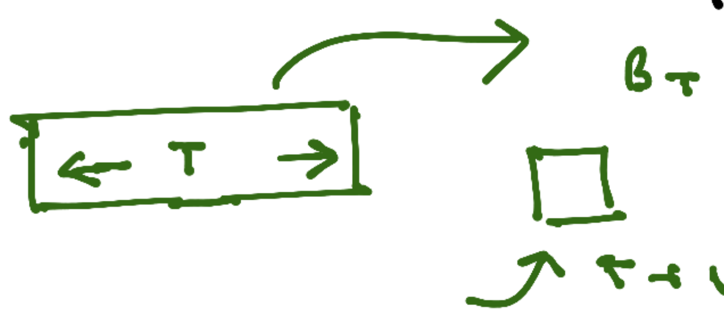
You can find the posterior.

$$\boxed{\beta \mid \text{data} \sim N_p(E_n, C_n)}$$

$$E_n = E(\beta \mid \text{data}) = \beta_0 + C_0 X^T (X C_0 X^T + V)^{-1} \cdot (Y - X\beta_0)$$

$$C_n = (X' V^{-1} X + C_0^{-1})^{-1}$$

Suppose you have data upto T.



$$\beta_T \sim N(\beta_T^{*}, C_T^{*})$$

Posterior at time T,

$$\beta_T \sim N(\hat{\beta}_T, \hat{C}_T)$$

Use the post at time T & prior for time T+1.

Use that to update the model.

[For Kalman filter : Paper → "Understanding Kalman filter" (1982) American Statistician.

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