

Probability and Statistics

Assignment -7 [13 - Nov - 2018]

1. (i) Let X_1, X_2, \dots be independent random variables with Bernouli(p) distribution. It is desired to test the hypothesis $H_0 : p = 0.3$ against the alternative $H_1 : p = 0.7$. It has been decided that the level of significance is to be 0.01. From first principles, derive the most powerful test based on X_1, X_2, \dots, X_n . Write down the critical region. For $n = 64$, find the power of the test (*the answer could be written in terms of R-functions pnorm / qnorm, such as power=pnorm(0.4) or qnorm(0.4)*). (ii) Write down the conclusion if the observed number of successes in 64 trials is 32.
2. (i) Let X_1, X_2, \dots be independent random variables with normal distribution, mean 0 and variance θ , $0 < \theta < \infty$. It is desired to test the hypothesis $H_0 : \theta = 4$ against the alternative $H_1 : \theta = 1$. It has been decided that the level of significance is to be 0.01. From first principles, derive the most powerful test based on X_1, X_2, \dots, X_n . Write down the critical region. For $n = 36$, find the power of the test. (*the answer could be written in terms of R-functions pchisq / qchisq*). (ii) Write down the conclusion if the observed value of $\sum_{i=1}^n X_i^2$ is 48.
3. (i) Let X_1, X_2, \dots be independent random variables with common density, for $\theta > 0$

$$f(x, \theta) = \frac{1}{\theta} \exp\left\{-\frac{x}{\theta}\right\}, \quad 0 < x < \infty.$$

It is desired to test the hypothesis $H_0 : \theta = 1.5$ against the alternative $H_1 : \theta = 1$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on X_1, X_2, \dots, X_n . Write down the critical region. (ii) For $n = 100$, find the power of the test. (*the answer could be written in terms of R-functions pgamma / qgamma*). Write down the conclusion if the observed value of $\sum_{i=1}^n X_i$ is 233.

4. In the previous problem, write down the critical region, power and conclusion using central limit theorem instead of gamma distribution.
5. (i) Let X_1, X_2, \dots be independent random variables with Poisson distribution, mean θ , $\theta > 0$. It is desired to test the hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 1.5$. It has been decided that the level of significance is to be 0.05. From first principles, derive the most powerful test based on X_1, X_2, \dots, X_n . Write down the critical region. For $n = 100$, find the power of the test. (*the answer could be written in terms of R-functions ppois / qpois, or using CLT in terms of pnorm / qnorm*) (ii) Write down the conclusion if the observed value of $\sum_{i=1}^n X_i$ is 233.
6. It is desired to test if movies are equally popular among men than women in a university campus or is there a gender bias? A random sample of 200 students was selected and each selected student was asked if they see one movie a week on the average. Out of 119 men in the sample 98 said yes while out of 81 women, 72 said yes. Write down the conclusion, if 0.01 level of significance is to be used. It is given that $\text{pchisq}(0.99, \text{df}=1)=0.6802576$.