PERSONAL STATEMENT

ARUL SHANKAR, DATE OF BIRTH NOVEMBER 2, 1986, APPLYING TO THE DEPARTMENT OF MATHEMATICS

I am applying for admission to the Ph.D program in Mathematics because I wish to pursue a career in research and teaching. My primary field of interest is Number Theory. I am also interested in Galois Theory Representation Theory, and Algebraic Geometry.

My interest in Mathematics was first kindled when in an attempt to understand the RSA algorithm I read the first few chapters from *A Course in Number Theory and Cryptography* by N. Koblitz. Two years later in 2002, I was selected to attend the IMOTC (the summer camp where the Indian team for the IMO is selected) as also in 2003 and 2004. I learnt a lot in these camps, and in the process, my interest in Mathematics grew substantially. At the end of high school I decided to enroll in the BSc Honours programme at the Chennai Mathematical Institute (CMI) because it provided me with the opportunity of learning mathematics from people actively engaged in research. I would then also have the chance to learn from the faculty of the Institute of Mathematical Sciences (IMSc).

In my second year at CMI, I took two optional courses in Number Theory under Professor R.Balasubramanian at IMSc viz. *Algebraic Number Theory*, and *Elliptic Curves and Modular Forms*. In the course on Algebraic Number Theory, we proved the Prime Number Theorem and Dirichlet's theorem on primes in arithmetic progressions among other things. We were studying L functions in the other course and I was astounded by the fact that they encode such basic arithmetic properties. I also understood that there were many close connections between the two subjects. For instance, the question: "When is the ring of integers of an imaginary quadratic field a unique factorization domain" is best solved using the theory of Elliptic Curves.

In the course Elliptic Curves and Modular Forms, I was fascinated by Mordell's proofs of the first two Ramanujan τ conjectures. As Hecke's reinterpretation of the proofs led to the concept of Hecke Operators, I studied them from *Elliptic Curves* by A. Knapp and *A Course in Arithmetic* by J.P. Serre. As a continuation of the course, I am reading further from Knapp's book, and have studied eight chapters of it so far. I plan to finish reading through it in the next few months. I particularly want to read the proof of the fact that the Taniyama -Weil Conjecture implies Fermat's Last Theorem.

One of the things which I found particularly beautiful in the course Elliptic Curves and Modular Forms was the use of complex analysis. I enjoyed the Complex Analysis course I took in my fourth semester taught by Prof. S. Nayak very much and I have studied the book *Theory of Complex Functions* by R. Remmert. I am taking an advanced course on Complex Analysis offered by Prof. R.Balasubramanian this semester. In this course I will also study aspects of Analytical Number Theory.

In my fifth semester, I took a course on *Abelian Varieties* offered by Prof. S. Ramanan. Here we studied Elliptic Curves using the language of Algebraic Geometry. I understood connections between modular forms, and cohomology and line bundles. I studied the Algebraic Geometry required for this course from Griffiths and Harris' *Principles of Algebraic Geometry*. Fascinated by it, I have started studying Algebraic Geometry from Hartshorne's book.

Because of the central role that Representation Theory plays in many parts of Mathematics and Physics, I took two optional courses in it. As part of the course offered by Prof. S. Kannan, I read through the first two parts of *Linear Representations of Finite Groups by J.P Serre*. In the other course offered by Prof. A. Prasad, we studied modular representations. We finally proved the Brauer Nesbitt theorem which gives information about the Cartan matrix of $F[G]$. The course also involved a project of independent work in which I investigated modular representations of finite abelian groups over finite fields and calculated the radical of its group ring. That allowed me to describe all its irreducible representations.

I am also taking an independent study course with Prof. C.S. Seshadri on *Automorphic Functions* which I started last semester (the fifth). In this course, I am reading the lecture notes of a course he delivered in 1969 at Harvard.

As preperation for my Algebra IV course I studied the sections on field extensions and Galois theory from Lang's Algebra. I understood that there are connections between the theory of modular forms and the inverse Galois problem. As top of my class I will visit the Ecole Normale Superiore, Paris as part of the CMI ENS exchange program. There I hope to explore more of this connection and also to study Gaolois cohomology in detail.

Prof. Balasubramanian mentioned that the Taniyama -Weil conjecture was a special case of the *Langland's Programme*, which involved relating L functions which arise from Algebraic Geometry to L functions in Number Theory. I plan to take a reading course under him and try to understand more of this program. As Class Field Theory is an essential prerequisite, I am studying it from Weil's Basic Number Theory. I plan to further my study in it from Serre's book on it next semester.

I am also interested in problems of Complexity Theory, especially those that arise from Number Theory. In a course on Algorithms that I took in my third semester, we studied the algorithm given by M.Agrawal which solved the primality testing problem in polynomial time. Cryptography is also something I am very interested in. I am especially interested in the use of results in Algebraic Geometry like the Riemann-Roch theorem and the Tate and Weil pairings in Elliptic Curve Cryptography. This semester, I am taking a course each in Cryptography and Complexity Theory.

I am applying to the California institute of Technology because I belive that it has one of the best departments of Mathematics in the world, especially in my areas of interest. If selected I hope to contribute substantially to my fields of interest under the guidence of the Caltech faculty and I am confident of my abilities to so.