Statement of Purpose

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I am applying for Ph.D. in mathematics. I wish to pursue research in group theory and representation theory. My other interests include number theory, differential geometry, algebraic topology, and algebraic geometry. I also seek to share my mathematical knowledge and insights through teaching.

Since childhood, I have enjoyed playing with numbers and shapes. I often read topics outside the school syllabus, such as elementary number theory and combinatorics. Later, I studied Olympiad mathematics and represented India twice at the International Mathematical Olympiad, winning silver medals both times(IMO 2003 and IMO 2004). As leading faculty at the IMO camp recommended Chennai Mathematical Institute (CMI) for further studies, I enrolled there for B.Sc. (Hons) Mathematics and am currently completing my fifth semester.

Through courses at CMI, seminars, summer camps, and personal sessions with professors, I have built a solid foundation in group theory, representation theory, commutative algebra, analysis, number theory, differential geometry, and other areas. I have extended this by attending advanced graduate-level courses, such as Global Calculus, Abelian varieties, Elliptic curves and modular forms, and courses related to the theory of computation and computational complexity.

Areas of interest

Group theory/representation theory

Group theory and representation theory have fascinated me the most. In my first year, I studied a proof of the Nielsen-Schreier theorem on free groups (at Professor Nagarajan's suggestion) and the orthogonality theorems in the representation theory of finite groups. I also attended a summer camp titled Groups, Representations and Algebras at the Institute of Mathematical Sciences, where Dr. Amritanshu Prasad computed all the irreducible representations of $\mathrm{GL}_2(F_p).Thisprovidedmeahands-onfeelofcomputingrepresentations.Inanelectivecourseonrepres$

0.1 Number theory

Spurred by my exposure to the theory of prime numbers and factorization during my Olympiad preparation, I attended a lecture series on algebraic number theory by Professor Balasubramanian (Institute of Mathematical Sciences). I also audited a course on Elliptic Curves and Modular Forms under Professor Balasubramanian. In the Microsoft Research Summer School on Algorithms, Complexity and Cryptography, I learnt more about factorization methods and their relation to the theoretical structure of multiplicative groups and elliptic curve groups.

Algebraic geometry

The connections of Hilbert's nullstellensatz with "families of curves with given intersection points" intrigued me. Professor Ramanan explained to me that algebraic geometry aims to generalize notions from reals and complex numbers to arbitrary fields (which may not have the same topological or analytic structure). I then studied books by Eisenbud and by Atiyah and Macdonald to familiarize myself with the algebra-geometric dictionary that translates geometric intuition to algebraic rigour. My exposure to Professor Balaji's topology course, where he covered an elegant approach to developing concepts of fundamental group and covering spaces from an algebraic geometry approach, has whetted my interest further.

This summer, I attended the Visiting Students' Research Programme at Tata Institute of Fundamental Research where I studied Kostant's paper titled "Lie Group Representations of Polynomial Rings" under Professor Dipendra Prasad. My presentation on this was appreciated by the attending faculty and researchers. This intense work helped me develop persistence in reading and understanding mathematics, and I enjoyed exploring the link between affine varieties and invariant subrings. I hope to study more papers in invariant theory under Professor Seshadri's guidance.

Differential geometry

While studying Global Calculus under Professor Ramanan, I was fascinated by the use of linear algebra and ring theory to develop the notions of differentiability on manifolds. I therefore attended his seminars on the representation theory of Lie algebras where he introduced "differential graded commutative algebras" and studied their structure using techniques very similar to those in global calculus.

During Dr. Aravinda's course in Elementary Differential Geometry, I studied curve theory and the complete classification theory of surfaces. From a seminar by Dr. Aravinda, I learnt how Ricci flows are used in solving the Poincare conjecture. I plan to study papers on Ricci flows under Dr. Aravinda's guidance next semester.

Abelian varieties

In Prof. Ramanan's course on Abelian varieties, I am exploring linking results in algebraic geometry with results in complex analysis and differential geometry, such as the many avatars of the Riemann-Roch theorem from the complex analysis viewpoint, as well as its implications for algebraic geometry. Through private classes with Professor Ramanan, I am furthering my understanding of the Riemann-Roch theorem and why its many forms are equivalent.

Computer science

I have fast-forwarded and audited numerous courses in computer science, including Theory of Computation and Programming Language Concepts. I audited a course in computational complexity, and am now studying the relation between expander graphs, complexity classes, and group theory, and explored the elegant use of group actions to construct an AM-protocol for membership testing. In the summer school on Algorithms, Complexity and Cryptography, I interacted with researchers in academia and industry and obtained up-to-date knowledge on major cryptographical problems such as factoring, discrete logarithms, and computing primitive roots. Here I found applications of deep results in complex analysis, such as the Riemann-Roch theorem and the Weil reciprocity, which I had read earlier.

Original work/teaching experience

Original work

In junior school, I played around with prime numbers, trying to discover new patterns. My passion for discovering and solving new problems still continues. Often, while attempting to solve a problem I have myself posed, I locate and read exciting mathematical literature in related subjects. In my attempts to evolve a theory for the systematic study of subgroup properties, I came up with the following problem:

"An automorphism of a group is termed extensible if it can be lifted to an automorphism for any embedding of the group in a bigger group. Is every extensible automorphism inner?"

I have worked on this and discused it with Professor Ramanan, and corresponded with Dr. Isaacs of Wisconsin-Madison and J.L Alperin of Chicago. I am recording my progress at:

http://www.cmi.ac.in/ vipul/extensibleautomorphisms/

Though stating this extensible automorphisms problem did not require any knowledge of representation theory, all solution approaches (that I or others have come up with) use group actions and representations. This has made me appreciate that effective problem-solvers need a thorough grasp of theories in the specific area as well as related areas.

Teaching/sharing knowledge

Sharing my knowledge through lectures and workshops is an activity that interests me. I have taught students preparing for maths olympiads at the invitation of the regional and national level organizers. This involved about 15 sessions (years 2004, 2005 and 2006) covering number theory, geometry, combinatorics and algebra. I used these sessions to convey a sense of love for maths by designing lessons that were simple and interesting for the students. I also prepared study material that I circulated and made available on my website. One article, on combinatorial identities ¡plug in the name if you can find the journal¿, was published in Samasya, a mathematical problems journal.

At CMI, I initiated a system of holding presentations and student seminars in mathematics, physics, and computer science. I have given talks in group theory, number theory, and algebraic geometry. My algebraic geometry talk explained how high school geometry motivates Hilbert's nullstellensatz.

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