Regression

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Dynamic linear Regression.

$$y_t = \beta_0 + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p} + \varepsilon_t$$

Auto regressive model of order p.

t	y_t	y_{t-1}	y_{t-2}
1	y_1	NA	NA
2	y_2	y_1	NA
$\vdots$	$\vdots$	$\vdots$	$\vdots$
T	y_T	y_{T-1}	y_{T-2}
	Y	X	

Model

$$Y = X\beta + \varepsilon$$

Granger causality model. .

$$Y_t = \beta_0 + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p} + \gamma_1 x_{t-1} + \dots + \gamma_q x_{t-q} + \varepsilon.$$

Test.

H_0 : $\gamma_1 = \gamma_2 = \dots = \gamma_q = 0$

vs H_1 : atleast one is non zero.

If it rejects null, then we can say,

" x_t granger causes y_t "

~ Extremely popular in economics & neuro Science.

Assumption.

It assumes one extra condition,

Both y_t & x_t are covariance Stationary.

In AR, MA and even in GC we assume regression coefficient to be fix.

But if correlation between variable changes, then coefficient are most likely to change.

$$y_t = X_t \beta_t + \varepsilon.$$

[* Use moving correlation to decide]

$$\beta_t = G_t \beta_{t-1} + \vartheta_t .$$

when. $x_t = 1$

$$y_t = \beta_t + \varepsilon_t \quad \text{where } \varepsilon_t \sim N(0, \sigma^2)$$

$$\beta_t = \beta_{t-1} + \vartheta_t , \quad \vartheta_t \sim N(0, \gamma^2)$$

then.

$$y_t \sim N(\beta_t, \sigma^2)$$

$$\beta_t \sim N(\beta_{t-1}, \gamma^2)$$

After some calculations.

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \frac{\sigma^2}{\sigma^2 + \gamma^2} (y_t - \hat{\beta}_{t-1})$$

Have a educated guess on β_0. and then start updating.

This update is also known as.
Kalman Filter Update.

Stochastic control system.

$$x_t = [1, x_t]$$

$$f(y_1, \ldots, y_T \mid \theta) = f(y_0) f(y_1 \mid y_0) f(y_2 \mid y_1 . y_0) \ldots\ldots f(y_T \mid y_1 \ldots y_{T-1})$$

as they are not independent

In ML, it's known as "online learning"