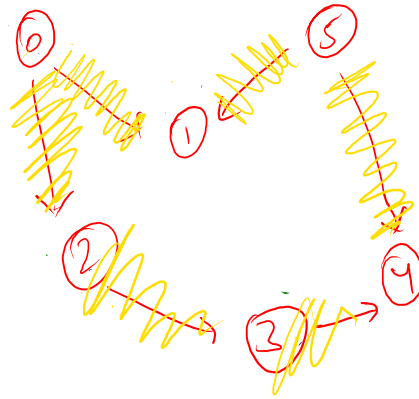


$O(V+E)$

Topological Sort  
↳ DAG

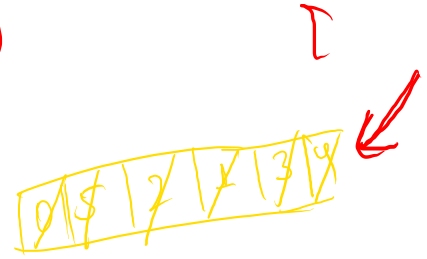
or

or



Kahn's algo

0	1	2	3	4	5
0	<del>0</del> 2	<del>0</del> 1	<del>0</del> 1	<del>0</del> 2	0



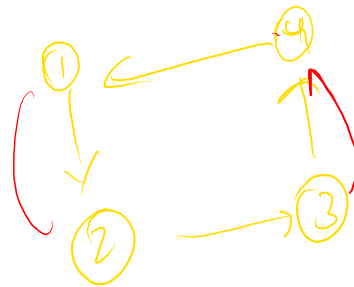
0 5 2 1 3 4

directed  
↓  
↓

topological sorting doesn't exist  
↓  
cycle detect



DAG  
↓  
directed → Acyclic



1 2 3 4  
→ 4 1 2 3  
1 2 → 2 3 1

{wot wof eo ett rftt}

t - ~~1~~  
w - ~~2~~  
o - ~~3~~  
e - ~~4~~

w - 0  
o - 10  
t - 10  
f - 10  
e - 10

~~w~~ ~~e~~ ~~t~~ ~~f~~ ~~t~~  
w e o t f

```
public String alienOrder(String[] words) {
    HashMap<Character, HashSet<Character>> graph = new HashMap<>();
    HashMap<Character, Integer> indegree = new HashMap<>();

    for (int i = 0; i < words.length; i++) {
        String st = words[i];
        for (int j = 0; j < st.length() - 1; j++) {
            indegree.put(st.charAt(j), 0);
        }
    }

    // ... (rest of the code) ...

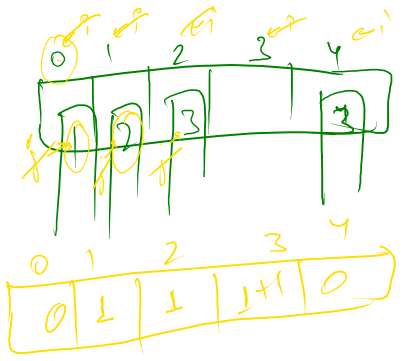
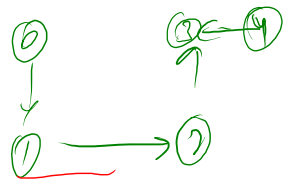
    for (char ch : indegree.keySet()) {
        if (indegree.get(ch) == 0) {
            queue.addLast(ch);
        }
    }

    let count = 0;
    while (queue.size() > 0) {
        char rem = queue.removeFirst();
        ans.append(rem);
        count++;

        if (graph.containsKey(rem) == true) {
            HashSet<Character> nbrs = graph.get(rem);

            for (char nbr : nbrs) {
                indegree.put(nbr, indegree.get(nbr) - 1);
                if (indegree.get(nbr) == 0) {
                    queue.addLast(nbr);
                }
            }
        }
    }
}
```

zy zn  
2 y → n  
y n z



a b c d

a b c  
d

$\{n + \text{word.length}\}$

y n z

(zy zn) 0  
y n z

2 n y

n 2 y → n y z

2 y → n y z

DSU  $\hookrightarrow$  Disjoint Set Union

Optimize

$\hookrightarrow$  Path Compression  
 $\hookrightarrow$  Union by Rank

Par:

0	1	2	3	4	5	6	7
1	1	2	2	4	4	6	7

Rank:

1	2	1	1	2	2	1	2
---	---	---	---	---	---	---	---

- (0,1)
- (1,2)
- (3,4)
- (6,7)
- (4,5)
- (0,3)

```

P S int find(int n) {
    if (Par[n] == n) {
        return n;
    }
    int temp = find(Par[n]);
    Par[n] = temp;
    return temp;
}

```

P S void union(int n, int y)

```

{
    int x = find(n);
    int y = find(y);
    if (x != y) {
        if (rank[x] > rank[y]) {
            Par[y] = x;
        } else if (rank[x] < rank[y]) {
            Par[x] = y;
        } else {
            Par[x] = y;
            rank[y]++;
        }
    }
}

```

Complexity

Without Opt:

$\hookrightarrow$  find  $\rightarrow O(n)$

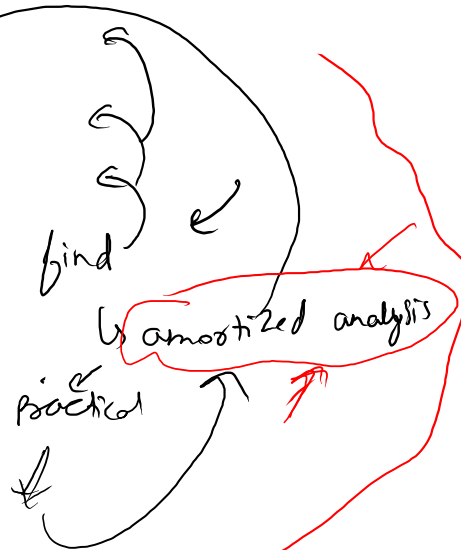
With Path Compression  $\rightarrow O(\log n)$

With union by rank  $\rightarrow O(\log n)$

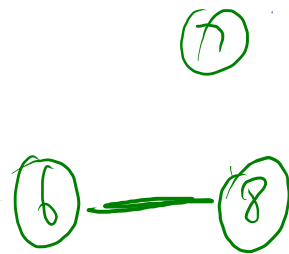
With both Opt  $\rightarrow O(1)$

$\hookrightarrow O(\alpha(n)) \rightarrow \alpha(n) < 4$

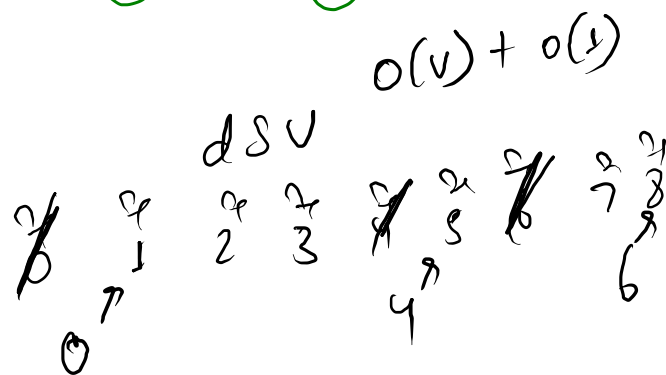
inverse ackerman function



d/s vs dsu



d/s	dsu
① $O(v+E_1)$	① $O(v+1)$
② $O(v+E_2)$	② $O(1)$
③ $O(v+E_3)$	③ $O(1)$



Set = ~~8~~ ~~8~~ ~~6~~

