CYCLES IN $kx + c$ FUNCTIONS

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Abstract. This paper treats cycles in $kx + c$ functions.

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Key words and phrases. 2-adic numbers, binary residue system.

Fundamentals short and sweet

cycle	A cycle is.
dead limb	A dead limb is.
unitary ring	A unitary ring is a ring with a multiplicative identity 1 (which differs from the additive identity $1 \neq 0$) such that $1a = a = a1$ for all elements a of the ring.
Ideal	Let $(R, +, \cdot)$ be a commutative unitary ring. Then the subset $I \subseteq R$ is called an ideal of R if $(I, +)$ is a commutative group and if $xI \subseteq I$ for all $x \in R$, see [2, p. 66-67].
quot. ring	Using an ideal of a ring $I \subseteq R$, we may define an equivalence relation $\sim$ on R by $a \sim b$ iff $a-b$ is in I [3, p. 69]. The equivalence class of a in R is given by $[a] = a + I := \{a + r \vert r \in I\}$ for $r \in R$ and referred to as "residue class of a modulo I", see [4, p. 122], [3, p. 70]. The set of all these equivalence classes becomes the quotient ring (residue class ring) modulo the ideal I, denoted by R/I.
compl. residue system	Let $I \subseteq R$ be an ideal and $[a]$ the residue classes of a modulo I, which means that $a+I = b+I$ when $a \equiv b \mod I$ or respectively $a-b \in I$ [3, p. 70]. R is the disjoint union of the different residue classes a modulo I. A subset $M \subseteq R$, which contains exactly one element from each of these residue classes, is called a complete residue system of R modulo I, see [3, p. 70].
$[a]_n$	The residue class (also termed congruence class) of the integers for a modulus n is the set $[a]_n = \{a + kn \vert k \in \mathbb{Z}\}$ and sometimes denoted by $\bar{a}_n$ or by $a + n\mathbb{Z}$, see [2, p. 15], [4, p. 122], [5, p. 25].
$\mathbb{Z}/n\mathbb{Z}$	The set of all residue classes $[a]_n$ is called the ring of integers modulo n and denoted by $\mathbb{Z}/n\mathbb{Z} = \{[a]_n \vert a \in \mathbb{Z}\}$ and trivially $\mathbb{Z}/0\mathbb{Z} = \mathbb{Z}$ and for all $n \neq 0$ we have $\mathbb{Z}/n\mathbb{Z} = \{[0], [1], \ldots, [n-1]\}$, see [2, p. 15], [5, p. 25].

direct prod.	If $R_1, R_2, \ldots, R_n$ are rings, the cartesian product $R_1 \times R_2 \times \ldots \times R_n$ forms the set of all ordered n-tuples $(r_1, r_2, \ldots, r_n)$, where $r_i \in R_i$. The addition and multiplication of these n-tuples is defined "coordinatewise" by components. The resulting ring is called a "direct product" of the original rings R_i [2, p. 51], [6, p. 169].
prod. of ideals	Let I, J be two ideals of a ring R. Their product IJ is defined as the set of all finite sums $a_1b_1 + \ldots + a_nb_n$ in which $n \geq 0$ and $a_1, \ldots, a_n \in I$ and $b_1, \ldots, b_n \in J$, see [7, p. 87].
power of an ideal	Let I be an ideal in R. The n-th power of I, denoted by I^n, is the n-times product of the ideal I with itself. I^n contains sums of elements of the form $a_1a_2 \cdots a_n$ where $a_1, a_2, \ldots, a_n \in I$ and the products refer to the multiplication defined in R. Consequently, $I^{n+1} \subseteq I^n$. Note that the product and power of ideals should not be confused with the direct product of rings.
filtra-tion	Let R be a ring. A sequence of ideals $I_0, I_1, I_2, \ldots$ is said to be a "filtration" on R if $I_0 = R$ and for each integer $j \geq 0$ applies that $I_j \supseteq I_{j+1}$ and if $I_jI_k \subseteq I_{j+k}$, see [8, p. 269].
princip. ideal	A "principle ideal" is an ideal in a ring R which is generated by a single element a of R through multiplication by every element of R. There are some rings in which every ideal is a principle ideal, so-called "principle ideal rings" [2, p. 68].
max. ideal	A proper Ideal M of a ring R is called "maximal ideal" of R if there is no other proper ideal N of R properly containing M [6, p. 247], [1, p. 37]. A Note on "proper containment": If R is any set, then R is the improper subset of R. Any other subset $N \neq R$ is a proper subset of R and denoted by $N \subset R$ or $N \subsetneq R$ [6, p. 2].
prime ideal	Let a and b are two elements of R and P a proper ideal such that their product ab is an element of P. P is called a prime ideal if at least one of a and b belongs to P, in other words from $ab \in P$ and $a \notin P$ always follows $b \in P$ [1, p. 9].
max. prime ideal	A proper prime ideal P is said to be a "maximal prime ideal" of the ring R, if there is no other proper prime ideal containing P [1, p. 23].
local ring	A commutative ring R is called a local ring if it has a unique maximal ideal M [9, p. 522].

1. Introduction

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