Cognitive Maps in Mice and Machines

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Research Scientist at DeepMind
MIND Summer School
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Extended version, with additional notes + citations for extra edification

Reinforcement Learning Problem

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- **Reinforcement learning problem:**
  Given some task, learn a policy to maximize expected cumulative reward, or value, over future states by trial and error

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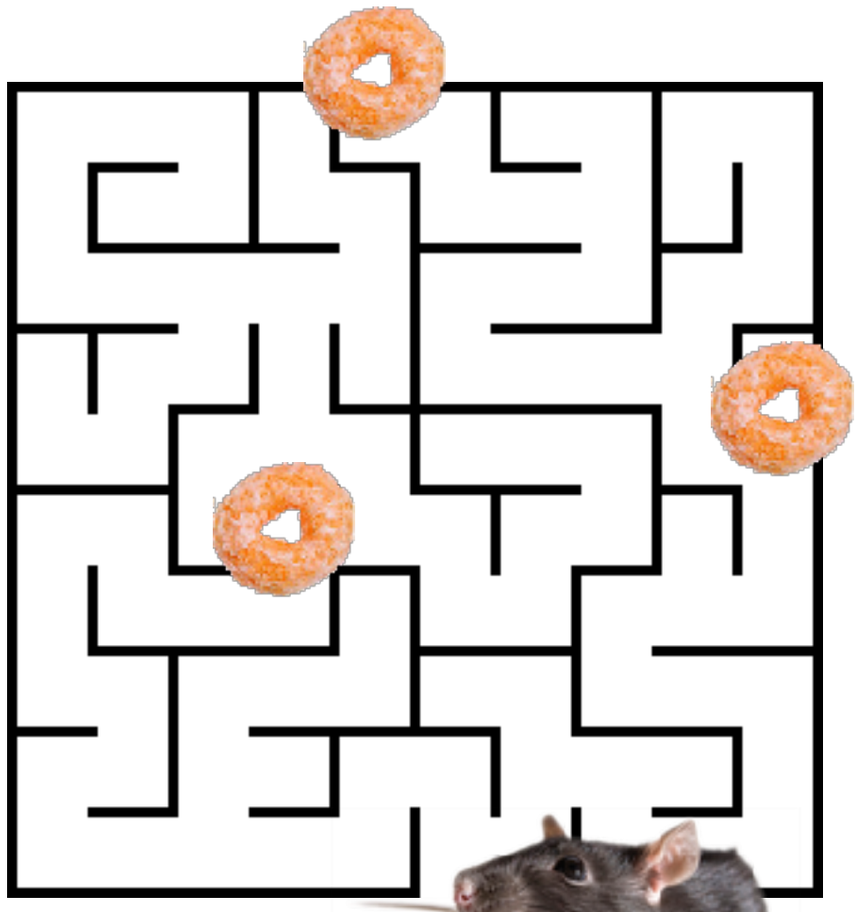

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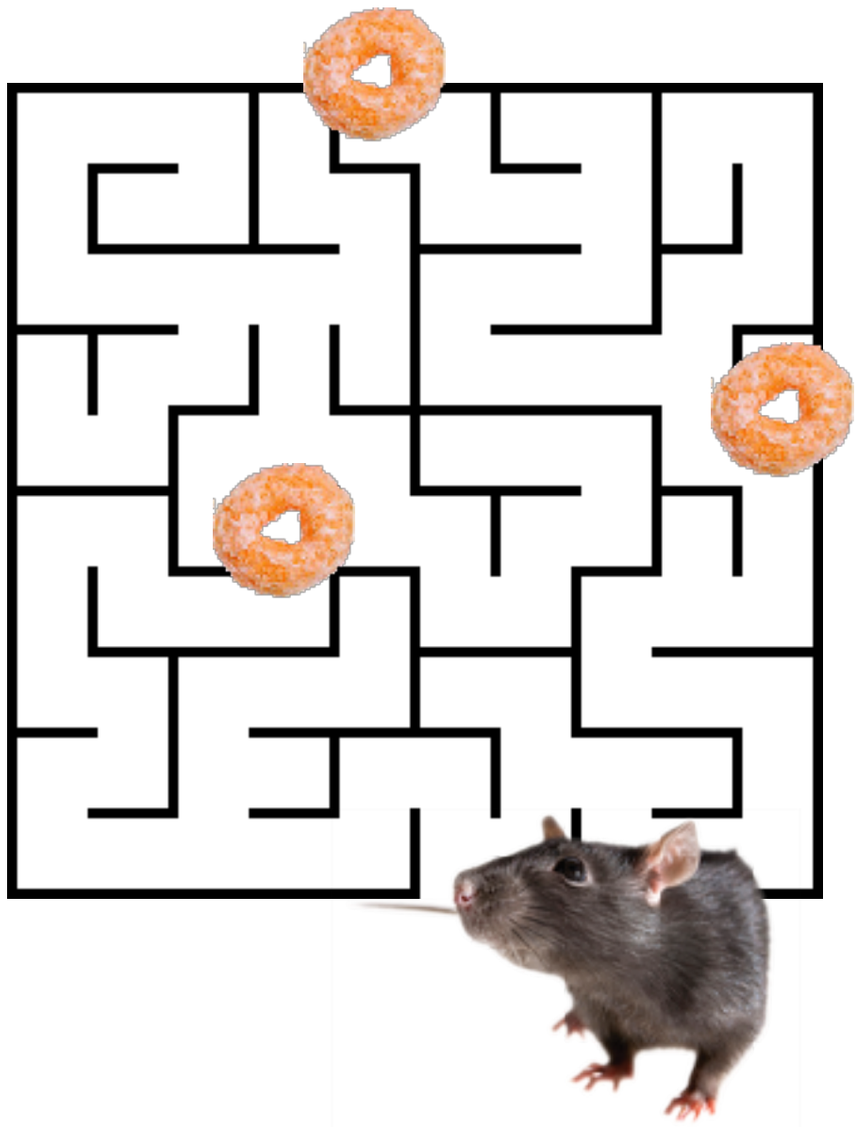

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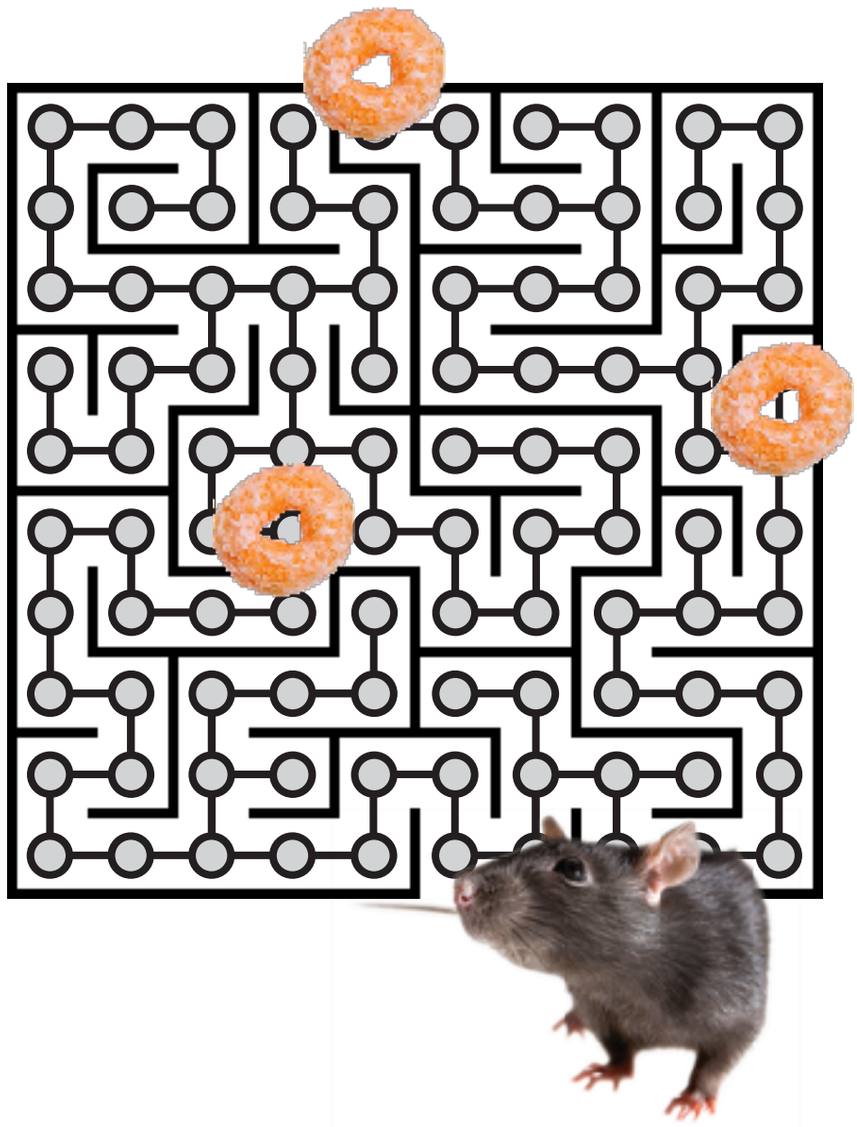

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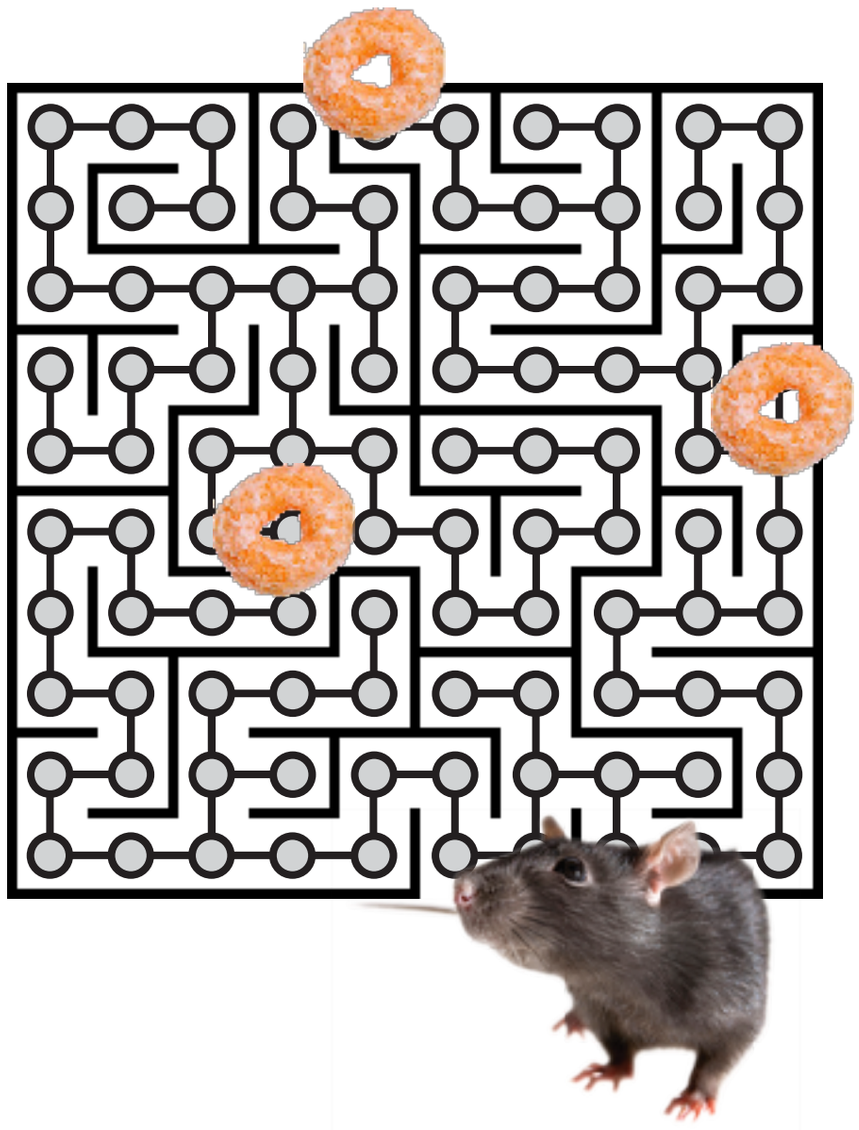

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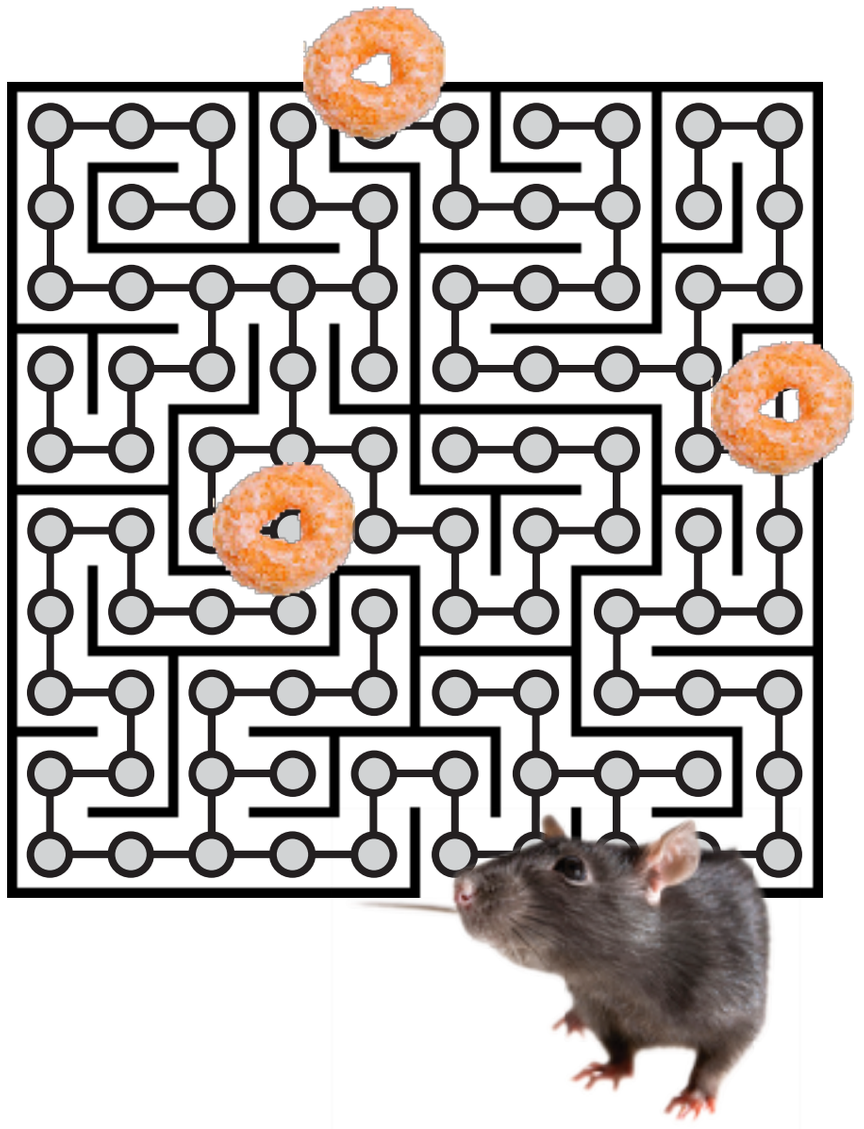

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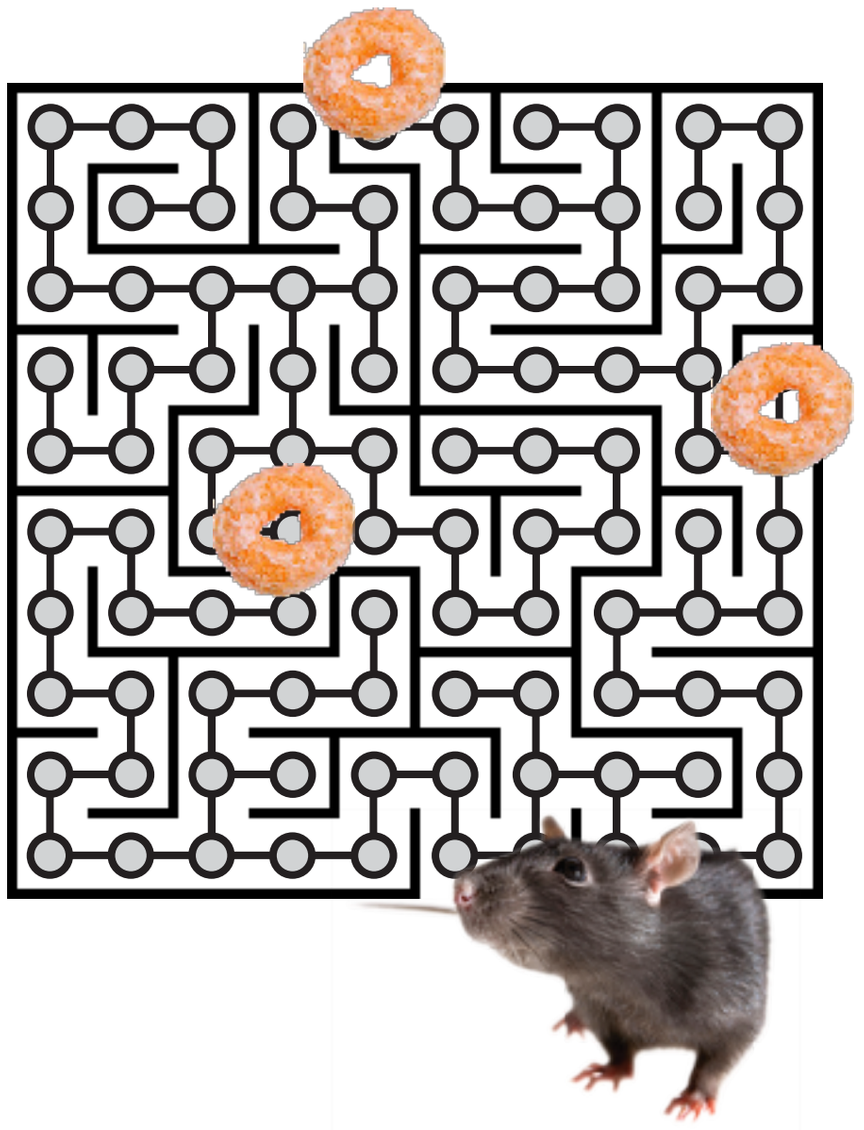

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- **Does the hippocampal cognitive map constitute a structured representation of task structure that supports efficient RL?**

Learning benefits from recognising structure

An inefficient thing to do:

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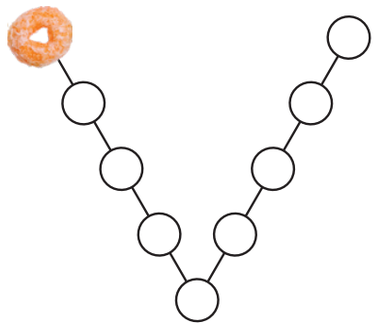

start

1st reward
“I guess that state over there is rewarding!”

Learning benefits from recognising structure

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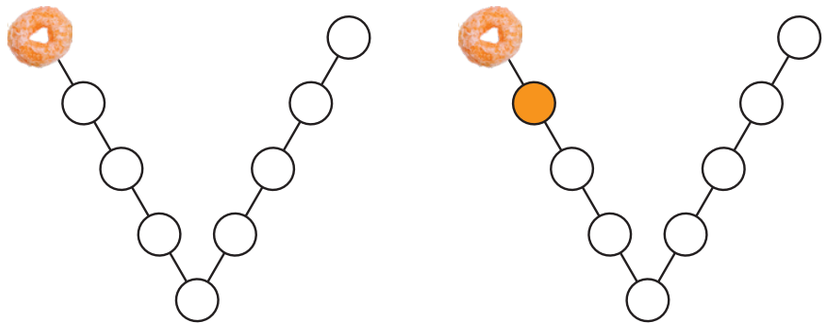

start

1st reward
“I guess that state over there is rewarding!”

2nd reward
“Neat, another valuable state!”

Learning benefits from recognising structure

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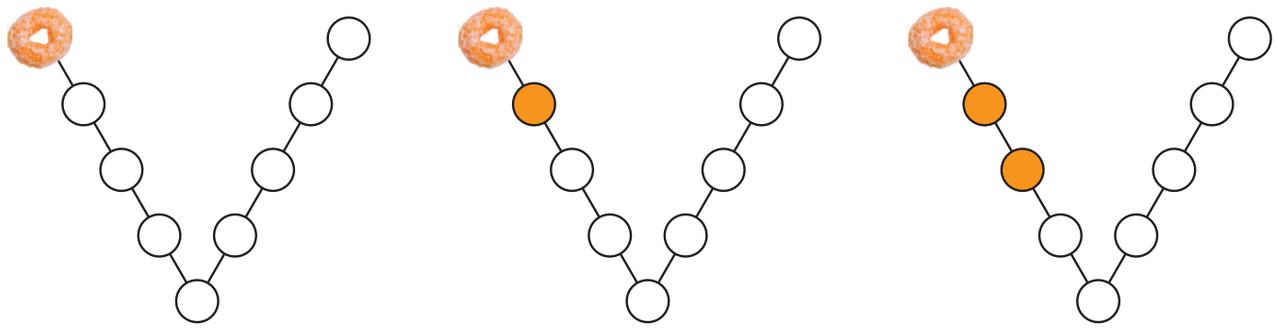

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3rd reward
“Yet another state that predicts value! What luck. If only there was some sort of pattern.”

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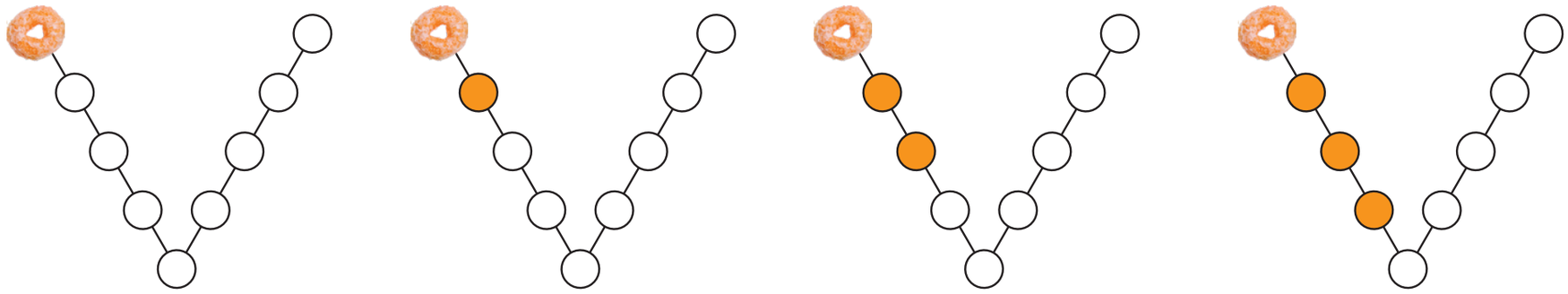

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"Yet another state that predicts value! What luck. If only there was some sort of pattern."

4th reward
…

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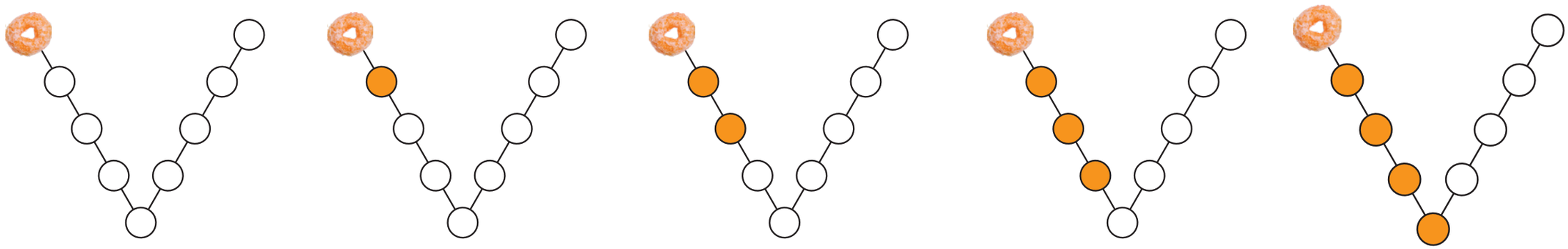

start

1st reward
“I guess that state over there is rewarding!”

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4th reward
…

5th reward
“I should go left.”

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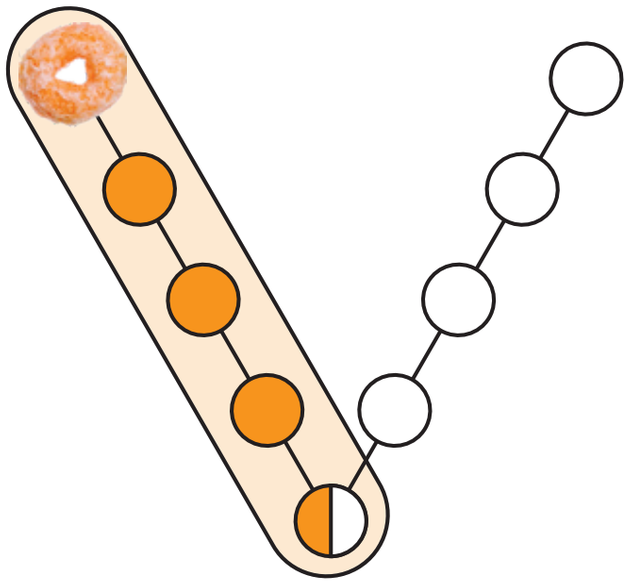
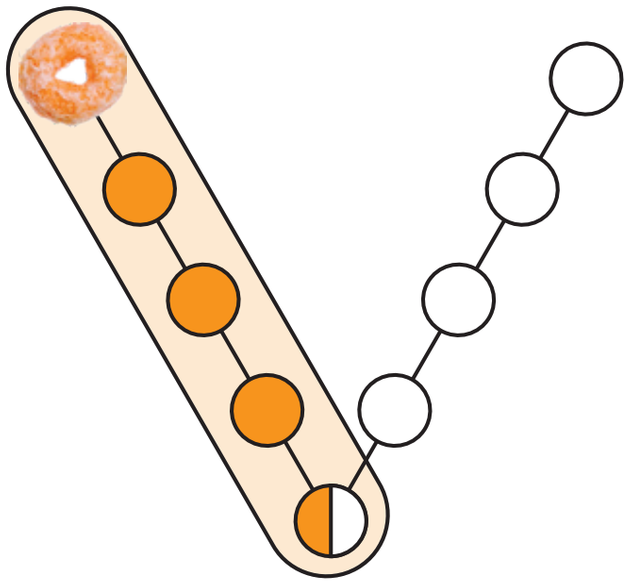


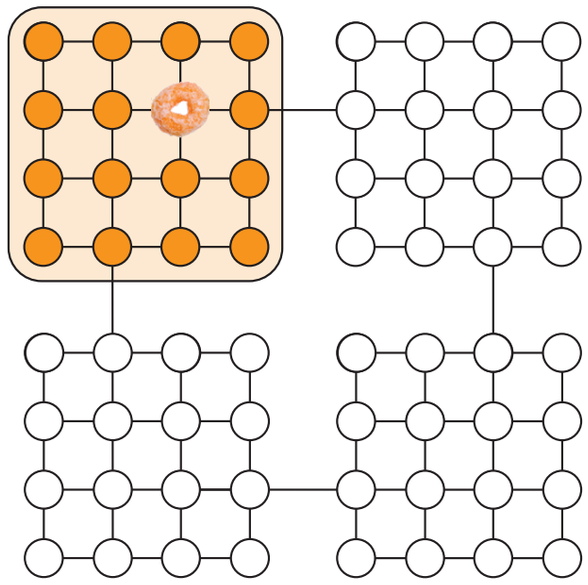
1st left
“I should go left.”

Learning benefits from recognising structure

A more reasonable thing to do:



1st left
“I should go left.”



1st visit to top left room
“I should go to the northwest room”

Exploration benefits from structure

Another inefficient thing to do:

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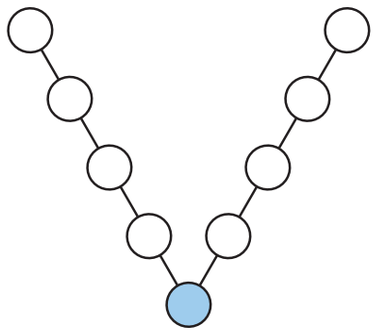

start

blue = visited state

Exploration benefits from structure

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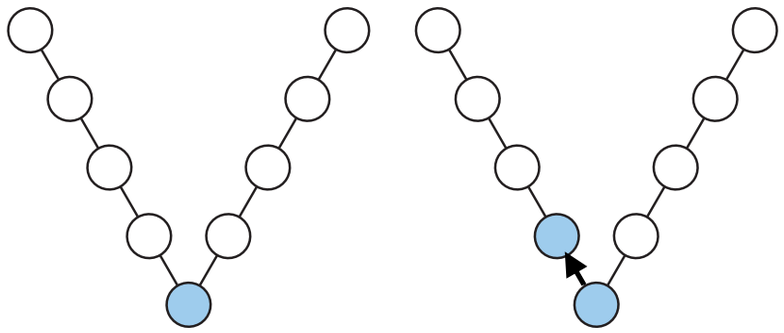

start **1st step**

blue = visited state

Exploration benefits from structure

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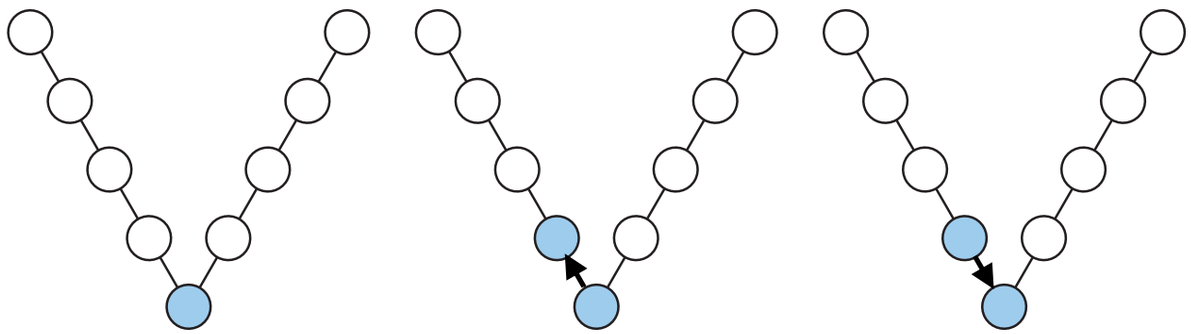

start **1st step** **2nd step**

blue = visited state

Exploration benefits from structure

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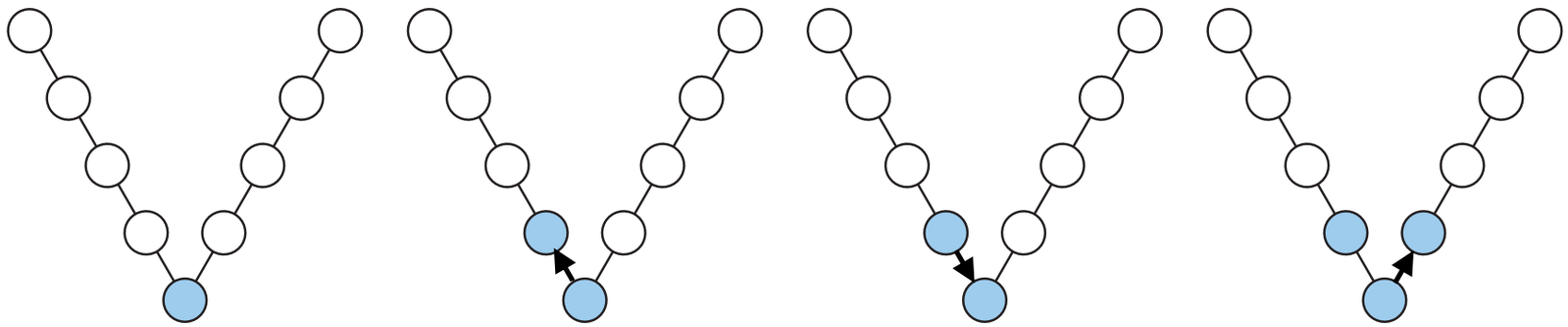

start **1st step** **2nd step** **3rd step**

blue = visited state

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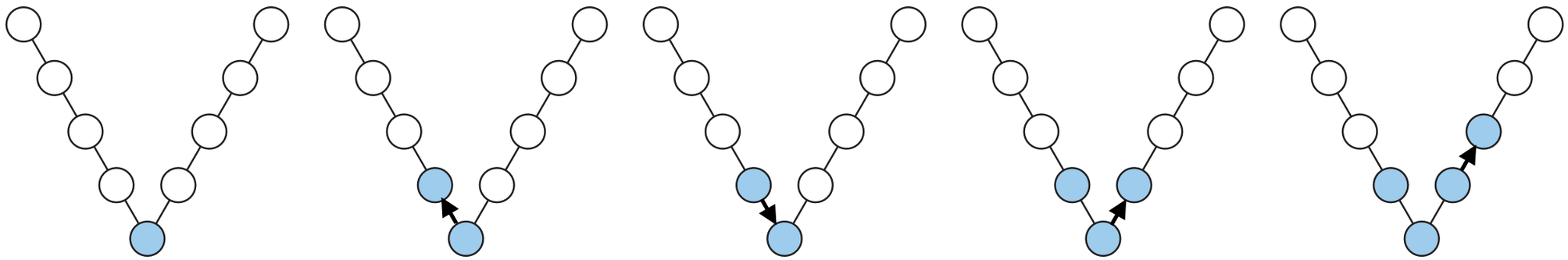

start **1st step** **2nd step** **3rd step** **4th step**

blue = visited state

Exploration benefits from structure

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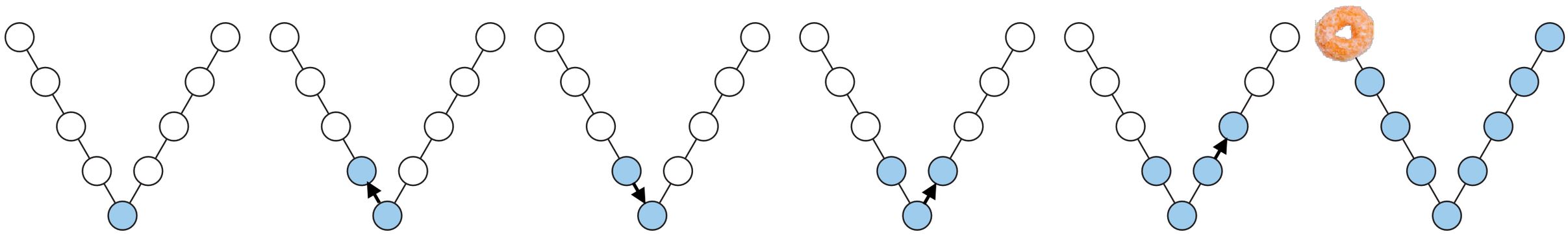

start **1st step** **2nd step** **3rd step** **4th step** **…eventually**

blue = visited state

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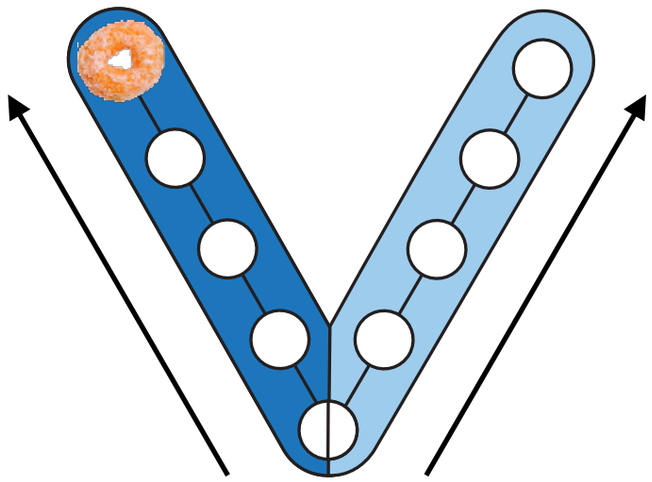

“I should check arm 1 and then arm 2”

Exploration benefits from structure

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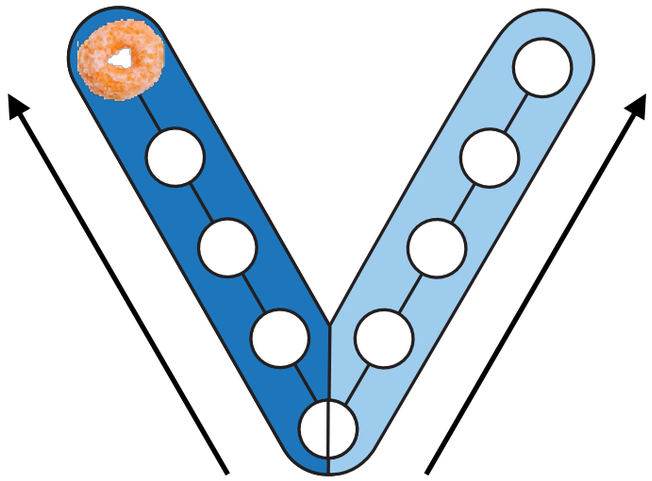

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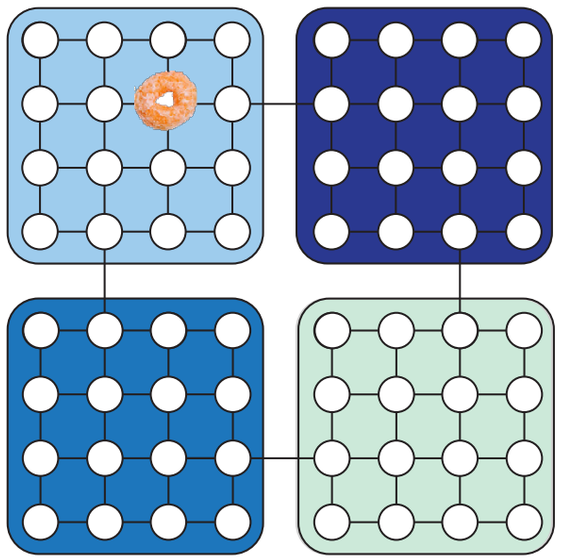

“I should check each room”

What makes a good representation?

- **Other examples of work using a "representation learning" perspective:** Banino et al (2018), Yamins et al (2014), Gardner et al (2018), Momennejad et al (2017), Gustafson & Daw (2011), Niv et al (2015), Wang et al (2015), Radulescu et al (2019), Gershman & Niv (2010), Doll et al (2012), Behrens et al (2018), Stachenfeld et al (2017)

- ^Not exhaustive

Abstract learning and exploration

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Learn in representation space

Value function approximation in representation space

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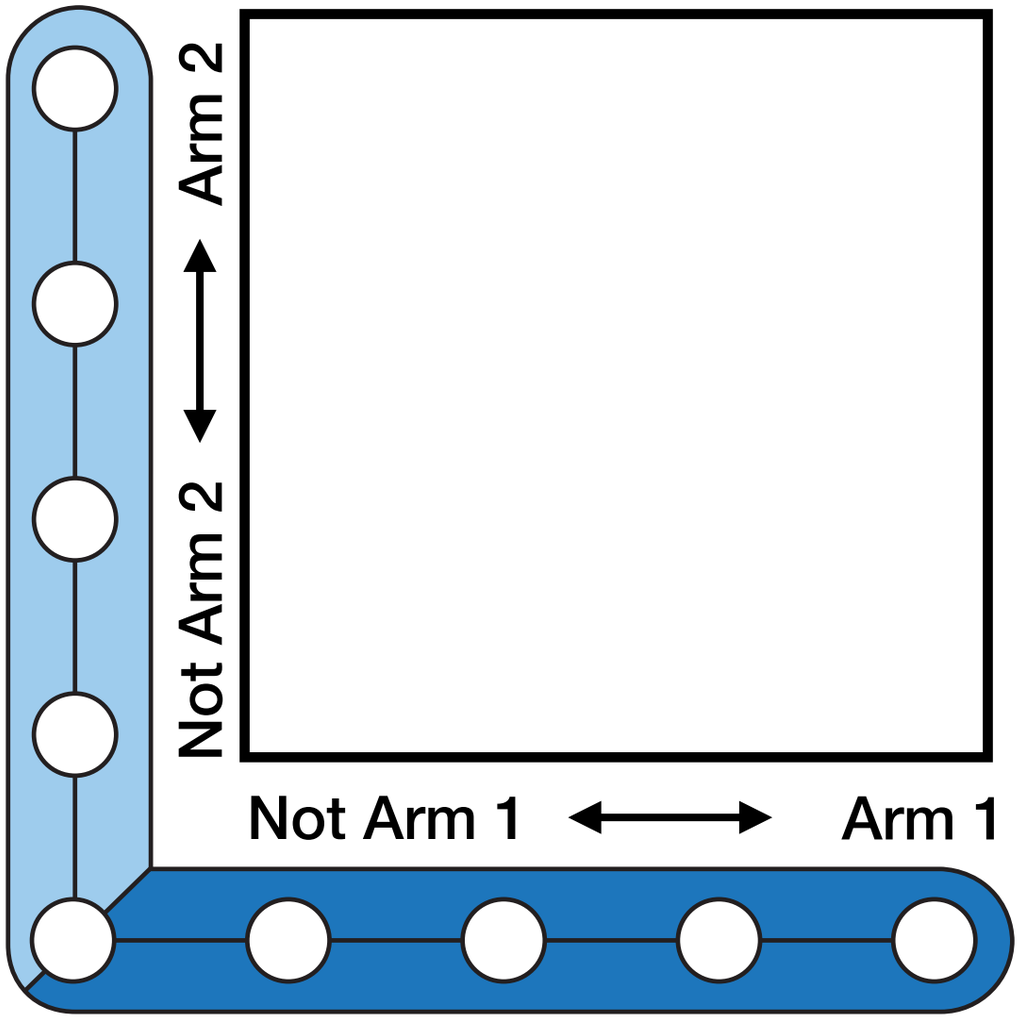

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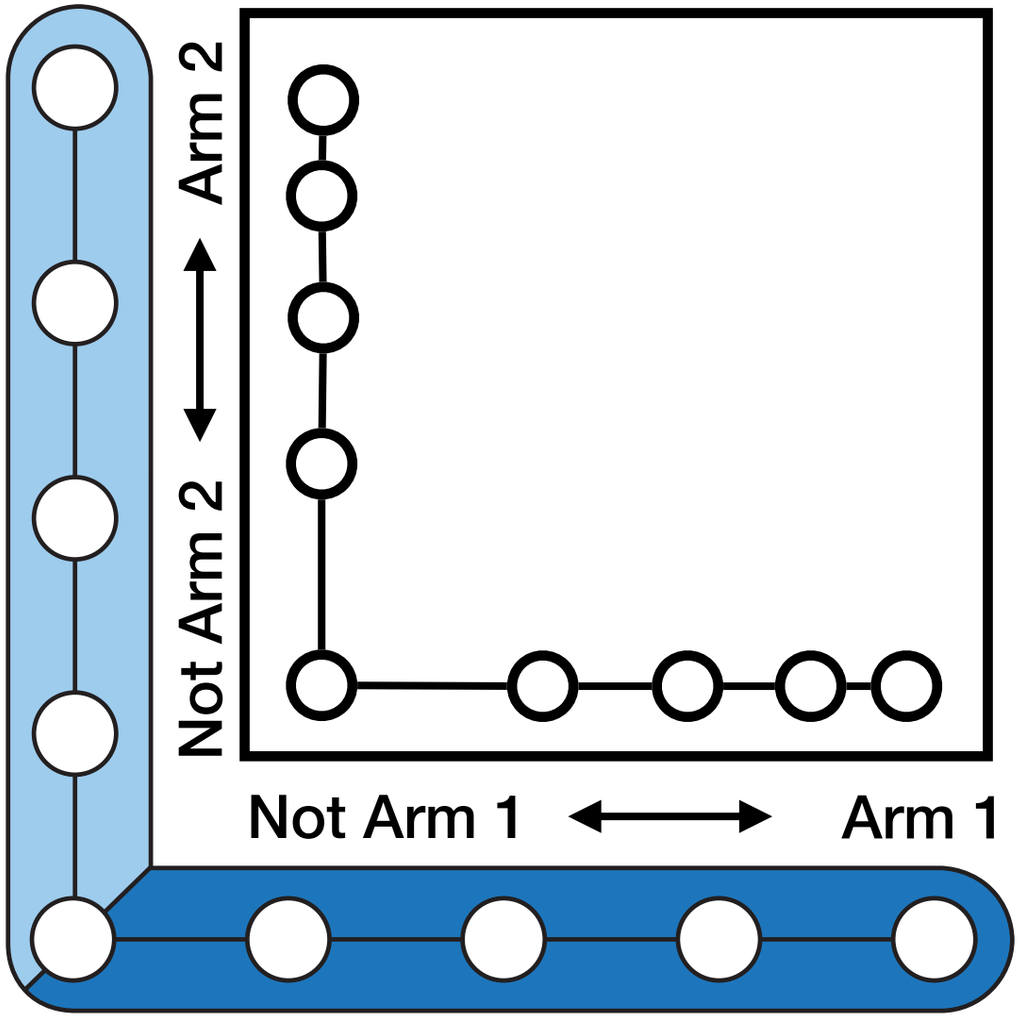

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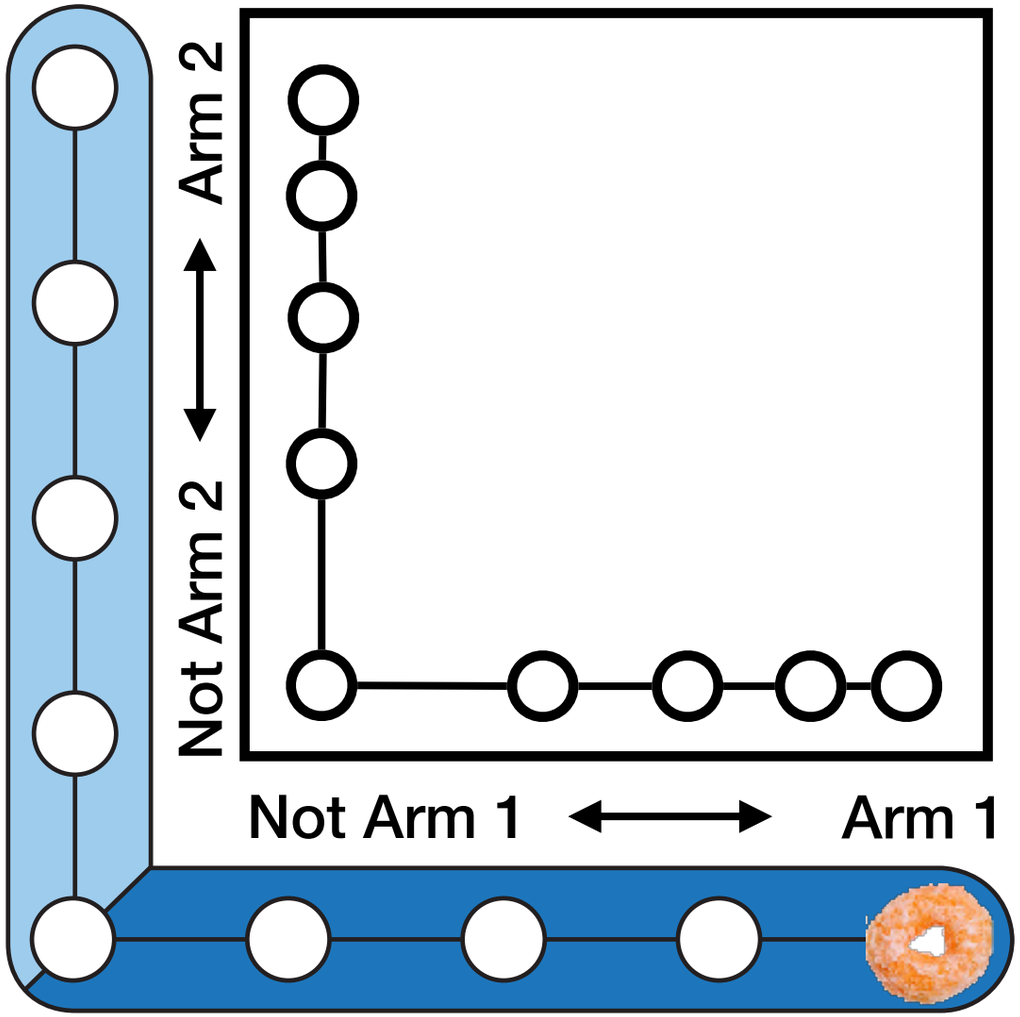

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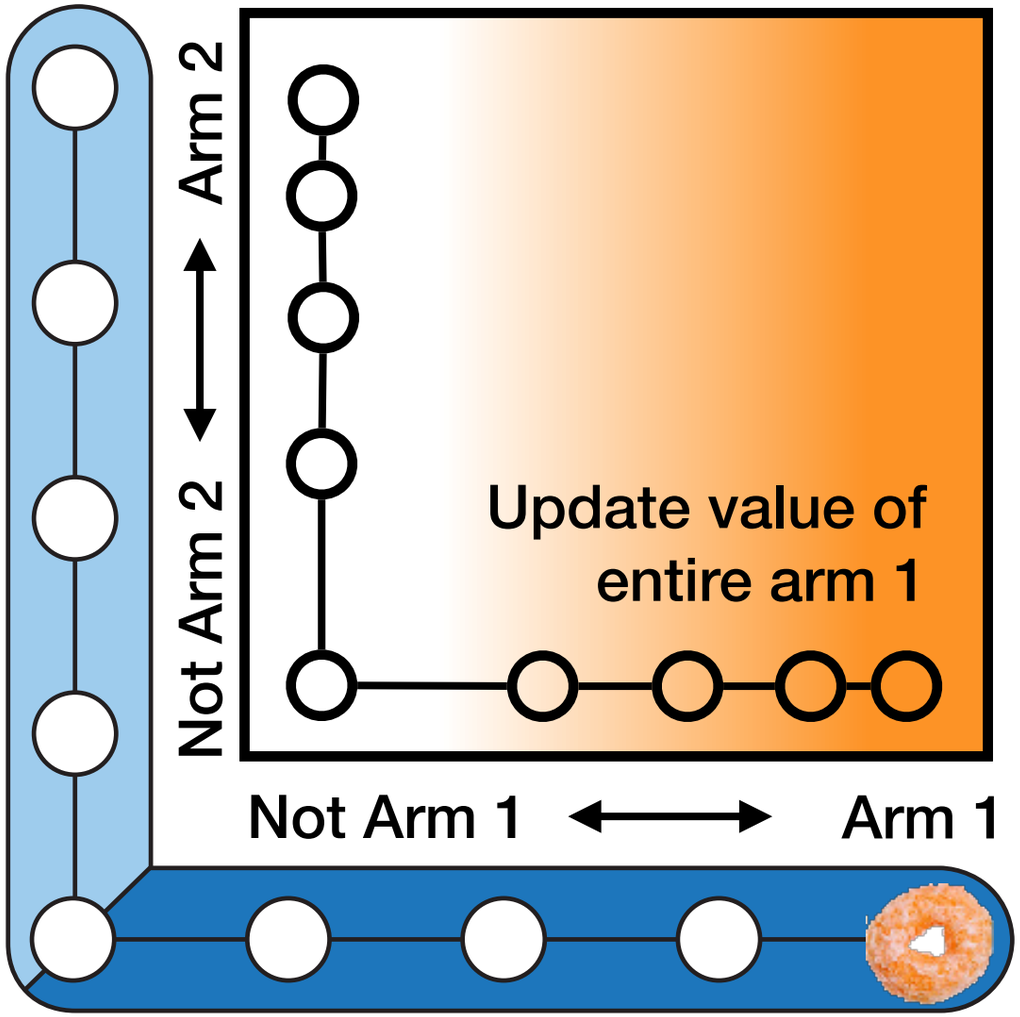

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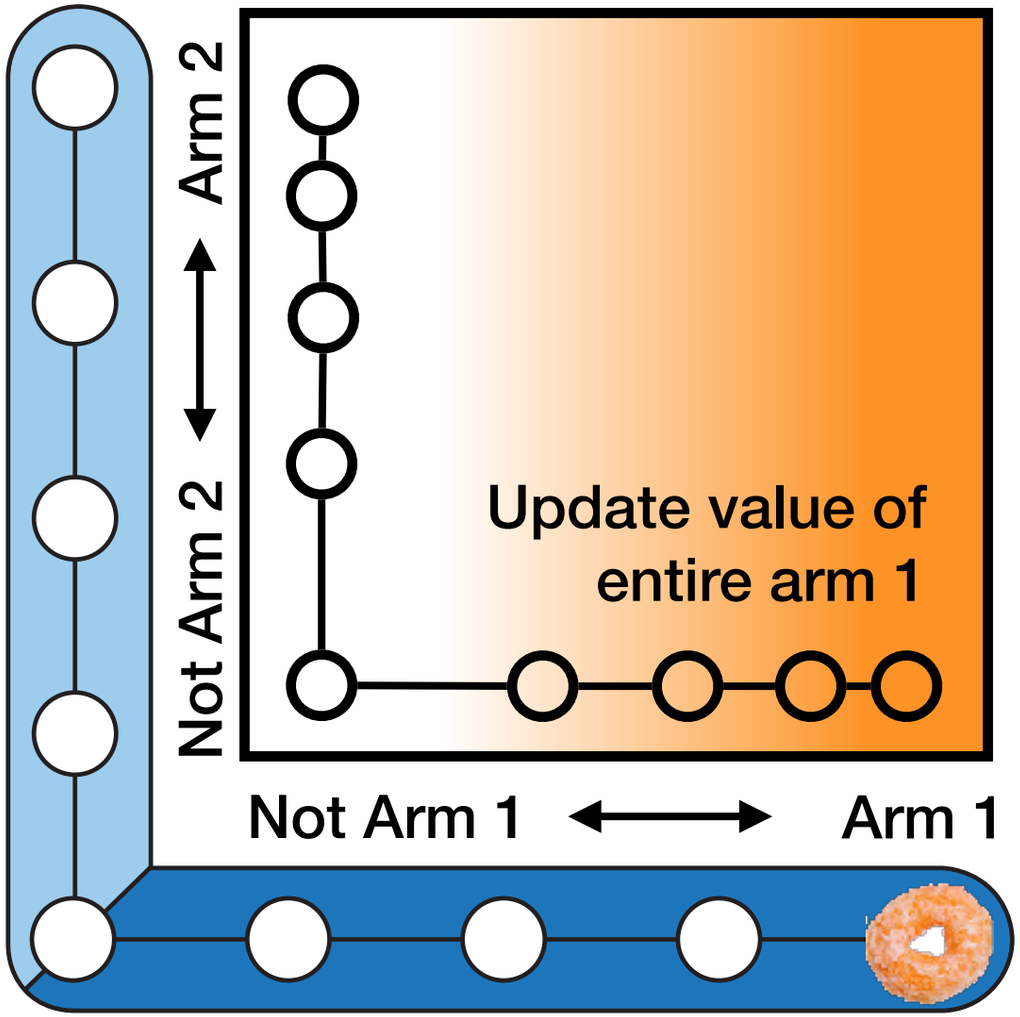
Explore representation space
Extract sequence of actions that explores an axis in representation space



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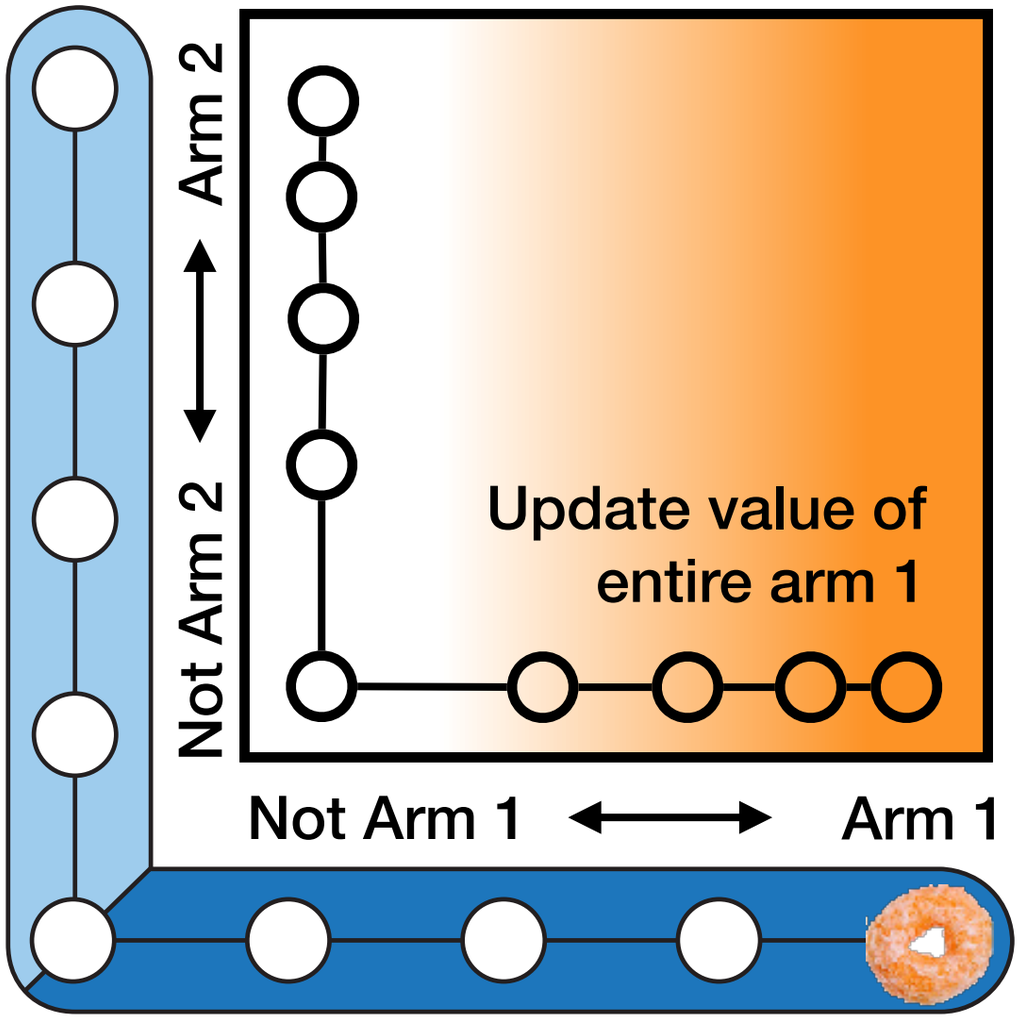
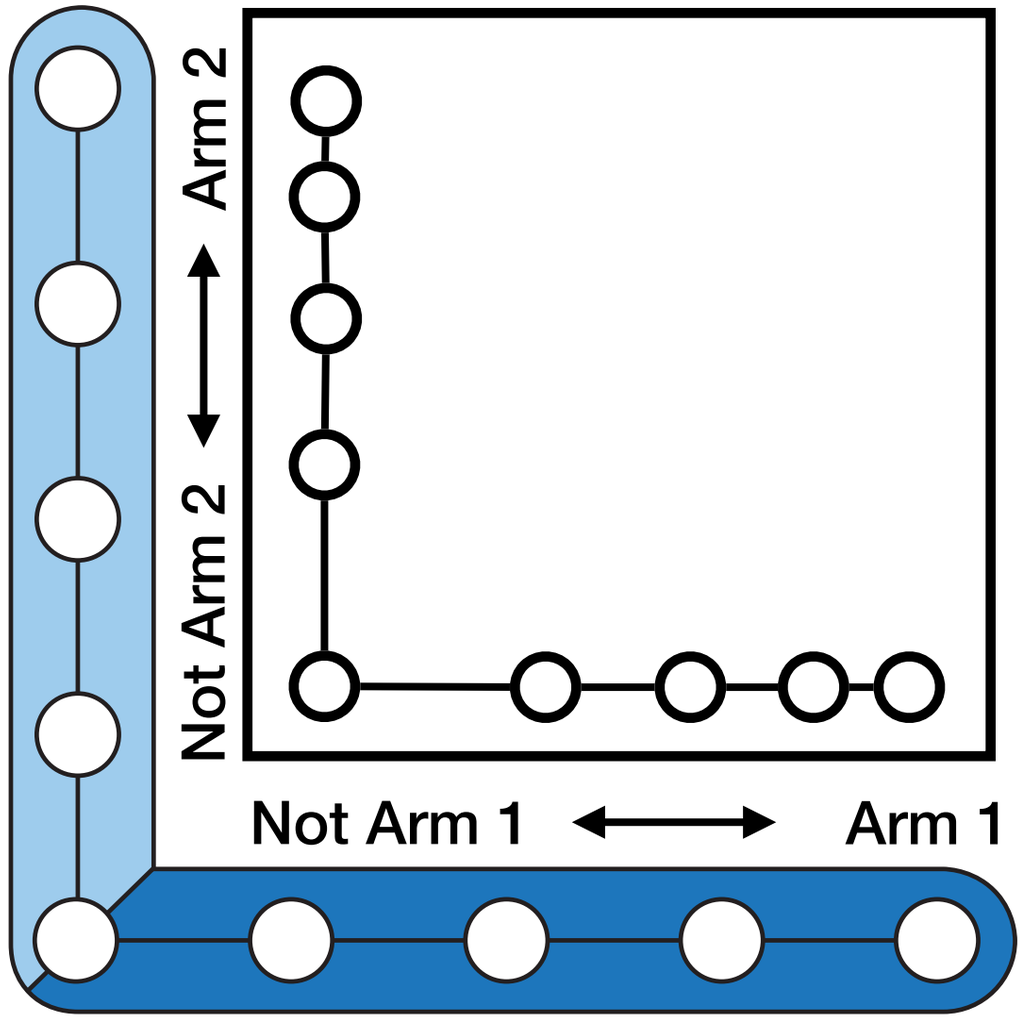


Explore representation space
Extract sequence of actions that explores an axis in representation space



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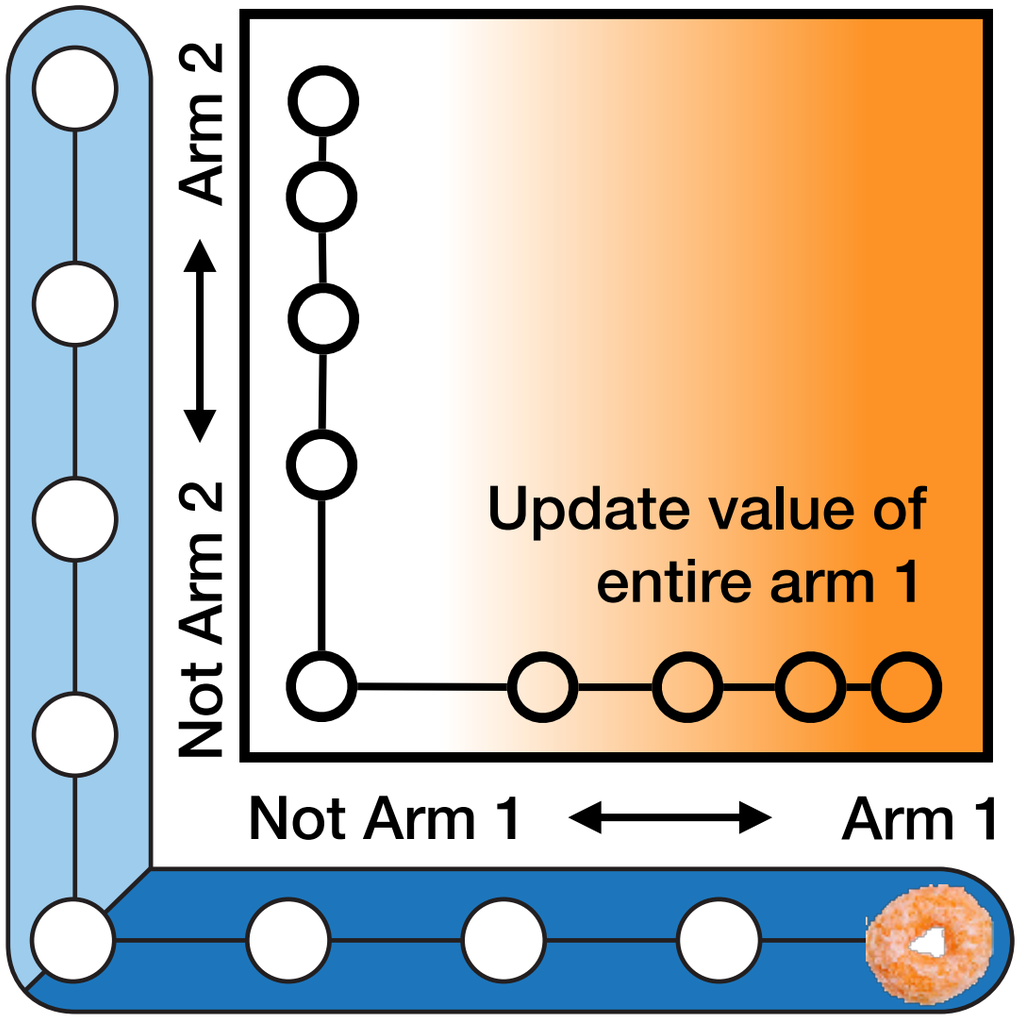

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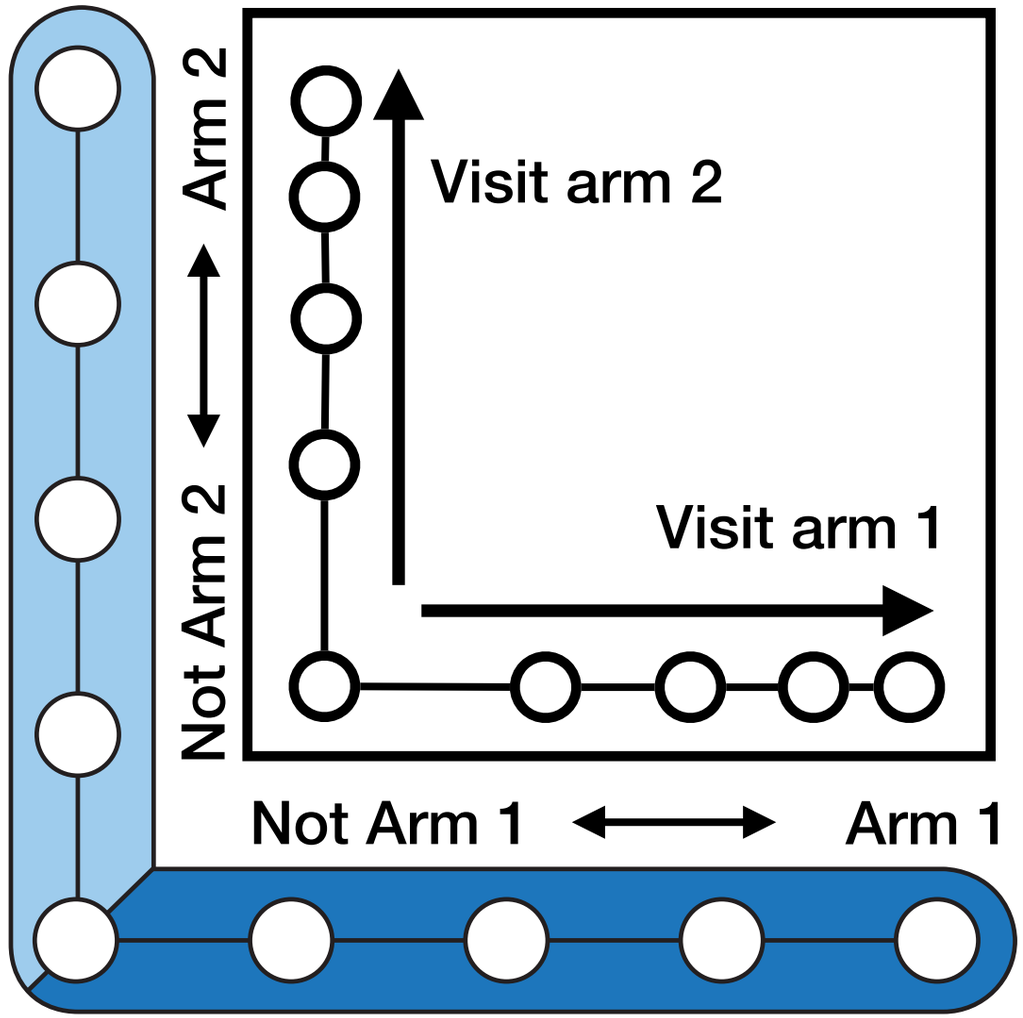

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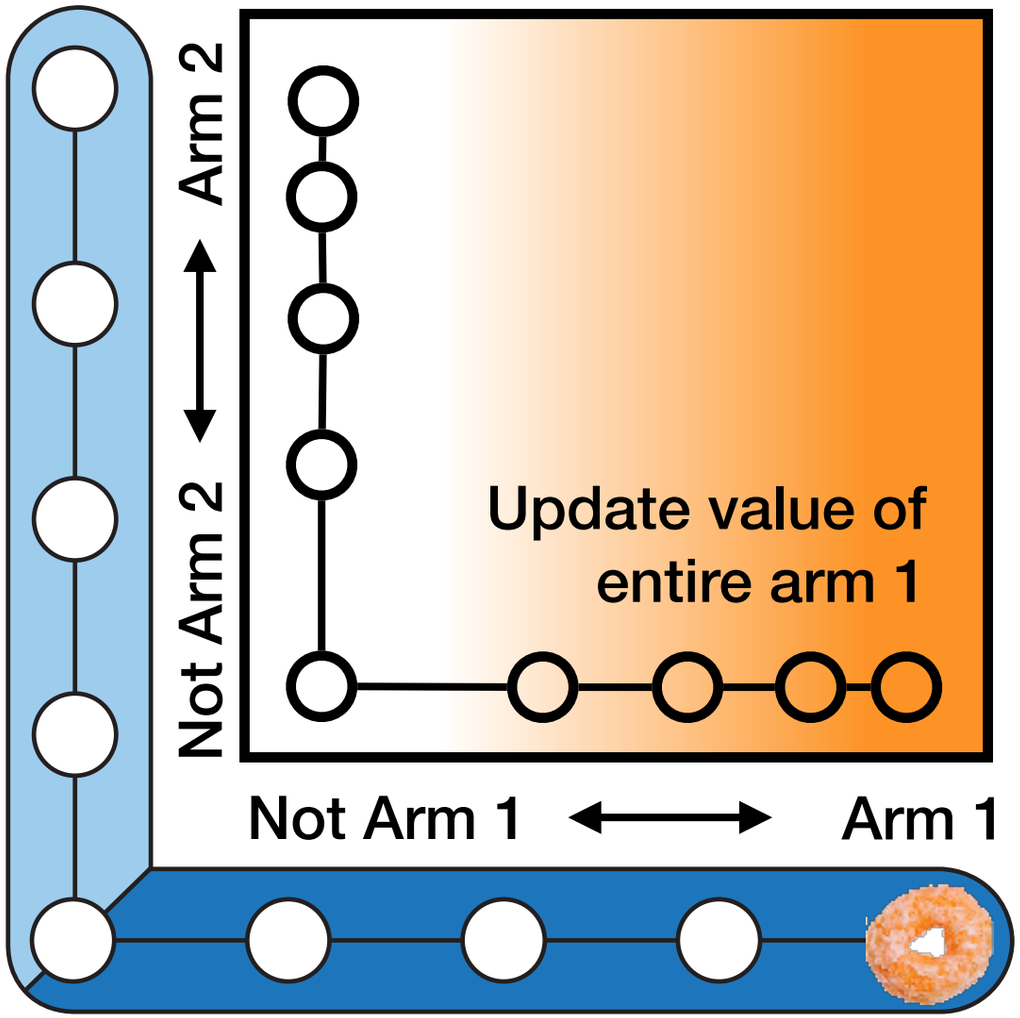

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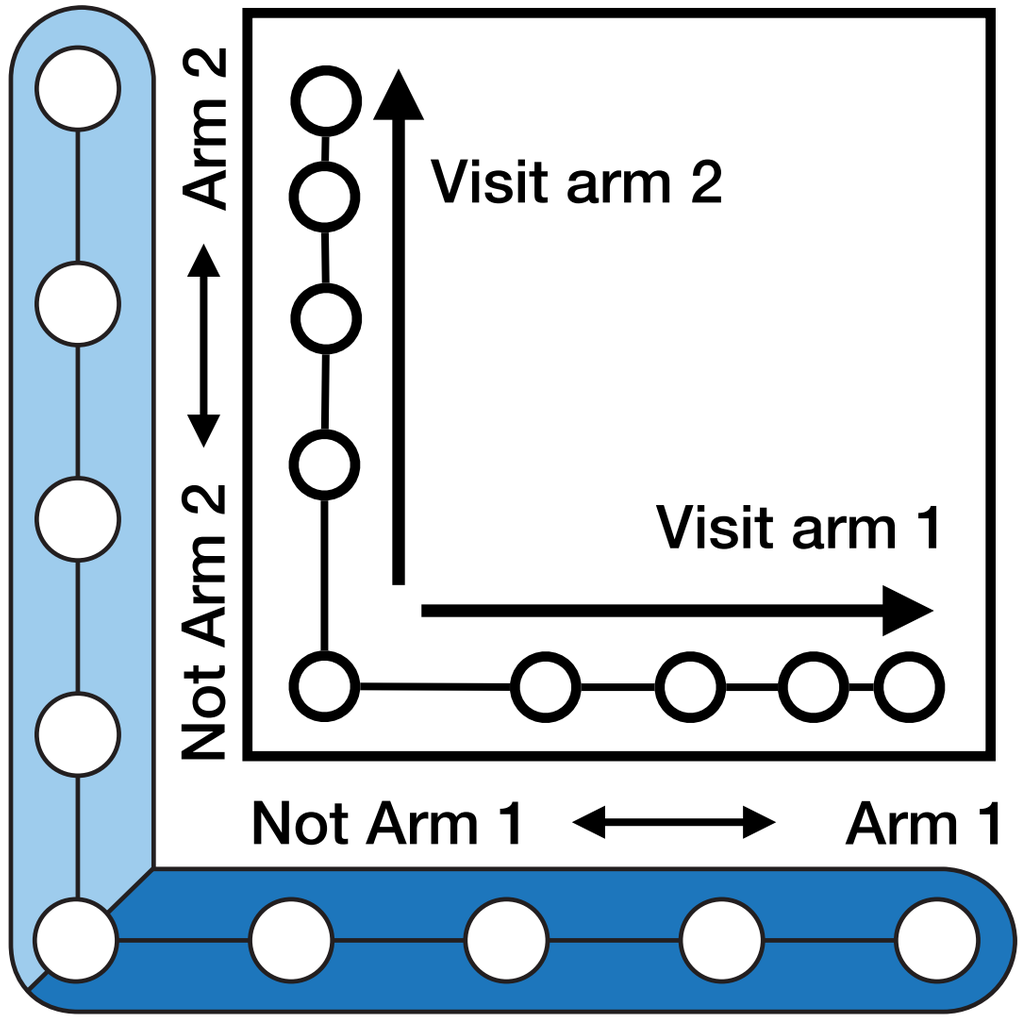
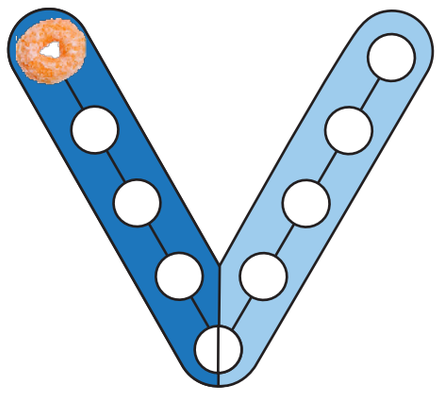


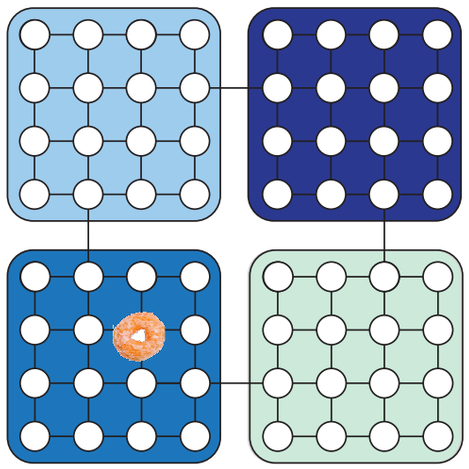
Representation learning goal for both:
cluster states that **generalize**
separate states that **do not**

What makes a good representation?

Maze arms



Rooms



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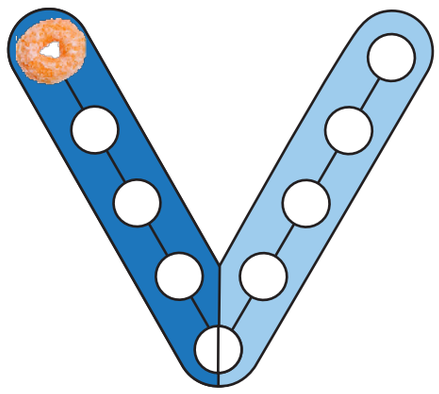

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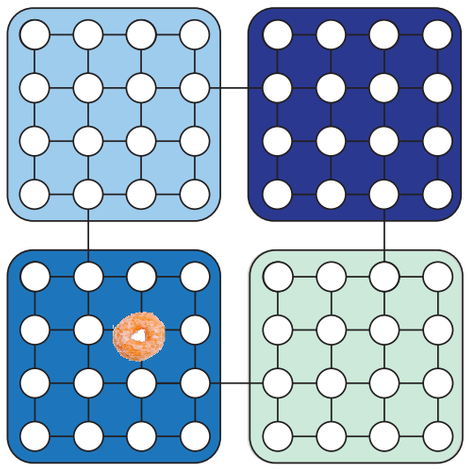
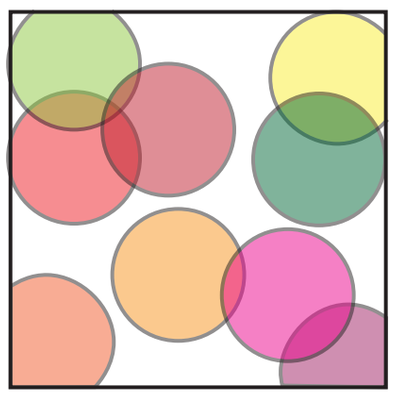
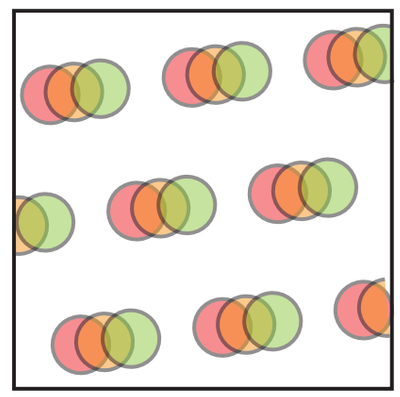


Place cells



Grid cells



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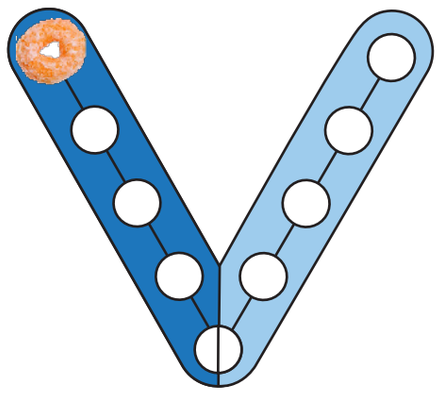
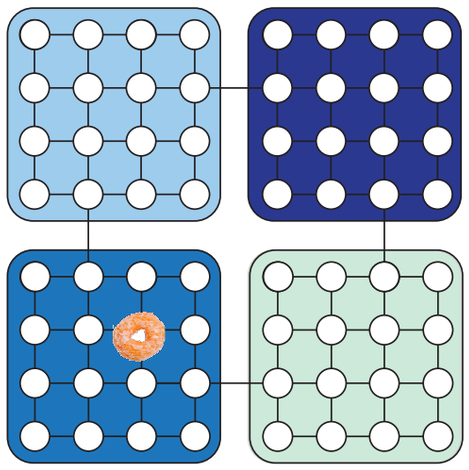
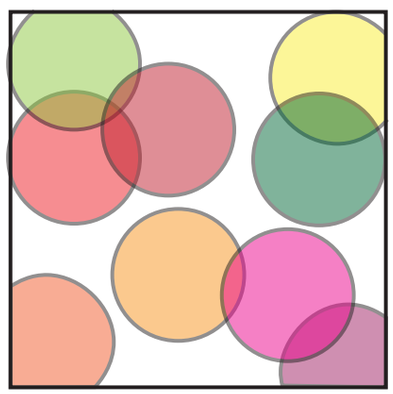


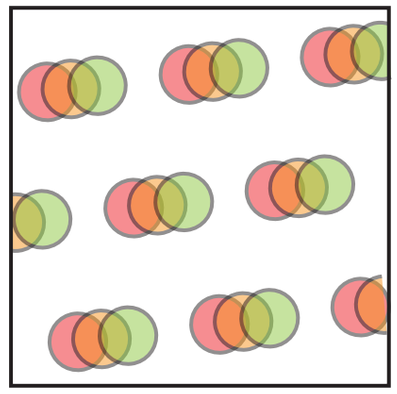
Rooms



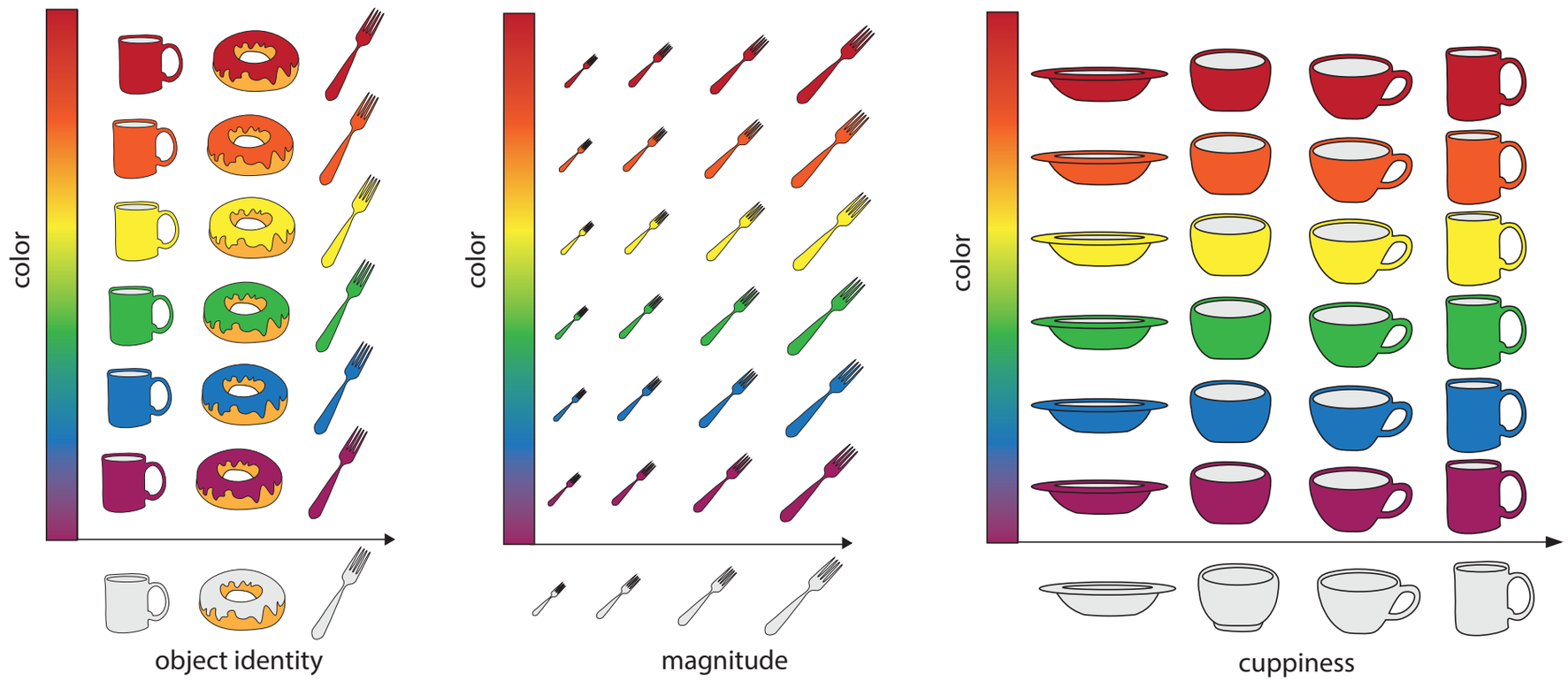
Place cells



Grid cells



Independent/disentangled/factorized components

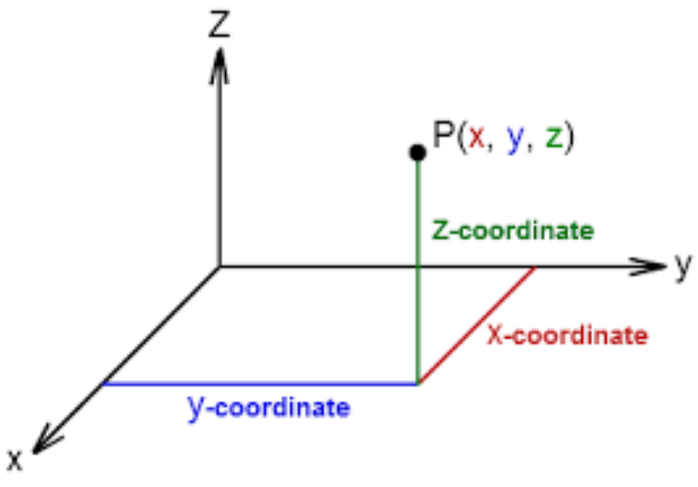


Behrens, Muller, Whittington, Mark, Baram, Stachenfeld, Kurth-Nelson (2018)

Spaces that aren’t Euclidean

Spaces that aren’t Euclidean

nD Euclidean
As in “of or pertaining to Euclid,”
that guy who did a lot of geometry
before fancy math was invented



hi

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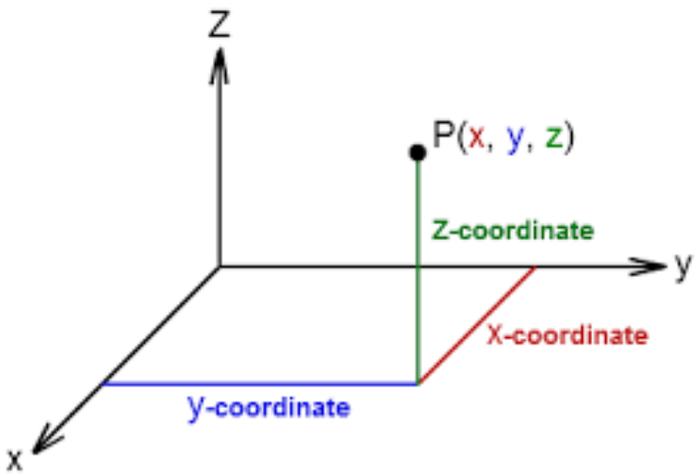
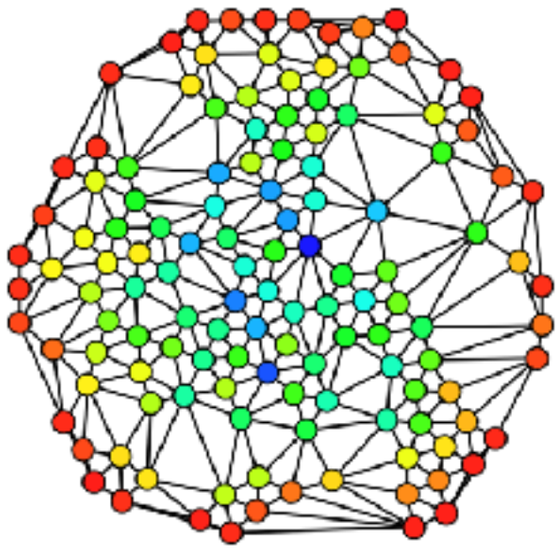
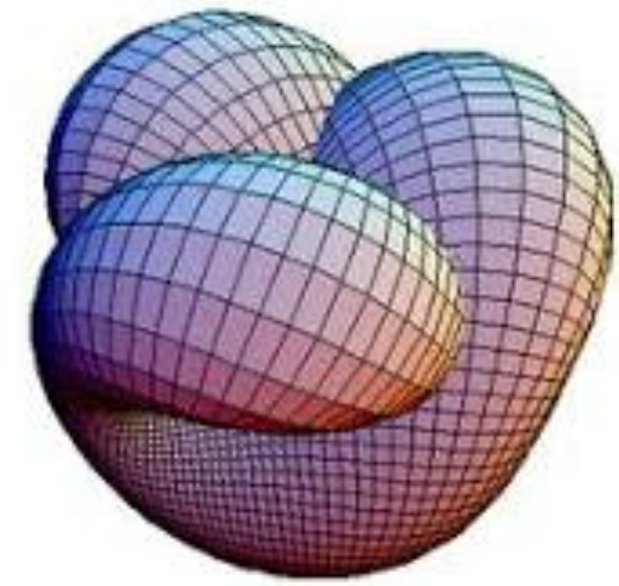


hi

Non-Euclidean
Fancy math spaces made out of more arbitrary relations



Graphs
Nodes connected by edges



Manifolds
"Locally Euclidean"

What makes a good representation?

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Image from www.jessicayung.com

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Successor representations in hippocampus

Stachenfeld, Botvinick, Gershman (2017)

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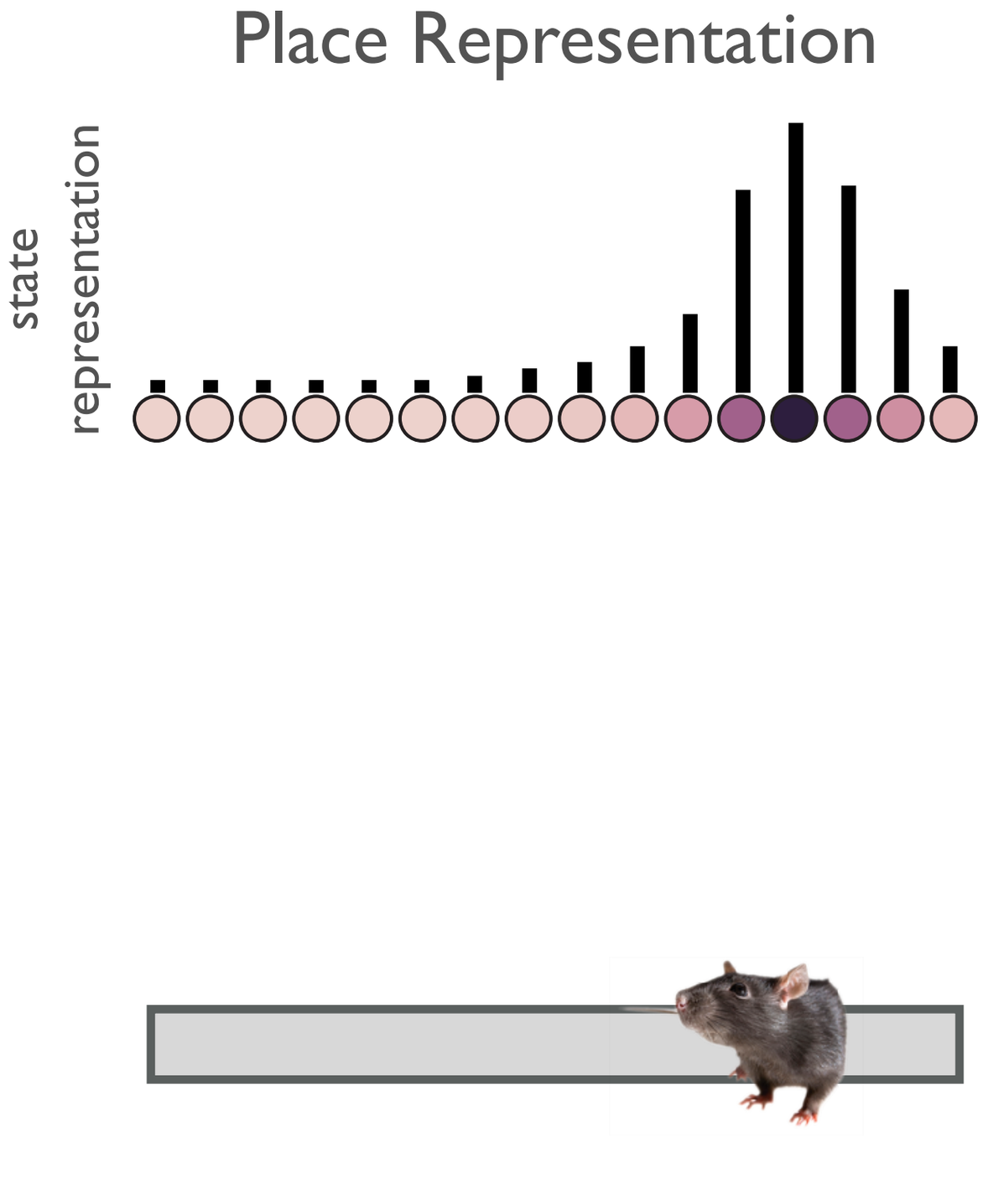

Stachenfeld, Botvinick, Gershman (2017)

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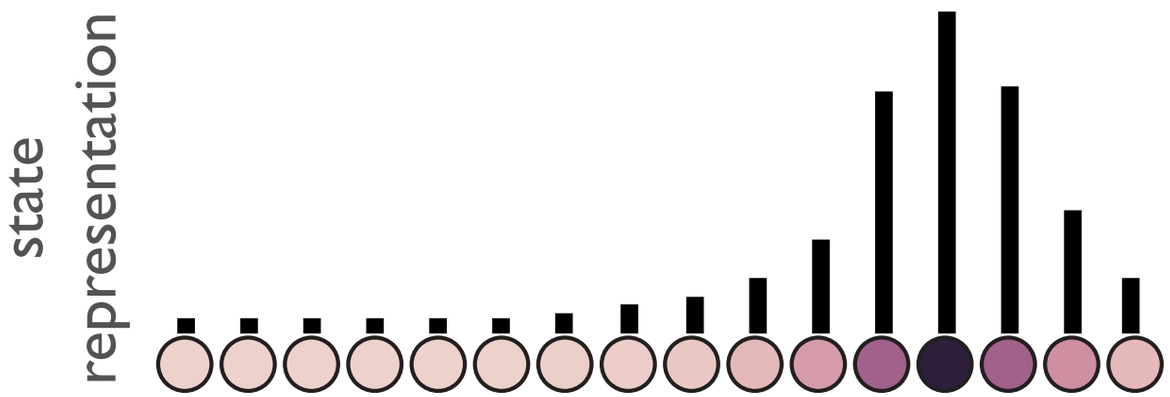
Place Representation

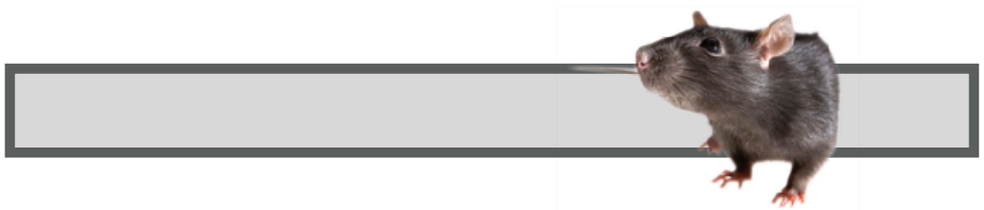


Firing of cell i
At location j

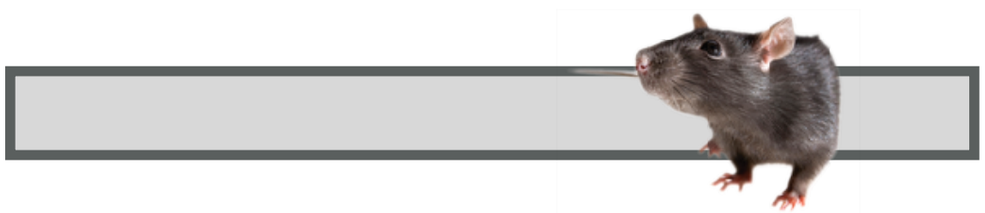
Cell i encoding distance from μ_i

$$Ae^{-\left(\frac{x_j-\mu_i}{\sigma}\right)^2}$$

Euclidean Gaussian with center i
centered at μ_i, width σ, height A



x_j

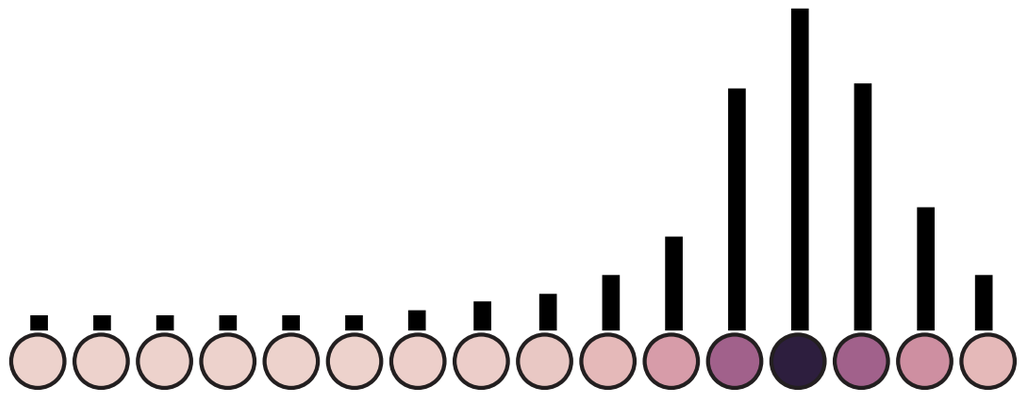


S_j

Successor representations in hippocampus

Place Representation

state representation



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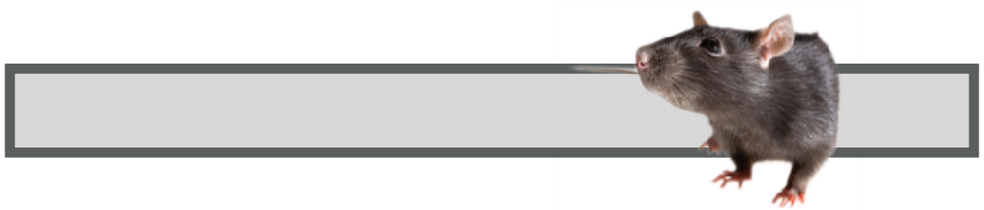
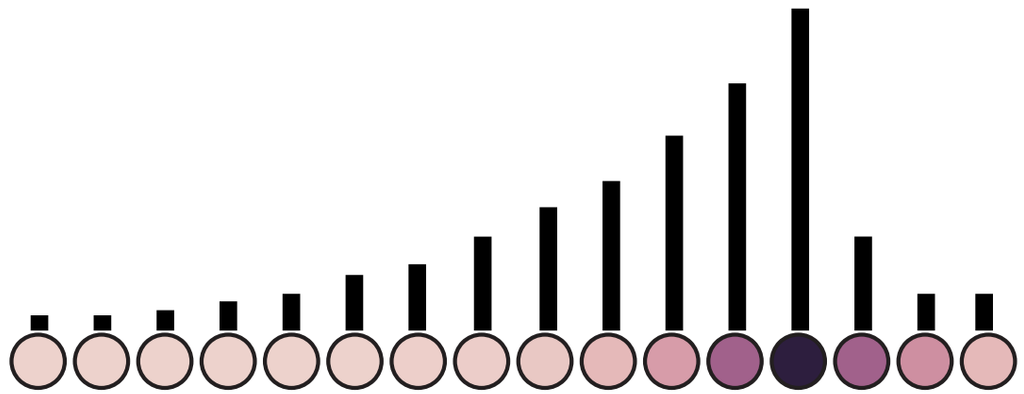


x_j

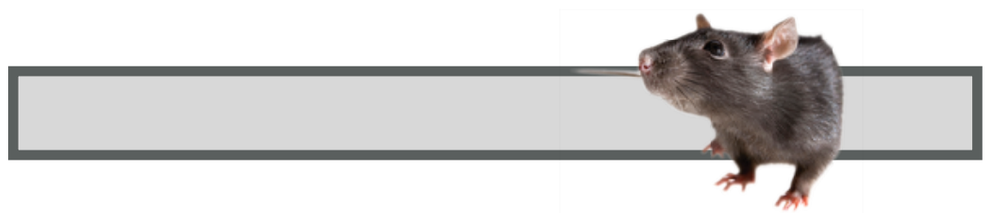
Successor Representation



Cell i encoding predictions about state i

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\gamma^t\mathbb{I}(s_t=i)\mid s_0=s_j\right]$$

Expected #visits to state i
Discount γ



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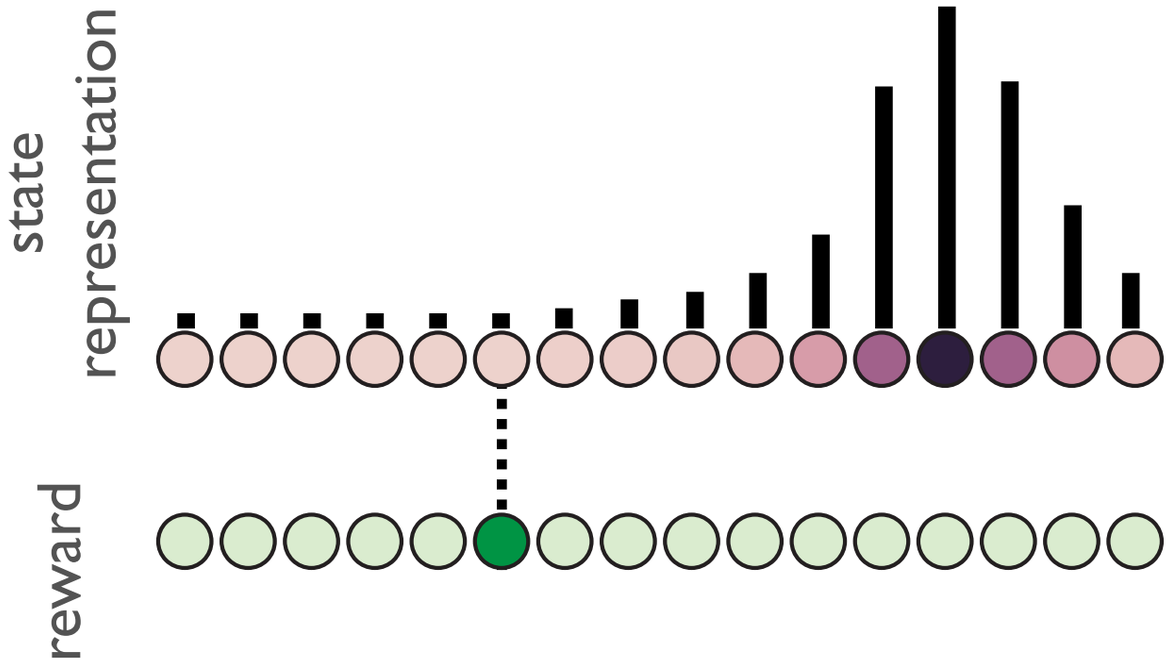
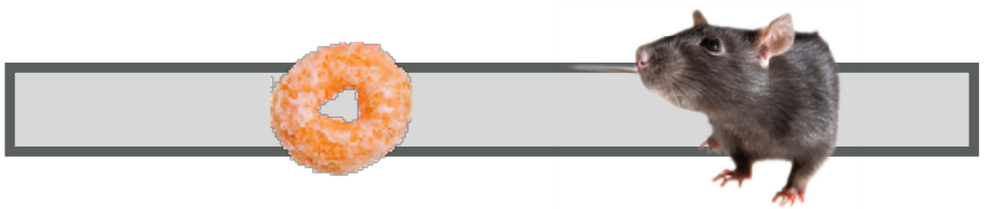


rep x reward =
current reward



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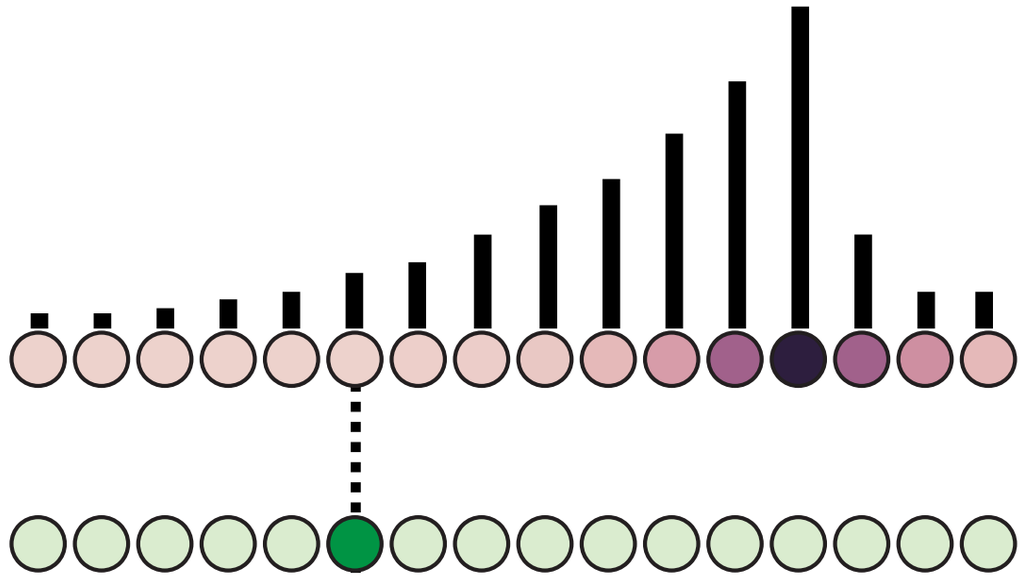
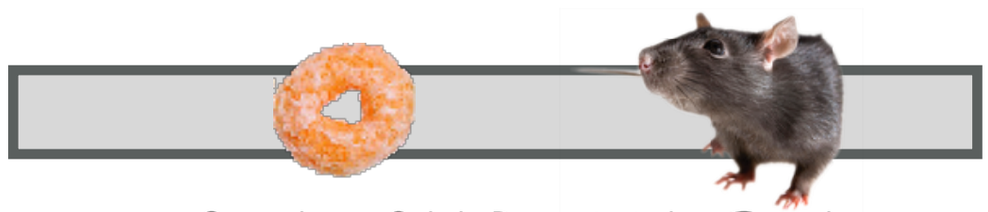


rep x reward =
exp. **future** reward =
Value



Stachenfeld, Botvinick, Gershman (2017)

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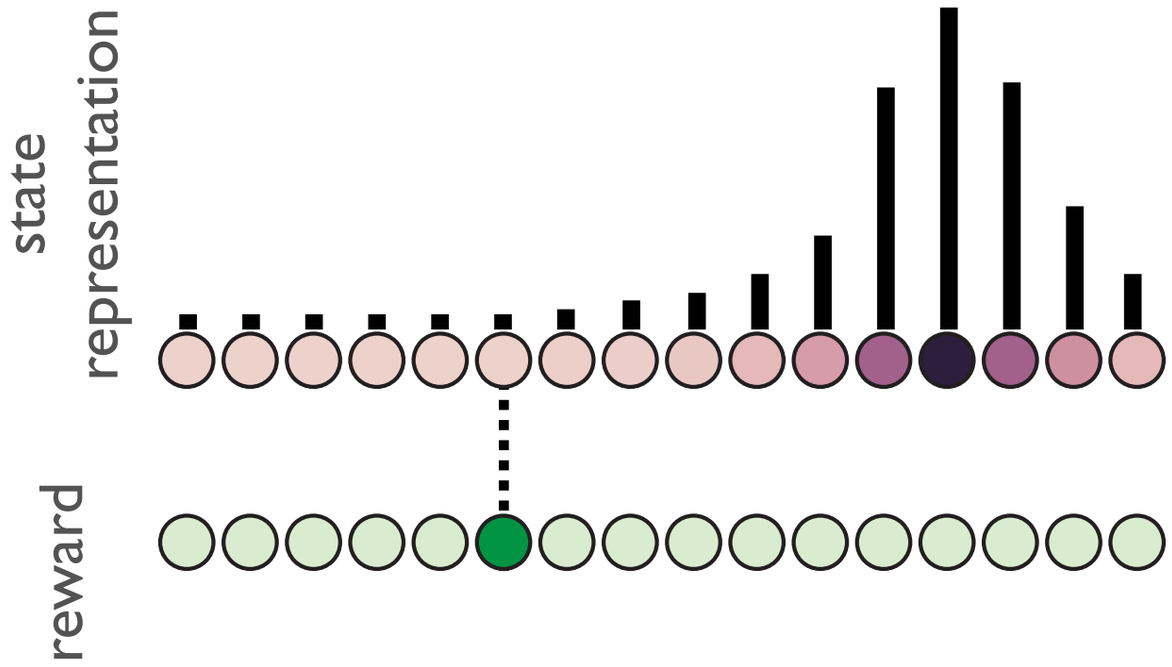

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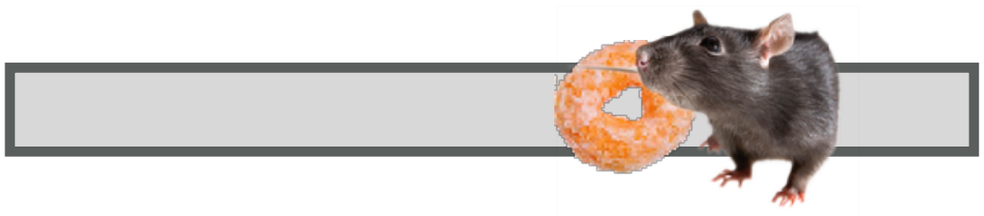

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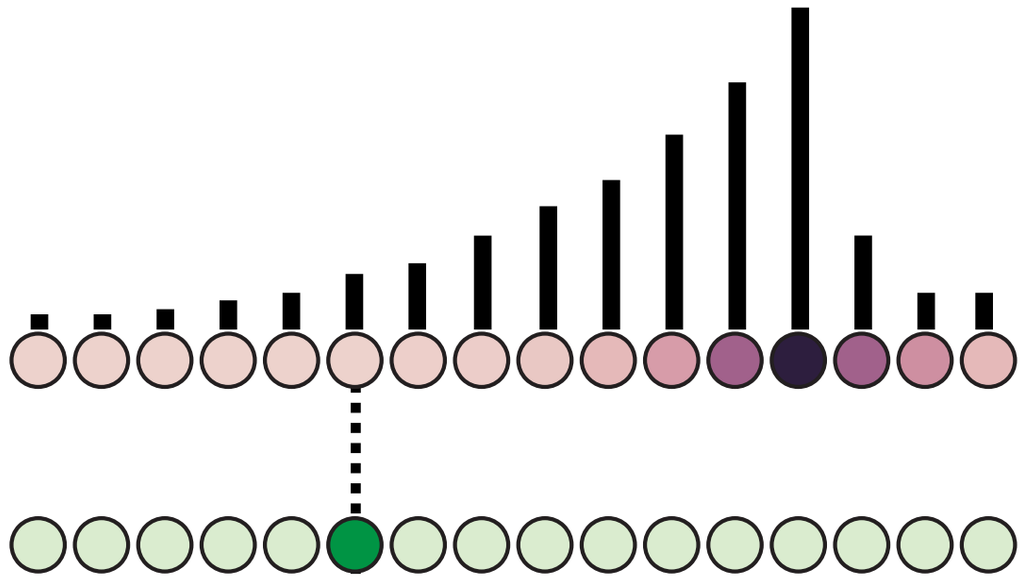

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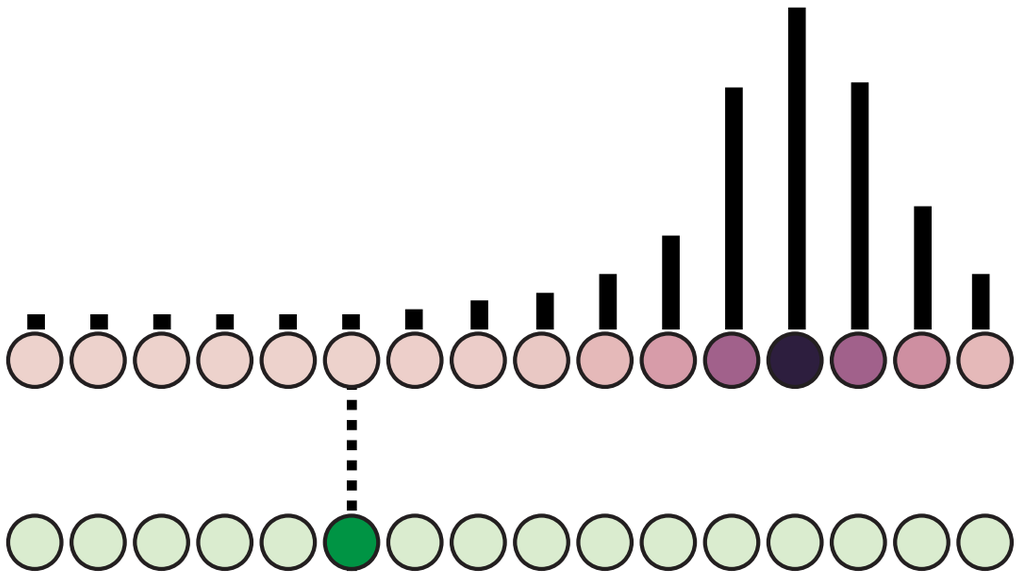

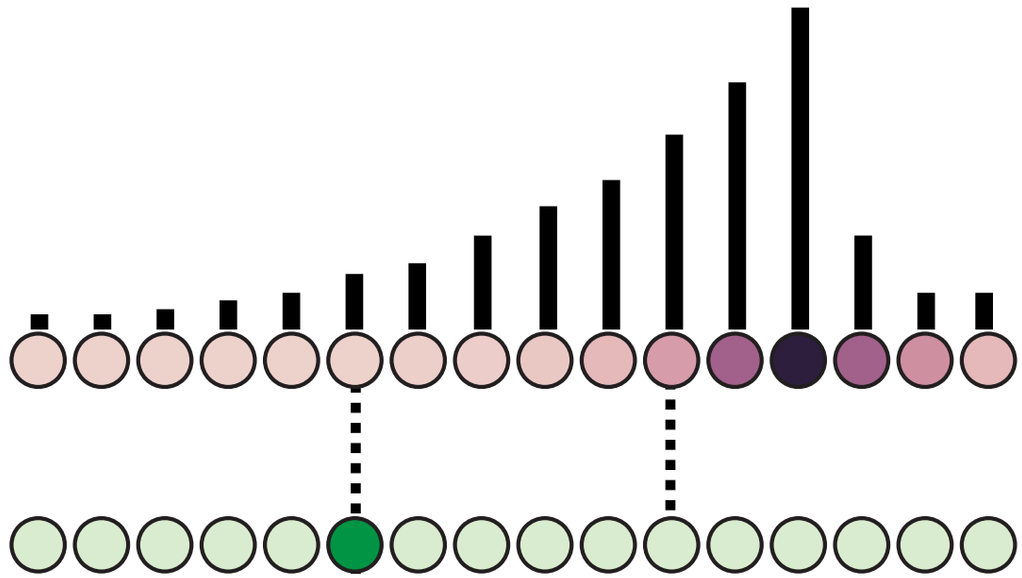

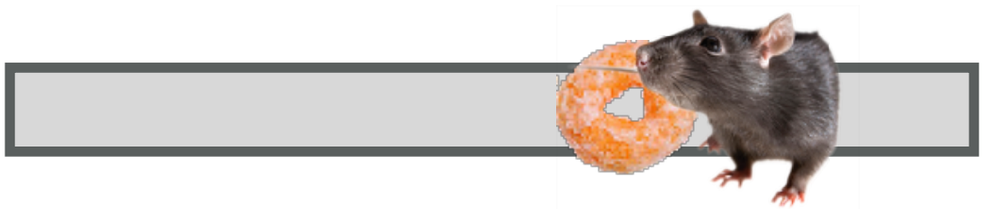

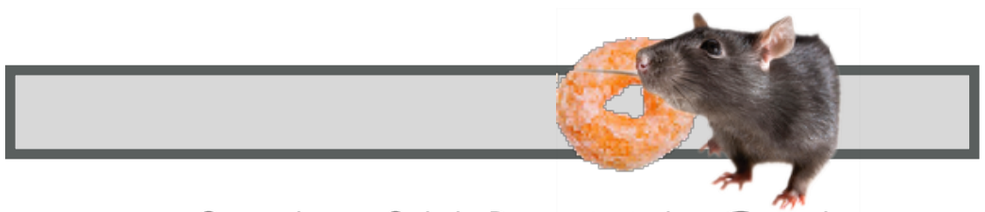

Stachenfeld, Botvinick, Gershman (2017)

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- **Important limitation:** still not as flexible as model-based planning

Some simulations

Stachenfeld, Botvinick, Gershman (2017)

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- Asymmetric experience dependent expansion (Mehta et al 2000)

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- See also: Russek et al (2017), Momennejad et al (2017), Garvert et al (2017), Bellmund et al (in prep)

Stachenfeld, Botvinick, Gershman (2017)

What else is happening with the SR?

- Make it deeper! (Kulkarni et al 2016)
- Make it transfer better! (Barreto et al 2016, “USFAs” Borsa et al 2019, Geerts et al (2019) CCN, Madarasz (2019) Cosyne)
- Integrate it with model-based rollouts and/or replay! (Momennejad et al (2017), Russek et al (2017))
- Make it probabilistic! (Geerts et al (2019) CCN, Vertesz et al (2019) Cosyne)
- Make it multiscale! (Stachenfeld et al 2017, Momennejad & Howard 2018 biorXiv)

Make it transfer better! cont.

There's been a lot of recent work about improving SR transfer when policy changes following a reward change. Here are some quick notes on these (added posthoc). Note that replay/SR-DYNA is also a way to do this — roughly the tradeoff is that replay takes more time to update but the below take more space.

Barreto et al 2016: procedure for learning a whole bunch of successor feature matrices and approximating a new one by linearly combining the old ones as a function of the new reward vector. a successor feature matrix is like a successor representation, except each row/column is a feature, not a state, and each entry says how much each feature predicts each other feature.
https://arxiv.org/abs/1606.05312

Universal Successor Feature Approximators ("USFAs") Borsa et al 2019: basically learn a set of features that can be recombined (linearly or by neural network) to approximate a new successor feature representation as a function of the new goal. Kind of like the previous one, except you're learning the best features to keep rather than just making the best of whatever library of successor feature matrices you've kept.
https://arxiv.org/abs/1812.07626 (about USFAs)
https://arxiv.org/abs/1901.10964 (about using them)
https://arxiv.org/abs/1804.03758 (related: Universal successor representations)

Madarasz (2019) Cosyne): learn a bunch of successor representations corresponding to different rewards, cluster the rewards, use a new reward to infer the appropriate successor representation (similar to USFAs and USRs but set up differently, more probabilistically rather than neural networkily/linearly, and related to hippocampal splitting data).
https://arxiv.org/abs/1906.07663

Geerts et al (2019) CCN abstract <-- done by a student I'm advising, so minor plug: a probabilistic approach that infers a distribution over SRs (rather than just the mean expected SR) using Kalman Filtering. Uses covariance matrix for updates, which can capture both uncertainty about an entry and dependencies between different entries in the matrix, meaning you can update correlated predictions together.

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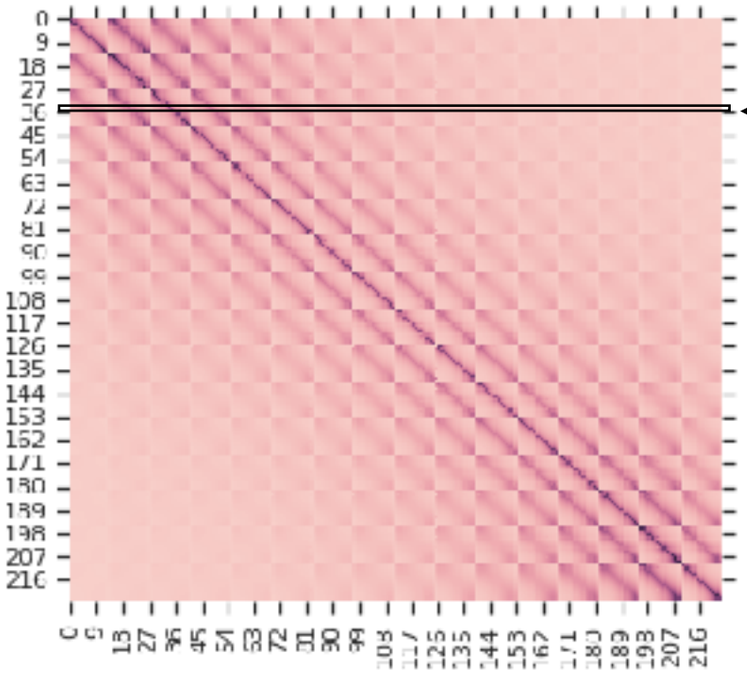
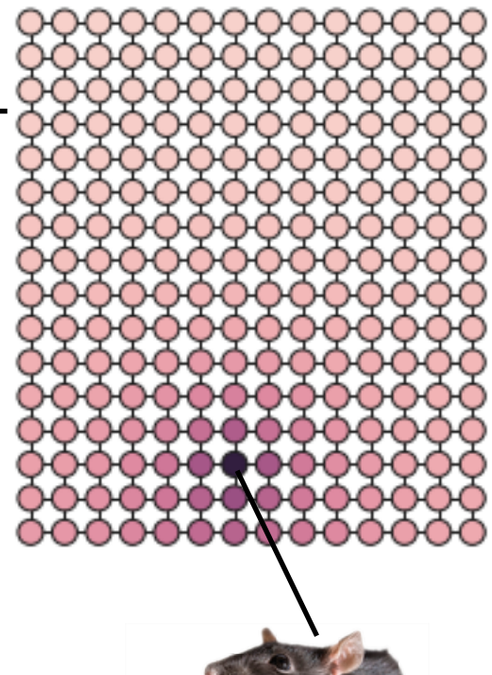
Dimensionality reduction

Successor Representation Matrix

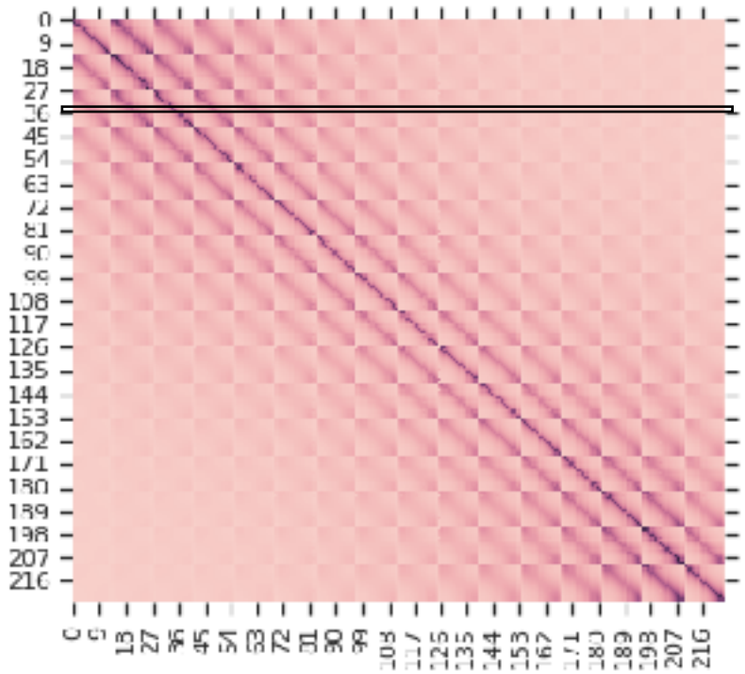


e.g. SR at state 34



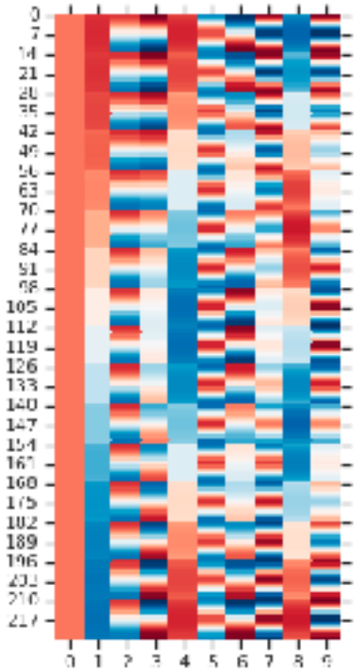
Dimensionality reduction

Successor Representation Matrix M

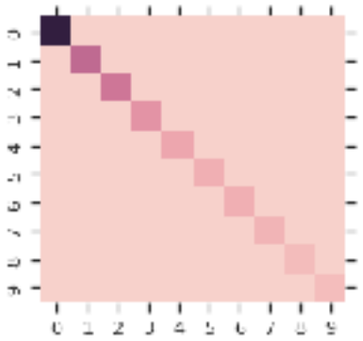


≈

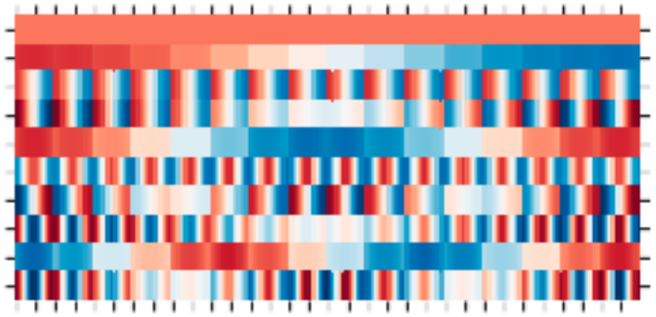
Eigenvectors



Eigenvalues

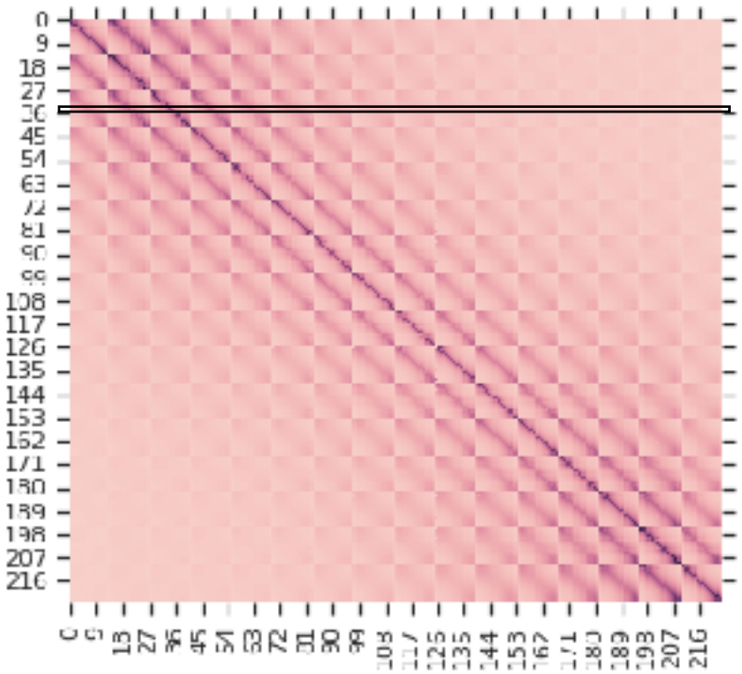


Eigenvectors



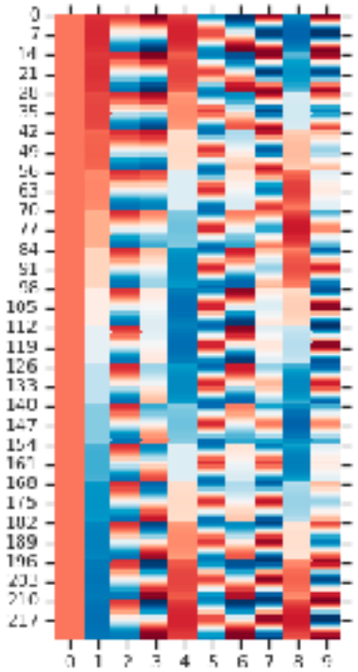
Dimensionality reduction

Successor Representation Matrix M

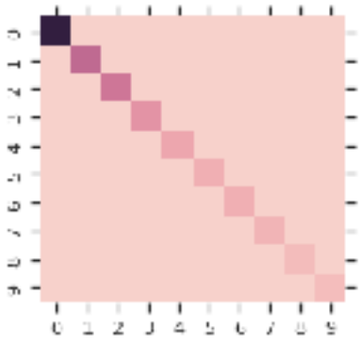


≈

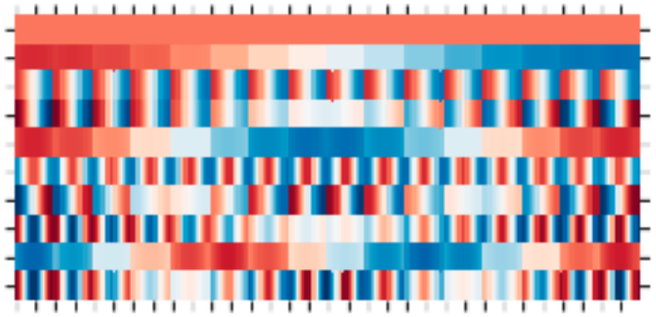
Eigenvectors



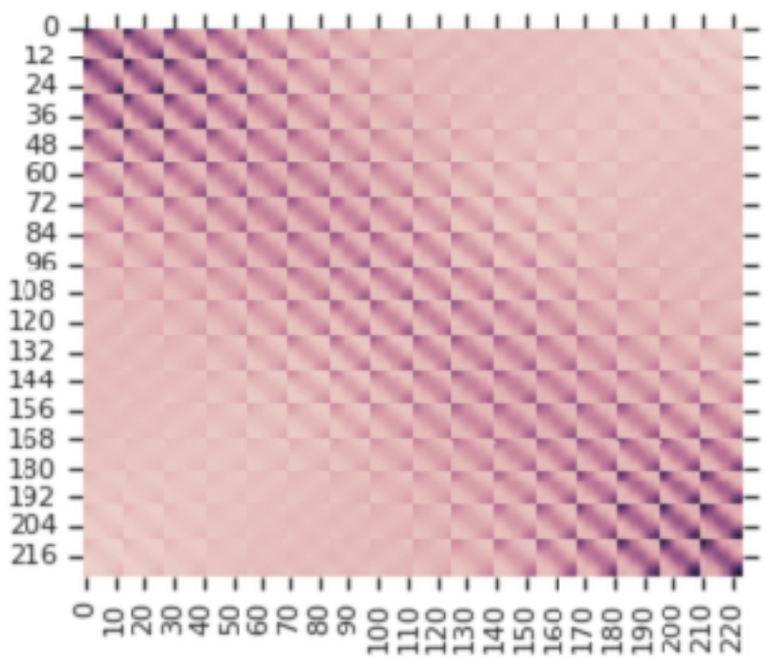
Eigenvalues



Eigenvectors

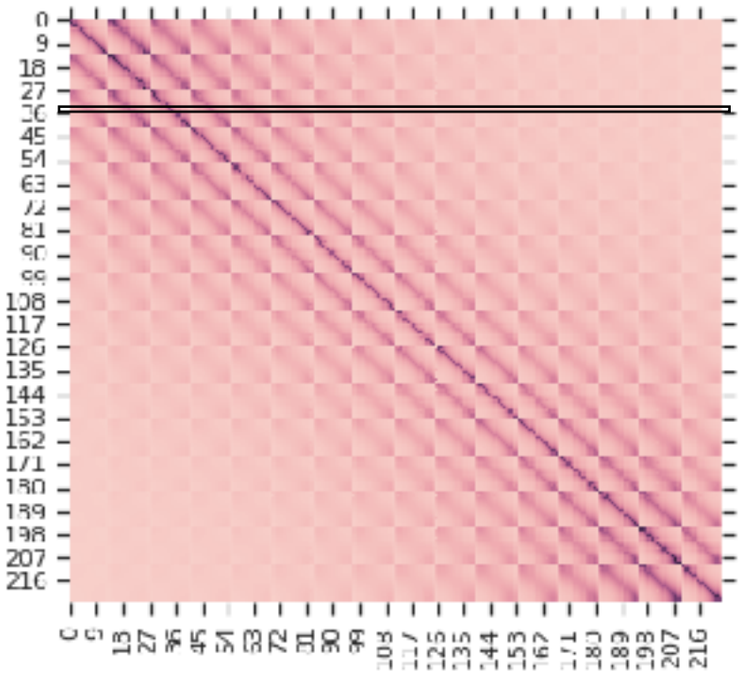


Reconstructed low-rank M



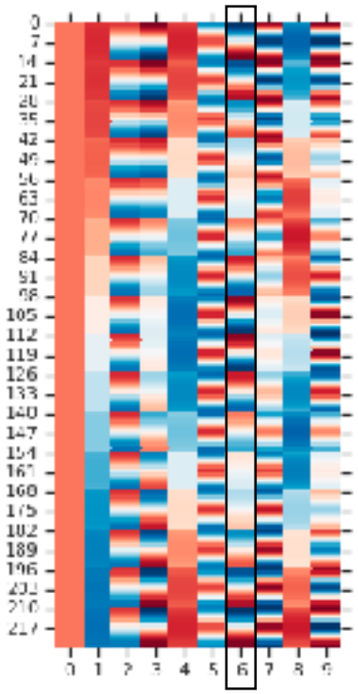
Dimensionality reduction

Successor Representation Matrix M

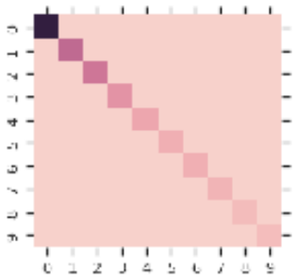


≈

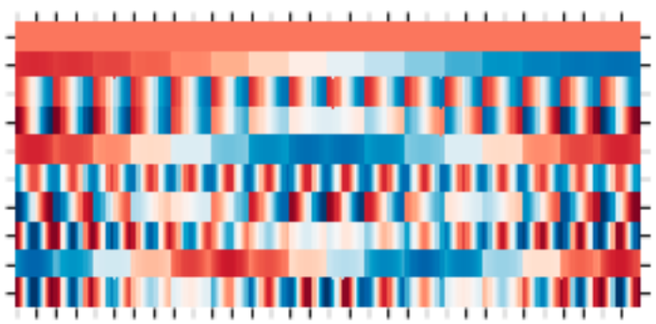
Eigenvectors



Eigenvalues



Eigenvectors



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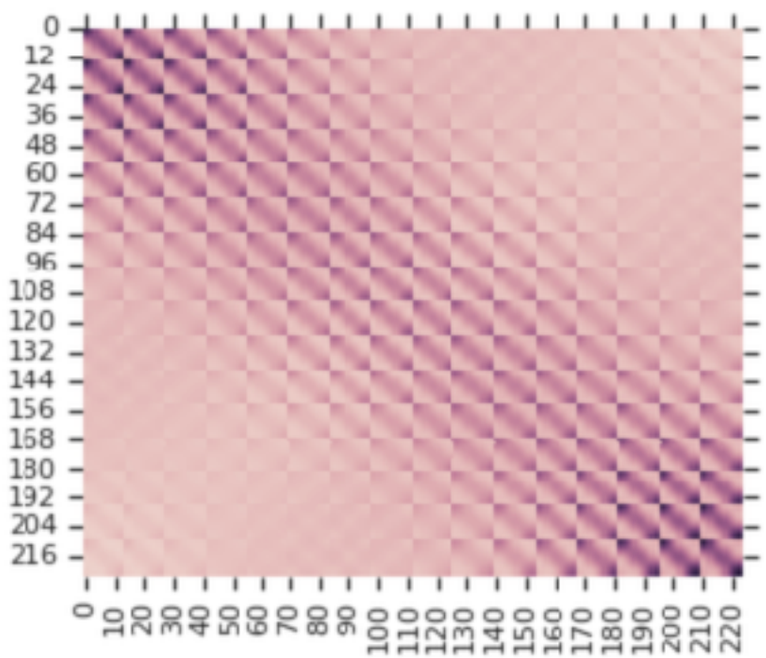
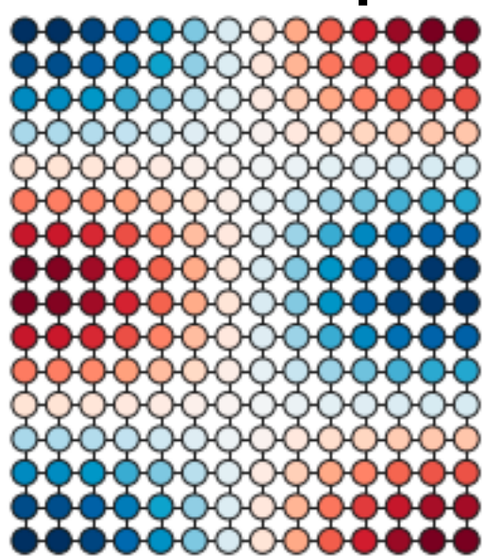


Eigenvector “receptive field”

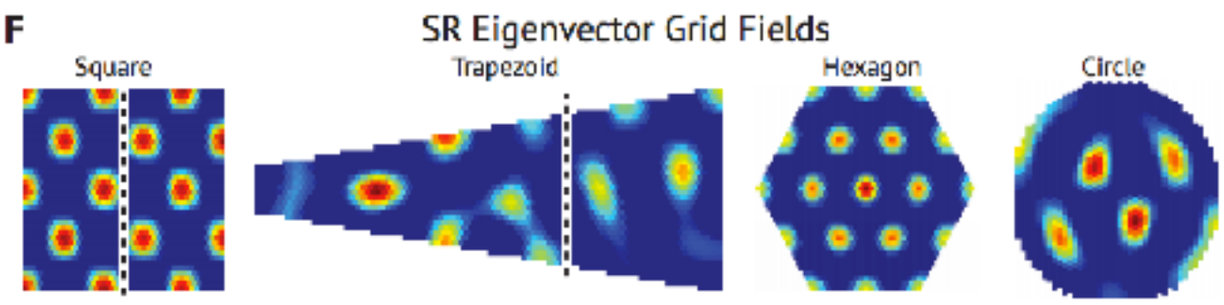


Simulating Grid Cells

Stachenfeld, Botvinick, Gershman (2017)

Simulating Grid Cells

- Grid fields in geometric environments
  Krupic et al. (2013)



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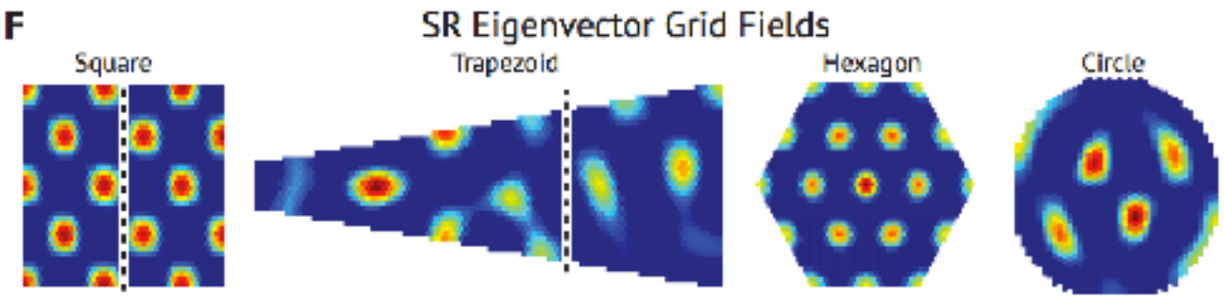

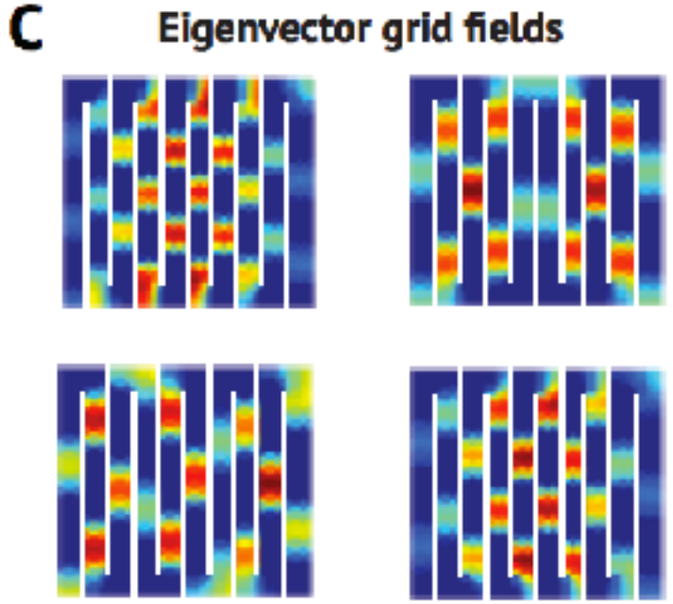

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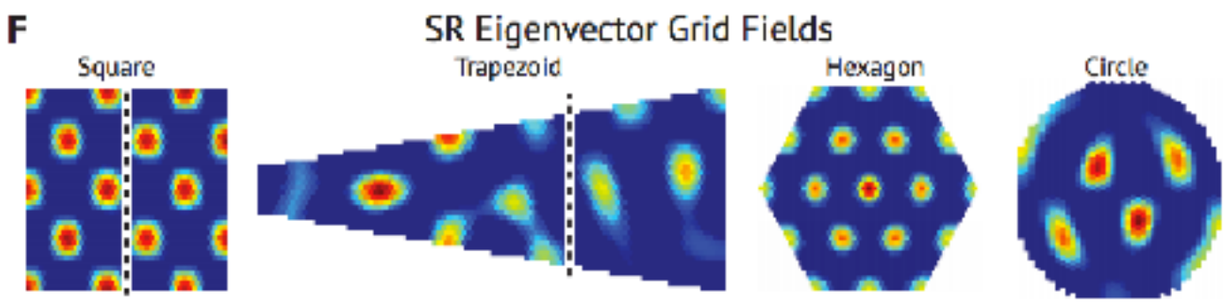

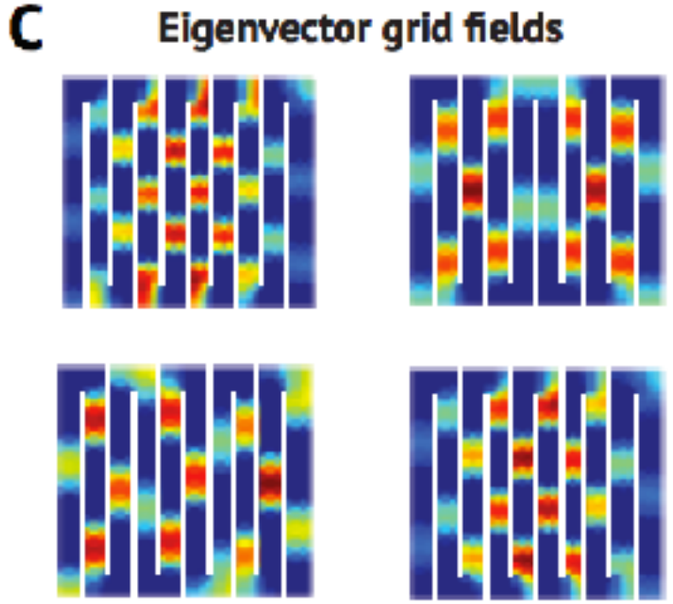

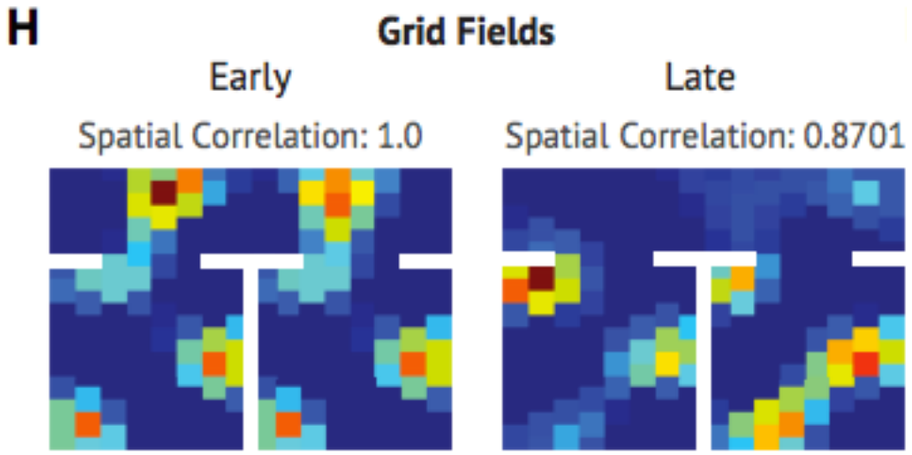

Stachenfeld, Botvinick, Gershman (2017)

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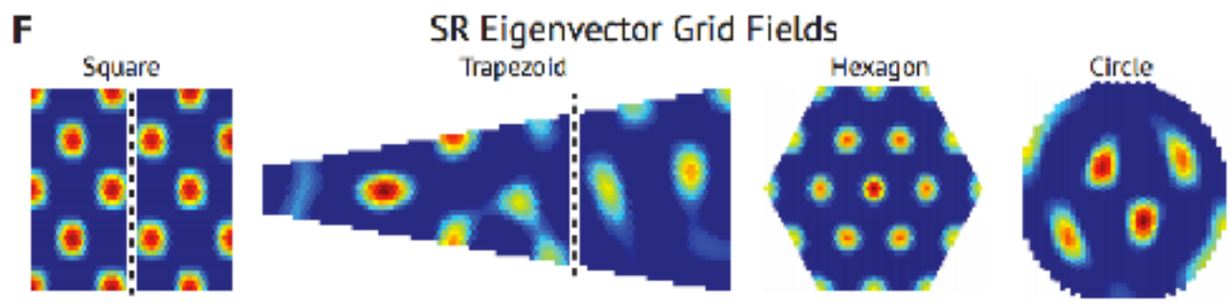
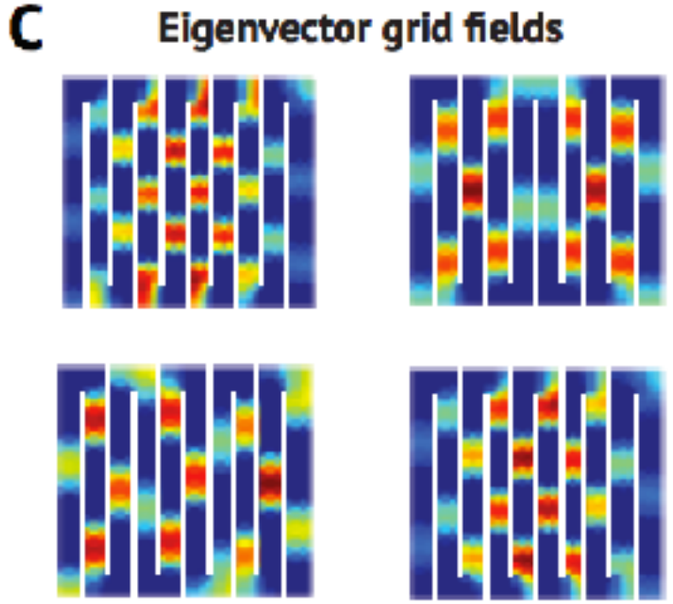
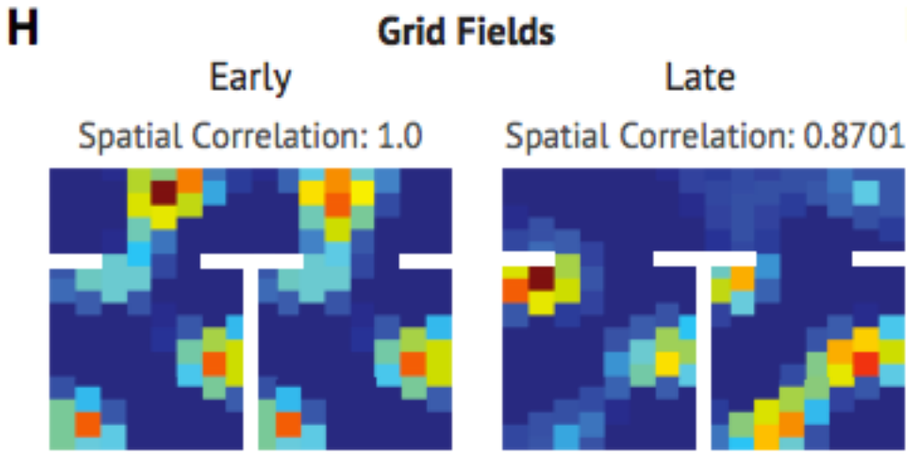
Takeaway: we capture many aspects of how environmental geometry and topology are known to affect grid cells.

Still missing some constraints



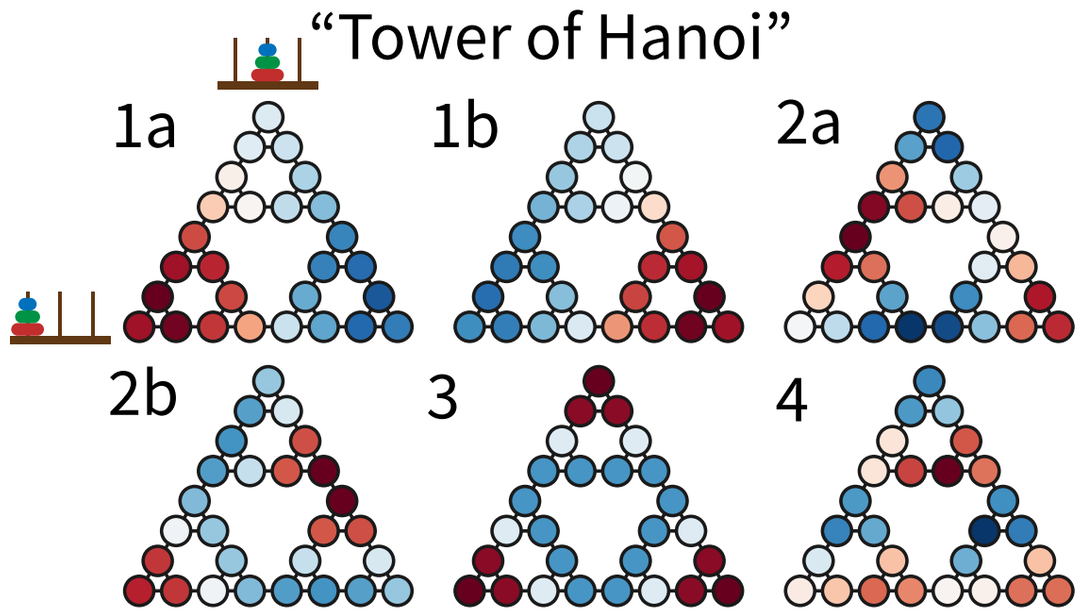


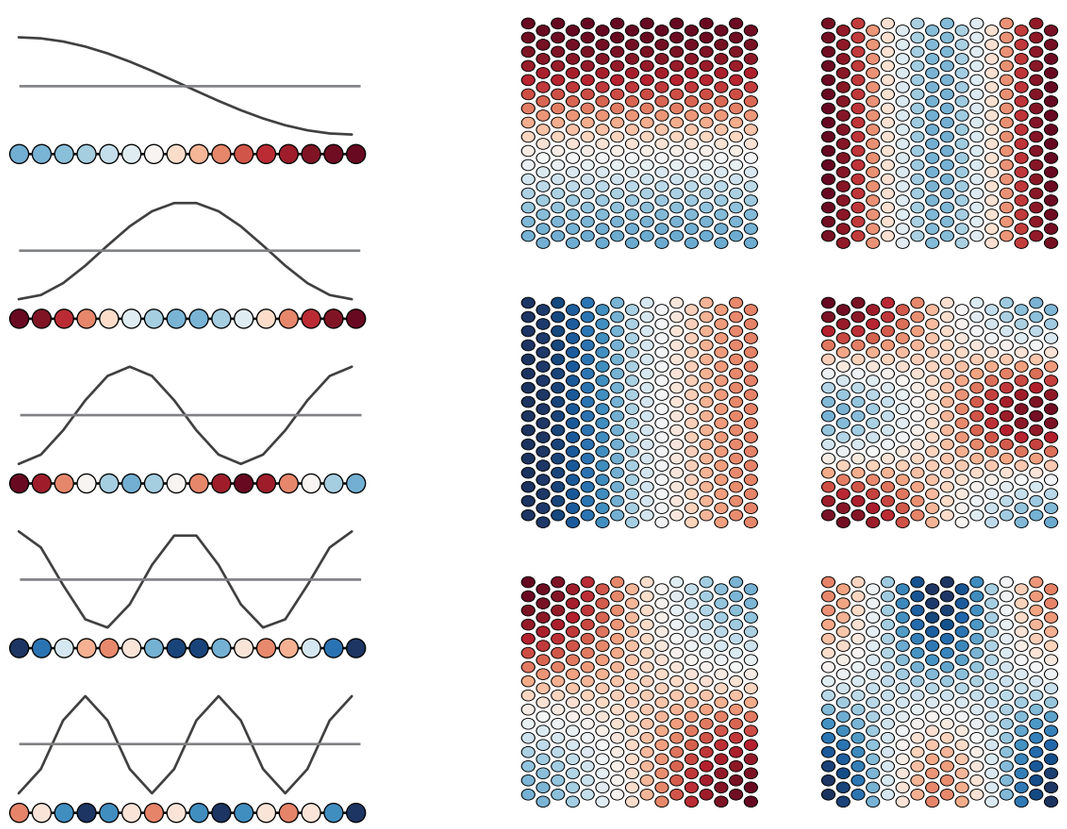


Stachenfeld, Botvinick, Gershman (2017)

These eigs have a lot of nice applications

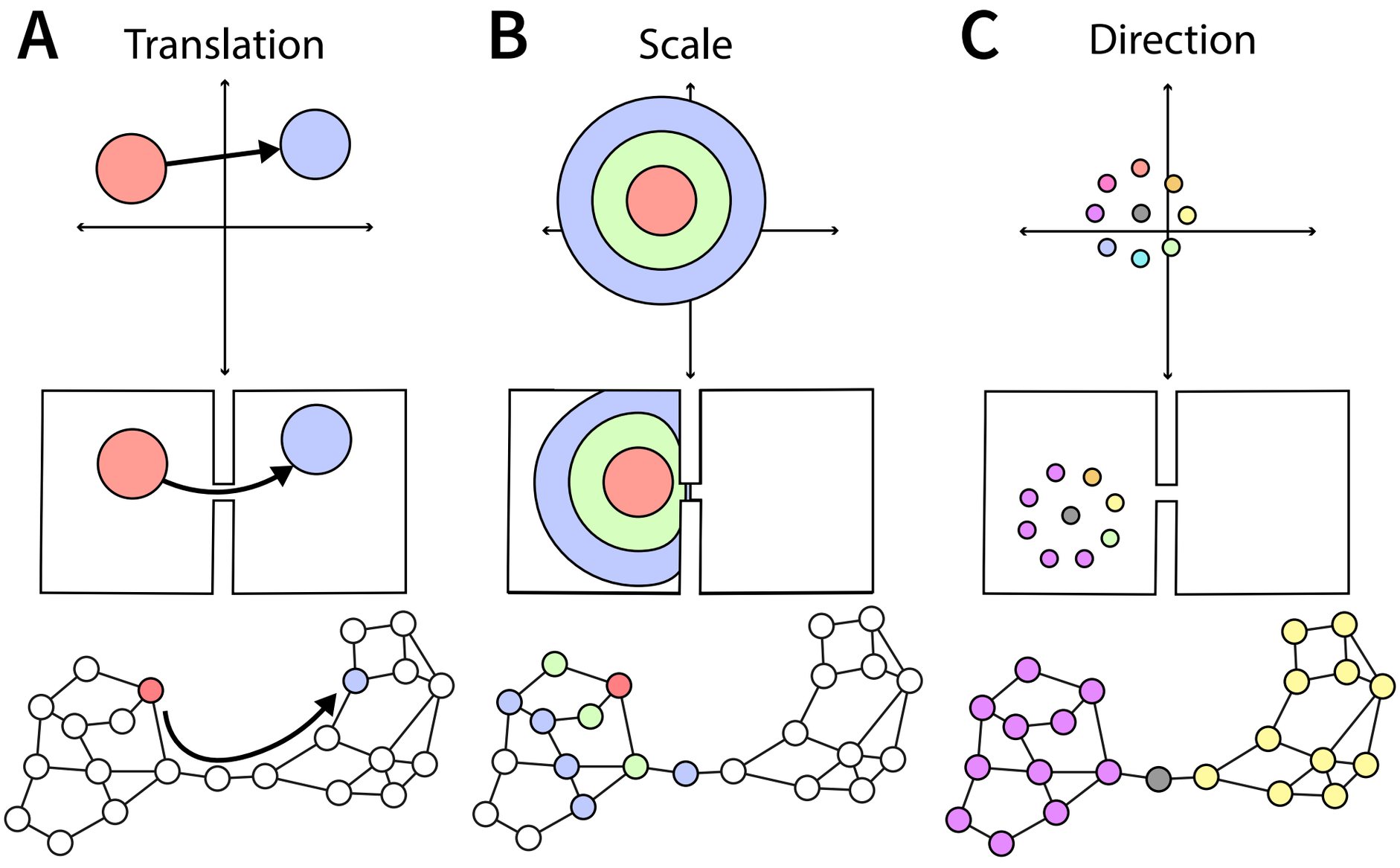
- **Sort of like PCA over transition structure**
  - Can help you smoothly approximate transition structure
- **Generalization of Fourier to graphs**
  Generalize spatial features to arbitrary geometries





Stachenfeld, McNamee, Behrens, Gershman, Botvinick (review article in prep)

Generalizing map ideas beyond the plane



Not obvious how these spatial constructs should apply to non-Euclidean spaces

But they do have meanings in Fourier space

Since eigenvectors = “graph Fourier”, this is a definition that transfers

Stachenfeld, McNamee, Botvinick, Gershman (in prep)

These eigs have a lot of nice applications

- **Sort of like PCA over transition structure**
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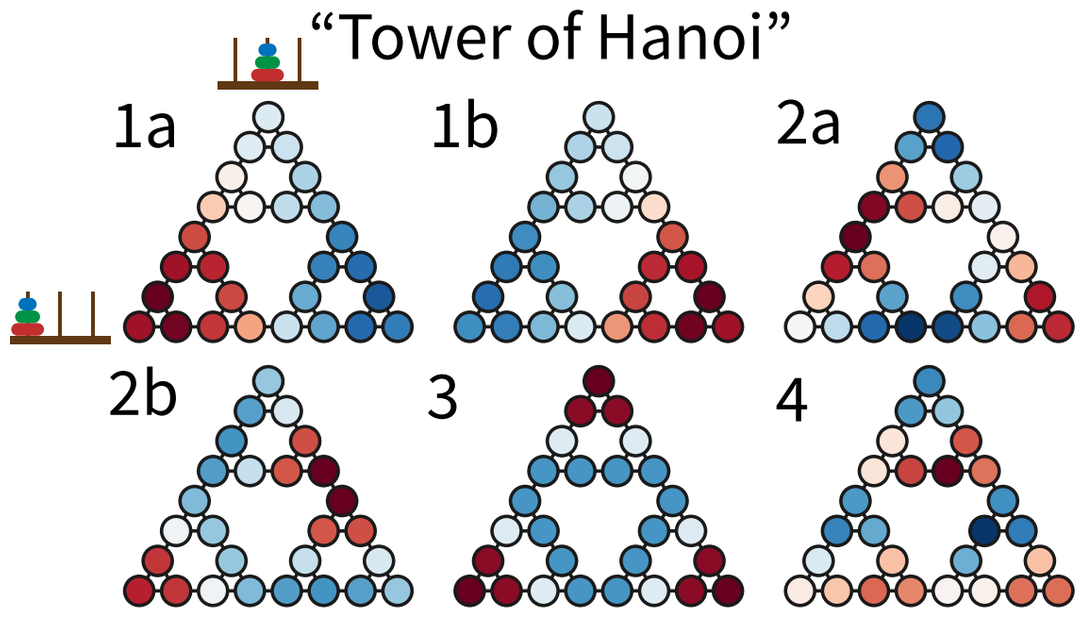
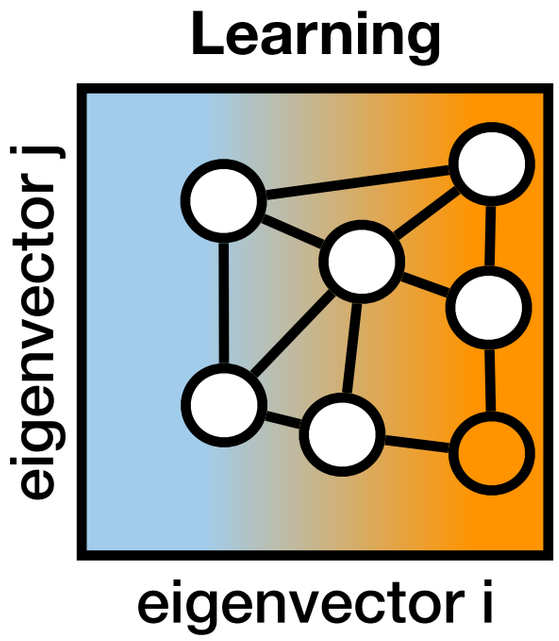
- **For supporting learning:** “Protovalue functions” (Mahadevan (2007))
  - Learn a value function over Laplacian eigenvectors instead of over states directly





Stachenfeld, McNamee, Behrens, Gershman, Botvinick (review article in prep)

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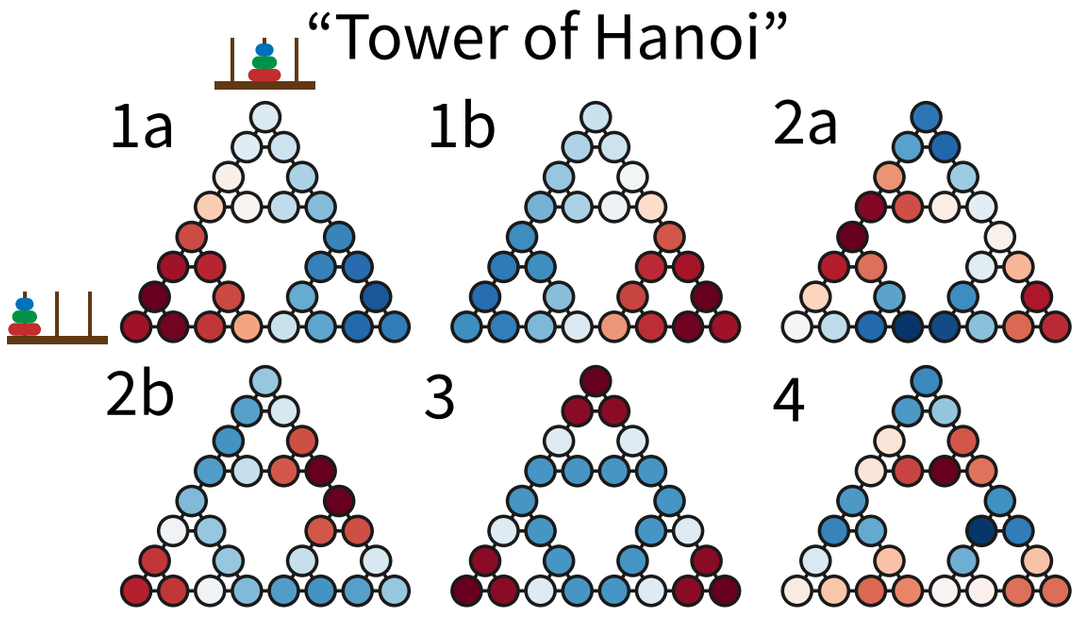
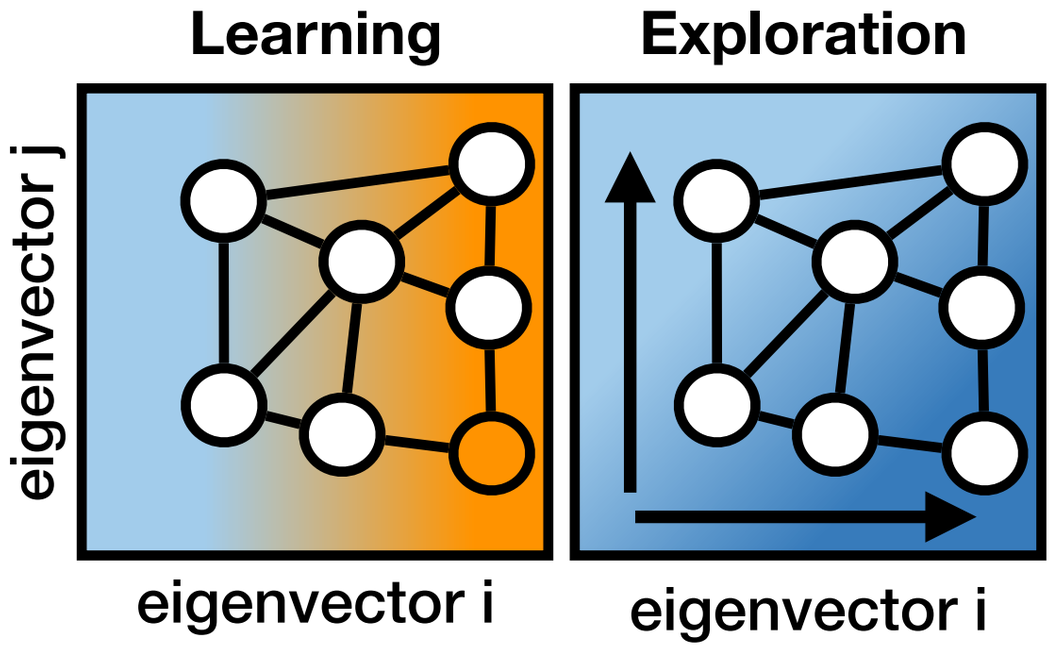
- **For supporting exploration:**
  “Eigenoptions”
  Machado et al (2017a,b—2018)
  - Use Laplacian eigenvectors as options for exploration

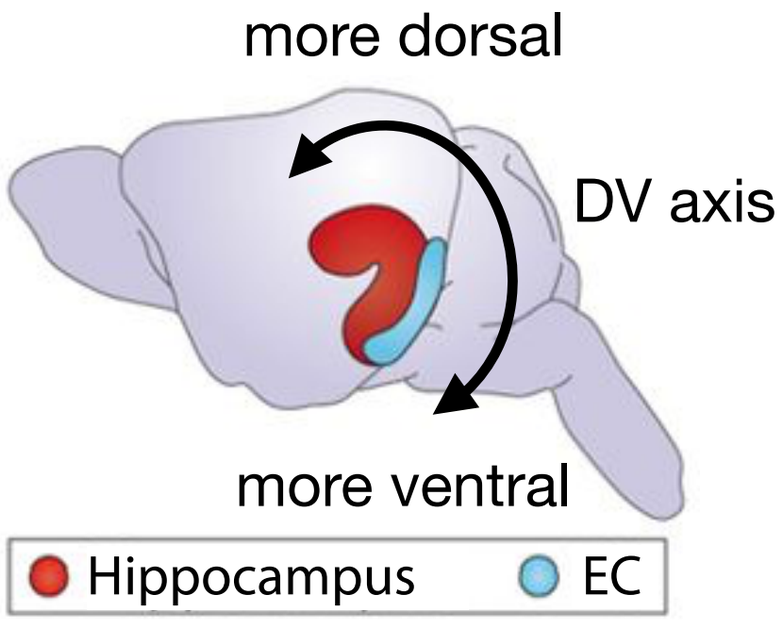




Stachenfeld, McNamee, Behrens, Gershman, Botvinick (review article in prep)

Multi-scale representations

Same eigenvectors support different coarseness

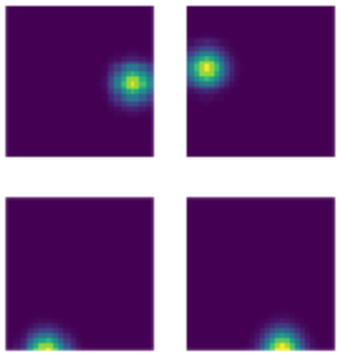


Stachenfeld, Botvinick, Gershman (2017)
also see Momennejad & Howard (2018) for things you can do with multi-scale SRs

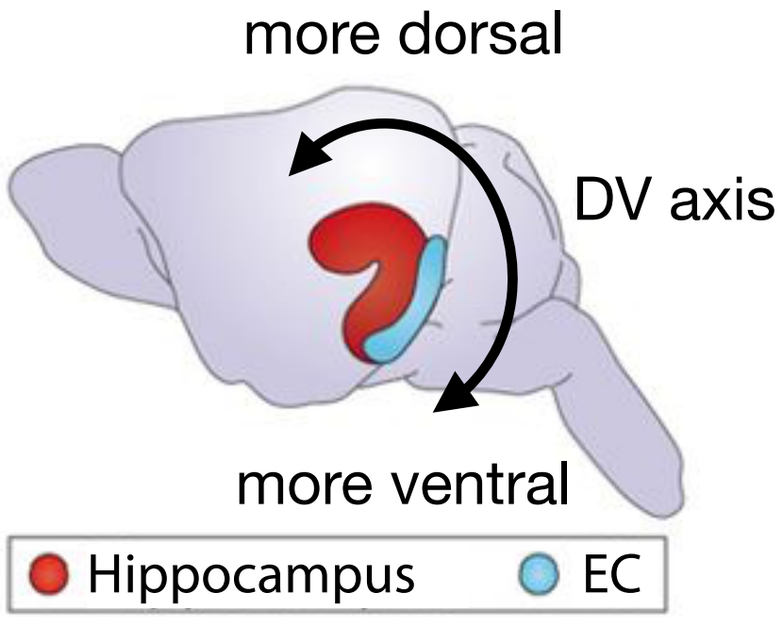
Multi-scale representations

Same eigenvectors support different coarseness

Simulated place fields with different scales



more dorsal
smaller discount factor



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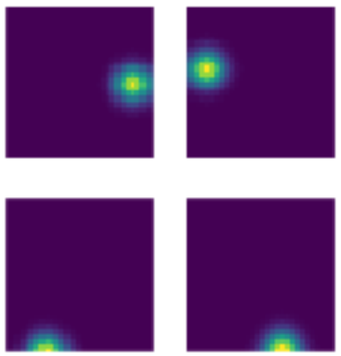
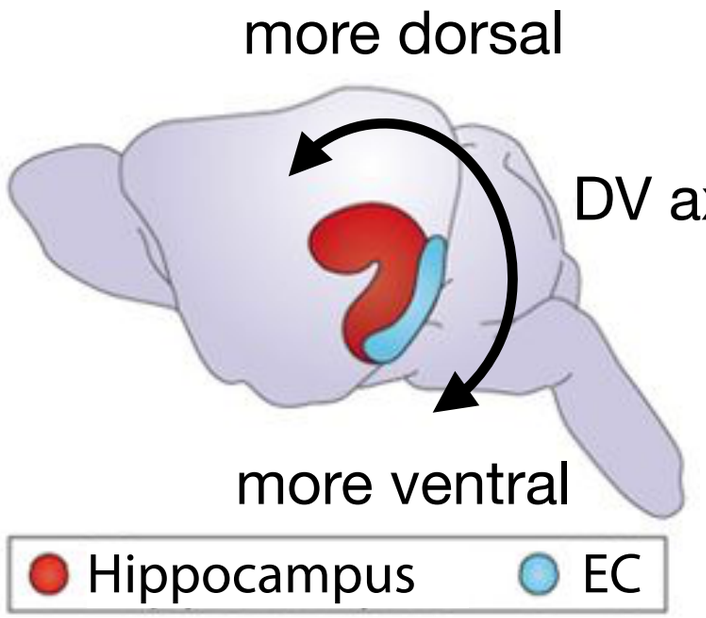
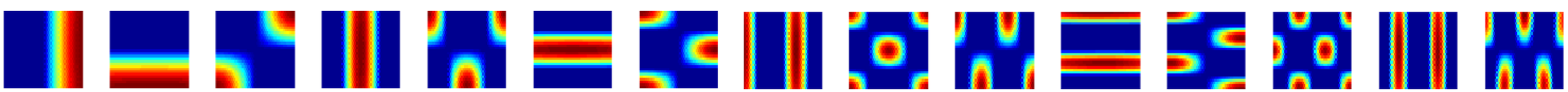


more dorsal
smaller discount factor



Simulated eigenvector grid fields

more ventral



more dorsal

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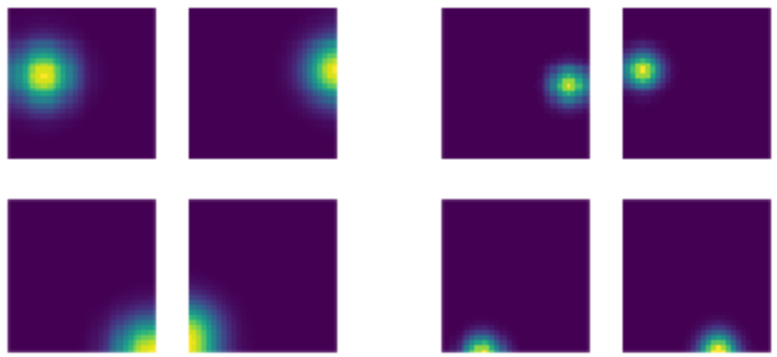
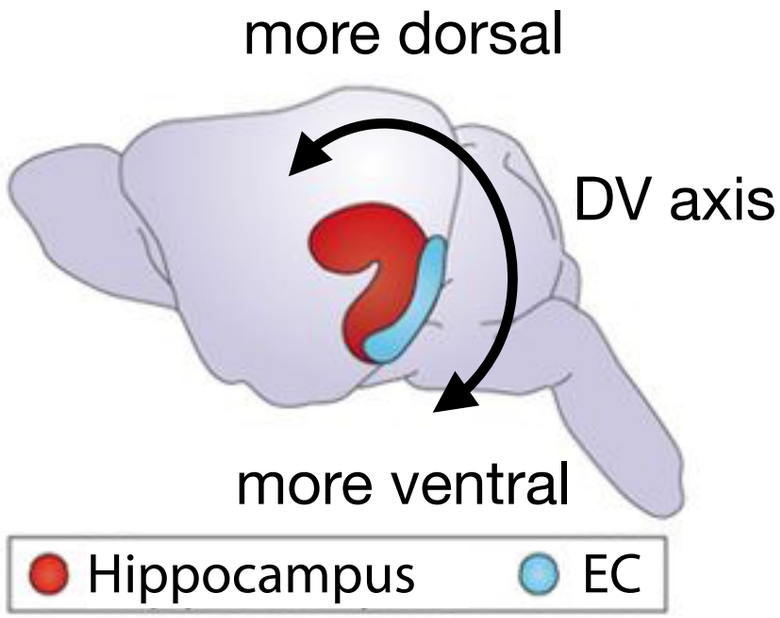
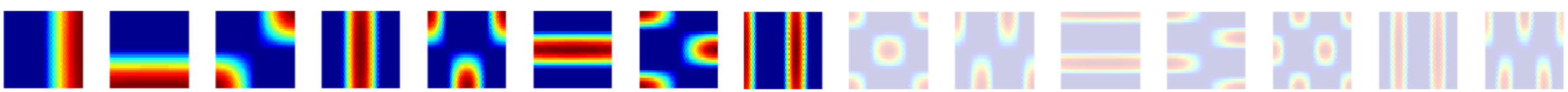


more dorsal
smaller discount factor



Simulated eigenvector grid fields

more ventral



more dorsal

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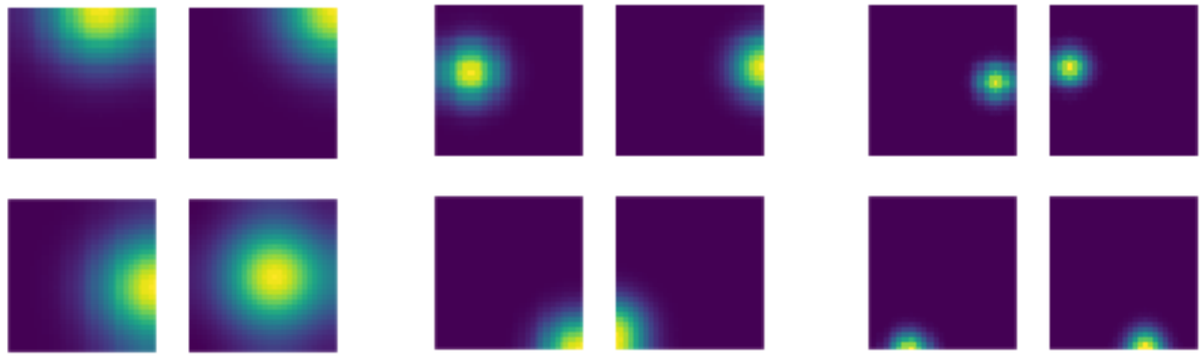
Multi-scale representations

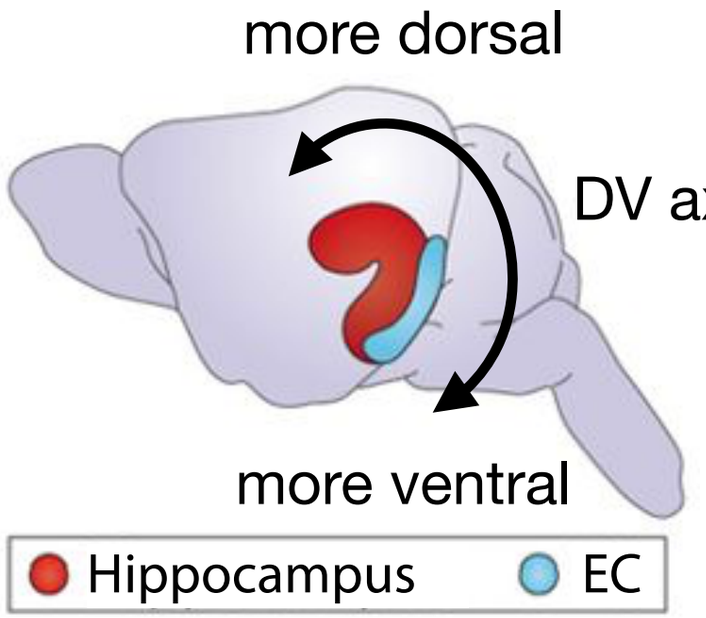
Same eigenvectors support different coarseness

Simulated place fields with different scales



more ventral
larger discount factor

more dorsal
smaller discount factor



Simulated eigenvector grid fields

more ventral

more dorsal

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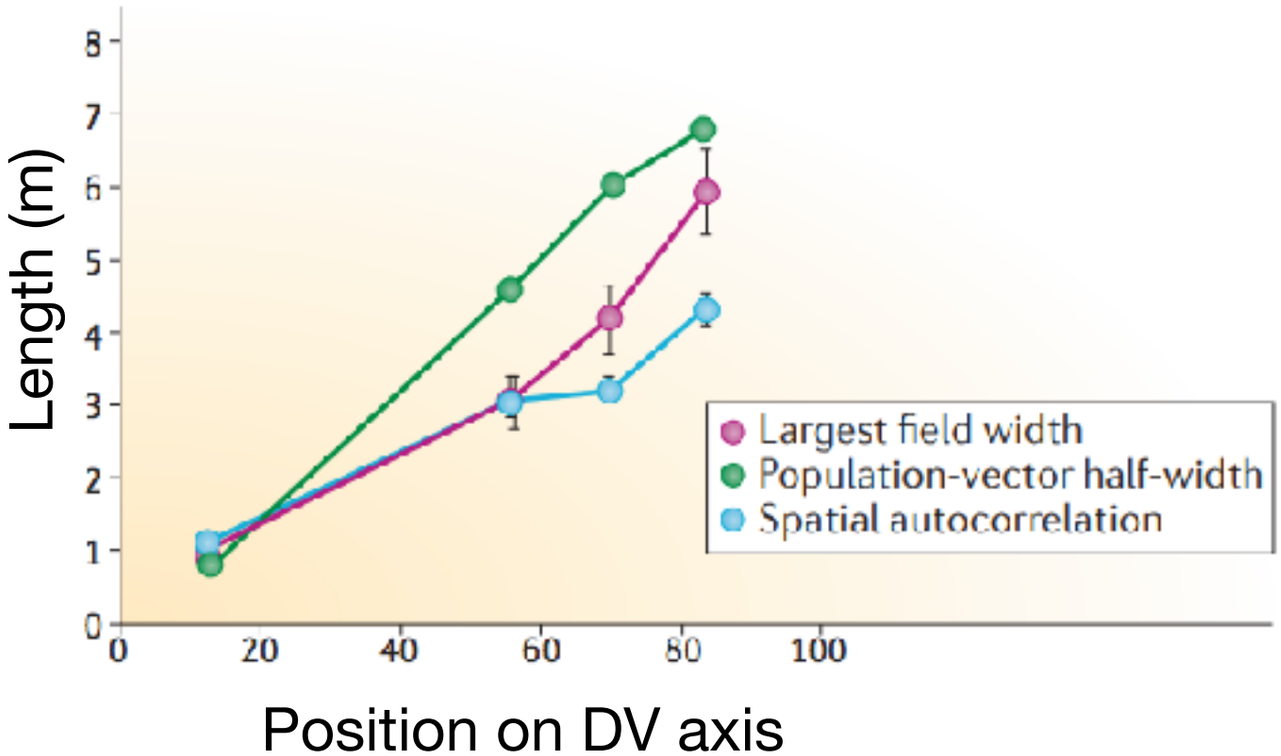
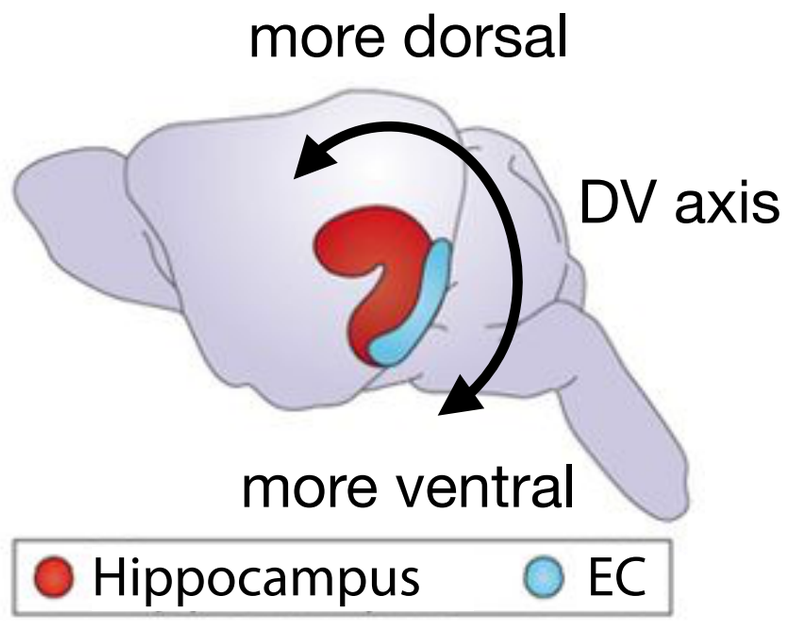
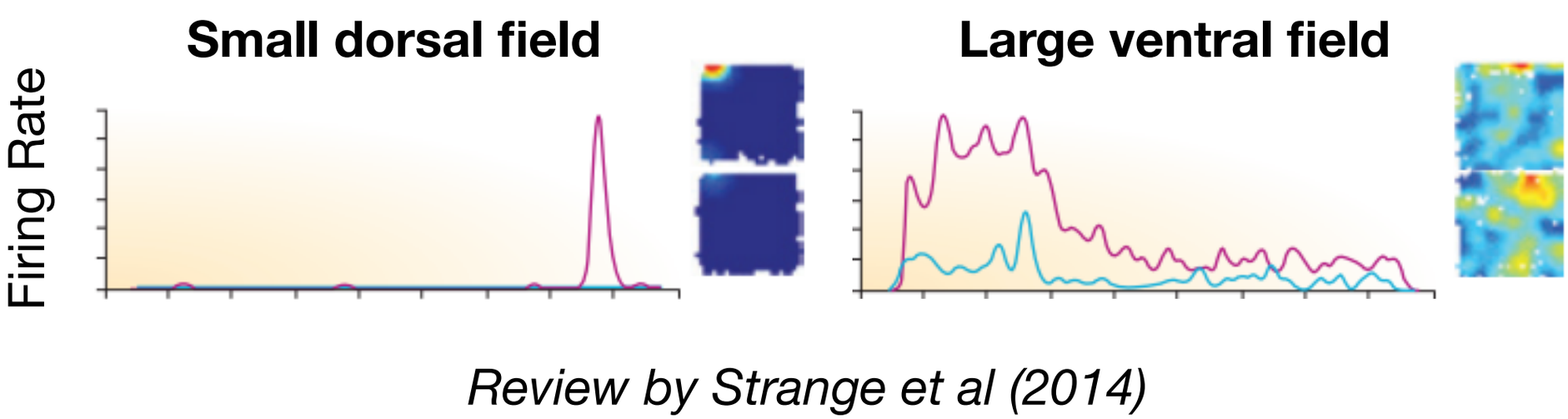
Multi-scale representations

Range of spatial scales along longitudinal axis of hippocampus + MEC

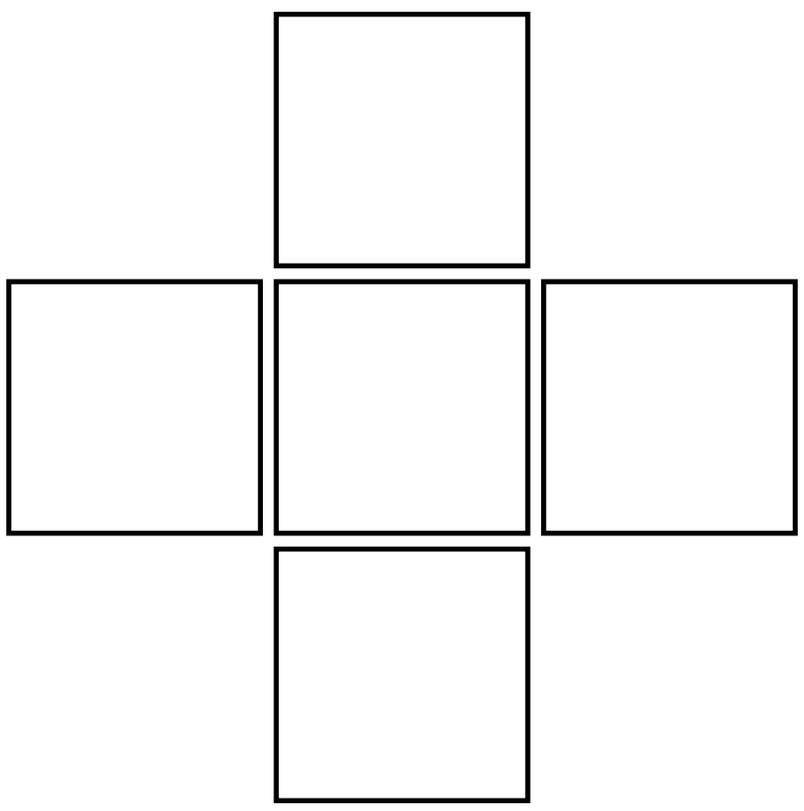






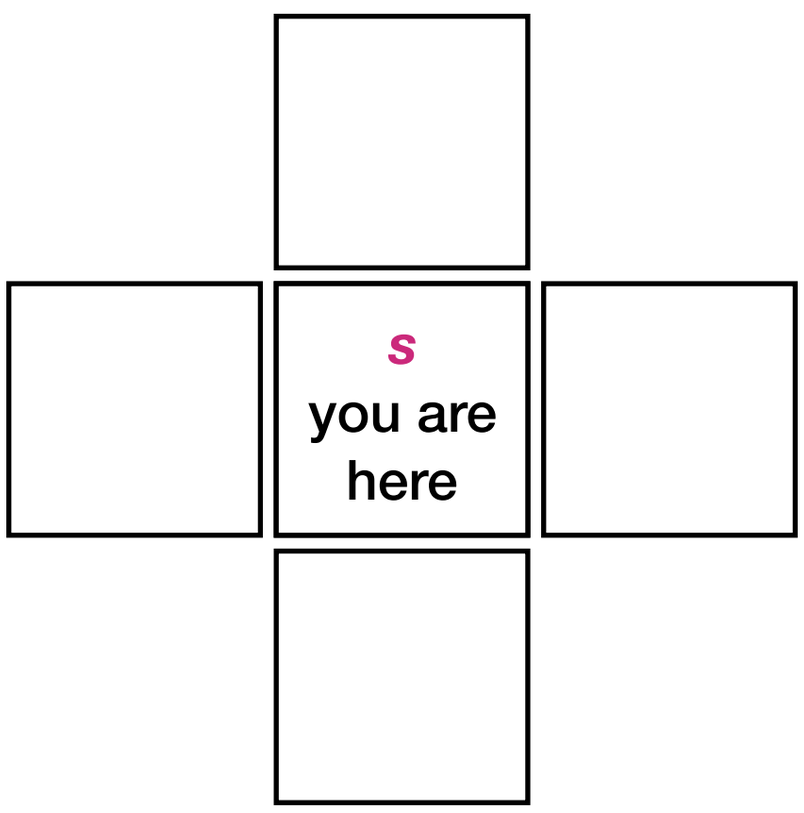
Review by Strange et al (2014)

Exploration with diffusion



McNamee, Stachenfeld, Botvinick, Gershman (in prep)

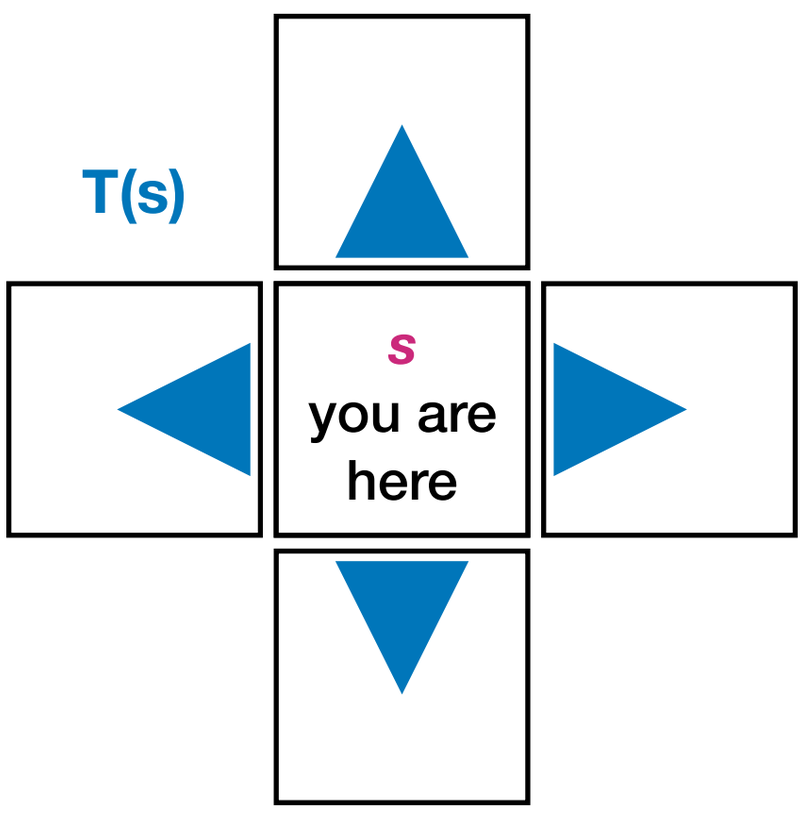
Exploration with diffusion



(1) Start in some state **s**

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with diffusion

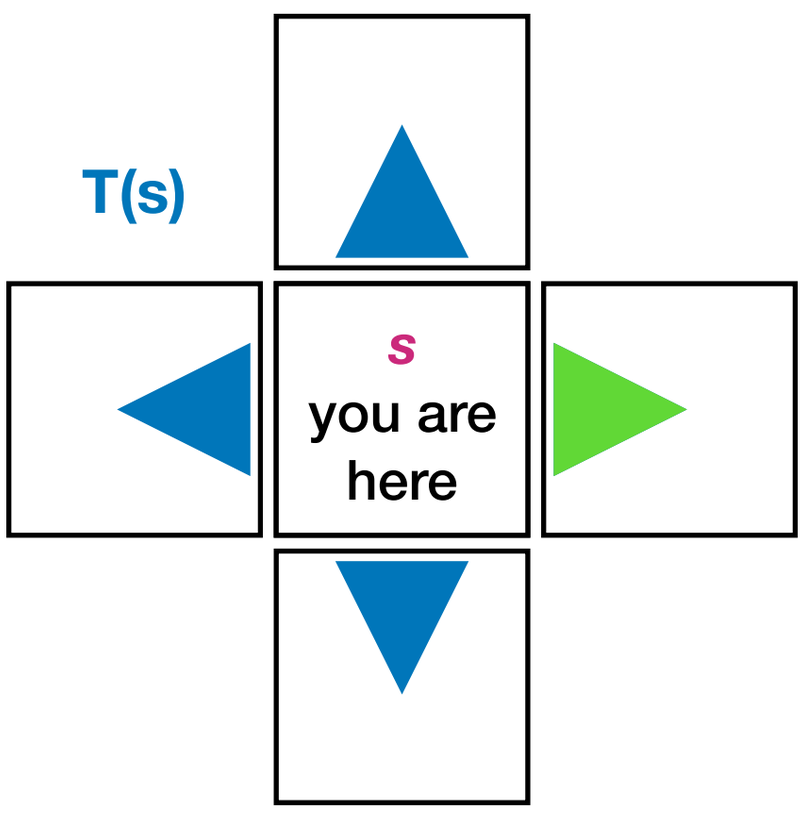


(1) Start in some state **s**

(2) Find which actions are available to you in one-step, **T(s)**

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with diffusion



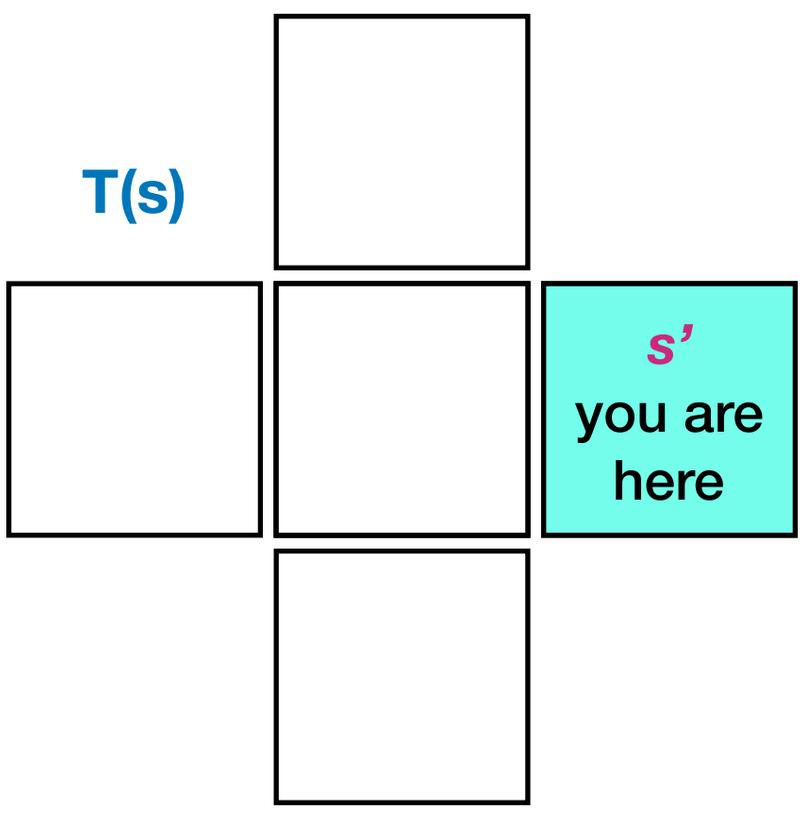
(1) Start in some state **s**

(2) Find which actions are available to you in one-step, **T(s)**

(3) Randomly sample **action** from **T(s)** that takes you to one of your neighbor states, **s'**

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with diffusion



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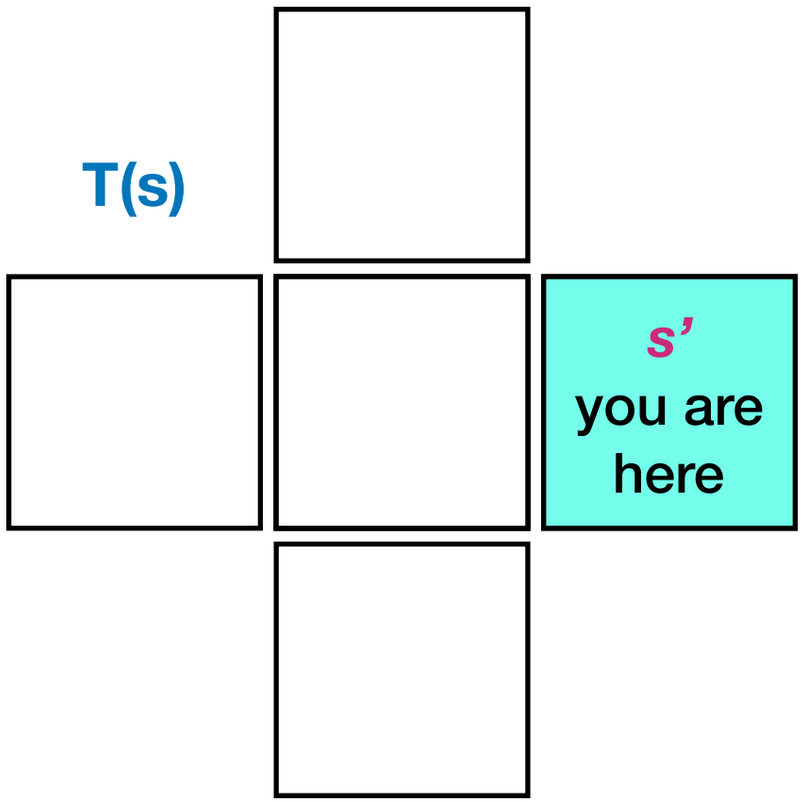

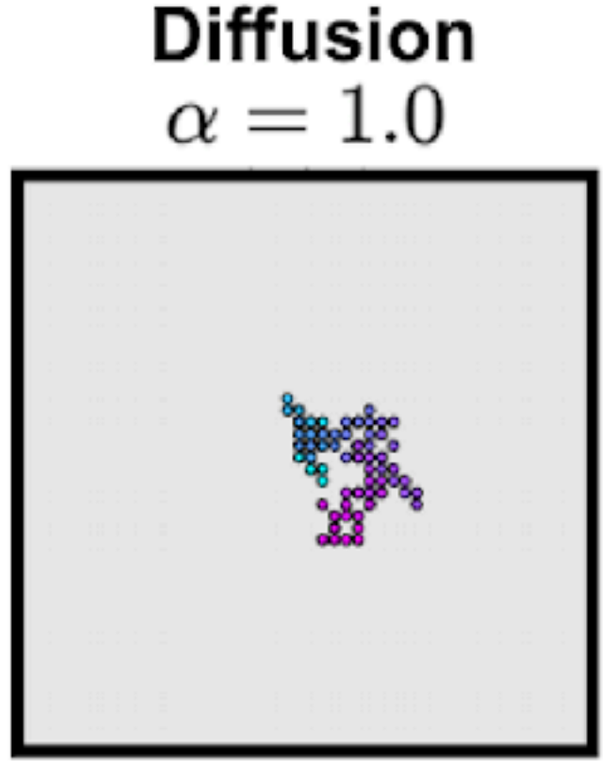

Eventually you cover some ground

(1) Start in some state **s**

(2) Find which actions are available to you in one-step, **T(s)**

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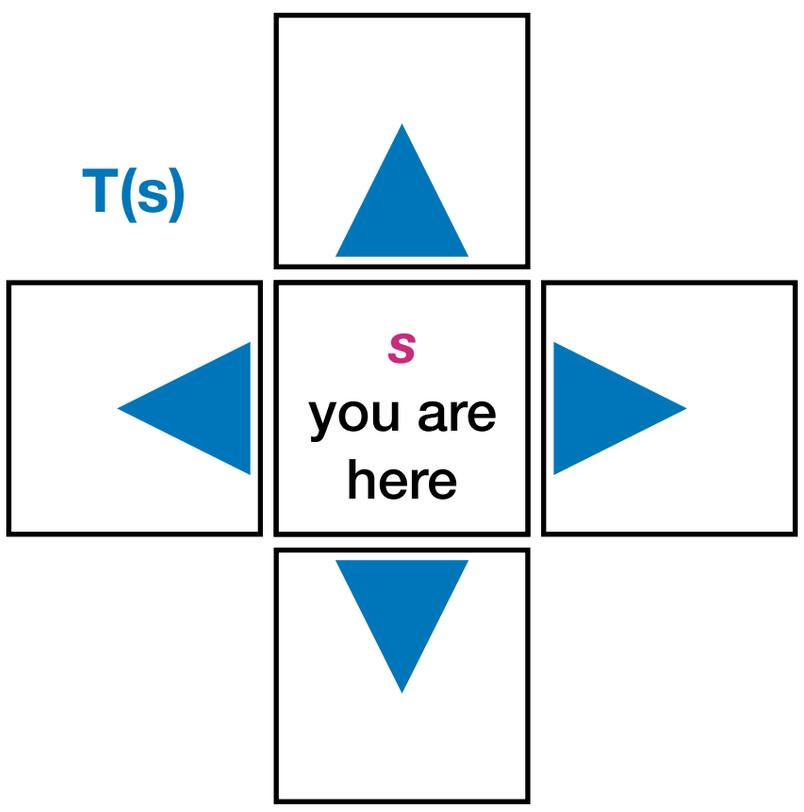
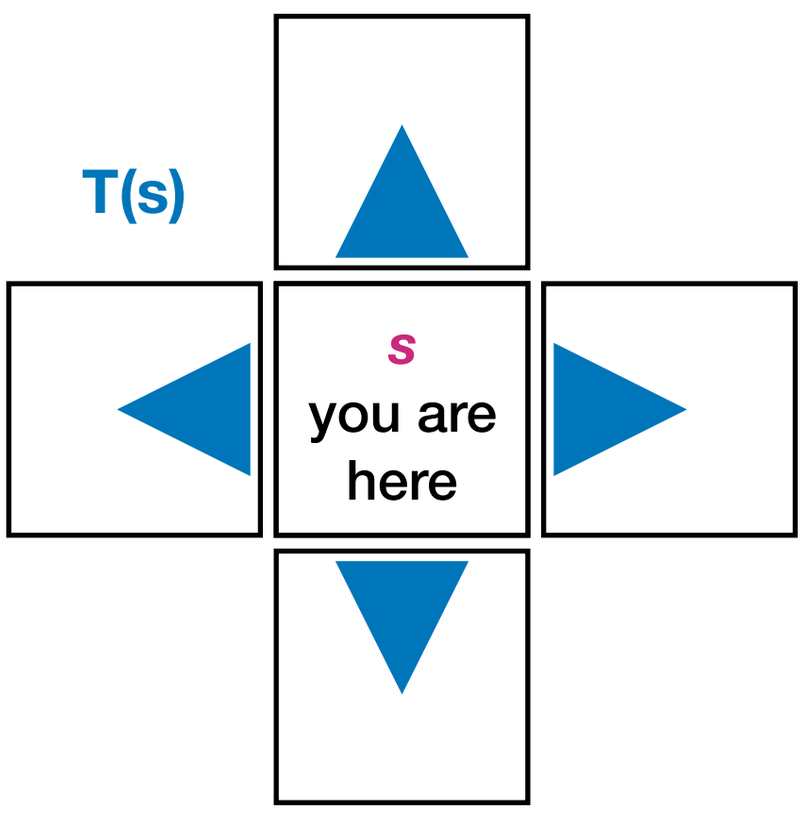
McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral diffusion



McNamee, Stachenfeld, Botvinick, Gershman (in prep)

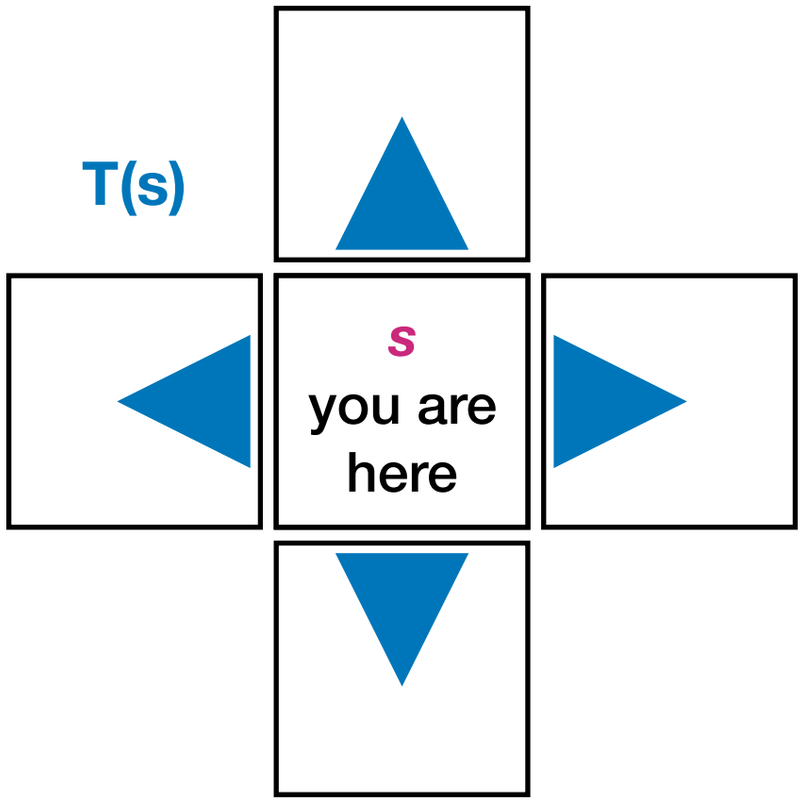
Exploration with spectral diffusion



(1) Start in some state **s** with neighbors **T(s)**

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral diffusion



(1) Start in some state **s** with neighbors **T(s)**

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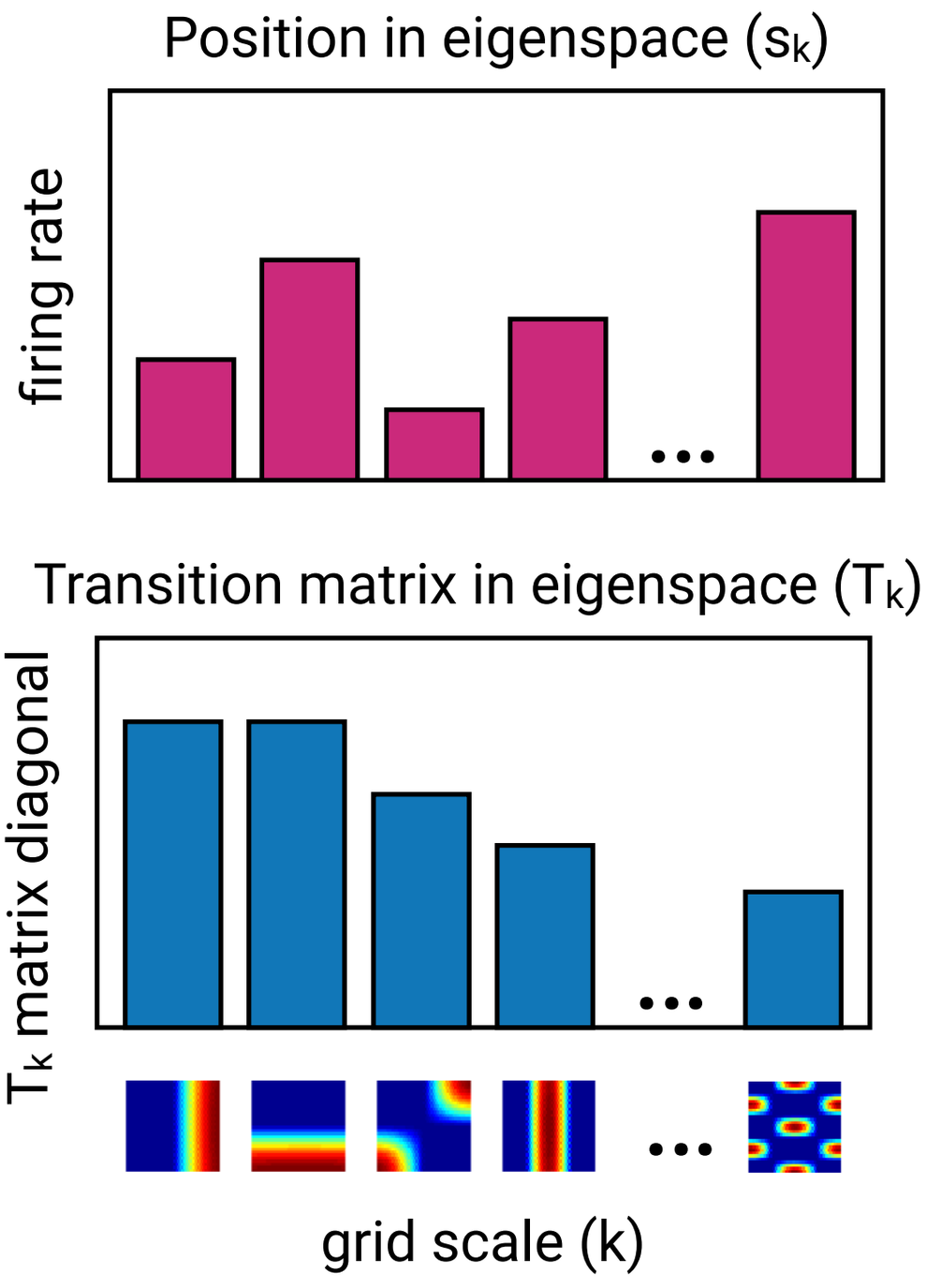
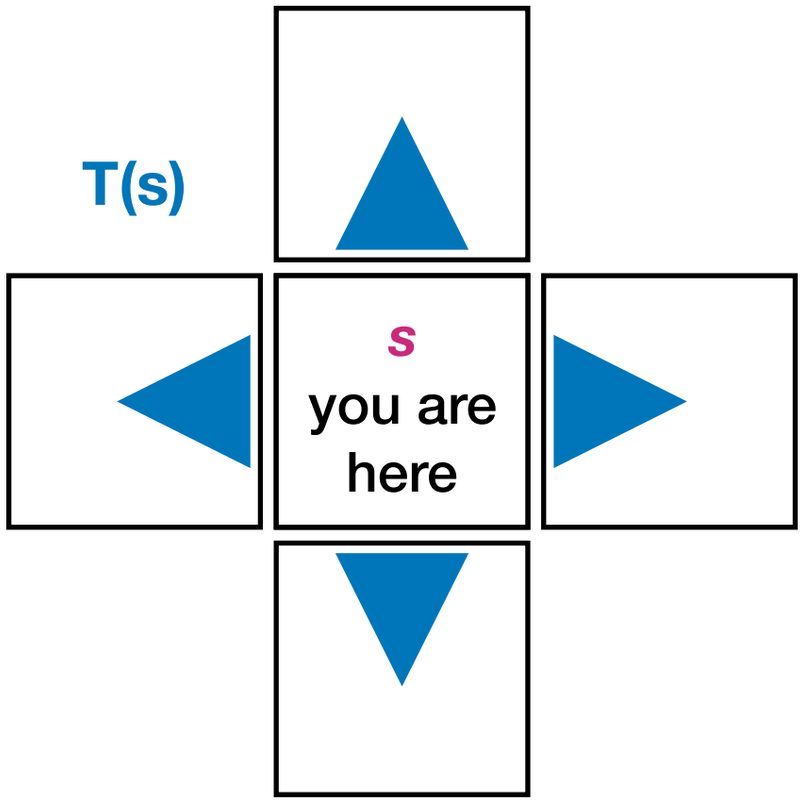


McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral diffusion



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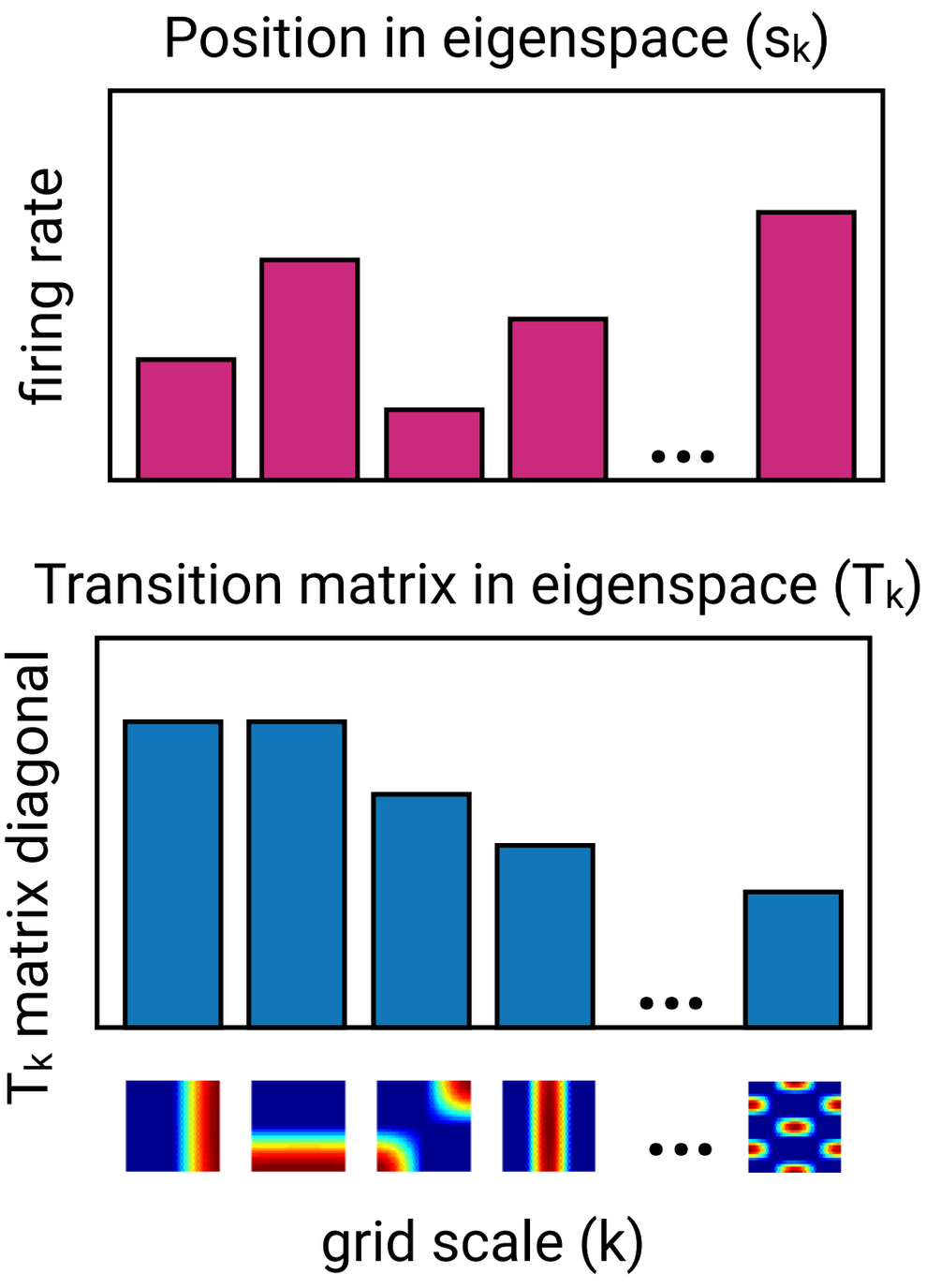
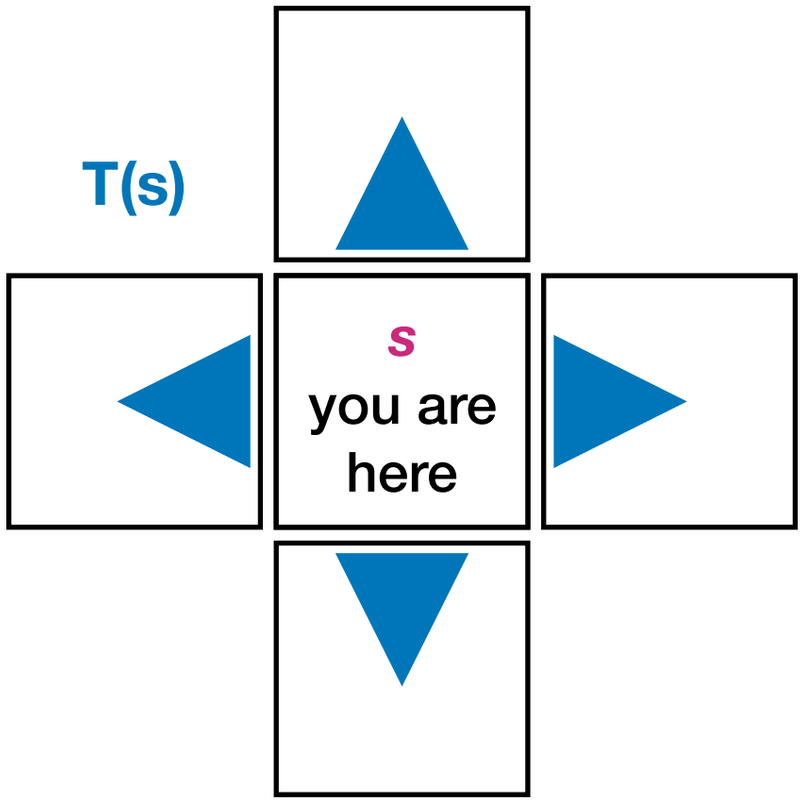


McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with <u>spectral</u> diffusion



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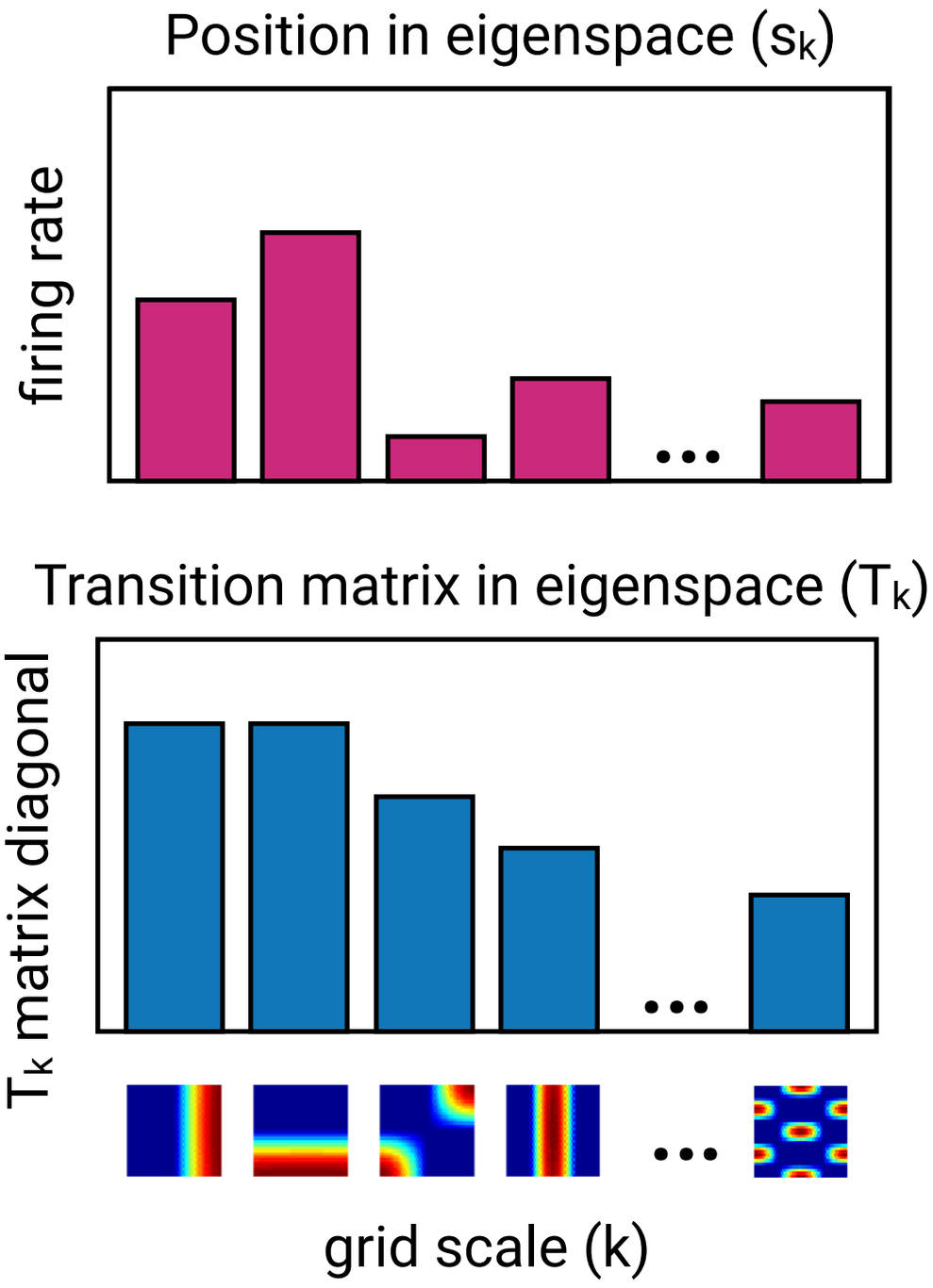
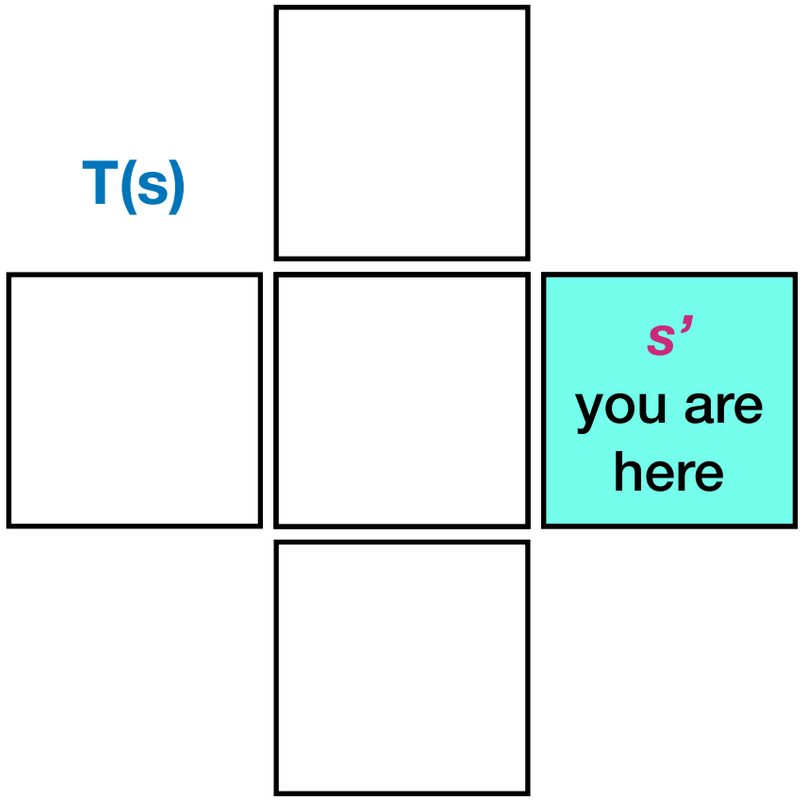
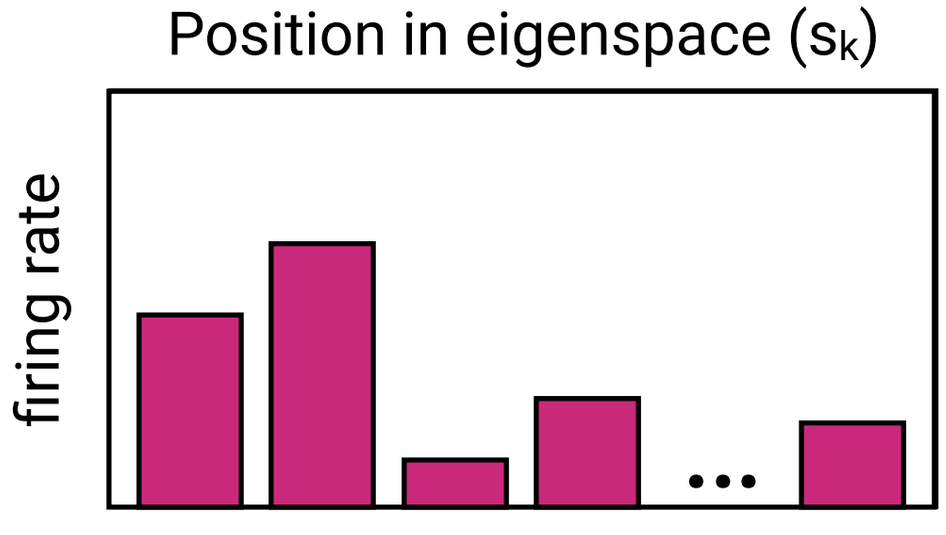
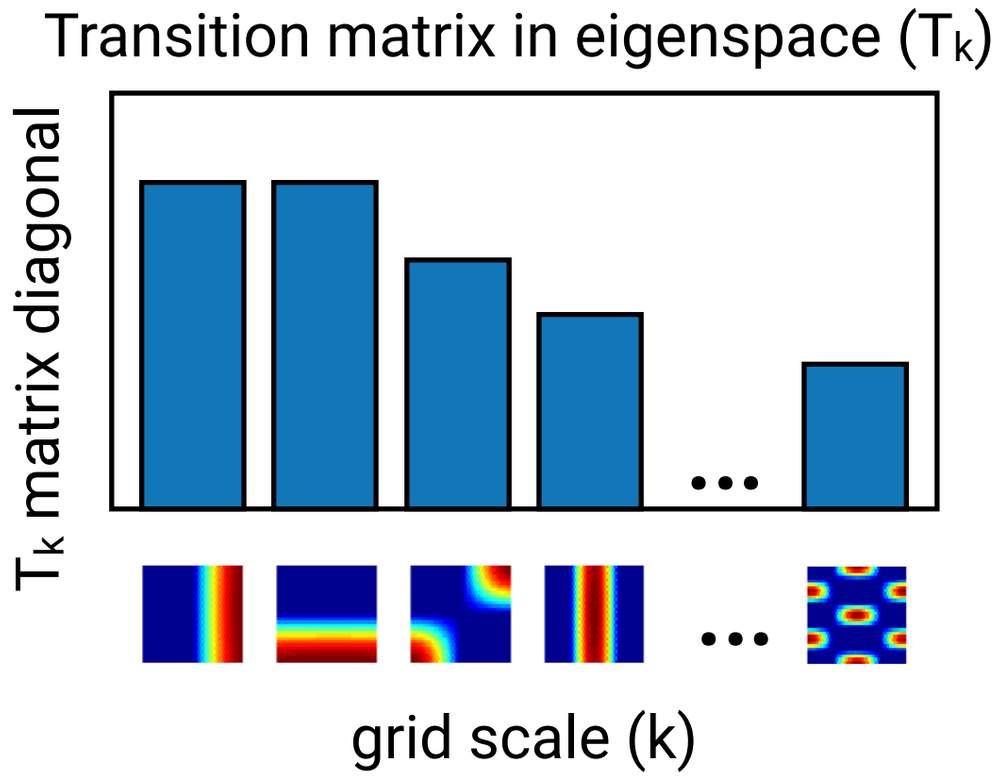


McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral diffusion







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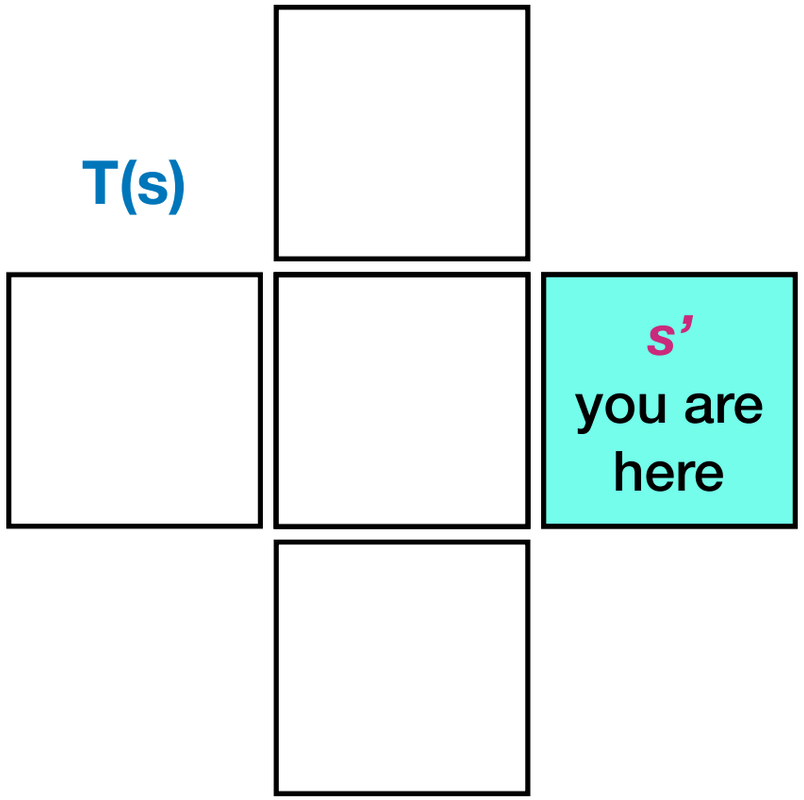
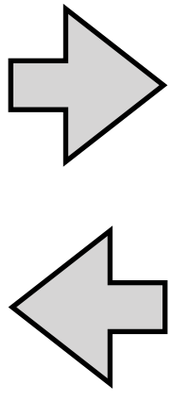
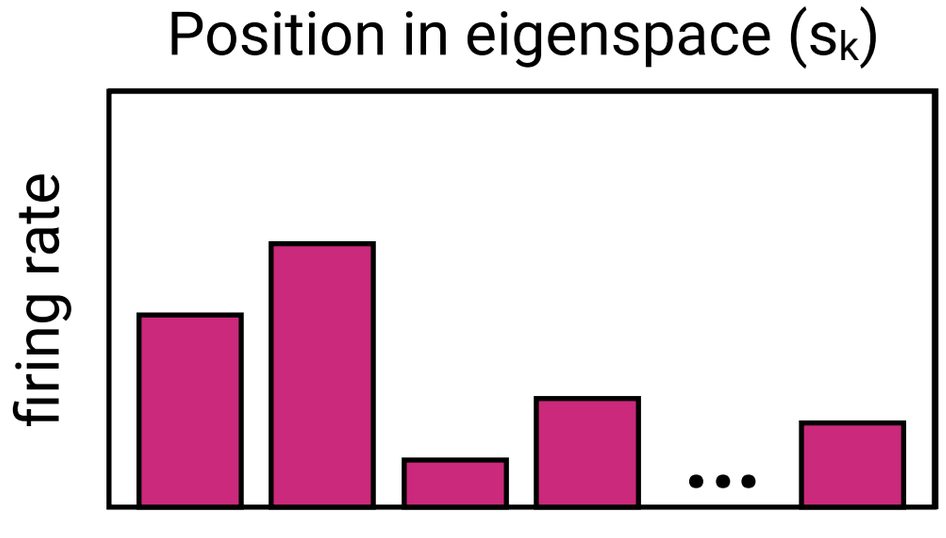
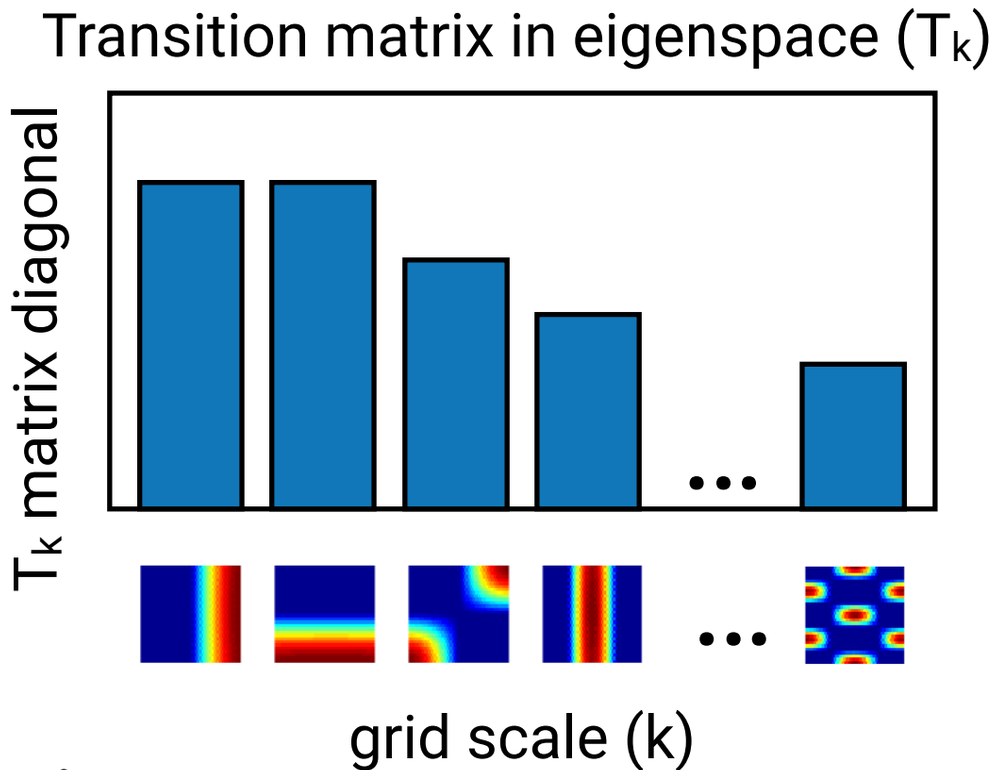
McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral diffusion









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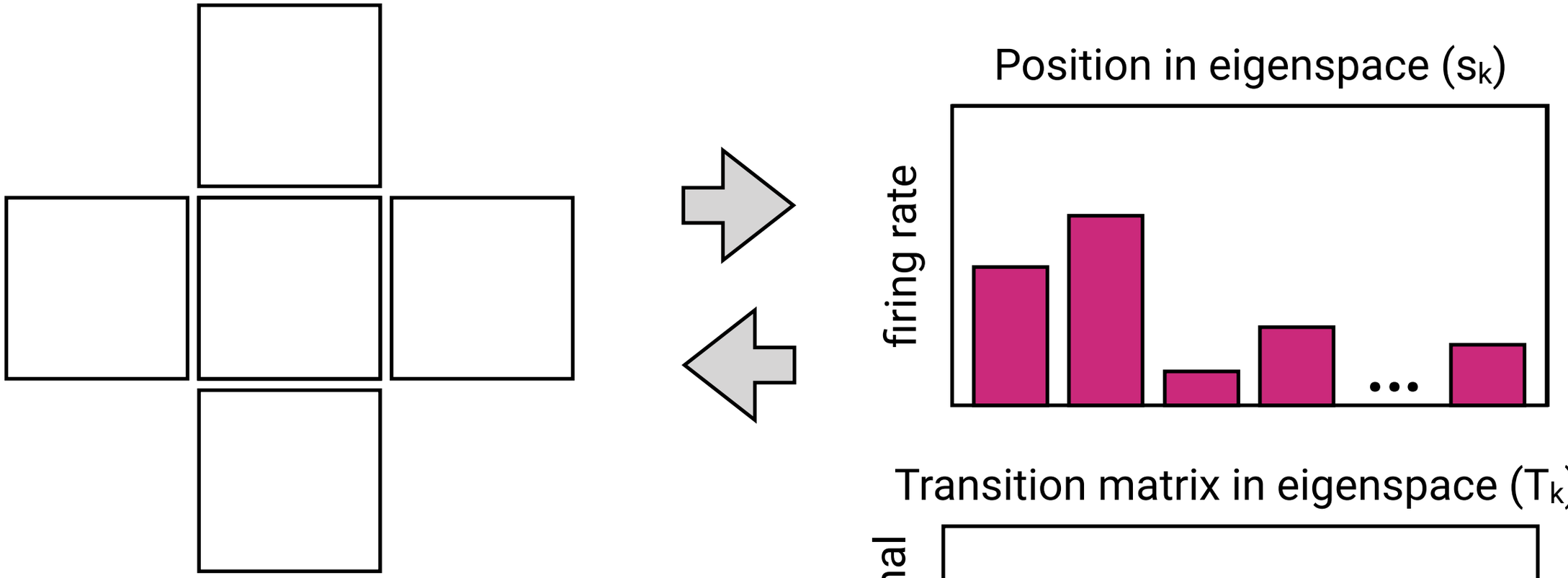
(5) Project back, sample **s'**

Also generates diffusion with most of the same statistics

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with <u>spectral</u> <u>super</u>diffusion



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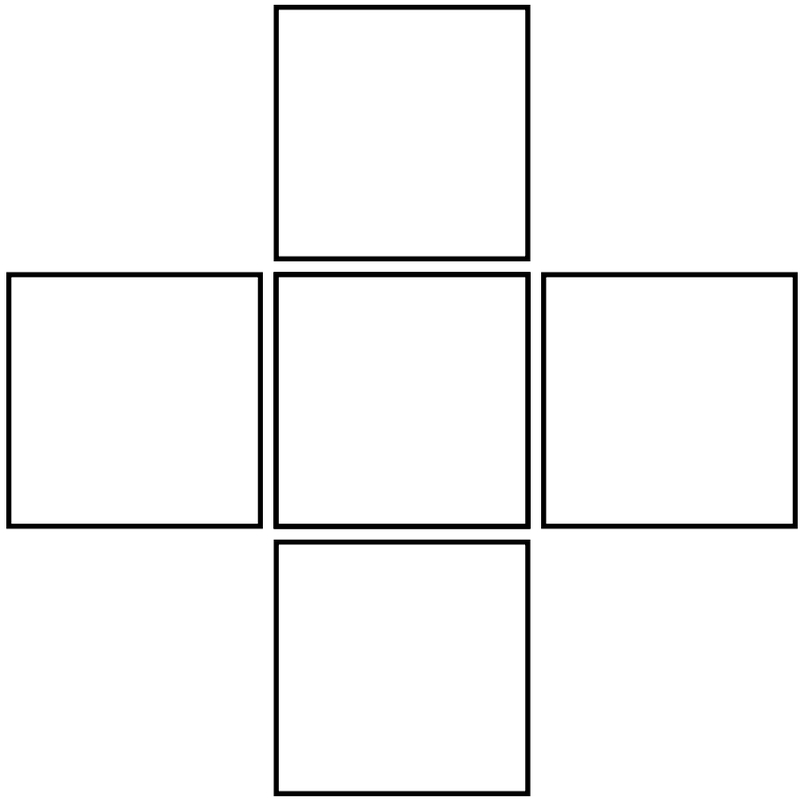

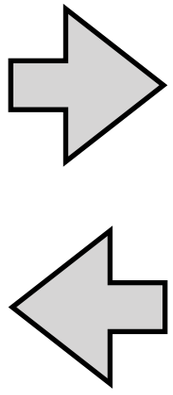

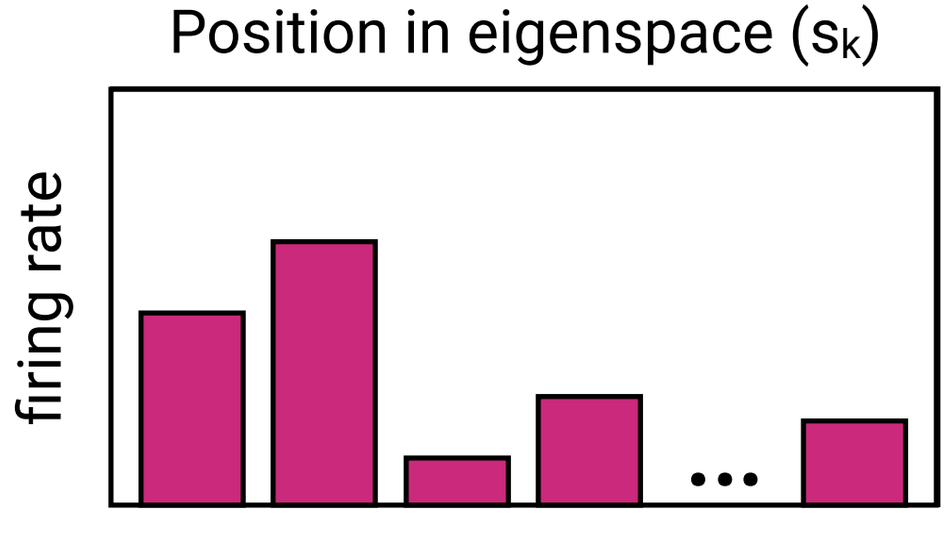

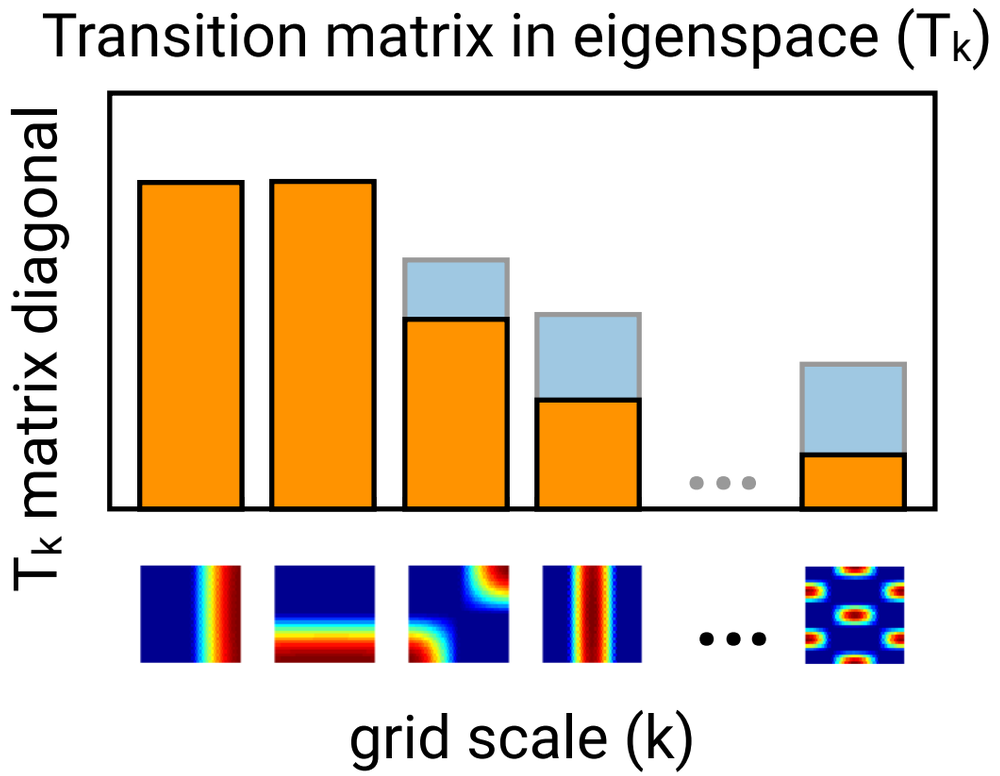

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McNamee, Stachenfeld, Botvinick, Gershman (in prep)

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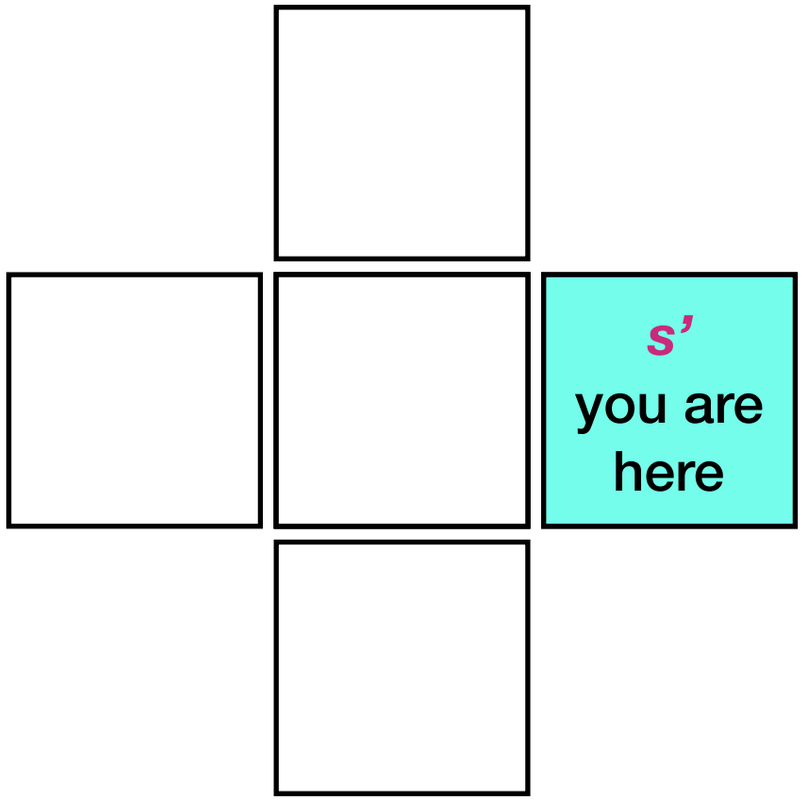

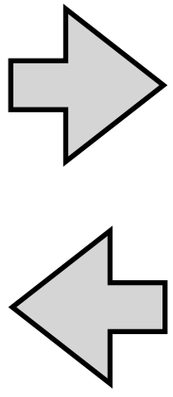

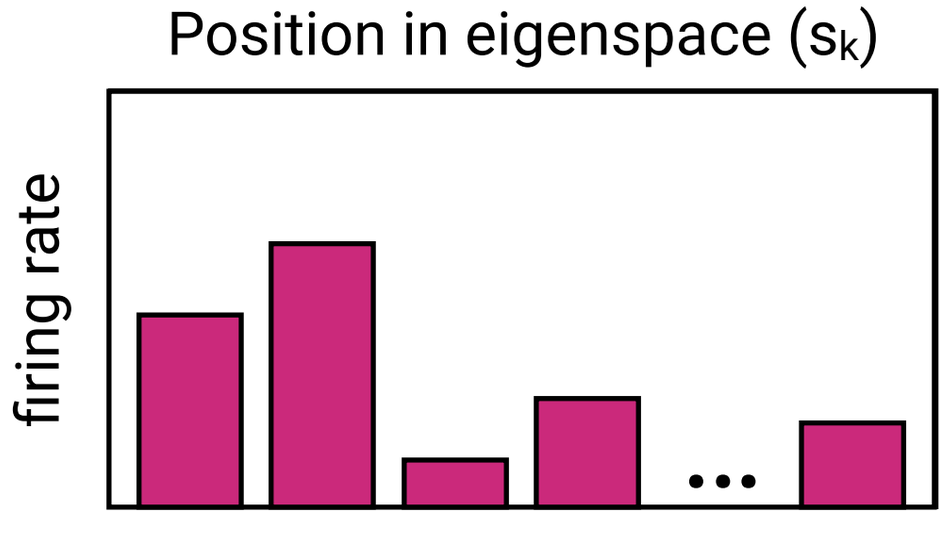

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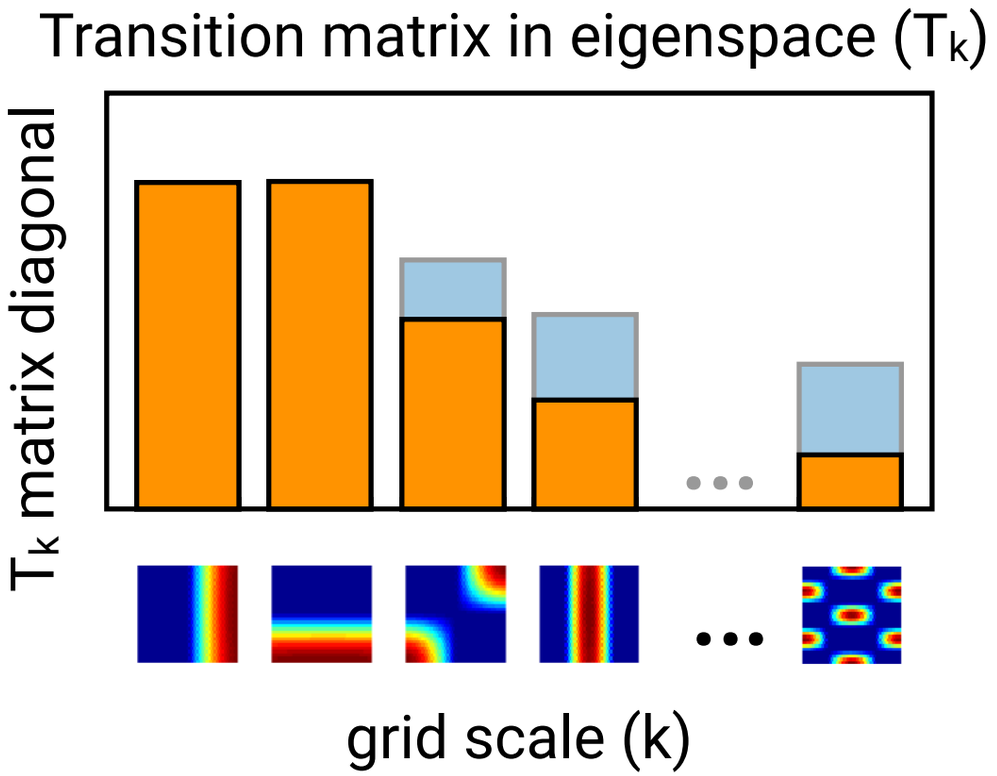
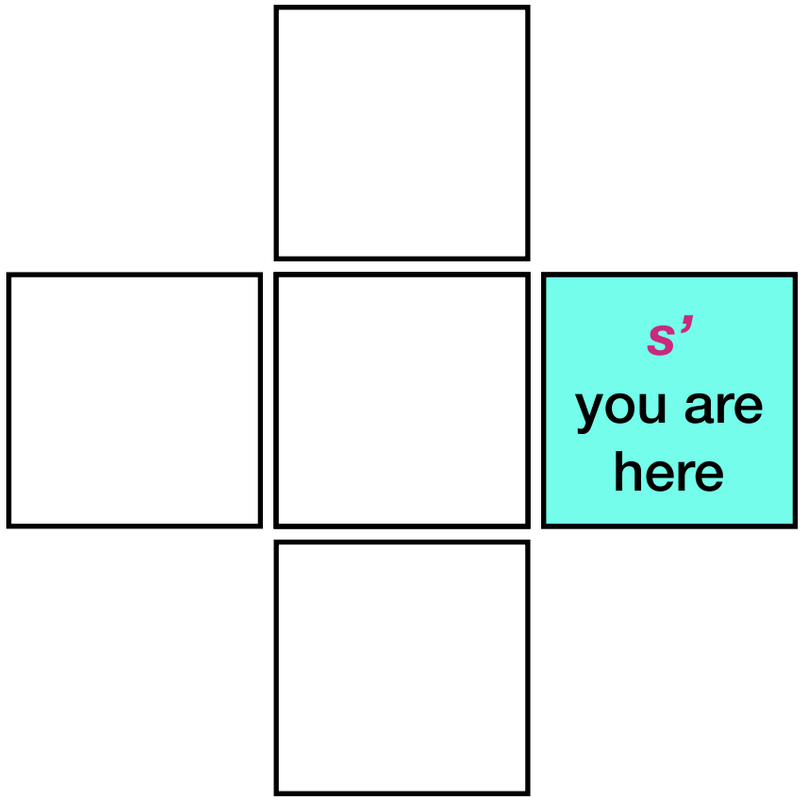
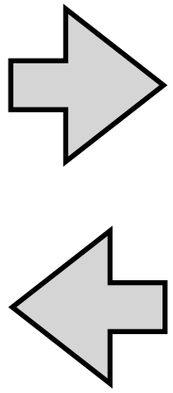
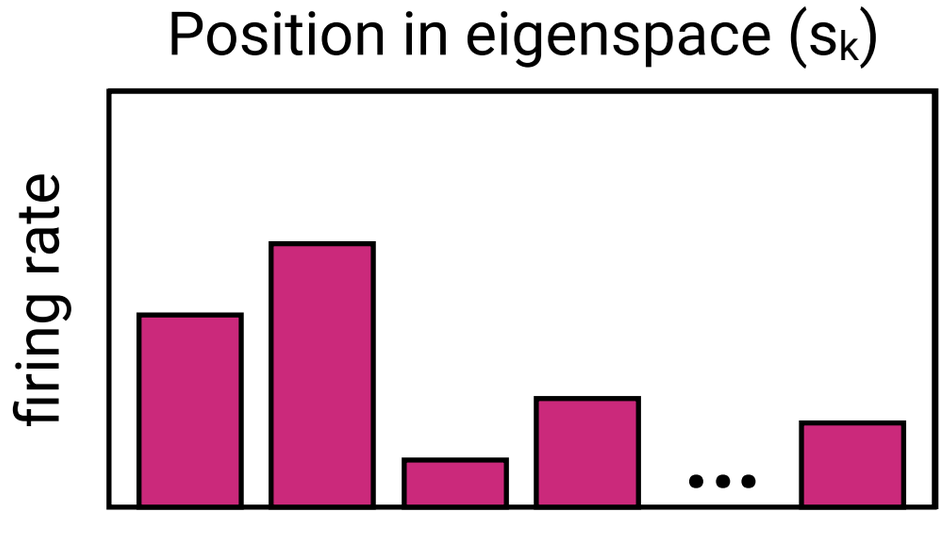


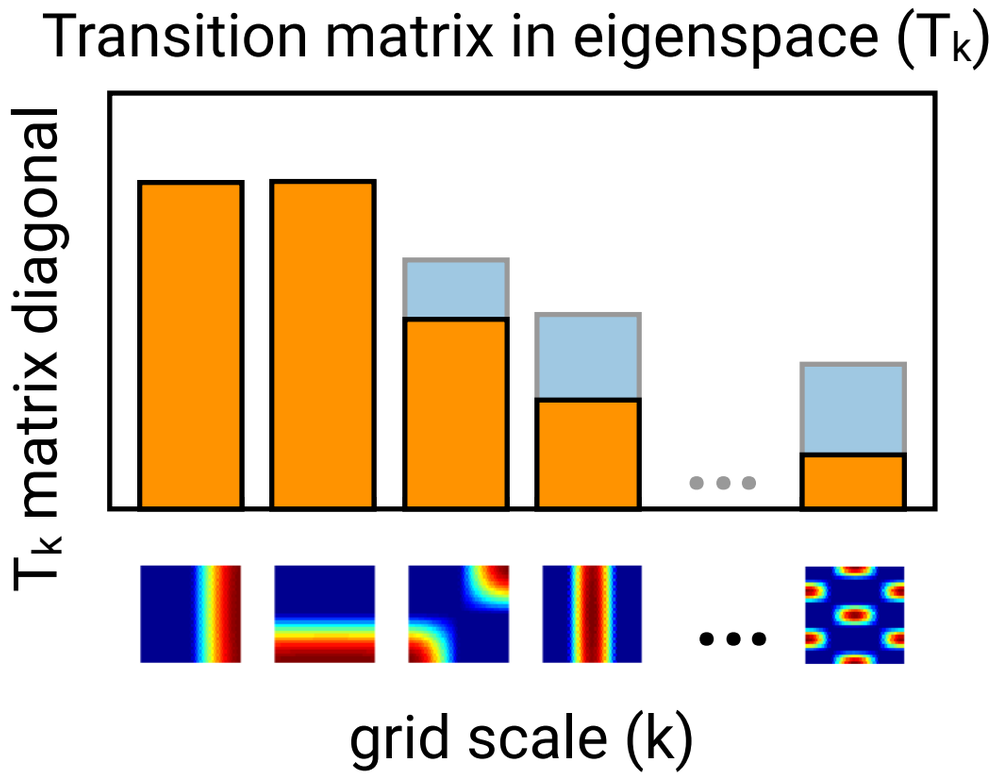
McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral superdiffusion









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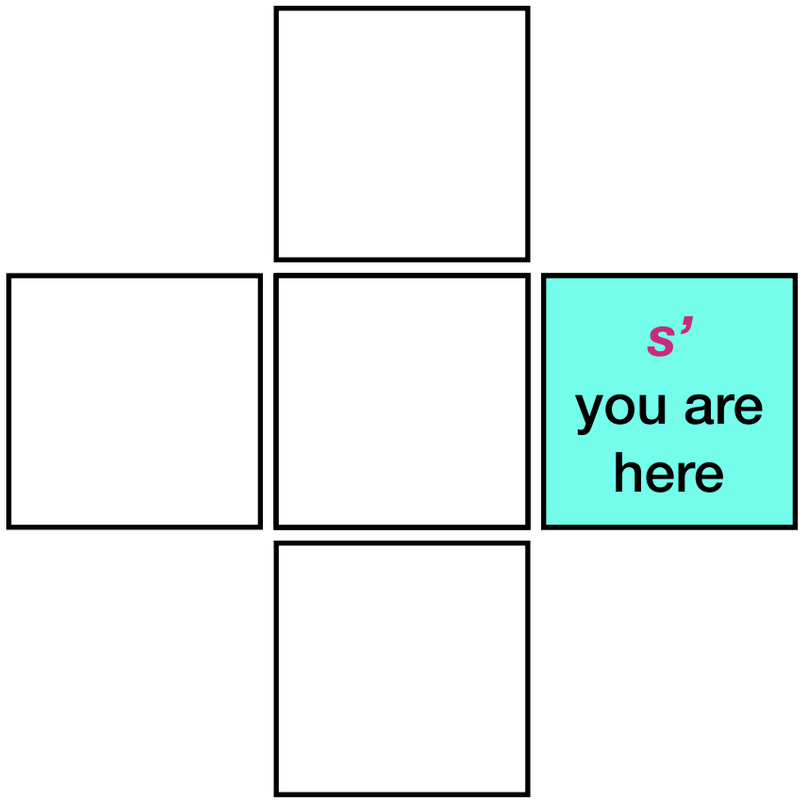
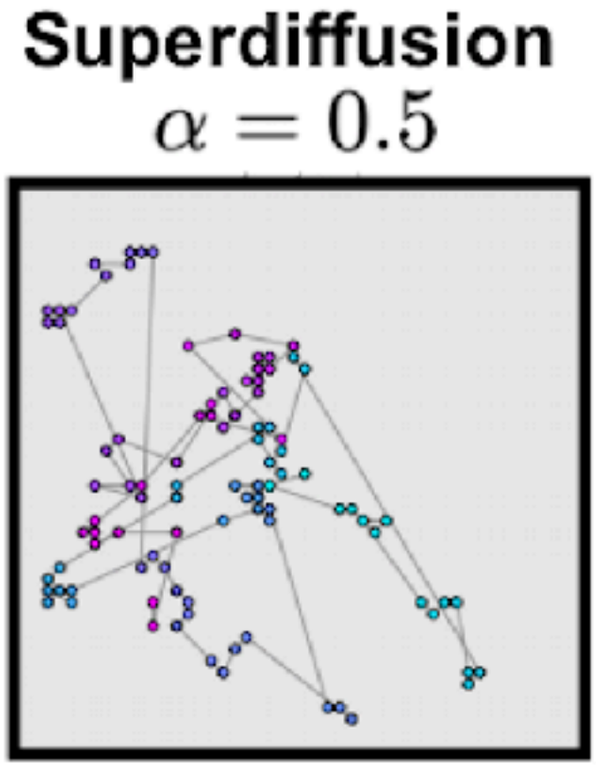
McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral superdiffusion





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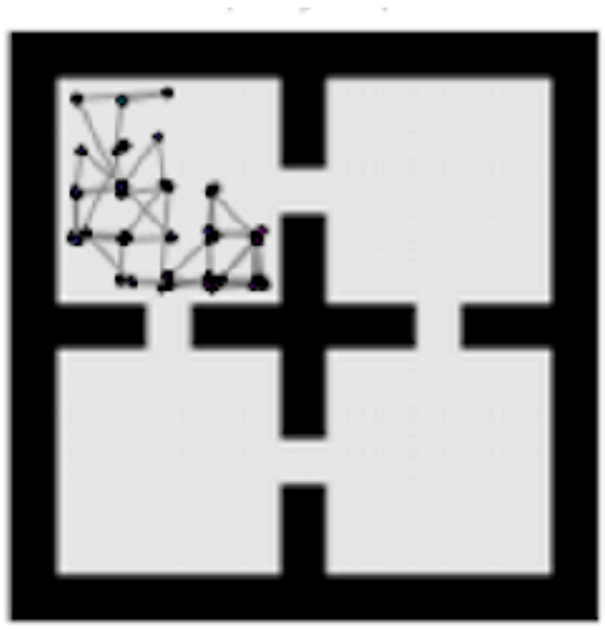
Covers more ground

Factoring into spatial + temporal terms can let you flexibly modulate timescale of exploration

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral superdiffusion in multicompartment environments

Diffusion



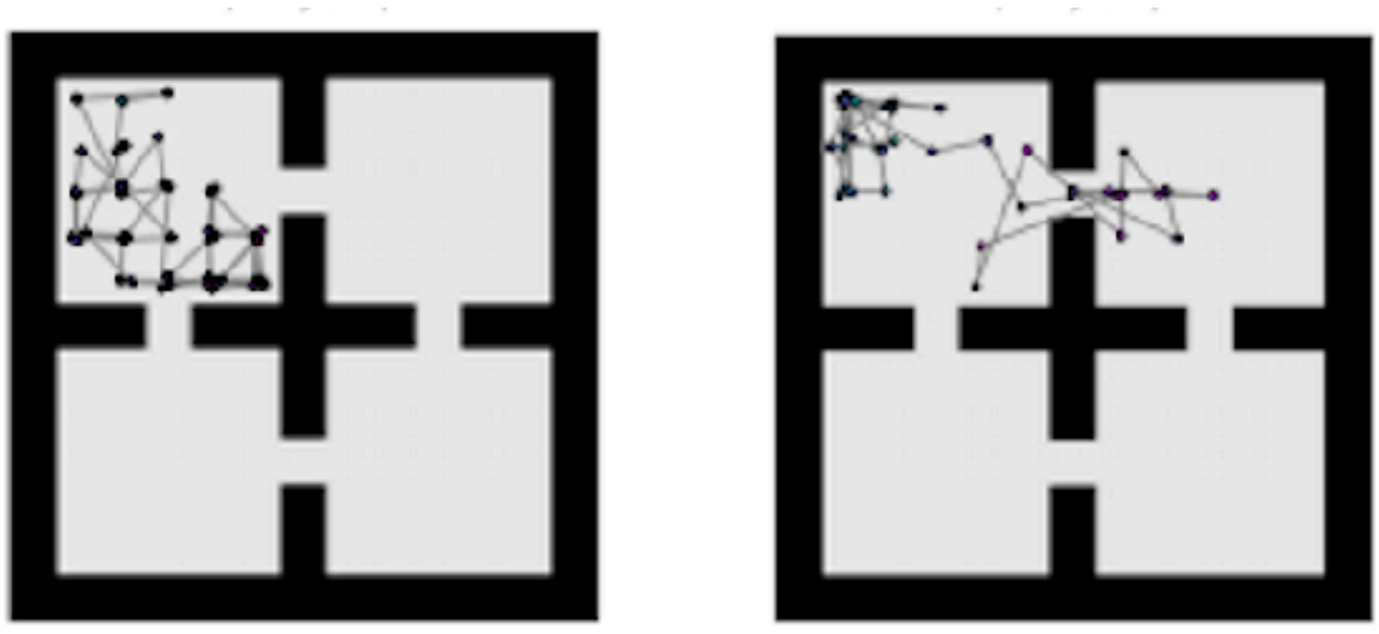
Agent gets stuck in room

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Exploration with spectral superdiffusion in multicompartment environments

Diffusion

Super-diffusion

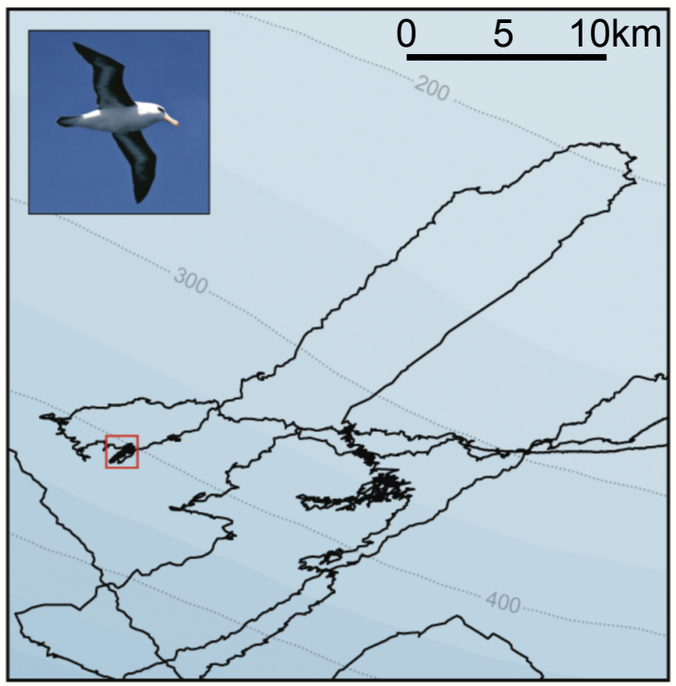


Agent gets stuck in room

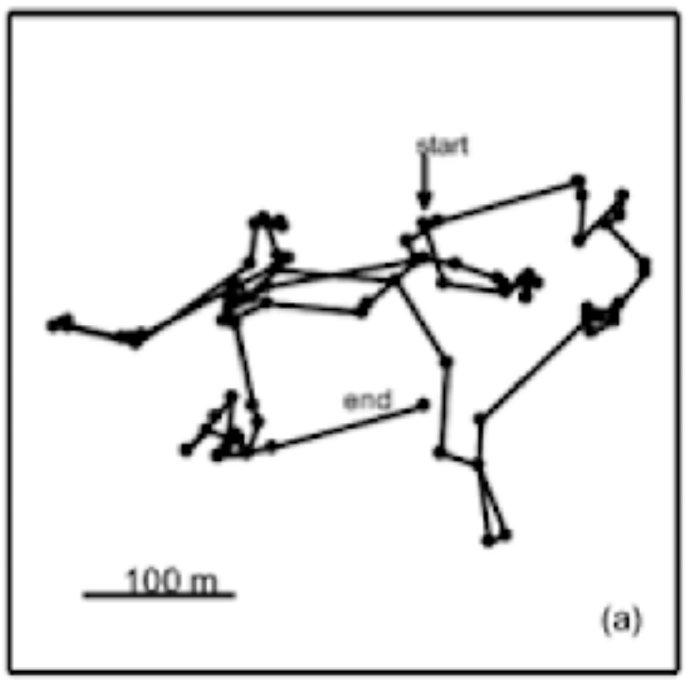
Agent frees itself from room, allowing it to find reward faster

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

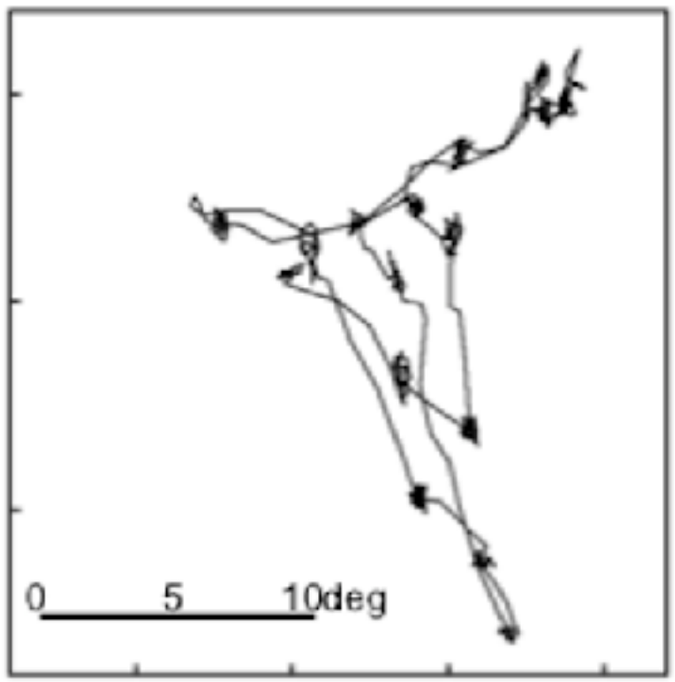
Super diffusion + Lévy Walks



Albatrosses searching for prey
Ornes et al (2013)



Foraging spider monkeys
Ramos-Fernandez et al (2014)



Human view paths for novel fractals
Marlow et al (2015)

Super-diffusion produces Lévy Walks

Ubiquitously observed in animal foraging (right?)

Better coverage time than diffusion

McNamee, Stachenfeld, Botvinick, Gershman (in prep)

Neuron

Available online 25 February 2019

In Press, Corrected Proof ?

Article

Hippocampal Reactivation of Random Trajectories Resembling Brownian Diffusion

Federico Stella [1], Peter Baracskay [1, 2], Joseph O'Neill [1, 3], Jozsef Csicsvari [1, 4]

Highlights

- Hippocampal replay can represent Brownian diffusion-like random trajectories
- Reactivated trajectories cover positions over wide ranges of spatiotemporal scales
- Replay event statistics are incompatible with actual behavioral trajectories
- Expression dynamics of replayed assemblies was linked to specific oscillatory bands

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replay has **diffusive** coefficient, behavior has **superdiffusive** coefficient

"the data indicate that hippocampal circuits have **the built-in ability to generate sequences that link assemblies together across the entire cognitive map, on all temporal and spatial scales**."

Take-aways

- Temporally-organized, spatially-extended state representations can turn support efficient learning + exploration
- Intelligent representation + simple learning -> intelligent learning
- We can use principles from representation learning to understand neural representations

Next steps

- Neuroscience: the data part!
- Machine learning: the neural network part!

Lab

- Work with some graphs
- Look at some of these representations
- See how they generalise differently

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