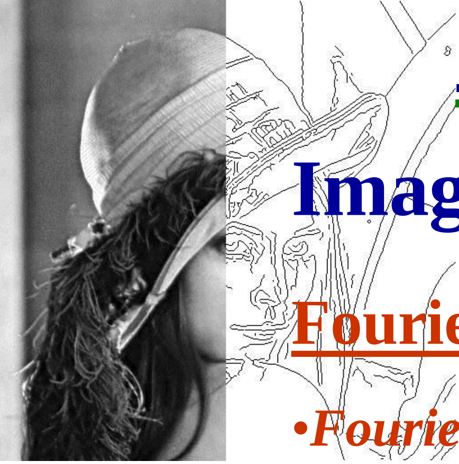


Image Enhancement in the *Frequency Domain*

Fourier Transform

- ***Fourier Series:*** *Any function that periodically repeats itself can be expressed as the sum of sines/cosines of different frequencies, each multiplied with a different coefficient.*

- ***Fourier Transform:*** *Functions that are not periodic, whose area under the curve is finite, can be expressed as the integral of sines and/cosines multiplied by a weighting function.*

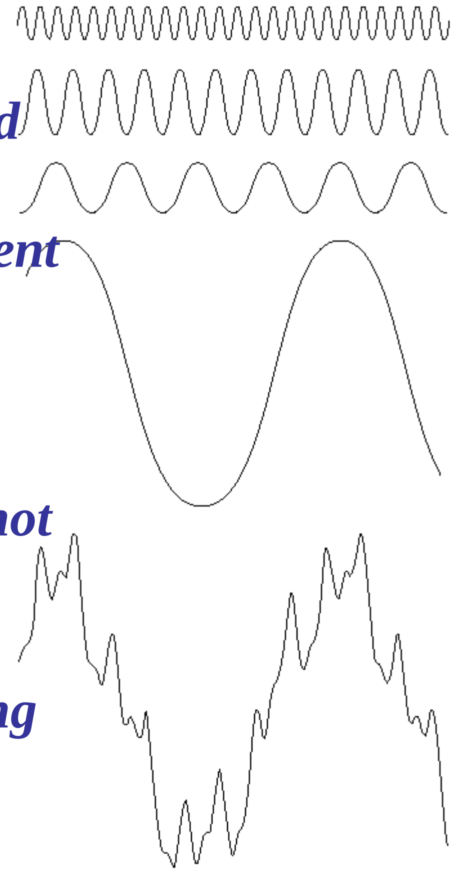


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

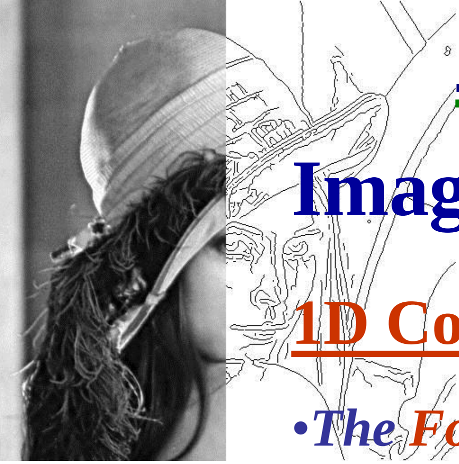


Image Enhancement in the *Frequency Domain*

1D Continuous Fourier Transform

•*The Fourier Transform is an important tool in Image Processing, and is directly related to filter theory, since a filter, which is a convolution in the spatial domain, is a simple multiplication in the frequency domain.*

•*1-D Continuous Fourier Transform*

The Fourier transform, F(u), of a single variable continuous function, f(x), is defined by:

$$F(u)=\int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}dx$$

Given Fourier transform of a function F(u), the inverse Fourier transform can be used to obtain, f(x), by:

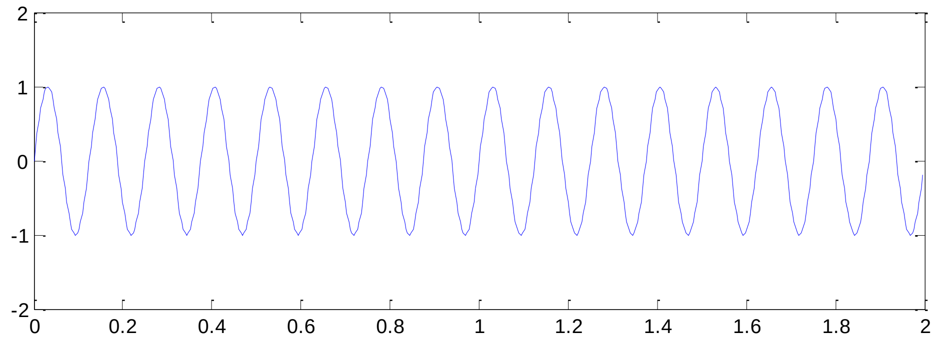
$$f(x)=\int_{-\infty}^{\infty} F(u)e^{j2\pi ux}du$$

Note: F(u), is the frequency spectrum, where, u represents the frequency, and f(x) is the signal where x represents time/space. $j=\sqrt{-1}$

Image Enhancement in the *Frequency Domain*

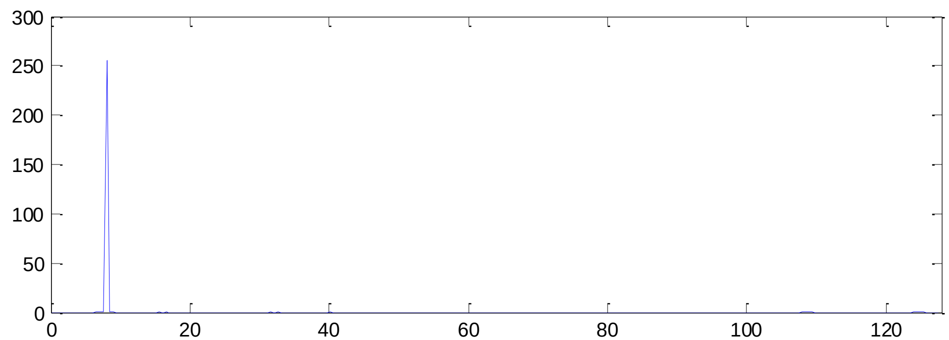
1D Continuous Fourier Transform

Time/Space domain



Sine wave

Frequency domain

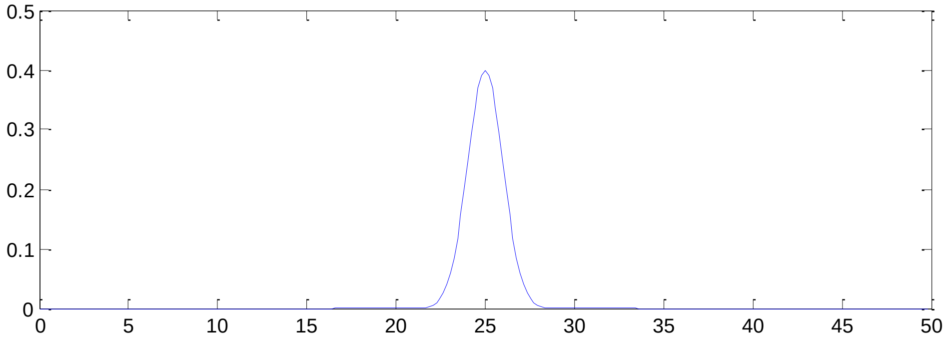


Delta function

Image Enhancement in the *Frequency Domain*

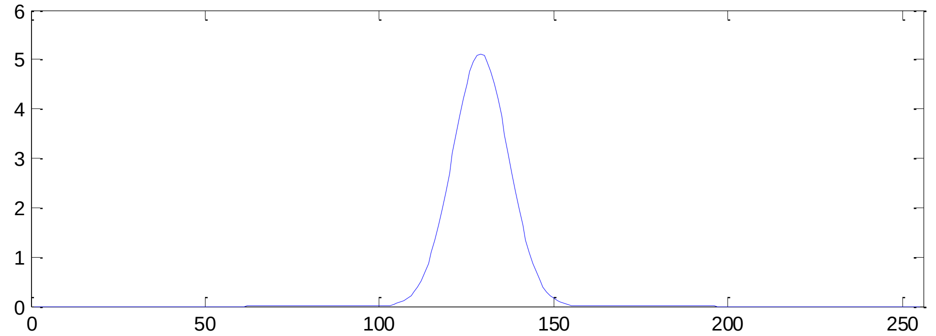
1D Continuous Fourier Transform

Time/Space domain



Gaussian

Frequency domain

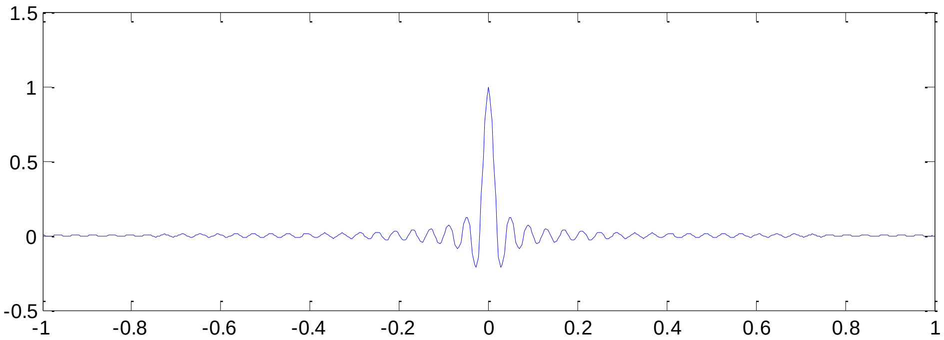


Gaussian

Image Enhancement in the *Frequency Domain*

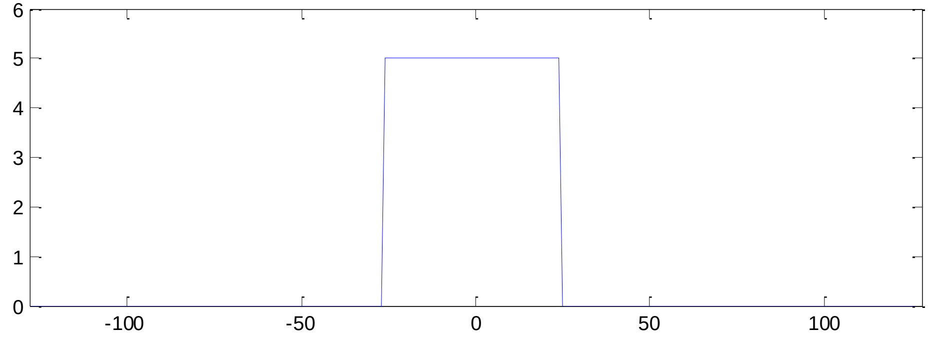
1D Continuous Fourier Transform

Time/Space domain



Sinc function

Frequency domain



Square wave

Image Enhancement in the *Frequency Domain*

Fourier Transform pairs (spatial versus Frequency)

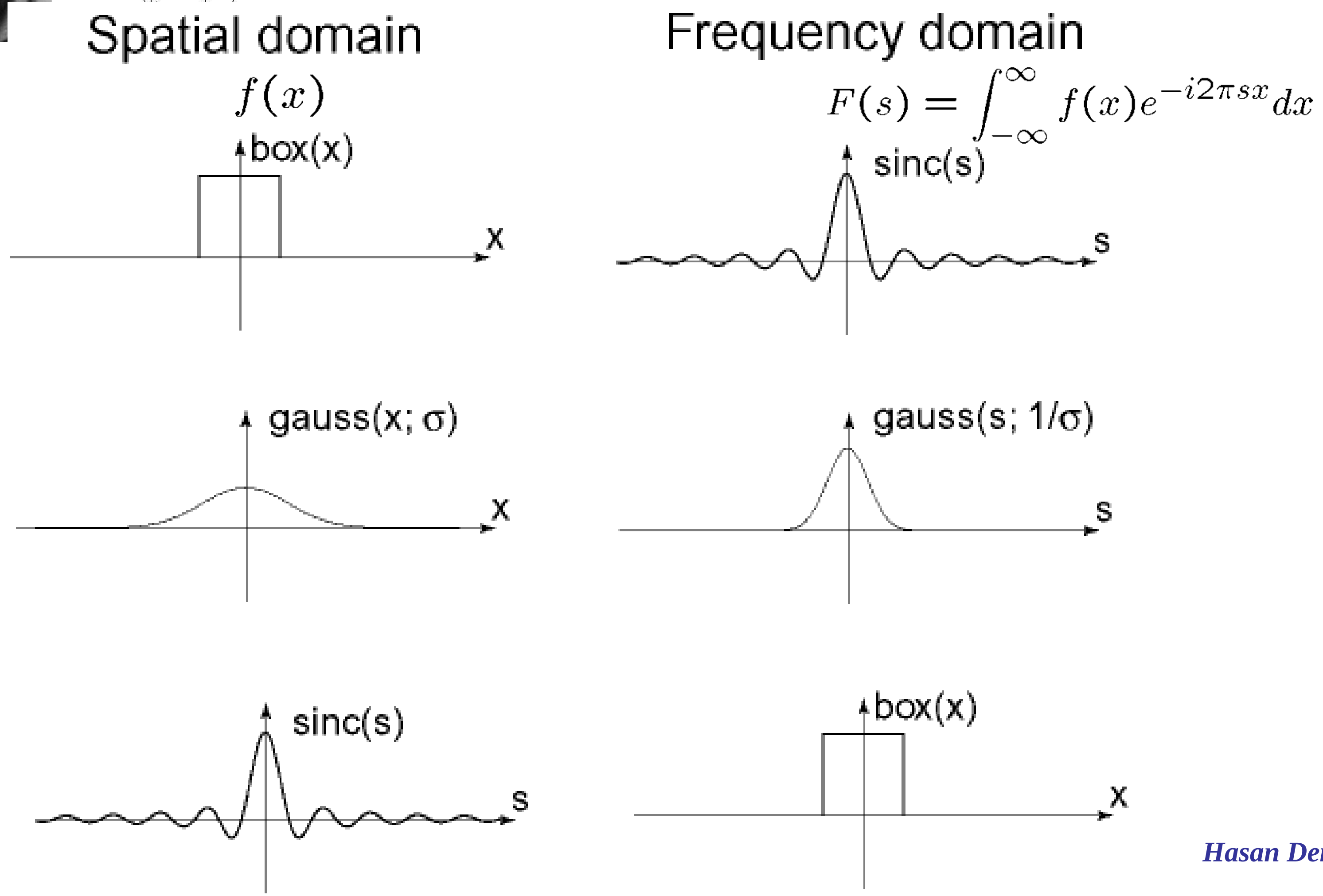


Image Enhancement in the *Frequency Domain*

1D Discrete Fourier Transform

•*1-D Discrete Fourier Transform*

The Fourier transform, F(u), of a discrete function of one variable, f(x), x=0, 1, 2, …, M-1, is given by:

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M}$$

The inverse Discrete Fourier Transform can be used to calculate f(x), by:

$$f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M}$$

Note: F(u), which is the Fourier transform of f(x) contains discrete complex quantities and it has the same number of components as f(x).

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Image Enhancement in the *Frequency Domain*

1D Discrete Fourier Transform

• ***1-D Discrete Fourier Transform***

Then;

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)[\cos 2\pi ux / M - j\sin 2\pi ux / M]$$

• ***The Fourier Transform generates complex quantities. The magnitude or the spectrum of the Fourier transform is given by:***

$$\left|F(u)\right| = \left[R^2(u) + I^2(u)\right]^{1/2}$$

R(u) is the Real Part and

I(u) is the Imaginary Part

• ***The Phase Spectrum of the transform refers to the angles between the real and imaginary components and it is denoted by:***

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right]$$

Image Enhancement in the *Frequency Domain*

1D Discrete Fourier Transform

•*1-D Discrete Fourier Transform*

•*The Power Spectrum/spectral density is defined as the square of the Fourier spectrum and denoted by*

$$P(u) = \left|F(u)\right|^2 = R^2(u) + I^2(u)$$

•*The power spectrum can be used, for example to separate a portion of a specified frequency (i.e. low frequency) power from the power spectrum and monitor the effect. Typically used to define the cut off frequencies used in lowpass and highpass filtering.*

- *We primarily use the Fourier Spectrum for image enhancement applications.*

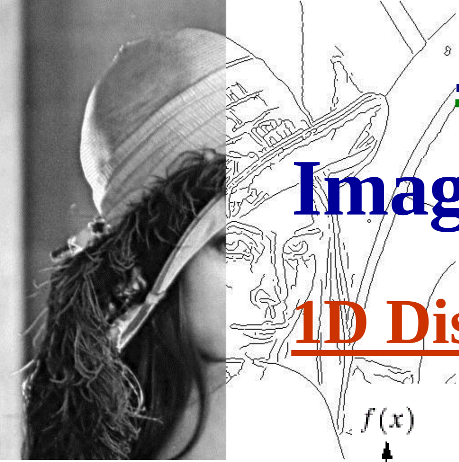


Image Enhancement in the *Frequency Domain*

1D Discrete Fourier Transform

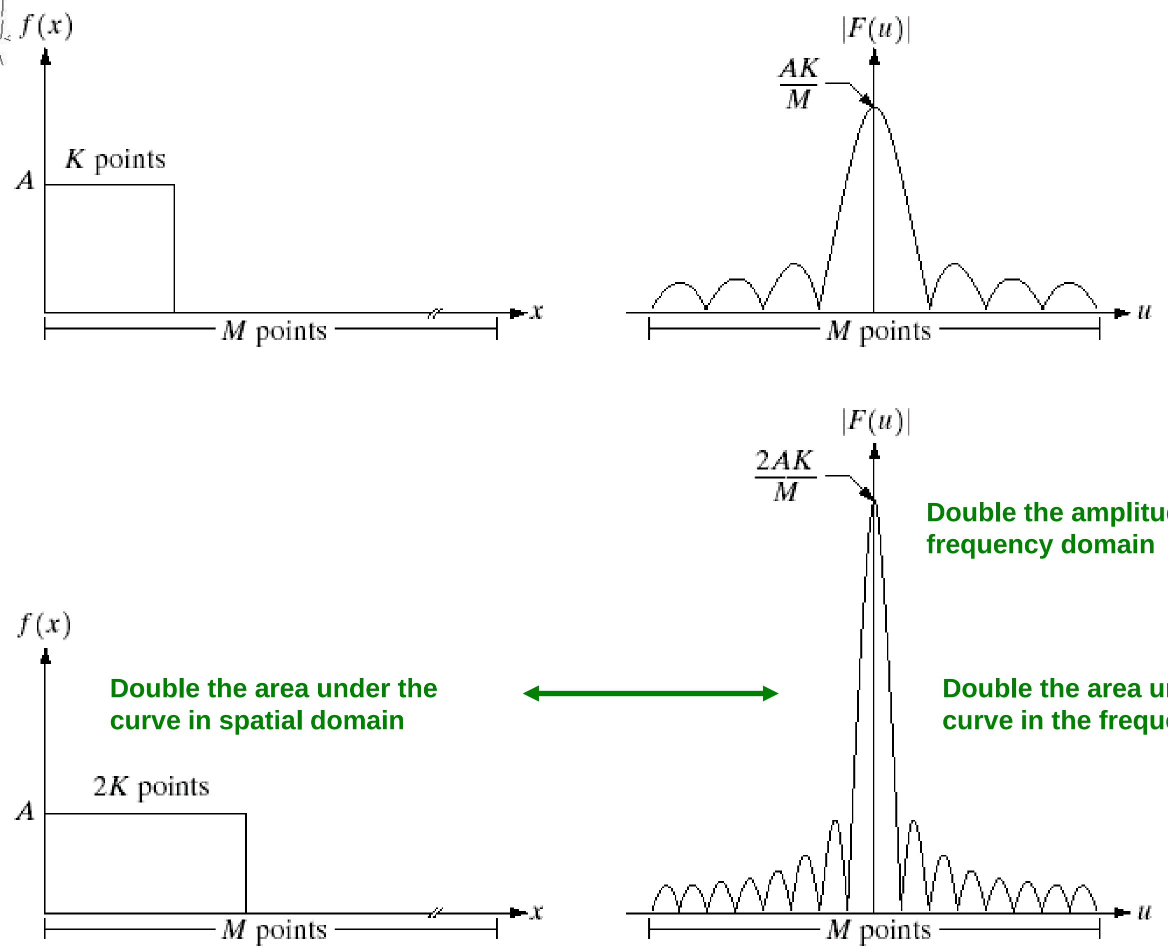


FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

Double the amplitude in the frequency domain

Double the area under the curve in spatial domain ↔ **Double the area under the curve in the frequency domain**

Prepared By: ***Dr. Hasan Demirel, PhD***

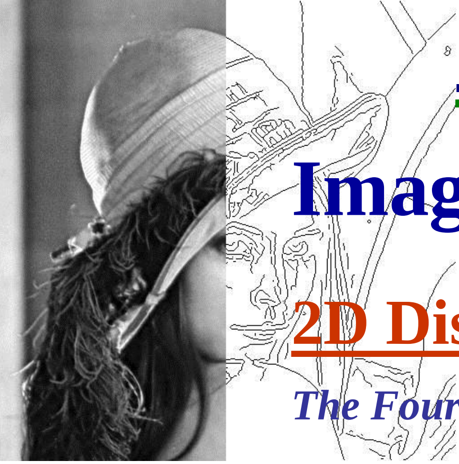


Image Enhancement in the *Frequency Domain*

2D Discrete Fourier Transform

The Fourier transform of a 2D discrete function (image) f(x,y) of size MxN is given by:

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

u=0, 1, 2, …, M-1, and v=0, 1, 2, …, N-1 and the inverse 2D Discrete Fourier Transform can be calculated by:

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

x=0, 1, 2, …, M-1, and y=0, 1, 2, …, N-1.

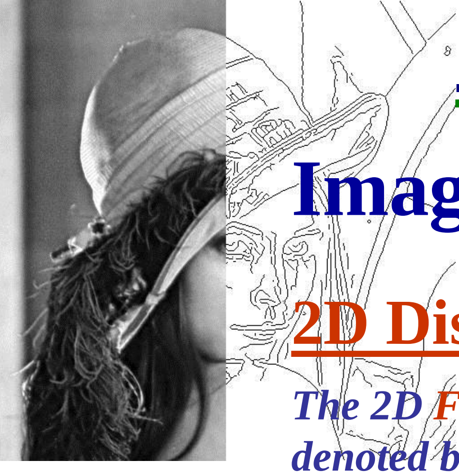


Image Enhancement in the *Frequency Domain*

2D Discrete Fourier Transform

The 2D Fourier Spectrum, Phase Spectrum and Power Spectrum can be respectively denoted by:

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$ ***Magnitude/Fourier Spectrum***

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$ ***Phase Spectrum***

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$ ***Power Spectrum***

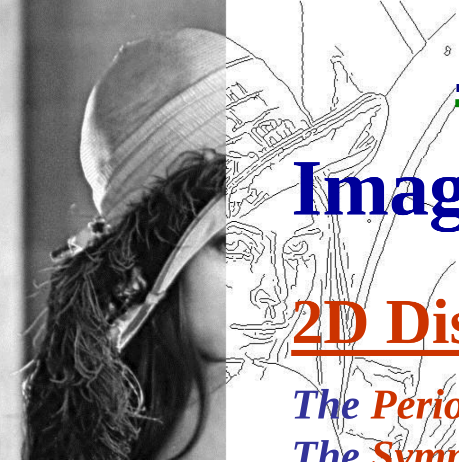
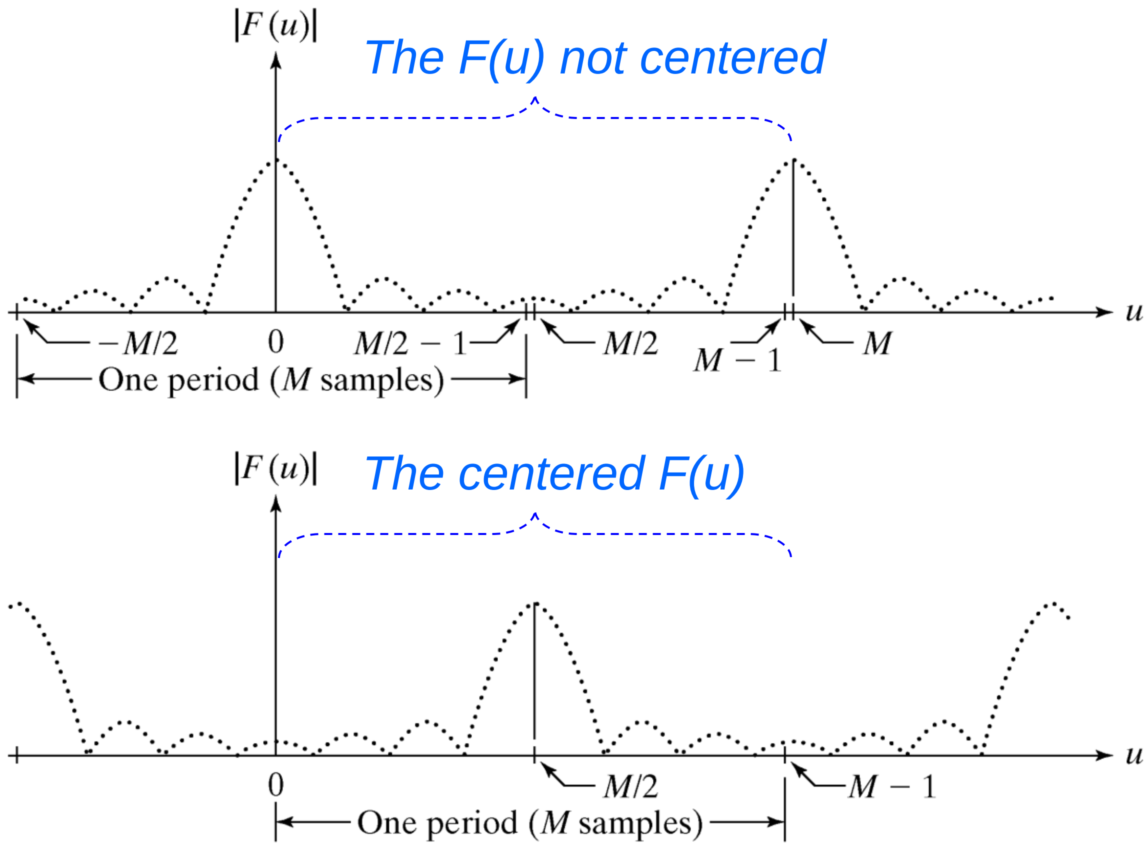


Image Enhancement in the *Frequency Domain*

2D Discrete Fourier Transform

The Periodicity property: F(u) in 1D DFT has a period of M

The Symmetry property: The magnitude of the transform is centered on the origin.



a
b

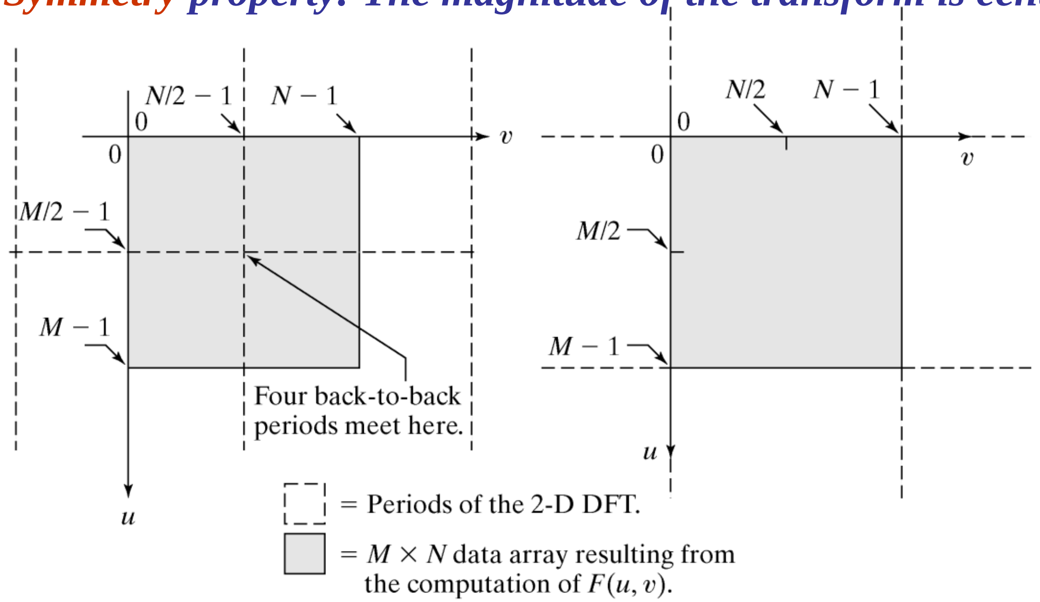
FIGURE 4.1 (a) Fourier spectrum showing back-to-back half periods in the interval $[0, M-1]$. (b) Centered spectrum in the same interval, obtained by multiplying $f(x)$ by $(-1)^x$ prior to computing the Fourier transform.

Image Enhancement in the *Frequency Domain*

2D Discrete Fourier Transform

*The **Periodicity** property: F(u,v) in 2D DFT has a period of N in horizontal and M in vertical directions*

*The **Symmetry** property: The magnitude of the transform is centered on the origin.*



a b

FIGURE 4.2 (a) $M \times N$ Fourier spectrum (shaded), showing four back-to-back quarter periods contained in the spectrum data. (b) Spectrum obtained by multiplying $f(x, y)$ by $(-1)^{x+y}$ prior to computing the Fourier transform. Only one period is shown shaded because this is the data that would be obtained by an implementation of the equation for $F(u, v)$.

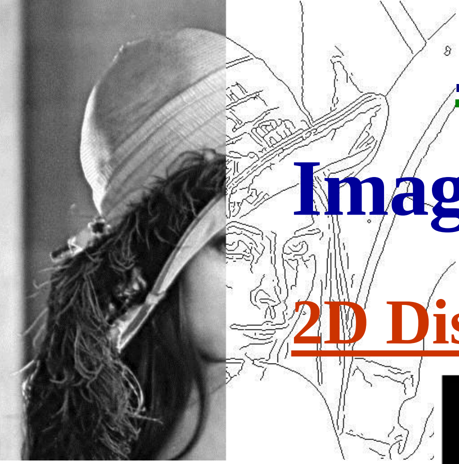
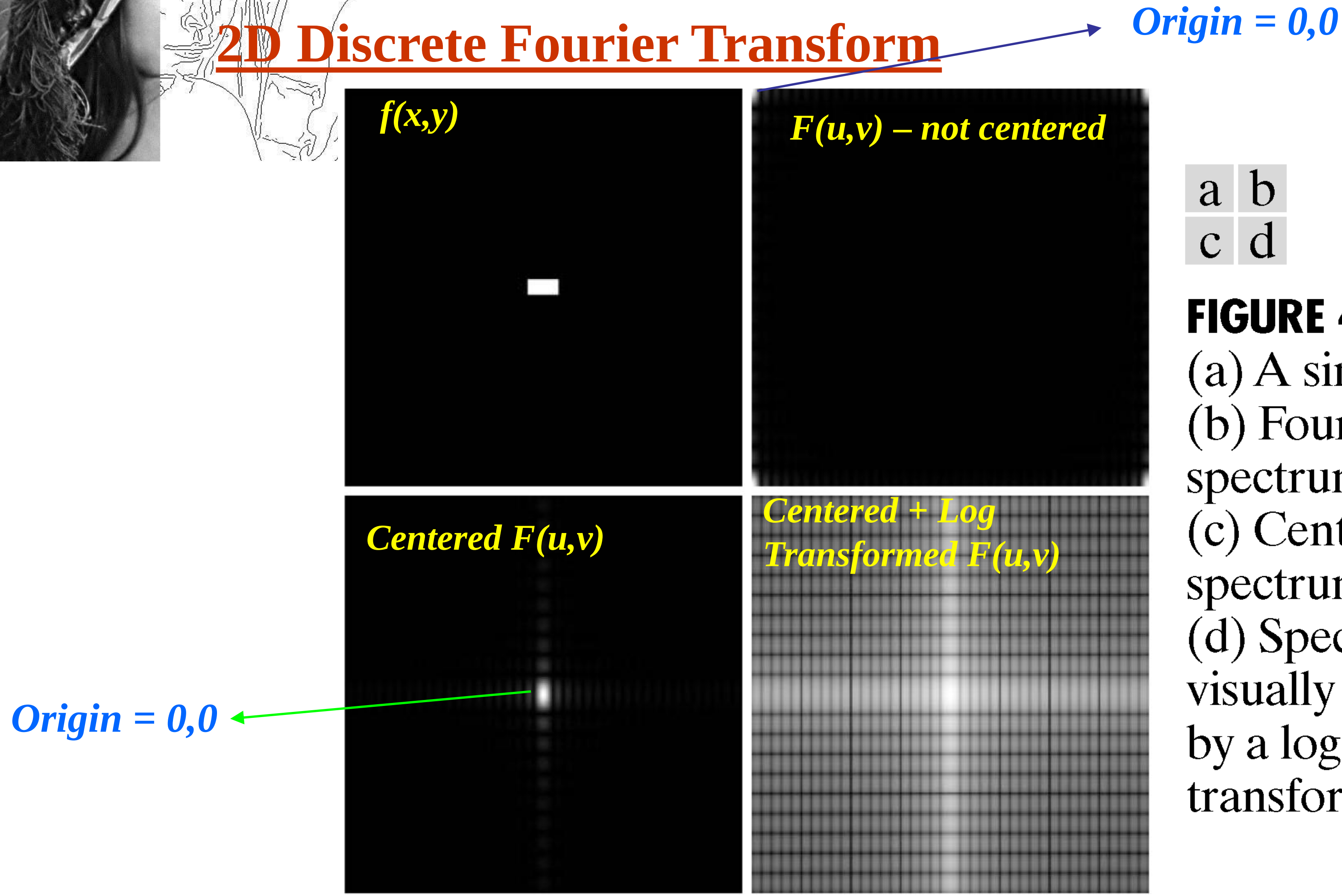


Image Enhancement in the *Frequency Domain*

2D Discrete Fourier Transform



a	b
c	d

FIGURE 4.3
(a) A simple image.
(b) Fourier spectrum.
(c) Centered spectrum.
(d) Spectrum visually enhanced by a log transformation.

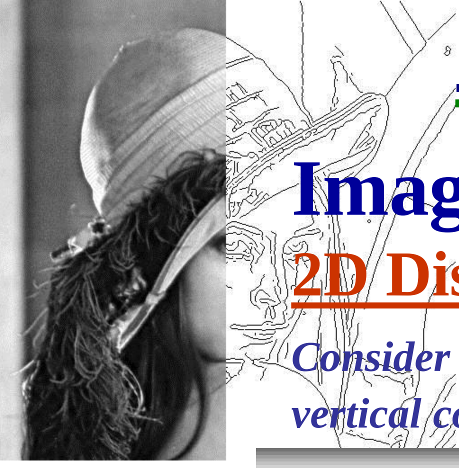
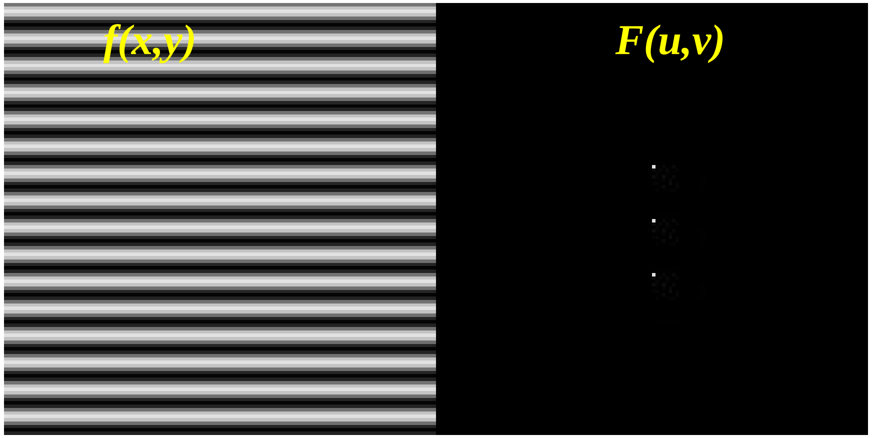


Image Enhancement in the *Frequency Domain*

2D Discrete Fourier Transform

Consider the following 2 left images which are pure vertical cosine of 4 cycles and a pure vertical cosine of 32 cycles.

Notice that the Fourier Spectrum for each image at the right contains just a single component, represented by 2 bright spots symmetrically placed about the center.



The dot at the center that represents the (0,0) frequency term or average value of the image. Images usually have a large average value/DC component. Due to low frequencies Fourier Spectrum images usually have a bright blob of components near the center.

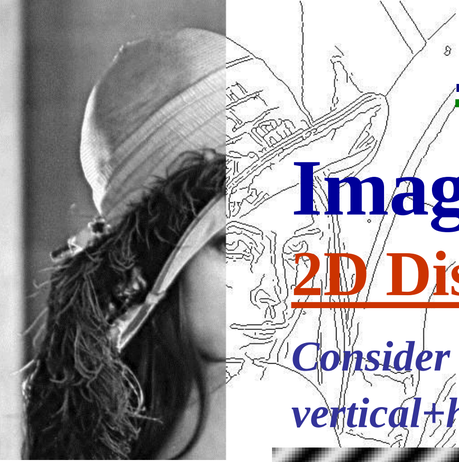
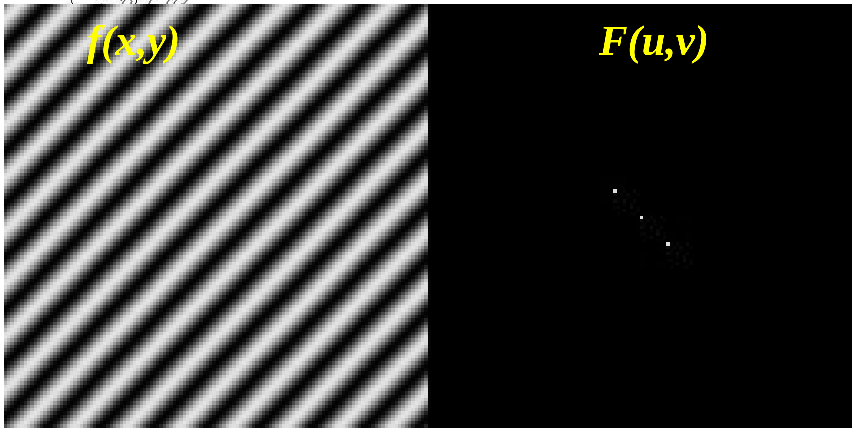


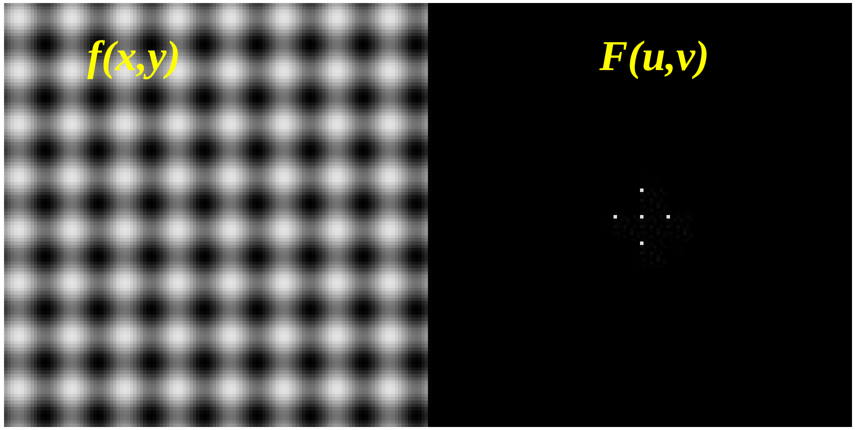
Image Enhancement in the *Frequency Domain*

2D Discrete Fourier Transform

Consider the following 2 left images with pure cosines in pure forward diagonal and mixed vertical+horizontal.



One of the properties of the 2D FT is that if you rotate the image, the spectrum will rotate in the same direction



The sum of 2 sine functions, each in opposite (vertical and horizontal) direction.

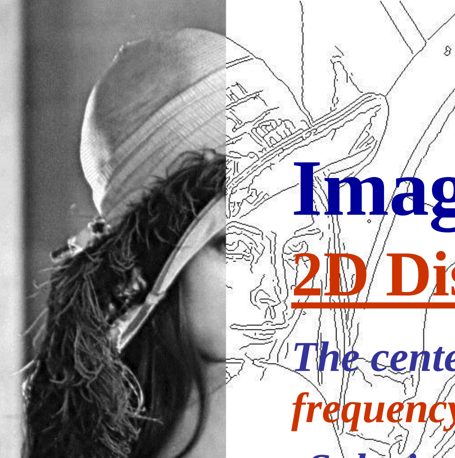


Image Enhancement in the *Frequency Domain*

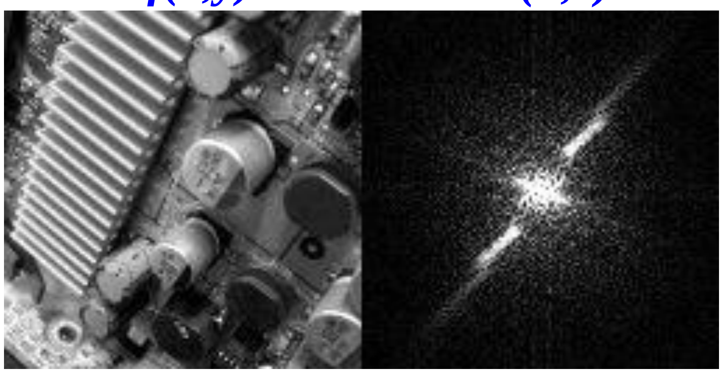
2D Discrete Fourier Transform

The center value (at the origin) of the Frequency Spectrum corresponds to the ZERO frequency component which also referred to as the DC component in an image:

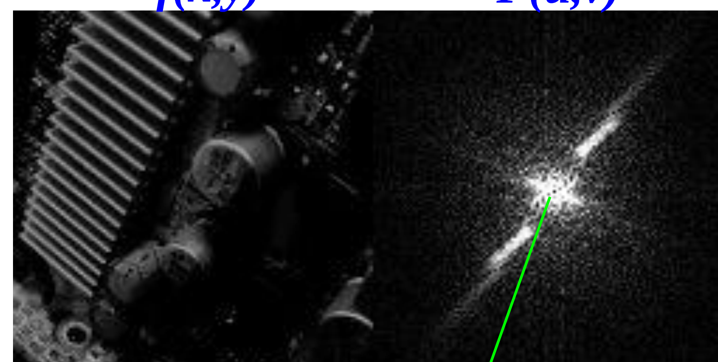
Substituting 0,0 to the origin, the Fourier transform function yields to the average/DC component value as follows:

$$F(0,0) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$$

f(x,y) **F(u,v)**



f(x,y) **F(u,v)**



The center/zero frequency component (DC component) is removed

f(x,y)
(Average image)

F(u,v)
(DC Component)

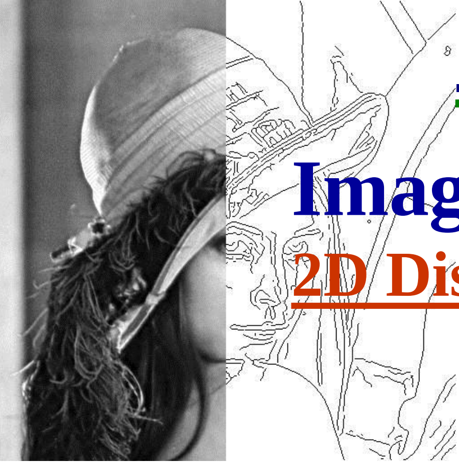
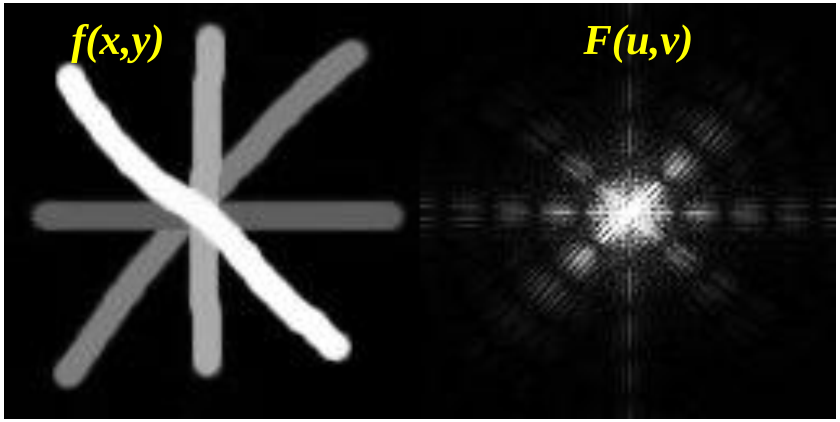


Image Enhancement in the *Frequency Domain*

2D Discrete Fourier Transform



The lines in an image often generate perpendicular lines in the spectrum

The sloped lines in the spectrum are due to the sharp transition from the sky to the mountain

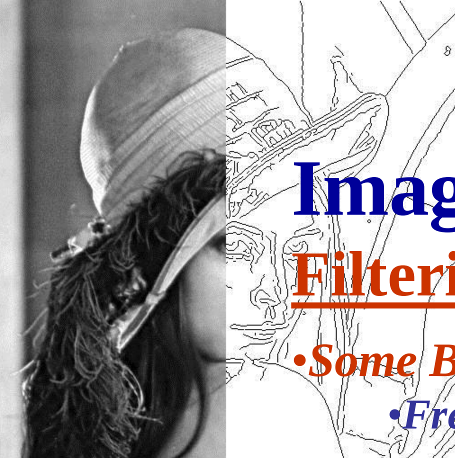
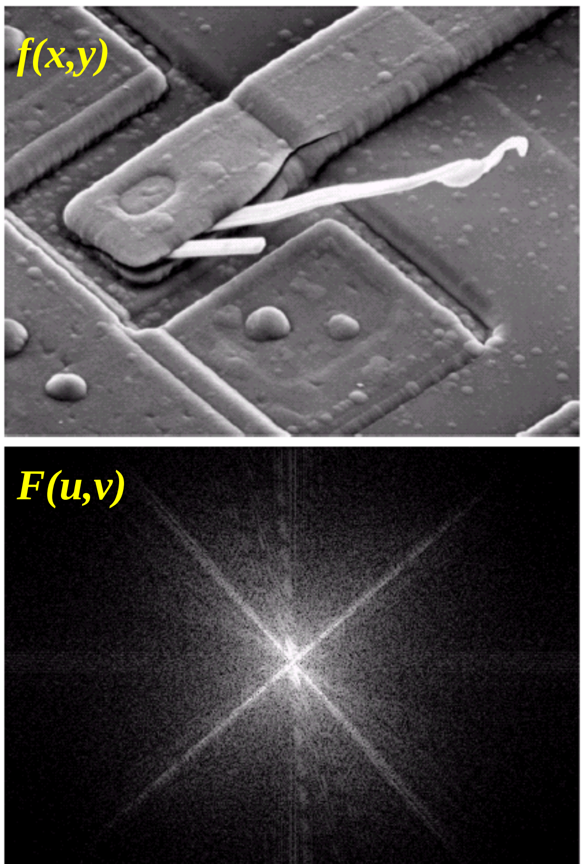


Image Enhancement in the *Frequency Domain*

Filtering in the Frequency Domain

•Some Basic Properties of the Frequency Domain:

•Frequency is directly related to the rate of change. Therefore, slowest varying component (u=v=0) corresponds to the average intensity level of the image. Corresponds to the origin of the Fourier Spectrum.



a
b

FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

•Higher frequencies corresponds to the faster varying intensity level changes in the image. The edges of objects or the other components characterized by the abrupt changes in the intensity level corresponds to higher frequencies.

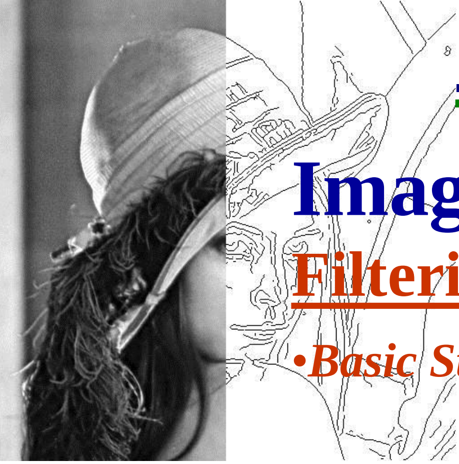


Image Enhancement in the *Frequency Domain*

Filtering in the Frequency Domain

•Basic Steps for Filtering in the Frequency Domain:

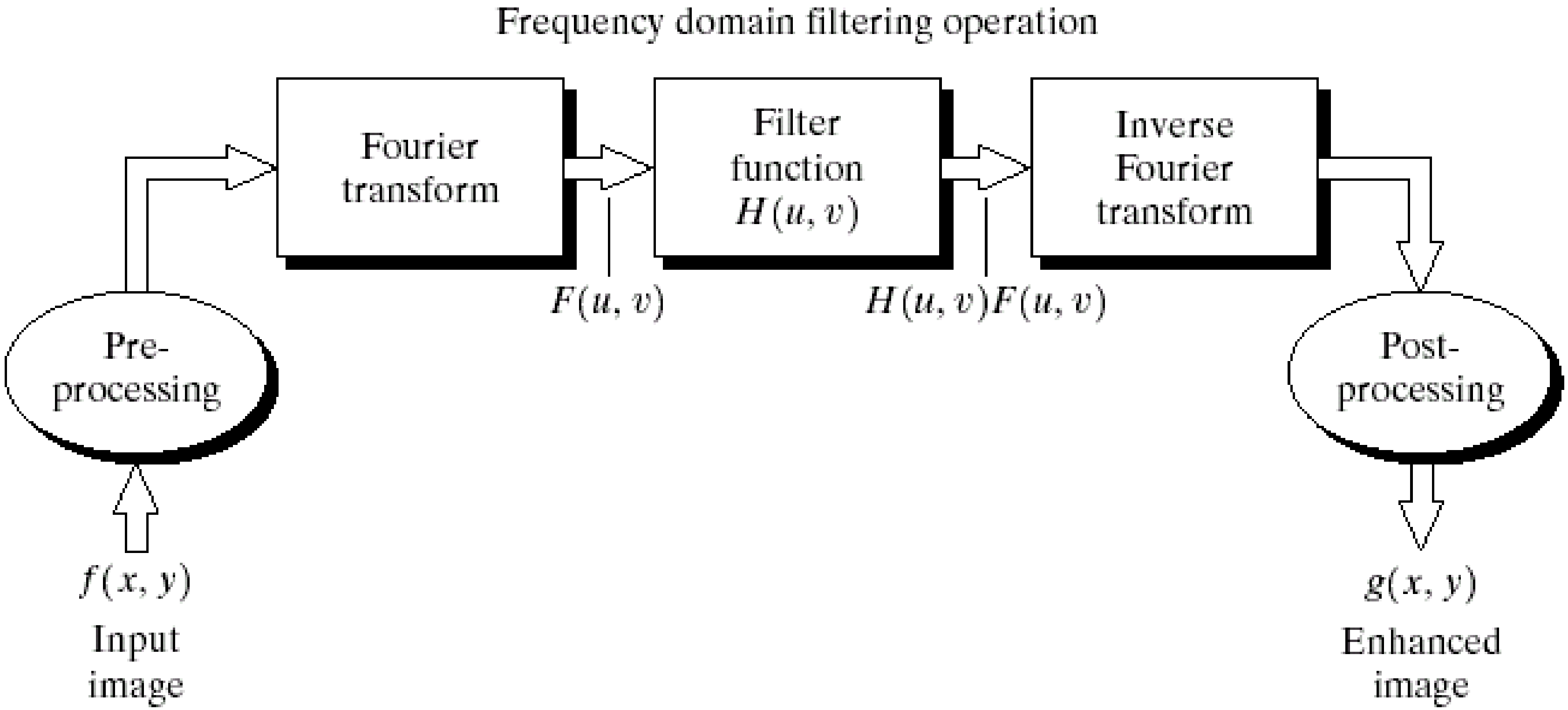


FIGURE 4.5 Basic steps for filtering in the frequency domain.

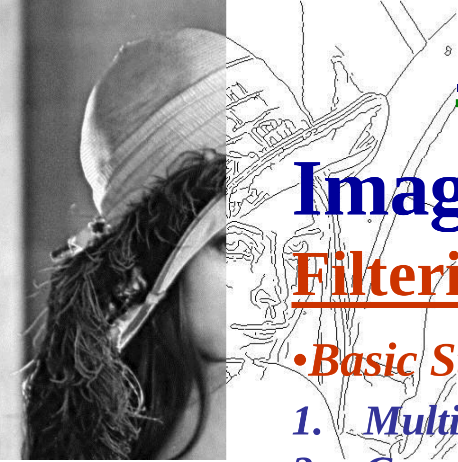


Image Enhancement in the *Frequency Domain*

Filtering in the Frequency Domain

•Basic Steps for Filtering in the Frequency Domain:

1. *Multiply the input image by $(-1)^{x+y}$ to center the transform.*
2. *Compute F(u,v), the DFT of the image from (1).*
3. *Multiply F(u,v) by a filter function H(u,v).*
4. *Compute the inverse DFT of the result in (3).*
5. *Obtain the real part of the result in (4).*
6. *Multiply the result in (5) by $(-1)^{x+y}$.*

Given the filter H(u,v) (filter transfer function) in the frequency domain, the Fourier transform of the output image (filtered image) is given by:

$$G(u,v) = H(u,v)F(u,v) \qquad \textit{Step (3)}$$

The filtered image g(x,y) is simply the inverse Fourier transform of G(u,v).

$$g(x,y) = \Im^{-1}\left[G(u,v)\right] \qquad \textit{Step (4)}$$

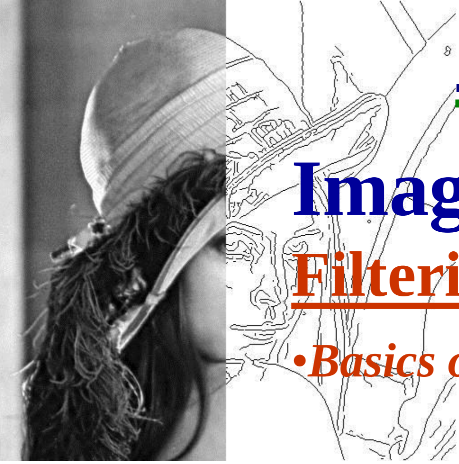
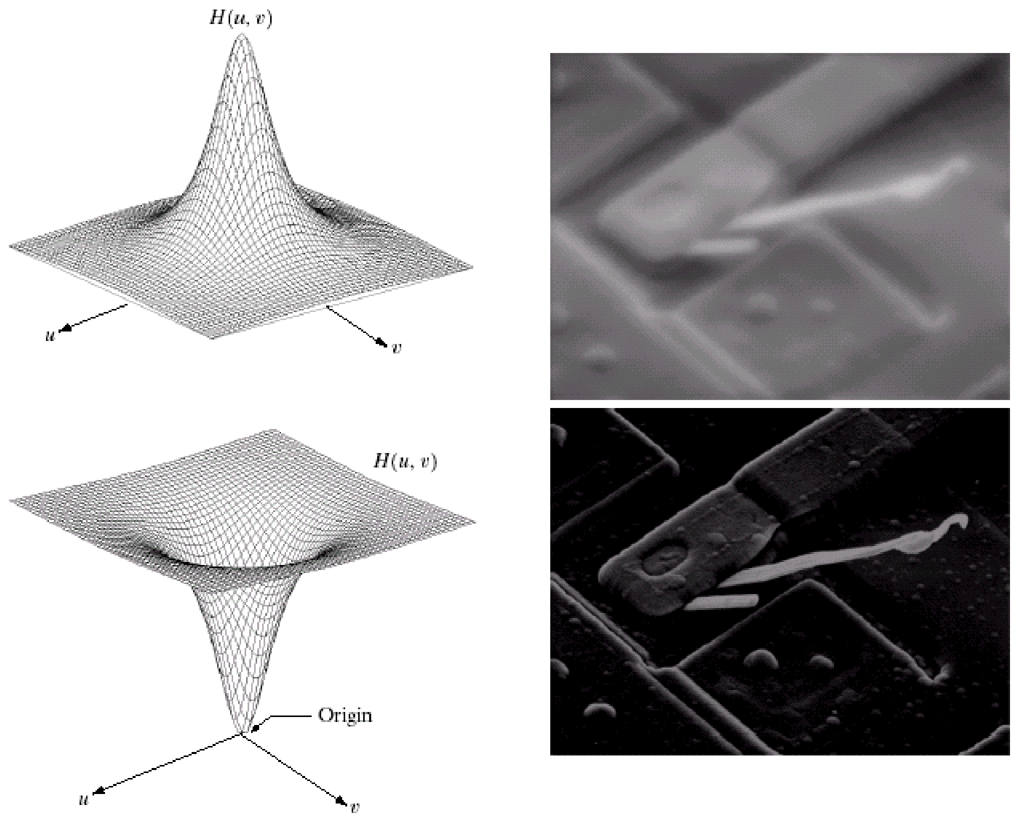


Image Enhancement in the *Frequency Domain*

Filtering in the Frequency Domain

•Basics of Low Pass Filters in the Frequency Domain:



a b
c d

FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

•lowpass filter: A filter that attenuates high frequencies while passing the low frequencies.

•Low frequencies represent the gray-level appearance of an image over smooth areas.

•highpass filter: A filter that attenuates low frequencies while passing the high frequencies.

•High frequencies represents the details such as edges and noise.

Image Enhancement in the *Frequency Domain*

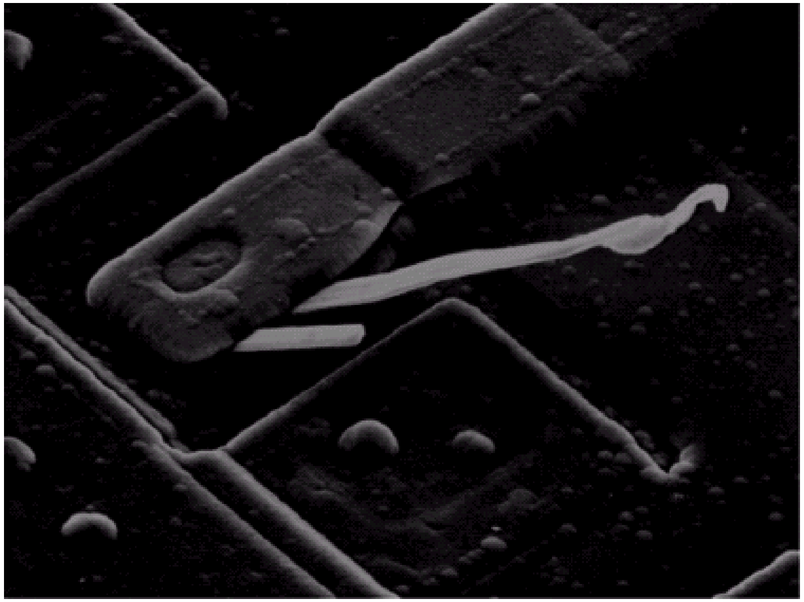
Filtering in the Frequency Domain

•*Consider the following filter transfer function:*

$$H(u,v)=\begin{cases}0 & if\ \ (u,v)=(M/2,N/2)\\ 1 & otherwise\end{cases}$$

•*This filter will set F(0,0) to zero and leave all the other frequency components. Such a filter is called the notch filter, since it is constant function with a hole (notch) at the origin.*

FIGURE 4.6 Result of filtering the image in Fig. 4.4(a) with a notch filter that set to 0 the $F(0,0)$ term in the Fourier transform.



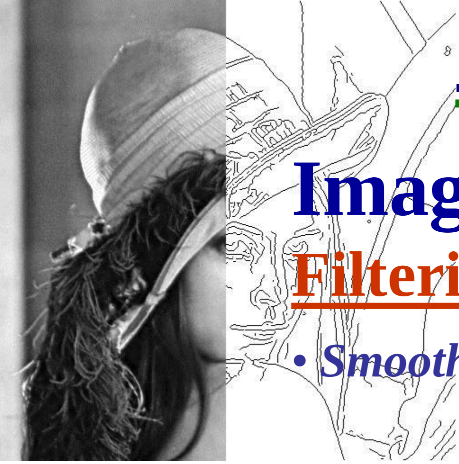


Image Enhancement in the *Frequency Domain*

Filtering in the Frequency Domain

- ***Smoothing Frequency Domain Filters: Ideal Lowpass Filters***

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)\le D_0\\ 0 & \text{if } D(u,v)>D_0\end{cases}$$

$D(u,v)$ is the distance from the origin

D_0 is the cutoff frequency

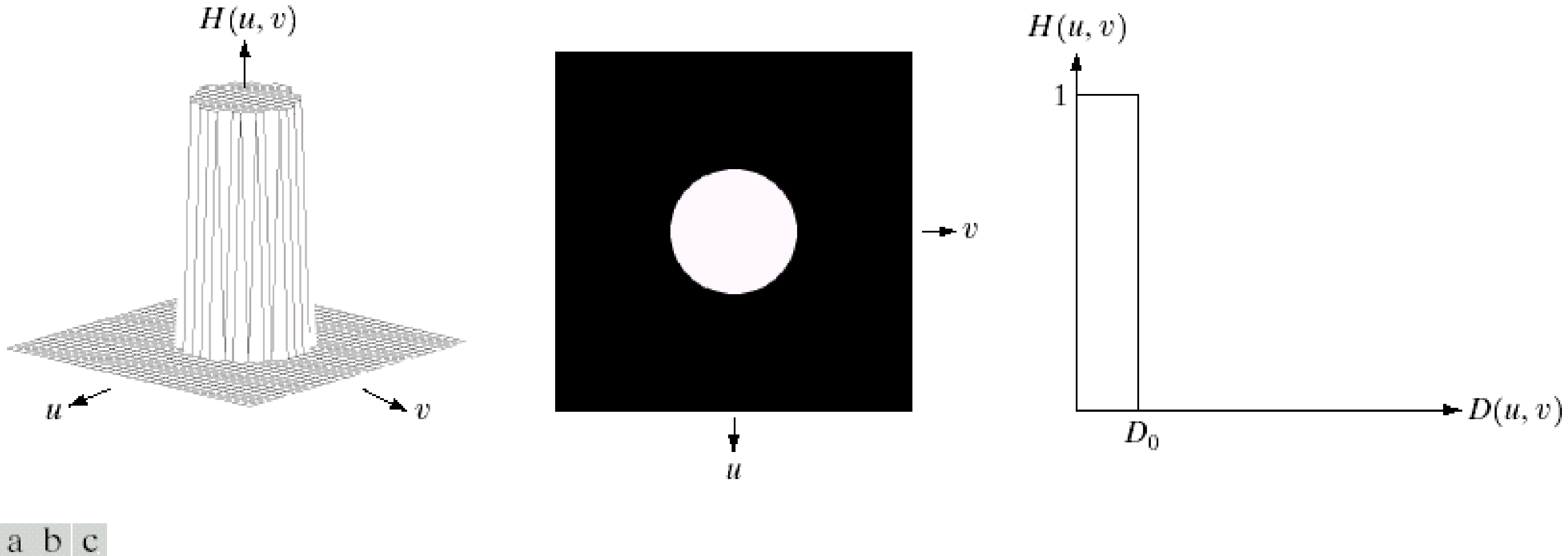


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

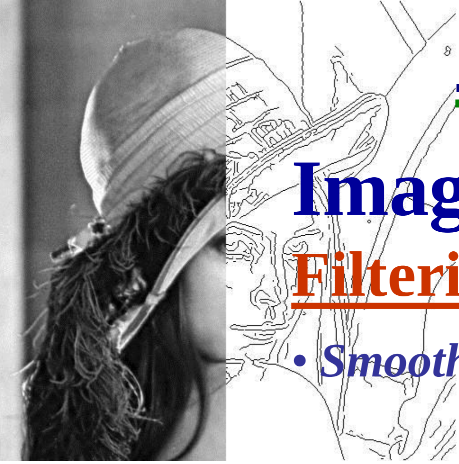


Image Enhancement in the *Frequency Domain*

Filtering in the Frequency Domain

- *Smoothing Frequency Domain Filters: Butterworth Lowpass Filters*

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$

$D(u,v)$ is the distance from the origin
D_0 is the cutoff frequency.
n is the order of the filter

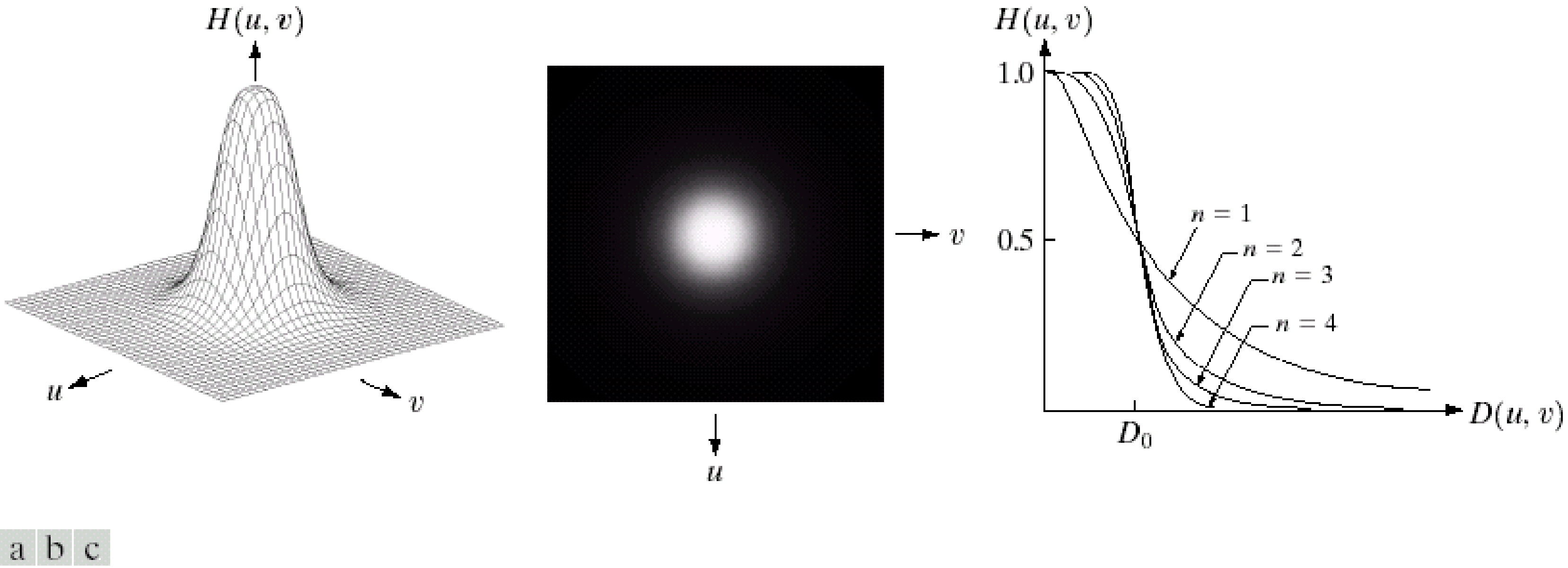


FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

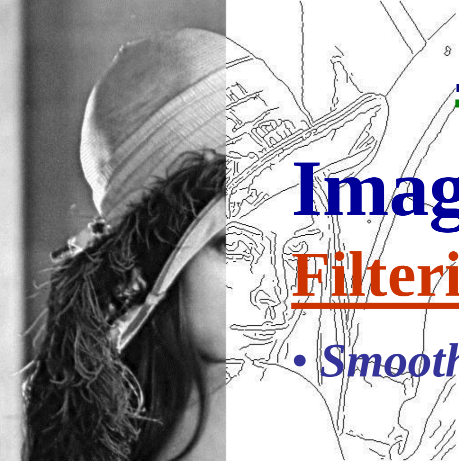


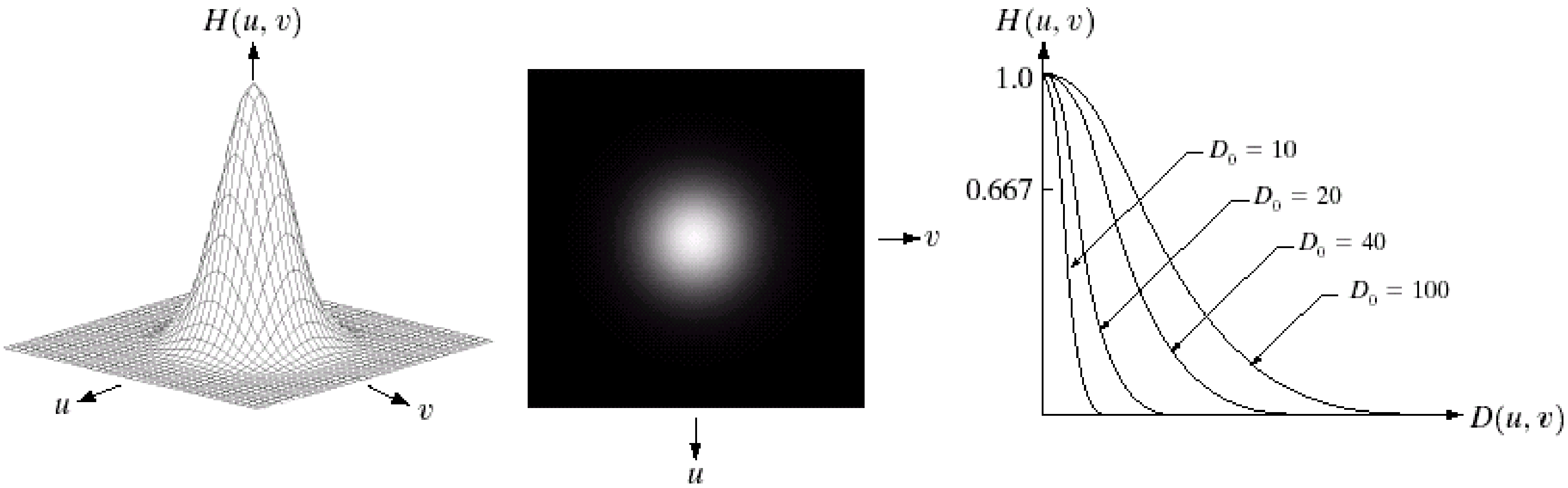
Image Enhancement in the *Frequency Domain*

Filtering in the Frequency Domain

• *Smoothing Frequency Domain Filters: Gaussian Lowpass Filters*

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

$D(u,v)$ is the distance from the origin
D_0 is the cutoff frequency.



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

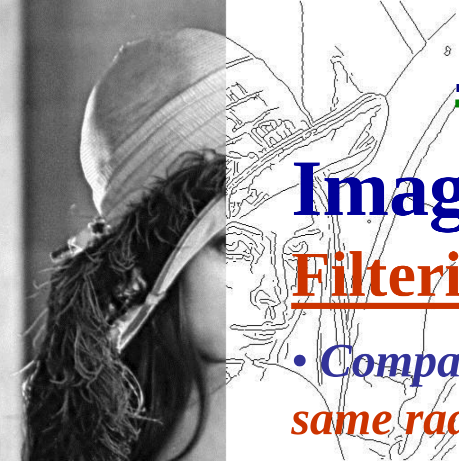
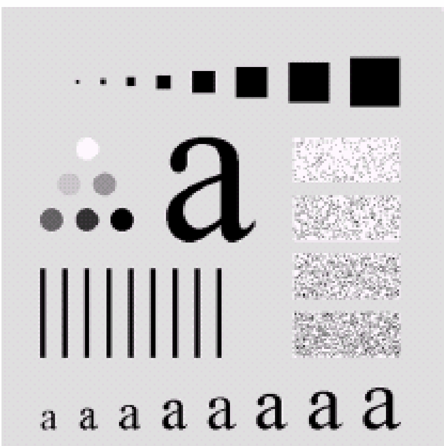


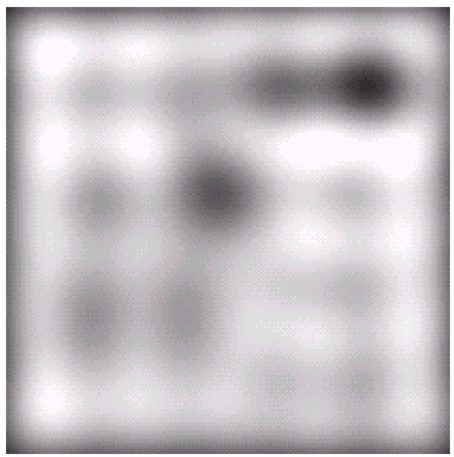
Image Enhancement in the *Frequency Domain*

Filtering in the Frequency Domain

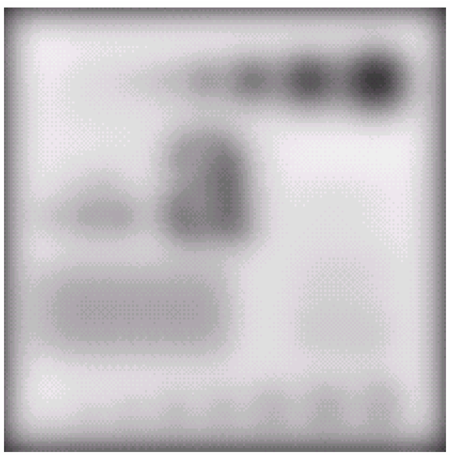
• *Comparison of Ideal, Butterworth and Gaussian Lowpass Filters having the same radii (cutoff frequency) value.*



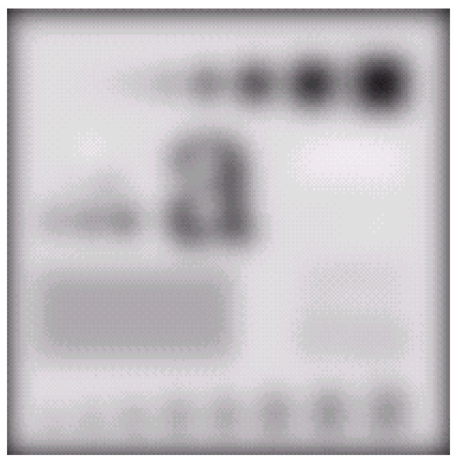
Original image



Ideal LP Filtered



Butterworth LP Filtered



Gaussian LP Filtered

Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

- ***The high-frequency components are: edges and sharp transitions such as noise.***

- ***Smoothing/blurring can be achieved by attenuating a specified range of high-frequency components in the frequency domain.***

- ***Given the Fourier transformed image F(u) , the filtered image G(u) can be obtained by:***

$$G(u,v) = H(u,v)F(u,v)$$

- ***Where H(u) is the filter transfer function.***

- ***Smoothing can be achieved by lowpass filters. We will consider only 3 types of lowpass filters:***
  - ***Ideal Lowpass filters,***
  - ***Butterworth Lowpass filters,***
  - ***Gaussian Lowpass filters***

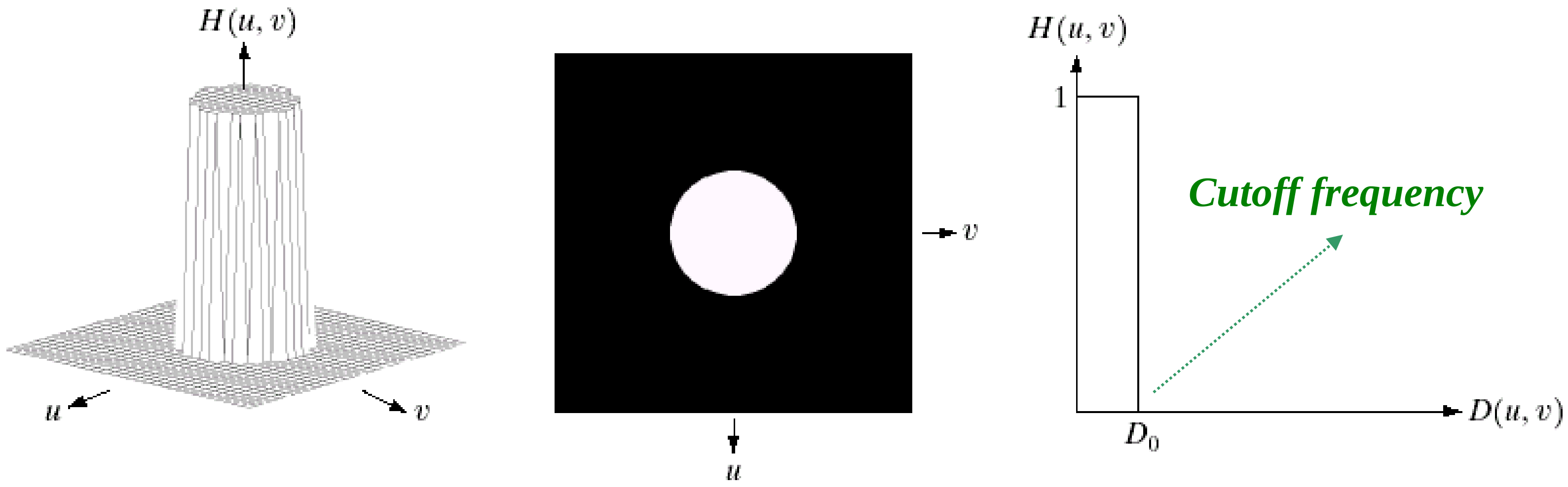
Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

• **Ideal Lowpass Filter (ILPF):** *Simply cuts off all the high frequencies higher than the specified cutoff frequency. The filter transfer function:*

$$H(u,v)=\begin{cases}1 & \text{if } \; D(u,v)\le D_0\\ 0 & \text{if } \; D(u,v)> D_0\end{cases}$$

$D(u,v)$ *is the distance from the origin*
D_0 *is the cutoff frequency*



a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

- **Cutoff Frequency of an Ideal Lowpass Filter:** *One way of specifying the cutoff frequency is to compute circles enclosing specified amounts of total image power.*
- ***Calculating P_T which is the total power of the transformed image:***

$$P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u,v) \qquad u = 0, 1, ..., M-1 \quad , v = 0, 1, ..., N-1$$

- ***The cutoff frequency D_o can be determined by specifying the α percent of the total power enclosed by a circle centered at the origin. The radius of the circle is the cutoff frequency D_o.***

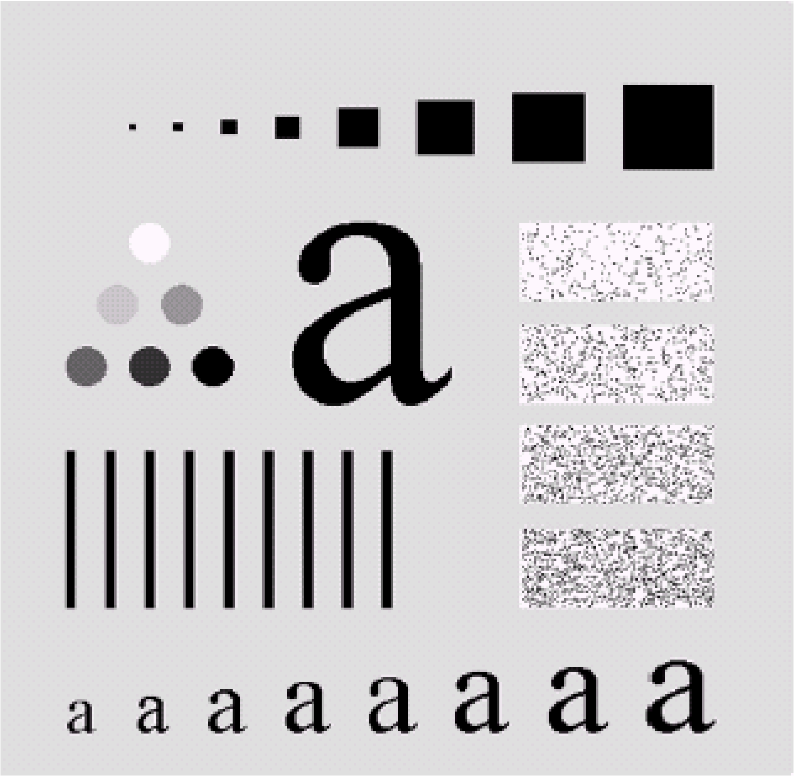
$$\alpha = 100 \left[\sum_u \sum_v P(u,v) / P_T \right] \qquad u \leq radius(D_o) \quad , v \leq radius(D_o)$$

Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

• Cutoff Frequency of an Ideal Lowpass Filter:

Original Image



Fourier Spectrum

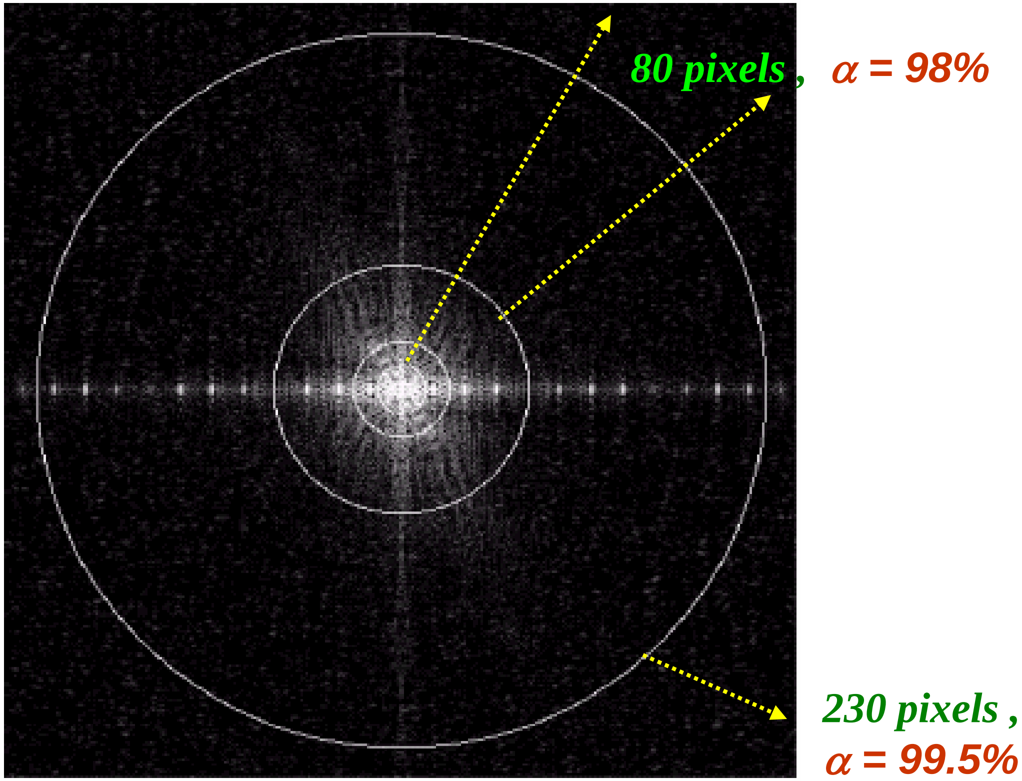
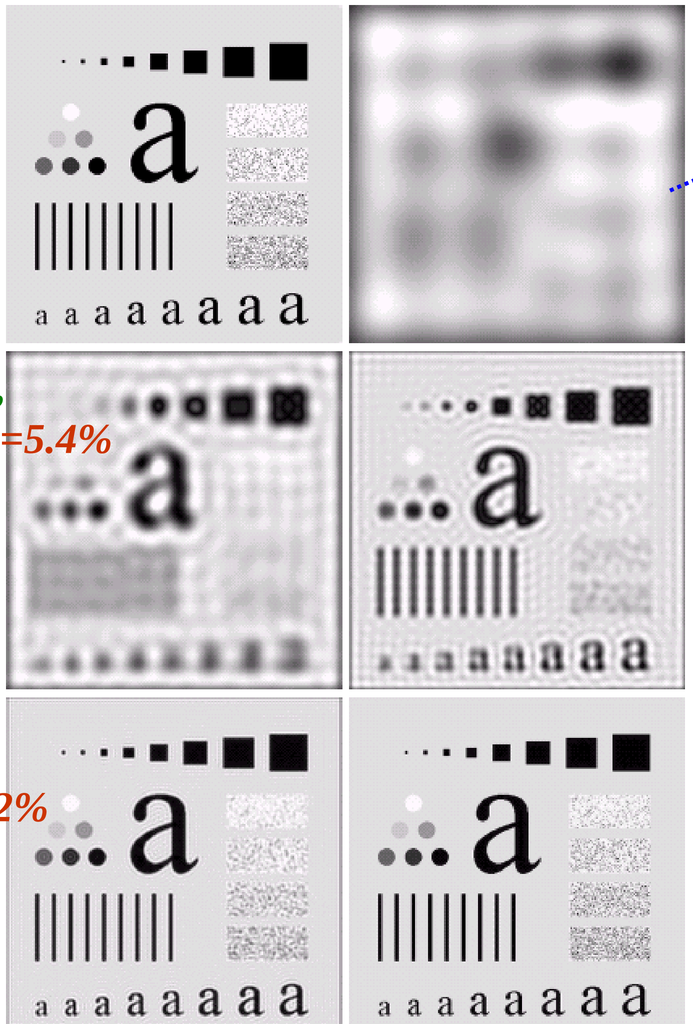


Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

- **Cutoff Frequency of an Ideal Lowpass Filter:**



Original Image

ILPF with cutoff=5, removed power=8%

Blurring effect is the result of LPIF

cutoff=15, r.d. power=5.4%

ILPF with cutoff=30, removed power=3.6%

Ringing effect is the problem of LPIF

cutoff=80, r.d. power=2%

ILPF with cutoff=230, removed power=0.5%

Prepared By: ***Dr. Hasan Demirel, PhD***

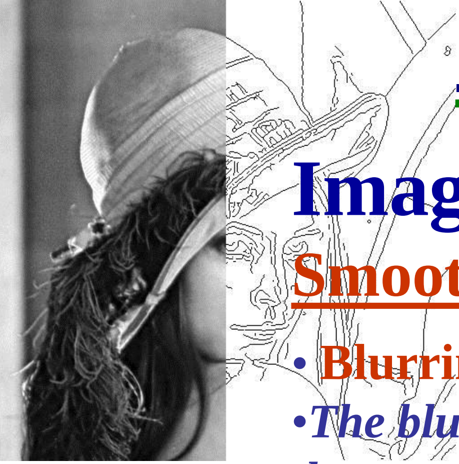


Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

• **Blurring and Ringing properties of *ILPF*:**

•*The blurring and ringing properties of the ILPF can be explained by the help of the convolution theorem:*

•*Given the Fourier transformed input image* ***F(u)*** *and output image* ***G(u) and*** *the filter transfer function* ***H(u)***,

$$G(u,v) = H(u,v)F(u,v)$$

•*The corresponding process in the spatial domain with regard to the convolution theorem is given by:*

$$g(x,y) = h(x,y) * f(x,y)$$

•*h(x,y) in the spatial domain, is the inverse Fourier transform of the filter transfer function H(u,v).*

•*The spatial filter h(x,y) has two major characteristics:*

- *dominant component at the origin*
- *Concentric circular components about center component.*

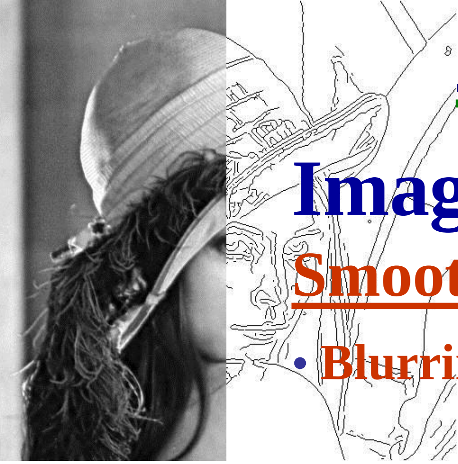
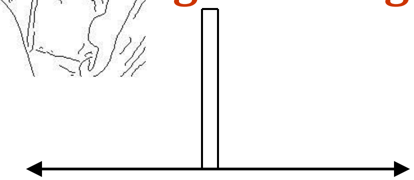


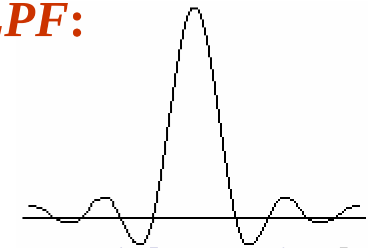
Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

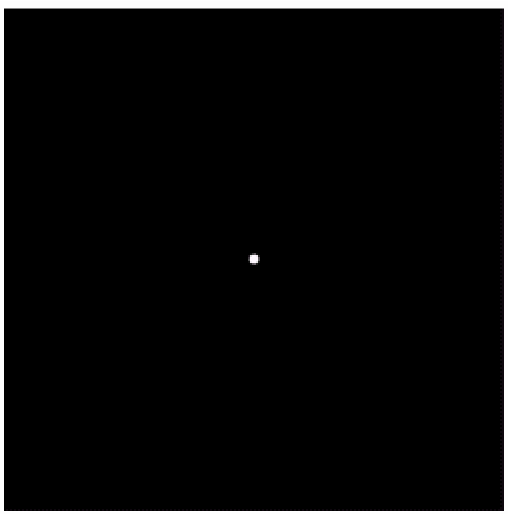
- **Blurring and Ringing properties of *ILPF*:**



*1D Frequency Domain- **H(u)***

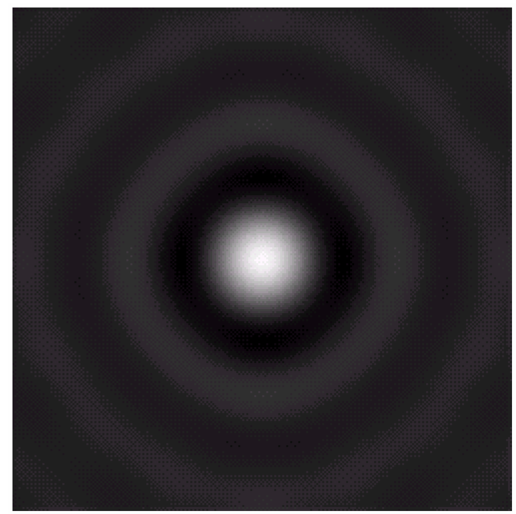


*1D Spatial Domain- **h(x)***



*2D Frequency Domain- **H(u,v)***

inverse DFT



*2D Spatial Domain – **h(x,y)***

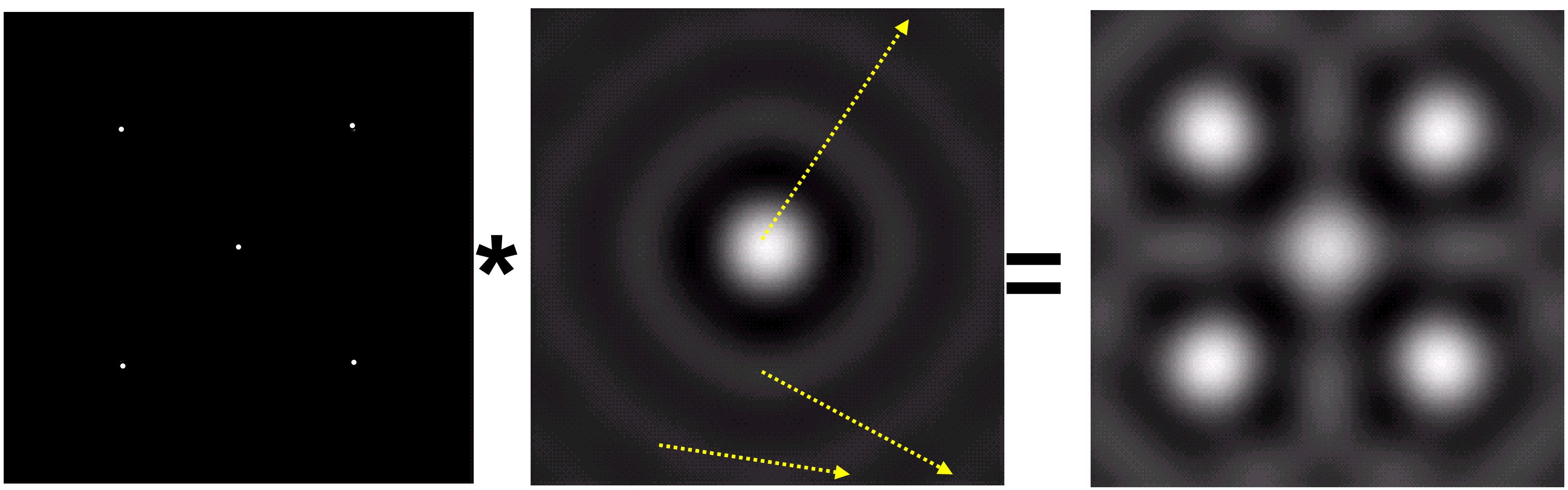
- ***The spatial domain filters center component is responsible for blurring.***
- ***The circular components are responsible for the ringing artifacts.***

Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

• **Blurring and Ringing properties of *ILPF*:** ***Lets consider the following convolution in the spatial domain:***

Central components: causes blurring



circular components: causes ringing effect

Input image - f(x,y) *Spatial Filter – h(x,y)* *filtered image – g(x,y)*

Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

• **Butterworth Lowpass Filter (BLPF):** ***The transfer function of BLPF of order n and with a specified cutoff frequency is denoted by the following filter transfer function:***

$$H(u,v)=\frac{1}{1+[D(u,v)/D_0]^{2n}}$$

***D(u,v)* is the distance from the origin**
***D_0* is the cutoff frequency.**
***n* is the order of the filter**

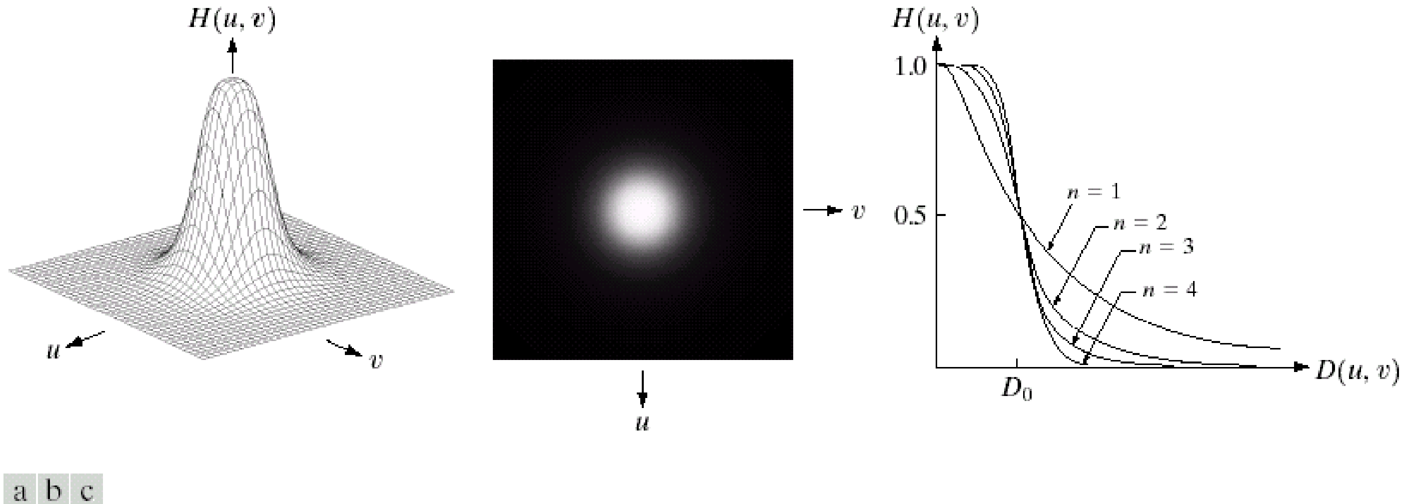


FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

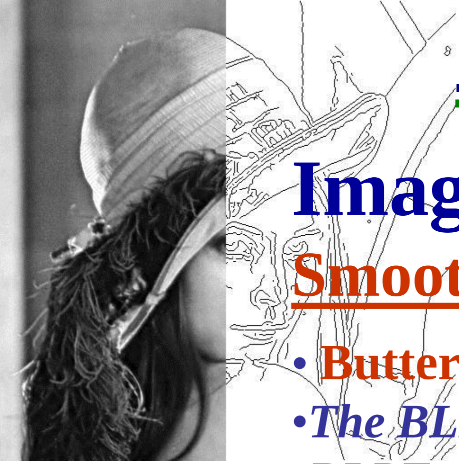


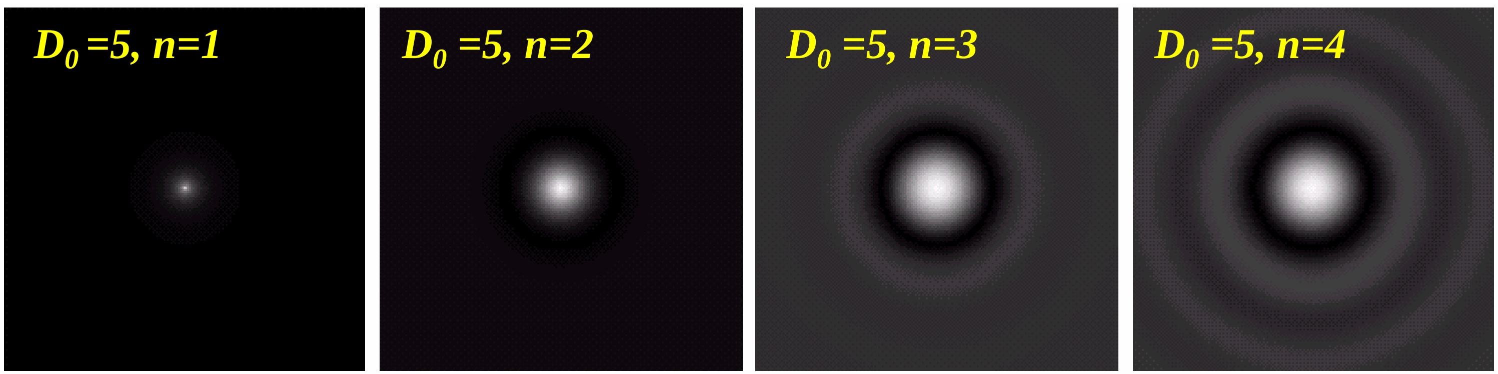
Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

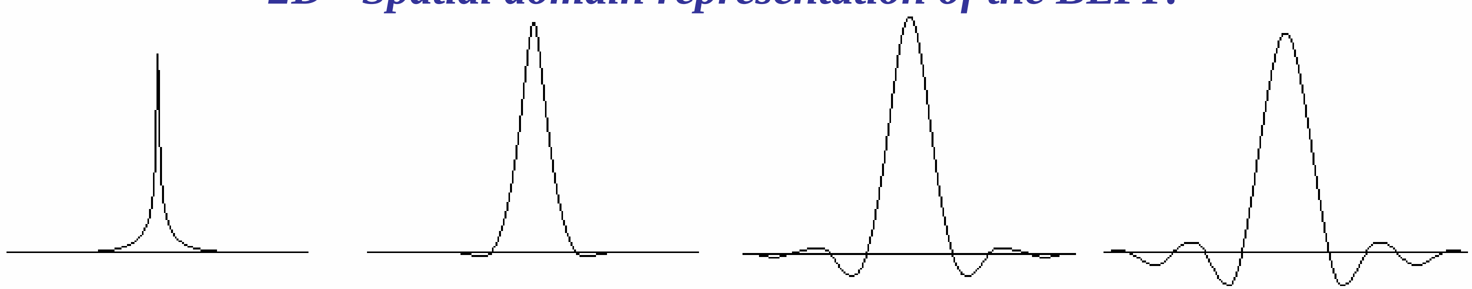
- **Butterworth Lowpass Filter (BLPF):**

•*The BLPF with order of 1 does not have any ringing artifact.*

•*BLPF with orders 2 or more shows increasing ringing effects as the order increases.*



2D - Spatial domain representation of the BLPF.



1D - Profile through the centre of the Spatial domain filters.

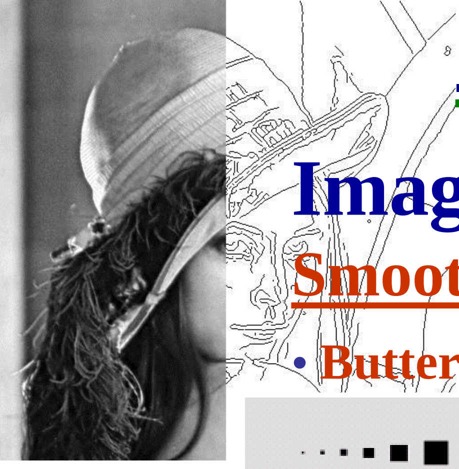
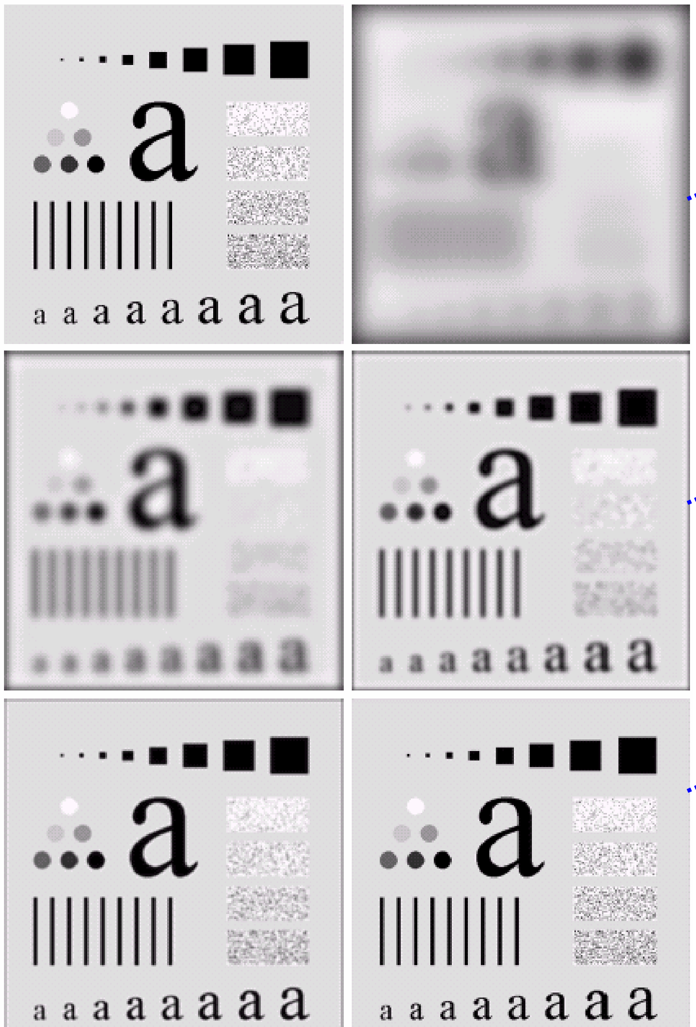


Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

- **Butterworth Lowpass Filter (BLPF):**

Original Image



BLPF with cutoff=5, order=2

BLPF with cutoff=30, order=2

BLPF with cutoff=230, order=2

cutoff=15, order=2

cutoff=80, order=2

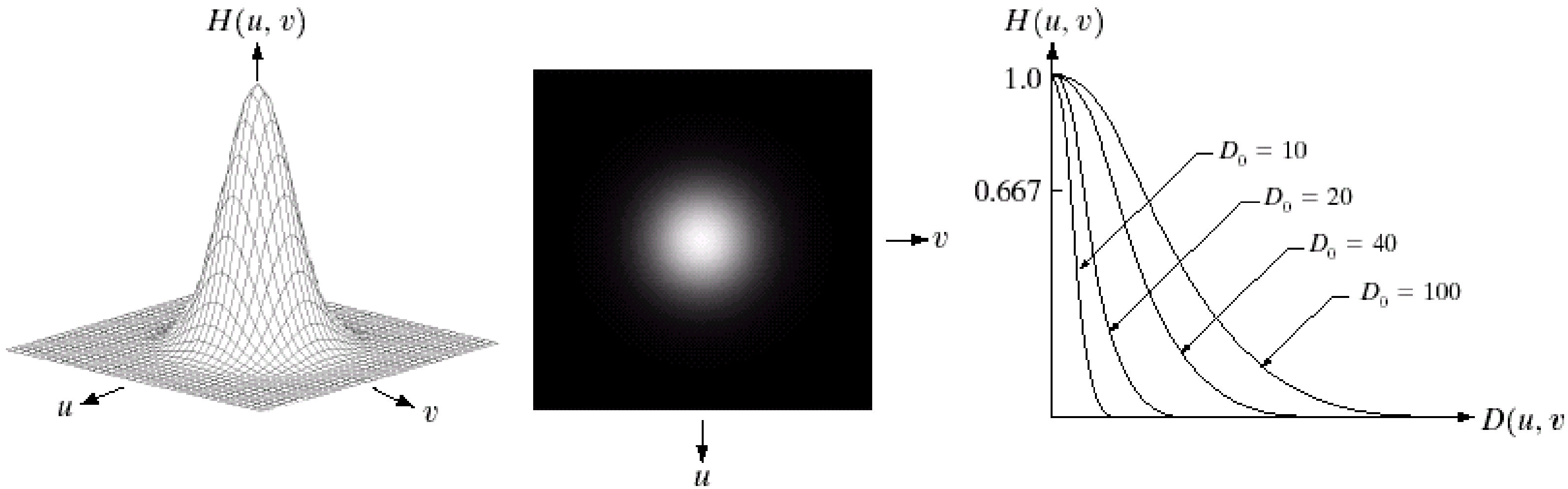
Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

• **Gaussian Lowpass Filter (GLPF):** *The transfer function of GLPF is given as follows:*

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

D(u,v) *is the distance from the origin*
D_0 *is the cutoff frequency.*



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

Original Image

cutoff=5

cutoff=15

cutoff=30

cutoff=80

cutoff=230

- **(GLPF):** *The inverse Fourier transform of the Gaussian Lowpass filter is also Gaussian in the Spatial domain.*

- *Therefore there is no ringing effect of the GLPF. Ringing artifacts are not acceptable in fields like medical imaging. Hence use Gaussian instead of the ILPF/BLPF.*

Prepared By: *Dr. Hasan Demirel, PhD*

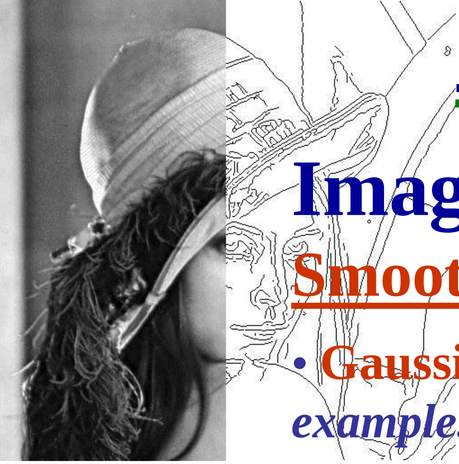
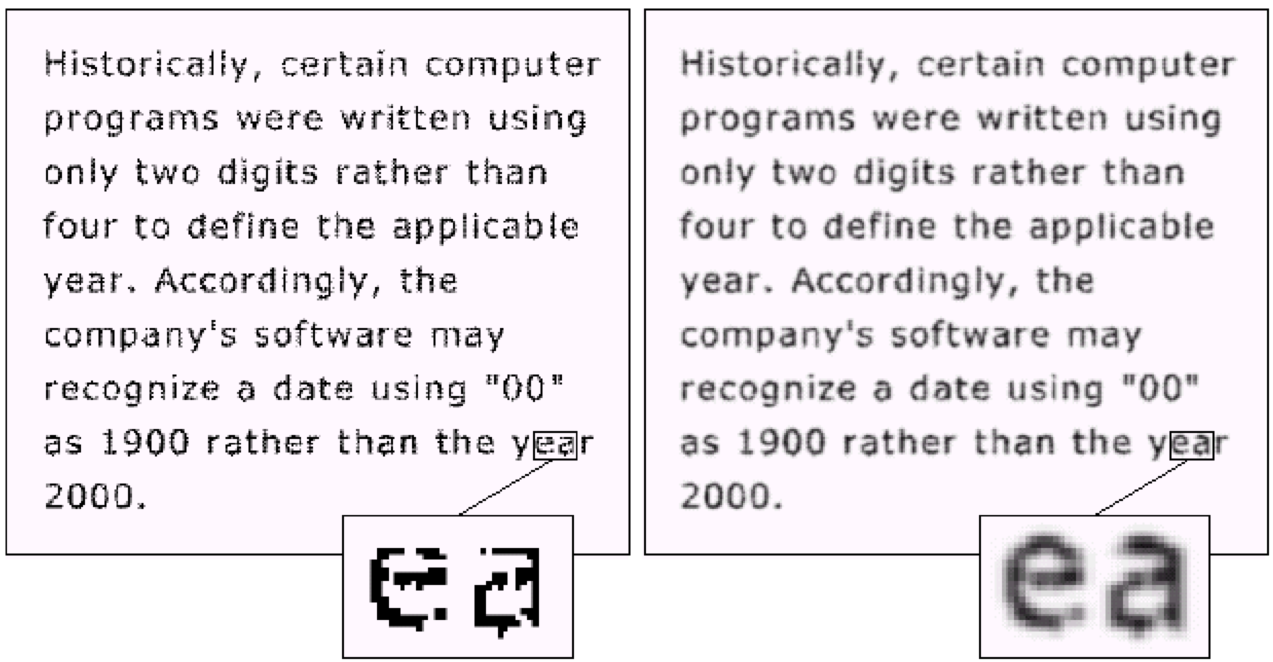


Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

• **Gaussian Lowpass Filter (GLPF):** ***Refer to the improvement in the following example.***



a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

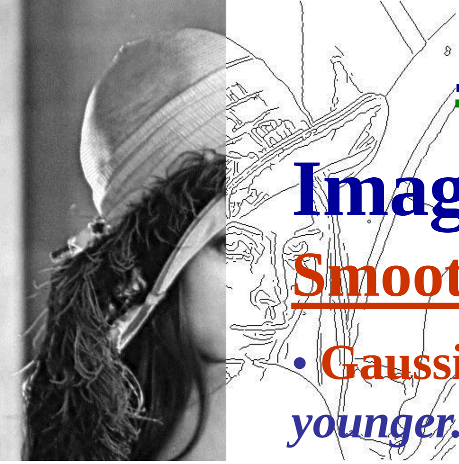


Image Enhancement in the *Frequency Domain*

Smoothing Frequency-Domain Filters

• **Gaussian Lowpass Filter (GLPF):** *The following example shows a lady younger. How? By using GLPF!*

a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

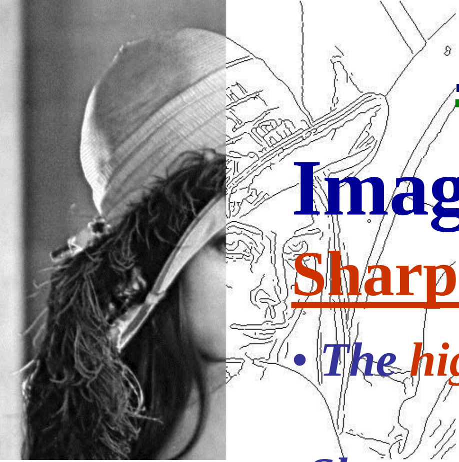


Image Enhancement in the *Frequency Domain*

Sharpening Frequency-Domain Filters

- *The high-frequency components are: edges and sharp transitions such as noise.*

- *Sharpening can be achieved by highpass filtering process, which attenuates low frequency components without disturbing the high-frequency information in the frequency domain.*

- *The filter transfer function, **$H_{hp}(u,v)$**, of a highpass filter is given by:*

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

- *Where **$H_{lp}(u,v)$**, is the transfer function of the corresponding lowpass filter.*

- *We will consider only 3 types of sharpening highpass filters :*
  - *Ideal Highpass filters,*
  - *Butterworth Highpass filters,*
  - *Gaussian Highpass filters*

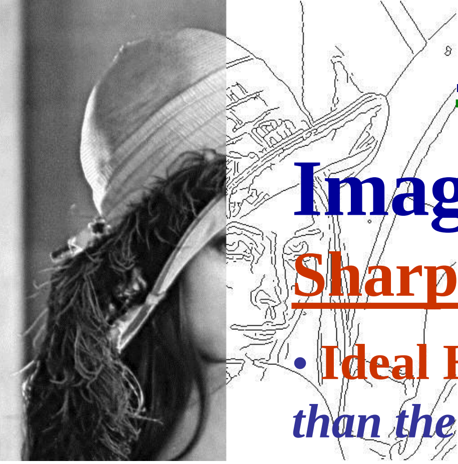


Image Enhancement in the *Frequency Domain*

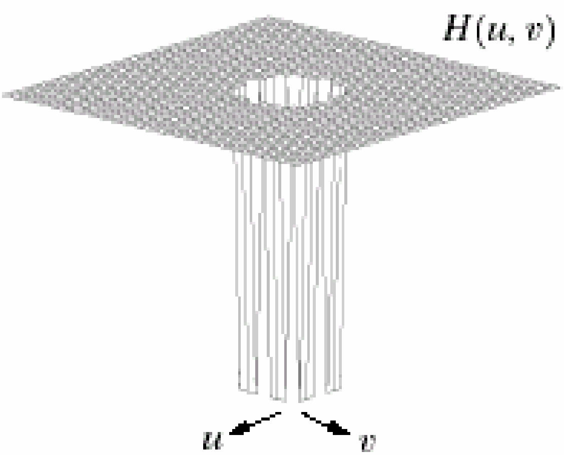
Sharpening Frequency-Domain Filters

• **Ideal Highpass Filter (IHPF):** *Simply cuts off all the low frequencies lower than the specified cutoff frequency. The filter transfer function:*

$$H(u,v)=\begin{cases}0 & \text{if } D(u,v)\le D_0\\ 1 & \text{if } D(u,v)>D_0\end{cases}$$

***D(u,v)** is the distance from the origin*

D_0 is the cutoff frequency



Perspective plot of IHPF

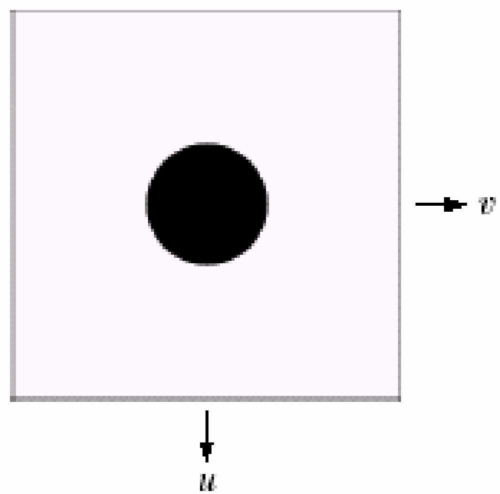
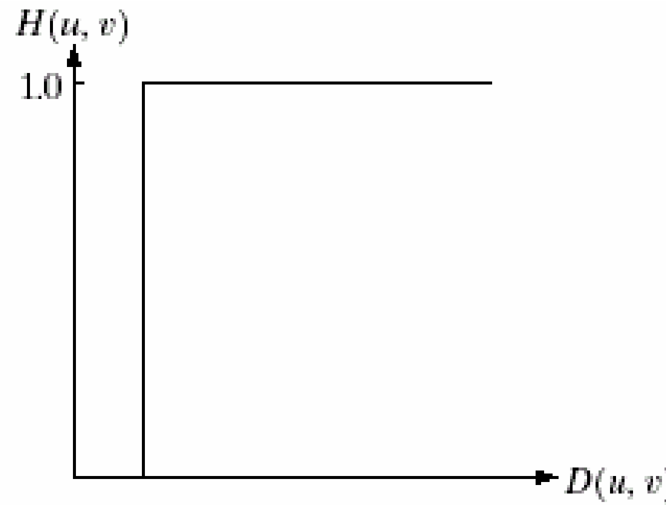


Image representation of IHPF



Cross section of IHPF

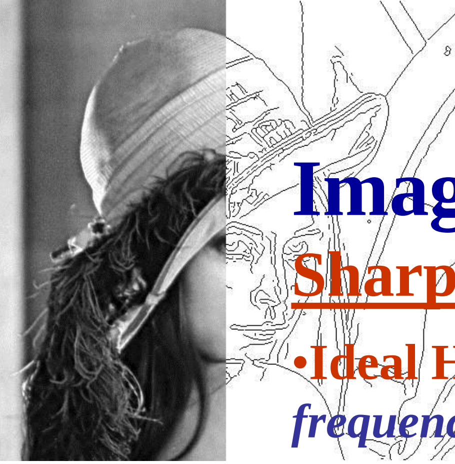


Image Enhancement in the *Frequency Domain*

Sharpening Frequency-Domain Filters

•**Ideal Highpass Filter (IHPF):** *The ringing artifacts occur at low cutoff frequencies*

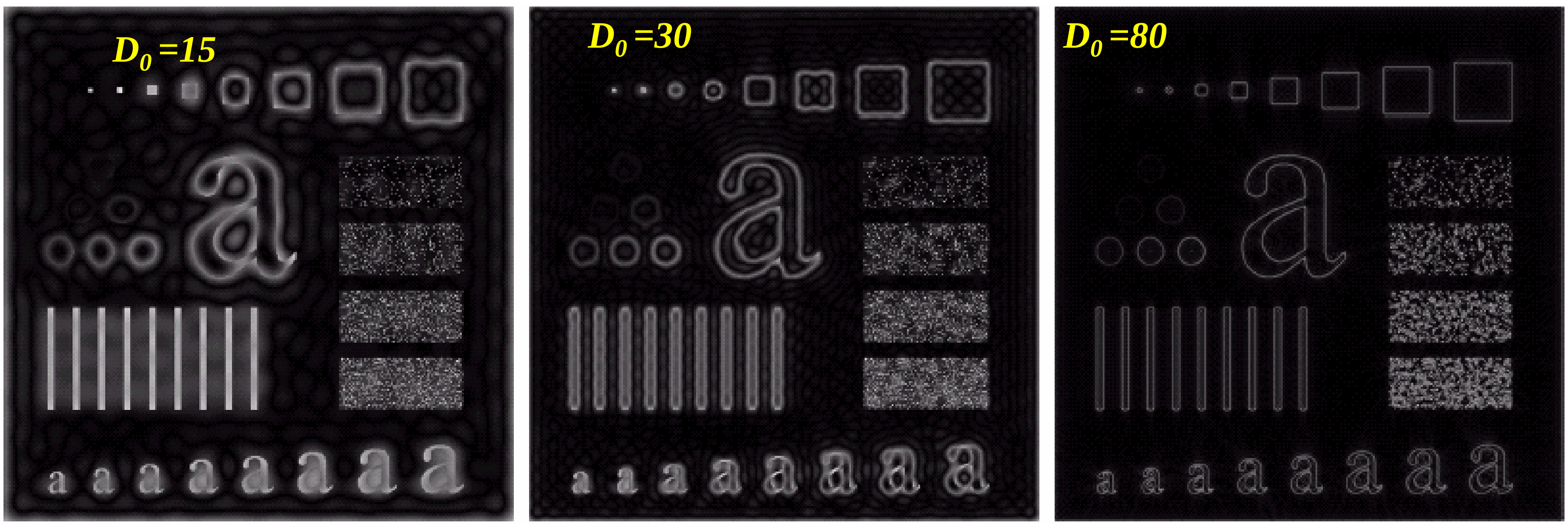


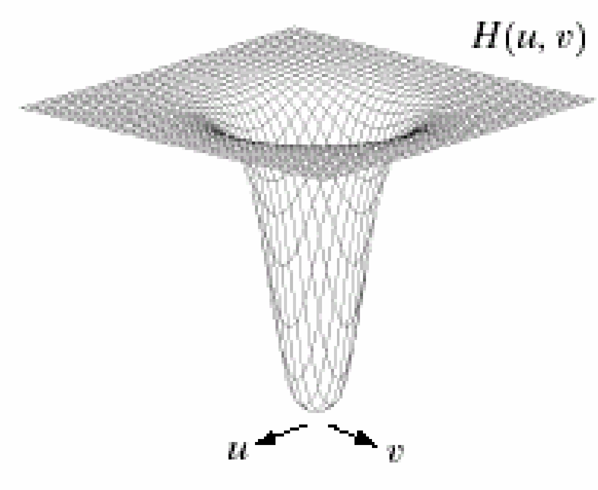
Image Enhancement in the *Frequency Domain*

Sharpening Frequency-Domain Filters

• **Butterworth Highpass Filter (BHPF):** *The transfer function of BHPF of order n and with a specified cutoff frequency is given by:*

$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

D(u,v) *is the distance from the origin*
D_0 *is the cutoff frequency.*
n *is the order of the filter*



Perspective plot of BHPF

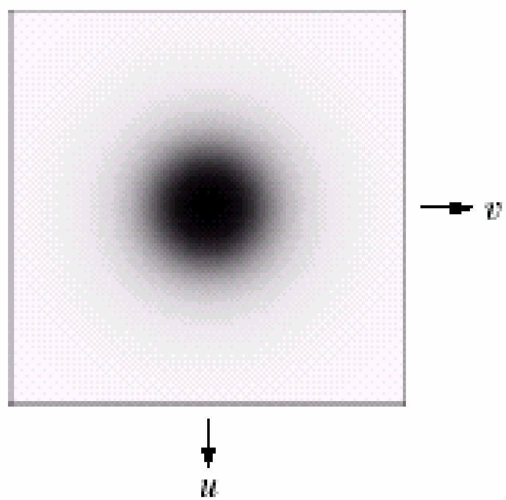
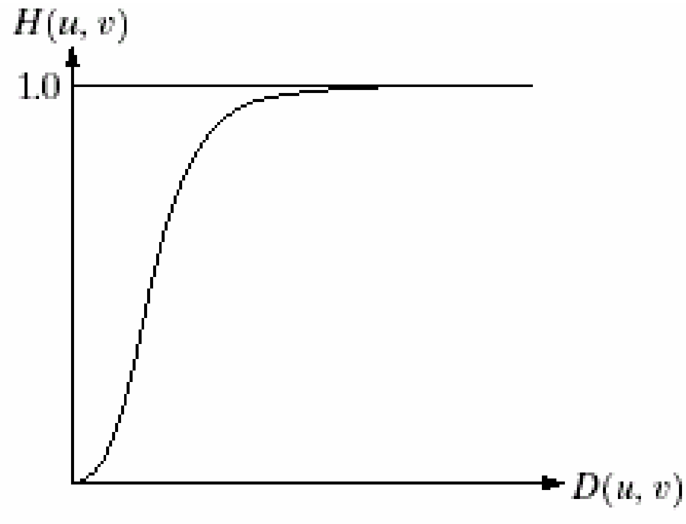


Image representation of BHPF



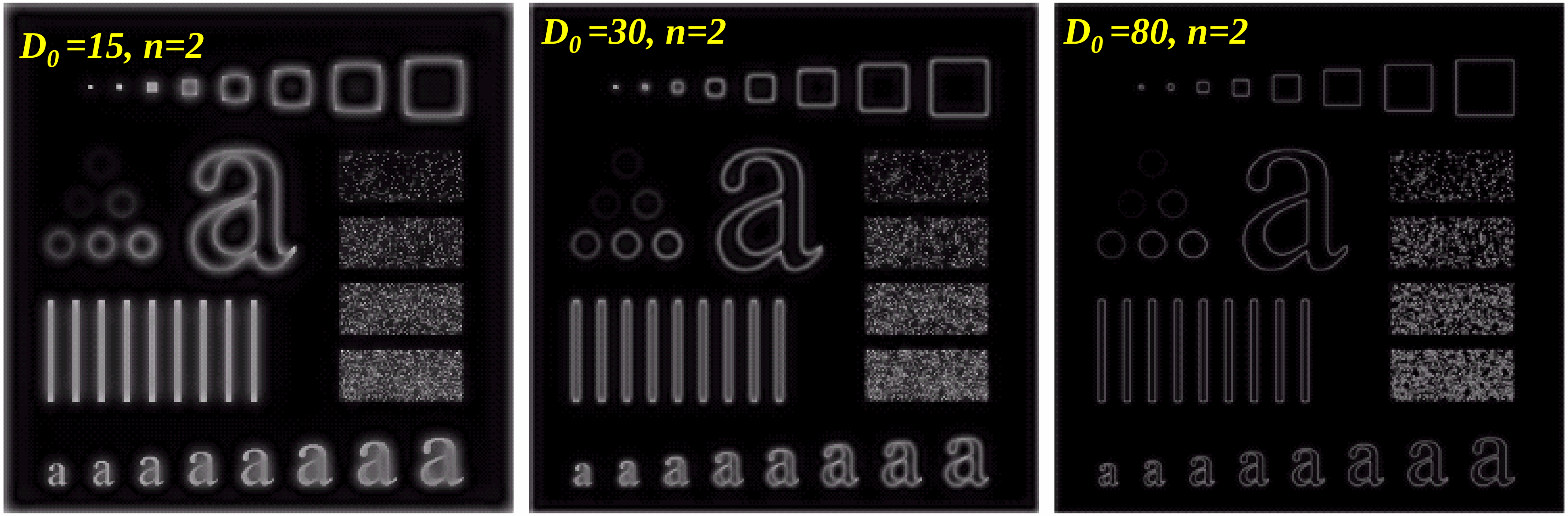
Cross section of BHPF

Prepared By: *Dr. Hasan Demirel, PhD*

Image Enhancement in the *Frequency Domain*

Sharpening Frequency-Domain Filters

•**Butterworth Highpass Filter (BHPF):** ***Smoother results are obtained in BHPF when compared IHPF. There is almost no ringing artifacts.***



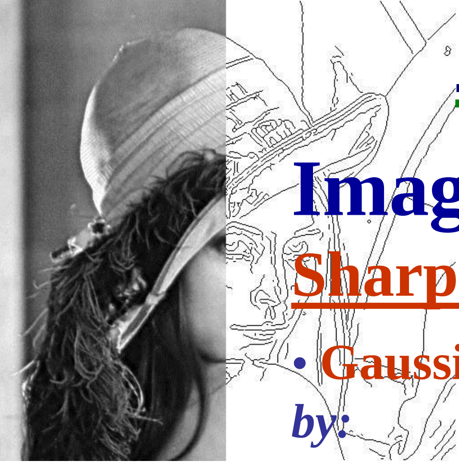


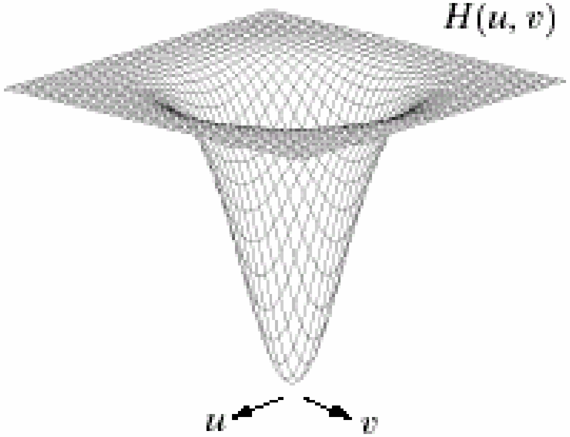
Image Enhancement in the *Frequency Domain*

Sharpening Frequency-Domain Filters

• **Gaussian Highpass Filter (GHPF):** *The transfer function of GHPF is given by:*

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

D(u,v) *is the distance from the origin*
D_0 *is the cutoff frequency.*



Perspective plot of BHPF

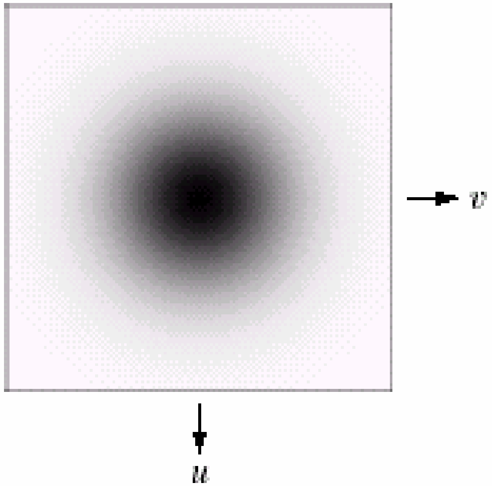
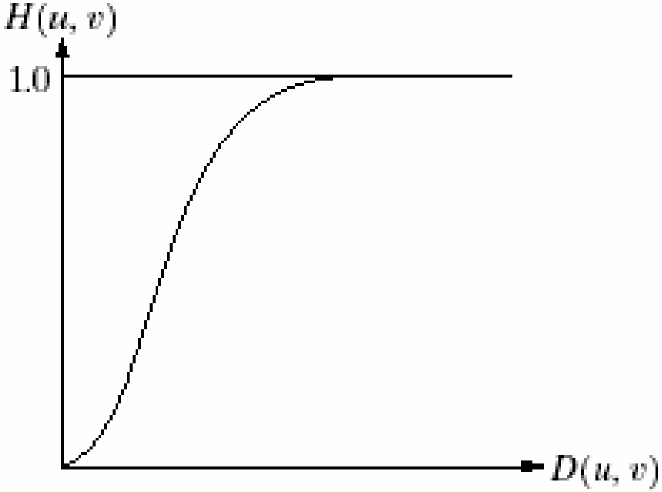


Image representation of BHPF



Cross section of BHPF

Prepared By: *Dr. Hasan Demirel, PhD*

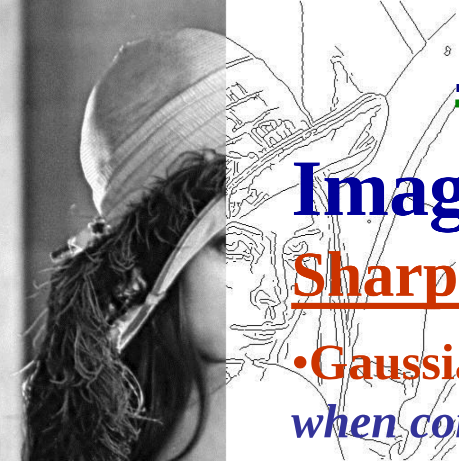


Image Enhancement in the *Frequency Domain*

Sharpening Frequency-Domain Filters

•**Gaussian Highpass Filter (GHPF):** *Smoother results are obtained in GHPF when compared BHPF. There is absolutely no ringing artifacts.*

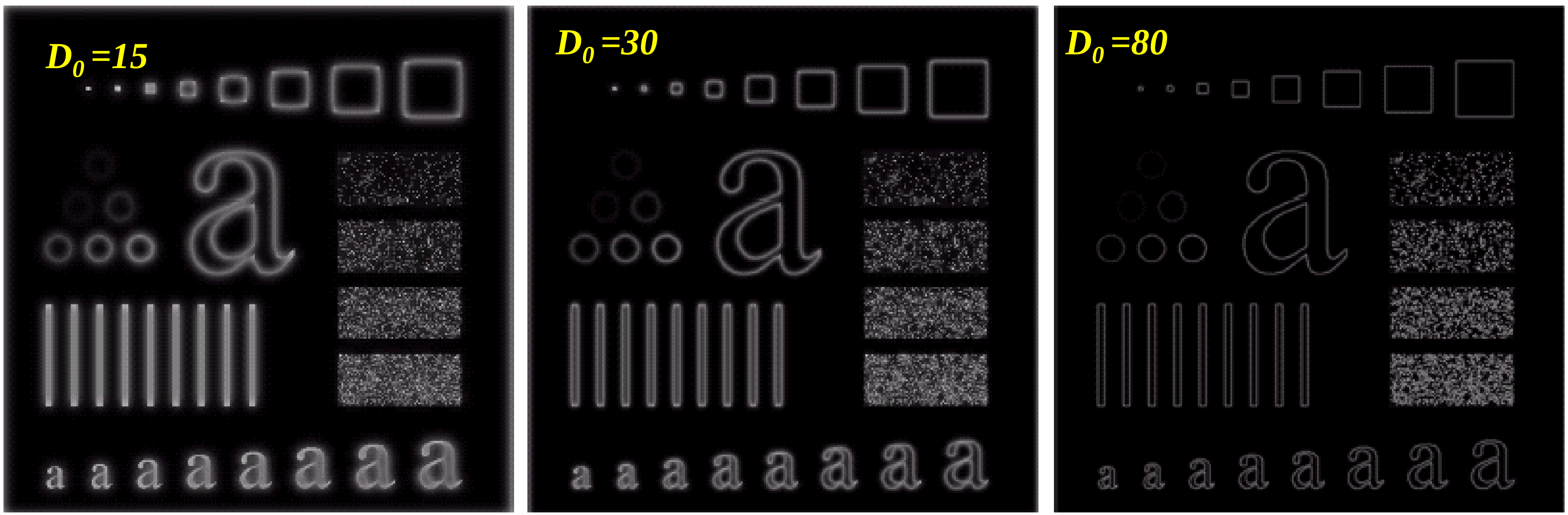
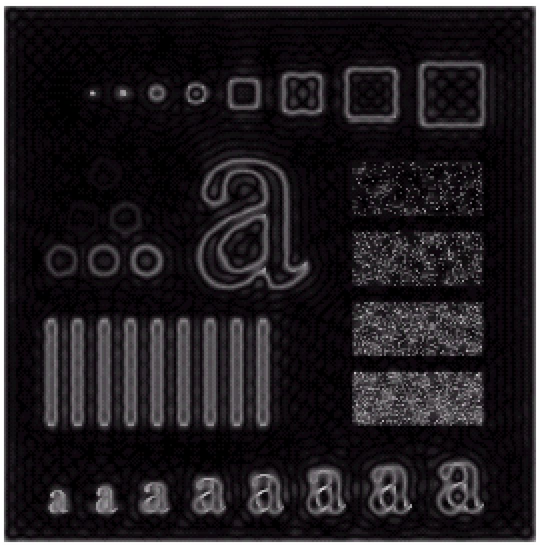


Image Enhancement in the *Frequency Domain*

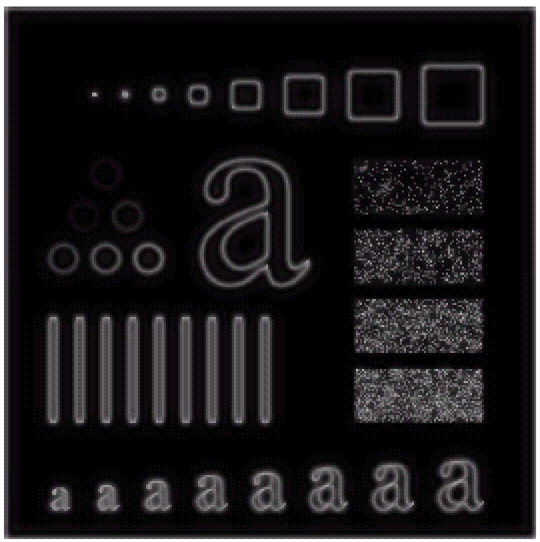
Sharpening Frequency-Domain Filters

•Comparison of Ideal, Butterworth and Gaussian High pass Filters

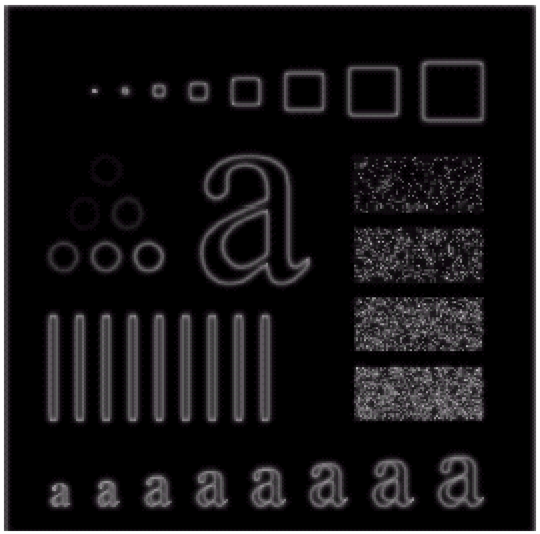
(D_0 =30)



Ideal High pass Filter



Butterworth High pass Filter



Gaussian High pass Filter