Graph Pooling Operation

$$\begin{cases} A \in \{0,1\}^{n \times n} \\ X \in \mathbb{R}^{n \times d} \end{cases} \Longrightarrow \begin{cases} A_p \in \{0,1\}^{n_p \times n_p} \\ X_p \in \mathbb{R}^{n_p \times d_{new}} \\ n_p < n \end{cases}$$

gPool

Downsample by selecting the most importance nodes

Importance Measure:

$$V_i \longrightarrow y_i$$

$$y_i = \frac{h_i^T p}{\|p\|}$$

Select top the n_p nodes:

$$idx = rank(y, n_p)$$

Generate A_p and intermediate H_{inter}

$A_p = A[idx, idx]$

$H_{inter} = H[idx, :]$

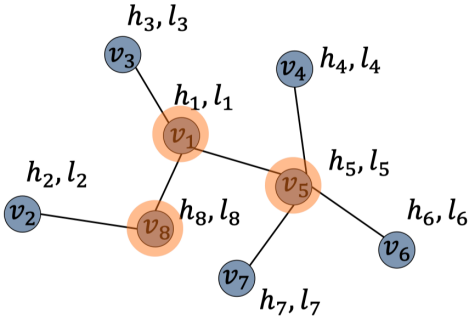
Generate H_p:

$\tilde{y} = sigmoid(y[idx])$

$H_p = H_{inter} \odot \tilde{y}$

gPool

Downsample by selecting the most importance nodes



$\mathbf{A} \in \{0,1\}^{n \times n}, \mathbf{H} \in \mathbb{R}^{n \times d}$

$\mathbf{A}_p \in \{0,1\}^{n_p \times n_p}, \mathbf{H}_p \in \mathbb{R}^{n_p \times d_{new}}, n_p < n$

Importance Measure

$v_i \rightarrow y_i \qquad y_i = \frac{h_i^T p}{||p||}$

Select top the n_p nodes

$idx = rank(\mathbf{y}, n_p)$

Generate A_p and intermediate H_{inter}

$A_p = A[idx, idx]$

$H_{inter} = H[idx, :]$

Generate H_p

$\tilde{y} = sigmoid(y[idx])$

$H_p = H_{inter} \odot \tilde{y}$

DiffPool

Downsample by clustering the nodes using GNN

Filter 1:

Generate a soft-assign matrix

$A \in \{0,1\}^{n \times n}$

$H \in \mathbb{R}^{n \times d}$ $\longrightarrow$ $A \in \{0,1\}^{n \times n}$, $H_a \in \mathbb{R}^{n \times n_p}$

Filter 2:

Generate new features

$A \in \{0,1\}^{n \times n}$

$H \in \mathbb{R}^{n \times d}$ $\longrightarrow$ $A \in \{0,1\}^{n \times n}$, $H_f \in \mathbb{R}^{n \times d_{new}}$

Generate A_p:

$$A_p = H_a^T A H_a$$

Generate H_p:

$$H_p = H_a^T H_f$$

Eigenpooling

Learn A_p using clustering methods

Focus on learning better H_p

Capture both feature and graph structure

Truncated Fourier Coefficients

1. Eigenvectors (fourier modes) of the subgraph
2. GFT : fourier coefficients
3. Truncated fourier coefficients
4. New features for the subgraph (a node in the smaller graph)