Boston University
Department of Electrical and Computer Engineering
EC522 Computational Optical Imaging
Homework No. 1

Issued: Monday, Jan. 22, 2024 Due: 11:59 pm Monday, Feb. 5, 2024

Problem 1: 2D Continuous Fourier Transform (FT), Discrete Fourier transform (DFT), and the concepts of spatial frequency

Consider the following definition of 2D FT

$$F(u,v) = \iint_{-\infty}^{\infty} f(x,y) e^{-j2\pi(f_x x + f_y y)}\, dx\, dy. \tag{1}$$

i) Derive the FT of the following 2D signals.

ii) Compute their DFTs in Matlab / Python.

iii) Plot the original 2D signal within the range: $x = [-2 : 0.01 : 1.99]$ and $y = [-2 : 0.01 : 1.99]$ and the corresponding DFTs.

(a) $\cos(20\pi x)$

(b) $\cos(40\pi y)$

(c) $\cos(20\pi x + 40\pi y)$

(d) $\cos(20\pi x) + \cos(40\pi y)$

Problem 2: Convolution and Zero Padding

Consider an object f defined at $x = [-50 : 1 : 49]$ and $y = [-50 : 1 : 49]$. It consists of a few "point sources" with unit amplitude at the following coordinates (0, 0), (5, 0), (0, 5), (-48, 0), (0, 48), (48, 48). Assume the point spread function (PSF) for a linear shift invariant (LSI) system h is a circular function with a diameter of 15.

Compute the convolution $g = f * h$ by using the following methods in Matlab / Python:

(a) Direct method in the spatial domain.

(b) Compute the convolution by using the DFT and the convolution property.

(c) Discuss why the result of (b) is different from that of (a).

(d) Repeat (b) by first performing "zero-padding" on both f and h in order to remove the artifacts observed in (b).

Problem 3: Digital filtering of image

Select an image and perform low-pass and high-pass filtering using an ideal circular filter a on this image in Matlab/Python. Discuss your results.

Problem 4: Noise and Signal-to-Noise Ratio (SNR)

Consider a 2D signal $f = \cos(20\pi x)$ defined within the range: $x = [-2 : 0.01 : 1.99]$ and $y = [-2 : 0.01 : 1.99]$.

(a) Plot f and its DFT.

(b) Construct a noisy signal f_n by corrupting f with an additive Gaussian noise n with zero mean and a standard deviation (std) of 0.2. Plot f_n and its DFT (hint: use log-scale to visualize the DFT to better see the effects of noise). Discuss the effects of noise in the spatial domain and in the Fourier domain.

(c) Compute the signal-to-noise ratio (SNR) based on the following definition:

$$\mathrm{SNR} = \frac{\text{total energy in } f}{\text{total energy in } n}. \tag{2}$$