

Mean intensity gradient: An effective global parameter for quality assessment of the speckle patterns used in digital image correlation

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ABSTRACT

Digital image correlation (DIC) is an image-based optical metrology for full-field deformation measurement. In DIC technique, the test object surface must be covered with a random speckle pattern, which deforms together with the object surface as a carrier of deformation information. In practice, the speckle patterns may show distinctly different intensity distribution characteristics and have an important influence on DIC measurements. How to assess the overall quality of different speckle patterns with a simple yet effective parameter is an interesting but confusing problem, and is also helpful to the optimal use of the technique. In this paper, a novel, simple, easy-to-calculate yet effective global parameter, called mean intensity gradient, is proposed for quality assessment of the speckle patterns used in DIC. To verify the correctness and effectiveness of the new concept, five different speckle patterns are numerically translated, and the displacements measured with DIC are compared with the exact ones. The errors are evaluated in terms of mean bias error and standard deviation error. It is shown that both mean bias error and standard deviation of the measured displacement are closely related to the mean intensity gradient of the speckle pattern used, and a so-called good speckle pattern should be of large mean intensity gradient.

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1. Introduction

Digital image correlation (DIC) [1–3] is an effective and practical tool for full-field deformation measurement, which has been commonly accepted and widely used in the field of experimental mechanics. In essence, DIC is an image-based deformation measuring technique based on digital image processing and numerical computation. In the most widely used subset-based DIC technique, a reference square subset is selected from the reference image (or source image) and used to track its location in the deformed image (or target image) to determine the subset center's displacement vector. To obtain reliable and accurate matching, the selected subset must contain sufficient intensity variations to ensure that it can be uniquely and accurately identified in the deformed image, which means that the test object surface has to be covered with speckle pattern (or more exactly, random gray level intensity variation). The speckle pattern deforms together with the specimen surface during deformation, and will be further used as a faithful carrier of surface deformation information in the subsequent matching process of DIC.

Generally speaking, the speckle pattern on test object surface can be naturally occurred speckle patterns (i.e., the texture of the object surface) or artificial speckle patterns prepared by spraying white and/or black paints using an airbrush. Besides, it is worth noting that fabrication of small speckle pattern for microscale deformation measurement in scanning electron microscope (SEM) has also been reported in [4]. Although the speckle pattern can be made with easy, a fact is that the speckle pattern made by various techniques or by different people may demonstrate distinctly different gray distribution characteristics. As a consequence, the histogram distribution, image contrast and other statistical parameters of these speckle patterns may be entirely different.

It is observed that the errors of measured displacements using DIC are closely related to the quality of the speckle pattern [5–8]. In other words, the measured displacements of different speckle patterns using the DIC technique may be different even though the deformation state of the specimen, the calculation parameters (e.g., the correlation criteria [9,10], sub-pixel registration algorithm [11], subset size [5–8], subset shape function [12], the interpolation scheme [13], and calculation path [14]) are the same. Consequently, how to assess the quality of the speckle pattern is undoubtedly an important but confusing problem to the users of DIC. In addition, an effective criterion for quality assessment of speckle pattern will provide clear guidance for sample surface preparation and is therefore helpful to the optimal use of the DIC

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technique. To study the influence of speckle patterns on the displacement measurement accuracy of DIC as well as to assess the quality of speckle pattern, various parameters, including both local parameter and global parameter, have been proposed. Local parameter, such as the subset entropy proposed by Sun and Pang [7] and sum of square of subset intensity gradient (SSSIG) proposed by Pan et al. [8], assesses the local speckle pattern quality of each subset separately, which is limited to the quality assessment of the local speckle pattern within an individual subset. However, for most speckle patterns used in DIC, the speckle granules contained in the image are normally evenly distributed, thus the local parameters computed for various subsets are normally of very little differences. Due to this reason, a global parameter rather than the aforementioned local parameter will be more convenient and useful in practical use, as it directly indicates the overall quality of the entire speckle pattern.

Recently, an image morphology method was used by Lecompte et al. [5] to determine the mean speckle size of a speckle pattern. It is also shown that the mean speckle size in combination with subset size do have an influence on the accuracy of the measured displacements. As a global parameter of a speckle pattern, mean speckle size is both simple and concise in concept. However, there exist several deficiencies while using mean speckle size to assess the speckle pattern quality. First, the concept of speckle size, which was originated from laser speckle technique [15], cannot be directly used for some digital images of the specimen surface (see Fig. 3(E) of this paper). Second, the use of mean speckle size for quality assessment of speckle pattern seems to lack a substantial theoretical foundation. Third, the computation of mean speckle size using image morphology is not only cumbersome but also relies on the practitioner's subjective experience. Therefore, at present, there still lacks a more simple and effective global parameter to evaluate the quality of the various speckle patterns used in DIC. The problem becomes more challenging considering the diversity of speckle patterns used in actual experiments.

In this paper, a novel, simple yet effective global parameter, called mean intensity gradient, is proposed for quality assessment of the speckle patterns used in DIC as well as estimating displacement measurement error related to speckle patterns. Different with the mean speckle size, the mean intensity gradient is based on the theoretical model derived for quantifying the accuracy and precision of measured displacements using DIC, and thus has solid theoretical basis. Besides, as will be shown later, the mean intensity gradient of a speckle pattern can be easily computed using a certain gradient operator, which is normally used in digital image processing for edge detection. To validate the correctness and effectiveness of the novel parameter for speckle pattern quality assessment, five different speckle patterns taken from actual experiments with distinctly different intensity distribution are used in numerical studies.

These speckle patterns are numerically translated to isolate the possible errors associated with experimental conditions. The displacements of the translated speckle patterns are computed with DIC and further compared with the imposed ones. The difference between the measured displacements and the exact ones are evaluated in terms of mean bias error and standard deviation error. It is shown that both the systematic error (mean bias error) and random error (standard deviation) of the measured displacements are closely related to the mean intensity gradient of the speckle pattern used. The speckle pattern with large mean intensity gradient will produce small displacement measurement errors.

2. Digital image correlation

Fig. 1 is a schematic illustration of typical experimental setup using optical imaging device for the DIC method. The test planar object surface must have a random speckle pattern, which deforms together with the specimen surface as a faithful carrier of deformation information. The speckle pattern can be natural texture of the specimen surface or artificially made by spraying black and/or white paints. The camera is placed with its optical axis normal to the specimen surface, imaging the planar specimen surface of different loading states onto its sensor plane. The digital images of test object surface are saved in computer and will be processed by DIC software to retrieve the surface deformation information.

DIC relies on the speckle pattern of specimen surface to obtain surface displacement fields. The basic principle of the standard subset-based DIC method is shown in Fig. 2. A square subset of $N \times N$ pixels centered at the interrogated point P from the reference image is chosen and used to determine its corresponding location in the deformed image. To evaluate the similarity between the reference subset and the target subset, a cross-correlation (CC) criterion or sum of squared differences (SSD) correlation criterion is predefined. The matching procedure is completed through searching the peak position of the distribution of correlation coefficients. Once the maximum or minimum (dependent on the correlation function used) correlation coefficient is detected, the shape and position of the target subset can be determined. The differences of the positions of the reference subset center and the target subset center yield the displacement vector at point $P(x, y)$. More details of the DIC technique can be found in a recent review paper written by Pan et al. [3].

3. Theoretical error analysis of DIC measurements

In this section, theoretical analysis of DIC measurement errors related to random noise, subset size and subset intensity is

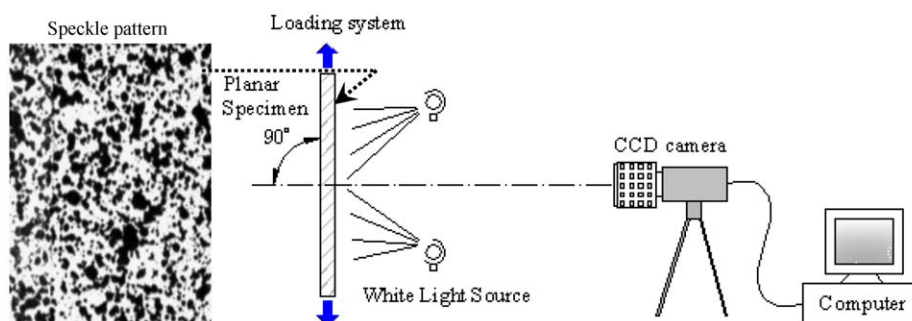


Fig. 1. Experimental measurement system of DIC method and a typical speckle pattern.

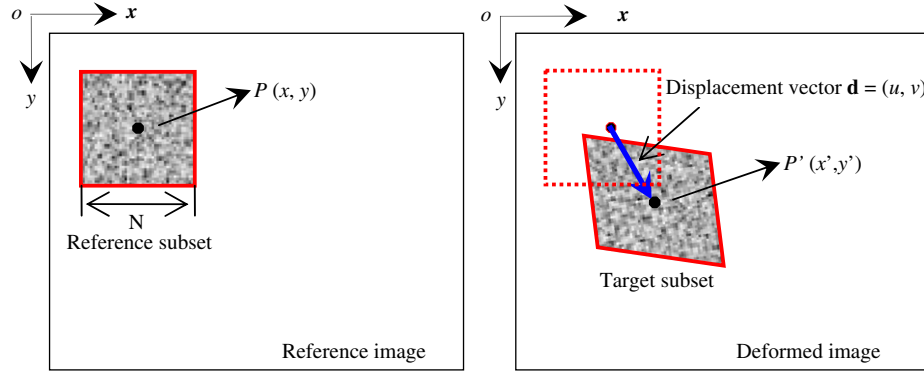


Fig. 2. Schematic illustration of reference square subset before deformation and the target (or deformed) subset after deformation. The differences of the positions of the reference subset center and the target subset center yield the desired displacement vector.

described briefly. It is emphasized that the theoretical derivation has been appeared in various works [8,16,17], and its originality is not claimed here. To keep the paper concise, only the final formulations of previous works are described in the following, and detailed mathematic derivation is not given here. The theoretical analysis in previous works was performed between a reference speckle pattern (denoted by $f(\mathbf{x}_{ij})$) and a translated speckle pattern (denoted by $g(\xi_{ij})$). For purpose of simplicity, only the analysis of one-dimensional translation is provided below. However, it is noted here that the following derivation can also be extended to two-dimensional cases with a similar conclusion. First of all, let us define the measured displacement of pixel x_{ij} to be u' , and the exact displacement to be u . Besides, the difference between the estimated and exact displacement is defined as $u_e = u' - u$. In the theoretical analysis, additive Gaussian noise $n_1(\mathbf{x}_{ij})$, $n_2(\xi_{ij})$ with mean value of zero and a variance of σ^2 are added to the reference speckle pattern and translated speckle pattern to simulate the effect of random noise.

The least square difference (also known as the SSD correlation criterion) [8,17] between the reference subset and translated subset is written as

$$C_{SSD} = \sum_{i=1}^N \sum_{j=1}^N [g(\xi'_{ij}) + n_2(\xi'_{ij}) - f(\mathbf{x}_{ij}) - n_1(\mathbf{x}_{ij})]^2 \quad (1)$$

where $\xi'_{ij} = \mathbf{x}_{ij} + u'$ is measured position of pixel \mathbf{x}_{ij} , N is the subset size used for calculation.

By minimizing the least square difference with respect to u_e , the results obtained by Pan et al. [8] and Wang et al. [17] show that the estimated displacements are unbiased if no interpolation error is introduced:

$$E(u_e) = 0 \quad (2)$$

However, a biased expectation of measured displacement can be deduced if interpolation error presents. For example, if a linear interpolation scheme is used, the mathematical expectation of the measured displacement [17] will be a function of the following: (a) the interpolation difference $h(\mathbf{x}_{ij})$ between translated and reference images, (b) magnitude of the noise σ^2 , (c) sub-pixel displacement τ_x and (d) sum of square of subset intensity gradient (SSSIG) $\sum_{i=1}^N \sum_{j=1}^N [f_x(\mathbf{x}_{ij})]^2$:

$$E(u_e) \cong \frac{\sum_{i=1}^N \sum_{j=1}^N [-h(\mathbf{x}_{ij})f_x(\mathbf{x}_{ij})] + (1 - 2\tau_x)N^2\sigma^2}{\sum_{i=1}^N \sum_{j=1}^N [f_x(\mathbf{x}_{ij})]^2} \quad (3)$$

where $h(\mathbf{x}_{ij}) = g(\xi_{ij}) - f(\mathbf{x}_{ij})$ with $\xi_{ij} = \mathbf{x}_{ij} + u$ is the exact position of pixel \mathbf{x}_{ij} .

It is also noted that the theoretical analysis conducted by Pan et al. [8] and Wang et al. [17] both show that the standard deviation of measured displacement is a function of the following: (1) the magnitude of the white noise and (d) sum of square of subset intensity gradient (SSSIG):

$$std(u_e) \cong \frac{\sqrt{2}\sigma}{\sqrt{\sum_{i=1}^N \sum_{j=1}^N [f_x(\mathbf{x}_{ij})]^2}} \quad (4)$$

We should note that the above relationship remains true when a more advanced interpolation scheme (e.g., the cubic interpolation) is used [17]. It is clear from Eqs. (3) and (4) that both the mean bias error and standard deviation error of the estimated displacement are in inverse proportion to the SSSIG value of the subset. Thus, SSSIG can be used as an effective local parameter for assessing the local speckle pattern quality within a specified subset [8].

4. Mean intensity gradient

Although SSSIG is effective, it is a local indicator of each individual subset and cannot be applied to the quality assessment of the entire speckle pattern. In practice, just like the mean speckle size proposed by Lecompte et al. [5], a more straightforward, easy-to-calculate global parameter to assess the quality of the entire speckle pattern would be more preferable to the users of DIC. Based on this consideration and also inspired by the SSSIG, a novel global parameter, called mean intensity gradient of speckle pattern (denoted by δ_f), is defined to evaluate the quality of the entire speckle pattern as follows:

$$\delta_f = \sum_{i=1}^W \sum_{j=1}^H |\nabla f(\mathbf{x}_{ij})| / (W \times H) \quad (5)$$

where W and H (in unit of pixels) are image width and height, $|\nabla f(\mathbf{x}_{ij})| = \sqrt{f_x(\mathbf{x}_{ij})^2 + f_y(\mathbf{x}_{ij})^2}$ is the modulus of local intensity gradient vector with $f_x(\mathbf{x}_{ij})$, $f_y(\mathbf{x}_{ij})$ are the x - and y -directional intensity derivatives at pixel (\mathbf{x}_{ij}) , which can be simply computed using the commonly used gradient operator (e.g., a central difference algorithm or a prewitt operator).

Once the mean intensity gradient of the entire speckle pattern is determined, the SSSIG within a local subset of $N \times N$ pixels can

be approximated as

$$\sqrt{\sum_{i=1}^N \sum_{j=1}^N [f_x(\mathbf{x}_{ij})]^2} \cong N \times \delta_f \quad (6)$$

It is obvious that a large subset size leads to more accurate approximation of Eq. (6). Substituting Eq. (6) into Eqs. (3) and (4), it is found that the displacement measurement accuracy (i.e., mean bias error) and precision (i.e., standard deviation error) of DIC are in inverse proportion to the product of the subset size N and the mean intensity gradient of the speckle pattern. Since mean intensity gradient is a statistical parameter of the speckle pattern and directly affects the displacement measurement error

of DIC when the subset size is fixed, it can be used as an effective global parameter for the quality assessment of the whole speckle pattern.

5. Numerical experiments

5.1. Speckle patterns

To verify the correctness and effectiveness of the novel parameter for speckle pattern quality assessment, numerical experiments are utilized in this study to isolate the possible

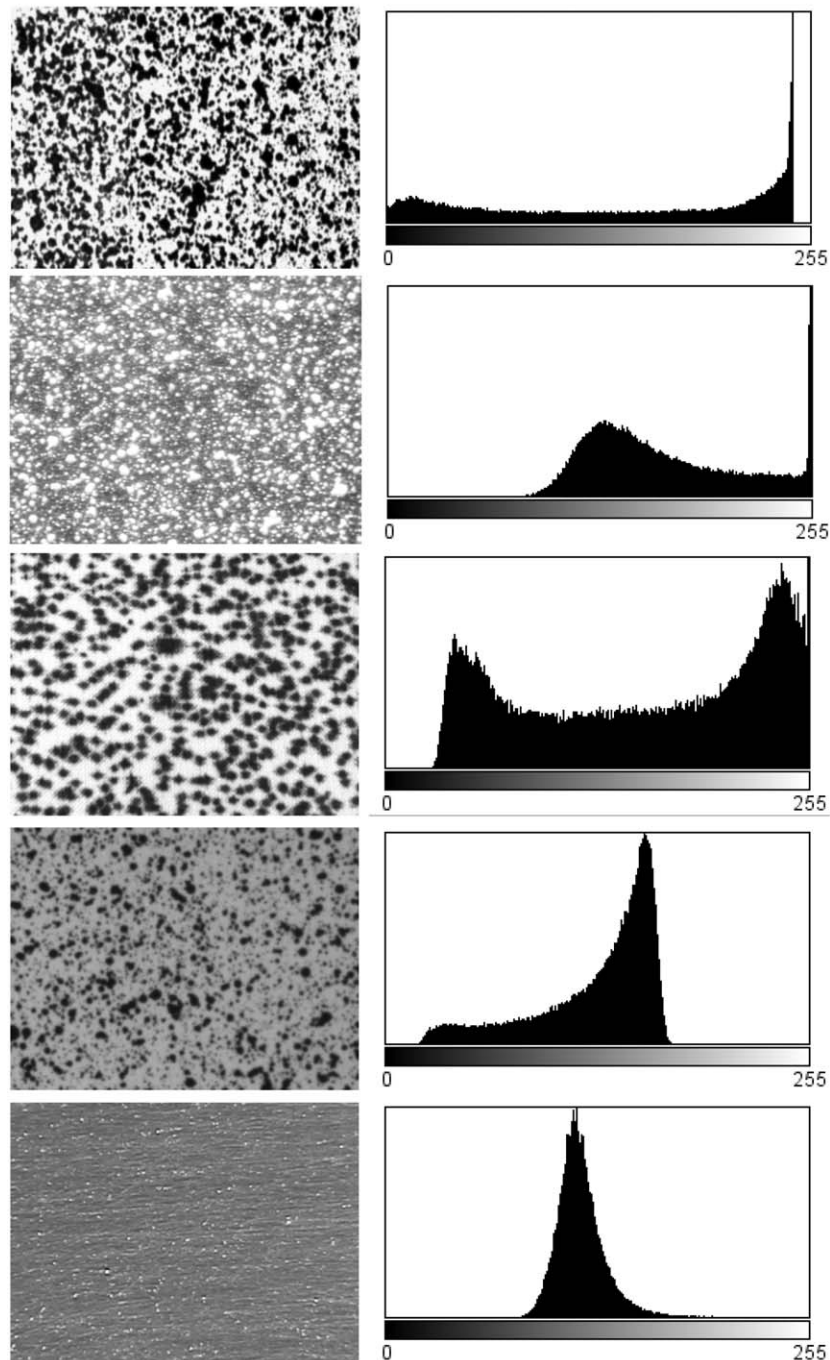


Fig. 3. Speckle patterns A–E (from top to bottom) used in the numerical experiments and their corresponding histograms.

errors caused by the image acquisition system (e.g., the imaging lens distortion), imperfect loading, out-of-plane motion of the specimen and illumination lighting fluctuation during image capturing. In the following numerical studies, only the influences of random image noise, speckle pattern and subset size are considered.

Fig. 3 shows the five speckle patterns used in the following numerical experiments along with their histograms. The five 8-bit (0–255 gray level range) speckle patterns with a resolution of 768×576 pixels were taken from our previous experiments. The speckle patterns A, B and D were acquired by randomly spraying black and white paints on flat specimen surfaces. The speckle pattern C was simply made by use of a black marker pen on a composite film surface, while the speckle pattern E was obtained just by simply polishing metal surface using sandpaper. It can be seen from Fig. 3 that the contrast, brightness and histogram distributions of these five test speckle images are distinctly different.

According to the definition of mean intensity gradient as given in Eq. (5), the mean intensity gradient value of each speckle pattern is computed and listed in Table 1. We see that the mean intensity gradient of speckle pattern A is much larger than that of other speckle patterns, while the speckle pattern E shows the smallest mean intensity gradient.

In the following numerical studies only pure in-plane translation tests were performed. A series of translated speckle images for each speckle pattern were generated by applying the appropriate shift in Fourier domain according to shift theorem [13]. The sub-pixel displacements applied in the x direction range from 0 to 1 pixel, corresponding to a shift of 0.05 pixels between two successive images. Moreover, to simulate the image noise, additive random Gaussian noise with mean value of 0, variance of 4, was added to each image to simulate the influence of noise (i.e., $\sigma=2$). The displacements of each translated speckle pattern were computed at regularly distributed $2501 (=61 \times 41)$ points (the distance between neighboring points is 10 pixels) using the

Newton–Raphson method with the simplest SSD correlation function and zero-order shape function.

5.2. Mean bias error and standard deviation error

To quantitatively evaluate the calculated displacements of the DIC method and compare them with the imposed ones, the errors of the computed displacements are decomposed into two components: mean bias error (or systematic error) and standard deviation error (or random error). The mean bias error of the measured displacement is defined as

$$u_e = u_{mean} - u_{imp} \quad (7)$$

where $u_{mean} = (1/N) \sum_{i=1}^N u_i$ represents the mean of the N estimated displacements and u_{imp} denotes the actual imposed sub-pixel displacement.

The standard deviation error of the measured displacement can be defined as

$$\sigma_u = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (u_{mean} - u_i)^2} \quad (8)$$

The standard deviation error reflects the deviation of the measured displacement corresponding to the mean value.

6. Results

6.1. Systematic errors

Fig. 4 shows the mean bias errors of detected displacements as a function of applied sub-pixel displacements for the five speckle patterns using 31×31 pixels subset (left) and 61×61 pixels subset (right). Some important observations can be made from Fig. 4 as follows: (1) the mean bias error of DIC is a function of sub-pixel displacement, which is approximately sinusoidal distribution with a period of 1 pixel. This kind of systematic error distribution can be attributed to positional intensity interpolation error on sub-pixel reconstruction [13]. It is interesting to note that the same phenomena have also been observed using other algorithms in the work of performed by Wattrisse et al. [18] and Pan et al. [11]. (2) The mean bias error of DIC does not depend on the size of the calculated subset. It is seen that the calculated systematic errors using 31×31 and 61×61 subset almost show the same distribution. (3) The magnitudes of

Table 1

Mean image intensity gradient of each speckle pattern used in numerical experiment.

Speckle pattern	A	B	C	D	E
Mean intensity gradient	34.6444	21.5143	20.0398	12.3400	9.0379

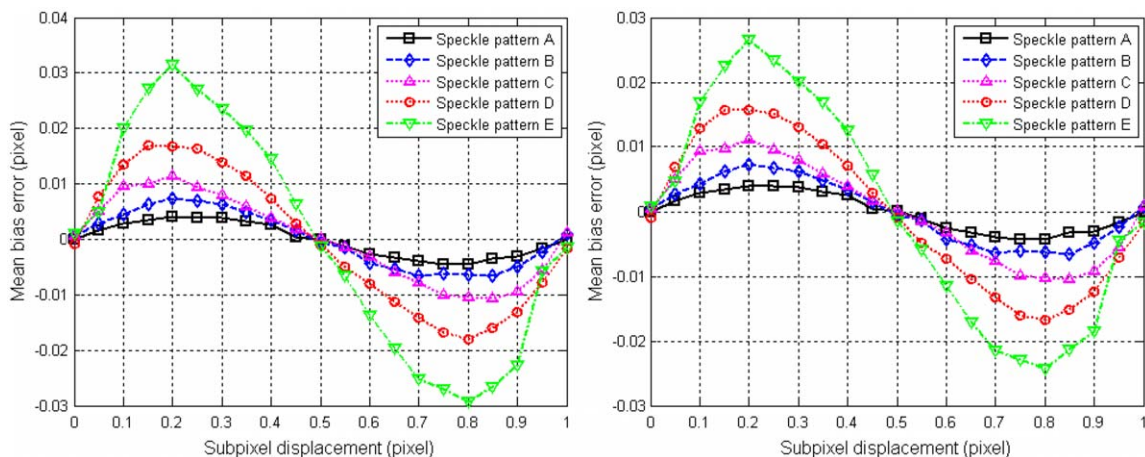


Fig. 4. Mean bias error of measured u -displacement as a function of sub-pixel displacement for the five speckle patterns using 31×31 pixels subset (left) and 61×61 pixels subset (right). It is clear that the speckle pattern with large mean intensity gradient produces small mean bias error.

the mean bias error (e.g., the amplitude of the sinusoidal curve) for different speckle patterns are different. It is clear from Fig. 4 that the speckle pattern with large mean intensity gradient produces small mean bias error. The observation is in good accordance with theoretical prediction. The above observations clearly show that mean intensity gradient can be an effective parameter for evaluating the mean bias error associated with speckle patterns.

6.2. Standard deviation error

Fig. 5 are the standard deviation errors of measured displacement as a function of pre-assigned sub-pixel displacements for the five speckle patterns using 31×31 pixels subset (left) and 61×61 pixels subset (right). It is observed from Fig. 5 that (1) the standard deviation errors of DIC are approximately stable and do not depend on the imposed sub-pixel displacement. (2) The standard deviation errors do depend on the subset size used. The calculated standard deviation errors using 31×31 are approximately two times larger than those computed with 61×61 pixels subset. (3) The standard deviation errors are closely related to the quality of speckle patterns. The speckle pattern with higher mean intensity gradient produces smaller standard deviation

error. These three observations can also be well explained by the above theoretical model of Eq. (4), which further demonstrate that mean intensity gradient can be used as an effective parameter for estimating the standard deviation error associated with speckle patterns.

6.3. Prediction of displacement measurement precision using mean intensity gradient

To further demonstrate the correctness of the theoretical model for prediction of the standard deviation errors of the DIC method, translated images with sub-pixel displacement of 0.5 pixel of each speckle pattern is calculated with subset sizes ranging from 17×17 to 71×71 pixels at an increment of 6 pixels. Fig. 6(a) shows the standard deviation errors of the u -displacement as a function of subset size for the five speckle patterns. Even though the same subset is used in all DIC calculations, it is seen that the standard deviation errors are different for the five speckle patterns. This can be easily explained by the fact that the mean intensity gradients of the various speckle patterns are different. For example, speckle pattern A with maximum mean intensity gradient produces smallest standard deviation error, whereas speckle pattern E with minimum mean

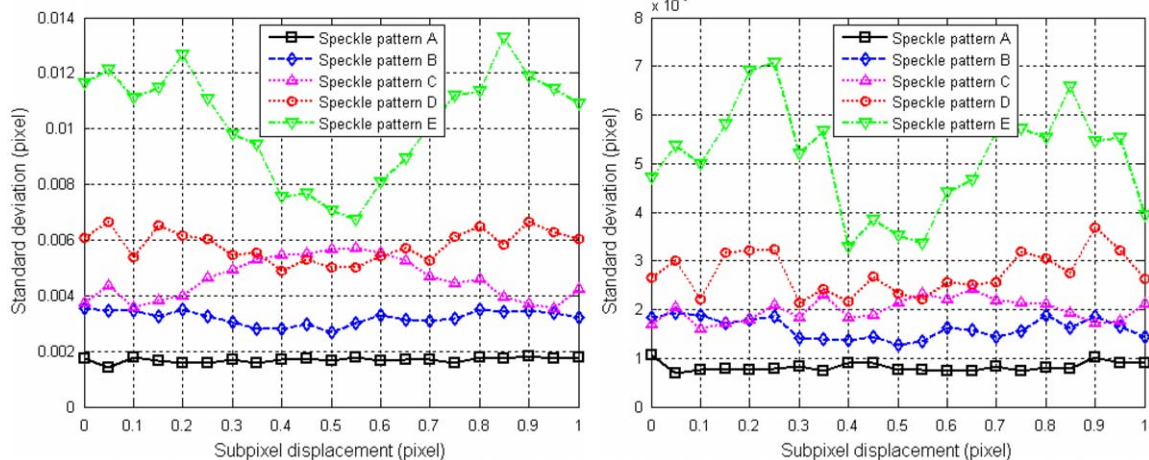


Fig. 5. Standard deviation errors of measured u -displacement for the five speckle patterns using 31×31 pixels subset (left) and 61×61 pixels subset (right). It is clear that the speckle pattern with large mean intensity gradient produces small standard deviation error.

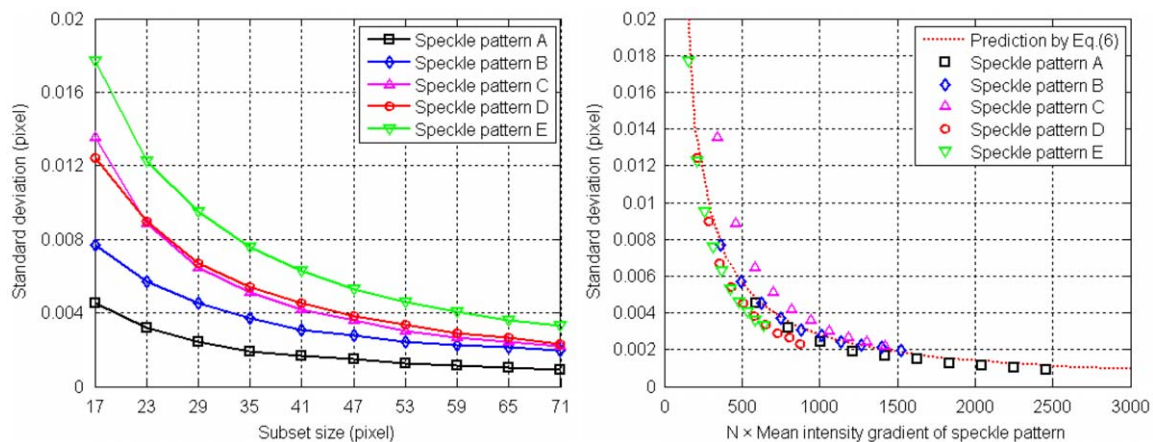


Fig. 6. Standard deviation errors for the u -displacements as a function of subset size (left) and the product of subset size and mean intensity gradient of the speckle pattern (right) for five speckle patterns.

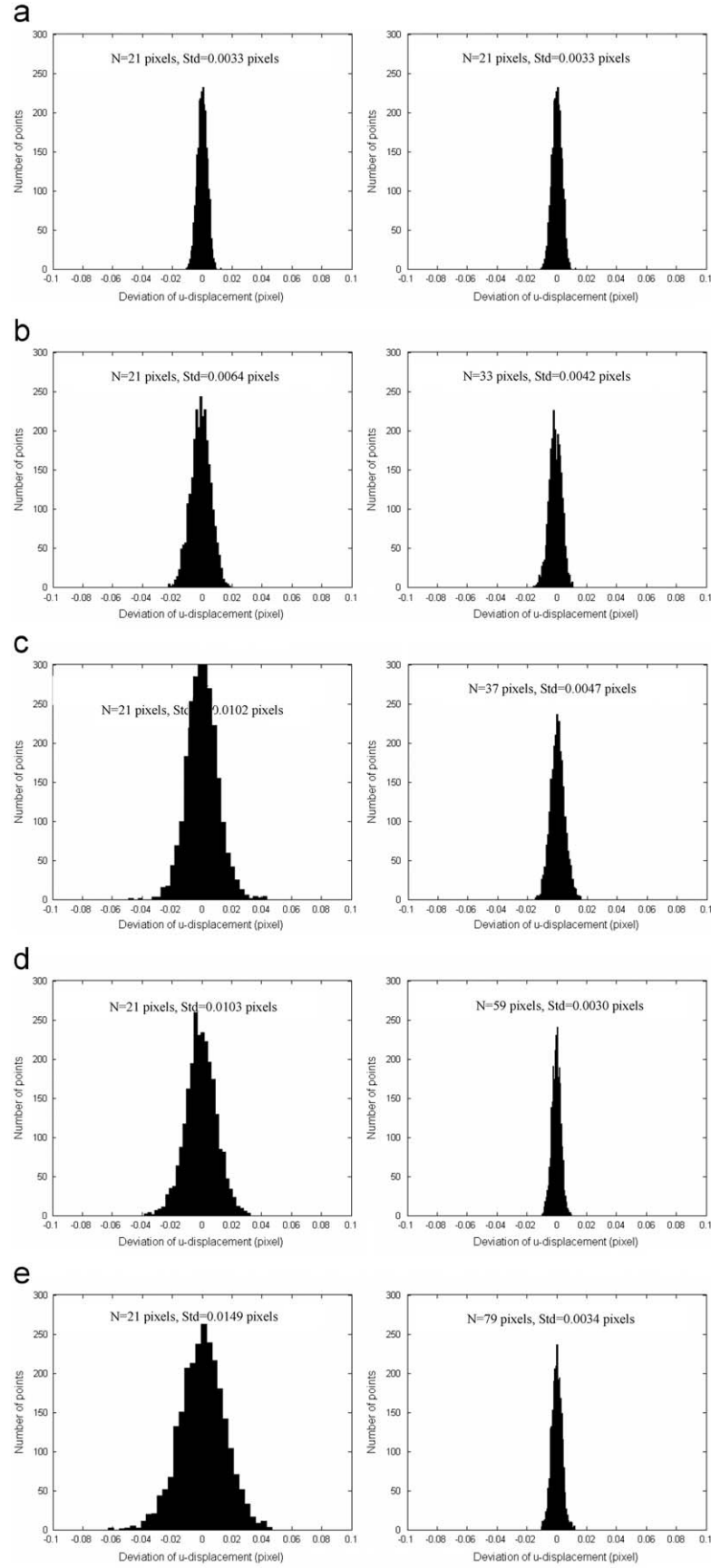


Fig. 7. Deviation of the measured displacement and the exact displacement for the speckle patterns A–E (from top to bottom) using a fixed 21×21 subset (left) and various subset estimated using Eq. (6) (right).

intensity gradient always yields largest standard deviation error. Fig. 6(b) replots the standard deviation errors of the u -displacement for the five speckle patterns as a function of the product of subset size (varying from 17×17 pixels to 71×71 pixels) and mean intensity gradient of the speckle pattern. As expected, perfect agreements are obtained between the calculated standard deviation errors and the prediction estimated by combining Eq. (4) and Eq. (6).

6.4. Subset size selection for speckle patterns based on mean intensity gradient

For different speckle patterns, it is possible for us to achieve the same standard deviation error by adjusting the subset size according to Eqs. (4) and (6). For example, for the speckle pattern A, if the subset size is chosen as 21×21 pixels and the noise variance is 4, the theoretical predicted standard deviation error is computed as 0.0039 pixels using Eq. (6). To achieve the same precision, the equivalent subset size should be used for the other four speckle patterns can be deduced using Eq. (6), and are 33 pixels (for speckle pattern B), 37 pixels (for speckle pattern C), 59 pixels (for speckle pattern D) and 79 pixels (for speckle pattern E), respectively. Fig. 7 shows the deviation between the measured displacement and the exact displacement for the five speckle patterns using a fixed 21×21 subset (left) and the various subset size estimated using Eq. (6) (right). By comparison, we see that almost same standard deviation error is obtained by using equivalent subset size derived from Eqs. (4) and (6) for the five speckle patterns. The good agreement between predicted standard deviation error and measured ones proves the effectiveness of the mean intensity gradient once again.

7. Discussion

In the above two subsections, the effectiveness of the mean intensity gradient is demonstrated by predicting the displacement measurement precision and also by using as a criterion for proper subset size selection. However, we should mention that only translated speckle patterns are used in the above demonstrations. Apparently, the prediction of displacement measurement precision merely considers the influence of speckle pattern, and the presented criterion only results in a lower limit for subset size selection. Another important factor, i.e., the actual deformation state within the subset, is assumed to be well approximated by the used shape function. Nevertheless, it is expected that the above precision prediction model and subset size selection criterion still hold true if no apparent large heterogeneous deformation presents in the deformed image.

However, in certain situations, the specimen may undergo a large and complex deformation, so the prediction of the displacement measurement accuracy as well as the proper selection of subset size is not as straightforward as that shown above. For a deformed image with large complicated nonhomogeneous deformation, on the one hand, the subset size should be enough so that the subset contains sufficiently distinctive intensity variations to distinguish itself from other subsets and achieve certain matching accuracy. On the other hand, a too larger subset size will introduce additional systematic errors in measured displacements, as the underlying deformation within a large subset may not be accurately represented by the used shape function [8]. In this case, the subset size must be chosen with care to get a balance between the random errors caused by subset size and the systematic errors related to unmatched shape functions. A notable example regarding this dilemma can be found in a recent work by Sun and Pang [7]. Through processing the speckle image pairs

with prescribed quadratic deformation field using first-order shape function and various subset sizes, the example clearly reveals that there exists an optimal subset size producing minimum standard deviation errors.

8. Conclusion

A novel, simple yet effective global parameter, called mean intensity gradient, is proposed to assess the quality of the entire speckle patterns used in DIC in this work. The correctness and effectiveness of the new parameter is demonstrated by numerical experiments using five different speckle patterns. It has been shown that both mean bias error and standard deviation of the measured displacement are closely related to the mean intensity gradient of the speckle pattern, and the speckle with larger the mean intensity produces the smaller mean bias error and standard deviation error. Hence, the so-called good speckle pattern should be of large mean intensity gradient. It is also shown by the present work that the novel parameter for speckle pattern quality assessment can be used for at least the following three purposes in DIC measurements: (1) it can be used as guidance for practical sample surface preparation; (2) it can be used for predicting the precision of the measured displacements; (3) combined with the desired displacement measurement precision, the mean intensity gradient can be used for subset size selection for various speckle patterns.

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References

- [1] Peters WH, Ranson WF. Digital imaging techniques in experimental stress analysis. *Opt Eng* 1981;21:427–31.
- [2] Chu TC, Ranson WF, Sutton MA, et al. Applications of digital-image-correlation techniques to experimental mechanics. *Exp Mech* 1985;25(3):232–44.
- [3] Pan B, Qian KM, Xie HM, Asundi A. Two-dimensional digital image correlation for in-plane displacement and strain measurement: a review. *Meas Sci Technol* 2009;20:062001.
- [4] Scrivens WA, Luo Y, Sutton MA, et al. Development of patterns for digital image correlation measurements at reduced length scales. *Exp Mech* 2007;47(1):63–77.
- [5] Lecomte D, Smits A, Bossuyt S, et al. Quality assessment of speckle patterns for digital image correlation. *Opt Lasers Eng* 2006;44(11):1132–45.
- [6] Haddadi H, Belhabib S. Use of rigid-body motion for the investigation and estimation of the measurement errors related to digital image correlation technique. *Opt Lasers Eng* 2008;46:185–96.
- [7] Sun YF, Pang HJ. Study of optimal subset size in digital image correlation of speckle pattern images. *Opt Lasers Eng* 2007;45:967–74.
- [8] Pan B, Xie HM, Wang ZY, Qian KM, Wang ZY. Study on subset size selection in digital image correlation for speckle patterns. *Opt Express* 2008;46(3):033601.
- [9] Tong W. An evaluation of digital image correlation criteria for strain mapping applications. *Strain* 2005;41(4):167–75.
- [10] Pan B, Asundi A, Xie HM, Gao JX. Digital image correlation using iterative least squares and pointwise least squares for displacement field and strain field measurements. *Opt Lasers Eng* 2009;47(7–8):865–74.
- [11] Pan B, Xie HM, Xu BQ, Dai FL. Performance of sub-pixel registration algorithms in digital image correlation. *Meas Sci Technol* 2006;17(6):1615–21.
- [12] Schreier HW, Sutton MA. Systematic errors in digital image correlation due to undermatched subset shape functions. *Exp Mech* 2002;42(3):303–10.
- [13] Schreier HW, Braasch JR, Sutton MA. Systematic errors in digital image correlation caused by intensity interpolation. *Opt Eng* 2000;39(11):2915–21.

- [14] Pan B. Reliability-guided digital image correlation for image deformation measurement. *Appl Opt* 2009;48(8):1535–42.
- [15] Dainty C, editor. *Laser speckle and related phenomena*. Berlin: Springer-Verlag; 1984.
- [16] Wang ZY, Li HQ, Tong JW, Ruan JT. Statistical analysis of the effect of intensity pattern noise on the displacement measurement precision of digital image correlation using self-correlated images. *Exp Mech* 2007;47:701–7.
- [17] Wang YQ, Sutton MA, Bruch HA, Schreier HW. Quantitative error assessment in pattern matching: effects of intensity pattern noise, interpolation, strain and image contrast on motion measurement. *Strain* 2009;45:160–78.
- [18] Wattrisse B, Chrysochoos A, Muracciole JM, et al. Analysis of strain localization during tensile tests by digital image correlation. *Exp Mech* 2001;41(1):29–39.