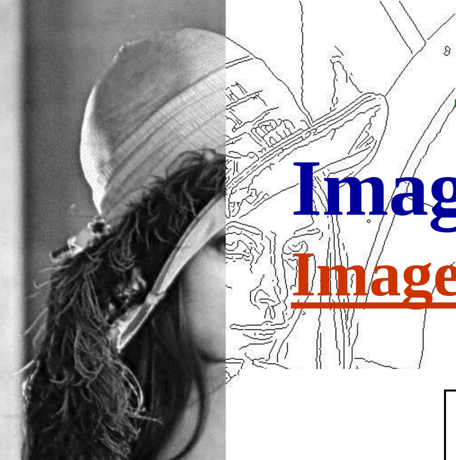


Image Restoration

Image Degradation and Restoration

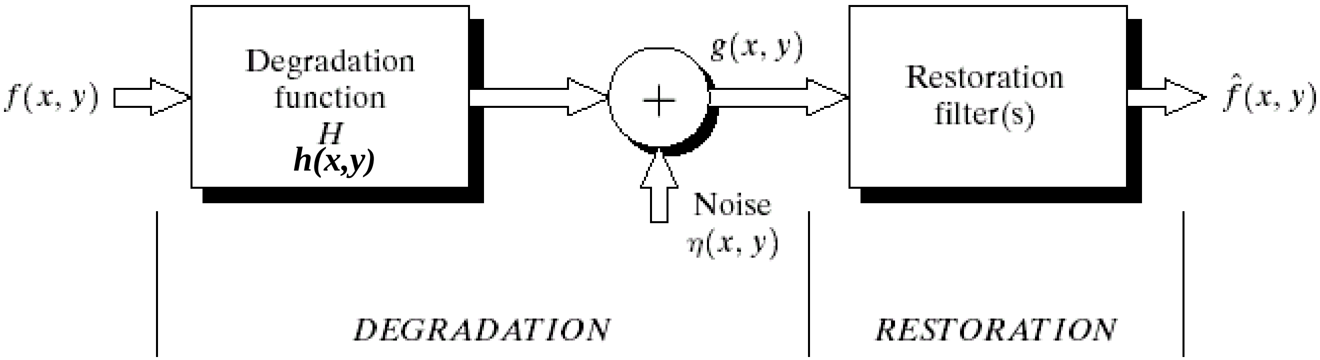


FIGURE 5.1 A model of the image degradation/restoration process.

- **Image Degradation Model:** ***Spatial domain representation can be modeled by:***

$$\boxed{g(x,y) = h(x,y) * f(x,y) + \eta(x,y)}$$

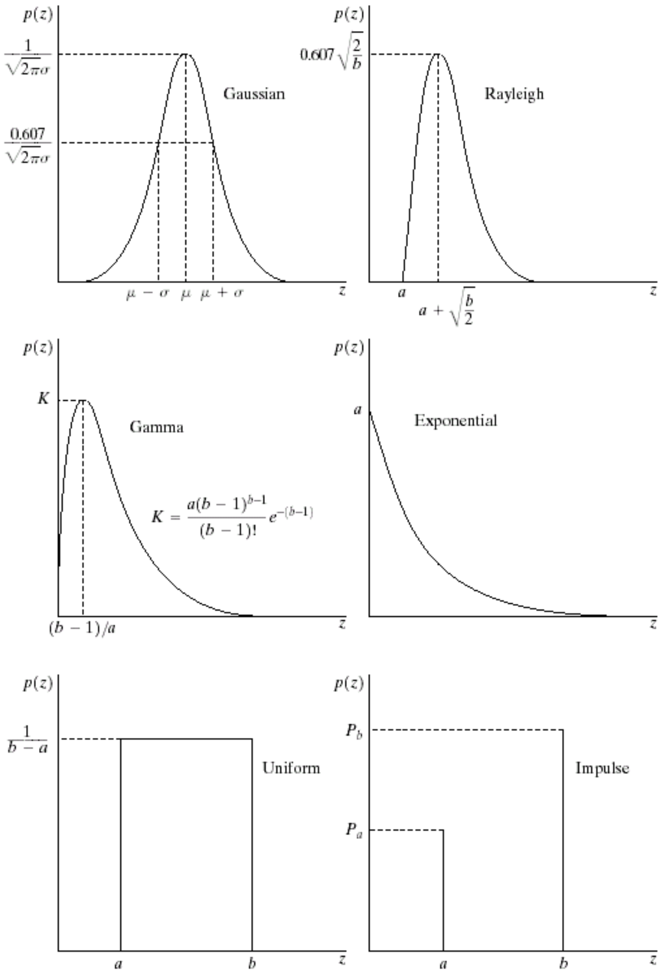
- ***Frequency domain representation can be modeled by:***

$$\boxed{G(u,v) = H(u,v)F(u,v) + N(u,v)}$$

Image Restoration

Common Noise Models:

- *Most types of noise are modeled as known probability density functions*
- *Noise model is decided based on understanding of the physics of the sources of noise.*
  - **Gaussian**: *poor illumination*
  - **Rayleigh**: *range image*
  - **Gamma**/**Exp**: *laser imaging*
  - **Impulse**: *faulty switch during imaging,*
  - **Uniform** *is least used.*
- *Parameters can be estimated based on histogram on small flat area of an image*



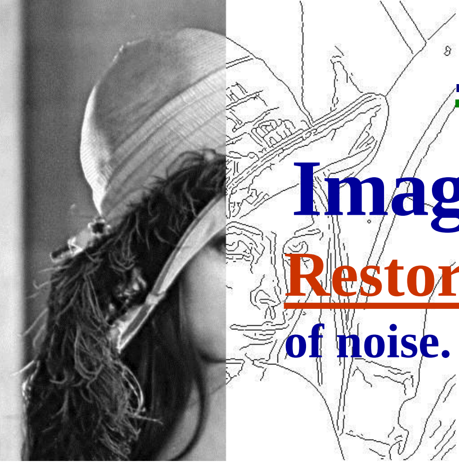


Image Restoration

Restoration methods: The following methods are used in the presence of noise.

- **Mean filters**
  - *Arithmetic mean filter*
  - *Geometric mean filter*
  - *Harmonic mean filter*
  - *Contra-harmonic mean filter*

- **Order statistics filters**
  - *Median filter*
  - *Max and min filters*
  - *Mid-point filter*

- **Adaptive filters**
  - *Adaptive local noise reduction filter*
  - *Adaptive median filter*

Image Restoration

Restoration in the presence of Noise:

Adaptive Local Noise Reduction Filter: ***Mean and variance are the simplest statistical measures of a random noise.***

- ***Mean*** ***gives the measure of the average gray level in a local region,***
- ***Variance*** ***gives the measure of the average contrast in the region.***

•Consider a filter operating in a local region, S_{xy}, where the response of the filter at any point (x,y) depends on:

a) $g(x,y)$, the value of the noisy image at (x,y)
b) σ^2_η, the additive noise variance,
c) m_L, local mean of pixels in S_{xy}.
d) σ^2_L, the local variance in S_{xy}.

Image Restoration

Restoration in the presence of Noise:

Adaptive Local Noise Reduction Filter:

•Consider an adaptive filter where, the following conditions are satisfied:

1. *If $\sigma^2_\eta = 0$, the filter should return g(x,y), zero-noise case [f(x,y)=g(x,y)].*
2. *If $\sigma^2_L >> \sigma^2_\eta$, the filter should return a value close to g(x,y). High local variance is associated with edges and should be preserved.*
3. *If $\sigma^2_L = \sigma^2_\eta$, return arithmetic mean of S_{xy}. This occurs when local noise has the same properties of the entire image. Averaging simply reduces the noise.*

• According to the preceding assumptions the filter response can be modeled as:

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

•The only unknown parameter in the above adaptive filter model is σ^2_η, The other parameters can be calculated at the local neighborhood of S_{xy}.

Image Restoration

Restoration in the presence of Noise:

Adaptive Local Noise Reduction Filter:

a b
c d

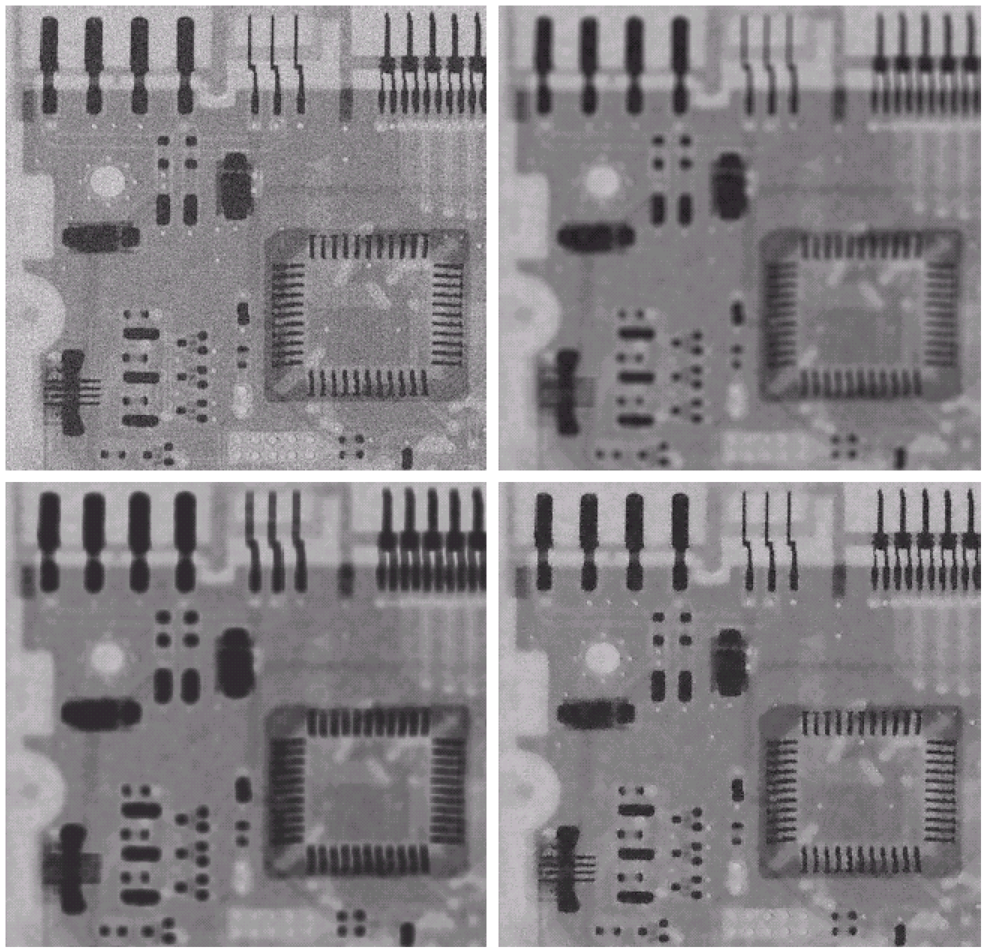


Image corrupted by Gaussian Noise with

$\sigma^2_\eta = 1000$

$\mu = 0$

7x7 Arithmetic Mean Filter

7x7 Geometric Mean Filter

7x7 Adaptive Filter

Image Restoration

Restoration in the presence of Noise:

Adaptive Median Filter: *Adaptive median filter can filter impulse noise with very high probabilities. Additionally smoothes the nonimpulse noise which is not the feature of a traditional median filter.*

- *The filter uses the following parameters in the neighborhood of S_{xy} :*

z_{min} = *minimum gray level value in* S_{xy}.
z_{max} = *maximum gray level value in* S_{xy}.
z_{med} = *median of gray levels in* S_{xy}.
z_{xy} = *gray level at coordinates (x,y).*
S_{max} = *maximum allowed size of* S_{xy}.

- *Note that unlike the other filters the size of S_{xy} increases during the filtering operation.*

- *Changing size of the filter mask does not change the fact that the output of the filter is still a single value centering the mask.*

Image Restoration

Restoration in the presence of Noise:

Adaptive Median Filter:

•*The Adaptive median filtering Algorithm: Two levels exist (Levels A and B)*

Level A:

$A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

if A1>0 and A2<0, goto Level B

else increase the window size

if window size<S_{max} *repeat Level A*

else output z_{xy}.

Level B:

$B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

if B1>0 and B2<0, output z_{xy}.

else output z_{med}.

Image Restoration

Restoration in the presence of Noise:

Adaptive Median Filter:

Undesired discontinuities

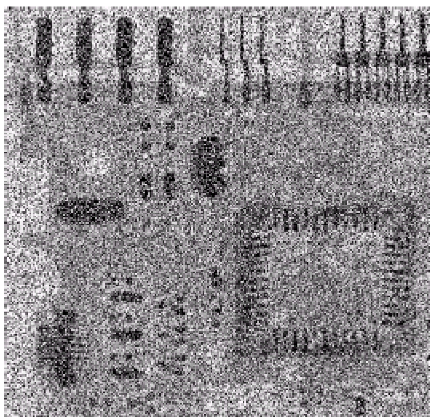
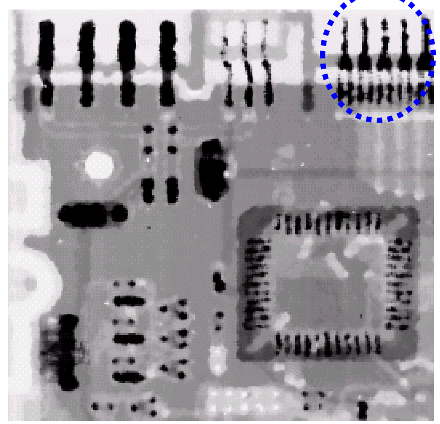
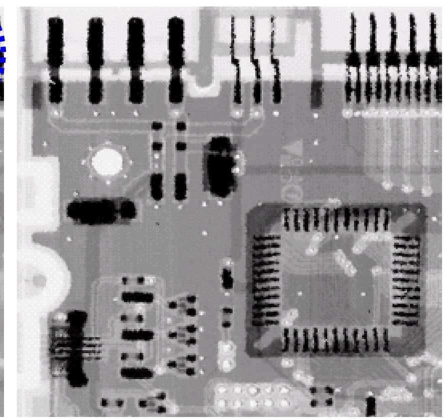


Image corrupted by salt & pepper noise with $P_a=P_b=0.25$



7x7 Median Filter



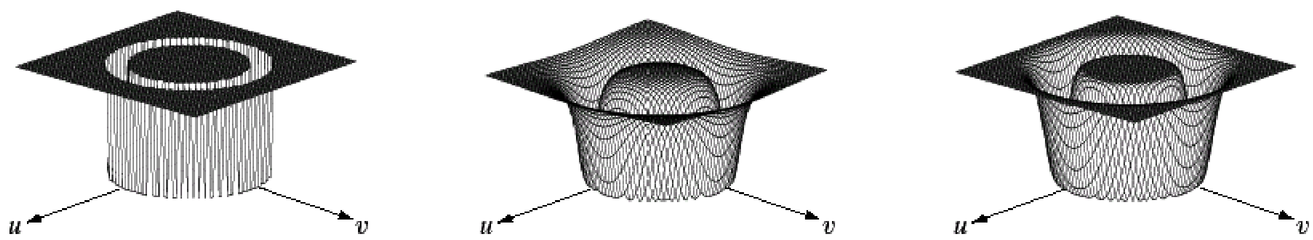
Adaptive Median Filter with $S_{max}=7$

Image Restoration

Restoration in the presence of Noise:

Periodic Noise Removal by Frequency Domain Filtering:

•*Bandreject, bandpass and notch filters can be used for periodic noise removal.*
•*Bandreject filters remove/attenuate a band of frequencies about the origin of the Fourier transform.*



a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Image Restoration

Restoration in the presence of Noise:

Periodic Noise Removal by Frequency Domain Filtering:

•*An Ideal Bandreject filter is given by:*

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v) < D_0-\dfrac{W}{2}\\[2mm] 0 & \text{if } D_0-\dfrac{W}{2}\le D(u,v)\le D_0+\dfrac{W}{2}\\[2mm] 1 & \text{if } D(u,v)\ge D_0+\dfrac{W}{2}\end{cases}$$

•*W is the width of the band*
•*D_0 is the radial center*
•*D(u,v) distance from the origin.*

•*Butterworth Bandreject filter is given by:*

$$H(u,v)=\frac{1}{1+\left[\dfrac{D(u,v)W}{D^2(u,v)-D_0^{\,2}}\right]^{2n}}$$

•*W is the width of the band*
•*D_0 is the radial center*
•*D(u,v) distance from the origin.*
•*n is the order of the filter*

Prepared By: *Dr. Hasan Demirel, PhD*

Image Restoration

Restoration in the presence of Noise:

Periodic Noise Removal by Frequency Domain Filtering:

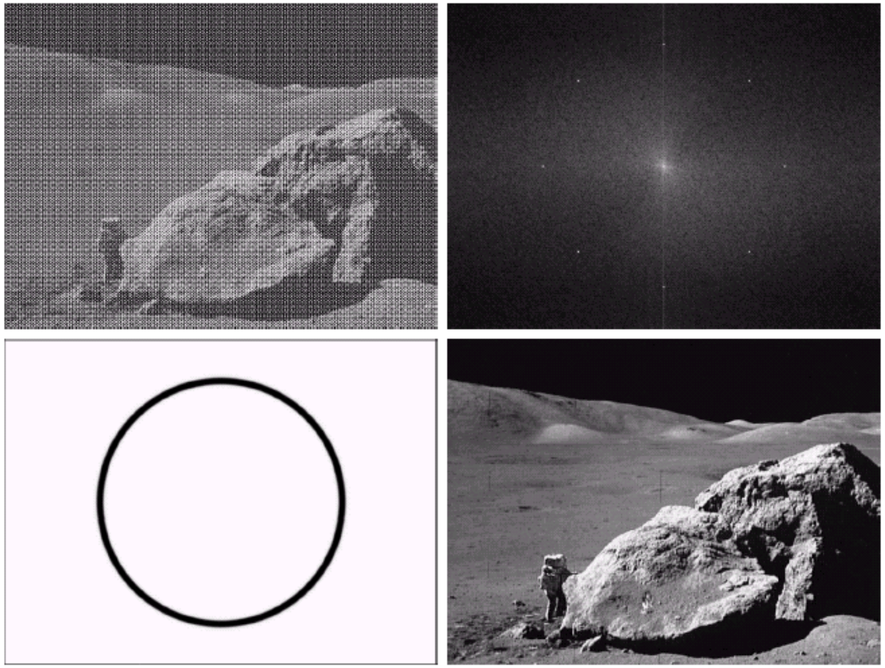
•*Gaussian Bandreject filter is given by:*

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^{\,2}}{D(u,v)W}\right]^2}$$

- *W is the width of the band*
- *D_0 is the radial center*
- *D(u,v) distance from the origin.*

Image corrupted by sinusoidal noise

Spectrum of corrupted image



Butterworth Bandreject Filter (n=4)

Filtered Image

Image Restoration

Restoration in the presence of Noise:

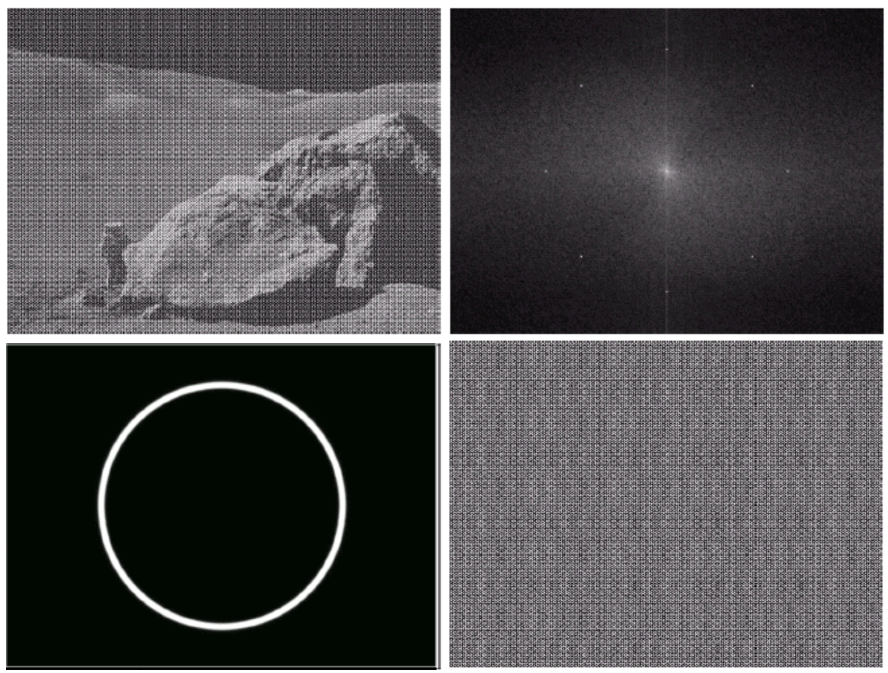
Periodic Noise Removal by Frequency Domain Filtering:

•***Bandpass filters*** *perform the opposite function of the bandreject filters and the filter transfer function of a bandpass filter is given by:*

$$H(u,v)_{bp} = 1 - H(u,v)_{br}$$

Image corrupted by sinusoidal noise

Spectrum of corrupted image



Butterworth Bandpass filter

Noise Image

Image Restoration

Restoration in the presence of Noise:

Periodic Noise Removal by Frequency Domain Filtering:

- ***Notch filters** rejects/passes frequencies in a predefined neighborhoods about the center frequency.*
- *Notch filters appear in symmetric pairs due to the symmetry of the Fourier transform.*
- *The transfer function of **ideal notch filter** of radius of D_0, with centers at (u_0,v_0) and by symmetry at $(-u_0,-v_0)$, is given by:*

$$H(u,v)=\begin{cases} 0 & if \quad D_1(u,v)\le D_0 \quad or \quad D_2(u,v)\le D_0 \\ 1 & otherwise \end{cases}$$

$$D_1(u,v)=\left[(u-M/2-u_0)^2+(v-N/2-v_0)^2\right]^{1/2}$$

$$D_2(u,v)=\left[(u-M/2+u_0)^2+(v-N/2+v_0)^2\right]^{1/2}$$

Image Restoration

Restoration in the presence of Noise:

Periodic Noise Removal by Frequency Domain Filtering:

•*Notch filters:*

•*The transfer function of Butterworth notch filter of order n and of radius of D_0, with centers at (u_0,v_0) and by symmetry at $(-u_0,-v_0)$, is given by:*

$$H(u,v)=\frac{1}{1+\left[\dfrac{D_0^{\,2}}{D_1^{\,2}(u,v)D_2^{\,2}(u,v)}\right]^n}$$

The transfer function of Gaussian notch filter of radius of D_0, with centers at (u_0,v_0) and by symmetry at $(-u_0,-v_0)$, is given by:

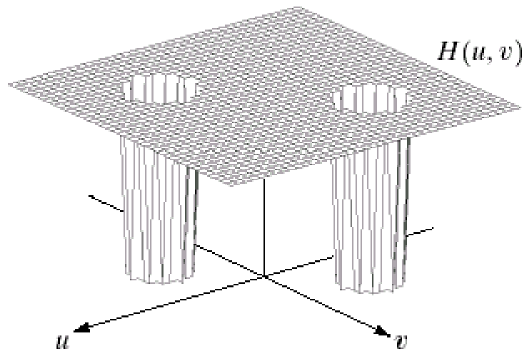
$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D_1^{\,2}(u,v)-D_2^{\,2}(u,v)}{D_0^{\,2}}\right]}$$

Image Restoration

Restoration in the presence of Noise:

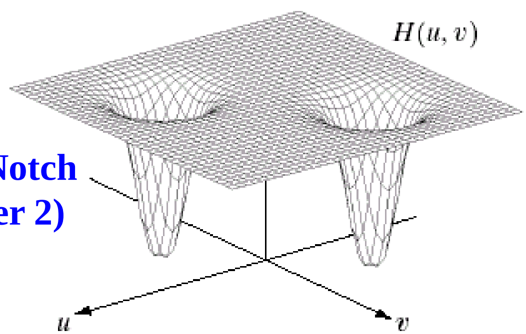
Periodic Noise Removal by Frequency Domain Filtering:

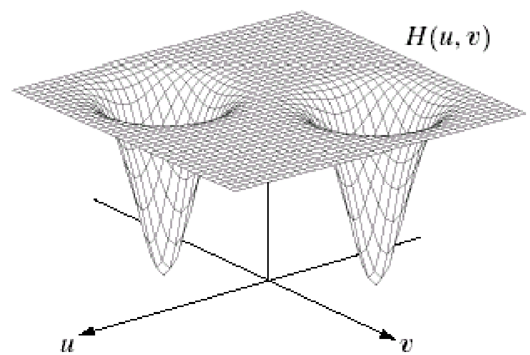
• ***Notch filters:***



Ideal Notch Filter

Butterworth Notch Filter (of order 2)





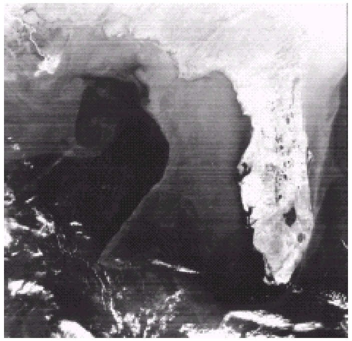
Gaussian Notch Filter

Image Restoration

Restoration in the presence of Noise:

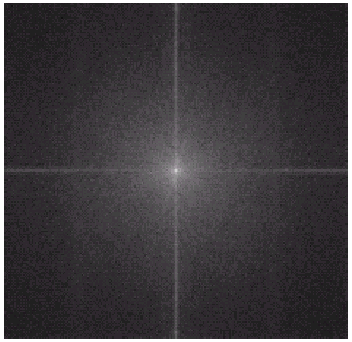
Periodic Noise Removal by Frequency Domain Filtering:

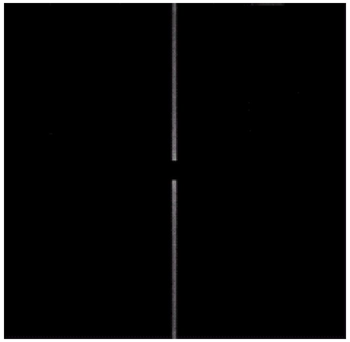
•*Notch filters:*



Original Noisy image with undesired horizontal scanning lines

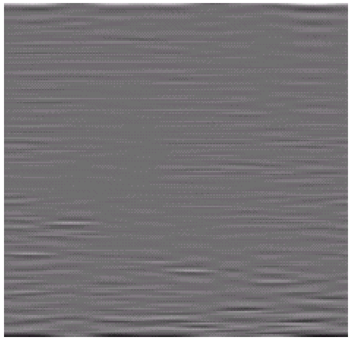
Spectrum of the image

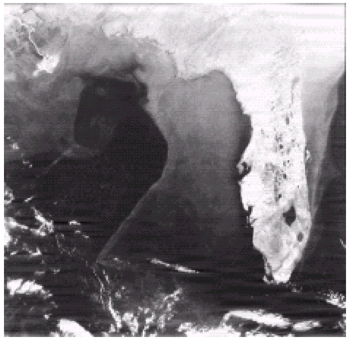




A simple ideal Notch filter along the vertical axis

The filtered noise pattern Corresponding the horizontal artifacts





filtered image free of horizontal scanning lines.

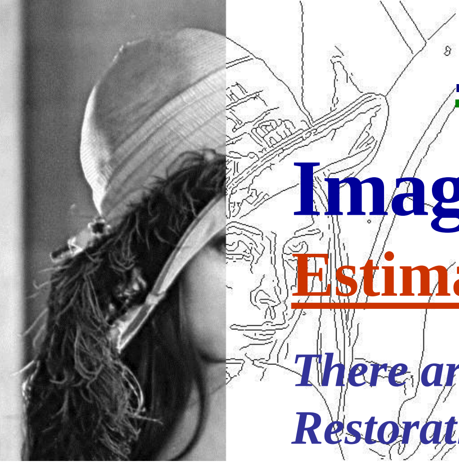


Image Restoration

Estimating the Degradation Function :

There are 3 principal methods of estimating the degradation function for Image Restoration: 1) Observation, 2) Experimentation, 3) Mathematical Modeling.

The degradation function H can be estimated by visually looking into a small section of the image containing simple structures, with strong signal contents, like part an object and the background. Given a small subimage $g_s(x,y)$, we can manually (i.e. filtering) remove the degradation in that region with an estimated subimage $\hat{f}_s(x,y)$ and assuming that the additive noise is negligible in such an area with a strong signal content.

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

Having $H_s(u,v)$ estimated for such a small subimage, the shape of this degradation function can be used to get an estimation of H (u,v) for the entire image.

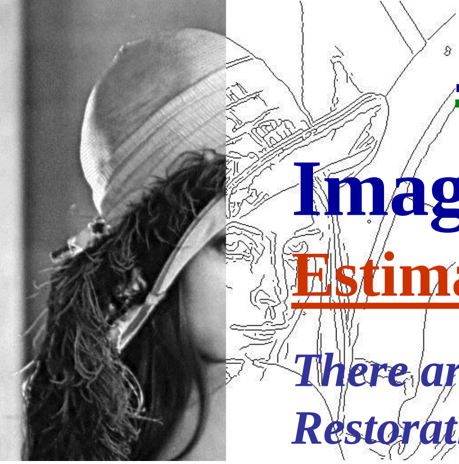


Image Restoration

Estimating the Degradation Function :

There are 3 principal methods of estimating the degradation function for Image Restoration: 1) Observation, 2) Experimentation, 3) Mathematical Modelling.

• ***Estimation by Image Experimentation:***

- ***If we have the acquisition device producing degradation on images, we can use the same device to obtain an accurate estimation of the degradation.***

- ***This can be achieved by applying an impulse (bright dot) as an input image . The Fourier transform of an impulse is constant, therefore.***

$$H(u,v) = \frac{G(u,v)}{A}$$

Where, A is a constant describing the strength of the impulse. Note that the effect of noise on an impulse is negligible.

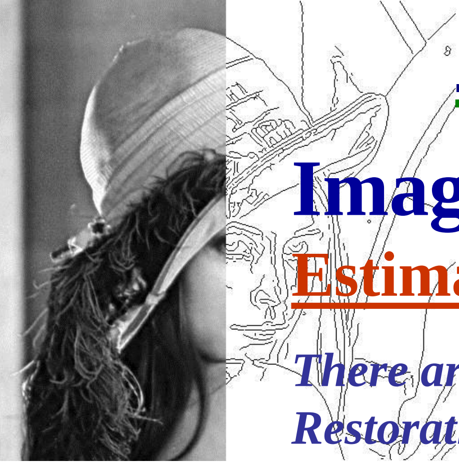
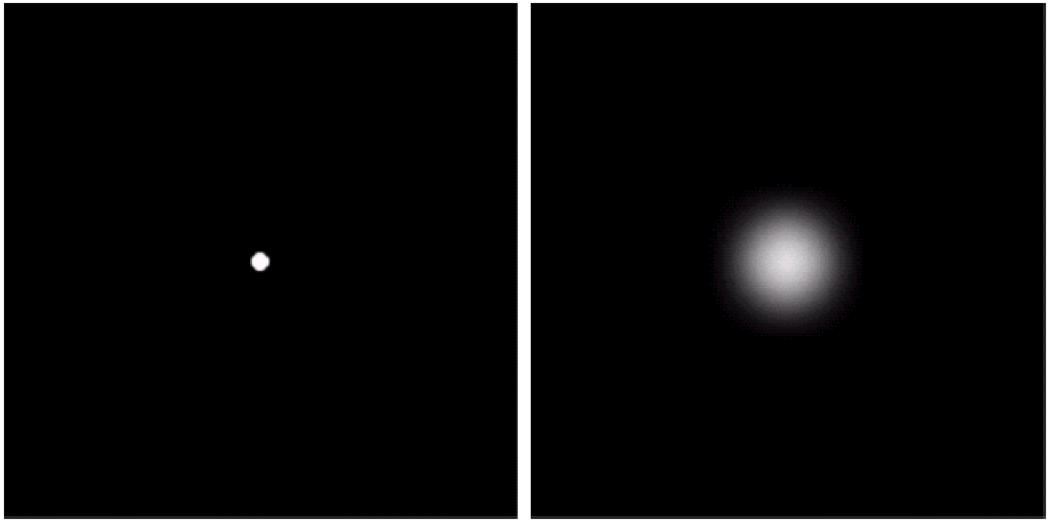


Image Restoration

Estimating the Degradation Function :

There are 3 principal methods of estimating the degradation function for Image Restoration: 1) Observation, 2) Experimentation, 3) Mathematical Modelling.

•***Estimation by Image Experimentation:***



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Impulse Image

Degraded Impulse Image consider as h(x,y)

Simply take the Fourier transform of the degraded image and after normalization by a constant A, use it as the estimate of the degradation function H(u,v).

Image Restoration

Estimating the Degradation Function :

There are 3 principal methods of estimating the degradation function for Image Restoration: 1) Observation, 2) Experimentation, 3) Mathematical Modelling.

•***Estimation by Mathematical Modeling:*** *Sometimes the environmental conditions that causes the degradation can be modeled by mathematical formulation. For example the atmospheric turbulence can be modeled by:*

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

k **is a constant that depends on the nature of the Turbulence**

• *This equation is similar to Gaussian LPF and would produce blurring in the image according to the values of k. For example if k=0.0025, the model represents severe turbulence, if k=0.001, the model represents mild turbulence and if k=0.00025, the model represents low turbulence.*

• *Once a reliable mathematical model is formed the effect of the degradation can be obtained easily.*

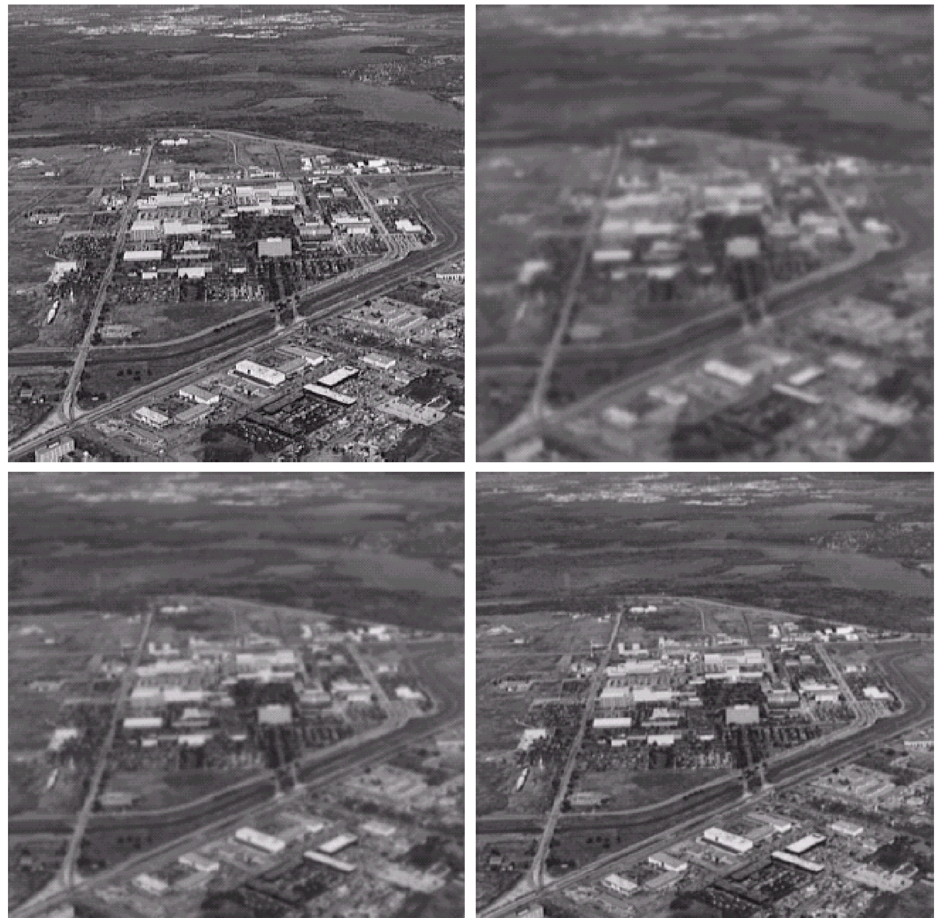
Image Restoration

Estimating the Degradation Function :

•*Estimation by Mathematical Modeling: Illustration of the atmospheric turbulence model*

Negligible turbulence

Severe turbulence
k=0.0025



Mild turbulence
k=0.001

Low turbulence
k=0.00025

Image Restoration

Estimating the Degradation Function :

•*Estimation by Mathematical Modeling:* ***In some applications the mathematical model can be derived by treating that the image is blurred by uniform linear motion between the image and the sensor during image acquisition. The motion blur can be modeled as follows:***

- ***Let f(x,y) be subject to motion in x- and y-direction by time varying motion components $x_0(t)$ and $y_0(t)$.***
- ***The total exposure is obtained by integrating the instantaneous exposure over the time interval during the shutter of the imaging device is open.***
- ***If T is the duration of the exposure, than***

$$g(x,y) = \int_0^T f\left[x - x_0(t), y - y_0(t)\right]\, dt$$

g(x,y) is the blurred image

$$G(u,v) = \int_{-\infty}^{-\infty}\int_{-\infty}^{-\infty} g(x,y) e^{-j2\pi(ux+vy)} dx\, dy$$

$$= \int_{-\infty}^{-\infty}\int_{-\infty}^{-\infty}\left[\int_0^T f\left[x - x_0(t), y - y_0(t)\right]\, dt\right] e^{-j2\pi(ux+vy)} dx\, dy$$

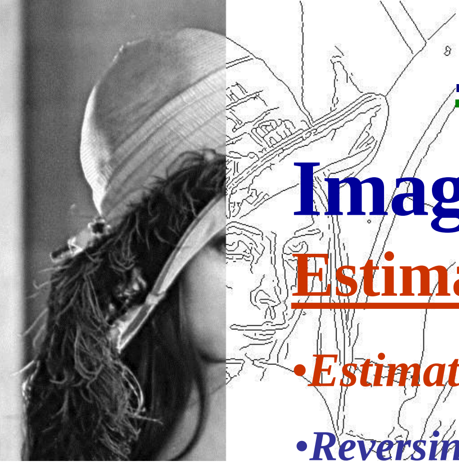


Image Restoration

Estimating the Degradation Function :

•*Estimation by Mathematical Modeling:*

•*Reversing the order of integration yields:*

$$G(u,v) = \int_0^T \left[\int_{-\infty}^{-\infty} \int_{-\infty}^{-\infty} f\left[x - x_0(t), y - y_0(t)\right] e^{-j2\pi(ux+vy)} dx\, dy \right] dt$$

•*Using the translation property of the Fourier Transform, the inner part can be simplified,*

$$G(u,v) = \int_0^T F(u,v) e^{-j2\pi(ux_0(t)+vy_0(t))}\ dt =$$

$$= F(u,v) \int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}\ dt$$

•*Then,*

$$H(u,v) = \int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}\ dt$$

Image Restoration

Estimating the Degradation Function :

•Estimation by Mathematical Modeling:

•By assuming that the linear uniform motion is in x-direction only at a rate of $x_0(t)=at/T$, the image covers a distance , when $t=T$.

$$H(u,v)=\int_0^T e^{-j2\pi u x_0(t)}\,dt=\int_0^T e^{-j2\pi u at/T}\,dt$$

$$=\frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$$

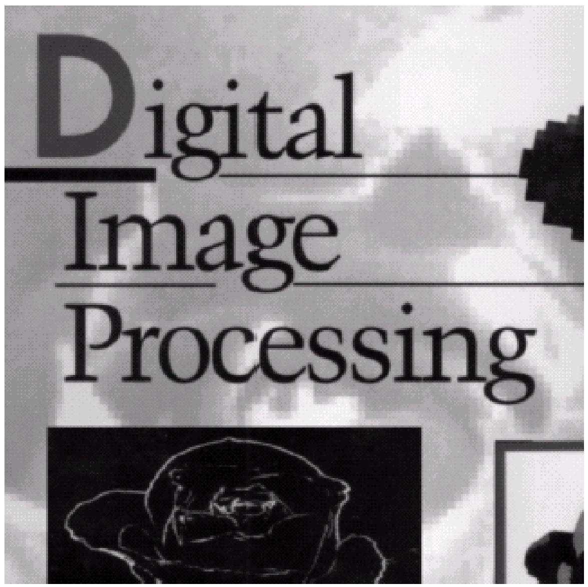
•If we allow the motion in y-direction, with $y_0(t)=bt/T$, the model becomes,

$$H(u,v)=\frac{T}{\pi(ua+vb)}\sin\left[\pi(ua+vb)\right]e^{-j\pi(ua+vb)}$$

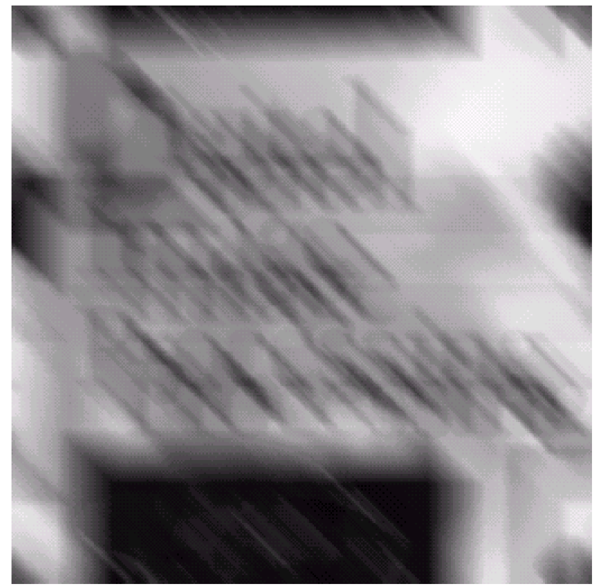
Image Restoration

Estimating the Degradation Function :

•*Estimation by Mathematical Modeling: The result of the modeled motion blur is demonstrated in the following example:*



Original Image



Blurred image with a=b=0.1 and T=1

$$H(u,v)=\frac{T}{\pi(ua+vb)}\sin\left[\pi(ua+vb)\right]e^{-j\pi(ua+vb)}$$

Image Restoration

Inverse Filtering:

•*Until now our focus was the calculation of degradation function H(u,v). Having H(u,v) calculated/estimated the next step is the restoration of the degraded image. The simplest way of image restoration is by using Inverse filtering:*

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} \quad , \ \hat{F}(u,v)$$

is the Fourier transform of the restored image

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Unknown random function

Must not be very small. Otherwise the noise dominates

•*In Inverse filtering, we simply take H(u,v) such that the noise does not dominate the result. This is achieved by including only the low frequency components of H(u,v) around the origin. Note that, the origin, H(M/2,N/2), corresponds to the highest amplitude component.*

Image Restoration

Inverse Filtering:

- *Consider the degradation function of the atmospheric turbulence for the origin of the frequency spectrum,*

$$H(u,v) = e^{-k\left[(u-M/2)^2+(v-N/2)^2\right]^{5/6}}$$

- *If we consider a Butterworth Lowpass filter of H(u,v) around the origin we will only pass the low frequencies (high amplitudes of H(u,v)).*
- *As we increase the cutoff frequency of the LPF more smaller amplitudes will be included. Therefore, instead of the degradation function the noise will be dominating.*

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Must not be very small. Otherwise the noise dominates

Image Restoration

Inverse Filtering:

•*Consider the degradation function of the atmospheric turbulence for the origin of the frequency spectrum,*

Input image with
Severe turbulence
k=0.0025
480x480 pixels

Result of full filter/degradation

Cutoff outside of radius 40

Cutoff outside of radius 70

Cutoff outside of radius 85

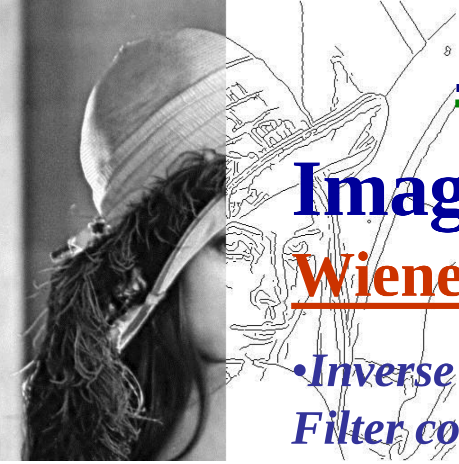


Image Restoration

Wiener (Min Mean Square Error) Filtering:

•*Inverse filtering does not consider the additive noise for restoration. The Wiener Filter consider both the degradation function and the statistical characteristics of the noise in the restoration process.*

• *The method tries to minimize the mean square error (MSE) between the uncorrupted image and the estimate of the image by:*

$$e^2 = E\left\{(f - \hat{f})^2\right\}$$

E{.} is the expected value of the argument

•*The noise and the image are assumed to be uncorrelated. The minimum of the error function given above is achieved in the frequency domain by the following expression.*

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2 + S_\eta(u,v)}\right]G(u,v)$$

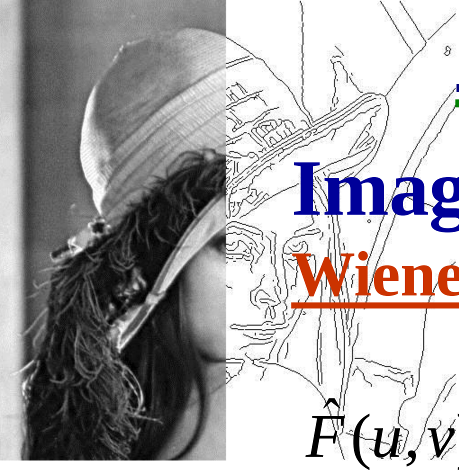


Image Restoration

Wiener (Min Mean Square Error) Filtering:

$$\hat{F}(u,v) = \left[\frac{H^*(u,v) S_f(u,v)}{S_f(u,v) \left| H(u,v) \right|^2 + S_\eta(u,v)} \right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{\left| H(u,v) \right|^2 + S_\eta(u,v) / S_f(u,v)} \right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \frac{\left| H(u,v) \right|^2}{\left| H(u,v) \right|^2 + S_\eta(u,v) / S_f(u,v)} \right] G(u,v)$$

$H(u,v) =$ *Degradation function.*

$H^*(u,v) =$ *Complex conjugate of H(u,v)*

$\left| H(u,v) \right|^2 = H^*(u,v) H(u,v)$

$S_\eta(u,v) = \left| N(u,v) \right|^2 =$ *Power spectrum of the noise.*

$S_f(u,v) = \left| F(u,v) \right|^2 =$ *Power spectrum of the undegraded image.*

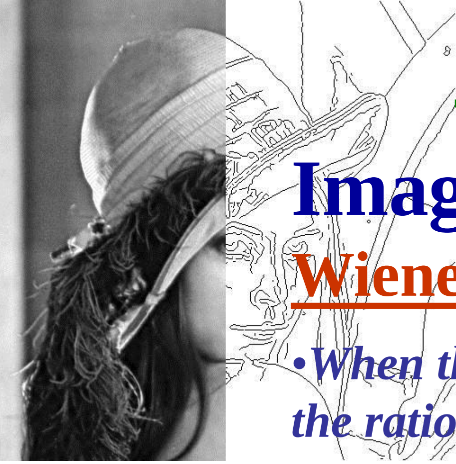


Image Restoration

Wiener (Min Mean Square Error) Filtering:

•*When the power spectrum of the undegraded image and noise are not known, the ratio of the power spectrums of the noise and image is assumed to be constant.*

$$\hat{F}(u,v)=\left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

When not known. Assumed to be constant

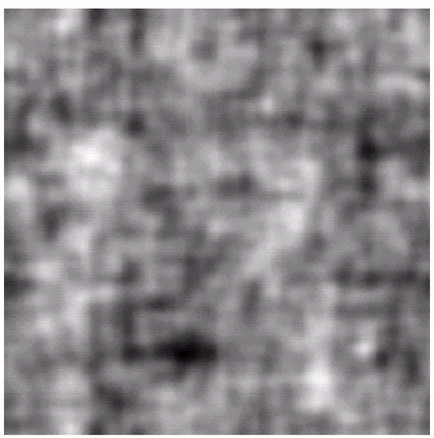
$$\hat{F}(u,v)=\left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\right]G(u,v)$$

•*Typically different values of K are chosen and the image quality is measured by MSE. The value of K is chosen in such a way that the MSE is minimized.*

Image Restoration

Wiener (Min Mean Square Error) Filtering:

Input image with Severe turbulence blur , k=0.0025 480x480 pixels



Result of Full inverse filtering

Radially limited Inverse Filtering with a cutoff radius of 70

Result of Wiener filtering with and optimized K

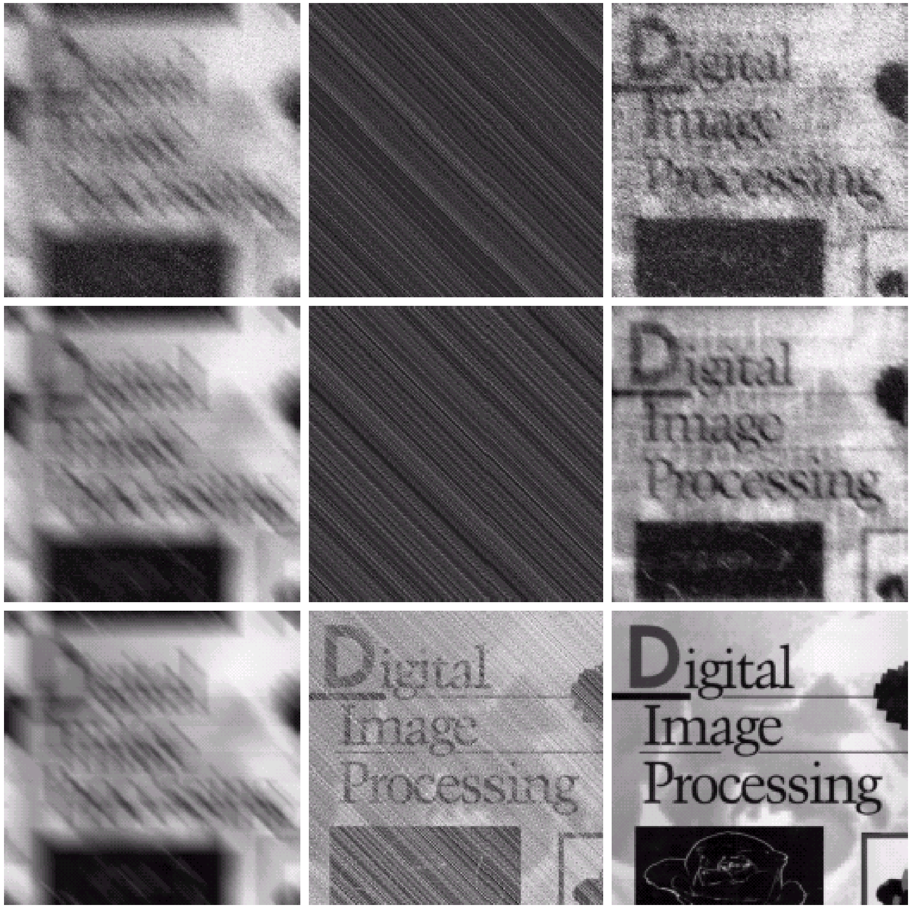
Image Restoration

Wiener (Min Mean Square Error) Filtering:

Image corrupted by motion blur and additive noise

Variance of the noise is one order of magnitude less

Variance of the noise is five order of magnitude less



Degraded Input Image

Result of Inverse filtering

Result of Wiener filtering