Imaging through turbulence (1)

EC 522 Computational Optical Imaging

Lei Tian

Admins

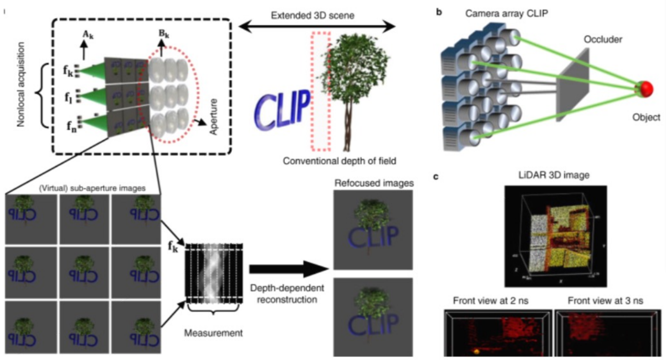
- HWs
  - HW 6: posted, due 4/17
- **Group lecture**
  - Reach out to me & your student mentor for discussions if needed
  - All questions are good questions
  - Grades are based on the demonstration of technical comprehension during the presentation

Group lectures begin					
3/18 (M)	Selected Topic in computational imaging		Group 1	**HW 5: inverse problem 2**	**HW 4**
3/20 (W)	Selected Topic in computational imaging		Group 2		
3/25 (M)	Selected Topic in computational imaging		Group 3		
3/27 (W)	Selected Topic in computational imaging		Group 4		
4/1 (M)	Selected Topic in computational imaging		Group 5	**HW 6: inverse problem 3**	**HW 5**
4/3 (W)	Selected Topic in computational imaging		Group 6		
4/8 (M)	Selected Topic in computational imaging		Group 7		
4/10 (W)	Selected Topic in computational imaging		Group 8		
4/15 (M)	**Patriots' Day Holiday**				
4/17 (W)	Selected Topic in computational imaging		Group 9		**HW 6**
4/22 (M)	Cancelled				
4/24 (W)	**Final Projects**		**Group 1-3**		
4/29 (M)	**Final Projects**		**Group 4-6**		
5/1 (W)	**Final Projects**		**Group 7-9**		

Check the updated syllabus on Blackboard

Theme: Computational photography

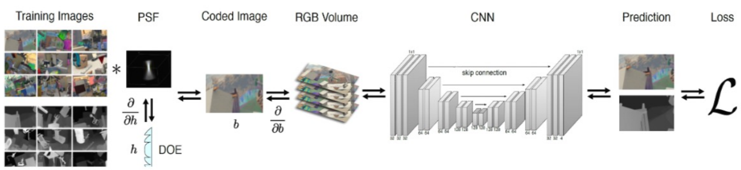
Group Lecture / Class Project 3: Lightfield imaging



Team 4 - **3/27 (W) / / 4/29 (M)**
Paper: CLIP
- Saini Ye
- Chen Qian
- Yuxiang Su

Student mentor:
Qianwan Yang

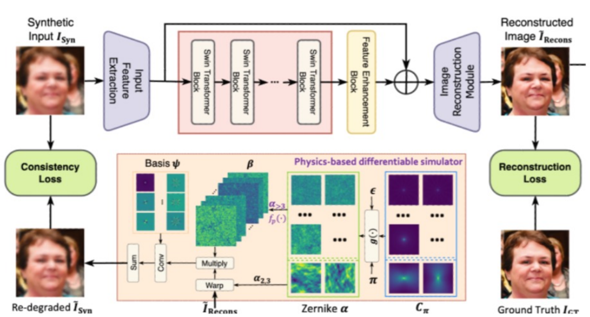
Group Lecture / Class Project 5: Computational 3D photography



Team 5 - **4/1 (M) / 4/29 (M)**
Paper: Depth from defocus
- Ian Lee
- Susan Zhang
- Wangyi Chen

Student Mentor:
Qianwan Yang

Group Lecture / Class Project 9: Computational imaging in complex media



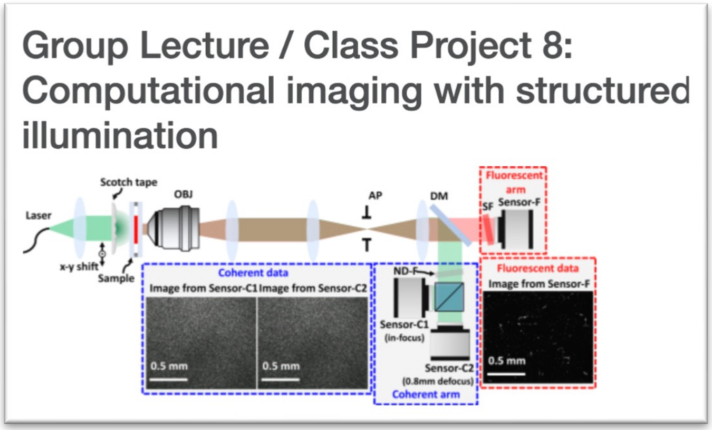
Team 6 - **4/3 (W) / 4/29 (M)**
Paper: Turbulence
- Shuyue Jia
- Hua Tong
- Yujie Zheng

Student mentor:
Tongyu Li

Team 7 - **4/8 (M) / 5/1 (W)**
Paper: NeuWS
- Noa Margolin
- Nolan Vild
- Adhithi Ramasubramanian

Student Mentor:
Hao Wang

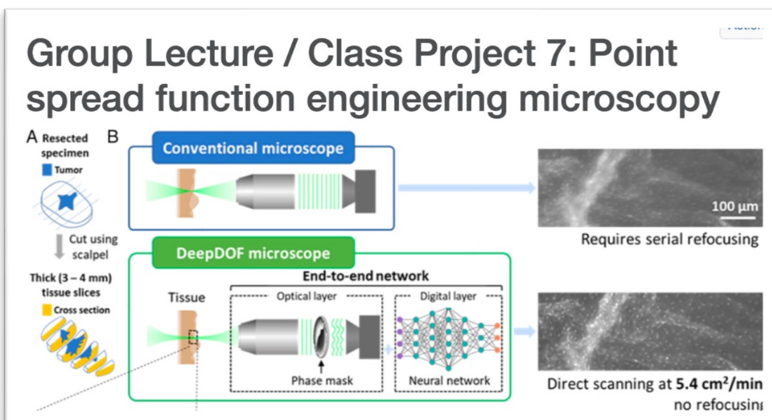
Theme: computational microscopy



Team 8 - **4/10 (W) / 5/1 (W)**

- Yi Shen
- Deming Li
- Kara Stratton

Student mentor:
Tongyu Li

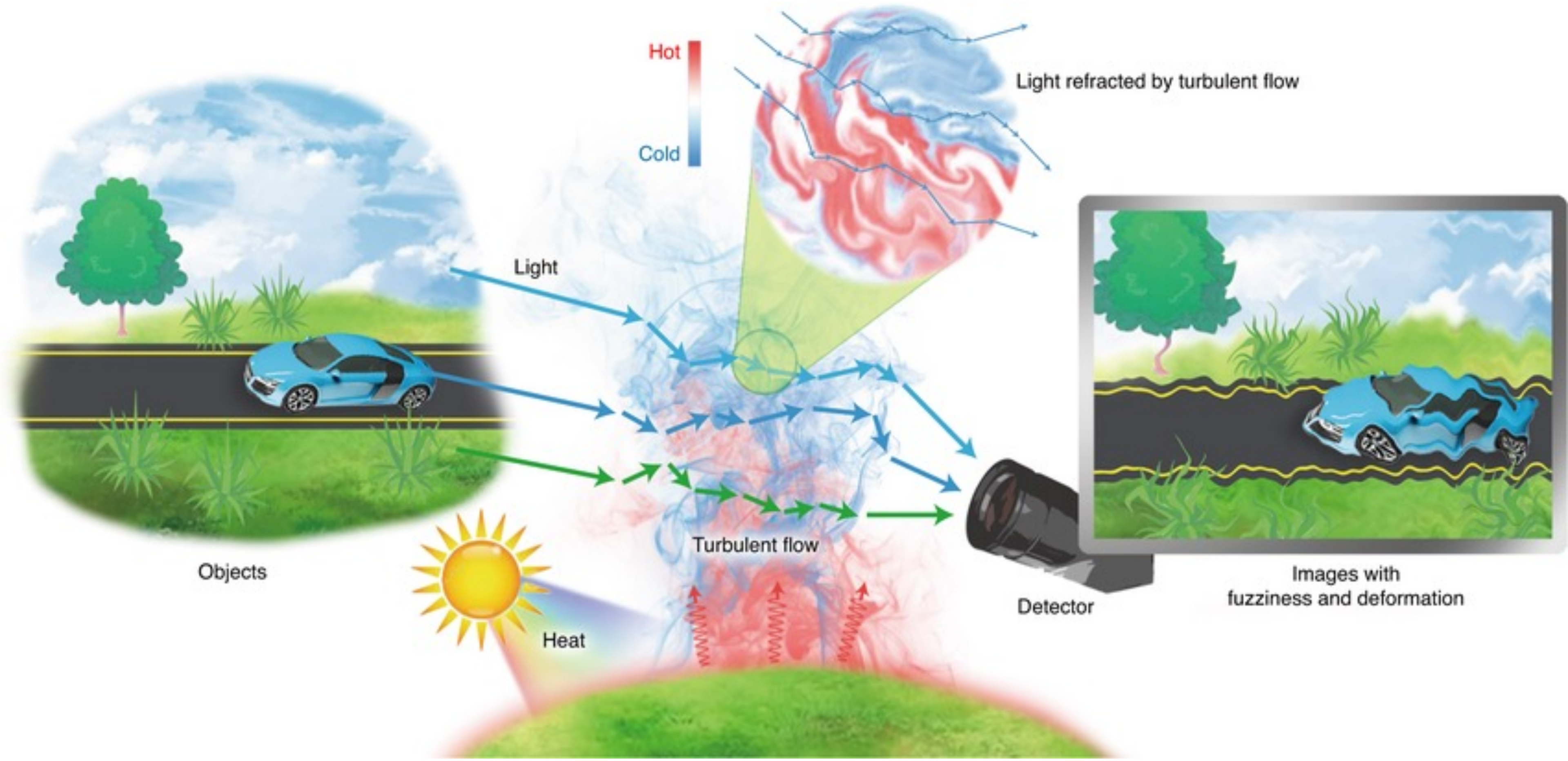


Team 9 - **4/17 (W) / 5/1 (W)**

- Rachel Chan
- Qilin Deng

Student Mentor:
Joe Greene

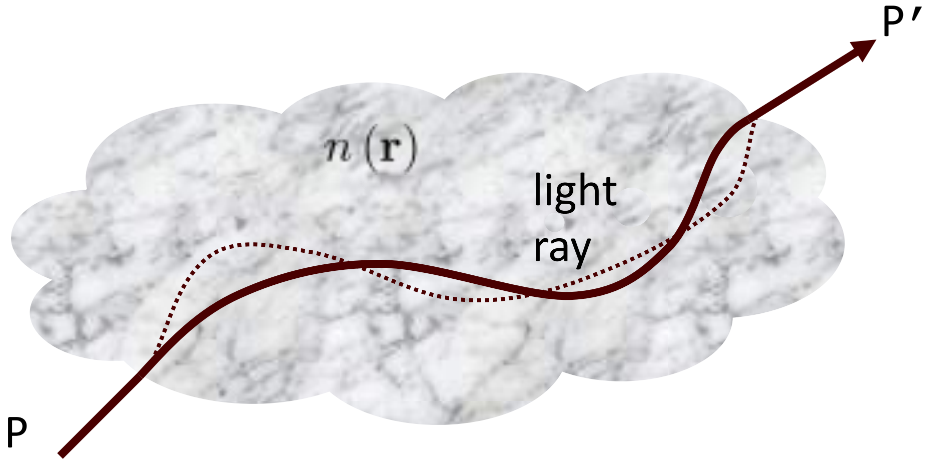
Why imaging through turbulence



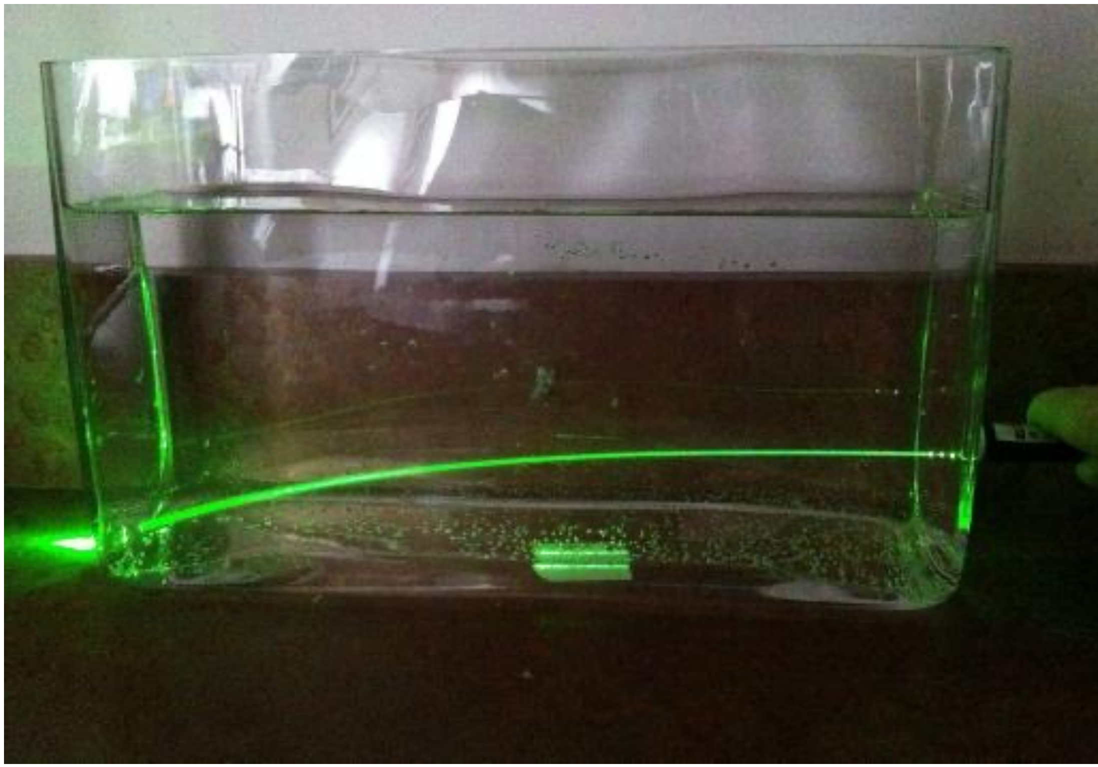
https://www.nature.com/articles/s42256-021-00392-1

Light follows a curved path in an inhomogeneous medium!

material with variable optical "density"
(i.e. inhomogeneous refractive index (index of refraction))



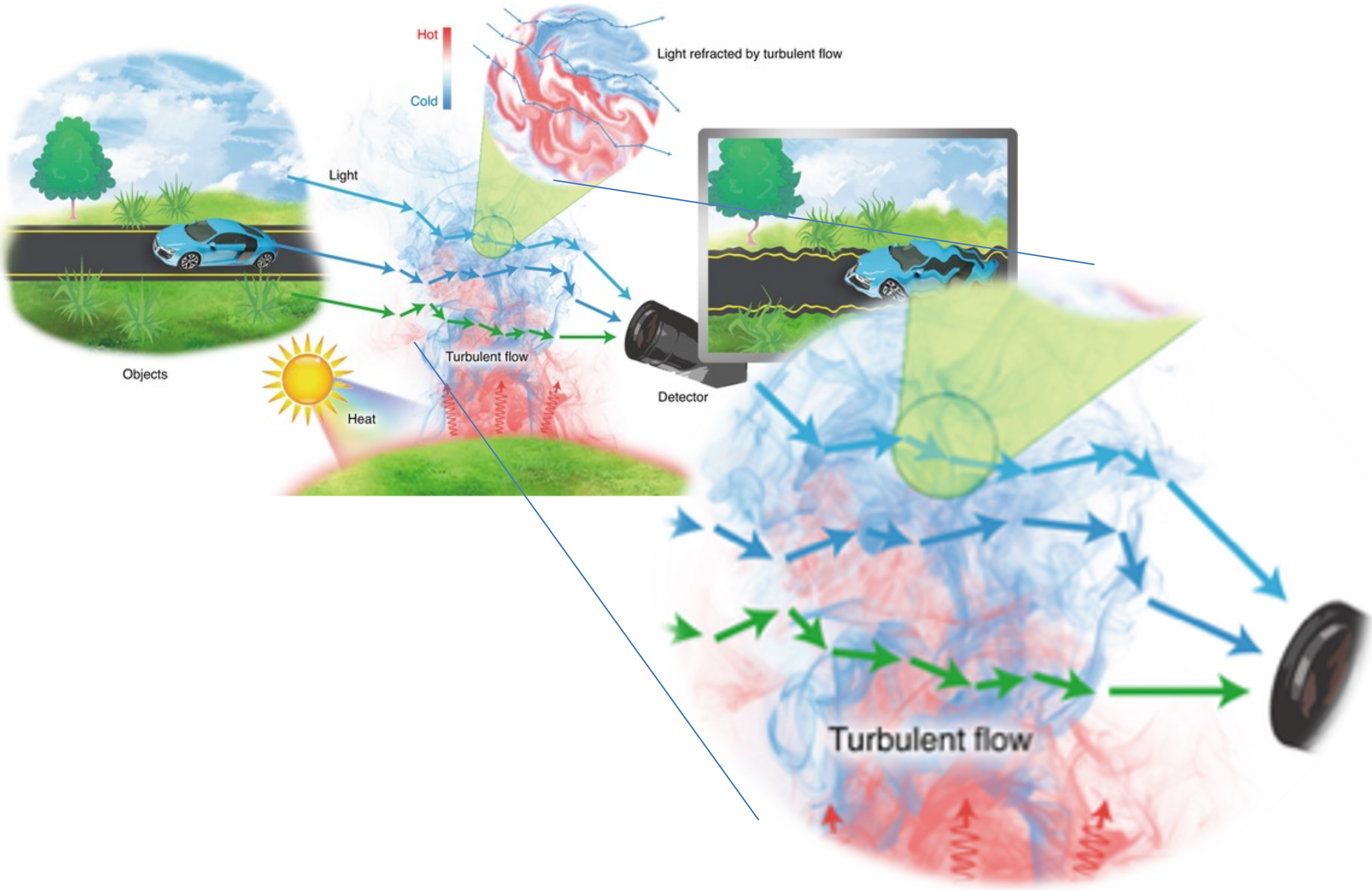
Example: Non-uniform medium with 1D non-uniformity



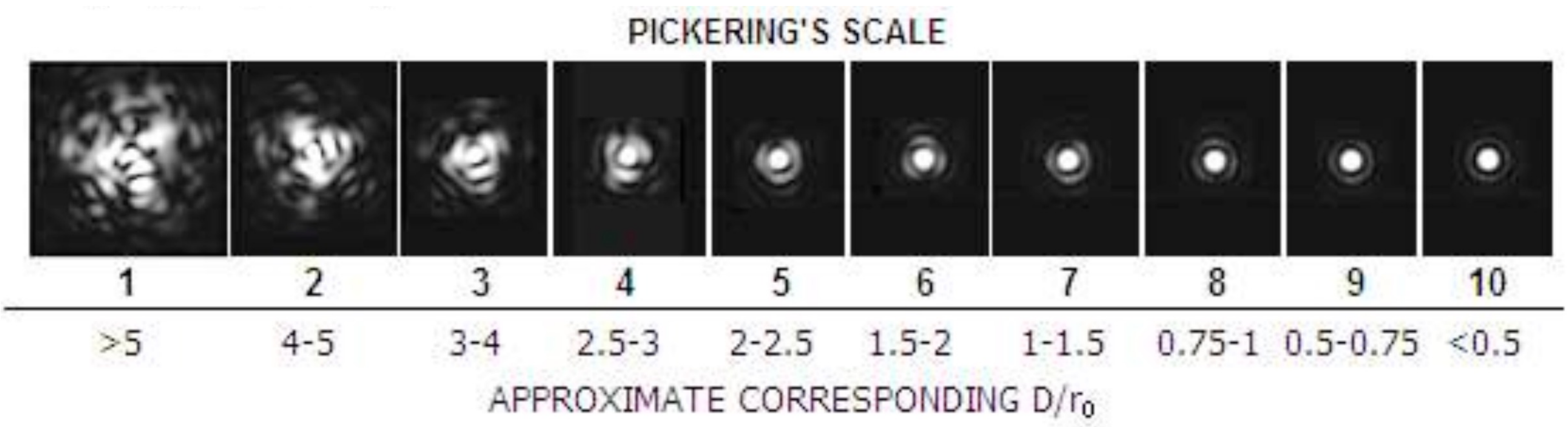
Refractive index increases in depths

”stratified medium”

Turbulence = 3D non-uniformity



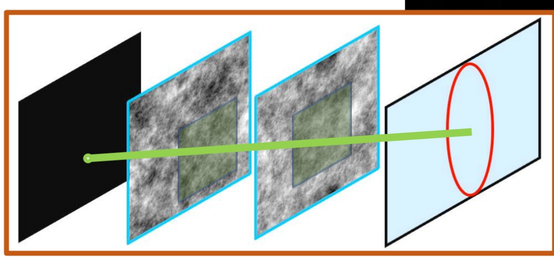
Turbulence *distorts* PSFs

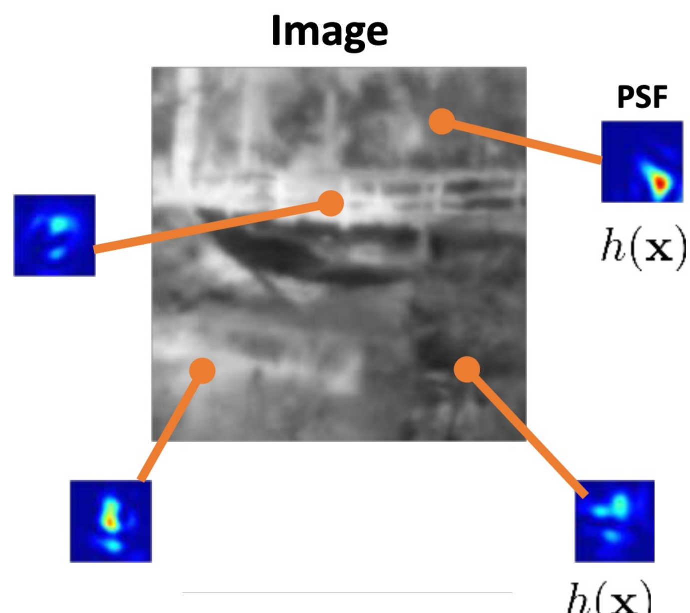


Strooooonger turbulence

Turbulence leads to linear shift variant (LSV) imaging

A collection of point spread functions





https://engineering.purdue.edu/ChanGroup/project_turbulence.html

An introduction to LSV imaging

The classical approach

Linear shift variant system

» $\mathbf{g} = \mathbf{A}\mathbf{f} \leftrightarrow \mathbf{g}_m = \sum_{n=1}^{N} \mathbf{a}_{mn} \mathbf{f}_n$

PSF at x_2

$$\mathbf{g} = \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \cdots & \mathbf{a}_{1N} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \cdots & \mathbf{a}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_{N1} & \mathbf{a}_{N2} & \cdots & \mathbf{a}_{NN} \end{pmatrix} \begin{pmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_N \end{pmatrix}$$

Computational requirement?

Linear operator and matrix

» A linear operator A: $\mathbb{R}^{\mathrm{N}} \rightarrow \mathbb{R}^{\mathrm{M}}$
(assume only real components in A, f, g*)

$$Af = g$$

» Described by a *M*x*N* matrix **A**

$$\mathbf{Af} = \mathbf{g}$$

**Same framework also works for complex numbers:*

- Split a complex number into real and imag. parts
- CR (Wirtinger)-Calculus

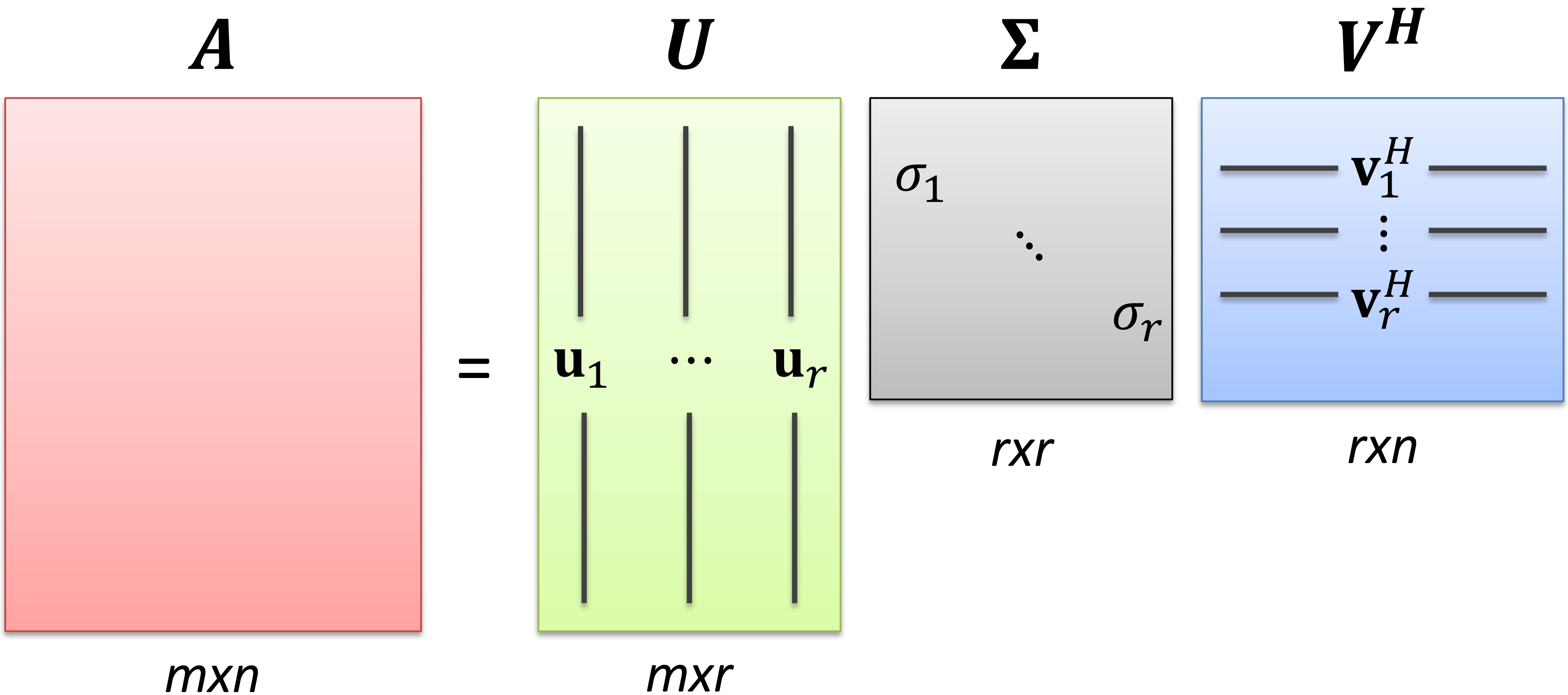
Singular value decomposition (SVD)

$$A = U\Sigma V^{H}$$

$U = [u_1, \ldots, u_r] \in \mathbb{R}^{m\times r}$ $u_i \in \mathbb{R}^m$ left singular vectors

$V = [v_1, \ldots, v_r] \in \mathbb{R}^{n\times r}$ $v_i \in \mathbb{R}^n$ right singular vectors

$\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_r) \in \mathbb{R}^{r\times r}$ $\sigma_1 \geq, \ldots, \geq \sigma_r$ singular values

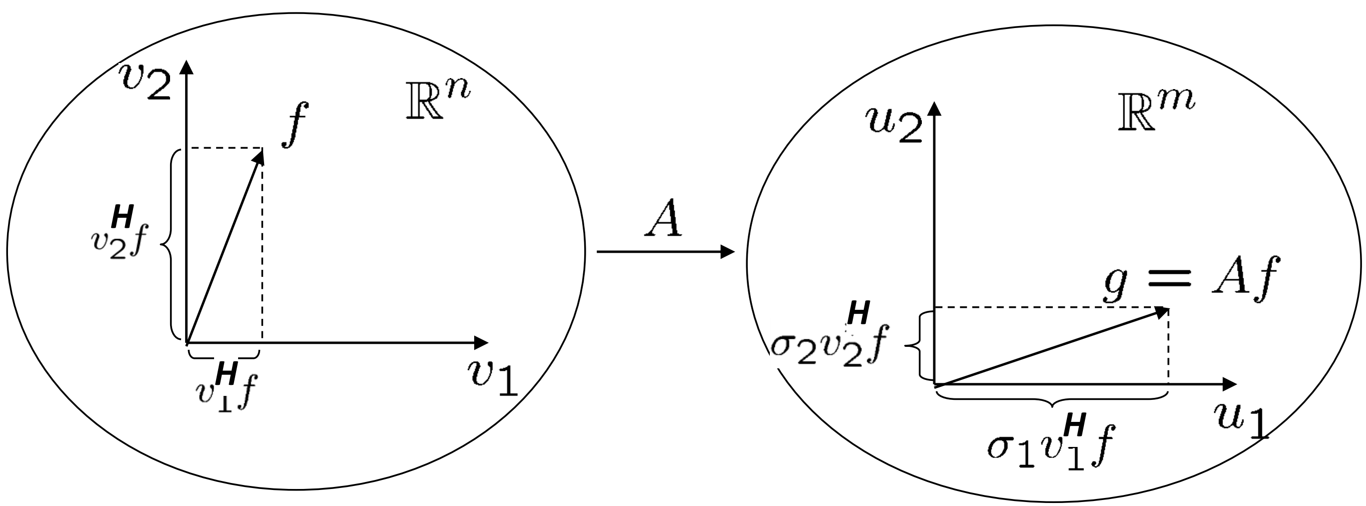


Spectral representation in terms of Singular value decomposition (SVD)

$$A = \mathrm{U}\Sigma V^H = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \, \mathbf{v}_k^H$$

$$A\mathbf{f} = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \, (\mathbf{v}_k^H \mathbf{f})$$

1. projects the input vector along v_k : $f_k = v_k^{\boldsymbol{H}} f$
2. synthetizes $g = Af$ by the linear combination $g = \sum_{k=1}^{r} \sigma_k f_k u_k$



Introduction to inverse problems in imaging Chap 9.2

Range of *A*

$$A\mathbf{f} = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \left(\mathbf{v}_k^H \mathbf{f}\right)$$

» $R(\mathbf{A})$ contains all the vectors $\mathbf{g}$, spanned by the ***column*** space in the form $\mathbf{g} = \mathbf{Af}$

» $R(\mathbf{A})$ is spanned by left singular vectors $\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_r$

Null space of *A*

$$A\mathbf{f} = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \left(\mathbf{v}_k^H \mathbf{f}\right)$$

» The null space $\mathcal{N}(\mathbf{A})$ is the subspace spanned by all the vectors $\mathbf{Af} = 0$

» Dimensionality? ***n-r***

Interim summary

- » Singular values act like the “discrete transfer function” for linear shift-variant system
  - » *The quality of a LSV system can be characterized by distribution of SVs

* More in inverse problem

Adjoint

$$A = \mathrm{U}\Sigma V^H = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \, \mathbf{v}_k^H$$

$$A\mathbf{f} = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \, (\mathbf{v}_k^H \mathbf{f})$$

$$A^H = V\Sigma U^H = \sum_{k=1}^{r} \sigma_k \mathbf{v}_k \, \mathbf{u}_k^H$$

$$A^{\mathbf{H}}\mathbf{g} = \sum_{k=1}^{r} \sigma_k \mathbf{v}_k \, (\mathbf{u}_k^H \mathbf{g})$$

Range of A^H

$$A^{\mathbf{H}}\mathbf{g} = \sum_{k=1}^{r} \sigma_k \mathbf{v}_k \left(\mathbf{u}_k^H \mathbf{g}\right)$$

» $R\left(\mathbf{A}^{\mathbf{H}}\right)$ is spanned by right singular vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_r$

Null space of A^H

$$A^{\mathbf{H}}\mathbf{g} = \sum_{k=1}^{r} \sigma_k \mathbf{v}_k \left(\mathbf{u}_k^H \mathbf{g}\right)$$

» Dimensionality? ***m-r***

Relation between left & right singular vectors

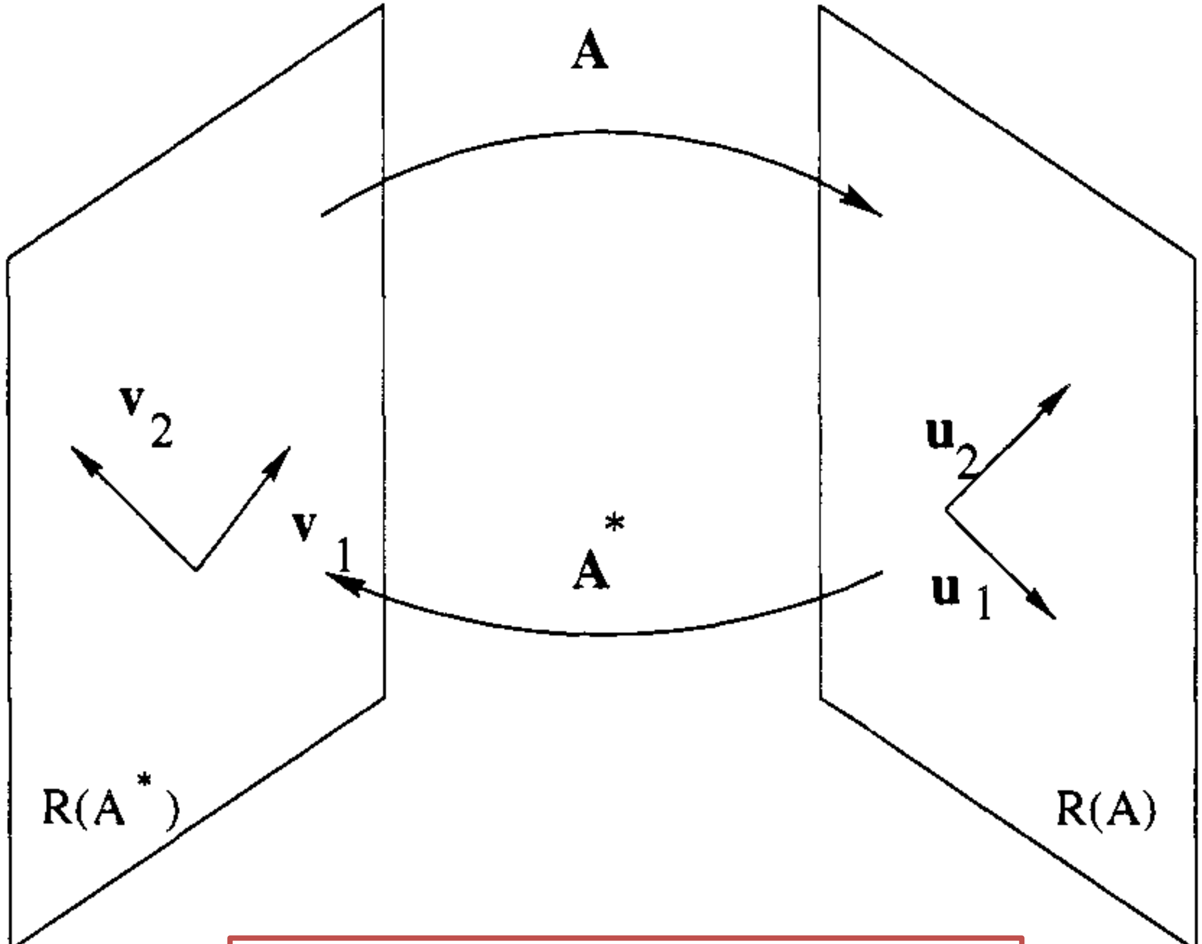
$$A = \mathrm{U}\Sigma V^H = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \, \mathbf{v}_k^H$$

$$A\mathbf{v}_k ? \qquad A\mathbf{v}_k = \sigma_k \mathbf{u}_k$$

$$A^H = V\Sigma U^H = \sum_{k=1}^{r} \sigma_k \mathbf{v}_k \, \mathbf{u}_k^H$$

$$A^H\mathbf{u}_k ? \quad A^H\mathbf{u}_k = \sigma_k \mathbf{v}_k$$

Relation between left & right singular vectors

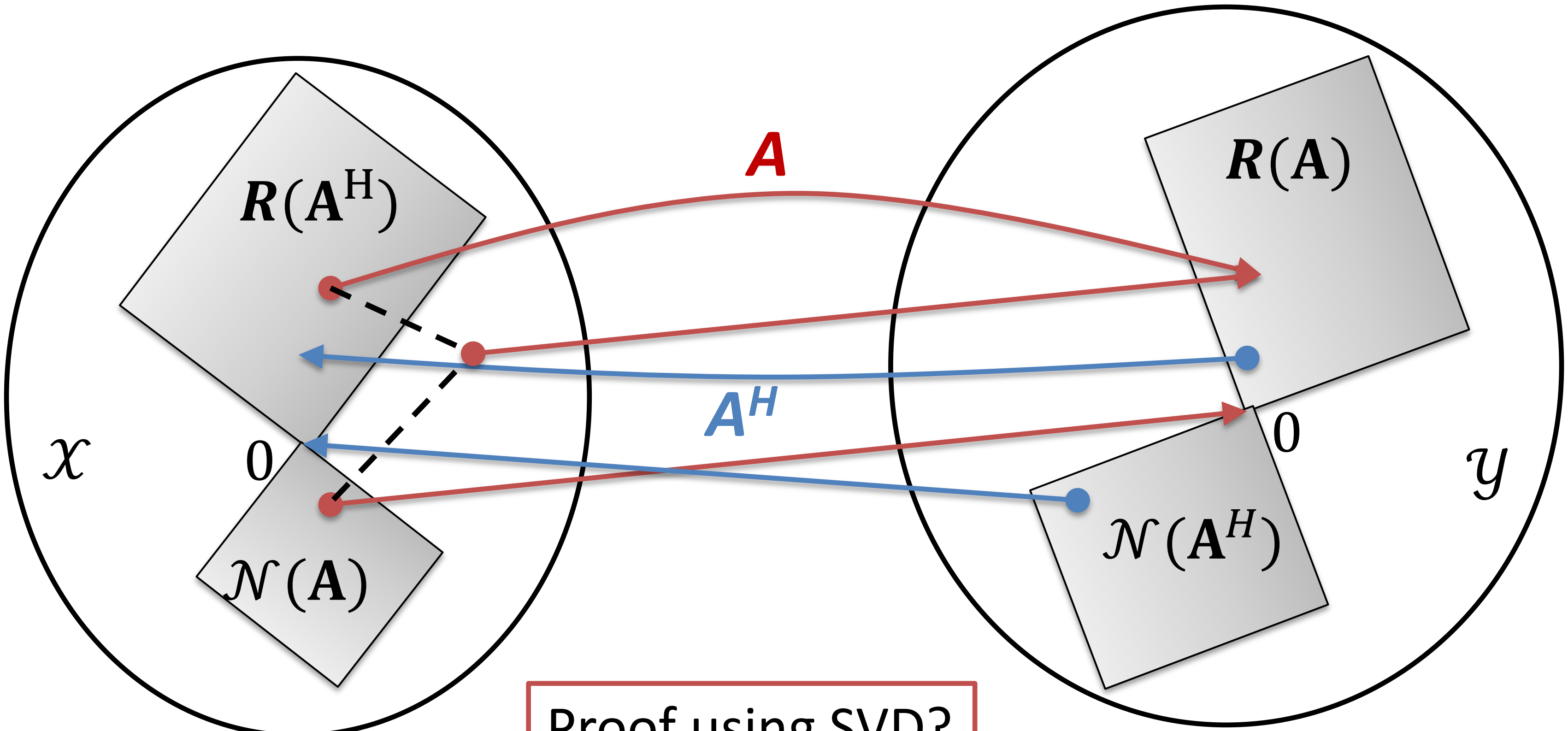


Physical meaning of Adjoint?

Introduction to inverse problems in imaging Chap 9.2

Review: relation between null space and range

$$R(\mathbf{A}) = \mathcal{N}(\mathbf{A}^H)^\perp \qquad R(\mathbf{A}^H) = \mathcal{N}(\mathbf{A})^\perp$$



Proof using SVD?

$$A\mathbf{f} = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \left(\mathbf{v}_k^H \mathbf{f}\right) \qquad A^{\mathbf{H}}\mathbf{g} = \sum_{k=1}^{r} \sigma_k \mathbf{v}_k \left(\mathbf{u}_k^H \mathbf{g}\right)$$