Boston University
Department of Electrical and Computer Engineering
EC522 Computational Optical Imaging
Homework No. 4

Issued: Monday, Mar. 4 Due: 11:59 pm Monday, Mar. 18

Problem 1: Deconvolution in imaging with defocused PSFs

This problem will continue our discussions on computational imaging problems related to defocus and depth of focus (DOF) from HW1. We will look into the inverse problem using the deconvolution techniques learned in the lectures.

Briefly, in an ideal camera, the defocus effect can be well approximated by a LSI (convolution) model. The output intensity image g is related to the input object f by

$$g(x, y) = f(x, y) * h(x, y), \tag{1}$$

where $*$ denote the 2D convolution, the defocus point-spread-function (PSF) h is determined by the "circle of confusion", as illustrated in Fig. 1, the larger the defocus distance z is, the wider the PSF is, which corresponds to more blur in the image.

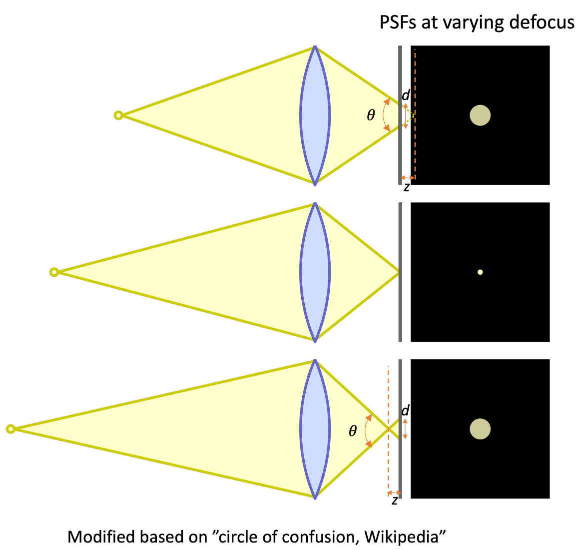

Figure 1: Circle of confusion due to defocus.

For this problem, we will consider a 2D imaging system, so the PSF can be modeled as a uniform circle with a diameter d. We will further assume that the total intensity (energy) within each circular PSF is always 1 (which implies lower intensity value as d increases). To formulate the fully discrete model, we will assume that the camera implements ideal impulse sampling. We will further use the delta-function basis for the discretization of the object.

We will assume the pixel pitch is 1μm, and the sampling rate for the object is also 1μm. We also assume that the imaging system has a unit magnification: $m = -d_o/d_i = -1$.

Write Matlab scripts to complete the following questions. Submit both your scripts as well as the output results.

The following Matlab files are provided.

a) The object, f, is in the mat-file `I1.mat`.

b) We will consider two defocus distances, including $z_1 = 0.1$mm and $z_2 = -0.2$mm. The corresponding PSFs, h_1 and h_2 are provided in `psf1.mat` and `psf2.mat`, respectively.

(1) Simulate the output images for the two cases, assuming no noise is present.

(2) What is the condition number of each case?

(3) Using the deconvolution techniques discussed in the lectures, including **a) min-norm solution, and b) Tikhonov deconvolution** to find the "best" deblurred images in each case. Can the original object be recovered with high accuracy?

The main point of this problem is to practice how to "translate" the matrix formulation into a computationally efficient code.

Specifically, for a 2D LSI system, the forward model takes the following matrix form

$$\mathbf{g} = \mathbf{A}\mathbf{f} = \frac{1}{N^2}\mathbf{W}_{2\mathrm{D}}^H \mathrm{diag}(\widehat{\mathbf{h}})\mathbf{W}_{2\mathrm{D}}\mathbf{f}, \tag{2}$$

computationally, this reads

1. $\mathbf{W}_{2\mathrm{D}}\mathbf{f} \rightarrow$ `F = fft2(f)`,
2. $\mathrm{diag}(\widehat{\mathbf{h}}) \rightarrow$ `G = fft2(h).*F` (element-wise multiplication)
3. $\mathbf{W}_{2\mathrm{D}}^H \rightarrow$ `g = ifft2(G)`.

For a 2D LSI system, the min-norm / generalized solution (GS) and Tikhonov regularization solution (TR) takes the following matrix form

$$\mathbf{f}_k = \frac{\mathbf{h}_k^*}{|\mathbf{h}_k|^2 + \mu}\mathbf{g}_k, \tag{3}$$

where GS corresponds to $\mu \rightarrow 0$. computationally, this reads

1. $\mathbf{g}_k \rightarrow$ `G = fft2(g)`,
2. $\mathbf{h}_k \rightarrow$ `H = fft2(h)`,
3. $\frac{\mathbf{h}_k^*}{|\mathbf{h}_k|^2+\mu} \rightarrow$ `F = conj(H).*G./(abs(H).^2 + mu)` (element-wise multiplication/division)
4. $\mathbf{f}_k \rightarrow$ `f = ifft2(F)`.

Next, we will investigate the effect of noise. White Gaussian noise (WGN) is commonly encountered in imaging. The commonly used assumptions is that the noise is independent of the measured intensity. The WGN assumes the mean as the noise-free image $I_{\text{noisefree}}$. We further assume the same standard deviation (std) n_{std} for all the pixels. The noisy image under this WGN assumption, I_{noisy}, can be generated using the following Matlab script:

```
I_noisy = normrnd(I_noisefree, n_std, Ny, Nx);
```

(where `Ny, Nx` are the dimensions of the image along vertical and horizontal directions)

(4) Consider different WGN levels by adjusting the values for $n_{\text{std}} = 1, 10$. Simulate the corresponding noisy image.
(5) Using the deconvolution techniques discussed in the lectures, including **a) min-norm solution, and b) Tikhonov deconvolution** to find the best deblurred images in each noisy case.

(6) How does the noise level affect the reconstruction quality? Qualitatively explain the artifacts you see in the reconstruction.

The primary artifacts should correspond to features at frequencies in which the transfer function values (eigenvalues of $\mathbf{A}$) are close to zero.

(7) How does the noise level affect the optimal value of Tikhonov regularization parameter?

In general, one should expect that the higher the noise level is, the higher the regularization parameter μ is needed. At the same time, the approximation error increases, which means the reconstruction will be more deviated from the true object.

At a fixed noise level, one should observe that the higher μ is, the approximation error increases, the noise-propagation error reduces monotonically, the total error first reduces then increases reflecting the trade-off between deviation from the true forward model and the noise amplification due to poor conditioning of $\mathbf{A}$.