Maximize cumulative future reward

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots \mid s_t = s,\ a_t = a,\ \pi\right]$$

CNN Model

Problem:

① correlations present in the sequence of observations

"small update" to Q ⟶ "significantly change" the policy

↓

"change data" distribution

Experience Replay (randomize over the data)

② Correlation between action-values Q and target value $r + \gamma \max Q(s', a')$

iterative update that adjust the action-values

Q toward target values

$Q(s, a; \theta_i)$ CNN model

$e_t = (s_t, a_t, r_t, s_{t+1})$

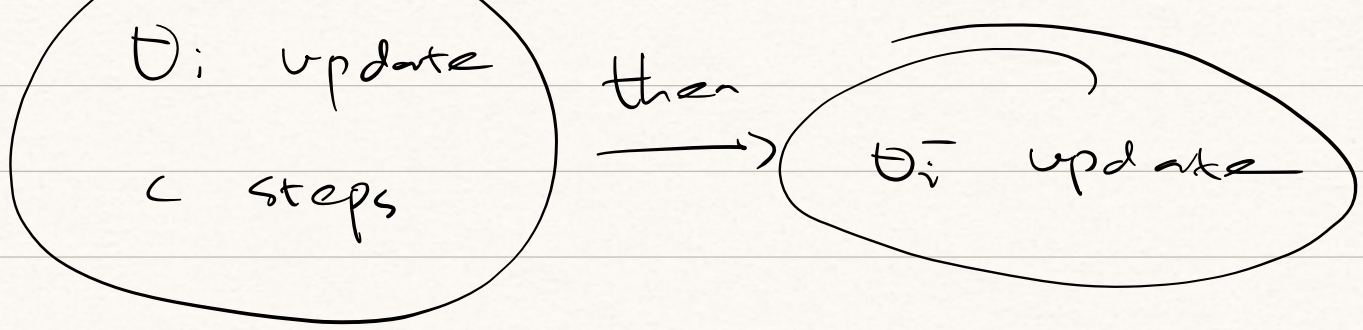
$D_t = \{e_1, e_2, \ldots, e_t\}$

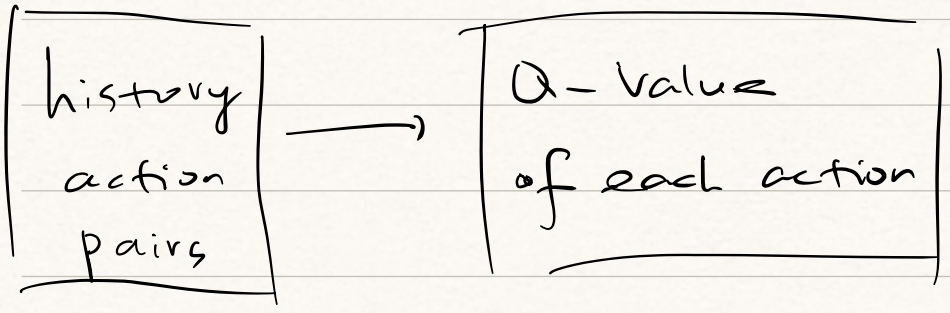
$(s, a, r, s') \sim U(D)$

$$L_i(\theta_i) = \mathbb{E}_{(s,a,r,s') \sim U(D)} \left[\left(r + \gamma \max_{a'} Q(s', a'; \theta_i^-) - Q(s, a; \theta_i) \right)^2 \right]$$

network parameters used to compute the target at iteration i

parameters of the Q-network at iteration i


θ_i update
C steps
then
θ_i^- update


history
action
pairs
Q-value
of each action

$$R_t = \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'}$$

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q^*(s',a') \mid s,a \right]$$

$$\boxed{Q(s,a;\theta) \approx Q^*(s,a)}$$

"use a function approximator to estimate the action-value function"

$$Q(s,a;\theta) \approx Q^*(s,a)$$

$$y = r + \gamma \max_{a'} Q(s',a';\theta_i^-)$$

$$L_i(\theta_i) = \mathbb{E}_{s,a,r}\left[\left(\mathbb{E}_{s'}[y \mid s,a] - Q(s,a;\theta_i) \right)^2 \right]$$

$$= \mathbb{E}_{s,a,r,s'}\left[\left(y - Q(s,a;\theta_i) \right)^2 \right] + \mathbb{E}_{s,a,r}\left[\mathbb{V}_{s'}[y] \right]$$

$$\nabla_{\theta_i} L(\theta_i) = \mathbb{E}_{s,a,r,s'}\Big[\big(r + \gamma \max_{a'} Q(s', a'; \theta_i^-)\big) - Q(s, a; \theta_i)\Big) \nabla_{\theta_i} Q(s, a; \theta_i)\Big]$$

$\theta_i^- = \theta_{i-1}$ from previous iteration

$$Q(s, a) \leftarrow Q(s, a) + \alpha\Big[r + \gamma \max_{a'} Q(s', a') - Q(s, a)\Big]$$

$$L(\theta) = \mathbb{E}\Big[\big(r + \gamma \max_{a'} Q(s', a'; \theta)\big) - Q(s, a; \theta)\big)^2\Big]$$

Algorithm 1: deep Q-learning with experience replay.

Initialize replay memory D to capacity N

Initialize action-value function Q with random weights θ

Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$

For episode = 1, M **do**

  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

  For $t = 1$,T **do**

    With probability ε select a random action a_t

    otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

    Execute action a_t in emulator and observe reward r_t and image x_{t+1}

    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D

    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D

    Set $y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^{-}) & \text{otherwise} \end{cases}$

    Perform a gradient descent step on $\left(y_j - Q(\phi_j, a_j; \theta)\right)^2$ with respect to the network parameters θ

    Every C steps reset $\hat{Q} = Q$

  End For

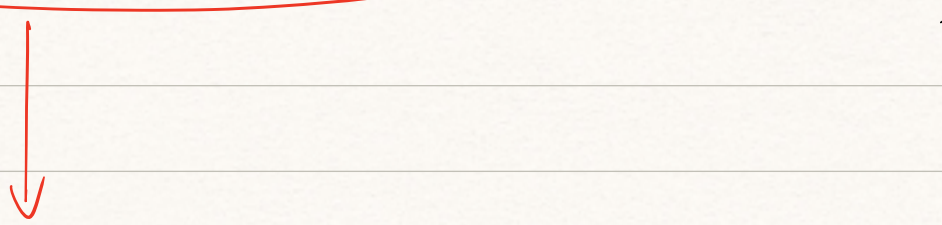
End For

$$\left[r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^{-}) \right] - \left[Q(\phi_j, a_j; \theta) \right]$$

Target Network θ^{-} — Q-network θ

$$L_i(\theta_i) = \mathbb{E}_{(s,a,r,s') \sim U(D)}$$

$$\left[\left(r + \gamma \max_{a'} Q(s', a'; \theta_i^{-}) \right) - \left(Q(s, a; \theta_i) \right) \right]^2$$



$$r + \gamma \max_{a'} Q^*(s', a')$$