





$$\text{net}_{h_1} = W_1 i_1 + W_2 i_2 + b_1$$

$$\text{out}_{h_1} = \frac{1}{1 + e^{-\text{net}_{h_1}}}$$

$$\text{net}_{h_2} = W_3 i_1 + W_4 i_2$$

$$\text{out}_{h_2} = \frac{1}{1 + e^{-\text{net}_{h_2}}}$$

$$\text{net}_{o_1} = W_5 \text{out}_{h_1} + W_6 \text{out}_{h_2}$$

$$\text{out}_{o_1} = \frac{1}{1 + e^{-\text{net}_{o_1}}}$$

$$\text{net}_{o_2} = W_7 \text{out}_{h_1} + W_8 \text{out}_{h_2}$$

$$\text{out}_{o_2} = \frac{1}{1 + e^{-\text{net}_{o_2}}}$$

Back Propagation

$$E_{\text{total}} = \sum \frac{1}{2} (\text{target} - \text{output})^2$$

$$E_{\text{total}} = E_{o1} + E_{o2}$$

$$E_{o1} = \frac{1}{2} (\text{target}_{o1} - \text{out}_{o1})^2$$

$$E_{o2} = \frac{1}{2} (\text{target}_{o2} - \text{out}_{o2})^2$$

$$\begin{aligned} \frac{\partial E_{\text{total}}}{\partial w_5} &= \frac{\partial E_{o1}}{\partial \text{out}_{o1}} \times \frac{\partial \text{out}_{o1}}{\partial \text{net}_{o1}} \times \frac{\partial \text{net}_{o1}}{\partial w_5} \\ &= \frac{1}{2} \times 2 (\text{target}_{o1} - \text{out}_{o1}) \times (-1) \\ &\quad \times \text{out}_{o1} (1 - \text{out}_{o1}) \\ &\quad \times \text{out}_{h1} \\ &= -(\text{target}_{o1} - \text{out}_{o1}) \times \text{out}_{o1} (1 - \text{out}_{o1}) \times \text{out}_{h1} \\ w_5^+ &= w_5 - \eta \times \frac{\partial E_{\text{total}}}{\partial w_5} \end{aligned}$$

$$\frac{\partial \bar{E}_{total}}{\partial W_1} = \frac{\partial \bar{E}_{total}}{\partial out_{h1}} \times \frac{\partial out_{h1}}{\partial net_{h1}} \times \frac{\partial net_{h1}}{\partial W_1}$$

$$= \left(\frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}} \right) \times \frac{\partial out_{h1}}{\partial net_{h1}} \times \frac{\partial net_{h1}}{\partial W_1}$$

$$= \left(\frac{\partial E_{o1}}{\partial net_{o1}} \times \frac{\partial net_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial net_{o2}} \times \frac{\partial net_{o2}}{\partial out_{h1}} \right) \times \frac{\partial out_{h1}}{\partial net_{h1}} \times \frac{\partial net_{h1}}{\partial W_1}$$

$$W_1^+ = W_1 - \eta \times \frac{\partial \bar{E}_{total}}{\partial W_1}$$

