Transitive Vertex Alignment Method

Graphs G : (n vertices $\times$ k dim)

Vectorial representation prototype representation

$$\arg\min_{\Omega} \sum_{j=1}^{M} \sum_{R_i^k \in C_j} \| R_i^k - \mu_j^k \|^2$$

$\Omega = (C_1, C_2, \dots, C_M)$ M clusters

μ_j^k : Mean of vertex representations of C_j

k-level affinity matrix (w.r.t. each graph)

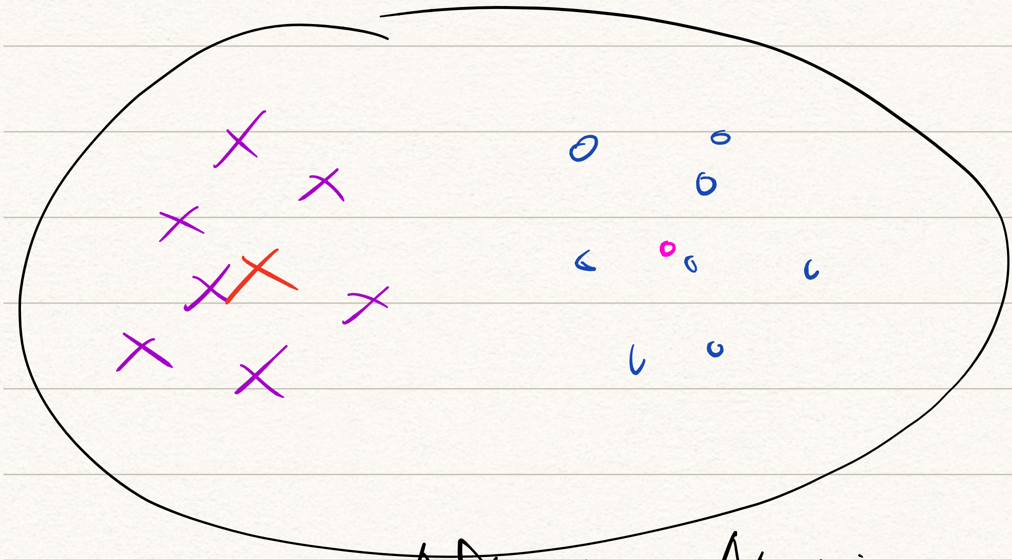
$$A_p^k(i,j) = \| R_{p;i}^k - \mu_j^k \|_2$$

$(|V_p| \times M)$

if $A_p^k(i,j)$ is the smallest one in row i)
Vectorial representation $R_{p;i}^k$ of $v \in V_p$ is aligned to the j-th prototype representation

$\mu_j^k \in \mathbb{R}^k$

(vertex V_i is aligned to j-th prototype representation)



Affinity Matrix

k-level correspondence matrix $C_p^k \in \{0,1\}^{|V_p| \times N}$

$$C_p^k(i,j) = \begin{cases} 1, & \text{if } A_p^k(i,j) \text{ is the smallest in row } i \\ 0, & \text{otherwise} \end{cases}$$

V_p → same prototype representation

↕ also aligned

V

V_q

Aligned Grid Structures

$$\boxed{\begin{array}{l} G_p(V_p, E_p, \tilde{A}_p) \\ X_p \in \mathbb{R}^{n \times c} \end{array}}$$

k-level vertex correspondence matrix

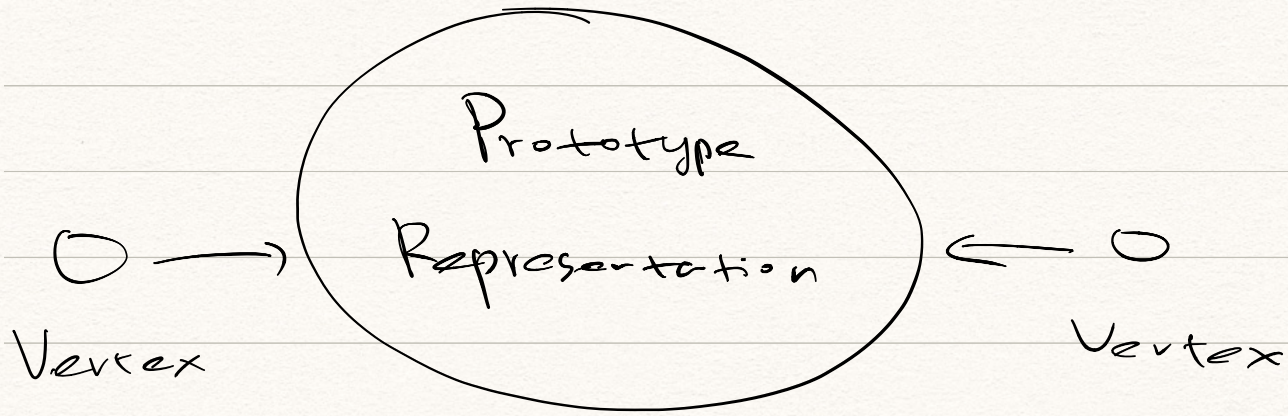
C_p^k

k-level aligned vertex feature matrix

$$\boxed{\bar{X}_p^k = (C_p^k)^T X_p}$$

k-level aligned vertex adjacency matrix

$$\boxed{\bar{A}_p^k = (C_p^k)^T (\tilde{A}_p)(C_p^k)}$$



$$G_R(V_R, E_R)$$

$$S(\mu_j^k, \mu_k^k) = \exp\left(-\frac{\|\mu_j^k - \mu_k^k\|_2}{k}\right)$$

DB (depth-based) representations :

(Vectorial vertex representations)

Aligned vertex grid structure :

$$\bar{X}_p = \sum_{k=1}^{L} \frac{X_p^k}{L}$$

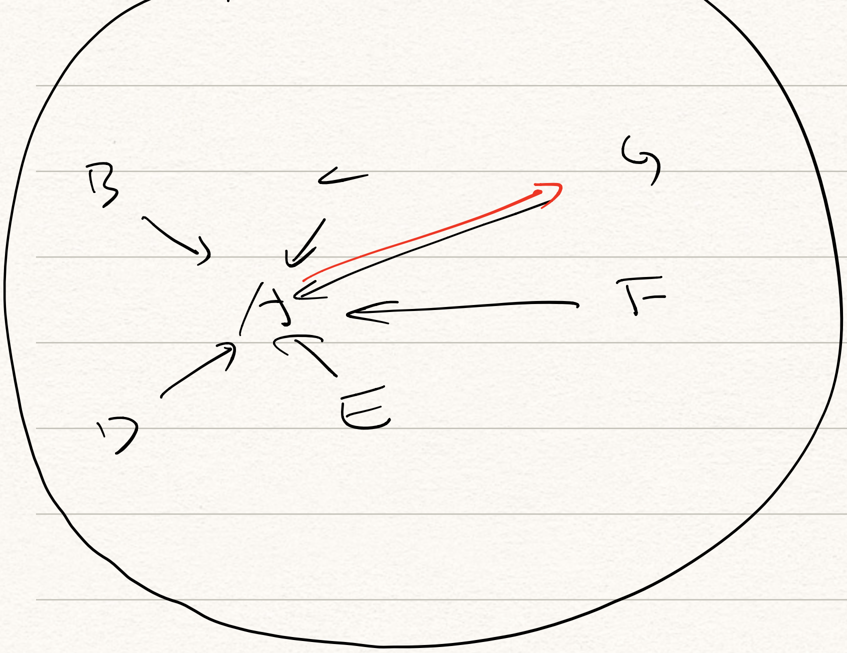
$$\bar{A}_p = \sum_{k=1}^{L} \frac{\bar{A}_p^k}{L}$$

Undirected graph

$$\bar{D}_p^D(i) = \sum_j \bar{A}_p^D(i,j)$$

$$P(i) = \frac{\bar{D}_p^D(i)}{\sum_j \bar{D}_p^D(j)}$$

$$A_p^D(V_i, V_j) = \begin{cases} \bar{A}_p(V_i, V_j), & \text{if } P(i) \le P(j) \\ 0, & \text{otherwise} \end{cases}$$



$$\frac{14+2p-f}{s}+1=10$$

$$\frac{14+2p-f}{s}=9$$

$$14+2p-f=9$$