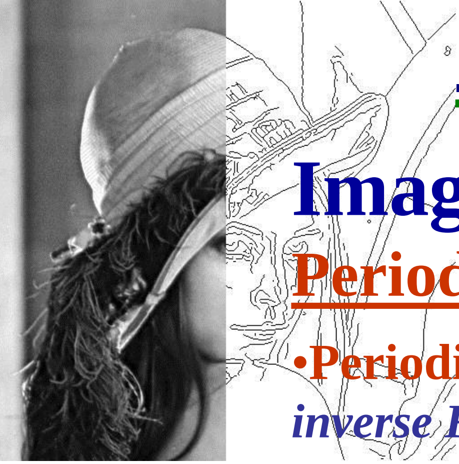


# Image Enhancement in the *Frequency Domain*

## Periodicity and the need for Padding:

•**Periodicity property of the DFT:** ***The discrete Fourier Transform and the inverse Fourier transforms are periodic. Hence:***

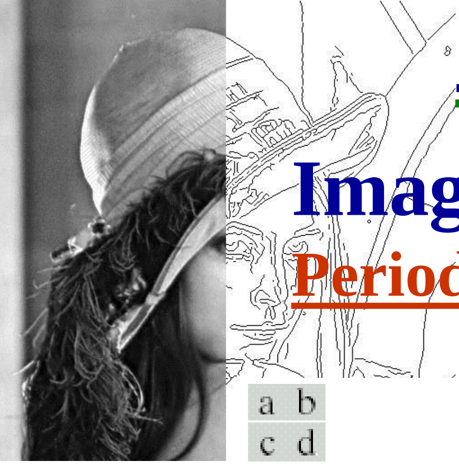
$$F(u,v) = F(u+M,v) = F(u,v+N) = F(u+M,v+N)$$

$$f(x,y) = f(x+M,y) = f(x,y+N) = f(x+M,y+N)$$

•**Conjugate Symmetry property :** ***The DFT is conjugate symmetric. The `*` indicates the conjugate operation on a complex number.***

$$F(u,v) = F^{*}(-u,-v)$$

$$|F(u,v)| = |F(-u,-v)|$$

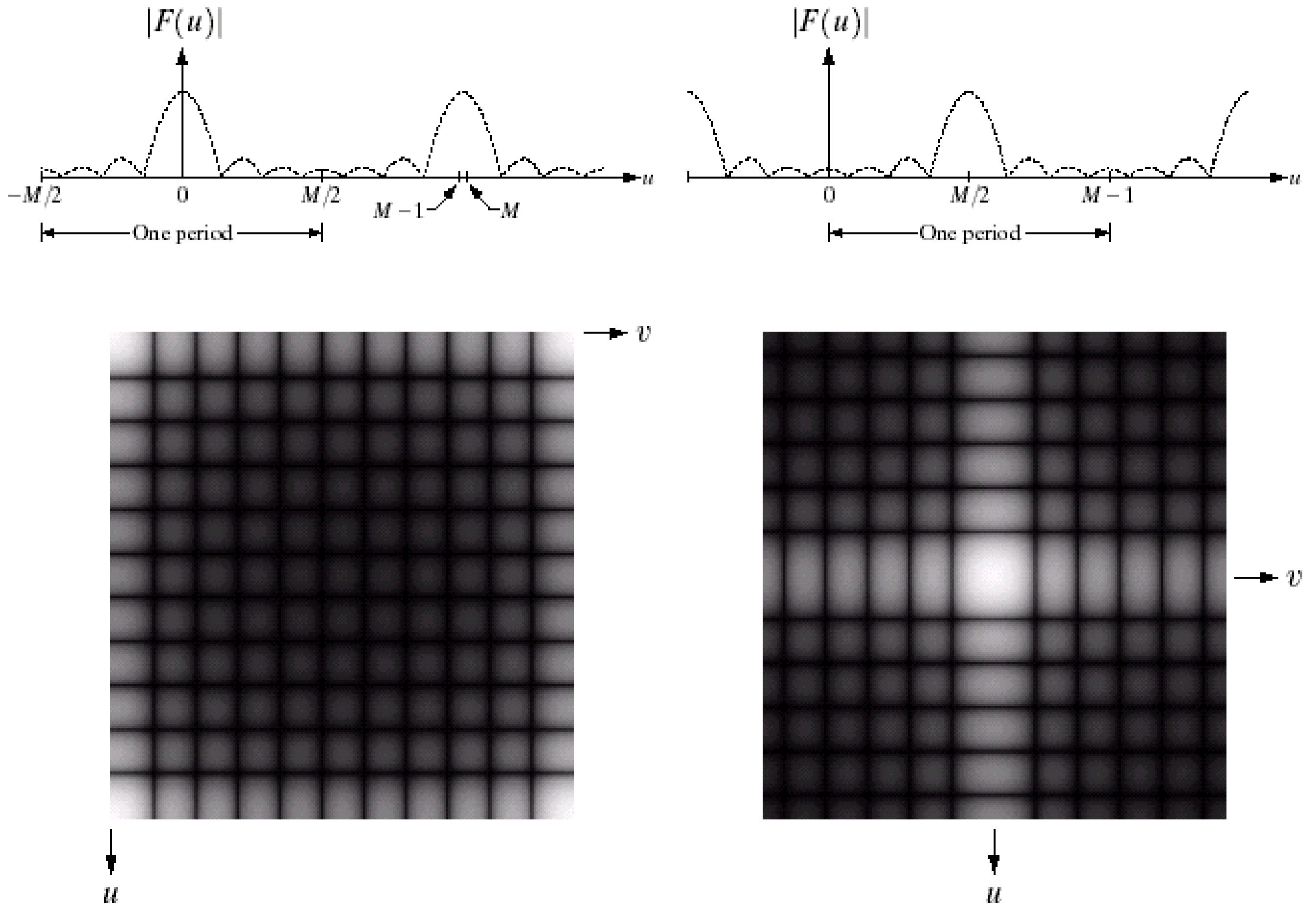


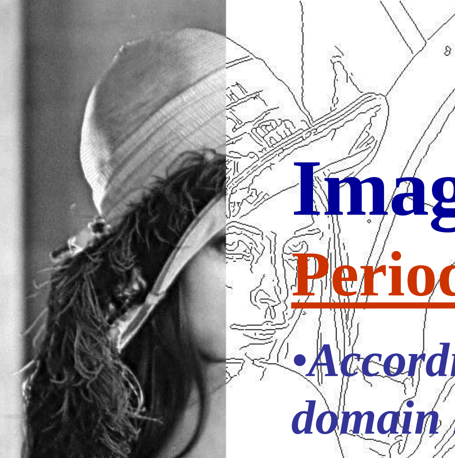
# Image Enhancement in the *Frequency Domain*

## Periodicity and the need for Padding:

a b
c d

**FIGURE 4.34**
(a) Fourier spectrum showing back-to-back half periods in the interval $[0, M-1]$.
(b) Shifted spectrum showing a full period in the same interval.
(c) Fourier spectrum of an image, showing the same back-to-back properties as (a), but in two dimensions.
(d) Centered Fourier spectrum.





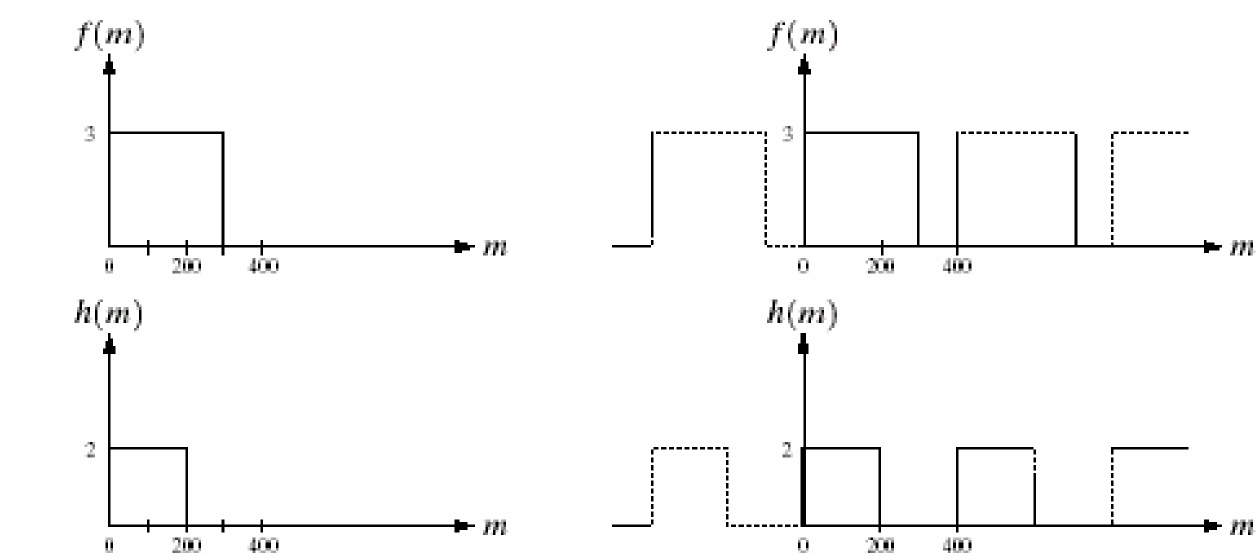
# Image Enhancement in the *Frequency Domain*

## Periodicity and the need for Padding:

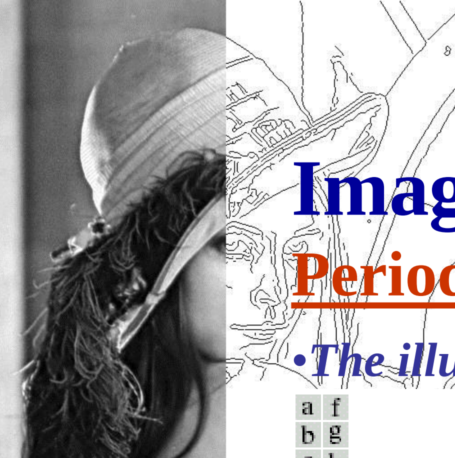
***•According to the convolution theorem, the multiplication in the frequency domain is the convolution in the spatial domain.***

***•Consider the 1D convolution of f(x) and h(x),***

$$f(x) * h(x) = \frac{1}{M}\sum_{m=0}^{M-1} f(m)h(x-m)$$




*discrete functions* *with periodicity property*



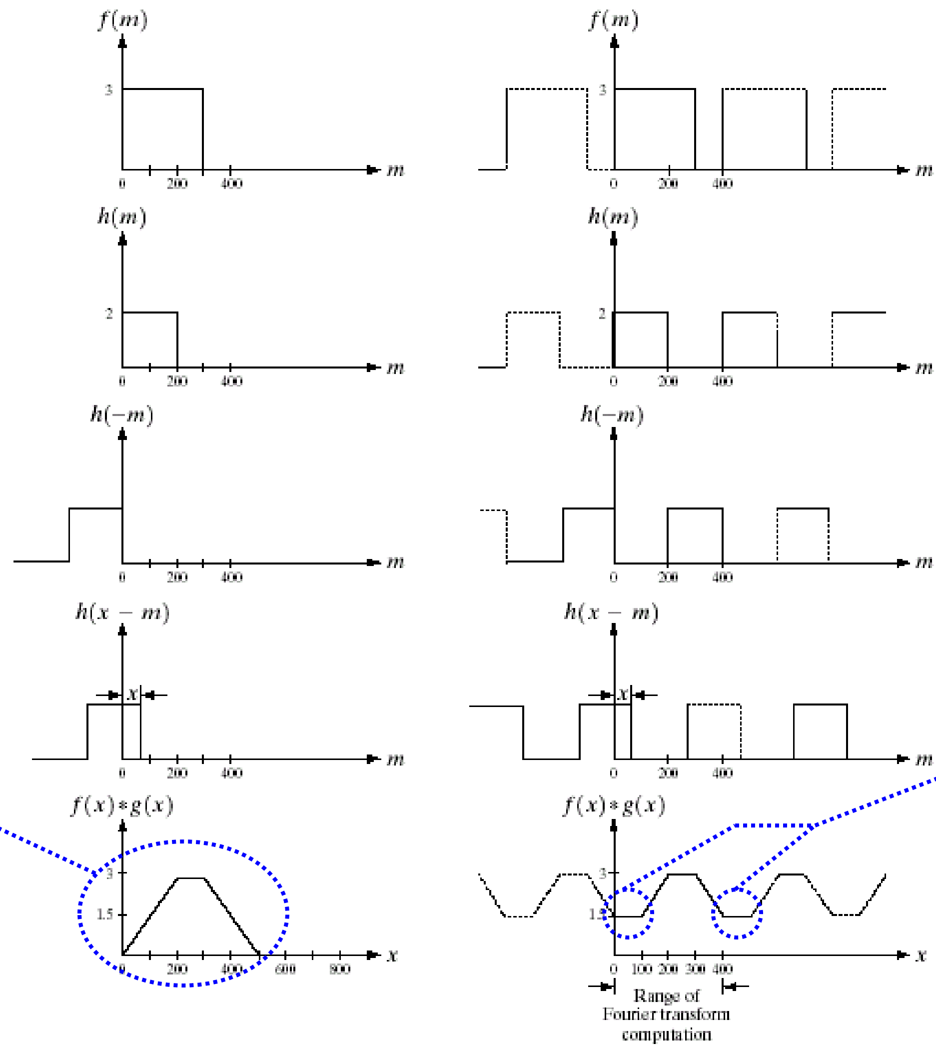
# Image Enhancement in the *Frequency Domain*

## Periodicity and the need for Padding:

• *The illustration of the 1D convolution:*

| | |
|---|---|
| a | f |
| b | g |
| c | h |
| d | i |
| e | j |

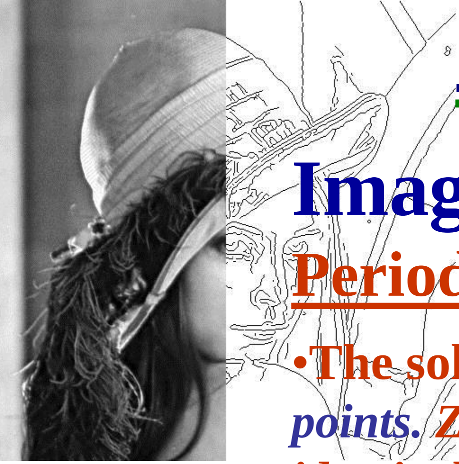
**FIGURE 4.36** Left: convolution of two discrete functions. Right: convolution of the same functions, taking into account the implied periodicity of the DFT. Note in (j) how data from adjacent periods corrupt the result of convolution.




*Correct convolution*

*Wraparound error*
*Due to periodicity*

*: Dr. Hasan Demirel, PhD*





# Image Enhancement in the *Frequency Domain*

## Periodicity and the need for Padding:

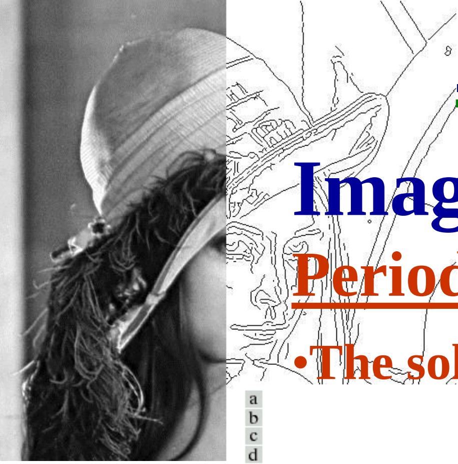
•**The solution of the wraparound error:** *Given f and h consist of A and B points. Zeros are appended to both functions so that both of the functions have identical periods, denoted by P.*

• *The 1D extended/padded functions are given by:*

$$f_e(x) = \begin{cases} f(x) & 0 \le x \le A-1 \\ 0 & A \le x \le P \end{cases}$$

$$h_e(x) = \begin{cases} h(x) & 0 \le x \le B-1 \\ 0 & B \le x \le P \end{cases}$$

$$P \ge A + B - 1$$



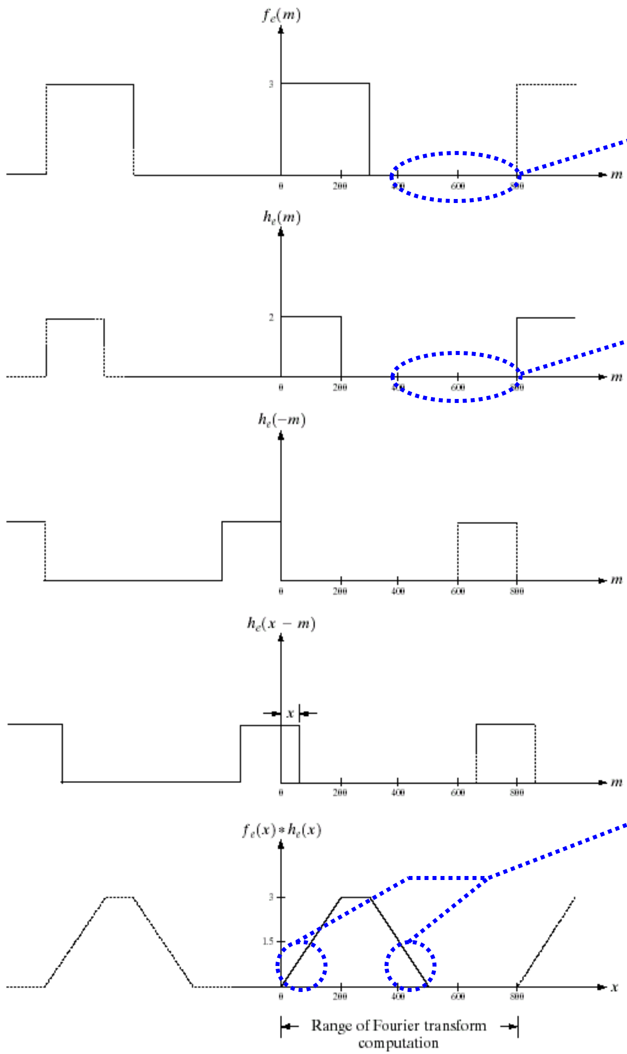
# Image Enhancement in the *Frequency Domain*

## Periodicity and the need for Padding:

- The solution of the wraparound error: *extended/padded functions.*

a
b
c
d
e

**FIGURE 4.37** Result of performing convolution with extended functions. Compare Figs. 4.37(e) and 4.36(e).




*Extended/padded region in f(x)*

*Extended/padded region in h(x)*

*The padding provides enough separation of the 2 signals. So there is no Wraparound error problem*

**Prepared By:** *Dr. Hasan Demirel, PhD*

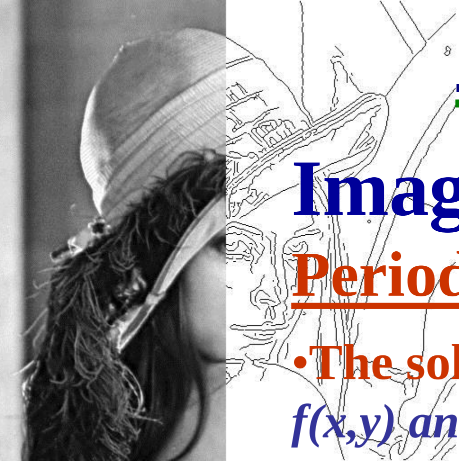
# Image Enhancement in the *Frequency Domain*

## Periodicity and the need for Padding:

•**The solution of the wraparound error:** ***The 2D extended/padded functions f(x,y) and h(x,y) with sizes AxB and CxD respectively can be defined by.***

$$f_e(x,y)=\begin{cases} f(x,y) & 0\le x\le A-1 \text{ and } 0\le y\le B-1 \\ 0 & A\le x\le P \text{ or } B\le y\le Q \end{cases}$$

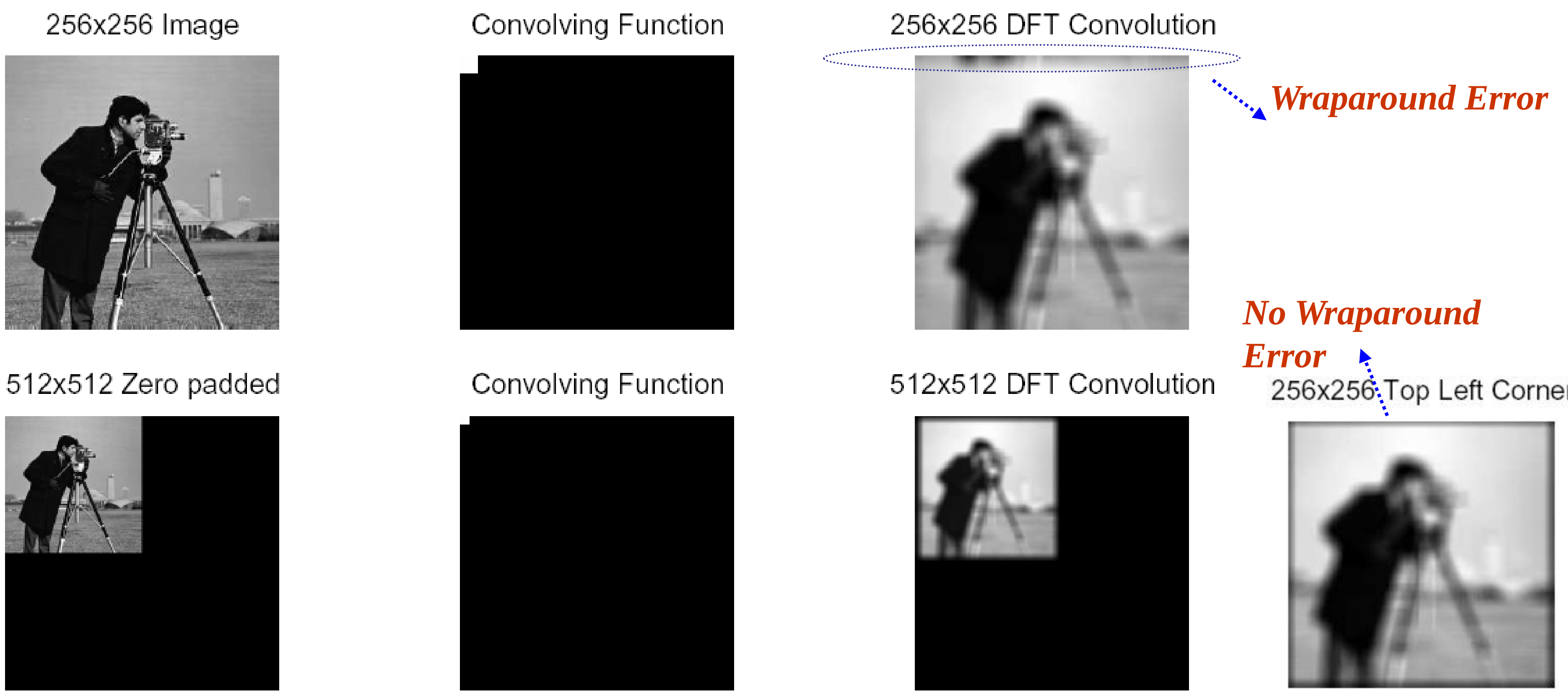
$$h_e(x,y)=\begin{cases} h(x,y) & 0\le x\le C-1 \text{ and } 0\le y\le D-1 \\ 0 & C\le x\le P \text{ or } D\le y\le Q \end{cases}$$

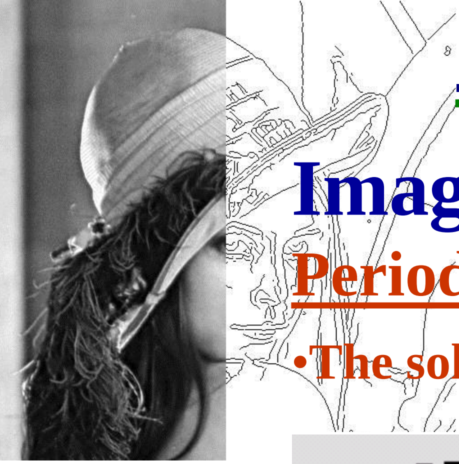


# Image Enhancement in the *Frequency Domain*

## Periodicity and the need for Padding:

•**The solution of the wraparound error:** *The 2D extended/padded functions f(x,y) and h(x,y) with sizes AxB and CxD respectively can be defined by.*



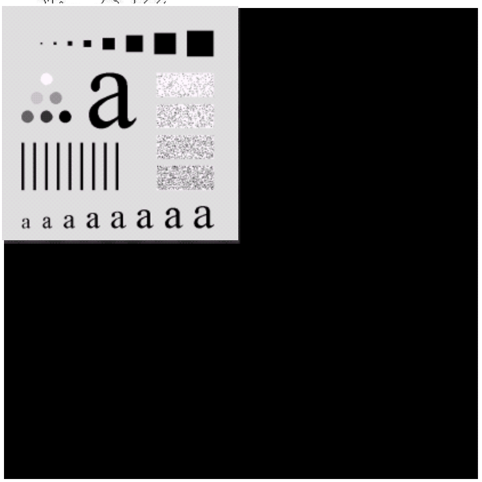


# Image Enhancement in the *Frequency Domain*

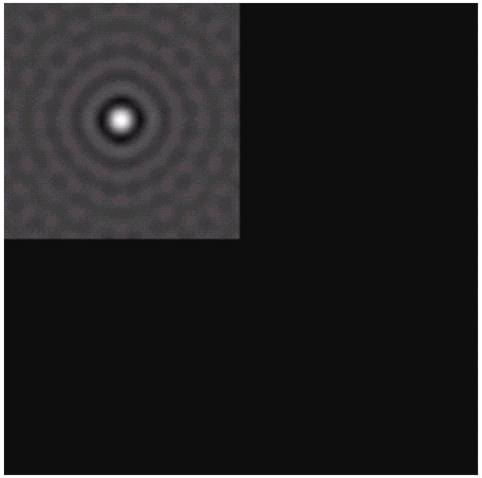
## Periodicity and the need for Padding:

•The solution of the wraparound error:

*Padded input image f(x,y)*

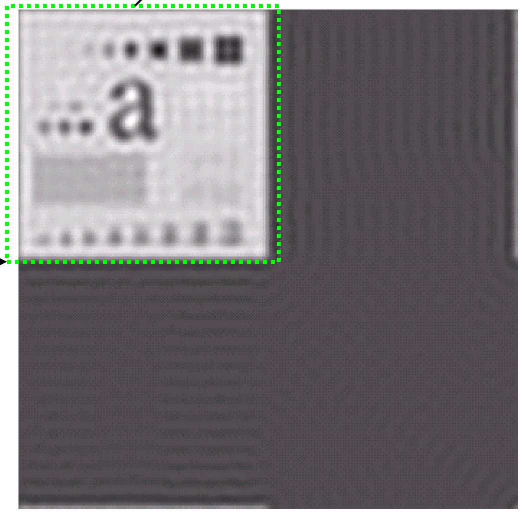


*Padded spatial Filter h(x,y)*



** (Convolution)*

*Needs to be cropped for the final answer*



*g(x,y)= f(x,y) * h(x,y)*

# Image Enhancement in the *Frequency Domain*

## Convolution and Correlation:

•**Convolution:** ***Discrete convolution of two functions f(x,y) and h(x,y) with sizes MxN is defined by.***

$$f(x,y)*h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f(m,n)h(x-m,y-n)$$

$$f(x,y)*h(x,y)\Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y)\Leftrightarrow F(u,v)*H(u,v)$$

•**Correlation:** ***Discrete correlation of two functions f(x,y) and h(x,y) with sizes MxN is defined by.***

$$f(x,y)\circ h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f^*(m,n)h(x+m,y+n)$$

$$f(x,y)\circ h(x,y)\Leftrightarrow F^*(u,v)H(u,v)$$

$$f^*(x,y)h(x,y)\Leftrightarrow F(u,v)\circ H(u,v)$$

# Image Enhancement in the *Frequency Domain*

## Convolution and Correlation:

- **Convolution:** ***The most important application of the convolution is the filtering in the spatial and frequency domains.***

- **Correlation:** ***The principal application of the correlation is matching.***
- ***In matching, f(x,y) is the image containing objects/regions and the h(x,y) is the object/reqion that we are trying to locate.***
- ***h(x,y) is called the template.***
- ***If there is a match the correlation of the two functions will be maximum at the location where the template h(x,y) finds the highest similarity in function f.***

- **Note*****: Image padding is also required in Correlation as well as in convolution.***

# Image Enhancement in the *Frequency Domain*

## Convolution and Correlation:

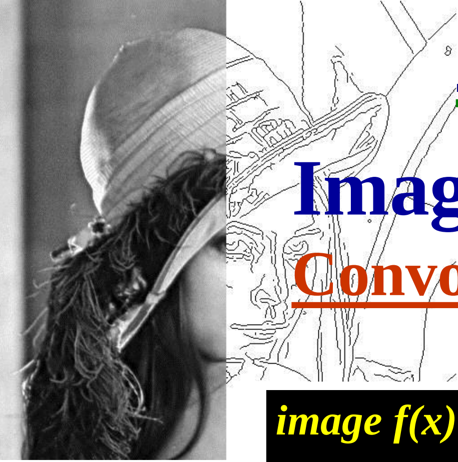
•**Correlation:** ***Cross correlation* is the special term given to the correlation of two different images.**

• ***In autocorrelation both images are identical, where:***

$$\boxed{f(x,y)\circ f(x,y) \Leftrightarrow F^{*}(u,v)F(u,v)=\left|F(u,v)\right|^{2}}$$

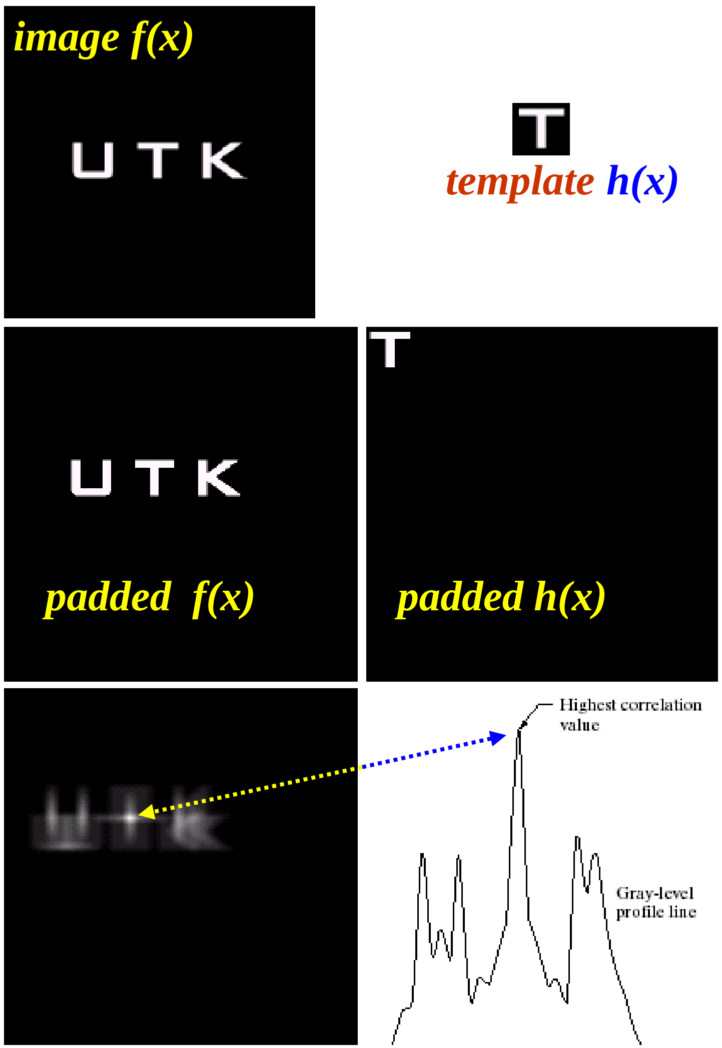
•***The spatial autocorrelation is identical to the power spectrum in the frequency domain.***

$$\boxed{\left|f(x,y)\right|^{2} \Leftrightarrow F(u,v)\circ F(u,v)}$$



# Image Enhancement in the *Frequency Domain*

## Convolution and Correlation:




a b
c d
e f

**FIGURE 4.41** (a) Image. (b) Template. (c) and (d) Padded images. (e) Correlation function displayed as an image. (f) Horizontal profile line through the highest value in (e), showing the point at which the best match took place.

- **Correlation:** *The cross correlation of a template h(x,y) with an input image f(x,y). This process is also known as the template matching.*

- **Note:** *The padded images are transformed into the Frequency Domain with DFT.*
- *Complex conjugate of one of the images is multiplied with the other.*
- *The resulting function is inverse transformed.*

**Prepared By:** *Dr. Hasan Demirel, PhD*

# Image Restoration

• *Restoration is to attempt to reconstruct or recover an image that has been **degraded** by using a **a priori knowledge** of the degradation .*

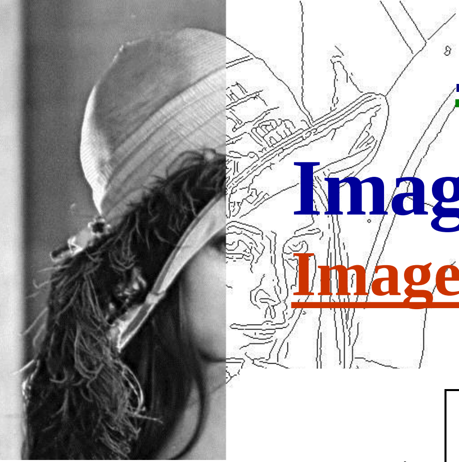
•*The restoration approach involves the **modeling of the degradation** and applying the **reverse process** to recover the original image.*

• **Image Degradation Model:** *Given some knowledge about the degradation function h(x,y), and some knowledge about the additive noise $\eta(x,y)$, the degraded output image g(x,y) can be obtained by:*

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

• *Frequency domain representation can be modeled by:*

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$



# Image Restoration

## Image Degradation/Restoration Process

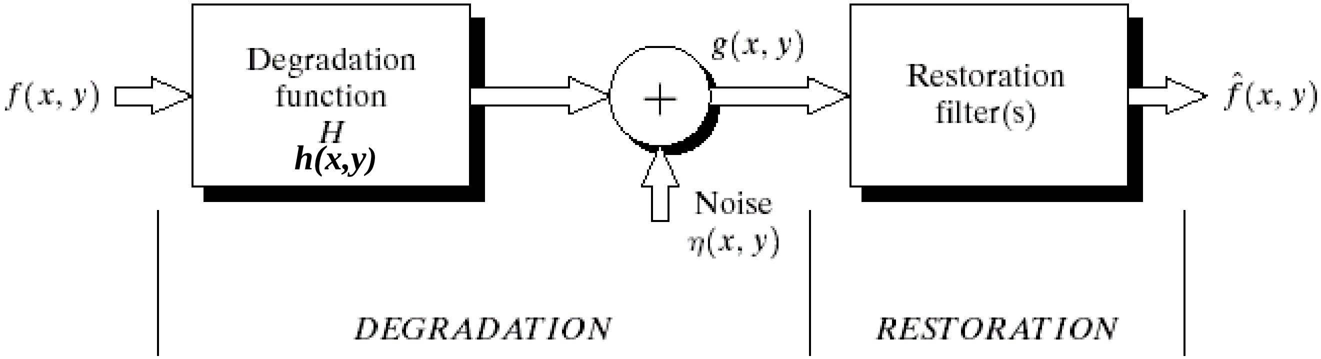



**FIGURE 5.1** A model of the image degradation/restoration process.

• **Noise Models:** ***Image acquisition (digitization) and the transmission processes are the primary sources of the noise.***

***Assumption: In almost all considerations we will assume that the noise is uncorrelated with the image, which means that there is no correlation between the pixel values of the image and the noise components.***

# Image Restoration

## Noise models: Some important Noise PDFs

*•The statistical behaviors of the noise components can be considered as the **random variables**, characterized by the probability density function (**PDF**).*

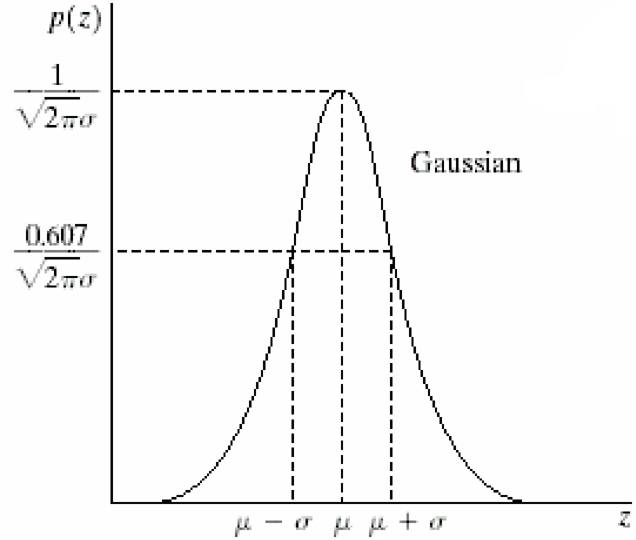
*•The following noise PDFs are common for image processing applications:*

**Gaussian Noise:**

*•Gaussian Noise models are frequently used in practice. Because this type of noise model is easily tractable in both spatial and frequency domains.*

*•The PDF of the Gaussian random variable, z is given by:*

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$

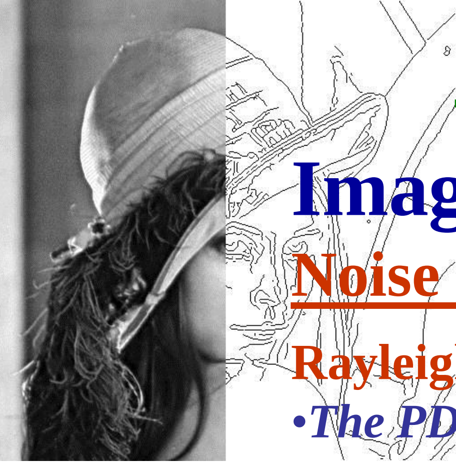



*•Where z represents gray level, $\mu$ is the average value of* **z,**

*•and the $\sigma$ is the standard deviation and $\sigma^2$ is the variance.*

*•Approximately 68% of the values will be in the range of*

• **[($\mu-\sigma$), ($\mu+\sigma$)]** *and about 95% will be in the range of* **[($\mu-2\sigma$), ($\mu+2\sigma$)].**



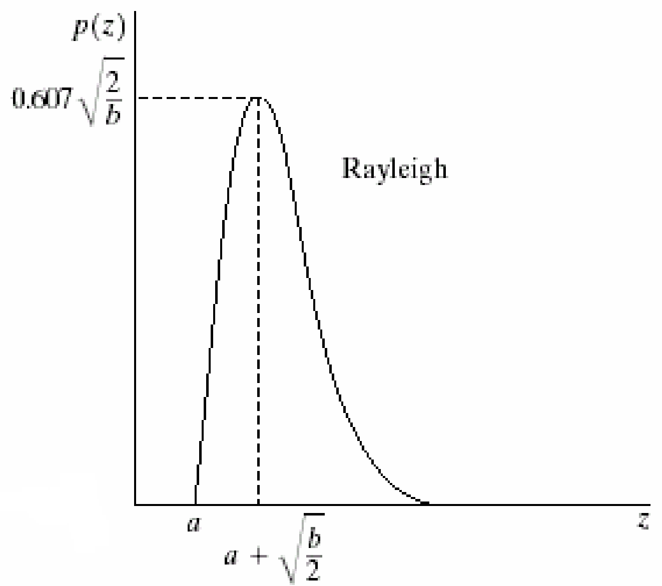
# Image Restoration

## Noise models: Some important Noise PDFs

**Rayleigh Noise:**

•*The PDF of Rayleigh noise is given by:*

$$p(z) = \begin{cases} \dfrac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$




•*The mean of the density is:*

$$\mu = a + \sqrt{\pi b / 4}$$

•*The variance of the density is:*

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

•*Because of its skewed distribution, it can be useful for approximating the distribution of the images characterized by skewed histograms.*

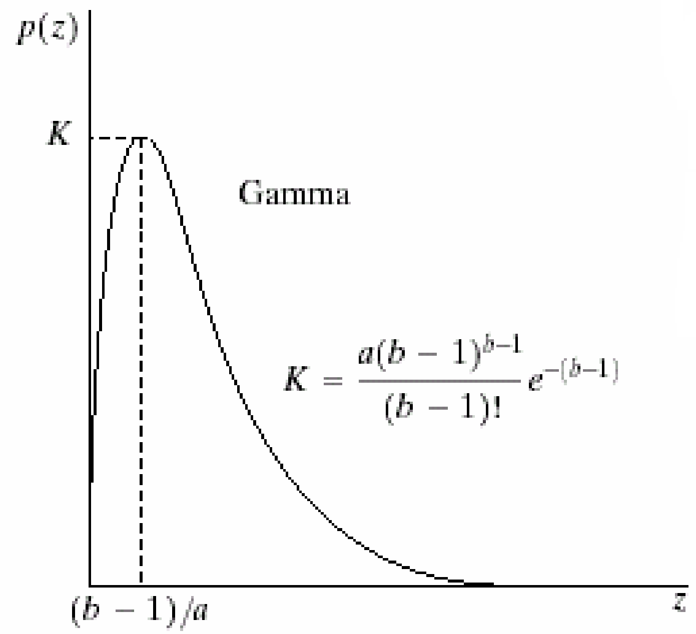
# Image Restoration

## Noise models: Some important Noise PDFs

**Erlang (Gamma) Noise:**

• *The PDF of Gamma noise is given by:*

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & for \quad z \geq 0 \\ 0 & for \quad z < 0 \end{cases}$$



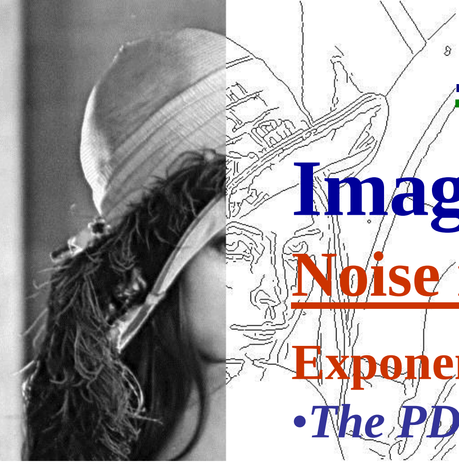

• *The mean of the density is:*

$$\mu = \frac{b}{a}$$

• *The variance of the density is:*

$$\sigma^2 = \frac{b}{a^2}$$

Prepared By: *Dr. Hasan Demirel, PhD*



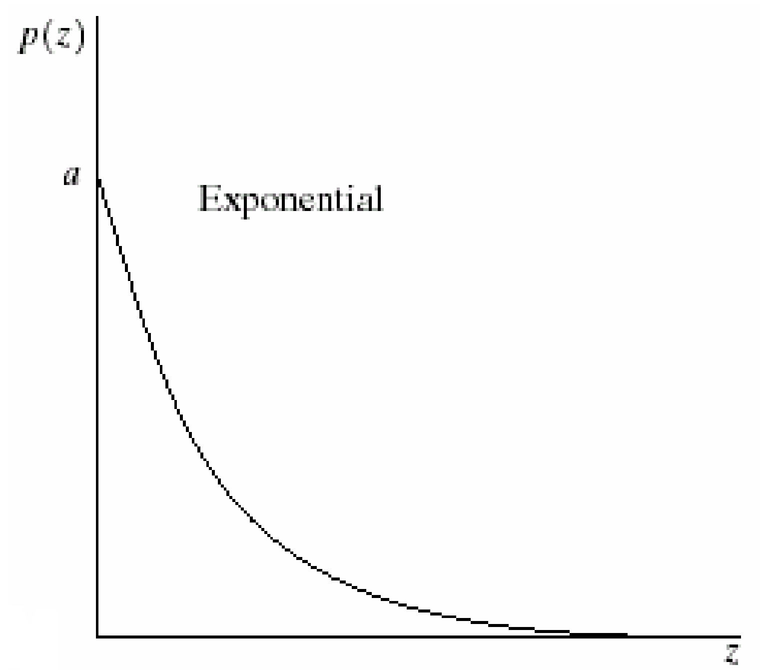
# Image Restoration

## Noise models: Some important Noise PDFs

**Exponential Noise:**

• *The PDF of Exponential noise is given by:*

$$p(z) = \begin{cases} ae^{-az} & for \quad z \geq 0 \\ 0 & for \quad z < 0 \end{cases}$$



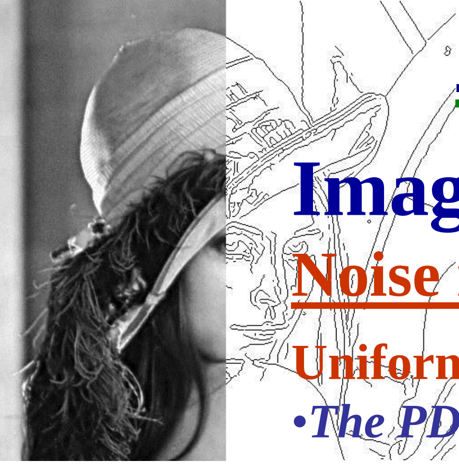

• *The mean of the density is:*

$$\mu = \frac{1}{a}$$

• *The variance of the density is:*

$$\sigma^2 = \frac{1}{a^2}$$

• *The PDF of the Exponential noise is the special case of the Gamma PDF, where b=1.*



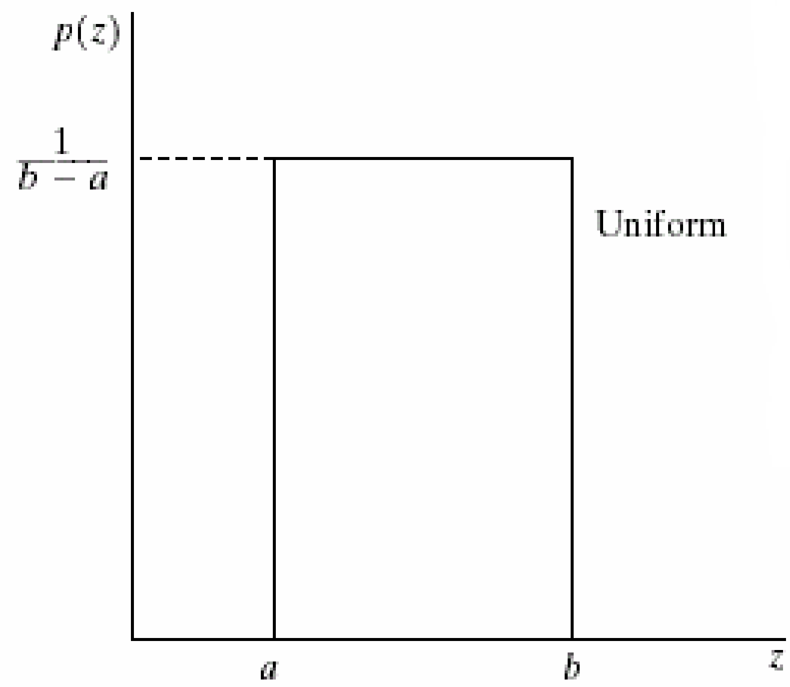
# Image Restoration

## Noise models: Some important Noise PDFs

**Uniform Noise:**

•*The PDF of Uniform noise is given by:*

$$p(z)=\begin{cases}\dfrac{1}{b-a} & if \quad a\le z\le b\\ 0 & otherwise\end{cases}$$

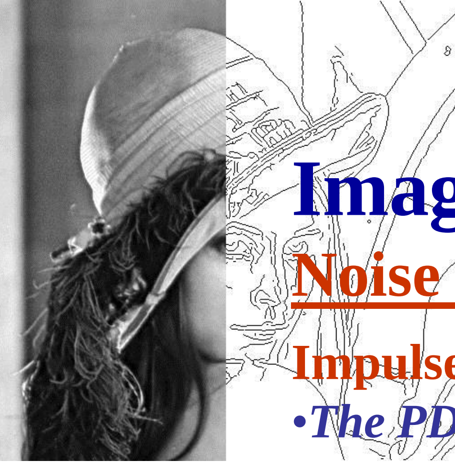



•*The mean of the density is:*

$$\mu=\frac{a+b}{2}$$

•*The variance of the density is:*

$$\sigma^2=\frac{(b-a)^2}{12}$$



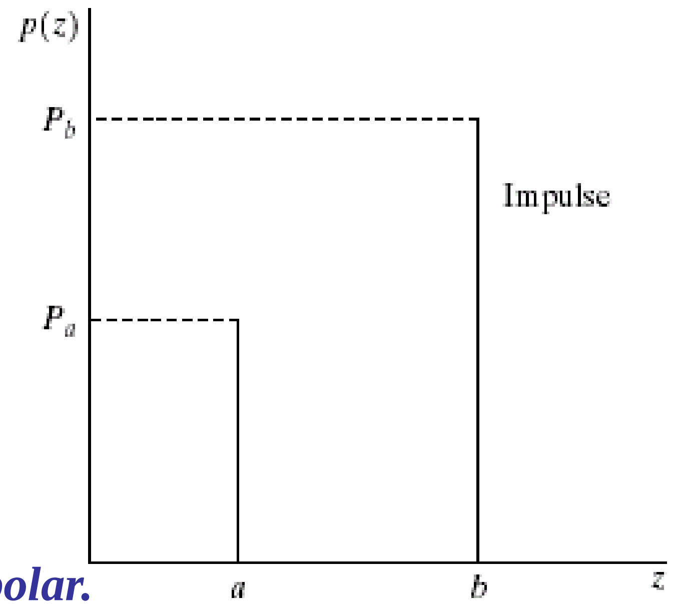
# Image Restoration

## Noise models: Some important Noise PDFs

**Impulse (salt-and-peper) Noise:**

•*The PDF of (bipolar) impulse noise is given by:*

$$p(z) = \begin{cases} P_a & for \quad z = a \\ P_b & for \quad z = b \\ 0 & otherwise \end{cases}$$

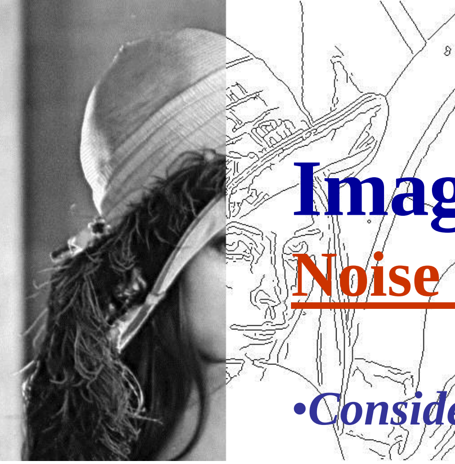



•*If b>a then, the gray-level b will appear as a light dot in the image.*

•*Otherwise the a will appear like a dark dot.*

•*If $P_a$ or $P_b$ is zero, than impulse noise is called unipolar.*

•*Typically impulse noise can be positive or negative. The negative impulse is quantized as zero which is black (pepper) and the positive impulse is quantized as max intensity value (i.e. 255) which is white (salt).*



# Image Restoration

## Noise models: Some important Noise PDFs

•*Consider the following image which contains 3 gray levels.*

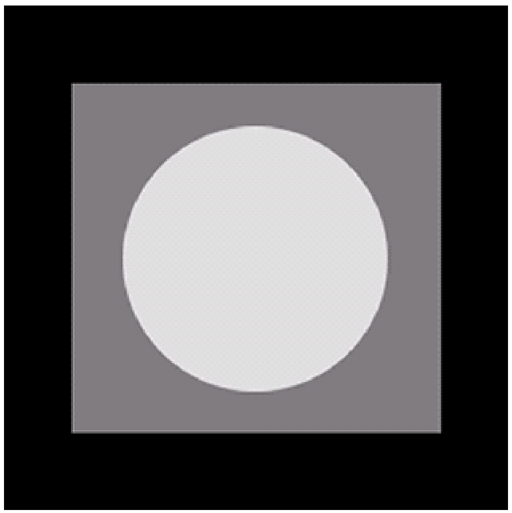
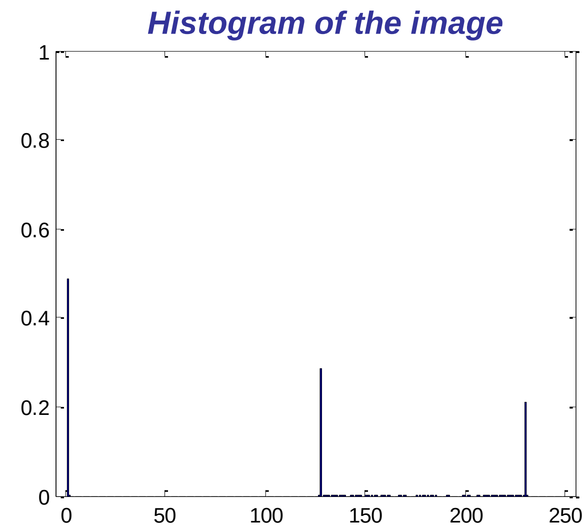
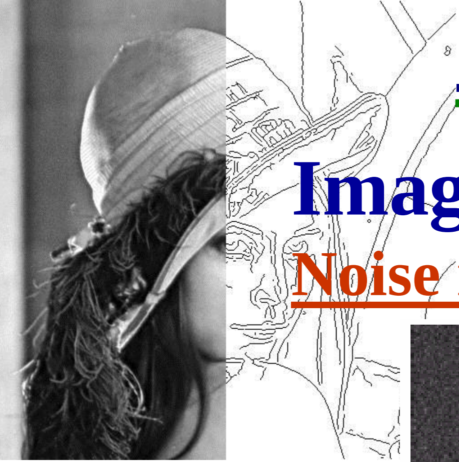


**FIGURE 5.3** Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.




•*If the only degradation in the image is additive noise then the image degraded by the additive noise is given by. Degradation function h(x,y) is ignored.*

$$g(x,y) = f(x,y) + \eta(x,y)$$

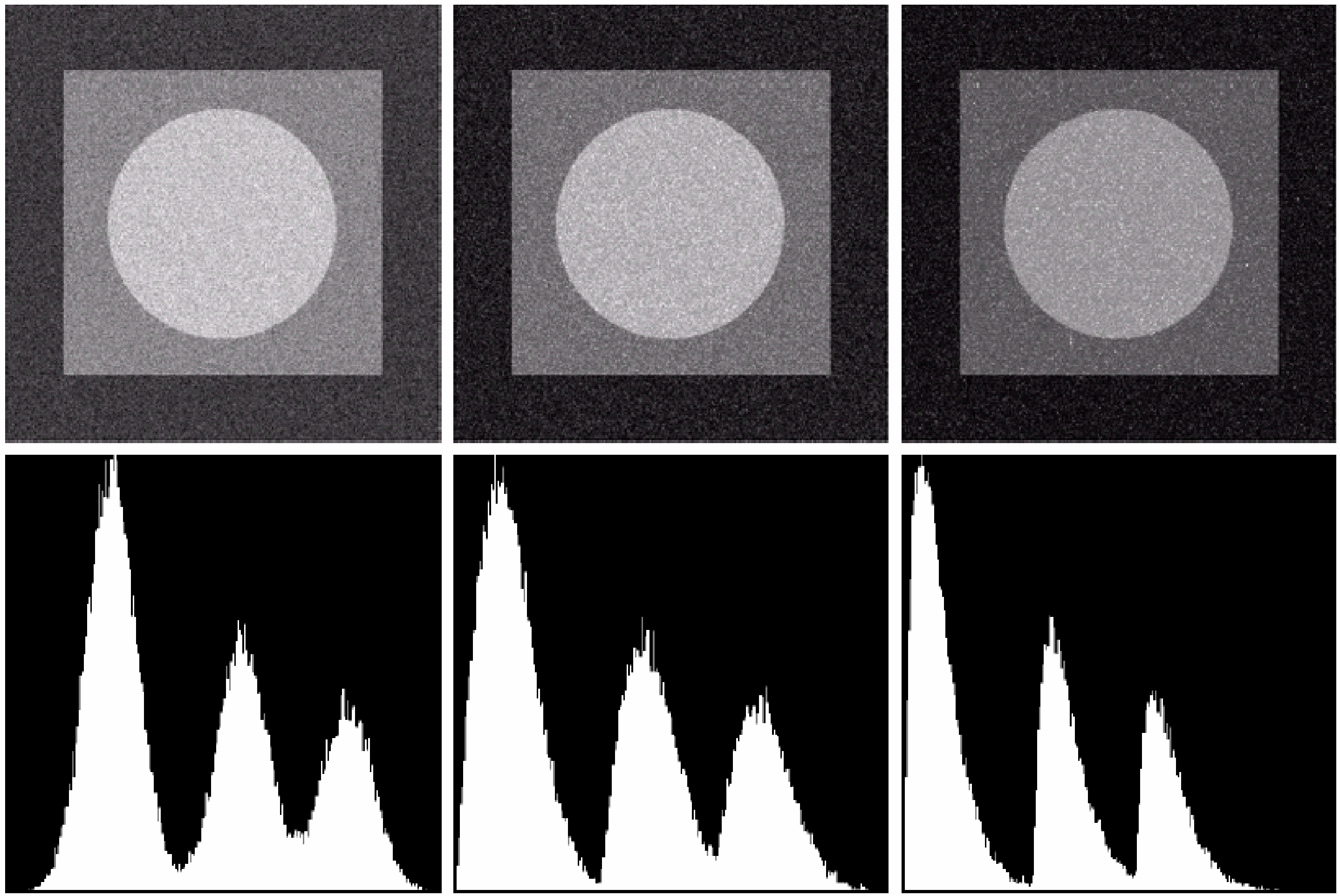


# Image Restoration

## Noise models: Some important Noise PDFs

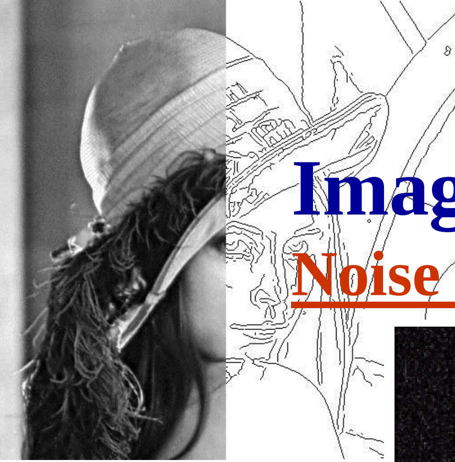
*Noisy Image*

*Histogram of the Noisy Image*



**Gaussian** **Rayleigh** **Gamma**

**Prepared By:** ***Dr. Hasan Demirel, PhD***

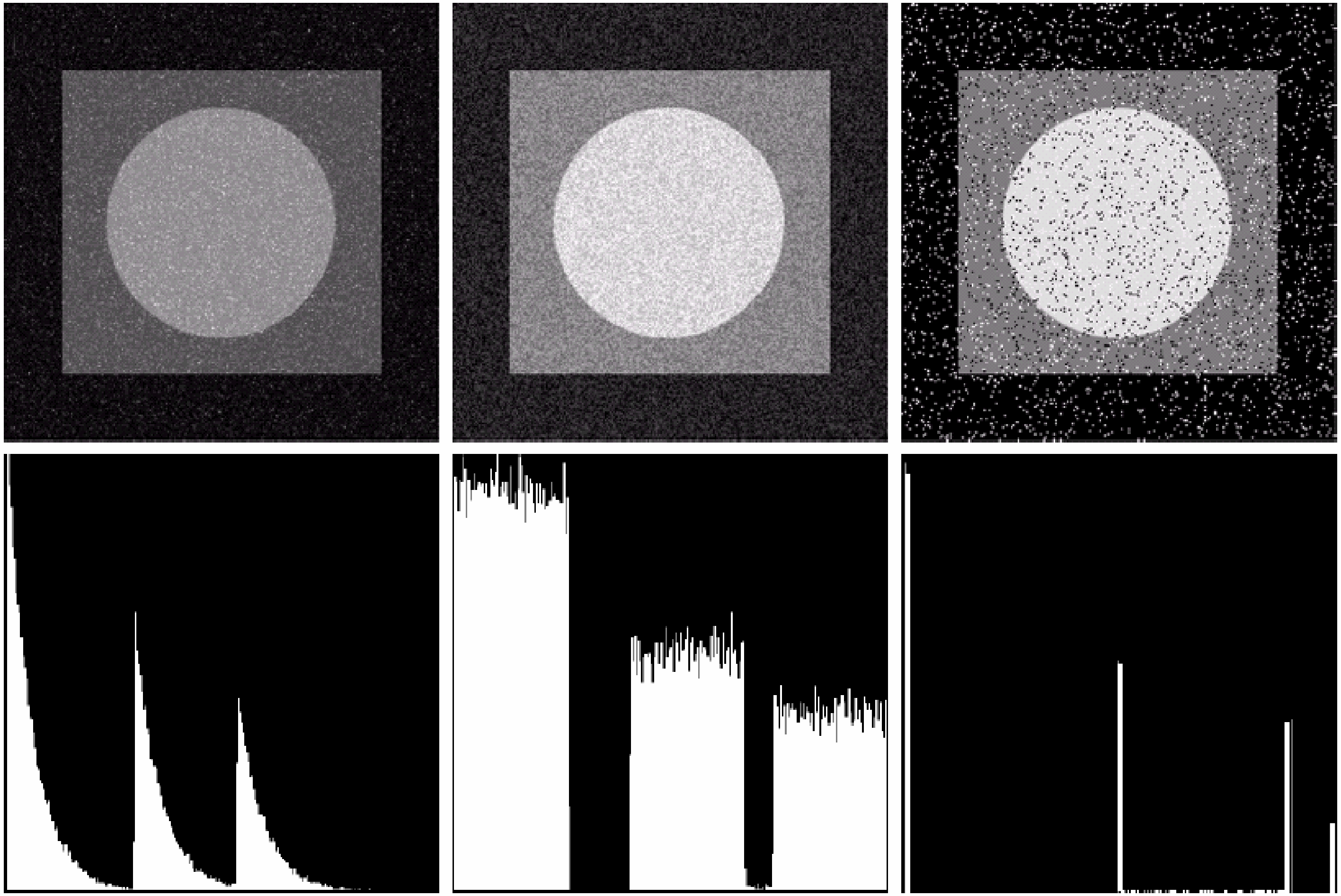


# Image Restoration

## Noise models: Some important Noise PDFs

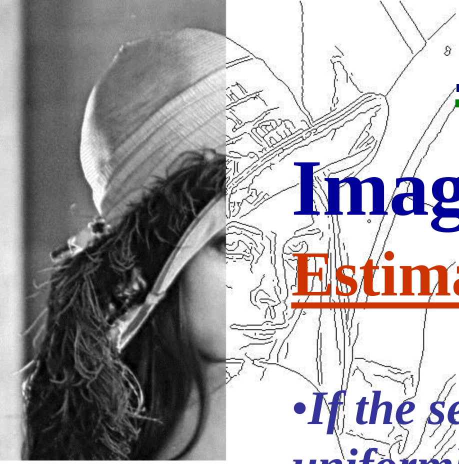
*Noisy Image*

*Histogram of the Noisy Image*



**Exponential** **Uniform** **Salt & Pepper**

**Prepared By:** ***Dr. Hasan Demirel, PhD***

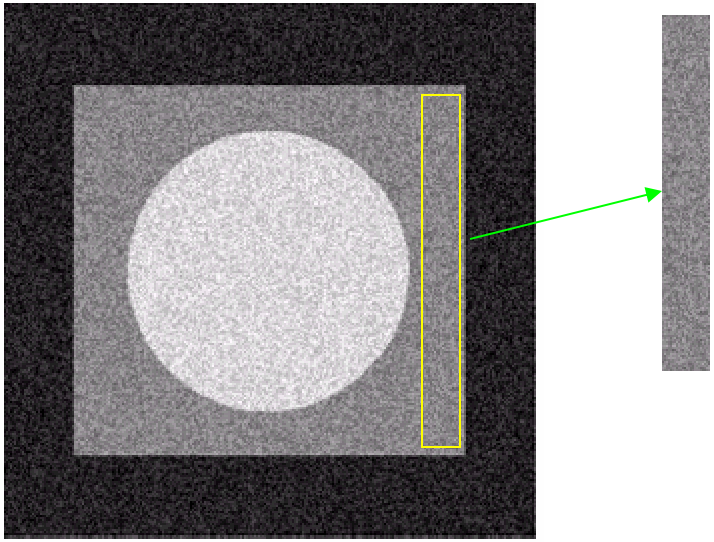


# Image Restoration

## Estimation of Noise Parameters models:

•*If the sensor device (i.e. camera) is available, then the a solid grey area which is uniformly illuminated is acquired. Then, the noise can be estimated by analyzing the histogram of this area.*

•*If the sensor is not available and already generated images are to be considered, then PDF parameters can be obtained from small patches of reasonable constant gray level.*



•*This image strip can be used to analyze the histogram.*

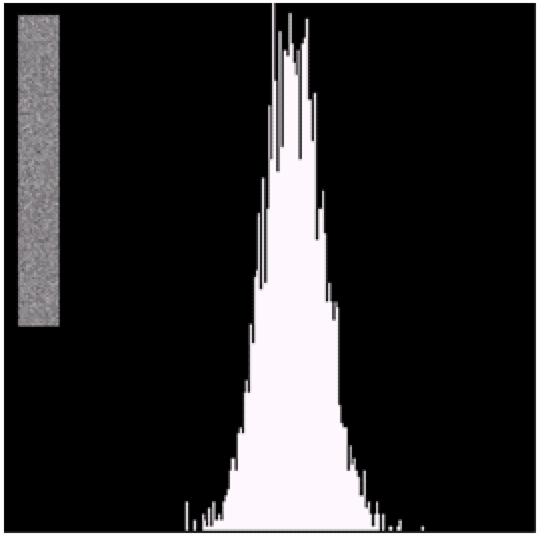
•*Then the mean and variance approximations can be extracted from this image strip.*

# Image Restoration

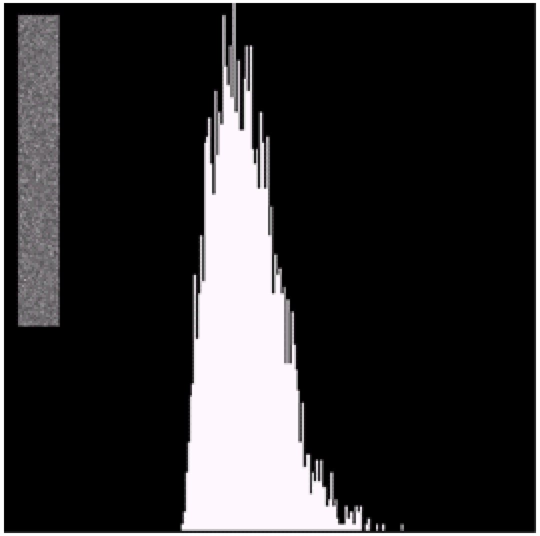
## Estimation of Noise Parameters models:

***•The following histograms are obtained from the image strips extracted from 3 different images.***

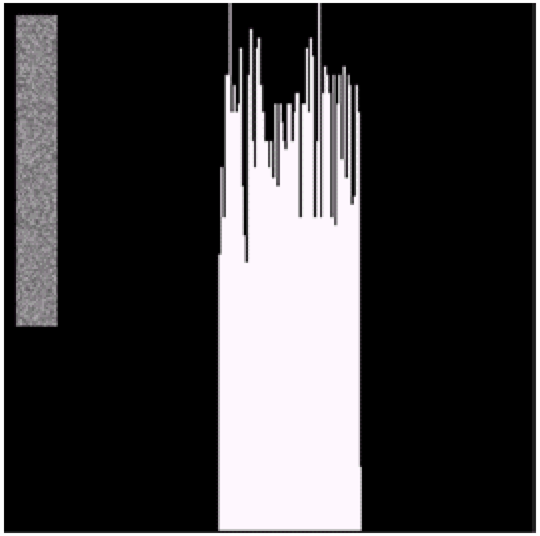
***•The shapes of the histograms gives an idea about the type of the noise distribution.***



**Gaussian**



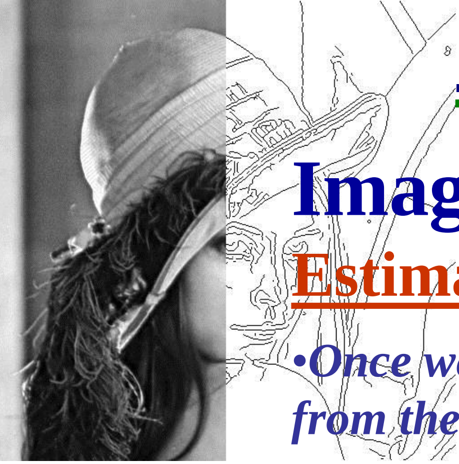
**Rayleigh**



**Uniform**

***•Based on observation, we can say that the first image strip contains Gaussian noise, the second one is Rayleigh and the third one is Uniform.***

**Prepared By:** ***Dr. Hasan Demirel, PhD***



# Image Restoration

## Estimation of Noise Parameters models:

*•Once we know the PDF type, then we can estimate the mean and variance from the basic statistics by:*

$$\mu = \sum_{z_i \in S} z_i \, p(z_i)$$

*and*

$$\sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 \, p(z_i)$$

*• where, $z_i$'s are the gray-level values of the pixels in image strip S, and $p(z_i)$ are the corresponding normalized histogram values.*

*•The PDF parameters of other noise models can be estimated by using the mean and variance approximations. The variables a and b can easily be calculated from the mean and variance.*

*• In the case of salt&pepper the mean and variance is not needed. The heights of the peaks corresponding to the white and black pixels are the estimates of the $P_a$ and $P_b$ respectively.*

**Prepared By:** *Dr. Hasan Demirel, PhD*

# Image Restoration

## Restoration in the presence of Noise:

*•If the only degradation in an image is the noise then:*

$$g(x,y) = f(x,y) + \eta(x,y)$$

*and*

$$G(u,v) = F(u,v) + N(u,v)$$

*• When, the distribution additive noise is estimated, we cannot simply subtract the noise terms from the noisy image.*

*•The method is to use spatial filtering for noise removal/reduction. Note that there is no point to transform into the frequency domain, as there is no convolution in the spatial domain.*

*•Only if there is a periodic noise, we can transform into the frequency domain.*

# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

•**Mean Filters**: ***There are four main types of noise reduction mean filters that can be used for image restoration/enhancement.***

• ***Arithmetic Mean Filter: Let $S_{xy}$ be the coordinates in a subimage window of size $m \times n$ centered at point $(x,y)$. The value of the restored image at any point $(x,y)$ is given by:***

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{s,t \in S_{xy}} g(s,t)$$

• ***This operation can be implemented by a convolution mask in which all its components have a value 1/mn.***

• ***Local variations in the image is smoothed and noise is reduced as a result of blurring.***

# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

**•Mean Filters:**

• ***Geometric Mean Filter:*** *Let $S_{xy}$ be the coordinates in a subimage window of size **m x n** centered at point **(x,y)**. The value of the restored image at any point (x,y) is given by:*

$$\hat{f}(x,y) = \left[ \prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

• ***In this filter, each pixel is given by the product of the pixels in the subimage windows, raised to the power of 1/mn.***

•***Achieves more smoothing, but looses less image details.***

# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

**•Mean Filters:**

• ***Harmonic Mean Filter:*** *Let* $S_{xy}$ *be the coordinates in a subimage window of size* $m \times n$ *centered at point* $(x,y)$*. The value of the restored image at any point* $(x,y)$ *is given by:*

$$\hat{f}(x,y) = \frac{mn}{\sum_{s,t \in S_{xy}} \frac{1}{g(s,t)}}$$

• *Harmonic mean filter works well with the salt noise but fails for the pepper noise.*

•*It works well with other types of noise as well, such as Gaussian noise.*

# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

**•Mean Filters:**

• ***Contraharmonic Mean Filter:*** *Let $S_{xy}$ be the coordinates in a subimage window of size m x n centered at point (x,y). The value of the restored image at any point (x,y) is given by:*

$$\hat{f}(x,y) = \frac{\sum_{s,t \in S_{xy}} g(s,t)^{Q+1}}{\sum_{s,t \in S_{xy}} g(s,t)^{Q}}$$

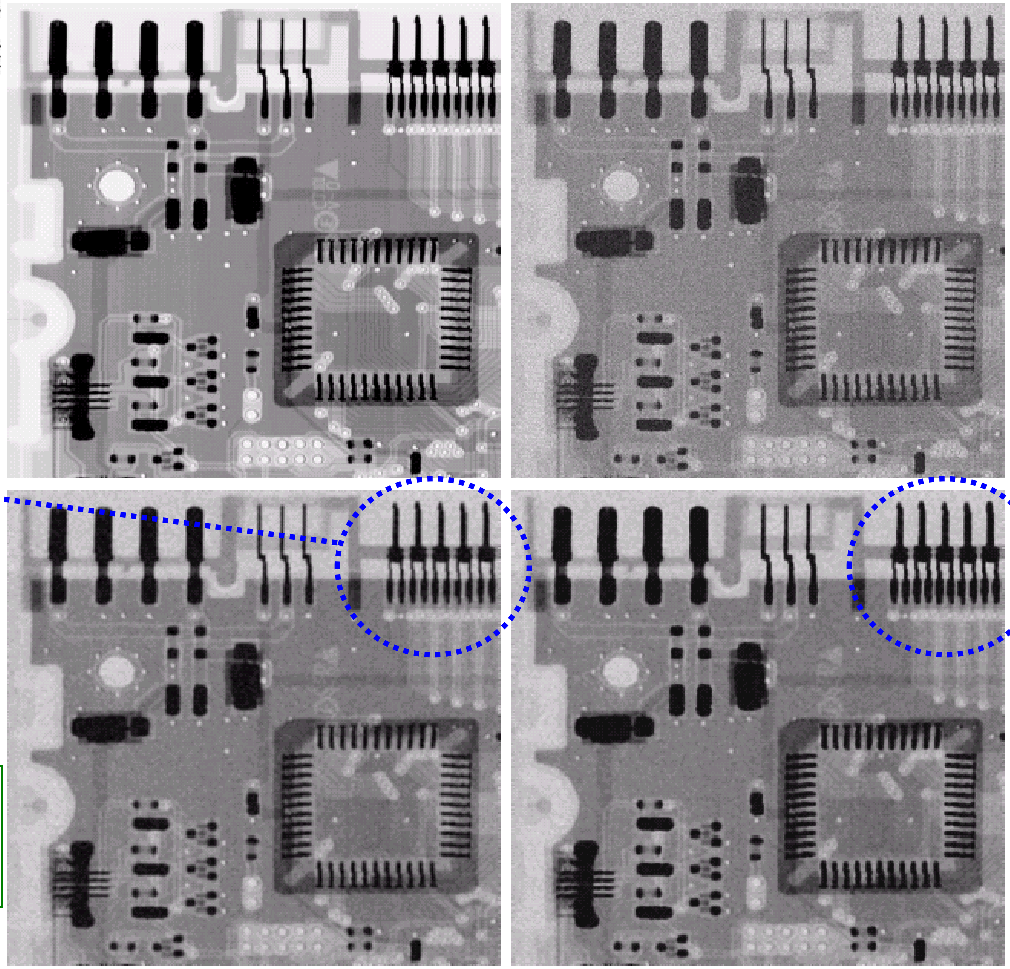
- *Q is the order of the filter. This filter is well suited for reducing the effects of salt-and-pepper noise.*
- *For negative values of Q, it eliminates the salt noise and for positive values of Q, it eliminates the pepper noise.*
- *The filter becomes the arithmetic mean filter for Q=0, and harmonic mean filter for Q= -1.*

# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

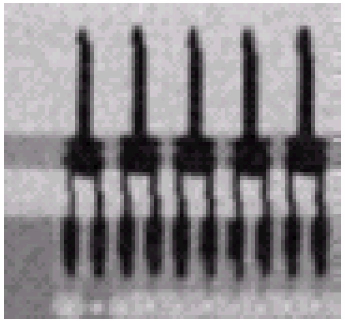
•**Mean Filters:**

Original



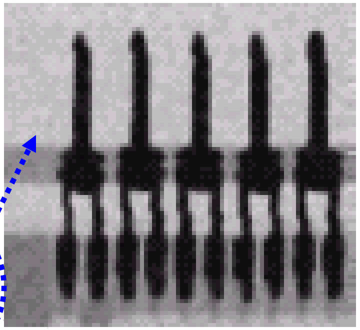
Additive Gaussian noise with

$\mu$=0, $\sigma^2$=400.



3x3 Arithmetic Mean Filter

$$\hat{f}(x,y)=\frac{1}{mn}\sum_{s,t\in S_{xy}} g(s,t)$$



3x3 Geometric Mean Filter

(Sharper details)

$$\hat{f}(x,y)=\left[\prod_{s,t\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$$

# Image Restoration

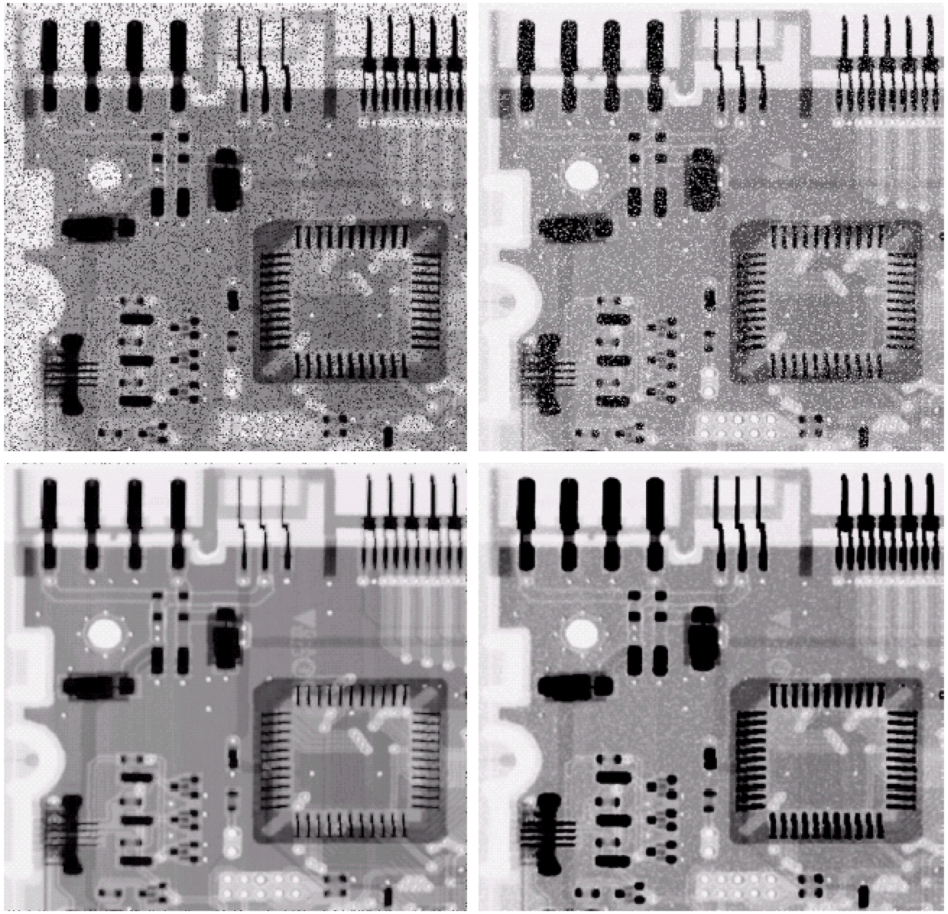
## Restoration in the presence of Noise: Only-Spatial Filtering

**•Mean Filters:**

$$\hat{f}(x,y)=\frac{\sum_{s,t\in S_{xy}} g(s,t)^{Q+1}}{\sum_{s,t\in S_{xy}} g(s,t)^{Q}}$$

**Pepper Noise with $P_p$=0.1**

**Salt Noise with $P_s$=0.1**



**3x3 Contraharmonic With Q=1.5 (good for Pepper)**

**3x3 Contraharmonic With Q=-1.5 (good for Salt)**

# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

•**Order Statistics Filters**: ***The response of these spatial filters is based on the ordering/ranking the pixels in the filter mask.***

• ***Median Filter: Let $S_{xy}$ be the coordinates in a subimage window of size m x n centered at point (x,y). The value of the restored image at any point (x,y) is given by:***

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

• ***The value of a pixel in is replaced by the median of the gray level in the neighborhood characterized by $S_{xy}$ subimage.***

•***Provides less blurring than the other linear smoothing filters.***

•***Median filters are very effective in bipolar or unipolar impulse (Salt&Pepper) noise.***

# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

**•Order Statistics Filters:**

• ***Max and Min Filters :*** *Let $S_{xy}$ be the coordinates in a subimage window of size **m x n** centered at point **(x,y)**. The value of the restored image at any point (x,y) is given by:*

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

• *Useful for finding the brightest points in the image. Also **reduces the pepper noise.***

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

• *Useful for finding the darkest points in the image. Also **reduces the salt noise.***



# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

### •Order Statistics Filters:

• *Midpoint filter : Let $S_{xy}$ be the coordinates in a subimage window of size m x n centered at point (x,y). The value of the restored image at any point (x,y) is given by:*

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

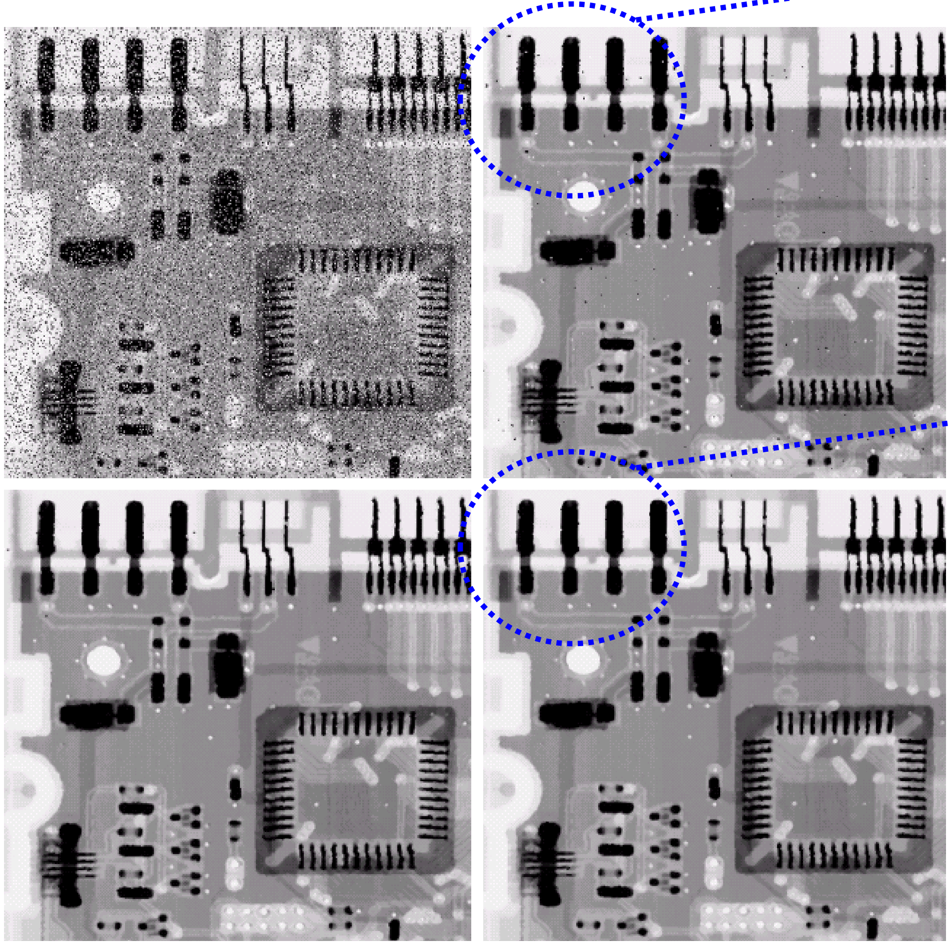
• *This filter combines the order statistics and averaging. Works best for Gaussian and uniform noise.*

# Image Restoration

## Restoration in the presence of Noise: Only-Spatial Filtering

- **Order Statistics Filters:**

**Salt & Pepper Noise with $P_a = P_b = 0.1$**



**Some traces of pepper noise**

**3x3 Median Filter (first pass)**

**Completely removed**

**3x3 Median Filter (second pass)**

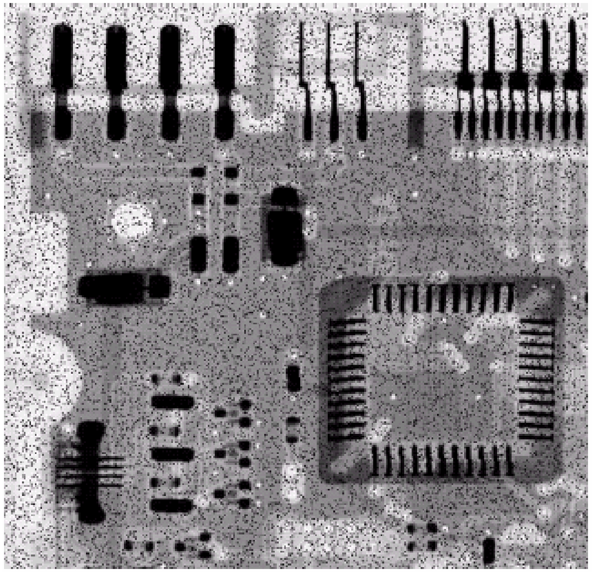
**3x3 Median Filter (third pass)**

# Image Restoration

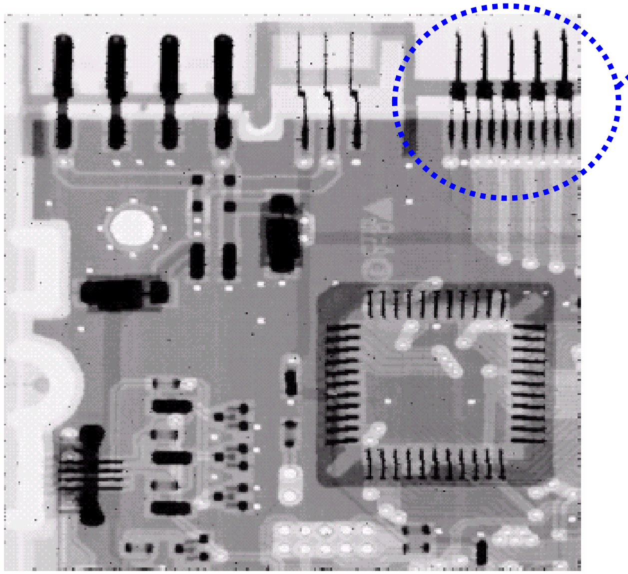
## Restoration in the presence of Noise: Only-Spatial Filtering

- **Order Statistics Filters:**

Some dark border pixels are removed



Pepper Noise with $P_b$=0.1



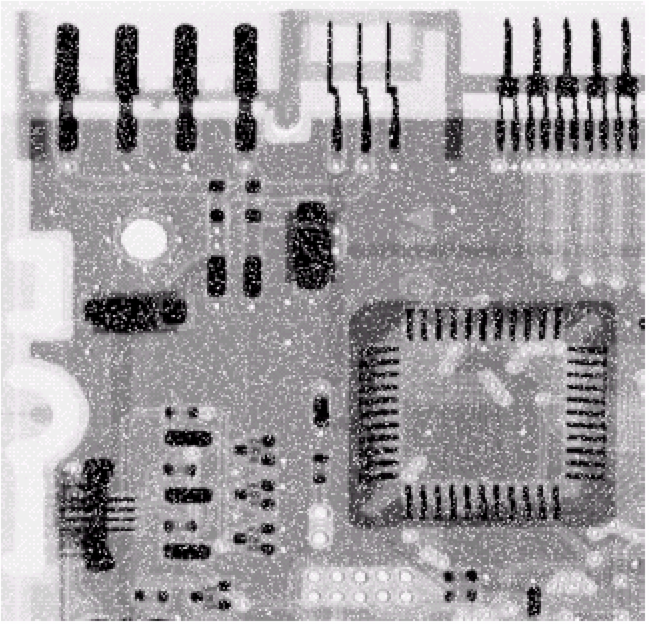
3x3 Max Filter Good for pepper

# Image Restoration

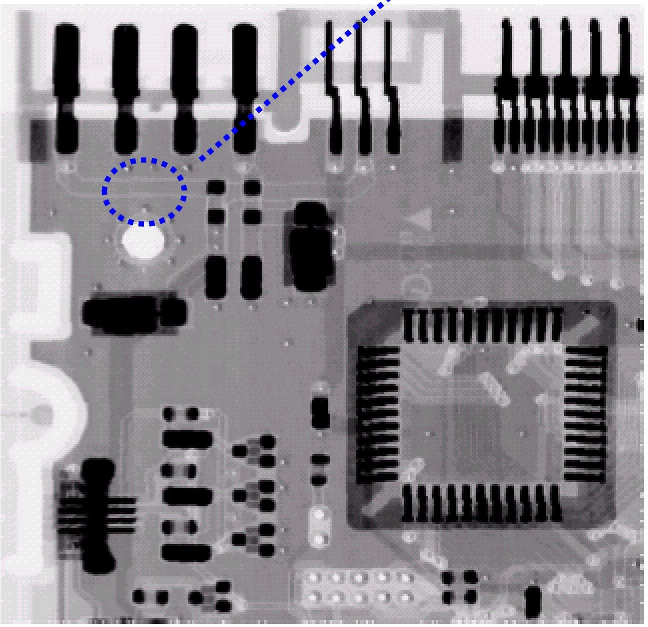
## Restoration in the presence of Noise: Only-Spatial Filtering

•Order Statistics Filters:

Some white dots are darker now.



Salt Noise with $P_a$=0.1



3x3
Min Filter
Good for Salt