Gravitationally Lensed Black Hole Emission Tomography

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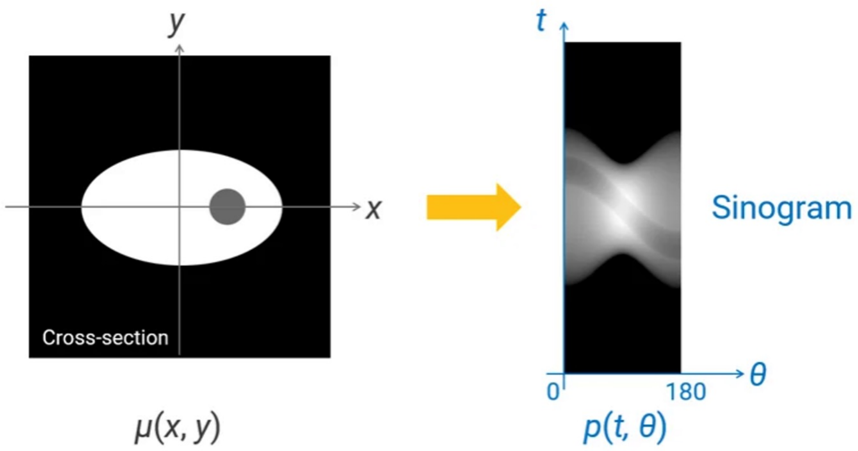
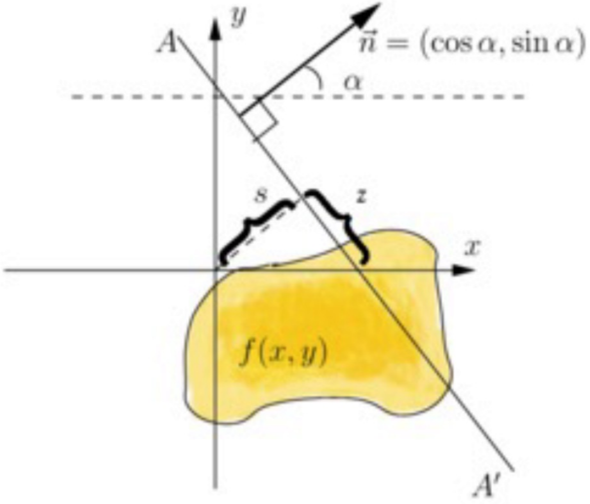
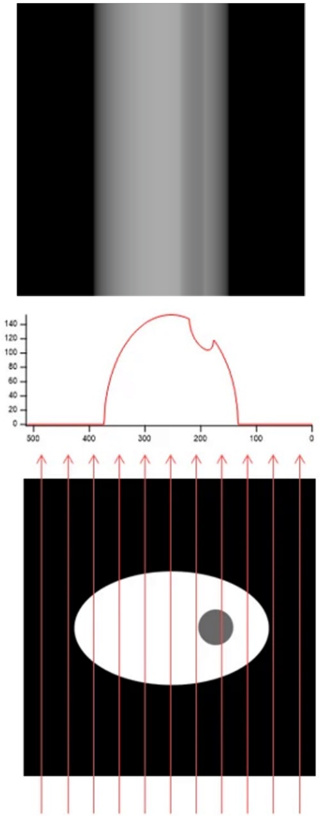
Related Works (Tomography 3D Reconstruction)

- Reconstructing 3D volume and density by collecting 2D projections from multiple different directions.

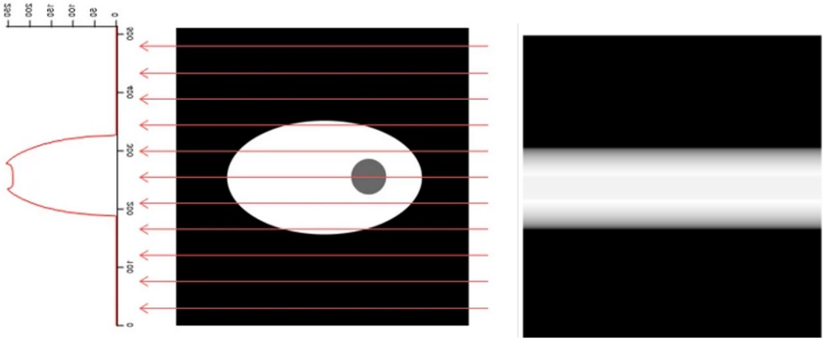




Related Works (Tomography 3D Reconstruction)



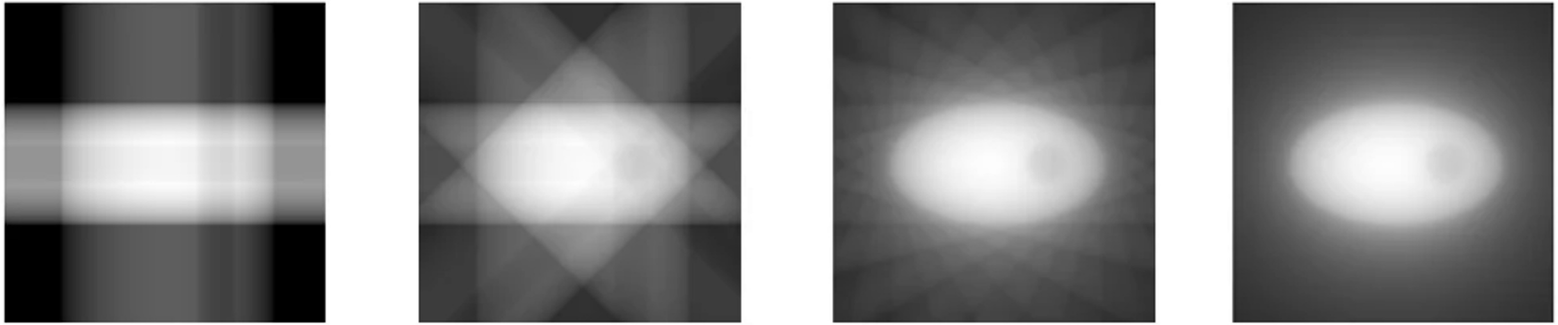
Projection at 0°



Projection at 90°

Related Works (Tomography 3D Reconstruction)

- As the number of projections increases, the reconstructed cross-sections become more refined



4 projections | 8 projections | 24 projections | 120 projections

Related Works (Tomography 3D Reconstruction)

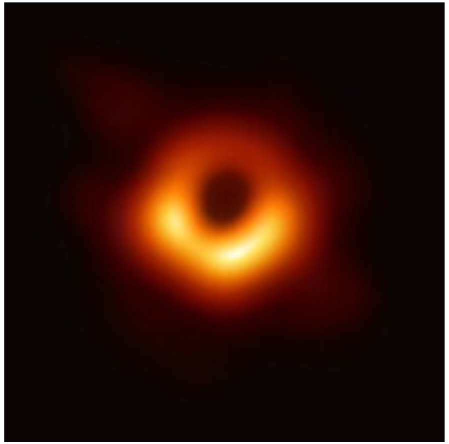
- In the paper “Spacetime Tomography Using the Event Horizon Telescope”
  - Observe the motion and changes of hot spots around black holes - these hot spots are high-energy regions generated by magnetic reconnection events in the accretion disk near the black hole - to detect and map the space-time structure around the black hole.
  - This process can be considered a special form of tomography because it relies on data obtained from different angles (i.e., the location and brightness of hot spots at different points in time) to recover a three-dimensional structure that we cannot directly observe (i.e. the geometry of spacetime around a black hole)
  - This article essentially performs the task of recovering high-dimensional structural information from low-dimensional data, which is similar to CT scanning in medical imaging and tomography in other fields. The goals of technology are similar.

Related Works (Coordinate Neural Representations)

- This method employs neural network (usually a multilayer perceptron MLP) to map directly from 3D coordinates to scene.
- Neural Radiance Fields (NeRF)
  - Traversing successive spatial points in the scene and integrating color and density information along a direction using a process similar to ray casting.
    - Similar to ray-tracing,interaction of light is indirectly represented through neural networks.
    - Rely on neural networks to learn continuous representations of scenes from existing data.
  - Able to process active scenes
    - Conventional methods rely on sparsity and rigidity of object deformation, or the autocorrelation nature of known objects.
      - – Not suitable for unknown and complex applications such as black holes.
    - Learn volume density and perspective-dependent radiance of the scene
    - Simulate and render dynamically changing celestial environments while reducing reliance on specialized prior knowledge.

Black Hole Imaging

- The Event Horizon Telescope (EHT) is a global network of radio telescopes synchronized to observe black holes
- First image of a black hole was published by EHT in 2019
- “Although a black hole itself is not visible, its imprint or “shadow” on the emission from the surrounding hot gas is what the EHT aims to image”
- Have previously been all 2D images, assuming the emissions were remaining the same over a whole night of measurements



Summary of novel BH-NeRF (Black Hole Neural Emission Radiance Fields)

- Unlike traditional imaging methods, which rely on static assumptions about the emission field, BH-NeRF aims to capture the evolving nature of emission around black holes, including dynamic features such as flares and hot spots
- Leveraging gravitational lensing effects to account for the curved paths of light rays around black holes, enabling accurate reconstruction of 3D emission fields from 2D observations.
- Using coordinate-based neural network representations to model the continuous volumetric distribution of emission, allowing for high-fidelity reconstruction of dynamic phenomena.
- Incorporating knowledge of Keplerian orbital dynamics to establish correspondence between 3D emission points over time, enabling the tracking of evolving features such as orbiting hot spots and accretion flows.

Emission Dynamics

- A significant source of emission around black holes is the accretion disk
  - A disk-shaped structure of gas and dust spiraling inward due to gravitational forces
- As material falls toward the black hole, it heats up and emits radiation across the electromagnetic spectrum.
- The intense gravitational field of the black hole causes the accretion disk to emit high-energy radiation, including X-rays and gamma-rays, making it observable to telescopes.
- Black holes often exhibit powerful jets of plasma that stream away from the vicinity of the black hole at relativistic speeds
  - Powered by the rotational energy of the black hole's accretion disk and magnetic fields

Gravitational Lensing

- The gravitational field around a black hole is so intense that it warps the fabric of spacetime, causing light rays to follow curved paths
- Strong Lensing: In regions close to the black hole, where the gravitational field is most intense, light rays can be bent significantly, leading to strong gravitational lensing effects. This can result in the formation of multiple images, Einstein rings, or even complete distortion of background objects.
- Weak Lensing: Farther away from the black hole, the gravitational lensing effects become weaker, but still observable. Weak lensing can cause subtle distortions in the shapes and orientations of background objects, which can be measured statistically to infer the mass distribution of the lensing object.

Ray Tracing Method in Gravitational Lensing

- "Backward" integration along the ray path to calculate the radiation intensity of the radiation source.
    - For a discrete image plane of N×N pixels, it is defined by the vector I(t) = [p1(t), ..., pN^2(t)], where each pn(t) represents a pixel point at time t radiation intensity.
    - Bended ray trace due to gravitational lensing
        - Integrate radiation to calculate the radiation intensity of the pixel, which approximates the continuous physical process as a discrete summation.
        - Emission intensity can be calculated as $\mathrm{pn(t)} = \int_{\Gamma_n} e(t,x)\,ds \approx \sum_{x_i \in \Gamma_n} e(t,x_i)\Delta s_i$
- Calculating the light contribution of each segment, and finally summing these contributions to calculate the total intensity.
    - To estimate the cumulative contribution to light rays from gravitational lensing,

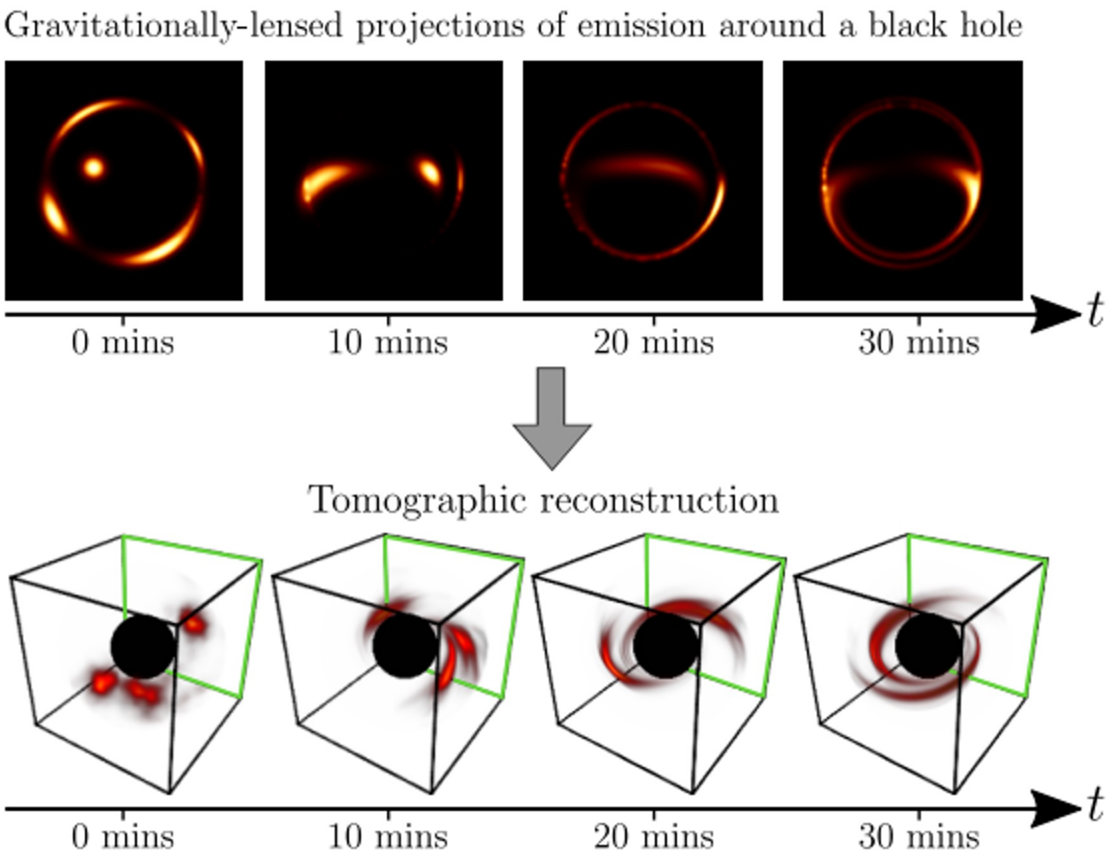
Keplerian Dynamics

$$\omega(r) = \frac{1}{2\pi\sqrt{r^3/GM}} \propto r^{-3/2}.$$

- The angular velocity of material in the accretion disk decreases with radial distance from the black hole
- The material undergoes periodic changes in velocity and density, leading to variations in emission intensity over time.
  - These temporal variations can manifest as flares, oscillations, and quasi-periodic oscillations (QPOs)
- Spatial variation in orbital velocity leads to shearing and stretching of emission features in the accretion disk, resulting in complex morphologies observed in black hole images

- angular velocity $\omega(r)$
- radius r
- mass M
- Gravitational constant G
- With 0 angular momentum

Emission Dynamics



Effects of Gravitational Lensing

- Gravitational lensing can distort the appearance of emission features around black holes, making them appear larger, brighter, or more elongated than they actually are. Failure to correct for these distortions can lead to biased measurements of black hole properties, such as mass, spin, and accretion rate.

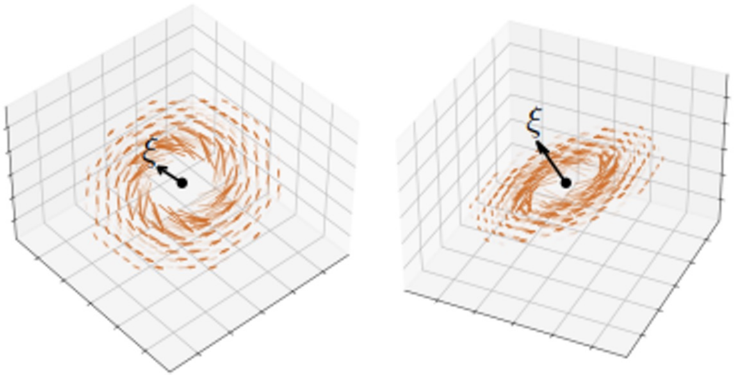


Figure 2.

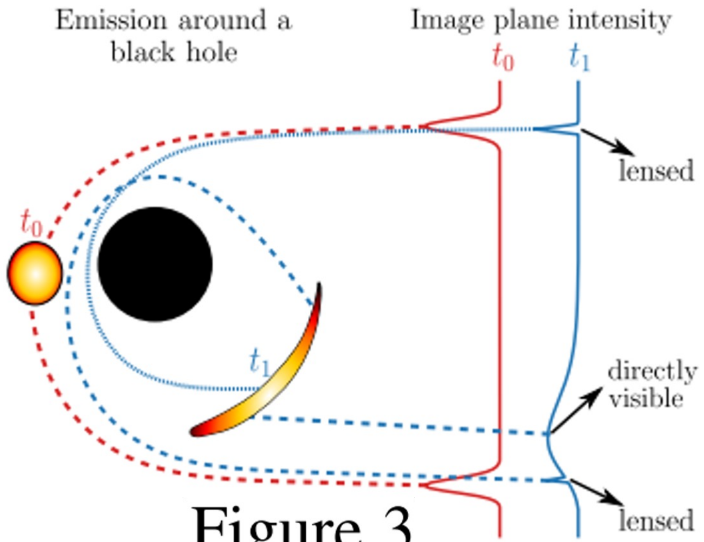


Figure 3

The continuous change in angular velocity with radial distance causes a *shearing* of structures (Fig. 2). The shearing operation can be represented by a coordinate transformation of a canonical emission at time $t = 0$:

$$e\left(t, \mathbf{x}\right) = e_0\left(\mathbf{R}_{\boldsymbol{\xi},\phi}\mathbf{x}\right), \tag{2}$$

where $\mathbf{R}_{\boldsymbol{\xi},\phi}$ is a rotation matrix of angle ϕ about the axis $\boldsymbol{\xi}$[1]. The position and time dependent angle ϕ is given by

$$\phi\left(t, r\right) = t\omega(r) \propto tr^{-3/2}. \tag{3}$$

Benefits of BH-NeRF

- By accurately modeling the curved paths of light rays using principles from general relativity, BH-NeRF can trace the distorted trajectories of photons and correct for the gravitational distortions in the observed data, ensuring faithful reconstruction of the underlying emission field
- Utilizes knowledge of Keplerian orbital dynamics to track the movement of dynamic emission features orbiting around the black hole
- By understanding the gravitational forces and angular velocities governing the orbits of emission sources, BH-NeRF establishes correspondence between 3D emission points over time, enabling accurate reconstruction of temporal variations in the emission field

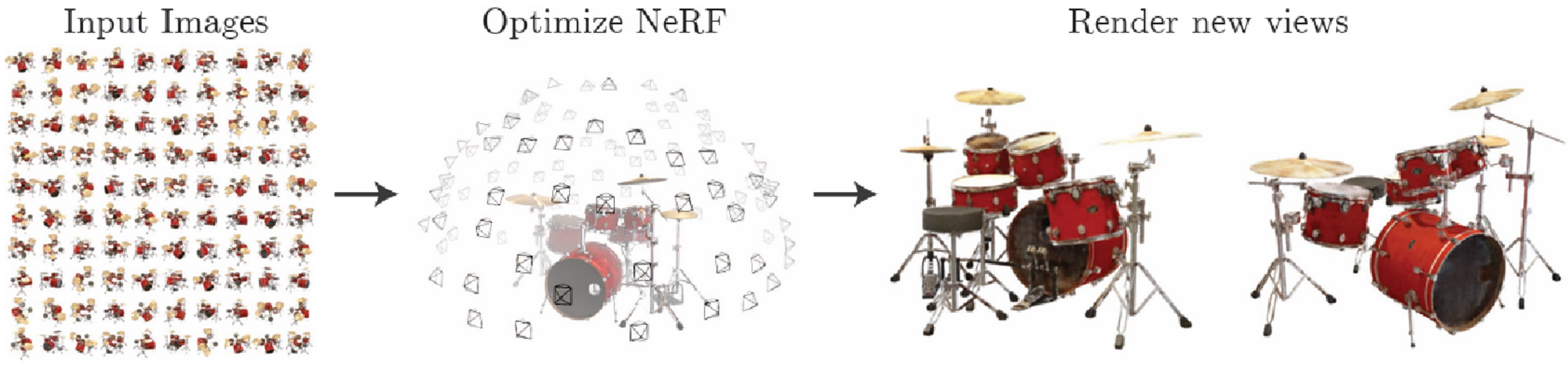
Benefits of a Continuous Volumetric Representation

- Enables BH-NeRF to capture a wide range of emission features, from compact hot spots to extended accretion disks, with high fidelity
- Helps reduce artifacts and discontinuities in the reconstructed emission field, resulting in visually appealing and physically plausible results
- Coordinate-based neural network representations offer efficient parameterization of the emission field, reducing the computational complexity of reconstruction algorithms

BH-NeRF

NeRF

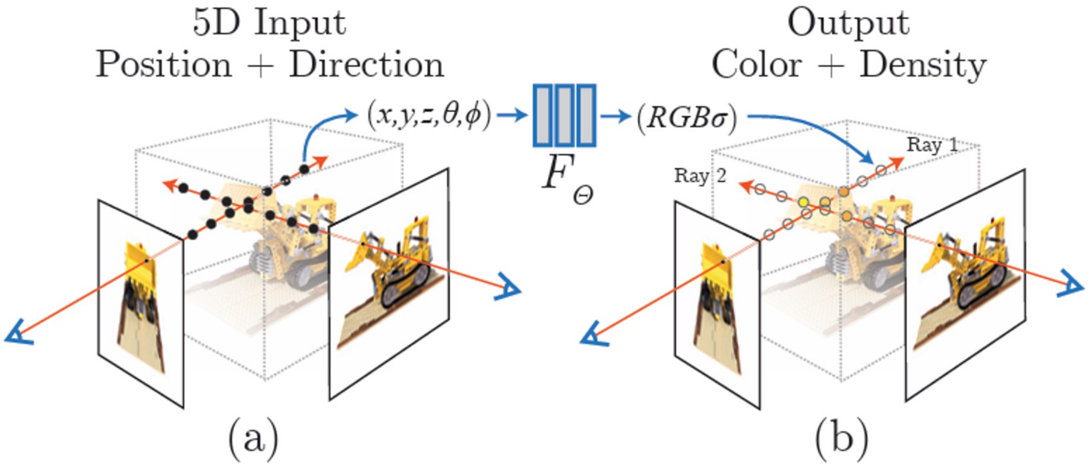
- Neural Radiance Fields: builds a continuous volumetric scene using a sparse set of input views



Paper: Ben Mildenhall, Pratul P. Srinivasan, Matthew Tancik, Jonathan T. Barron, Ravi Ramamoorthi, and Ren Ng. NeRF: Representing scenes as neural radiance fields for view syn thesis. ECCV, 2020.

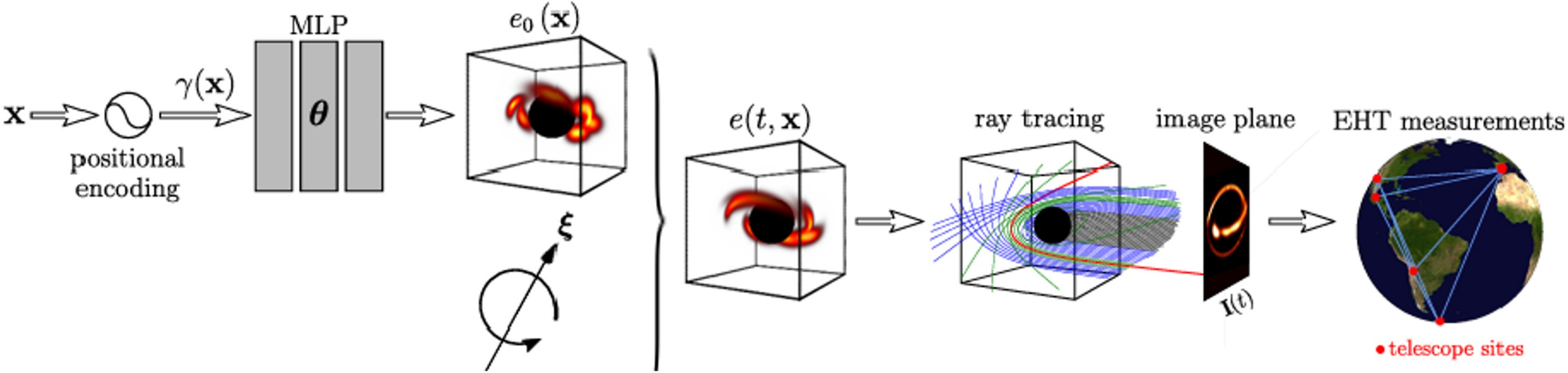
NeRF

- Model: multilayer perceptron
- 5D input
  - 3D location: (x, y, z)
  - 2D viewing direction: (θ, ϕ)
- 4D output
  - Color: (r, g, b)
  - Volume density/opacity: σ
- Positional encoding



BH-NeRF

- Black Hole Neural Radiance Fields
- Input: 3D coordinates **x** (i.e., (x, y, z))
- Output: 4D emission dynamics e(t, **x**)
- Dataset: Synthetic EHT observations (eht-imaging library)

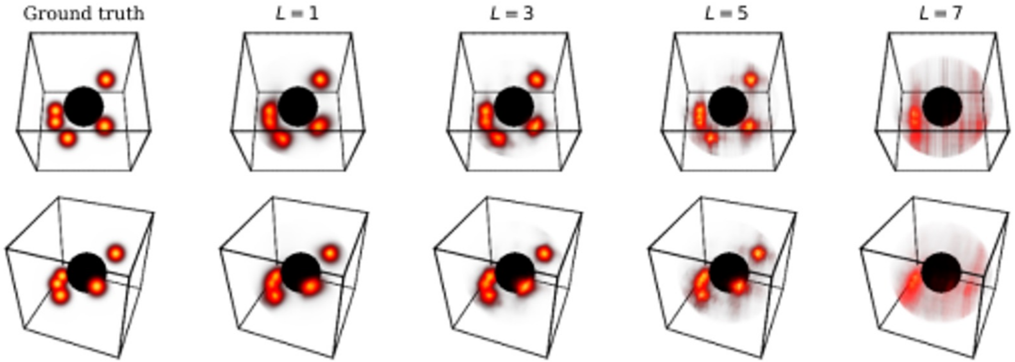


Positional Encoding

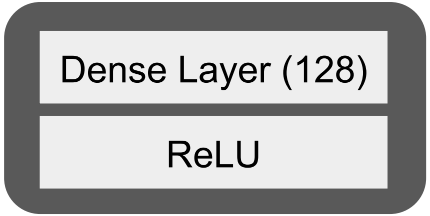
- Positional encoding: projects each input coordinate onto a set of sinusoids with exponentially-increasing frequencies

$$\gamma(\mathbf{x}) = \left[\sin(\mathbf{x}), \cos(\mathbf{x}), \ldots, \sin\left(2^{L-1}\mathbf{x}\right), \cos\left(2^{L-1}\mathbf{x}\right)\right]^{\mathrm{T}}$$

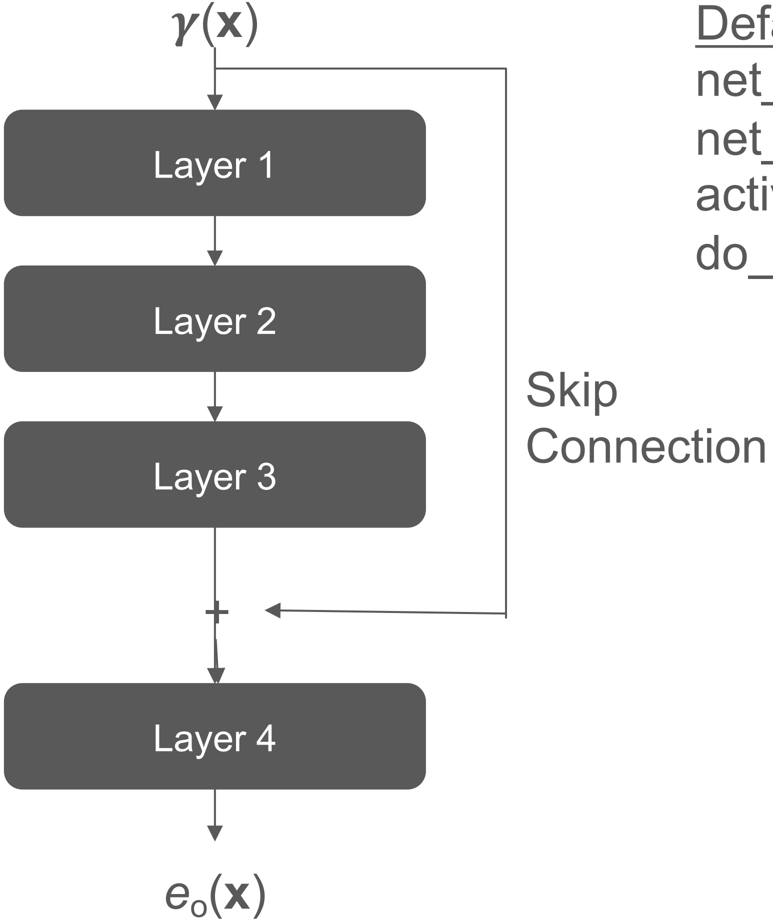
  - Maximum positional encoding degree of L = 3



MLP architecture

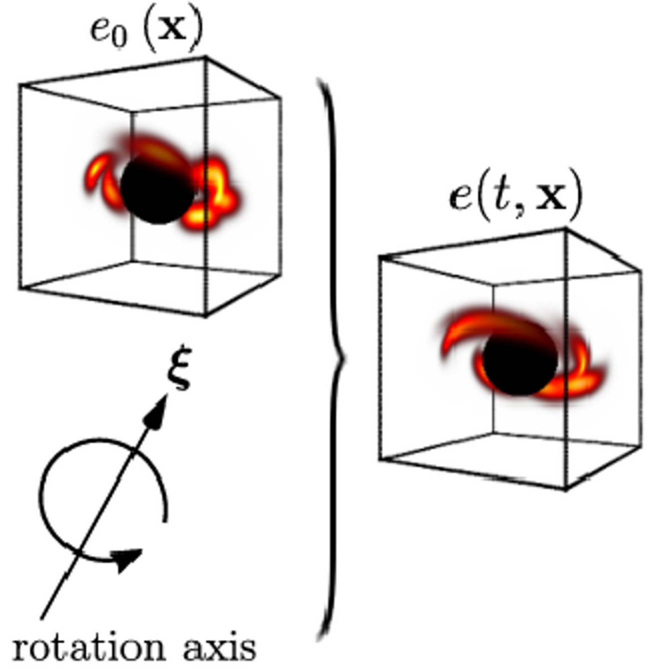


Each layer is composed of a fully-connected and an activation layer.



Default values:
net_depth = 4
net_width = 128
activation = relu
do_skip = True

Rotation Axis Estimation



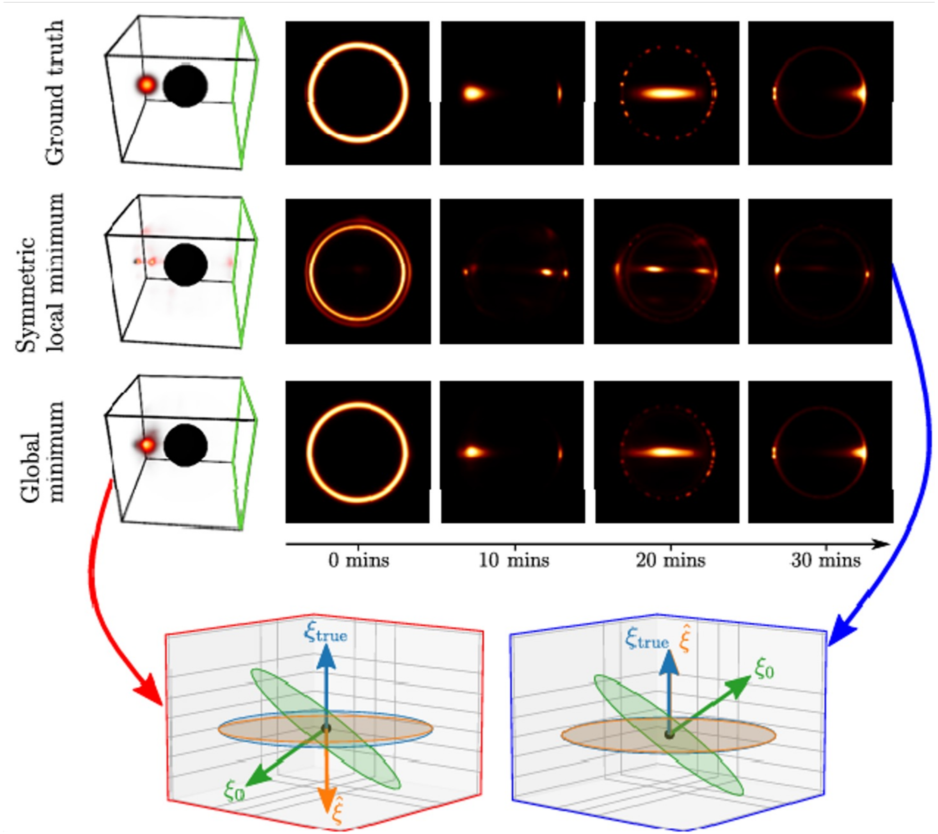
Equations estimating emission dynamics:

$$e(t, \mathbf{x}) = e_0(\mathbf{R}_{\boldsymbol{\xi}, \phi}\mathbf{x})$$

$$\phi(t, r) = t\omega(r) \propto tr^{-3/2}$$

$$\omega(r) = \frac{1}{2\pi\sqrt{r^3/GM}} \propto r^{-3/2}$$

Avoiding local minimum



Loss Function

$$\boxed{\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\xi}) = \sum_t \|\mathbf{y}(t) - \mathbf{F}_t \mathbf{I}_{\boldsymbol{\theta},\boldsymbol{\xi}}(t)\|^2_{\boldsymbol{\Sigma}}}$$

Loss Function

$$\mathbf{y}(t) = \mathbf{F}_t \mathbf{I}(t) + \boldsymbol{\varepsilon}$$

EHT measurement

$$\mathbf{I}(t) = [p_1(t), \ldots, p_{N^2}(t)]$$

NxN Image Plane

$$p_n(\boldsymbol{\theta}, \boldsymbol{\xi}) = \sum_{\mathbf{x}_i \in \Gamma_n} \mathrm{MLP}_{\boldsymbol{\theta}} \left[\gamma(\mathbf{R}_{\boldsymbol{\xi}} \mathbf{x}_i)\right] \Delta s_i$$

Pixel

$$\|\mathbf{v}\|^2_{\boldsymbol{\Sigma}} \equiv \mathbf{v}^\top \boldsymbol{\Sigma}^{-1} \mathbf{v}$$

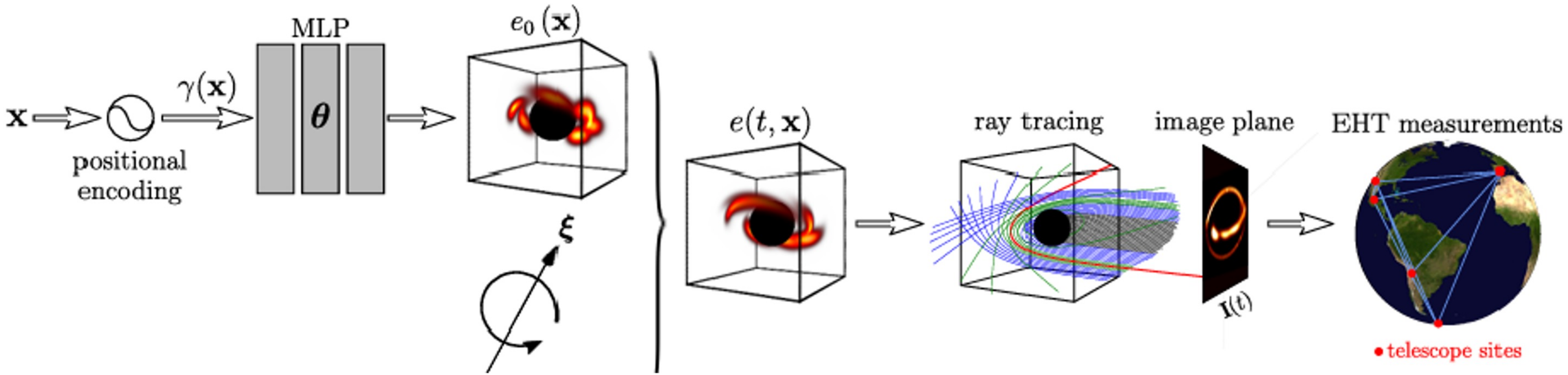
Norm

$$\boldsymbol{\Sigma} = \mathrm{diag}\left(\sigma_1^2, \ldots, \sigma_K^2\right)$$

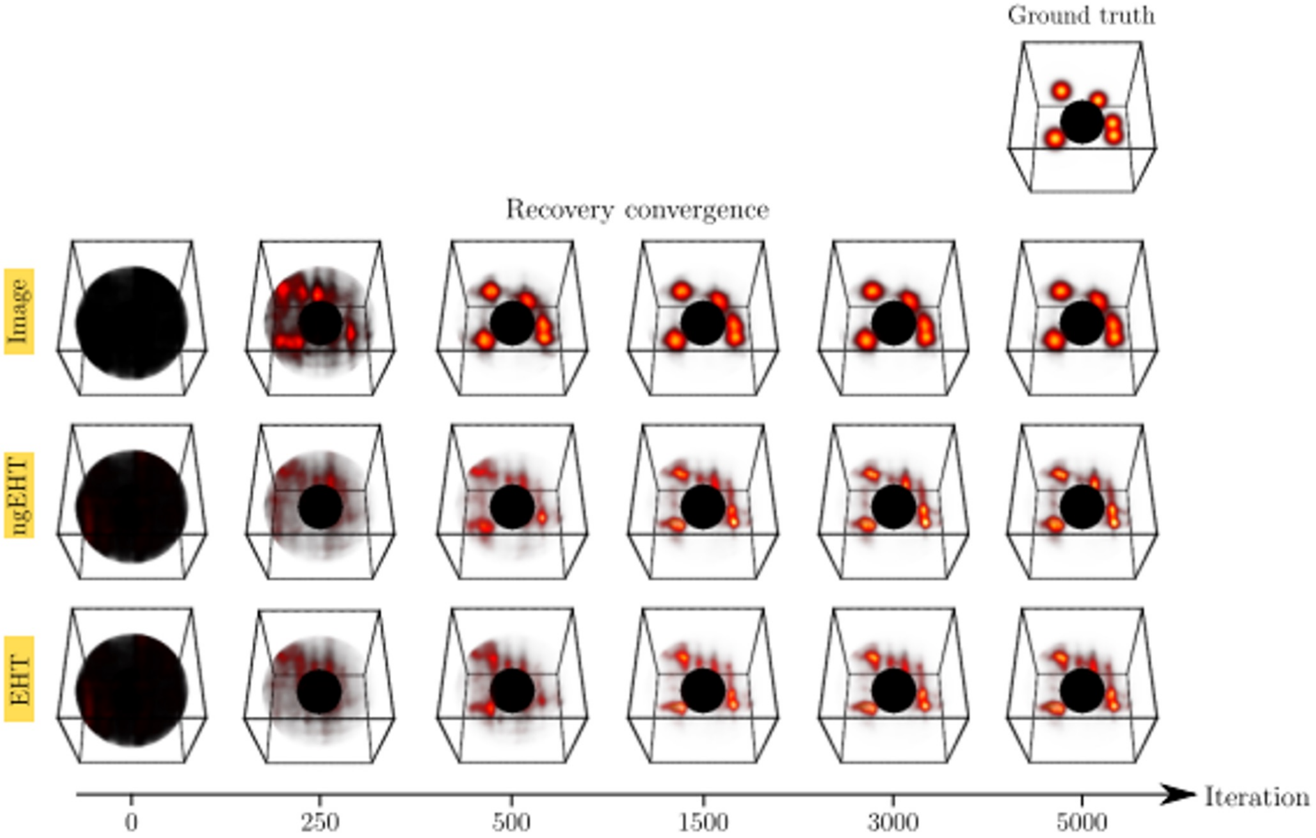
Thermal Noise Covariance Matrix

Optimization

- Adam optimizer
- Polynomial learning rate transitioning from 1e-4 -> 1e-6
- 5K iterations



Results



Ground truth
Recovery convergence
Image
ngEHT
EHT
Iteration
0
250
500
1500
3000
5000

Experiment Setup

- Target Blackhole SgrA*
  - Galactic centre black hole, 4 million solar masses
  - Assuming no angular momentum
  - Observed with EHT, data available
  - Restore its 3D emission field with BH-NeRF
- Emission Hotspot
  - Randomly placed at 1.16Rms radius
  - Emulate the effect of gravity on light rays
    - 128 time frames, 643 voxel grid resolution
    - JAX framework implementation
    - ADAM optimizer for fine tuning
    - EHT-Imaging toolbox for data emulation

Experiment

- 3D mesh
  - Discretize space into grid， voxels store information such as emission intensity
  - Depending on the sampling rate, details in continuous space may not be captured
  - Unable to effectively capture continuous changes in time due to separate modeling of time
- 4D MLP (four-dimensional multilayer perceptron)
  - Represent four-dimensional spatio-temporal data, able to handle temporal events
  - Accepts space-time coordinates as input and outputs its emission intensity
  - Hard to handle unknown and complex situation as no pattern algorithm available
- BH-NeRF
  - Utilizing the concept of Neural Radiation Fields (NeRF)
  - Strengthened to gravitational lensing effect and the dynamic distribution of matter
  - Provides modeling capabilities for unknown complex dynamic scenarios

Experiment

- Adding Hotspots
  - Increase of complexity in distinguishing hotspots, may cause confusion
  - Increase system complexity, making long-term calculations more complex
  - The data obtained from EHT are sparse, increasing difficulties of recovering
- Experiment Result
  - Even with increased complexity, hotspots are still accurately recovered
  - Taking into account the gravitational lensing, simulated the movement accurately
  - High-quality result recovered from sparse data in complex scenarios with multiple hot spots.
- BH-NeRF Performance
  - Utilizing the concept of Neural Radiation Fields (NeRF)
  - Strengthened to gravitational lensing effect and the dynamic distribution of matter
  - Provides modeling capabilities for unknown complex dynamic scenarios

Experiment

- Perturbations in Velocity Model
  - Astronomic dynamic behavior may differ from our model assumptions
  - Verify the adaptability of BH-NeRF to potential problems in this real-world application
  - Robustness testing, that is, the ability to recover under mismatching conditions
- Experiment Result
  - BH-NeRF can effectively capture emitting structures
  - Verified its robustness, especially when the dynamic model has errors in real application.
  - High-quality result recovered from sparse data in complex scenarios with multiple hot spots.
- Comprehensive Results
  - BH-NeRF demonstrates its superior performance in terms of accuracy, detail and dynamics.
  - High-level of PSNR confirms the accuracy of BH-NeRF in complex situations
  - Error in models has little effect on BH-NeRF restoring structures

Limitations

- Assumptions on Blackhole
  - BH-NeRF relies on assumptions about the black hole's spin and mass
    - Estimated by backpropagation of the ray tracing equations for gravitational lensing
  - Verify the adaptability of BH-NeRF to potential problems in this real-world application
  - Robustness testing, that is, the ability to recover under mismatching conditions
- Extra Flares in Observation
  - BH-NeRF did not count flares occurring during the observation window
    - Flares can be algorithmically identified and isolated
  - High-quality result recovered from sparse data in complex scenarios with multiple hot spots.
- Simplified Experimental Conditions
  - Not including challenging conditions,i.e. instrumental errors or challenging atmospheric noise
    - Improve models, ensuring its ability to handle real-world data with noise and errors.