Tracking Control of Quadrotor Using Sliding Mode Control via Backstepping Approach

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***Abstract*— Quadrotor is one of VTOL UAV that have capability to hover, agile manuevers. But also, it is instable system that requires active control. Nonlinear control techinque, sldiing mode control based on backstepping approach is adopted. Designed sliding mode controller for attitude & position of the vehicle is presented. Also, using sliding mode attitude controller. Compared performance of classical PD(Proportional - Derivative) & sliding position controller by helix path command in simulation. PD controller shows better performance than sliding mode controller.**

I. INTRODUCTION

Unmanned Aerial Vehicle(UAV)s are conventionally used in military purpose for intelligence surveillance & reconnaissance(ISR) missions. Recently UAVs are widely used in non-military purposes like package delivery, crop dusting, aerial photography, search & rescue(SAR) operations. One of those UAV is the helicopter which has one main rotor and one tail rotor. And more variations like co-axial helicopter, multi-rotor type helicopters. The Quad-rotor type helicopter(Quadrotor) is one of multi-rotor type helicopter which only has four rotors on a simple frame structure. Quadrotor is mechanically simple, agile and can hover in place. on the other hand, quadrotor is inherently instable system and not suite for fast forward, lateral flight.

II. MATHEMATICAL MODEL

A. Quadrotor Dynamic Model

Most common quadrotor configuration is either '+' or 'X' due to it's simplicity. '+' configuration is more intuitive in terms of attitude control since x,y axes are coincide with body structure. For example, to quadrotor pitch forward, control foward rotor to rotate slower & backward rotor faster and roll control can be done in a same manner. While 'X' configuration requires all four rotor to control pitch & roll. but 'X' configuration has advantage than '+' configuration

- More agile, faster response on roll & pitch control
- Clear visibility of front side when camera is equipped

due to above advantages, commercial quadrotors are mostly 'X' configuration. Quadrotor model used in this paper also adopted 'X' Configuration as shown in Fig. 1.

The dynamic model of quadrotor can be obatined via Lagrange approach [2],[3] or Newton Approach[2],[4] with follow assumptions:

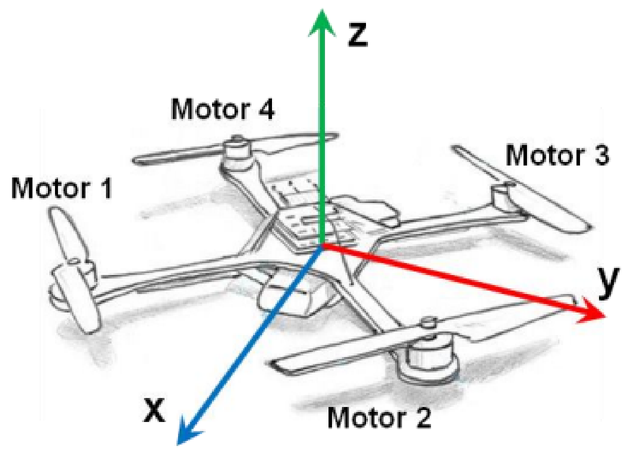

Fig. 1: Quadrotor Configuration - 'X' [1]

- The structure is rigid & symmetrical.
- The Center of Gravity(CG) and the body fixed frame origin are assumed to coincide.
- The propellers are supposed rigid.
- Thrust and drag are proportional to the square of propellers speed.

$$\begin{aligned}
\ddot{x} &= \{U_1(cos\phi sin\theta cos\psi + sin\phi sin\psi) - K_1\dot{x}\}/m \\
\ddot{y} &= \{U_1(sin\phi sin\theta cos\psi - cos\phi sin\psi) - K_2\dot{y}\}/m \\
\ddot{z} &= \{U_1(cos\phi cos\psi) - K_3\dot{z}\}/m - g \\
\ddot{\phi} &= \frac{1}{I_x}\{\dot{\theta}\dot{\psi}(I_y - I_z) - K_4\dot{\phi}^2 - J_r\Omega_r\dot{\theta} + U_2\} \\
\ddot{\theta} &= \frac{1}{I_y}\{\dot{\phi}\dot{\psi}(I_z - I_x) - K_5\dot{\theta}^2 + J_r\Omega_r\dot{\phi} + U_3\} \\
\ddot{\psi} &= \frac{1}{I_z}\{\dot{\phi}\dot{\theta}(I_x - I_y) - K_6\dot{\psi}^2 + U_4\}
\end{aligned} \tag{1}$$

Where (x,y,z) are position of the vehicle, (ϕ,θ,ψ) are Euler angles which represents roll, pitch, yaw respectively. (I_x,I_y,I_z) is moment of inertia with respect to each axes, J_r is moment of inertia of rotor, K_i is drag coefficient and in this paper, consider drag as a disturbance so neglected drag terms. Ω_r is gyroscopic forces produced by rotor, U_i is the control input.

$$\Omega_r = (\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4) \tag{2}$$

Ω_i represents each rotor's angular velocity. Rotor notation is as seen in Fig. 1.

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Thrust generated by a single motor/prop system can be modeled as follow.

$$\begin{aligned} T &= C_T \rho A_r r^2 \Omega^2 = c_T \Omega^2 \\ Q &= L C_T \rho A_r r^2 \Omega^2 = c_Q \Omega^2 \end{aligned} \tag{3}$$

Where C_T is thrust coefficient of the propeller, ρ is the air density, A_r is the cross-sectional area of the propeller's rotation, r is the radius of the rotor, L is distance between a motor/prop system and CG, and Ω is the angular velocity of the rotor.

For simple modelling, a lumped parameter(c_T, c_Q) is introduced since propeller does not change its shape while they rotating and also air density can be seen as constant because quadrotor operates near ground mostly[1]. In [4] similar parameters are introudced to simplify thrust & torque.

So, Eq. (3) can be used to model control input of the quadrotor as follow. Note that c_Q in (2) is introduced to account for yaw(heading) control input U_4

$$\begin{aligned} U_1 &= c_T\left(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2\right) \\ U_2 &= c_T\left(-\Omega_1^2+\Omega_2^2+\Omega_3^2-\Omega_4^2\right) \\ U_3 &= c_T\left(-\Omega_1^2-\Omega_2^2+\Omega_3^2+\Omega_4^2\right) \\ U_4 &= c_Q\left(-\Omega_1^2+\Omega_2^2-\Omega_3^2+\Omega_4^2\right) \end{aligned} \tag{4}$$

An actuator is modeled as simple delay and τ is time constant of motor/prop system.

$$TF_{actuator} = \frac{1}{\tau s+1} \tag{5}$$

III. Sliding Mode Control Design

Sliding mode control is one of non-linear control technique that ensures Lyapunov stability. And also known as nonlinear robust control technique that allows control of system with uncertainties.

State vector X can be defined as follow.

$$X = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}]^T \tag{6}$$

$$\begin{array}{l|l} x_1 = \phi & x_7 = x \\ x_2 = \dot{x}_1 = \dot{\phi} & x_8 = \dot{x}_7 = \dot{x} \\ x_3 = \theta & x_9 = y \\ x_4 = \dot{x}_3 = \dot{\theta} & x_{10} = \dot{x}_9 = \dot{y} \\ x_5 = \psi & x_{11} = z \\ x_6 = \dot{x}_5 = \dot{\psi} & x_{12} = \dot{x}_{11} = \dot{z} \end{array} \tag{7}$$

The quadrotor model (1) can be re-written into the state representation form as below with neglecting drag terms. And also, for quadrotor in hover flight model, Euler rate $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ can be simplified to body angular rate (p, q, r).

$$f(X,U) = \begin{pmatrix} \dot{\phi} \\ a_1\dot{\theta}\dot{\psi} + a_2\dot{\theta}\Omega_r + b_1 U_2 \\ \dot{\theta} \\ a_3\dot{\phi}\dot{\psi} - a_4\dot{\phi}\Omega_r + b_2 U_3 \\ \dot{\psi} \\ a_5\dot{\theta}\dot{\phi} + b_3 U_4 \\ \dot{x} \\ U_1(cos\phi sin\theta cos\psi + sin\phi sin\psi)]/m \\ \dot{y} \\ U_1(cos\phi sin\theta sin\psi - sin\phi cos\psi)/m \\ \dot{z} \\ g - (cos\phi cos\theta)\frac{1}{m}U_1 \end{pmatrix} \tag{8}$$

With simplified parameter a_i *and* b_i are :

$$\begin{array}{l|l} a_1 = (I_y - I_z)/I_x & b_1 = \frac{\sqrt{2}}{2}d/I_x \\ a_2 = J_r/I_x & b_2 = \frac{\sqrt{2}}{2}d/I_y \\ a_3 = (I_z - I_x)/I_y & b_3 = 1/I_z \\ a_4 = J_r/I_y & \\ a_5 = (I_x - I_y)/I_z & \end{array} \tag{9}$$

Sliding surface design using backstepping approach is suggested in [2][4]. New state variable for tracking control z_i as follows.

$$z_i = \begin{cases} x_{id} - x_i & \text{for i = } \{1,3,5,7,9,11\} \\ x_i - \dot{x}_{i-1d} - \alpha_{i-1}z_{i-1} & \text{for i = } \{2,4,6,8,10,12\} \end{cases} \tag{10}$$

Sliding surface design concept is similar to classical PD control with respect to error for tracking control.

$$\begin{cases} S_\phi = z_2 = x_2 - \dot{x}_{1d} - \alpha_1 z_1 \\ S_\theta = z_4 = x_4 - \dot{x}_{3d} - \alpha_3 z_3 \\ S_\psi = z_6 = x_6 - \dot{x}_{5d} - \alpha_5 z_5 \\ S_x = z_8 = x_8 - \dot{x}_{7d} - \alpha_7 z_7 \\ S_y = z_{10} = x_{10} - \dot{x}_{9d} - \alpha_9 z_9 \\ S_z = z_{12} = x_{12} - \dot{x}_{11d} - \alpha_{11} z_{11} \end{cases} \tag{11}$$

Lyapunov stability can be checked by setting augmented Lyapunov candidate function as follows.

$$V_i = \frac{1}{2}z_{i-1}^2 + \frac{1}{2}z_i^2 \qquad \text{where } i = \{2,4,6,8,10,12\} \tag{12}$$

First, By setting Lyapunov candidate function as :

$$V_1 = \frac{1}{2}z_1^2 \tag{13}$$

$$\dot{V}_1 = z_1\dot{z}_1 = z_1(\dot{x}_{1d} - x_2) \tag{14}$$

As for backstepping approach, taking x_2 as a virtual control input.

$$x_2 = \dot{x}_{1d} + \alpha_1 z_1 \qquad (\alpha_1 > 0) \tag{15}$$

With (15), Eq. (14) becomes as follows and satisfies Lyapunov stability.

$$\dot{V}_1 = -\alpha_1 z_1^2 \quad < 0 \tag{16}$$

As for sliding mode control, sliding condition($S\dot{S} < 0$) must be verified. For first sliding surface S_ϕ,

$$\begin{aligned} S_\phi\dot{S}_\phi &= z_2\dot{z}_2 \\ S_\phi\dot{S}_\phi &= z_2(a_1x_4x_6 + a_2x_4\Omega_r + b_1U_2 - \ddot{x}_{id} - \alpha_1 z_1) \end{aligned} \tag{17}$$

for $\dot{S}_\phi$,

$$\begin{aligned} \dot{S}_\phi &= -K_{\phi,1} sign(S_\phi) - K_{\phi,2} S_\phi \\ \dot{S}_\phi &= a_1 x_4 x_6 + a_2 x_4 \Omega_r + b_1 U_2 - \ddot{x}_{1d} - \alpha_1(\dot{x}_{1d} - x_2) \end{aligned} \quad (18)$$

The symbol *sign* is signum function, neglected $\ddot{x}_{1d}$ terms for simpification. Then with condition for control gains $K_{\phi,1}, K_{\phi,2} > 0$, control input U_2 can be extracted,

$$U_2 = \frac{1}{b1}\{K_{\phi,1} sign(S_\phi) + K_{\phi,2} S_\phi - a_1 x_4 x_6 - a_2 x_4 \Omega_r + \alpha_\phi(\dot{x}_{1d} - x_2)\} \quad (19)$$

This satisfies Lyapunov stability of candidate function $V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$. This can be done to any other sliding surface in Eq. (11) in a same manner. For all sliding surfaces, control inputs are as follows.

$$\begin{cases} U_2 = \frac{1}{b1}\{-K_{\phi,1} sign(S_\phi) - K_{\phi,2} S_\phi - a_1 x_4 x_6 - a_2 x_4 \Omega_r + \alpha_\phi(\dot{x}_{1d} - x_2)\} \\ U_3 = \frac{1}{b2}\{-K_{\theta,1} sign(S_\theta) - K_{\theta,2} S_\theta - a_3 x_2 x_6 - a_4 x_2 \Omega_r + \alpha_\theta(\dot{x}_{3d} - x_4)\} \\ U_4 = \frac{1}{b3}\{-K_{\psi,1} sign(S_\psi) - K_{\psi,2} S_\psi - a_5 x_2 x_4 + \alpha_\psi(\dot{x}_{5d} - x_6)\} \\ U_x = \frac{m}{U_1(c\phi s\theta c\psi + s\phi s\psi)}\{-K_{x,1} sign(S_x) - K_{x,2} S_x + \alpha_x(\dot{x}_{7d} - x_8)\} \\ U_y = \frac{m}{U_1(c\phi s\theta s\psi - s\phi c\psi)}\{-K_{y,1} sign(S_y) - K_{y,2} S_y + \alpha_y(\dot{x}_{9d} - x_{10})\} \\ U_1 = \frac{m}{(c\phi c\theta)}\{-K_{z,1} sign(S_z) - K_{z,2} S_z + \alpha_z(\dot{x}_{11d} - x_{12}) + g\} \end{cases} \quad (20)$$

Signum function is known as discrete function. With an actuator delay, signum function presents known chattering problem. To prevent chattering, replaced signum function with saturation fucntion as follows to make control input continous.

$$sat(x) = \left\{ \begin{array}{lll} 1, & \text{if} & x > 1 \\ x, & \text{if} & -1 \le x \le 1 \\ -1, & \text{if} & x < -1 \end{array} \right\} \quad (21)$$

A. Position Control

For the Quadrotor, x axis position control can be achived by pitch control, and also y position control can be acheived by roll control. To do that, need to convert U_x, U_y to roll & pitch commands [5].

$$\begin{aligned} U_x &= \frac{m}{U_1(c\phi s\theta c\psi + s\phi s\psi)}\{-K_{x,1} sign(S_x) - K_{x,2} S_x + \alpha_x(\dot{x}_{7d} - x_8)\} \\ U_y &= \frac{m}{U_1(c\phi s\theta s\psi - s\phi c\psi)}\{-K_{y,1} sign(S_y) - K_{y,2} S_y + \alpha_y(\dot{x}_{9d} - x_{10})\} \end{aligned} \quad (22)$$

Denominator of $U_x \& U_y$ has complex Euler angle rotation. Taking $\psi = 0$ to simpify it, then Eq. 21 becomes as follows.

$$\begin{aligned} U_x &= \frac{m}{U_1(c\phi s\theta)}\{-K_{x,1} sign(S_x) - K_{x,2} S_x + \alpha_x(\dot{x}_{7d} - x_8)\} \\ U_y &= \frac{m}{U_1(-s\phi)}\{-K_{y,1} sign(S_y) - K_{y,2} S_y + \alpha_y(\dot{x}_{9d} - x_{10})\} \end{aligned} \quad (23)$$

Then, roll & pitch command from position command can be extracted as follows.

$$\begin{aligned} \theta_d &= \ \ arcsin[(\frac{1}{U_1 c\phi})\{-K_{x,1} sign(S_x) - K_{x,2} S_x + \alpha_x(\dot{x}_{7d} - x_8)\}] \\ \phi_d &= -arcsin[(\frac{1}{U_1})\{-K_{y,1} sign(S_y) - K_{y,2} S_y + \alpha_y(\dot{x}_{9d} - x_{10})\}] \end{aligned} \quad (24)$$

Fig. 2: Interboard-Multwii Model

IV. Simulation

Simulations were made using designed sliding mode controller above. Simulation environment is based on MATLAB/Simulink using open-source 'Quadcopter Dynamic Modeling and Simulation (Quad-Sim) v1.00' simulator[1]. Simulation model is based on real model we have, Model informations are :

$$\begin{cases} m = \ 1.05 \ \ kg \\ I_x = \ 0.012706 \ \ kg \cdot m^2 \\ I_y = \ 0.012706 \ \ kg \cdot m^2 \\ I_z = \ 0.024253 \ \ kg \cdot m^2 \\ d = \ 0.25 \ \ m \\ c_t = \ 1.86 \, x \, 10^{-7} \ \ N/RPM^2 \\ c_q = \ 5.7 \, x \, 10^{-8} \ \ N \cdot m/RPM^2 \\ \tau = \ 0.1 \ \ sec \end{cases}$$

And all intiial conditions are zero (Euler angles, body rates, linear velocity & position), except all motor speed were set to 2,000 RPM as ready to take-off situation.

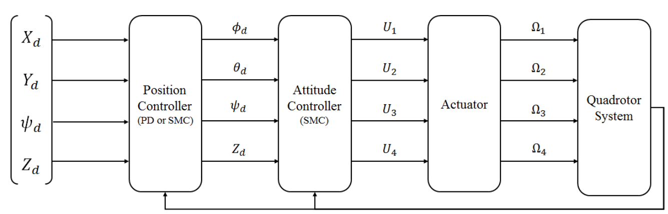
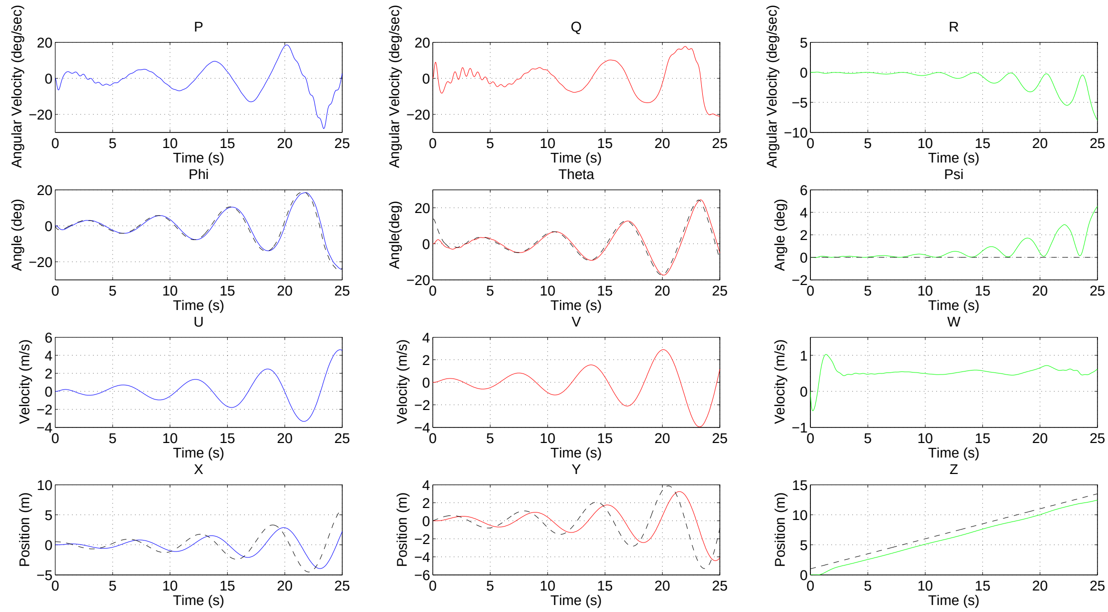


Fig. 3: Control Scheme of Proposed Controller

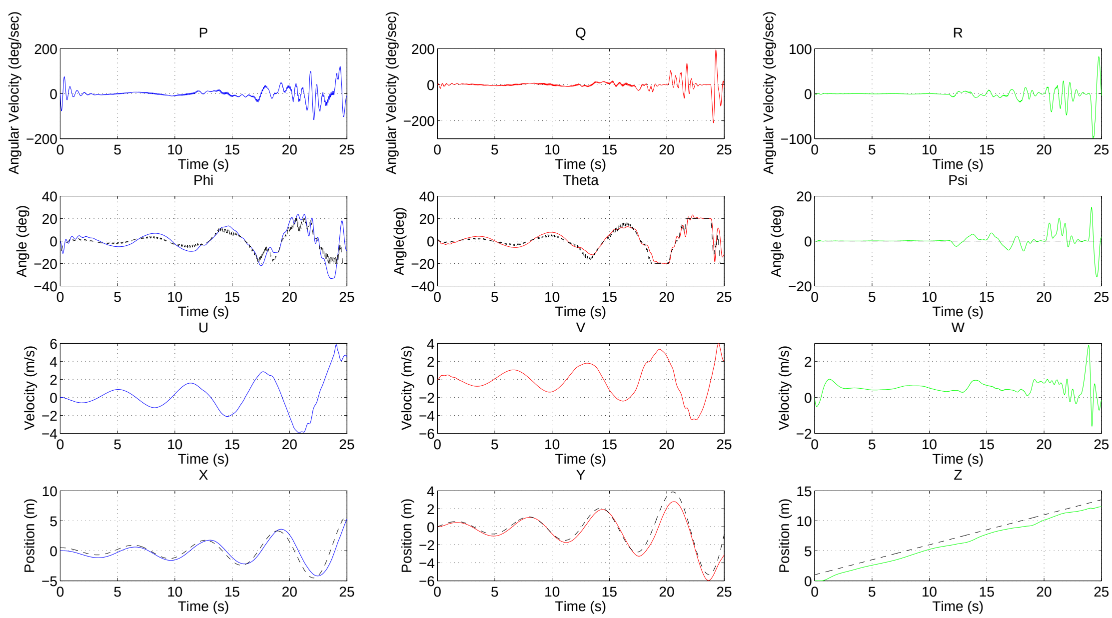
Fig. 3 shows control scheme, acutator(motor), quadrotor system. In simulation, attitude controller & altitude controller are proposed sliding mode controller, but position controllers(outer loop) are two schemes. First one is classical PD controller and second one is sliding mode position controller.

To ensure quadrotor stable flying, Euler angle command (ϕ_d, θ_d) is constrained limits to ± 30 degrees. For comparison, position command are given as exponential helix as follows.

$$\begin{aligned} X_{cmd} &= 0.5e^{0.1t} cos(t) \\ Y_{cmd} &= 0.5e^{0.1t} sin(t) \\ Z_{cmd} &= 0.5t + 1 \end{aligned} \quad (25)$$



(a) Simulation Result Using PD Controller - States



(b) Simulation Result Using Sliding Mode Controller - States

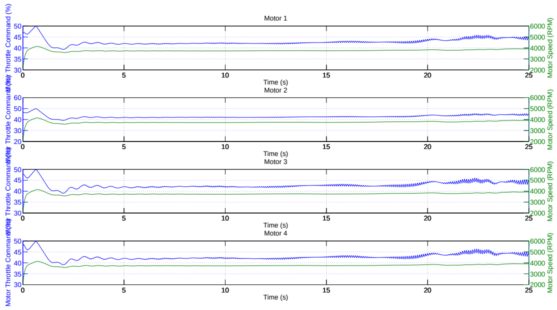
Result graphs of simulation is at next page through Fig. 4 to 7. First two figures(Fig. 4 to 5) shows states. Dashed line indicates command, and solid line indicates true states. Next two figures(Fig. 5 to 6) shows control inputs. Blue solid line indicates throttle, and green solid line indicates RPM of motors.

Simulation Results with PD position controller shows smooth response, follows command accordingly but with some delays. sliding mode altitude controller shows steady-state error regardless of control gains with proposed design.

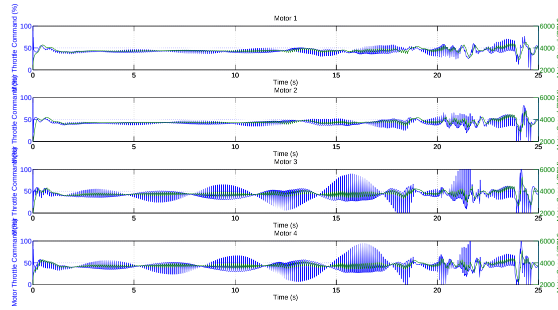
On the other hand, simulation using sliding mode position controller tracks command with less delay than PD control. But also requires much more control gains on attitude controller than PD one to try to track vibrating attitude commands.

V. CONCLUSIONS

In this paper, Adopted sliding mode control based on backstepping approach. Designed sliding mode controller for position & attitude control. Also compared classical



(a) Simulation Result Using PI Controller - RPM



(b) Simulation Result Using Slding Mode Controller - RPM

PD & sliding mode position controller with complex helix path command in simulation. classical PD controller shows smooth, better tracking performance than sliding mode controller. sliding mode position controller generates vibrating attitude command. Causes hard switching commands that attitude controller barely follows. From result of simulation, proposed design of sliding mode position controller also shows good tracking performance about position. but demands high angular velocity which would cause not ignorable aerodynamic drag in real environment. Aerodynamic drag would degrade performance. In short, proposed sliding mode position controller is not suite for real environment.

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