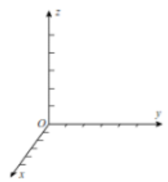
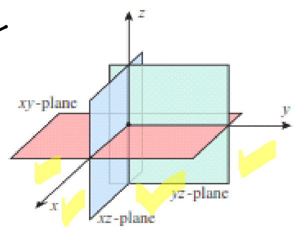
Chapter # 11

quadrant System and octant System

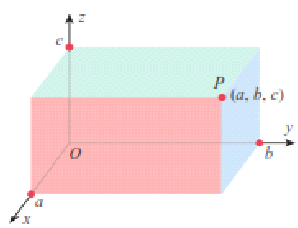


▲ Figure 11.1.1

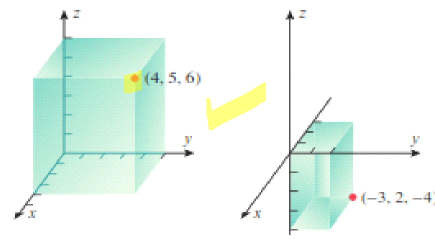
line in 3-Space → 11.5
plane 3-Space → 11.6



▲ Figure 11.1.3



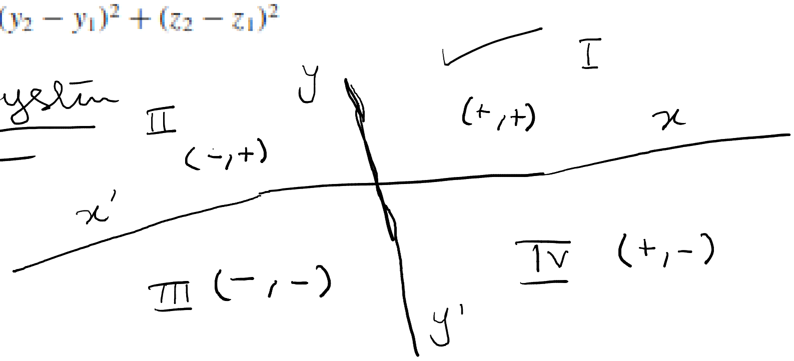
▲ Figure 11.1.4



▲ Figure 11.1.5

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Quadrant System



(−2,4)

(−1,−1)

Octant System

(3, 4, 2)

(x, y, 3)

(−3, −2, −4)

I (+, +, +)

II (−, +, +)

III (−, −, +)

IV (+, −, +)

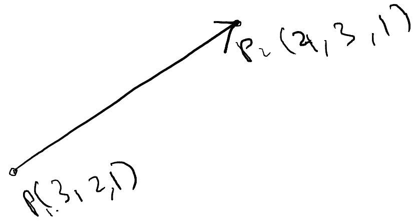
V (+, +, −)

VI (−, +, −)

VII (−, −, −)

VIII (+, −, −)

point in 3-space → $P_1(3, 2, 1)$



x_1, y_1, z_1

$P_2(4, 3, 1)$

$x_2 \quad y_2 \quad z_2$

Vector in 3-space

$\vec{P_1P_2} = \vec{V} = \underline{P_2 - P_1} = \langle 4-3, 3-2, 1-1 \rangle$

$= \underline{\langle 1, 1, 0 \rangle}$

$\vec{V} = \underline{1\hat{i} + 1\hat{j} + 0\hat{k}}$

norm of vector / Magnitude of vector

$|\vec{V}|$ ← $\|\vec{V}\| = \sqrt{(1)^2 + (1)^2 + 0^2}$

$= \sqrt{2}$

Dot product:

$\vec{V}_1 = \langle 1, 2, 3 \rangle$

$\vec{V}_2 = \langle 2, 3, 1 \rangle$

$\vec{V}_1 \cdot \vec{V}_2 = \langle 1, 2, 3 \rangle \cdot \langle 2, 3, 1 \rangle$

$= 2 + 6 + 3$

$= 11$

Cross product:

$\vec{V}_1 \times V_2 = | \; i \quad j \quad k \; |$

$$\underbrace{\vec{V}_1 \times V_2} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= i(2-9) - j(1-6) + k(3-4)$$

Vector → ? $\underbrace{\vec{V}_1 \times \vec{V}_2} = -7i + 5j - k$

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angle b/w vector

$$\underbrace{\vec{V}_1 \cdot \vec{V}_2} = \underbrace{\|V_1\| \|V_2\| \cos\theta}$$

$$\cos\theta = \frac{\vec{V}_1 \cdot \vec{V}_2}{\|V_1\| \|X_2\|}$$

$$\theta = \cos^{-1}\left(\frac{\vec{V}_1 \cdot \vec{V}_2}{\|V_1\| \|V_2\|}\right)$$

$$\begin{vmatrix} 2 & -3 & 1 \\ 4 & 1 & -3 \\ 0 & 1 & 5 \end{vmatrix} = \_\_\_\_$$

Scalar triple product

21–24 Find $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

21. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{w} = \mathbf{j} + 5\mathbf{k}$ — 80

22. $\mathbf{u} = \langle 1, -2, 2 \rangle$, $\mathbf{v} = \langle 0, 3, 2 \rangle$, $\mathbf{w} = \langle -4, 1, -3 \rangle$ — 29

$\boxed{U \cdot (\underline{V \times W})}$

$(V \times W) \cdot U$

two vector are parallel

they are scalar multiple

$V_1 = \langle 2, 6, 8 \rangle$

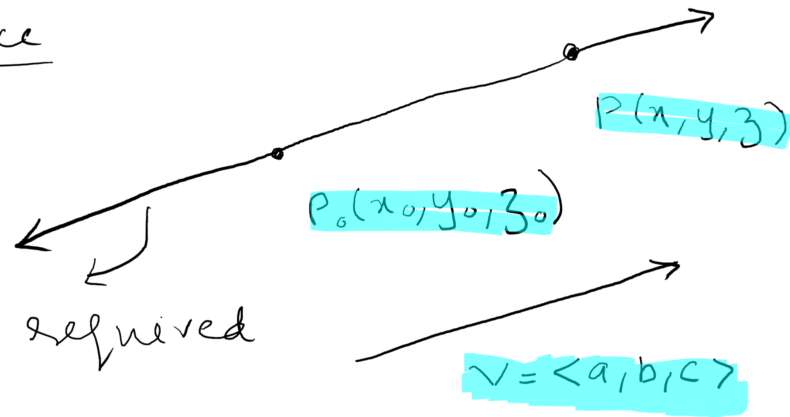
$V_2 = \langle 1, 3, 4 \rangle$

$\vec{V}_1 = 2\vec{V}_2$

$\langle 2, 6, 8 \rangle = 2\langle 1, 3, 4 \rangle$

$\langle 2, 6, 8 \rangle = \langle 2, 6, 8 \rangle$

line in 3-Space



$$\vec{P_0P} = \langle x - x_0, y - y_0, z - z_0 \rangle$$

$$\vec{v} = \langle a, b, c \rangle$$

Scalar multiple $\vec{P_0P} = t\,\vec{v}$

$$\langle x - x_0, y - y_0, z - z_0 \rangle = t \langle a, b, c \rangle$$

$$\langle x - x_0, y - y_0, z - z_0 \rangle = \langle at, bt, ct \rangle$$

$$\left.\begin{aligned} x - x_0 &= at \\ y - y_0 &= bt \\ z - z_0 &= ct \end{aligned}\right]$$

equations of line in 3-Space

or

parametric eq

$P(0, 1, 2)$, $v = \langle 1, 2, 3 \rangle$
(x_0, y_0, z_0) $\quad \langle a, b, c \rangle$

$x - 0 = 1t \longrightarrow x = t$

$y - 1 = 2t \longrightarrow y - 1 = 2t$

$x - 0 = 1t \longrightarrow x = t$

$y - 1 = 2t \longrightarrow y - 1 = 2t$

$z - 2 = 3t \longrightarrow z - 2 = 3t$

► **Example 1** Find parametric equations of the line

(a) passing through (4, 2) and parallel to $\mathbf{v} = \langle -1, 5 \rangle$;
(b) passing through (1, 2, −3) and parallel to $\mathbf{v} = 4\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$;
(c) passing through the origin in 3-space and parallel to $\mathbf{v} = \langle 1, 1, 1 \rangle$.

$x - x_0 = at$

$y - y_0 = bt$

$z - z_0 = ct$

$x_0 = 1$

$y_0 = 2$

$z_0 = -3$

$a = 4$

$b = 5$

$c = -7$

3–4 Find parametric equations for the line through P_1 and P_2 and also for the line segment joining those points.

3. (a) $P_1(3, -2)$, $P_2(5, 1)$ (b) $P_1(5, -2, 1)$, $P_2(2, 4, 2)$

4. (a) $P_1(0, 1)$, $P_2(-3, -4)$
(b) $P_1(-1, 3, 5)$, $P_2(-1, 3, 2)$

$\vec{P_1P_2} = \langle 5-3, 1+2 \rangle$

$\vec{v} = \langle 2, 3 \rangle$

$= \langle a, b \rangle$

(x_0, y_0)

$P_1(3, -2)$

$x - x_0 = at \longrightarrow x - 3 = 2t$

$y - y_0 = bt \longrightarrow y + 2 = 3t$

Line are Skew $\longrightarrow$ line are not parallel
line are not intersecting.

$\frac{x-1}{7} = t \rightarrow \frac{x-1}{7} = \frac{y-3}{1} = \frac{z-5}{-3} = t$ Symmetric eq of line

$y - 3 = t$

$\frac{z-5}{-3} = t$

31–32 Show that the lines L_1 and L_2 are skew.

31. L_1: $x = 1 + 7t$, $y = 3 + t$, $z = 5 - 3t$
L_2: $x = 4 - t$, $y = 6$, $z = 7 + 2t$

32. L_1: $x = 2 + 8t$, $y = 6 - 8t$, $z = 10t$
L_2: $x = 3 + 8t$, $y = 5 - 3t$, $z = 6 + t$

$x - x_0 = at$

$y - y_0 = bt$

$z - z_0 = ct$

$\frac{3-3}{-3}=t$

32. $L_1: x = 2 + 8t,\ \ y = 6 - 8t,\ \ z = 10t$
$L_2: x = 3 + 8t,\ \ y = 5 - 3t,\ \ z = 6 + t$

$z - z_0 = ct$

Q31

$L_1 \rightarrow P_1(1, 3, 5)$

$L_2 \rightarrow P_2(4, 6, 7)$

$V_1 = \langle 7, 1, -3 \rangle$

$V_2 = \langle -1, 0, 2 \rangle$

Step I (parallel check)

$V_1 = \langle 7, 1, -3 \rangle$

$V_2 = \langle -1, 0, 2 \rangle$ $\longrightarrow$ ✓

V_1 & V_2 are not parallel b/c
there V_1 & V_2 are not scalar Multiple

Step II: (Compare and find t_1 & t_2)

$1 + 7t_1 = 4 - t_2 \longrightarrow (1)$

$3 + t_1 = 6 \longrightarrow (2)$

$5 - 3t_1 = 7 + 2t_2 \longrightarrow (3)$

Consider eq (2)

$3 + t_1 = 6$

$\boxed{t_1 = 3}$

put $t_1 = 3$ in eq (1)

$1 + 7(3) = 4 - t_2$

$22 = 4 - t_2$

$\boxed{-18 = t_2}$

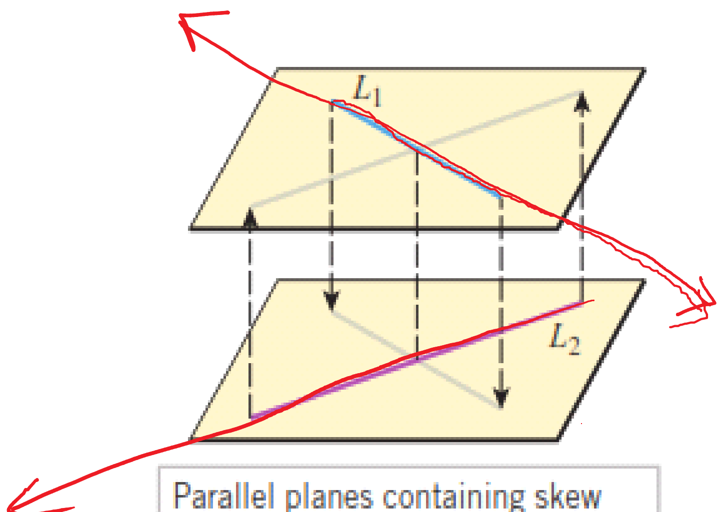
Step III (put $t_1 = 3$, $t_2 = -18$ in eq (3))

$5-3t_1 = 7+2t_2$

$5-3(3) = 7+2(-18)$

$-4 \neq -29$

Line are Skew.



Parallel planes containing skew lines L_1 and L_2 can be determined by translating each line until it intersects the other.

Figure 11.5.3

Q $V_1 = \langle 8, -8, 10 \rangle$

$V_2 = \langle 8, -3, 1 \rangle$

Step I:- (parallel check)

l_1 & l_2 are not parallel b/c V_1 & V_2 are not scalar multiple.

Step II:

32. $L_1: x = 2 + 8t,\ y = 6 - 8t,\ z = 10t$
$L_2: x = 3 + 8t,\ y = 5 - 3t,\ z = 6 + t$

$2 + 8t_1 = 3 + 8t_2 \rightarrow (1)$

$6 - 8t_1 = 5 - 3t_2 \longrightarrow$ 2

$10t_1 = 6 + t_2 \longrightarrow$ 3

Consider eq 1 ~~&~~ 2 add 1 & 2

$$2 + \cancel{8t_1} = 3 + 8t_2$$
$$6 - \cancel{8t_1} = 5 - 3t_2$$

$$8 = 8 + 5t_2$$

$$\boxed{0 = t_2}$$

put $t_2 = 0$ in eq 1

$$2 + 8t_1 = 3 + 8(0)$$

$$2 + 8t_1 = 3 + 0$$
$$8t_1 = 3 - 2$$

$$\boxed{t_1 = 1/8}$$

Step III (put $t_1 = 1/8$ and $t_2 = 0$ in eq 3)

$$10t_1 = 6 + t_2$$
$$10\left(\frac{1}{8}\right) = 6 + (0)$$
$$5/4 \neq 6$$

line are Skew