# OR

Date:

## Theory Of Games :

In 1928 (Von Neumann)
called the father of game theory

In 1944 (Neumann S. Morgenstren)

Theory of Games and Economics Behaviour

Competitive Games — is called C.G if it has following 6 properties.

i

ii — $n \geq 2$ ↓ no. of player

⋮

iv

payoff → is the outcome of playing the game.

A payoff (gain or game)

Player B

$$
\begin{array}{c|cccccccc|l}
 & 1 & 2 & 3 & \cdots & j & \cdots & n & \\
\hline
1 & a_{11} & a_{12} & a_{13} & \cdots & a_{ij} & \cdots & a_{1n} & \text{A's pay off matrix} \\
2 & & a_{22} & \cdots & & & & a_{2n} & \\
3 & \vdots & & & & & & \vdots & \\
i & a_{i1} & & \cdots & & & & a_{in} & \\
\vdots & \vdots & & & & & & \vdots & \\
m & a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mj} & \cdots & a_{mn} &
\end{array}
$$

Player A

Date:

Player B

Player A

$$
\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & \cdots & j & \cdots & n \\
\hline
1 & -a_{11} & -a_{12} & & & -a_{1j} & & -a_{1n} \\
2 & & & & & & & -a_{2n} \\
3 & & & & & & & \vdots \\
\vdots & & & & & & & \vdots \\
i & -a_{i1} & -a_{i2} & -a_{i3} & & & & -a_{in} \\
\vdots & & & & & & & \vdots \\
m & -a_{m1} & -a_{m2} & \cdots & & -a_{mj} & \cdots & -a_{mn}
\end{array}
$$

Player B

| Player A | Strategy 3 | Strategy 4 | |
|---|---|---|---|
| Strategy 1 | + 4 | + 6 | 4 |
| Strategy 2 | + 3 | + 5 | 3 |
| | 4 | 6 | |

Date:

## Q: Two Person Zero-Sum Game With Saddle Point:

In ~~a point~~ a certain game, player A has 3 possible choices L, M and N while player B has 2 possible choices P and Q. Payments are to be made a/c to the choices made?

| Choices | Payments |
|---|---|
| L, P | A pay B \$ 3 |
| L, Q | B pay A \$ 3 |
| M, P | A pay B \$ 2 |
| M, Q | B pay A \$ 4 |
| N, P | B pay A \$ 2 |
| N, Q | B pay A \$ 3 |

What are the best strategies for player A and B in this game? what is the value of the game for A and B?

Sol) Let +ve number represent a payment from B to A and negative number a payment from A to B then we have the pay off matrix shown in Table - 1.

Date:

Table-1

Player B

Plans (Choices)

| | | P | Q | min. of row |
|---|---|---|---|---|
| Plans Player A (choices) | L | -3 | 3 | -3 |
| | M | -2 | 4 | -2 |
| | N | 2 | 3 | 2 Maximin |
| max of col. | | 2 | 4 | |
| | | Minimax | | |

Zero Sum Game = 2.

{-3, -2, 2}
⇒ 2

{2, 4}
⇒ 2

(Player A, Player B)
(N, P) Strategies

Game Value = +2

Saddle Point (N, P)

Game Value = +2.

B = 0

$$A \begin{bmatrix} -4 & 3 \\ -3 & -7 \end{bmatrix}$$

No saddle point exists since there is no ele which is the lowest in its row and highest in column.

$$\begin{array}{c|cc|c} & B & & \\ \hline A & 3 & 2 & 2 \\ & -2 & -3 & -3 \\ & -4 & -5 & -5 \\ \hline & 3 & 2 & \end{array}$$

Strategies : A row 1 and B col 2.

Saddle Point : (1, 2)

Game Value : +2

Date:

$$\begin{array}{c c} & B \\ A & \begin{bmatrix} 1 & 13 & 11 \\ -9 & 5 & -11 \\ 0 & -3 & -3 \end{bmatrix} \end{array}$$

Saddle Point : (1, 1)

Strategy : A, row 1

B, col 1

Game Value : +1

$$\begin{array}{c c c} & B & \\ A & \begin{bmatrix} 16 & 4 & 0 & 14 & -2 \\ 10 & 8 & \boxed{6} & 10 & 12 \\ 2 & 6 & 4 & 8 & 14 \\ 8 & 10 & 2 & 2 & 0 \end{bmatrix} & \begin{matrix} \text{min} \\ -2 \\ 6 \\ 2 \\ 0 \end{matrix} \\ \text{Max} & \begin{matrix} 16 & 10 & 6 & 14 & 14 \end{matrix} & \end{array}$$

Saddle Point : (2, 3)

Strategy :

Row 2 : A

B : col 3

Game Value : +6.

If the value is 0 then we can say that the game is fair.

$$\{-2, \underset{\substack{\uparrow \\ \text{max}}}{6}, 2, 0\} = 6$$

$$\{16, 10, \underset{\substack{\uparrow \\ \text{min}}}{6}, 14, 14\} = 6$$

Date:

Q1:

Find ranges of values of P and Q which will render the entry (2,2) saddle point for the game.

$$\begin{array}{c|ccc} & & \text{Player B} & \\ & 2 & 4 & 5 \\ \text{Player A} & \left[\begin{array}{c}10\\4\end{array}\right. & \begin{array}{c}7\\p\end{array} & \left.\begin{array}{c}q\\6.\end{array}\right] \end{array}$$

Rule 2: Reduce by dominance.
If no pure strategies exist, the next step is to eliminate certain strategies (row and/or column) by dominance. The resulting game can be solved by; some mixed strategy.

Q2: (3x3 Game, Matrix Reduction by Dominance)
Two players, play a game. Each of them has to
P and Q
choose one of the three colors White (W), Black (B), and Red (R) individually
Shown below find the optimum strategies for P and Q and the value of game.

Sol:

Q1

~~Row Minimax~~ Date:

| | Player B | | | Row minimax |
|---|---|---|---|---|
| Player A | 2 | 4 | 5 | 2 |
| | 10 | 7 | q | 7 |
| | 4 | p | 6 | 4 |
| Col max | 10 | 7 | 6 | |

This improved cond. on p and q is

$q \le 7$ or $p \ge 7$.

$\therefore$ Maximin value = 7

Minimax value = 7.

Saddle Point (2, 2).

~~Rule 1~~

$q \le 7$ OR $p \ge 7$.

It is based col and row value choosing strategy and will vary w/ the pos. of variables in the table given.

Date:

| | | Col ured chosen by Q W | B | R | Row Min |
|---|---|---|---|---|---|
| Coloured chosen by P | W | 0 | -2 | 7 | -2 |
| | B | 2 | 5 | 6 | 2 |
| | R | 3 | -3 | 8 | -3 |
| Col Max | | 3 | 5 | 8 | |

No common Value, No saddle point.
Rule #01 failed so Rule #02 will be applied.

$R_2 > R_1$ ×

Dominance Rule For Col B:

Every value in the dominy col(s) must be less than or equal to the corresponding value of the dominated col.

| | W | B |
|---|---|---|
| W | 0 | -2 |
| B | 2 | 5 |
| R | 3 | -3 |

$R_2 \geq R_1$

Take it easy:
Koi ek bh row dosri row sai choti hai tu choti wali row ko delete kardia

Date:

# Method for Matrices:

Solve the given game of following table:

| | | B 1 | 2 | 3 | Min |
|---|---|---|---|---|---|
| A | 1 | 7 | 1 | 7 | 1 |
| | 2 | 9 | -1 | 1 | -1 |
| | 3 | 5 | 7 | 6 | 5 |
| Max. | | 9 | 7 | 7 | |

max: 5

min: 7

To solve 3x3 or higher mixed strategy (3x3) Game.

Game Value lies b/w 5 and 7.

No saddle point exist.

| | | B | | | $C_1-C_2$, | $C_2-C_3$ | |
|---|---|---|---|---|---|---|---|
| A | 1 | 7 | 1 | 7 | 6 | -6 | 7-1 |
| | 2 | 9 | -1 | 1 | 10 | -2 | 9-(-1) |
| | 3 | 5 | 7 | 6 | -2 | 1 | 5-7 |
| | | -2 | 2 | 6 | | | -1-1=-2 |
| | | 4 | -8 | -5 | | | |

Next, calculate the oddment for $A_1, A_2, A_3$ and $B_1, B_2$ and $B_3$.

Oddment for $A_1$ = det $\begin{vmatrix} 10 & -2 \\ -2 & 1 \end{vmatrix} = 10-4 \Rightarrow 6$

Date:

Oddment for $A_2 = \det \begin{vmatrix} 6 & -6 \\ -2 & 1 \end{vmatrix} = 12 - 6 \Rightarrow 6$

Oddment for $A_3 = \det \begin{vmatrix} 6 & -6 \\ 10 & -2 \end{vmatrix} = -12 + 60 = 48$

Oddment for $B_1 = \det \begin{vmatrix} 2 & 6 \\ -8 & -5 \end{vmatrix} =$ $\qquad \frac{\cancel{14}\,7}{\cancel{60}\,30}$

$B_2 = \det \begin{vmatrix} 2 & 6 \\ 4 & -5 \end{vmatrix} =$

$B_3 = \det \begin{vmatrix} -2 & 2 \\ 4 & -8 \end{vmatrix} =$ $\qquad \frac{\cancel{10}}{\cancel{10}} = 1$

B

| | | 1 | 2 | 3 | | | | |
|---|---|---|---|---|---|---|---|---|
| A | 1 | 7 | 1 | 7 | 6 | 1 | $\frac{1}{10}$ | $\frac{3}{30}$ |
| | 2 | 9 | -1 | 1 | 6 | 1 | $\frac{1}{10}$ | $\frac{3}{30}$ |
| | 3 | 5 | 7 | 6 | 48 | 8 | $\frac{8}{10}$ | $\frac{24}{30}$ |
| | | 38 | 14 | 8 | 60 | | | |
| | | 19 | 7 | 4 | | | | |
| | | $\frac{19}{30}$ | $\frac{7}{30}$ | $\frac{4}{30}$ | | | | |

$\frac{6}{60} = \frac{1}{10}$

$\frac{6}{60} = \frac{1}{10}$

$\frac{\cancel{48}\,8}{\cancel{60}\,10} = \frac{8}{10}$

$\frac{19}{30} \quad \frac{9}{30} \quad \frac{4}{30}$

$\frac{\cancel{32}\,16}{\cancel{30}\,15}$

Thus, optimal status are

$A\left(\frac{3}{30}, \frac{3}{30}, \frac{24}{30}\right) \qquad B\left(\frac{19}{30}, \frac{7}{30}, \frac{4}{30}\right)$

Date:

Game Value = $V = \dfrac{7\times1 + 9\times1 + 5\times8}{1+1+8} = \dfrac{28}{5} = 5.6$

## Graphical Method: (2×n or m×2 Games).

Solve the game given in the table 1 by graphical method.

| | $y_1$ | $y_2$ | $y_3$ | $y_4$ | Min |
|---|---|---|---|---|---|
| $y_1$ | 19 | 6 | 7 | 5 | 5 |
| $y_2$ | 1 | 3 | 14 | 6 | 1 |
| $y_3$ | 12 | 8 | 18 | 6 | 6 |
| $y_4$ | 8 | 7 | 15 | -1 | -1 |
| Max | 19 | 8 | 18 | 9 | b/w 5 & 6. |

Sol:

Check Saddle Point:

( 2×n OR m×2 Game )

Date:

## Graphical Method:

Solve the game given in table -1 by graphic method.

$$
\begin{array}{c|cccc|cc}
 & & & B & & & \\
 & y_1 & y_2 & y_3 & y_4 & \text{min} & \text{Minmax} \\
\hline
x_1 & 19 & 6 & 17 & 5 & 5 & 5 \\
x_2 & 7 & 3 & 14 & 6 & 3 & \\
x_3 & 12 & 8 & 18 & 4 & 4 & \\
x_4 & 8 & 7 & 13 & -1 & -1 & \\
\hline
\text{max} & 19 & 8 & 18 & 6 & \text{Maximin} & 6.
\end{array}
$$

A

No saddle point found.

Now use dominance Rule.

$C_1$ dominates $C_2$ and $C_4$.
$C_1$ deleted.

$C_2$ dominates $C_1$ and $C_3$
$C_2$ deleted

$$
\begin{bmatrix} 6 & 7 & 5 \\ 3 & 14 & 6 \\ 8 & 18 & 4 \\ 7 & 13 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 5 \\ 3 & 6 \\ 8 & 4 \\ 7 & -1 \end{bmatrix}
$$

Date:

Comparing $R_3$ and $R_4$
delete $R_4$.

$$\begin{array}{c|cc} & y_2 & y_4 \\ \hline x_1 & 6 & 5 \\ x_2 & 3 & 6 \\ x_3 & 8 & 4 \end{array}$$

Now, this matrix cannot be solved by dominance rule. Now use graphical method.

B

$$A \quad \begin{array}{c|cc} & y_2 & y_4 \\ \hline x_1 & 6 & 5 \\ x_2 & 3 & 6 \\ x_3 & 8 & 4 \end{array}$$

$6y^2 + 5(1-y^2)$
$3y^2 + 6(1-y^2)$.

$$\begin{array}{c|cc} & y_2 & y_4 = 1-y_2 \\ \hline x_1 & 6 & 5 \\ x_2 & 3 & 6 \\ x_3 & 8 & 4 \end{array}$$

B's expected payoff corresspending to A's pure strategies are given below.

$5 - 5y^2$

Date:

A's pure Strategies B's expected payoffs.

$$\left[\begin{array}{ll} 1 & 6y_2 + 5(1-y_2) = y_2 + 5 \\ 2 & 3y_2 + 6(1-y_2) = -3y_2 + 6. \\ 3 & 8y_2 + 4(1-y_2) = 4y_2 + 4. \end{array}\right]$$

The resulting 2×2 game is shown below:

|  |  | B $y_2$ | $y_4$ |  |  |
|---|---|---|---|---|---|
| A | $x_1$ | 6 | 5 | 3 | 3/4 |
|  | $x_2$ | 3 | 6 | 1 | 1/4 |
|  |  | 1 | 3 | 4 |  |
|  |  | 1/4 | 3/4 |  |  |

continued ...

Optimum Strategies are $A\left(\frac{3}{4}, \frac{1}{4}, 0, 0\right)$.

Optimum Strategies are $B\left(0, \frac{1}{4}, 0, \frac{3}{4}\right)$.

Value Of The Game, $V = \dfrac{6 \times 1 + 3 \times 5}{1+3}$

$$V = \frac{21}{4} = 5.25.$$

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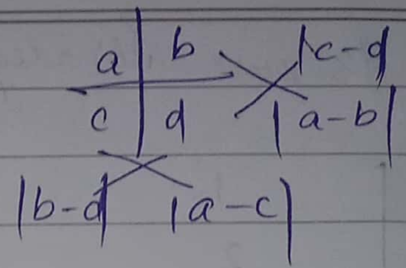

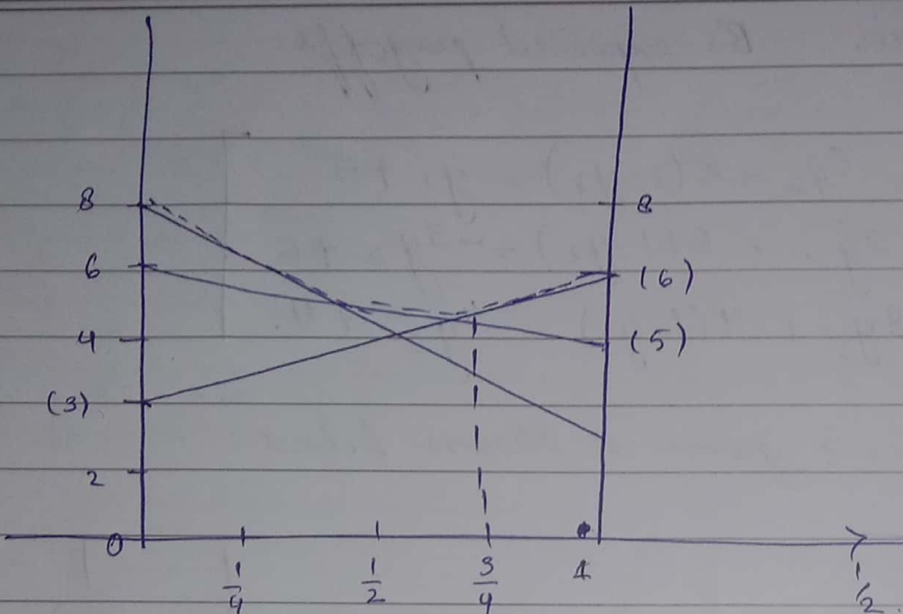

Continued.... Remaining

These 3 straight lines can be plotted as function of $y_2$ as follows:

Draw 2 lines $B_2$ and $B_4$ parallel to each other one unit apart & mark a scale on each of them (see fig). These 2 lines represent the 2 strategies available to B. ~~The~~ represent A's 1st strategies first mark 6 on $B_2$ with marks 5 on $B_4$; To represent A's second strategy join mark 3 on $B_2$ with mark 6 on $B_4$; and so on and bound the figure from above as shown. Since player B wishes to minimize his maximum expected losses, the two lines which intersect at the lowest point of the pure strategy i.e: $A_1$ and $A_2$. We can thus immidiately reduce the 3x2 game to 2x2 game which can be easily solved by arithmetic method. The resulting 2x2 game is shown the following in table.

Date:

Q

$$\begin{array}{cc} & \text{Caltex} \\ \text{PSO} & \begin{bmatrix} 4 & 1 & -3 \\ 3 & 1 & 6 \\ -3 & 4 & -2 \end{bmatrix} \end{array}$$

Determine the optimum strategies for 2 oil companies

$$\left[\begin{array}{ccc|c} 4 & 1 & -3 & -3 \\ 3 & 1 & 6 & 1 \\ -3 & 4 & -2 & -2 \end{array}\right]$$

4 4 6

4

1

No saddle point

The value will lie b/w 1 and 4

$$\begin{array}{cc} \text{Caltex} & \\ \begin{bmatrix} 4 & 1 & -3 \\ 3 & 1 & 6 \\ 3 & 4 & -2 \end{bmatrix} & \text{PSO} \end{array}$$

Dominance rule cannot be apply

Now apply method of matrix.

$$\begin{bmatrix} 4 & 1 & -3 \\ 3 & 1 & 6 \\ -3 & 4 & -2 \end{bmatrix} \begin{array}{cc} 3 & 4 \\ 2 & -5 \\ -7 & 6 \end{array}$$

1 0 -9

6 -3 8

27 62 3

92/116

Date:

Since, the sum of the oddments of A and B are not equal. The problem cannot be solved by the method of matrices. The method of linear prog will be used to solve it.

Odd $A_1 = \det \begin{vmatrix} 2 & -5 \\ -7 & 6 \end{vmatrix}$

$A_2 = \det \begin{vmatrix} 3 & 4 \\ -7 & 6 \end{vmatrix}$

$A_3 = \det \begin{vmatrix} 3 & 4 \\ 2 & -5 \end{vmatrix}$

$B_1 = \det \begin{vmatrix} 0 & -9 \\ -3 & 8 \end{vmatrix}$

$B_2 - \det \begin{vmatrix} 1 & -9 \\ 6 & 8 \end{vmatrix}$

$B_3 = \det \begin{vmatrix} \cancel{4} & 0 \\ 6 & -3 \end{vmatrix}$

Let the value of the game (to A) be V.
Consider the game from B's point of view.
B is trying to minimize V. Then;

Let,

against $A_1$ $\quad 4y_1 + y_2 - 3y_3 \leq V$

$A_2$ $\quad 3y_1 + y_2 + 6y_3 \leq V$

$A_3$ $\quad -3y_1 + 4y_2 - 2y_3 \leq V$

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$y_1 + y_2 + y_3 = 1$ (Sum of problem must be equal to 1).

where ; $y_1, y_2, y_3 \geq 0$.

Divide each of above relation by V.

$$4\frac{y_1}{V} + \frac{y_2}{V} - 3\frac{y_3}{V} \leq 1$$

$$3\frac{y_1}{V} + \frac{y_2}{V} + \frac{6y_3}{V} \leq 1$$

$$-3\frac{y_1}{V} + 4\frac{y_2}{V} - 2\frac{y_3}{V} \leq 1.$$

$$\frac{y_1}{V} + \frac{y_2}{V} + \frac{y_3}{V} = \frac{1}{V}$$

put $\frac{y_j}{V} = Y_j$. $j = 1, 2, 3$

we get subset to constants..

$$\left.\begin{aligned} 4Y_1 + Y_2 - 3Y_3 &\leq 1 \\ 3Y_1 + Y_2 + 6Y_3 &\leq 1 \\ -3Y_1 + 4Y_2 - 2Y_3 &\leq 1 \end{aligned}\right\} \text{— (1)}$$

$Y_1 + Y_2 + Y_3 = \frac{1}{V}$ — (2) → Objective fn

$Y_1, Y_2, Y_3 \geq 0$.

Since B is trying to minimize V. he must minimize $\frac{1}{V}$. Thus the prob is to maximize objective function (eq (2)) subj to constraints (1)

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which can be done by simplex method under following steps . .

Saturday :

## Markovian Decision Process :

Soil Condition

State (1) good (2) fair (3) poor.

State of the sys. next year.

State of sys this year

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & & & \\ 2 & & & \\ 3 & & & \end{array}\quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 0.1 \end{bmatrix} = p^{1}$$

$$p^{2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.3 & 0.6 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0.05 & 0.4 & 0.55 \end{bmatrix} \end{array}$$

Reward function :

$$R^{1} = \| r_{ij} \| = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 7 & 6 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & -1 \end{bmatrix} \end{array}$$

Date:

$$R_2 = \| r_{ij}^{2} \| = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 6 & 5 & -1 \\ 2 & 7 & 4 & 0 \\ 3 & 6 & 3 & -2 \end{array}$$

## Finite-Stage Dynamic Programming Mid.

Let $k=1$ and 2 represents the alternatives available to the gardner.

$P^k$ S $R^k$ – with transition probability & reward for

$$P^1 = \| p_{ij} \| = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}, \quad R^1 = \| r_{ij} \| = \begin{bmatrix} 7 & 6 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$P^2 = \| p_{ij}^{2} \| = \begin{bmatrix} 0.3 & 0.6 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0.05 & 0.4 & 0.55 \end{bmatrix}, \quad R^2 = \| R_{ij}^{2} \| = \begin{bmatrix} 6 & 5 & -1 \\ 7 & 4 & 0 \\ 6 & 3 & -2 \end{bmatrix}$$

Date:

Dynamic Programming:

Suppose that number of state for each stage (year) is $m$ (= 3 in gardener's example).

$f_n(i)$ = Optimal expected revenue of stages:

$n, n+1, \ldots\ldots, N$ given that the state of the system (soil condition) at the beginning of year $n$ is $i$.

The backward recursive equation relating $f_n$ and $f_{n+1}$ can be written as:

$$f_n(i) = \max_k \left\{ p_{ij}^k \left[ r_{ij}^k + f_{n+1}(j) \right] \right\}, \quad n = 1, 2, 3, \ldots$$

where;

$$f_{N+1}(j) = 0, \text{ for all } j$$

The equation is that the cummulative revenue, $r_{ij}^k + f_{n+1}(j)$, resulting from reaching state $j$ at stage $n+1$ from state $i$ at stage $n$ occurs w/ probabily $p_{ij}^k$

Letting,

$$v_i^k = \sum_{j=1}^{n} p_{ij}^k \; r_{ij}^k$$

Fertilizer is used

1 good
2 fair
3 poor

No fertilizer is used.

1 good
2 fair
3 poor

Date:

The DP recursive equation can be written as:

$$f_N(i) = \max_k \{ v_i^k \}$$

$$f_n(i) = \max_k \left\{ v_i^k + \sum_{j=1}^{m} P_{ij}^k f_{n+1}(j) \right\}$$

$n = 1, 2, \ldots, N-1.$

To illustrate the computation of $v_i^k$, consider the case in which no fertilizer is used $(k=1)$.

$$v_1^1 = 0.2 \times 7 + 0.5 \times 6 + 0.3 \times 3 = 5.3$$

$$v_2^1 = 0 \times 0 + 0.5 \times 5 + 0.5 \times 1 = 3$$

$$v_3^1 = 0 \times 0 + 0 \times 0 + 1 \times -1 = -1$$

Thus, if the soil cond is good, a single transition yields 5.3 for that year, if it is fair, the yield is 3, and if it is poor, the yield is -1.

Q

Date:

Solve the ~~following~~ gardener problem using the data summarized in the matrices ~~$P_1, P_2$ and $P_3$~~ $R_1$ $P^1$, $P^2$ and $R^1$, $R^2$ given a horizon of 3 years only $(N = 3)$.

Sol.

$$P^1 = \| P^1_{ij} \| = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^1 = \| R^1_{ij} \| = \begin{bmatrix} 7 & 6 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$P^2 = \| P^2_{ij} \| = \begin{bmatrix} 0.3 & 0.6 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0.03 & 0.4 & 0.55 \end{bmatrix} \qquad R^2 = \| R^2_{ij} \| = \begin{bmatrix} 6 & 5 & -1 \\ 7 & 4 & 0 \\ 6 & 3 & -2 \end{bmatrix}$$

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$$f_n(i) = \max_k \left\{ \sum_{j=1}^{n} P_{ij}^{k} \left[ r_{ij}^{k} + f_{n+1}(j) \right] \right\} \quad n = 1, 2, \dots N.$$

$$V_i^k = \sum_{j=1}^{m} P_{ij}^{k} \, r_{ij}^{k}.$$

$\overline{4.7}$

$V_1^2 = 0.3\times6 + 0.6\times5 + 0.1\times-1$

$V_2^2 = 0.1\times7 + 0.6\times4 + 0.3\times0 = 3.1$

$V_3^2 \quad 0.05\times6 + 0.4\times3 + 0.5\times-2$

$= 0.4$

| $i$ | $V_i^1$ | $V_i^2$ |
|---|---|---|
| 1 | 5.3 | 4.7 |
| 2 | 3 | 3 |
| 3 | −1 | 0.4 |

Stage 03

| | $V_i^k$ | | Optimal Schedule | |
|---|---|---|---|---|
| $i$ | $k=1$ | $k=2$ | $f_3(i)$ | $k^x$ |
| 1 | 5.3 | 4.7 | 5.3 | 1 |
| 2 | 3 | 3.1 | 3.1 | 2 |
| 3 | −1 | 0.4 | 0.4 | 2 |

Date:

$$f_n(i) = \max_k \left\{ V_i^k + \sum_{j=1}^{m} P_{ij}^k f_{n+1}(i) \right\}_{n=1,2,3,\dots,N-1.}$$

Stage 02:

$V_i^k + P_{i1}^k \times f_3(1) + P_{i2}^k \times f_3(2) + P_{i3}^k \times f_3(3)$

| $i$ | $k=1$ | $k=2$ | $f_2(i)$ | $k$ |
|---|---|---|---|---|
| 1 | $5.3 + 0.2\times5.3 + 0.5\times3.1 + 0.3\times0.4 = 8.03$ | ~~4.7 + 0.3 × 0.3~~ ~~4.7~~ $4.7 + 0.3\times5.3 + 0.6\times3.1 + 0.1\times0.4 = 8.19$ | 8.19 | 2 |
| 2 | $3 + 0\times5.3 + 0.5\times3.1 + 0.5\times0.4 = 4.75$ | $3.1 + 0.1\times5.3 + 0.6\times3.1 + 0.3\times0.4 = 5.61$ | 5.61 | 2 |
| 3 | ~~0× 4 + 0.5×3.1+~~ $-1 + 0\times5.3 + 0\times3.1 + 1\times0.4 = -0.6.$ | $0.4 + 0.05\times5.3 + 0.4\times3.1 + 0.55\times0.4 = 2.31$ | 2.31 | 2 |

~~Stage 01:~~

~~i~~

~~1~~

~~2~~

~~3~~

~~$V_i^k + P_{i1}^k + P$~~

Stage 01:

| i | $V_i^k + P_{i1}^k f(1) + P_{i2}^k f(2) + P_{i3}^k f(3)$ | | $f(i)$ | $k^*$ |
|---|---|---|---|---|
| | $k = 1$ | $k = 2$ | | |
| 1 | $5.3 + 0.2 \times 8.19 + 0.5 \times 5.61 + 0.3 \times 2.31 \approx 10.38$ | $4.7 + 0.3 \times 8.19 + 0.6 \times 5.61 + 0.1 \times 2.13 \approx 10.74$ | 10.74 | 2 |
| 2 | $3 + 0 \times 8.19 + 0.5 \times 5.61 + 0.5 \times 2.31 \approx 6.87$ | $3.1 + 0.1 \times 8.19 + 0.6 \times 5.61 + 0.3 \times 2.31 \approx 7.92$ | 7.92 | 2 |
| 3 | ~~$-1 \times 0 +$~~ $-1 + 0 \times 8.19 + 0 \times 5.61 + 1 \times 2.31 \approx 1.13$ | $0.4 + 0.05 \times 8.19 + 0.4 \times 5.61 + 0.55 \times 2.31 \approx 4.23$ | 4.23 | 2. |

Date:

The optimal solution shows that for years 1 and 2 ($k^* = 2$) regardless of the state of system (soil condition) as revealed by the chemical test in year 3, fertilizer should be applied if the system is in the state 2 or 3 (fair or poor soil cond).

The total respective revenues ~~resume~~ for the 3 years are $f(1) = 10.74$ if the state of the system is year (1) is good, $f_1(2) = 7.92$, if it's fair and $f_1(3) = 4.23$ if it's ~~in ye~~ poor.