Date: 29-4-21

CS211 - Discrete Structures
Assignment #4, Spring 2021
20K-1052 (S.M. Hassan Ali)

Q.1

a) Using product rule

$= 27 \cdot 37 = 999$ offices

b) 12 = shirts, 3 = sizes, 2 = gender

$= 12 \times 3 \times 2$

$= 72$ diff type of shirt

Q.2

a) Repitition allowed

$= 26 \times 26 \times 26 = 17576.$

b) Repitition not allowed

$= 26 \times 25 \times 24 = 15600.$

Q.3

a) For Hexadecimal consider.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F (16)

String(10) $= 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 = 16^{10}$

String(26) $= 16^{26}$

String(58) $= 16^{58}$

$= 16^{10} + 16^{26} + 16^{58}$

$= 6.9 \times 10^{69}$

b) $26 \times 26 \times 26 \times 26 = 26^4 = 456,976$ (with x)

$25^4 = 390,625$ (without x)

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string with at least x $= 26^4 - 25^4$
$= 66,351$

Q.4

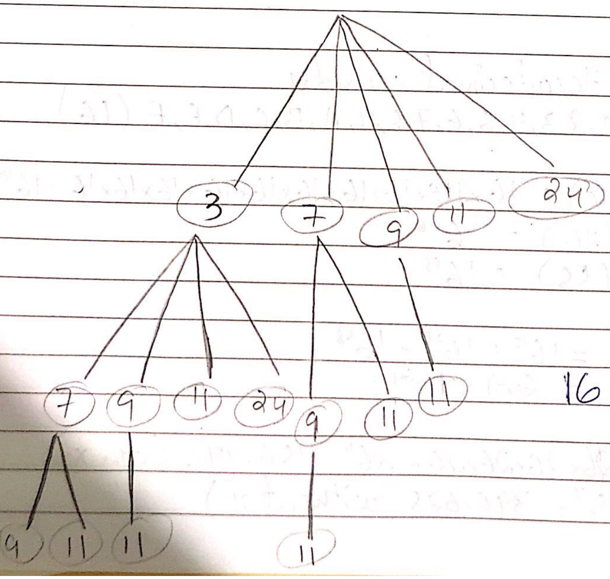
a) $\{1, 2, \dots m\}$ domain set $\{0,1\}$ image.

$f(1) = 0$ or 1
$f(2) = 0$ or 1
$f(m) = 0$ or 1

so $2 \times 2 \times 2 \dots 2 = 2^n$

b) set A with element 5 to set B with element 5.
$= 5 \times 4 \times 3 \times 2 = 120.$

Q.5 Tree diagram $\{3, 7, 9, 11, 24\}$ sum < 28



16 subsets

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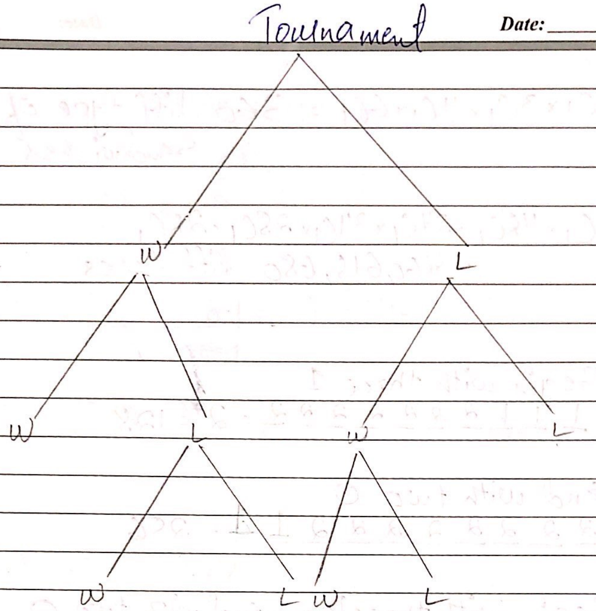
b)



Q.6

a) $^{8}C_{3} = 56$ ways.

b) $12C_{6} = 924$ ways

c) $^{9}C_{5} = 126$ ways

Q.7

a) $\underline{20}\times\underline{19}\times\underline{18}\times\underline{17}\times\underline{16}$ / $20P_5$ = 1,860,480 ways

b) $16\times15\times14\times13$ / $16P_4$ = 43,680 ways.

c) $15\times14 = 210$ ways

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Q.8

a) $5C_1 \times 3C_1 \times 4C_1 \times 6C_1 = 360$ diff type of sandwiches

b) $15C_1 \times 48C_1 \times 24C_1 \times 34C_1 \times 28C_1 \times 28C_1$
$= 460{,}615{,}680$ diff faces

Q.9

a) Begin with three 1

$\underline{1}\ \underline{1}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 2^7 = 128$

End with two 0

$\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1} = 256$

Begin with three 1 and end with two 0

$\underline{1}\ \underline{1}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1} = 32$

$= 128 + 256 - 32 = 352.$

b) Begin with zero or end with two 1

$\underline{1}.\underline{2}.\underline{2}.\underline{1}.\underline{1} = 4$

Begin with zero

$\underline{1}.\underline{2}.\underline{2}.\underline{2}.\underline{2} = 16$

end ~~Begin~~ with two 1

$\underline{2}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1} = 8$

$= 16 + 8 - 4$
$= 20$

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Q.10

a) Students with last name = 30

alphabets = 26

$\left\lceil \frac{30}{26} \right\rceil = 2.$ (atleast 2 students with same last letter name).

b) $\left\lceil \frac{8008278}{100000} \right\rceil = 9$ (proved)

c) $\left\lceil \frac{677}{38} \right\rceil = 18$ diff class rooms.

Q.11

a) x^5 in $(1+x)^{11}$

${}^{n}C_{r}(1)^{r}(x)^{n-r}$

${}^{11}C_{6}(1)^{6}(x)^{5} = \underline{462x^5}$

b) $a^7b^{17}(2a-b)^{24}$

$-{}^{24}C_{7}(2a)^{7}(b)^{24-7}$

$-44301312\ a^7b^{17}$

Q.12 a) if $a|b$ and $b|c$ then $a|c$

$b = ax$

$c = by$

$\frac{c}{a} = \frac{by}{a} = \frac{ax \times y}{a} = xy$ (hence proved).

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b) if $a|b$ and $a|c$ then $a|(b+c)$

$b = ax$

$c = ay$

$$\frac{b+c}{a} = \frac{ax+ay}{a} = \frac{\cancel{a}(x+y)}{\cancel{a}} = x+y$$

Q.13

a) $n > 5$ such that $2^n - 1$ is prime

at $n = 7$

$2^7 - 1 = 127$ which is prime

b) if $p|a$, $p \nmid (a+1)$

$a = px \quad a+1 = py$

Subtract eq/s

$(a+1) - a = py - px$

$1 = p(y-x)$

Hence p divides a and not a+1

Q.14 real no a,b $\sqrt{(a+b)} = \sqrt{a} + \sqrt{b}$

a)

$a = 16 \; b = 0 \quad / \quad a = 0 \; b = 9$

$\sqrt{16+0} = \sqrt{16} + \sqrt{0}$

$4 = 4$ Proved.

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b) if $|x| > 1$ then $x > 1$ or $x < -1$

$x = 2$ $\qquad$ $x = -2$

$|2| > 1$ $\qquad$ $|-2| = 2 > 1$ $\qquad$ Proved.

Q.15

a) product of two irrational numbers is an irrational number.

$x = \sqrt{2}$ (irrational)

$y = \frac{\sqrt{2}}{2}$ (irrational)

$x \times y = \frac{\sqrt{2} \times \sqrt{2}}{2} = \frac{2}{2} = 1$

statement not true at this condition.

b) Sum of rational and irrational number is irrational.

$x = \frac{a}{b}$ $\qquad$ $x + y = \frac{c}{d}$ irrational

rational

$\frac{a}{b} + y = \frac{c}{d}$

$y = \frac{c}{d} - \frac{a}{b}$

$y = \frac{cb - ad}{db}$ (rational)

Contradicts our assumption of b as irrational so sum of rational and irrational num is irrational

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Q.16

a) For every prime no, n+2 is prime

$n = 2$

$n + 2 = 2+2 = 4$ (not a prime)

b) Set of prime no is infinite

Using Contradiction.

prime nos are finite.

$P = P_1 \times P_2 \times P_3 \times \dots \times P_n + 1$

P can't be prime cause all $(P_1 \dots P_n)$ are part of it. So it would be divisible by any of the P_n where remainder $= 1$

That contradicts our assumption, hence original statement is True.

Q.17 If n and m are odd integers, then m+n is an even integer.

Using contradiction.

If n and m are odd integers, then m+n is odd integer.

$n = 2k+1$

$m = 2k+1$

$m+n = 2k+1+2k+1$

$= 4k+2$

$= 2(2k+1)$

$= 2r$ (Even)

Contradicts our assumption so original statement True.

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b) if m+n is even, then m and n are both even or m and n are both odd.

Using contrapositive.

If m+n are both odd or m+n are both even, then m+n is odd.

$m = 2K+1$ $\quad$ $m+n = 2K+1+2K$

$n = 2K$ $\quad$ $= 4K+1$

Hence sum of event and odd integer is odd.
So original statement is True.

Q.18 a) $6-7\sqrt{2}$ is irrational using contradiction.

assuming it is rational

$$x = 6-7\sqrt{2} = \frac{a}{b}$$

$$(6-7\sqrt{2})^2 = \left(\frac{a}{b}\right)^2$$

$$36-2(6)(7\sqrt{2})+(7\sqrt{2})^2 = \frac{a^2}{b^2}$$

$$134-84\sqrt{2} = \frac{a^2}{b^2}$$

$$134b^2 - 84\sqrt{2}b^2 = a^2$$

$$134b^2 - a^2 = 84\sqrt{2}b^2$$

b^2 is rational and so $84\sqrt{2}$

Contradicts our assumption hence original statement is True.

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b) $\sqrt{2} + \sqrt{3}$ is irrational.

assuming it as rational.

so $(\sqrt{2}+\sqrt{3})^2 = 2 + 2\sqrt{2\times3} + 3$

$= 5 + 2\sqrt{6}$

but here $\sqrt{6}$ is irrational.

contradicts the assumption hence original statement is True.

Q.19.

a) $36P_{36} = 36!$

$= 3.72\times10^{41}$

b) $36P_7 = 36\times35\times34\times33\times32\times31\times30$

$= 4.2\times10^{10}$

c) $20P_{20} \times {}^{16}P_{16}$

$20! \times 16! = 5.1\times10^{31}$

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Q.20 Mathematical Induction.

a) $1^2+2^2+3^2 \ldots n^2 = (n(n+1)(2n+1))/6$

Basic step

$n = 1$

$(1)^2 = (1(1+1)(2+1))/6$

$1 = 6/6$

$1 = 1$

Inductive step

$n = K$

$1^2+2^2+3^2 \ldots K^2 = (K(K+1)(2K+1))/6$

$n = K+1$

$(K+1)^2 = (K+1(K+1+1)(2K+2+1))/6$

$(K+1)^2 = (K+1(K+2)(2K+3))/6$

~~add~~ proof

$K^2+(K+1)^2 = (K(K+1)(2K+1))/6 + (K+1)^2$

$\frac{K(K+1)(2K+1)+6(K+1)^2}{6}$

$\frac{(K+1)[K(2K+1)+6(K+1)]}{6}$

$\frac{(K+1)(2K^2+7K+6)}{6}$

$\frac{(K+1)(K+2)(2K+3)}{6}$ Proved.

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b) $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$

$n = 0$ Basic step.

$2^0 = 2^{0+1} - 1$

$1 = 2 - 1$

$1 = 1$

Inductive step

$n = K$

$1 + 2 + 2^2 + \dots 2^K = 2^{K+1} - 1$

$n = K+1$

$2^{K+1} = 2^{K+1+1} - 1$

$2^{K+1} = 2^{K+2} - 1$

↳add ↳proof

$2^K + 2^{K+1} = 2^{K+1} - 1 + 2^{K+1}$

$= 2(2^{K+1}) - 1$

$= 2^1 \cdot 2^{K+1} - 1$

$= 2^{K+1+1} - 1$

$= 2^{K+2} - 1$ proved.

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c) $1^3 + 2^3 + 3^3 \ldots\ldots + n^3 = \frac{1}{4} n^2 (n+1)^2$

Basic step

$n = 1$

$(1)^3 = \frac{1}{4} (1)^2 (1+1)^2$

$(1)^3 = 4/4$

$1 = 1$

Inductive step

$n = k$

$1^3 + 2^3 + 3^3 \ldots + k^3 = \frac{1}{4} k^2 (k+1)^2$

$n = k+1$

$\underbrace{(k+1)^3}_{\text{add}} = \underbrace{\frac{1}{4} (k+1)^2 (k+2)^2}_{\text{proof}}$

$k^3 + (k+1)^3 = \frac{1}{4} k^2 (k+1)^2 + (k+1)^3 \times \frac{4}{4}$

$= \frac{1}{4} (k+1)^2 \cancel{[(k+1)+1]} (k^2 + 4(k+1))$

$= \frac{1}{4} (k+1)^2 (k^2 + 4k + 4)$

$= \frac{1}{4} (k+1)^2 (k+2)^2$ proved.