Probability And Stats

Date 29-4 2022.

Assignment-11

K20-1052
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BSE-4B

Q.1 $p = 0.7 \quad n = 13 \quad q = 0.3$

a) $P(H=5)$

$P(H=5) - P(H=4)$

$$\sum_{x=0}^{5} {}^{13}C_x (0.7)^x (0.3)^{13-x} - \sum_{x=0}^{4} {}^{13}C_x (0.7)^x (0.3)^{13-x}$$

$= 0.01822 - 0.000403$
$= 0.0142$

b) $P(H \leq 7)$

$$\sum_{x=0}^{7} {}^{13}C_x (0.7)^x (0.3)^{13-x} = 0.1654$$

c) $\mu = np$
$= 13 \times 0.7 = 9.1$

$\sigma^2 = npq$
$= 13(0.7)(0.3)$
$= \sqrt{2.73}$
$= 1.652$

Q.2 $\mu = 6$

a) $P(x=6)$

$\cancel{\Sigma}\ P(6;6) - P(5;6)$

$$\sum_{x=0}^{6} \frac{e^{-6} \times (6)^x}{x!} - \sum_{x=0}^{5} \frac{e^{-6} \times (6)^x}{x!}$$

$= 0.1606$

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b) $P(x < 5)$

$$= \sum_{x=0}^{4} \frac{e^{-6} \times (6)^{x}}{x!} = 0.285$$

c) $P(x \geq 4)$

$1 - P(3;6)$

$1 - 0.1512 = 0.849$

Q.3 $N = 9$ $n = 2$
$K = 3$ $x = ?$

$$= \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

a)
b)

x	0	1	2
$P(x)$	$5/12$	$1/2$	$1/12$

$$= \frac{3C_0 \times 6C_2}{9C_2} = 5/12$$

x represents no of defectives

c) $P(1 \leq x \leq 2)$

$P(\leq 2) - P(< 1)$

$1 - 5/12 = 7/12$

$P(x \leq 1)$

$5/12 + 1/2 = 11/12$

$P(0 < x \leq 2)$

$P(\leq 2) - P(x \geq 0)$

$1 - (1 - P(x < 0))$

$\cancel{1} + $

$7/12$

$f(2) = 1/12$

$f(5) = 1/12$

x	0	1	2
$f(x)$	$10/28$	$15/28$	$3/28$

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Q.4

a) $Var(x) = E(x^2) - (E(x))^2$

Mean $= 0(10/28) + 1(15/28) + 2(3/28)$
$= 0.75$

mean of squares $= 0(10/28) + 1(15/28) + 4(3/28)$
$= 0.9642$

$Var(x) = 0.9642 - (0.75)^2 = 0.4017$

b) $F(1) = P(x \leq 1)$

$= 10/28 + 15/28 = \frac{25}{28}$

c) $P(1 \leq x \leq 2)$

$P(x \leq 2) - P(x \leq 1)$

$\cancel{3/28} - \cancel{(1 - 10/28)} = 1 - 10/28 = \frac{18}{28} = \frac{9}{14}.$

Q.5 a) $p = 0.16$ $n = 15$ $q = 0.84$

$P(x \leq 1)$

$= \sum_{x=0}^{1} {}^{15}C_x (0.16)^x (0.84)^{15-x}$

$= 0.2821$

x = No of girls chosen

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b)

x	0	1	2	3
$f(x)$	$1/35$	$12/35$	$18/35$	$4/35$

$$\text{Mean}(E(x)) = 0(1/35) + 1(12/35) + 2(18/35) + 3(4/35)$$
$$= 12/7$$

$$E(x^2) = \frac{24}{7}$$

$$Var(x) = E(x^2) - (E(x))^2$$
$$= \frac{24}{7} - \left(\frac{12}{7}\right)^2 = \frac{24}{49}.$$

Q.6 $f(x) = \begin{cases} \frac{x^2}{9}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

a) $\int_{-1}^{2} \frac{x^2}{9} dx = 1$

b) $P(0 < x \leq 1)$

$\int_{0}^{1} \frac{x^2}{9} dx = 1/27$

c) $P(0 < x < 3)$

$\int_{0}^{2} \frac{x^2}{9} dx = 8/27$

$P(x=2) = 0$ since continuous.

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$\cancel{F(x) = \int_{-\infty}^{x} \frac{x^2}{9} dx}$

$\cancel{F(0.5) = \int_{0}^{0.5} \frac{x^2}{9} dx = 1/216}$

Q.7 $f(x,y) = \begin{cases} \frac{2}{7}(x+2y), & 0<x<1, 0<y<2 \\ 0, & \text{otherwise} \end{cases}$

$g(x) = \int_0^1 \frac{2}{7}(x+2y)dy$

$= \frac{2}{7}\left(x + y^2\right)_0^1 \Rightarrow \frac{2}{7}(x+1)$

$h(y) = \int_0^2 \frac{2}{7}(x+2y)dx$

$= \frac{2}{7}\left(\frac{x^2}{2} + 2y\right)_0^2 \Rightarrow \frac{2}{7}\left(2 + \frac{2}{y}\right)$

$\mu_x = \int_0^1 \frac{2}{7}x(x+1)dx = \frac{5}{21}$

$\mu_{x^2} = \int_0^1 \frac{2x^2}{7}(x+1)dx = 1/6$

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$$\mu_y = \int_0^2 \frac{2}{7} y\left(2 + \frac{2}{y}\right) dy \Rightarrow 16/7$$

$$\mu_{y^2} = \int_0^2 \frac{2}{7} y^2\left(2 + \frac{2}{y}\right) dy \Rightarrow 8/3$$

$$E(XY) = \int_0^2\int_0^1 \frac{2}{7} xy(x+2y)\,dx\,dy$$

$$= \frac{20}{21}$$

$$\sigma_{xy} = E(XY) - \mu_x \mu_y$$

$$= \frac{20}{21} - \left(\frac{5}{21}\right)\left(\frac{16}{7}\right) \Rightarrow \frac{20}{49}$$

$$\sigma_x^2 = E(x^2) - (\mu_x)^2$$

$$= \frac{1}{6} - \left(\frac{5}{21}\right)^2 \Rightarrow \sqrt{97/882}$$

$$\sigma_y^2 = \frac{8}{3} - \left(\frac{16}{7}\right)^2 \Rightarrow \sqrt{376/147}$$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{20/49}{\sqrt{97/882} \times \sqrt{\frac{376}{147}}}$$

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Q.7 b

x \ y	0	1	2	
0	0.12	0.04	0.04	0.2
1	0.08	0.19	0.05	0.32
2	0.06	0.12	0.30	0.48
	0.26	0.35	0.39	

$E(y) = 0(0.26) + 1(0.35) + 2(0.39)$
$= 1.13$

$E(y^2) = 0(0.26) + 1(0.35) + 4(0.39)$
$= 1.91$

$E(x) = 0(0.2) + 1(0.32) + 2(0.48)$
$= 1.28$

$E(x^2) = 0(0.2) + 1(0.32) + 4(0.48)$
$= 2.24$

$E(XY) = (0)(0)(0.12) + (0)(1)(0.04) + (0)(2)(0.04)$
$+ (1)(1)(0.19) + (1)(2)(0.05) + (2)(1)(0.12)$
$+ (2)(2)(0.30)$

$= 1.73$

$\sigma_{xy} = E(XY) - \mu_x \mu_y$
$= 1.73 - (1.28)(1.13)$
$= 0.284$

$\sigma_x^2 = 2.24 - (1.28)^2 = 0.6016$
$\sigma_y^2 = 1.91 - (1.13)^2 = 0.6331$

$$\rho_{xy} = \frac{0.284}{\sqrt{0.6016} \times \sqrt{0.6331}}$$

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Q.8 $P = 8/200 = 0.04$ $n = 8$ $q = 0.96$

a) ${}^{4}C_0 (0.04)^0 (0.96)^4$

$= 0.8493$

b) $P(x \geq 3) \Rightarrow 1 - P(\leq 2)$

$= 1 - {}^{8}C_x (0.04)^x (0.96)^{8-x}$

$= 1 - 0.99969$

$= 3.07 \times 10^{-3}$

c) $\mu = np$

$= 100(0.04) = 4$

Q.9 a. $\mu = 5$

i) $\dfrac{e^{-5} \times (5)^0}{0!} = 6.74 \times 10^{-3}$

ii) $\dfrac{e^{-5} \times (5)^4}{4!}$

iii) $P(x \leq 6)$

$= \sum_{x=0}^{6} \dfrac{e^{-5} \times (5)^x}{x!}$

$= 0.7622$

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b) $\mu = 5$

$P(x < 3)$

$= \sum_{x=0}^{2} \frac{e^{-5} \times (5)^x}{x!} = 0.125$

Q.10a $n = 1000$ $\mu = 174.5$
sd $= 6.9$.

a) $P(x < 160)$ $z = \frac{x - \mu}{\sigma}$

$= \frac{159.75 - 174.5}{6.9} = -2.1376$
(use of normal dist table).

$P(z \leq -2.1376) = 0.0166$

b) $P(171.5 \leq x \leq 182.0)$

$z = \frac{171.5 - 174.5}{6.9}$ $= \frac{182 - 174.5}{6.9} = 1.086$

$= -0.4347$

$P(-0.4347 \leq z \leq 1.086)$

$P(z \leq 1.086) - P(z < -0.4347)$

$0.85993 - 0.33724$
$= 0.5227$

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$P(x = 175)$

c) $z = \frac{175 - 174.5}{6.9} = 0.07246$

$P(z = 0.07246)$

$= 0.52790 - 0.52392 =$

d) $P(x \geq 188)$

$1 - P(x \leq 187.75)$

$z = \frac{187.75 - 174.5}{6.9} = 1.92$

$P(z \leq 1.92) = 0.97257$

$1 - 0.9725 = 0.0275$

(b) $\cancel{N} = 5000 \quad \mu = 1250 \quad \sigma = 250$

a) $P(x > 1500)$

$z = \frac{1499 - 1250}{250}$

$= 0.996$

$1 - P(x \leq 1499)$

$1 - P(z \leq 0.996)$

$1 - 0.83891 = 0.1611$

% of workers $= 0.1611 \times 100$

$= 16.11\%$

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b) $P(x \leq 750)$

$$z = \frac{750 - 1250}{250} = -2$$

$P(z \leq -2) = 0.97725$

% of workers = 97.72%

c) $P(750 \leq x \leq 1500)$

$$z = \frac{750 - 1250}{250}$$

$= -2$

$$z = \frac{1500 - 1250}{250}$$

$= \cancel{2} \; 1$

$P(-2 \leq z \leq 1)$

$P(z \leq 1) - P(z < -2)$

$0.84134 - \cancel{0.9767} \; 0.0222$

$= 0.8191$

% of workers = 81.91