

Course Code: CS211	Course Name: Discrete Structures
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Student Roll No:	Section No:

Instructions:

- Return the question paper together with the answer script. Read each question completely before answering it. There are **3 questions and 2 pages**.
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Attempt all the questions in given sequence of the question paper.

Total Time: 60 minutes

Maximum Marks: 26

Question # 1 (Propositional Logic and Rules of Inference)

[5x2=10 points]

(i) Let p and q be the propositions.

p : Swimming at the New Jersey shore is allowed.

q : Sharks have been spotted near the shore.

Write these propositions using p and q and logical connectives (including negations):

a) Swimming at the New Jersey shore is not allowed and either Swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore.

Solution: $\neg p \wedge (p \vee \neg q)$

b) Swimming at the New Jersey shore is allowed iff sharks have not been spotted near the shore.

Solution: $p \leftrightarrow \neg q$

(ii) Prove the following logical equivalence using the laws of logic:

$$\neg [c \vee (b \wedge (\neg c \rightarrow \neg a))] \cong \neg c \wedge (a \vee \neg b)$$

Solution:

$$\equiv \neg c \wedge \neg (b \wedge (\neg c \rightarrow \neg a))$$

$$\equiv \neg c \wedge \neg (b \wedge (\neg \neg c \vee \neg a))$$

$$\equiv \neg c \wedge \neg (b \wedge (c \vee \neg a))$$

$$\equiv \neg c \wedge (\neg b \vee \neg (c \vee \neg a))$$

$$\equiv \neg c \wedge (\neg b \vee (\neg c \wedge a))$$

$$\equiv (\neg c \wedge \neg b) \vee (\neg c \wedge (\neg c \wedge a))$$

$$\equiv (\neg c \wedge \neg b) \vee ((\neg c \wedge \neg c) \wedge a)$$

$$\equiv (\neg c \wedge \neg b) \vee (\neg c \wedge a)$$

$$\equiv \neg c \wedge (\neg b \vee a)$$

$$\equiv \neg c \wedge (a \vee \neg b)$$

Hence Proved.

De Morgan's law

conditional rewritten as disjunction

double negation law

De Morgan's law

De Morgan's law and double negation

distributive law

associative law

idempotent law

distributive law

commutative law

(iii) Determine using truth table that whether the following is a tautology, contradiction or a contingency.

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

Solution:

P	q	r	$p \rightarrow q$	$(q \rightarrow r)$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$(p \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Hence, it's a Tautology.

(iv) What relevant conclusion or conclusions can be drawn from the following premises? Also, explain the rules of inference used to obtain each conclusion from the premises.

"The file is either a binary file or a text file."

"My program won't accept the file if it's a binary file."

"My program will accept the file."

Assume,

p = "The file is a binary file."

q = "The file is a text file."

r = "My program will accept the file."

Solution:

Let p denote the proposition 'The file is a binary file', let q denote 'The file is a text file', and let r denote 'My program will accept the file'. Then:

$$P_1 \equiv p \vee q$$

$$P_2 \equiv p \rightarrow \neg r$$

$$P_3 \equiv r$$

$$Q \equiv q$$

The argument now takes the form:

$$[(p \vee q) \wedge (p \rightarrow \neg r) \wedge r] \rightarrow q$$

(v) Write the negation of the following sentences in English.

a) If Jaffer lives in Pakistan, then he lives in Karachi.

Solution: Jaffer lives in Pakistan and he does not live in Karachi ($p \wedge \neg q$).

b) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

Solution: n is divisible by 6 but n is not divisible by 2 or by 3 ($p \wedge \neg(q \vee r)$).

Question # 2 (Predicate and Quantifiers)

[3x2=6 points]

(i) Let $F(a, b)$ means " $a + 3b = ab$ ", where a and b are Positive integers. Determine the truth value of the statement.

a) $\forall a \exists b F(a, b)$. **Solution: False**

b) $\forall b \exists a \neg F(a, b)$. **Solution: True**

(ii) Let $B(x)$ be the statement " x has an Internet connection" and $C(x, y)$ be the statement " x and y have chatted over the Internet," where the domain for the variables x and y consists of all students in your class. Write the statement in good English without using variables in your answers.

a) $\neg \forall x B(x)$. **Solution: Not everyone in your class has an Internet connection.**

b) $\exists x B(x) \wedge \forall y \neg C(x, y)$

Solution: Someone in your class has an Internet connection but has not chatted with anyone else in your class.

(iii) Express the following sentences using logical expression with nested quantifiers:

a) The Sum of two negative integers numbers is negative.

Solution: $\forall x \forall y ((x < 0) \wedge (y < 0) \rightarrow (x + y < 0))$

b) The difference of two positive integers is not necessarily positive.

Solution: $\neg \forall x \forall y ((x > 0) \wedge (y > 0) \rightarrow (x - y > 0))$ or

(b) A value is positive, when it is larger than 0. A value is positive, when it is smaller than or equal to 0.

We can rewrite the given sentence as: "There exists an integer x and an integer y , such that x is positive and y is positive and their difference is not positive"

$$\exists x \exists y ((x > 0) \wedge (y > 0) \rightarrow (x - y \leq 0))$$

Question # 3 (Functions and Set theory)

[5x2=10 points]

(i) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by the formula $f(x) = 4x-1 \forall x \in \mathbb{R}$. Is f a bijective function? If no, give reason why? If yes, find its inverse.

Solution:

Then f is bijective, therefore f^{-1} exists. By definition of f^{-1} ,

$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

Now solving $f(x) = y$ for x

$$\Leftrightarrow 4x-1 = y \quad (\text{by definition of } f)$$

$$\Leftrightarrow 4x = y + 1$$

$$\Leftrightarrow x = \frac{y+1}{4}$$

Hence, $f^{-1}(y) = \frac{y+1}{4}$ is the inverse of $f(x)=4x-1$ which defines $f^{-1}: \mathbb{R} \rightarrow \mathbb{R}$.

(ii) Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ and $g: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(n) = n+1$ for $n \in \mathbb{Z}$ and $g(n) = n^2$ for $n \in \mathbb{Z}$.

a) Find the compositions $g \circ f$ and $f \circ g$.

b) Is $g \circ f = f \circ g$?

Solution:

SOLUTION:

a. By definition of the composition of functions

$$(g \circ f)(n) = g(f(n)) = g(n+1) = (n+1)^2 \text{ for all } n \in \mathbb{Z} \text{ and}$$

$$(f \circ g)(n) = f(g(n)) = f(n^2) = n^2 + 1 \text{ for all } n \in \mathbb{Z}$$

b. Two functions from one set to another are equal if, and only if, they take the same values.

In this case,

$$(g \circ f)(1) = g(f(1)) = (1 + 1)^2 = 4 \text{ where as}$$

$$(f \circ g)(1) = f(g(1)) = 1^2 + 1 = 2$$

Thus $f \circ g \neq g \circ f$

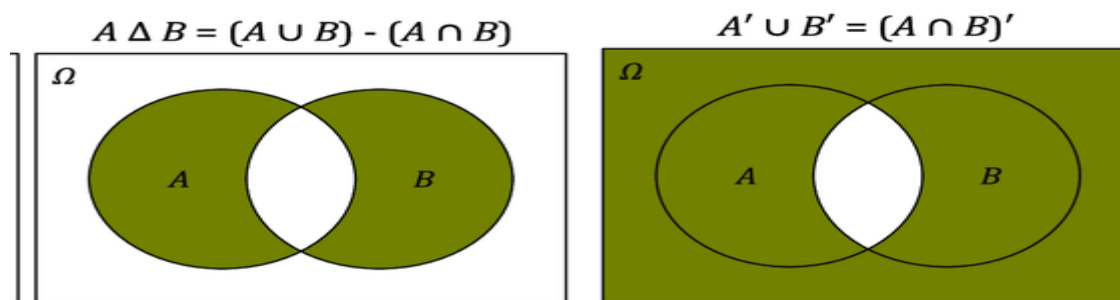
REMARK: The composition of functions is not a commutative operation.

(iii) Draw Venn Diagram of the following relationships between the sets:

a) $(A \cup B) - (A \cap B)$

b) $\bar{A} \cup \bar{B}$

Solution:



(iv) Using set-builder notation, prove or disprove the following set operations:

$$A - (B \cap C) = (A - B) \cap (A - C).$$

Solution:

$$A - (B \cap C) = (A - B) \cap (A - C)$$

$$A \cap \overline{B \cap C} = (A \cap \overline{B}) \cap (A \cap \overline{C})$$

$$A \cap \overline{B} \cup \overline{C} = (A \cap A) \cap (\overline{B} \cap \overline{C})$$

$$A \cap \overline{B} \cup \overline{C} \neq A \cap \overline{B} \cap \overline{C}$$

Hence, it a disproof.

$$A \cap \overline{B} = (A - B)$$

De-Morgan and Associative Law

Idempotent Law

(v) Among a group of 165 students, 8 are taking calculus, psychology, and computer science; 33 are taking calculus and computer science; 20 are taking calculus and psychology; 24 are taking psychology and computer science; 79 are taking calculus; 83 are taking psychology; and 63 are taking computer science. How many are taking none of the three subjects?

Solution:

SOLUTION Let CALC, PSYCH, and COMPSCI denote the sets of students taking calculus, psychology, and computer science, respectively. Let U denote the set of all 165 students (see Figure 1.1.6). Since 8 students are taking calculus, psychology, and computer science, we write 8 in the region representing $\text{CALC} \cap \text{PSYCH} \cap \text{COMPSCI}$. Of the 33 students taking calculus and computer science, 8 are also taking psychology; thus 25 are taking calculus and computer science but not psychology. We write 25 in the region representing $\text{CALC} \cap \overline{\text{PSYCH}} \cap \text{COMPSCI}$. Similarly, we write 12 in the region representing $\text{CALC} \cap \text{PSYCH} \cap \overline{\text{COMPSCI}}$ and 16 in the region representing $\overline{\text{CALC}} \cap \text{PSYCH} \cap \text{COMPSCI}$. Of the 79 students taking calculus, 45 have now been accounted for. This leaves 34 students taking only calculus. We write 34 in the region representing $\text{CALC} \cap \overline{\text{PSYCH}} \cap \overline{\text{COMPSCI}}$. Similarly, we write 47 in the region representing $\overline{\text{CALC}} \cap \text{PSYCH} \cap \overline{\text{COMPSCI}}$ and 14 in the region representing $\overline{\text{CALC}} \cap \overline{\text{PSYCH}} \cap \text{COMPSCI}$. At this point, 156 students have been accounted for. This leaves 9 students taking none of the three subjects. ◀

ALL THE BEST 😊