Assignment #3

Date: M T W T F S S

Design and Analysis of Algorithms.

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K20-1052
BSE-5B
Group members:
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Q.1 A = CABA

B = BCBDAB

		B	C	B	D	A	B
	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
C 1	0	0	1	1	1	1	1
A 2	0	0	1	1	1	2	2
B 3	0	1	1	2	2	2	3
A 4	0	1	1	2	2	3	3

Crossing diagonal elements = CAB.

HASSAN
EHTESHAM

		E	H	T	E	S	H	A	M
	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
H 1	0	0	1	1	1	1	1	1	1
A 2	0	0	1	1	1	1	1	2	2
S 3	0	0	1	1	1	2	2	2	2
S 4	0	0	1	1	1	2	2	2	2
A 5	0	0	1	1	1	2	2	3	3
N 6	0	0	1	1	1	2	2	3	3

if not match take prev and digonal else if match take left diagonal and add 1.

= HSA

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b) Shortest Common Supersequence.

		E	H	I	E	S	H	A	M
	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
H 1	0	0	1	1	1	1	1	1	1
A 2	0	0	1	1	1	1	1	2	2
S 3	0	0	1	1	1	2	2	2	2
S 4	0	0	1	1	1	2	2	2	3
A 5	0	0	1	1	1	2	2	3	3
N 6	0	0	1	1	1	2	2	3	3

nmashsaethe

enteahssamn

Logic is to eliminate the element from x and y as we move diagonally in LCS. Then reversing the whole string.

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Longest Common Subsequence

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c) Group member 1: 1655
Group member 2: 1695

5, 10, 2, 5, 20

```
while(i > j){
if(arr[i]) > arr[j])
{
Lis[i] = max(Lis[i],
  Ljs[j]+1);
} j++
else { i++; }
}
```

j = 0 ; if i = j

	j		i			
arr =	5	10	2	5	20	
Lis =	1	1	1	1	1	

1

	j →		i			
arr =	5	10	2	5	20	
Lis =	1	1	1	1	1	

2-

	j →		i			
arr =	5	10	2	5	20	
Lis =	1	2	1	1	1	

max(Lis[i], Lis[j+1])
max(1, 1+1)
max(1, 2)

3-

	j →			i		
arr =	5	10	2	5	20	
Lis =	1	2	1	2	1	

4-

	j →				i	
arr =	5	10	2	5	20	
Lis =	1	2	1	2	3	

1 2 3

Lis would be now => (5, 10, 20)
=> (2, 5, 20)

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Levenshtein-distance (edit-distance)

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d) str1 = "PLASMA"
str2 = "ALTRUISM"

	""	A	L	T	R	U	I	S	M
""	0	1	2	3	4	5	6	7	8
P	1	1	2	3	4	5	6	7	8
L	2	2	1	2	3	4	5	6	7
A	3	2	2	2	3	4	5	6	7
S	4	3	3	3	3	4	5	5	6
M	5	4	4	4	4	4	5	6	5
A	6	5	5	5	5	5	5	6	6

Total 6 operations need to perform that could be either update, add, delete for converting str1 to str2.

Logic is to choose previous, upper diagonal and upper no if the characters do not match. Choose the minimum of these and add 1.

If the element matches take upper diagonal only.

$C[i,j] = \min\{C[i,k] + C[k+1,j] + P_{i-1} \times P_k \times P_j\}$ $i \le k < j$

Matrix Chain.

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e) $A_1 \times A_2 \times A_3 \times A_4 \times A_5$

$2 \times 25 \times 25 \times 3 \times 3 \times 16 \times 16 \times 1 \times 1 \times 6$

P_0 P_1 P_2 P_3 P_4 P_5

Hassan

length = 6

	1	2	3	4	5
1	0	150	256	173	185
2		0	1200	123	273
3			0	48	66
4				0	96
5					0

$C[1,2] = \min^{k=1}\{C[1,1] + C[2,2] + P_0P_1P_2$

$0 + 0 + 2 \times 25 \times 3 = 150$

$C[2,3] = \min^{k=2}\{C[2,2] + [3,3] + d_1d_2d_3$

$0 + 0 + 25 \times 3 \times 16 = 1200$

$C[3,4] = \min^{k=3}\{C[3,3] + [4+4] + d_2d_3d_4$

$0 + 0 + 3 \times 16 \times 1 = 48$

$C[4,5] = 96$

$C[1,3] = \min^{k=1}\{C[1,1] + C[2,3] + d_0d_1d_3$

$0 + 1200 + 2 \times 25 \times 16 = 800$

$\checkmark k=2\{C[1,2] + C[3,3] + d_0d_2d_3$

$150 + 0 + 2 \times 3 \times 16 = 96$

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$$C[2,4] = \min \begin{cases} k=2 & C[2,2] + C[3,4] + d_1 d_2 d_4 \\ & 0 + 48 + 25 \times 3 \times 1 = 123 \\ k=3 & C[2,3] + C[4,4] + d_1 d_3 d_4 \end{cases}$$

$$C[2,5] = \min \begin{cases} k=2 & C[2,2] + C[3,5] + d_1 d_2 d_5 \\ & 0 + 66 + 25 \times 3 \times 6 = 516 \\ k=3 & C[2,3] + C[4,5] + d_1 d_3 d_5 \\ & 1200 + 96 + 25 \times 16 \times 6 = 3696 \\ k=4 & C[2,4] + C[5,5] + d_1 d_4 d_5 \\ & 123 + 0 + 25 \times 1 \times 6 = 273 \end{cases}$$

min

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0-1 Knap Sack Problem

f) W = [1, 2, 3, 4, 5] C = 5
V = [3, 5, 4, 8, 10]

bounded.

capacity →

V_i	W_i	0	0	1	2	3	4	5
		0	0	0	0	0	0	0
3	1	1	0	3	3	3	3	3
5	2	2	0	3	5	8	8	8
4	3	3	0	3	5	8	8	9
8	4	4	0	3	5	8	8	11
10	5	5	0	3	5	8	8	11

When considering 2nd object or more consider previous ones too.

$$V[i, w] = \max \{ V[i-1, w], V[i-1, w-w_i] + V[i] \}$$

Partition Problem

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g) Hassan Amna (special case of Knapsack)

$S = \{8, 1, 19, 1, 13, 14\}$

Sum = 56/2 = 28

- Find elements that will add up to sum 28, followed by others

```
if (nums[i-1] > j){
    T[i][j] = T[i-1][j];
}
else {
T[i][j] = T[i-1][j] || T[i-1][j-nums[i-1]];
}
```

Finding the subsets for equal sum.

$S_1 = \{8, 1, 19\}$ sum = 28

$S_2 = \{1, 13, 14\}$ sum = 28.

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Rod Cutting Problem.

(unbounded Knap Sack)

h)

Length[] = {1, 2, 3, 4, 5, 6, 7, 8}.

Price[] = {1, 5, 8, 9, 10, 16, 18, 20}

Rod length = 8.

len	1	2	3	4	5	6	7	8
$V_{(i)}$ price	1	5	8	9	10	16	18	20
$C_{(i)}$	1	5	8	10	13	17	18	22

$$C(i) = \max\{V_k + C(i-k)\}$$

$$C(1) = \max\{1 + C(0)\} = 1$$

$$C(2) = \max\begin{cases} 1 + C(2-1) = 1+1 = 2 \\ V_2 = 5 \end{cases} \quad \max = 5$$

$$C(3) = \max\begin{cases} 1 + C(3-1) = 1+5 \\ 5 + C(3-2) = 5+1 \\ V_3 = 8 \end{cases} \quad = 8$$

$$C(4) = \max\begin{cases} 1 + C(4-1) = 1+8 \\ 5 + C(4-2) = 5+5 \\ 8 + C(4-3) = 8+1 \\ V_4 = 9 \end{cases} \quad = 10$$

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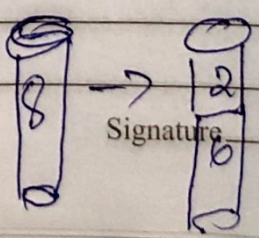
$$C(5) = \begin{cases} 1 + C(5-1) & = 1+10 \\ 5 + C(5-2) & = 5+8 \\ 8 + C(5-3) & = 8+5 \\ 9 + C(5-4) & = 9+1 \\ V_5 = 10 & = 10 \end{cases} = 13 \text{ max}$$

$$C(6) = \begin{cases} V_1 + C(5) & = 1+16 \\ V_2 + C(4) & = 5+10 \\ V_3 + C(3) & = 8+8 \\ V_4 + C(2) & = 9+5 \\ V_5 + C(1) & = 10+1 \\ V_6 & = 16 \end{cases} = 17 \text{ max}$$

$$C(7) = \begin{cases} V_1 + C(6) & = 1+17 \\ V_2 + C(5) & = 5+13 \\ V_3 + C(4) & = 8+10 \\ V_4 + C(3) & = 9+8 \\ V_5 + C(2) & = 10+5 \\ V_6 + C(1) & = 16+1 \\ V_7 & = 18 \end{cases} = 18 \text{ max}$$

$$C(8) = \begin{cases} V_1 + C(7) & = 1+18 \\ V_2 + C(6) & = 5+17 \\ V_3 + C(5) & = 8+13 \\ V_4 + C(4) & = 9+10 \\ V_5 + C(3) & = 10+8 \\ V_6 + C(2) & = 16+5 \\ V_7 + C(1) & = 18+1 \\ V_8 & = 20 \end{cases} = 22 \text{ max}$$



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Coin change Making

i) $S = \{1, 5, 6, 8\}$. change = 13.

		0	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13
5	1	0	1	2	3	4	1	2	3	4	5	2	3	4	5
6	2	0	1	2	3	4	1	1	2	3	4	2	2	2	3
8	3	0	1	2	3	4	1	1	2	1	2	2	2	2	2

```
if(coins[i] > j ){
  arr[i][j] = a[i-1][j] }
else {
arr[i][j] = min(a[i-1][j],
    1 + a[i][j-coins[i]]);
```

Min 2 coins required to make desired change of 13.
= {5, 8}.

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Word Break Problem

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i) S = {i, like, ~~sam, sung~~, mobile, apple, ice, icecream, ~~india~~, ~~go~~, ~~mango~~}

Input = "Ilikeapple" (length = 9)

	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	T	F	F	F	F	F
1		F	F	F	T	F	F	F	F	T
2			T	F	F	F	F	F	F	F
3				F	F	F	F	F	F	F
4					F	F	F	F	F	F
5						F	F	F	F	T
6							F	F	F	F
7								F	F	F
8									F	F
9										F

length of input = 1

Input(0,0) = i ✓
Input(1,1) = l X
Input(2,2) = i ✓
Input(3,3) = k X
Input(4,4) = e X
Input(5,5) = a X
Input(6,6) = p X
Input(7,7) = p X
Input(8,8) = [illegible] X
Input(9,9) = e X

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ilikeapple

length of Input = 2.

Input(0,1) = il ✗
Input(1,2) = li ✗
Input(3,4) = ik ✗
Input(5,6) = ke ✓
Input(6,7) = ea ✗
Input(6,7) = ap ✗
Input(7,8) = pp ✗
Input(8,9) = pl ✗
~~Input(10 = le~~ ✗

Input = 3

Input(0,2) = ili ✗
Input(1,3) = lik ✗
Input(2,4) = ike ✗
Input(3,5) = kea ✗
Input(4,6) = eap ✗
Input(5,7) = app ✗
Input(6,8) = ppl ✗
Input(7,9) = ple ✗

length of Input = 4

Input(0,3) = ilik ✗
Input(1,4) = like ✓
Input(2,5) = ikea ✓
Input(3,6) = keap ✗
Input(4,7) = eapp ✗
Input(5,8) = appl ✗
Input(6,9) = pple ✗

Input = 5

Input(0,4) = ilike ✓
Input(1,5) = likea ✗
Input(2,6) = ikeap ✗
Input(3,7) = keapp ✗
Input(4,8) = eappl ✗
Input(5,9) = apple ✓

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Date:

M	T	W	T	F	S	S

"ilikeapple"

length = 6

Input(0,5) = "ilikea" ×
Input(1,6) = "likeap" ×
Input(2,7) = "ikeapp" ×
Input(3,8) = "keappl" ×
Input(4,9) = "eapple" ×

length = 7

Input(0,6) = "ilikeap" ×
Input(1,7) = "likeapp" ×
Input(2,8) = "ikeappl" ×
Input(3,9) = "keapple" ✓

length = 8

Input(0,7) = "ilikeapp" ×
Input(1,8) = "likeappl" ×
Input(2,9) = "ikeapple" ×

length = 9

Input(0,8) = "ilikeappl" ×
Input(1,9) = "likeapple" ✓