

## VECTORS

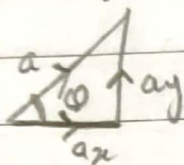
A vector is a quantity that has both magnitude and direction.

eg: velocity, displacement, acceleration and force

AC is the vector sum of AB and BC

- Components of vectors

Resolving vectors:  $a_x = a \cos \theta$        $a_y = a \sin \theta$



$$\text{magnitude of } a = \sqrt{a_x^2 + a_y^2}$$

$$\tan \theta = a_y / a_x$$

- Unit vector

①  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

②  $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

③ unit v.  $\hat{r} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{|\vec{r}|}$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j}$$

For,  $r_x = a_x + b_x$

$$r_y = a_y + b_y$$

$$r_z = a_z + b_z$$

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## Multiplying vectors.

$$\hat{a} \cdot \hat{b} = |\hat{a}| |\hat{b}| \cos \theta$$

- for non-zero vectors
- if  $\cos \theta = 0$ , vectors are perpendicular.

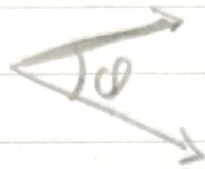


Fig. 21

## • Dot product.

$$A \cdot B = AB \cos \theta$$

$$A \cdot \hat{i} = A \cos \theta = A_x$$

$$A \cdot B = A_x B_x + A_y B_y + A_z B_z$$

## • Vector Product.

$$a \times b = ab \sin \theta$$

$$i \times j = k \quad i \times k = j \quad j \times k = i$$

$$A \times B = (A_y B_z - A_z B_y)i + (A_z B_x - A_x B_z)j + (A_x B_y - A_y B_x)k$$

## MOTION

### - Displacement

$$\Delta x = x_f(t_f) - x_i(t_i)$$

vector quantity, unit = meters (m)

Has +ve and -ve direction

Displacement  $\neq$  Distance

### - Speed

scalar quantity, unit m/s or km/h

$$\text{Avg. speed} = \frac{\bar{v} \cdot \text{dist}}{\bar{v} \cdot \text{time}}$$

- Instantaneous speed: At a given instant in time
- Avg speed: Avg of all inst. speeds

### - Velocity

vector quantity, m/s

$$\text{Avg. velocity} = \Delta x / \Delta t$$

$$\text{Inst. velocity} = \frac{dx}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

~~Velocity is slope of position-time graph.~~

- Inst. velocity: at a given instant
- Uniform velocity: const. velocity, inst. velocities are always same.



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## - Acceleration

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

• Inst. acc =  $a_x = \lim_{\Delta t \rightarrow 0} \Delta v_x / \Delta t = dv_x / dt$

• Avg. acc = • non-uniform acceleration

Vector quantity, Rate of change of velocity,  
Units =  $m/s^2$

## - Equations:

①  $v = u + at$

②  $\Delta x = \frac{1}{2}(u + v)t$

③  $s = ut + \frac{1}{2}at^2$

④  $v^2 = u^2 + 2as$

## MOTION IN 2D

Position vector:  $r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

Displacement vector:  $\Delta r = r_2 - r_1 = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$

- Avg velocity:  $V_{avg} = \frac{\Delta r}{\Delta t} = \frac{\Delta x}{\Delta t}\mathbf{i} + \frac{\Delta y}{\Delta t}\mathbf{j} + \frac{\Delta z}{\Delta t}\mathbf{k}$

- Inst. velocity:  $\vec{v} = d\vec{r}/dt$

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- Average acceleration:  $a_{avg} = \frac{\text{change in velocity}}{\text{time interval}}$

$$a_{avg} = \frac{v_2 - v_1}{\Delta t} = \frac{\Delta v}{\Delta t}$$

- Inst. acc =  $\frac{dv}{dt} = \frac{dv_x}{dt} i + \frac{dv_y}{dt} j + \frac{dv_z}{dt} k$

### FREE FALL / G. ACC

Terminal velocity, when the force of gravity on a falling object equals the force of air resistance going against gravity the obj stops accelerating

### Circular Motion

• centripetal acc =  $v^2/r$

$$T = 2\pi r / v$$

• Tangential and Radial acc:

Radial = $\frac{d v }{dt}$ ( $a_t$ )
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Tangential = $v^2/r$ ( $a_r$ )
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$$Total = a_r + a_t$$

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## Projectile Motion

- Types:

Horizontal:

- velocity = const

Vertical:

- Force due to gravity
- Vertical comp changes with time

$$V_{xf} = V \cos \theta$$

$$V_{yf} = V \sin \theta - gt$$

- Equations:

X - comp:

$$x_f = x_i + V_{xi} t$$

Y - comp:

$$y_f = y_i + V_{yi} t - \frac{1}{2}gt^2$$

$$V_{yf}^2 = V_{yi}^2 - 2g\Delta y$$

$$V_{yf} = V_{yi} - gt$$

Vectors:

$$V_{xi} = V_i \cos \theta$$

$$V_{yi} = V_i \sin \theta$$

$$\text{Equation path} = y = (\tan \theta_0)x - \frac{gx^2}{2(V_0 \cos \theta_0)^2}$$

$$\text{Horizontal Range: } R = \frac{v_0^2}{g} \sin 2\theta_0$$

$$\text{Max height: } h = \frac{V_i^2 \sin^2 \theta_i}{2g}$$