Assignment no 2

K20-1052 - S.M. Hassan Ali

Ex 4.1

Q.2. $u = (u_1, u_2)$ $v = (v_1, v_2)$

$u + v = (u_1 + v_1 + 1, u_2 + v_2 + 1)$

$ku = (ku_1, ku_2)$

a) $u + v = (2, 2)$

$ku = (0, 8)$

b) $(0, 0) + (u_1, u_2)$

$= (0, u_1 + 1, 0 + u_2 + 1)$

$\neq (u_1 + 1, u_2 + 1) \neq (u_1, u_2)$

c) $(-1, -1) + (u_1, u_2)$

$= (-1 + u_1 + 1, -1 + u_2 + 1)$

$= (u_1, u_2)$

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d) $-u = (-2-u_1, -2-u_2)$

$u + (-u) = (u_1 + (-2-u_1)+1,$
$u_2 + (-2-u_2)+1)$

$(-1,-1) = 0$

e) Axiom 7 does not apply.

$K(u+v) = K(u_1+v_1+1, u_2+v_2+1)$

$= (Ku_1, Kv_1, K, Ku_2 + Kv_2 + K)$

$Ku + Kv = (Ku_1, Ku_2) + (Kv_1, Kv_2)$

LHS $\neq$ RHS

Axiom 8

$(K+m)u = Ku + mu$

$(K+m)(u_1, u_2) = K(u_1, u_2) + m(u_1, u_2)$

$(Ku_1 + mu_1, Ku_2 + mu_2)$

$\neq (Ku_1 + mu_1 + 1, Ku_2 + mu_2 + 1$

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Q.a $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ 2x2

Axiom 1:

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} a_0 & 0 \\ 0 & b_0 \end{bmatrix} = \begin{bmatrix} a+a_0 & 0 \\ 0 & b+b_0 \end{bmatrix}$$

Axiom 2,3 as 1

4: $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

5: $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ of negative $\begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix}$

6: $k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix}$

7,8,9 as 1

10: $1\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

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Q.11 $(1, x)$

$(1,y)+(1,y')=(1,y+y')$

$K(1,y)=(1,Ky)$

1: $(1,y)+(1,y')=(1,y+y')$

2: $(1,y)+(1,y')$

$=(1,y'+y)=(y'+y,1)$

$=(1+y')+(1+y)$

3:

$(1,y)+((1,y')+(1,y''))$

$=(1,y)+(1,y'+y'')$

$=(1,y+y'+y'')$

$=(1,y)+(1,y+y')$

$=((1,y)+(1,y')+(1,y''))$

4: $0=(1,0)$

$(1,y)+(1+0)=(1,y)$

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5: $u=(1,y)$ $-u=(1,-y)$

$(1,y)+(1,-y)=(1,0)$

$(1,-y)+(1,y)=(1,0)$.

6: $k(1,y)=(1,ky)$

7: $k((1,y)+(1,y'))=k(1,y+y')$

$=(1,ky+ky')=(1,ky)+(1,ky')$

$=k(1,y)+k(1,y')$

8: $(k+m)(1,y)=(1,(k+m)y)$

$=(1,ky+my)=(1,ky)+(1,my)$

$=k(1,y)+m(1,y)$

9: $k(m(1,y))=k((1,my))$

$(1,kmy)=km(1,y)$

10: $1(1,y)=(1,y)$

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Q.12 $(a_0+a_1x)+(b_0+b_1x)$

$= (a_0+b_0x)+(a_1+b_1)x$

$k(a_0+a_1x) = (ka_0)+(ka_1)x$

1: $(a_0+a_1x)+(b_0+b_1x) =$

$(a_0+b_0x)+(a_1+b_1x)$

2: same as 1

3: $((a_0+b_0x)+((a_1+b_1x)+(a_2+b_2x$

$= (a_0+a_1+a_2)+(b_0+b_1+b_2)x$

$= ((a_0+b_0x)+(a_1+b_1x))+$

(a_2+b_2x)

4: $0 = 0+0x$

$(a+bx)+(0+0x) = a+bx$

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5: $u = a + bx \quad -u = -a - bx$

$(a+bx) + (-a-bx)$

$= 0 + 0x$.

6: $k(a+bx) = ka + (kb)x$

7 $k((a_0+b_0x) + (a_1+b_1x))$

$= k((a_0+a_1) + (b_0+b_1)x)$

$= k(a_0+b_0x) + k(a_1+b_1x)$

8: $(k+m)(a+bx)$

$= (k+m)a + (k+m)bx$

$= k(a+bx) + m(a+bx)$

9: $k(m(a+bx)) = k(ma + mbx)$

$(kma + kmbx) = km(a+bx)$

10: $1(a+bx) = a+bx$.

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Q.4.2.

Q.3

a)
$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 0 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ 0 & b_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+b_{11} & 0 & \cdots & 0 \\ 0 & a_{22}+b_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 0 \end{bmatrix}$$

$$K\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 0 \end{bmatrix} = \begin{bmatrix} Ka_{11} & 0 & \cdots & 0 \\ 0 & Ka_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 0 \end{bmatrix}$$

W is a subspace of M_{nn}

b)
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

det = 1

Not a subspace of M_{nn}.

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c) $A = [a_{ij}]$ $B = [b_{ij}]$

$tr(A) = a_{11} + a_{12} \dots a_{nn}$

$tr(B) = b_{12} + b_{12} \dots a_{nn}$

$tr(A+B) = (a_{11} + b_{11}) + (b_{12} + a_{12}) \dots$

$tr(KA) = Ka_{11} + Ka_{22} \dots a_{nn}$

subspace

d) $A^T = -A$ $B^T = -B$

$(A+B)^T = -A - B = -(A+B)$

$(KA)^T = K(-A) = -KA$

space of M_{nn}.

Q.4

a) $A^T = A$ $B^T = B$

$(A+B)^T = A+B$

$(KA)^T = K(A^T) = KA$

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b) $Ax = 0$ has only trivial solution.

Not closed under multiplication

So not a subspace of M_{nn}.

c) $AB = BA \quad CB = BC$

$(A+C)B = AB + CB$

$= BA + BC = B(A+C)$

$(KA)B = K(AB) = B(KA)$

subspace of M_{nn}.

d) For invertible $n \times n$ matrices the set is not closed under multiplication.

Not subspace of M_{nn}.

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Q.14 $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ -1 & 1 & 0 & 7 \end{bmatrix}$$

a) when $K = 0$ so set not closed under multiplication. Not subspace of R^4..

b) $Ax = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$A(x+y) - Ax + ay = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A(Kx) = K\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

W is a subspace of R^4.

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Even

Q.16

a) $P(x) = Ex^2$

$= Ex^2 + E'x^2$

$K(Ex^2) = KEx^2$

W is a subspace of P_∞

b) $(a_0 + a_1x - - a_nx^n) +$

$(b_0 + b_1x - - - b_mx^m)$

$(a_0+b_0) + (a_1+b_1) - - (a_m+b_m) = 0$

$K(a_0 + a_1x + a_2x^2 \ldots + a_nx^n)$

$= K(a_0 + a_1 . + - - a_n) = K0 = 0$

W is a subspace of P_∞

c) $(a_0 + a_1x + a_2x^2 - - a_nx^{2n}) +$

$(b_0 + b_1x + b_2x^2 - - b_mx^{2m})$

$= (a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2$

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$--- + (a_{2m} + b_{2m})x^{2m}$

$K(a_0 + a_1x + a_2x^2 + --- + a_nx^{2n})$

$= Ka_0 + Ka_1x + Ka_2x^2 + --- + Ka_nx^{2n}$

W is a subspace of P_∞.

Ex 4.3 K20-1951

Q.15 $Ax = 0$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

a) $u = (1, 0, -1, 0)$,
$v = (0, 1, 0, -1)$

RREF of A is

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} x_1 + x_3 = 0 \\ x_2 + x_4 = 0 \end{matrix}$$

$x_1 = -x_3 \qquad x_1 = -s$
$x_2 = -x_4 \qquad x_2 = -t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} +s \\ +t \\ -s \\ -t \end{bmatrix} = \begin{bmatrix} +s \\ 0 \\ -s \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ t \\ 0 \\ -t \end{bmatrix}$$

$$= s \underbrace{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}}_{u} + t \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}}_{v}$$

therefore span span W

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b) $u = (1, 0, -1, 0)$ $V = (1, 1, -1, -1)$

$$RREF = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$u' = (0, 1, 0, -1)$

$V = u + u'$

Hence will spane the space W.

Q.16 $Ax = 0$

$$A = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

a) $u = (1, 1, 1, 0)$ $v = (0, -1, 0, 1)$

$$RREF = \begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2 - x_3 + x_4 = 0$

$x_2 = r$ $\qquad$ $x_3 = s - t$

$x_3 = s$

$x_4 = t$

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} r \\ s-t \\ s \\ t \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -t \\ 0 \\ t \end{bmatrix}$$

$$= r\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Hence do not span W.

b)

$u = (0,1,1,0)$ $v = (1,0,1,1)$

when $(1,0,0,0)$ do not span so also $u = (0,1,1,0)$ is not in span of vectors.

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Q.17 $T_A : R^2 \longrightarrow R^2$

$u_1 = (1,2) \quad u_2 = (-1,1)$

a) $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

$T_A(1,2) = (-1,4)$

$T_A(-1,1) = (-2,2)$

An arbitrary vector $b = (b_1, b_2)$

$(b_1, b_2) = K_1(-1,4) + K_2(-2,2)$

$-K_1 - 2K = b_1$
$4K_1 + 2K_2 = b_2$

$\begin{bmatrix} -1 & -2 \\ 4 & 2 \end{bmatrix}$ $\det = 6 \neq 0$

so $T_A(u_1)$ and $T_A(u_2)$ span R^2.

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b) $\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

$T_A(u_1) = (-1, 2)$

$T_A(u_2) = (-2, 4)$

$(b_1, b_2) = k_1(-1, 2) + k_2(-2, 4)$

$$-k_1 - 2k_2 = b_1$$
$$2k_1 + 4k_2 = b_2$$

$$\begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix} = 0$$

Since det $= 0$ so donot span R^2

Ex 4.4 K20-1052.

Q 11 Linearly dependent set in R^3

$v_1 = (\lambda, -\frac{1}{2}, -\frac{1}{2})$ $v_2 = (-\frac{1}{2}, \lambda, -\frac{1}{2})$

$v_3 = (-\frac{1}{2}, -\frac{1}{2}, \lambda)$

$K_1(\lambda, -\frac{1}{2}, -\frac{1}{2}) + K_2(-\frac{1}{2}, \lambda, \frac{1}{2})$

$K_3(-\frac{1}{2}, -\frac{1}{2}, \lambda)$

$K_1\lambda - \frac{K_2}{2} - \frac{K_3}{2} = 0$

$-\frac{K_1}{2} + K_2\lambda \mp \frac{K_3}{2} = 0$

$-\frac{K_1}{2} + \frac{K_2}{2} + K_3\lambda = 0$

$$\begin{vmatrix} \lambda & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \lambda & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \lambda \end{vmatrix} = 0 \text{ (for linearly dependent)}$$

$\lambda^3 - \frac{3}{4}\lambda - \frac{1}{4}$

$(\lambda + \frac{1}{2})(\lambda^2 - \frac{\lambda}{2} - \frac{1}{2}) = (\lambda + \frac{1}{2})(\lambda + \frac{1}{2})(\lambda - 1)$

$\lambda = -\frac{1}{2}, 1$

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Q.14 $T_A : R^3 \to R^3$

$u_1 = (1,0,0)$, $u_2 = (2,-1,1)$

$u_3 = (0,1,1)$.

a) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2\cancel{0} & 0 \end{bmatrix}$

$T_A(u_1) = (1,1,2)$

$T_A(u_2) = \cancel{(2,-1,1)}\ (3,-1,2)$

$T_A(u_3) = (3,-3,2)$

$K_1(1,1,2) + K_2(3,-1,2) + K_3(3,-3,2) = 0$

$K_1 + 3K_2 + 3K_3 = 0$

$K_1 - K_2 - 3K_3 = 0$

$2K_1 + 2K_2 + 2K_3 = 0$

$\begin{vmatrix} 1 & 3 & 3 \\ 1 & -1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = -8 \neq 0$

Therefore, system has trivial solution and is linearly independent.

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b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix}$

$T_A(u_1) = (1,1,2)$
$T_A(u_2) = (2,-2,2)$
$T_B(u_3) = (2,-2,2)$

since $T_A(u_2) = T_A(u_3)$ so

$\{T_A(u_1), T_A(u_2), T_A(u_3)\}$ is linearly independent.

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