Date: 19-5-21

CS211-Discrete Structures
Assignment #6, Spring 2021
(20K-1052, S.M. Hassan Ali)

Q.1

i) Undirected edges, multiple edges, no loops so undirected Multi-graph.

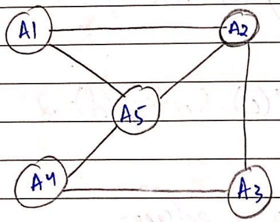
ii) Undirected edges, no multiple edges, no loops so simple graph.

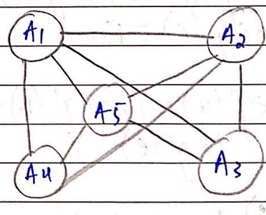
iii) Undirected edges, multiple edges, loops (3) so Undirected Pseudo graph.

iv) Directed edges, multiple edges, loops(2) so directed Multi-graph.

Q.2.

i)





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Q.3

a i) No of Vertices = 5   No of edges = 13
Degree of Vertices
deg(a) = 6, deg(b) = 6, deg(c) = 6, deg(d) = 5
deg(e) = 3
Neighbouring Vertices:
N(a) = {a, b, e}, N(b) = {a, c, d, e}, N(c) = {b, c, d}

N(d) = {b, c, e}, N(e) = {a, b, d}

ii) No of Vertices = 9   No of edges = 12.
Degree of vertices
deg(a) = 3, deg(b) = 2, deg(c) = 4, deg(d) = 0, deg(e) = 6,
deg(f) = 0, deg(g) = 4, deg(h) = 2, deg(i) = 3
Neighbouring Vertices:
N(a) = {c, e, i}, N(b) = {e, h}, N(c) = {a, e, g, i}, N(d) = ∅,
N(e) = {a, b, c, g}, N(f) = ∅, N(g) = {c, e}, N(h) = {b, i},
N(i) = {a, c, h}.

b i) No of Vertices = 4, No of edges =
$deg^-(a) = 6$, $deg^-(b) = 1$, $deg^-(c) = 2$, $deg^-(d) = 4$
$deg^-(e) = 0$.

$deg^+(a) = 1$, $deg^+(b) = 5$, $deg^+(c) = 5$, $deg^+(d) = 2$
$deg^+(e) = 0$.

ii) No of Vertices = 4   No of edges =

$deg^-(a) = 2$, $deg^-(b) = 3$, $deg^-(c) = 2$, $deg^-(d) = 1$

$deg^+(a) = 2$, $deg^+(b) = 4$, $deg^+(c) = 1$, $deg^+(d) = 1$

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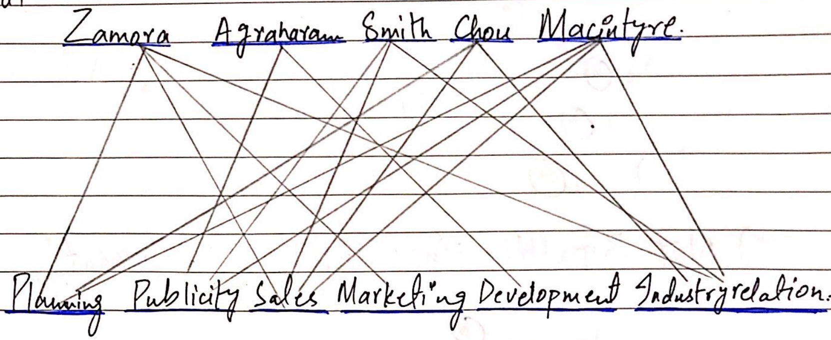
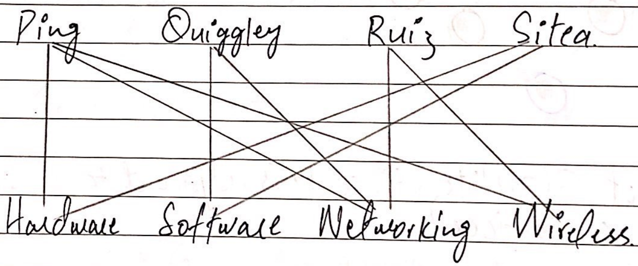
Q.4

a)

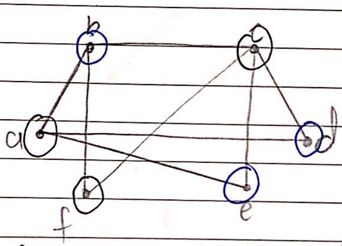


b)



Q.5

i) Not bipartite because b adj vertices becomes adjacent to a.



Same color of opposite vertices.

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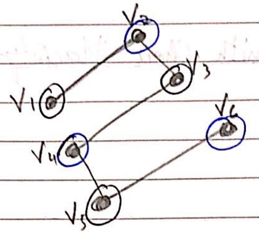
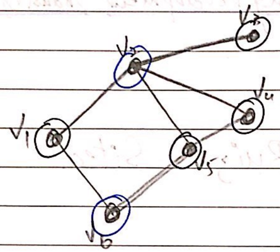
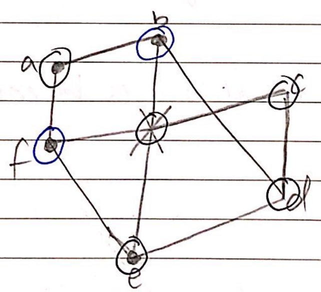
ii) Bipaltite ($A(V_1, V_3, V_5)$ & $B(V_2, V_4, V_6)$)



iii) Not Bipaltite since V_4 & V_5 are adjacent vertices.



iv) Not Bipartite (since b is adjacent to d and e vertices).



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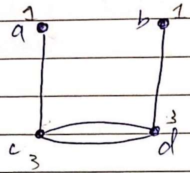
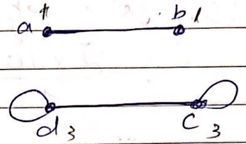
Q.6

a) No such graph is possible because the sum of vertices is odd $(1+1+2+3=7)$.

b)



OR



c) There is no simple graph with four vertices of degrees 1, 1, 3 and 3.

Q.7

a) Handshaking theorem.

$15 \times 3 = 45$

$45 \neq 2e$

So graph not possible.

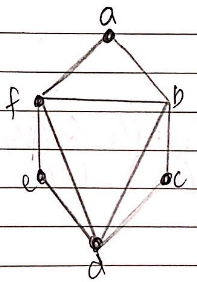
b) $4 \times 3 = 12$

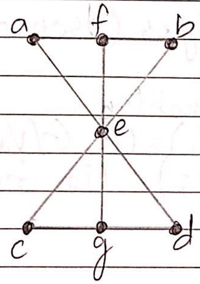
$12 = 2e$

$e = 6$ So graph is possible.

Q.8

a)





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b) degree = 4 edges = 10
vertices = ?

vertices × degree = 2 × edges (Handshaking theorem)

$$V = \frac{2 \times 10}{4} = 5$$

Q.9

i) Both G and G' have 5 vertices and 7 edges of same degrees (Isomorphic)

function:

$G(V_1) = W_2$, $G(V_2) = W_3$, $G(V_3) = W_1$
$G(V_4) = W_5$ $G(V_5) = W_4$.

ii) Both G and G' have 6 vertices and 6 edges. (Isomorphic)

function:

$G(V_1) = U_5$ $G(V_2) = U_2$ $G(V_3) = U_4$ $G(V_4) = U_3$
$G(V_5) = U_1$ $G(V_6) = U_6$

iii) Both G and G' have 7 vertices and 9 edges (Isomorphic).

function.

$G(V_1) = U_5$ $G(V_2) = U_4$ $G(V_3) = U_3$ $G(V_4) = U_2$.
$G(V_5) = U_7$ $G(V_6) = U_1$ $G(V_7) = U_6$

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iv) Both Graph G and G' are Isomorphic as they have 5 vertices and 7 edges.

$G(u_1) = V_5$, $g(u_2) = V_1$, $G(u_3) = V_2$, $G(u_4) = V_3$
$G(u_5) = V_4$.

Q.10
ii)

N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(z)
a	4,a	3,a	∞	∞	∞	∞	∞
ac			6,c	9,c	∞	∞	∞
ac~~d~~b			6,c		∞	∞	∞
acbd				7,d	11,d	∞	∞
acbde						12,e	∞
acbdef							18,f
acbdefg							16,g
acbdefgz	4,a	3,a	6,c	7,d	11,d	12,e	16,g

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N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(h)	D(i)	D(j)	D(k)	D(l)	D(m)	D(n)	D(o)	D(p)	D(q)	D(r)	D(s)	D(t)	D(z)
a	2,a.	4,a	1,a																	
ad	2,a	4,a			6,d	5,d														
adb		4,a		3,b	6,d	5,d														
adbe		4,a		3,b	6,d	5,d	6,e													
adbec					6,c	5,d	6,e													
adbeg					6,c		6,e			7,g										
adbegf							6,e	8,f	10,f	7,g										
adbegfh								8,f	10,f	7,g	7,h			14,h						
adbegfhk								8,f	10,f		7,h		11,k	14,h			9,k			
adbegfhkl								8,f	10,f			10,l	11,k	13,l			9,k			
adbegfhkli								8,f	10,f			10,l	11,k	13,l			9,k			
adbegfhklir									10,f			10,l	10,r	13,l		17,r			14,r	
adbegfhklirj									10,f			10,l	10,r	13,l		17,r			14,r	
adbegfhklirjm													10,r	13,l	12,m	17,r			14,r	
adbegfhklirmn														13,l	12,m	12,n			14,r	
adbegfhklirmnp														13,l		12,n		14,p	13,p	
adbeghklirmnpq														13,l				14,p	13,p	
adbeghklirmnpqo																		14,p	13,p	
adbeghklirmnpqot																		14,p		21,t
adbeghklirmnpqots																				16,s
adbeghklirmnpqotsz																				
	2,a	4a	1,a	3,b	6,c	5,d	6,e	8,f	10,f	7,g	7,h	10,l	10,r	13,l	12,m	12,n	9,k	14,p	13,p	16,s

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Q.11

i) Hamiltonian Circuit are:

ABCDA = 125

ABDCA = 140

ACBDA = 155.

So shortest way ABCDA = 125.

ii) Hamiltonian Circuit are:

ABCDA = 97

ABDCA = 108

ACBDA = 141

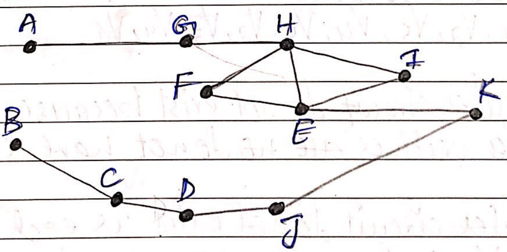
So ABCDA is the minimum distance travelled.

Q.12.

a) It can be done through.

Path: A —> H —> G —> B —> C —> D —> G —> F —> E.

b)



There is an euler path from A to B such that AGHIEFHEKJDCB.

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Q.13

i) Hamiltonian circuit:
$V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3, V_0$
Hamiltonian Path:
$V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3$

ii) Hamiltonian circuit:
does not exist
Hamiltonian Path:
d, a, b, c, f, e, h, g

iii) Hamiltonian circuit:
a, b, c, e, f, g, d, a.
Hamiltonian path:
a, b, c, e, f, g, d

Q.14

a) i) All 5 vertices have even degree.
Euler circuit:
$V_1, V_2, V_5, V_4, V_5, V_2, V_3, V_4, V_1$

ii) Euler circuit do not exist because all V_9 vertices ~~are no~~ do not have even vertices

b) ~~Euler circuit do not exist as each of the vertices do not have even degree.~~
ii) Euler path exists because exact two vertices have odd degree.
Path: $U, V_1, V_0, V_7, U, V_2, V_3, V_4, V_2, V_6, V_5, W, V_6, V_4, W$.

iv) Euler Path does not exist because four vertices have odd degree.

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Q.15

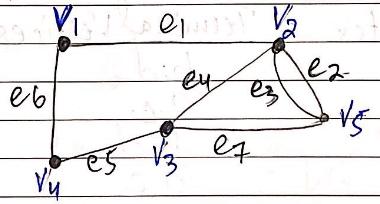
a) i)

$$\begin{array}{c|ccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \hline v_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ v_3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ v_5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ v_6 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

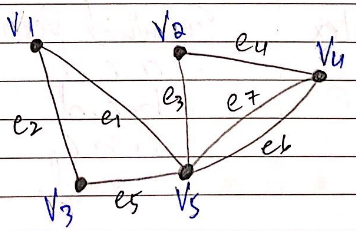
ii)

$$\begin{array}{c|cccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\ \hline v_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ v_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}$$

b) i



ii)



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Q.16

i)

Initial Vertex	Terminal Vertices
a	a, b, c, d
b	d
c	a, b
d	b, c, d

ii)

Initial Vertex	Terminal Vertices
a	b, d
b	a, c, d, e
c	b, c
d	a, e
e	c, e

iii)

Initial Vertex	Terminal Vertices
a	b, d
b	a, c
c	b, d
d	a, c

iv)

Initial Vertex	Terminal Vertices
a	a, c, d
b	b, c, d
c	a, b, c
d	a, c, d.

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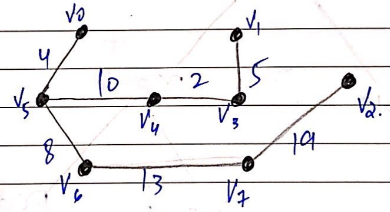
Q.17

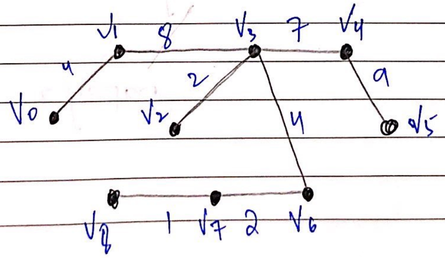
i) Level of n = 3
ii) Level of a = 0
iii) Tree hight = 5
iv) v, u children of n
v) parent of g is d.
vi) K and l siblings of j.
vii) m, s, t, x, y are descendants of f.
viii) a, b, e, K, c, f, m, t, d, h, i, n, o and v internal nodes
ix) v, n, h, d ancestors of z.
x). j, l, q, r, s, x, y, g, p, u, w & z are leaves.

Q.18 Minimum spanning tree cost.

i) $(V_0, V_5) = 4$ $(V_5, V_6) = 8$ $(V_4, V_5) = 10$
$(V_3, V_4) = 2$ $(V_1, V_3) = 5$ $(V_6, V_7) = 13$
$(V_2, V_7) = 19$ MST cost = 61



ii) $(V_0, V_1) = 4$ $(V_0, V_8) = 8$ $(V_7, V_8) = 1$
$(V_6, V_7) = 2$ $(V_3, V_6) = 4$ $(V_2, V_3) = 2.$
$(V_3, V_4) = 7$ $(V_4, V_5) = 9$



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Q.19 Kruskal's algorithm for minimum spanning tree.

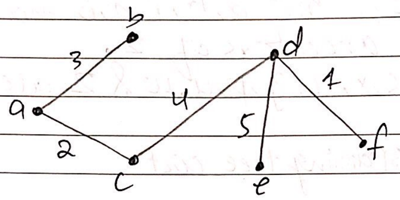
i) Order of edges:

(a,b) = 3 (d,f) = 1 (a,c) = 2

(c,d) = 4 ~~(e,f) = 6~~ (d,e) = 5

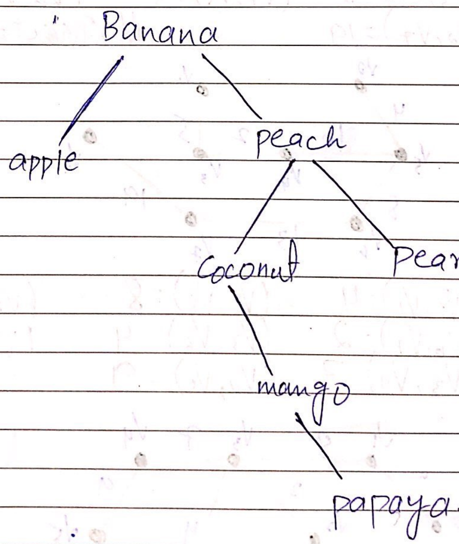
~~(b,c) = 3~~

~~(c,e) = 6.~~ MST cost = 15



Q.20

a) i)



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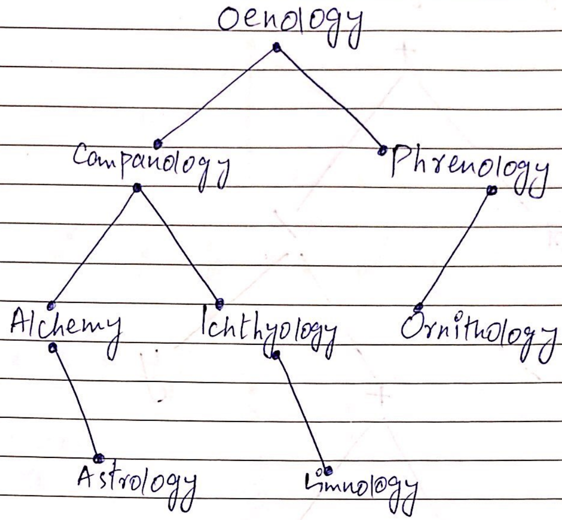
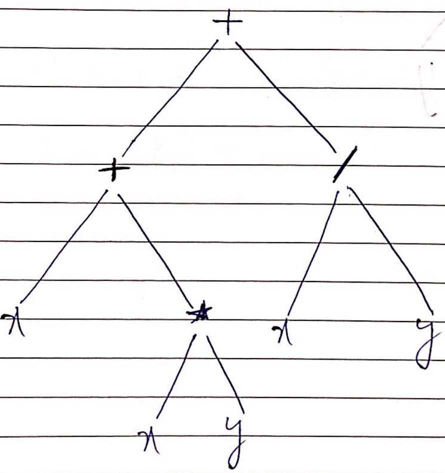
ii)



b i) $(x + xy) + (x/y)$



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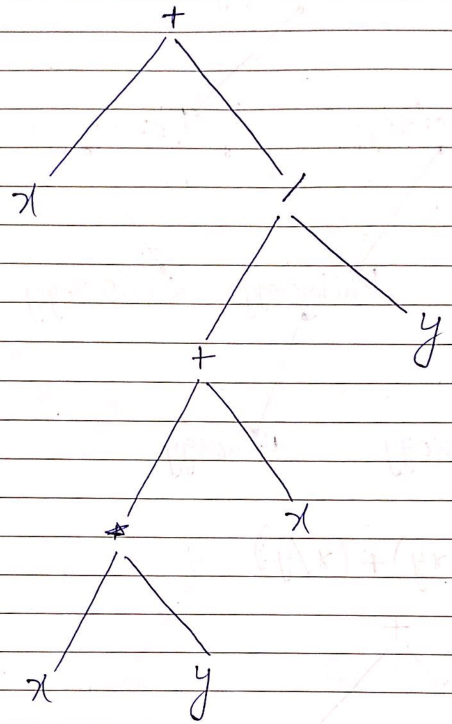
ii) $x+((xy+x)/y)$



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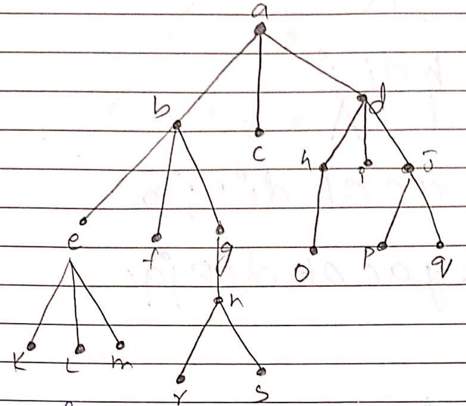
Q.21 Preorder, Inorder, Postorder.

i)



Preorder

a b c d

a b e f g c d h i j

a b e K l m f g n c d h o i j p q

a b e K l m f g n r s c d h o i j p q

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Inorder

iii) b a c d

e bf g ac h di j

k elmf ng ac oh di pjq

kelmf rns g ac oh di pjq

Postorder.

b c d a

efgbchijda

klmefngbcohipqjda

kl mef rs ng bcohipqjda

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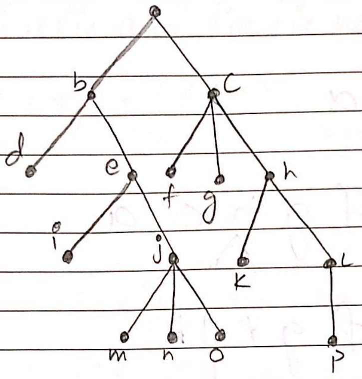
ii)



Preorder:

a b c

a b d e c f g h

a b d e i j c f g h k l

a b d e j m n o c f g h l p

Inorder:

b a c

d b e a f c g h

d b i e j a f c g k h l

d b i e m j n o a f c g k h p l

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Post order:

b c a

d e b f g h c a

d i j e b f g k l h

d i m n o j e b f g k p l h c a

Q.22.

a) Tree having 10000 vertices will have $n-1$ edge so 9999 edges.

b) Binary tree has two edges for each vertex. So it will have $1000 \times 2 = 2000$ edges

c) full m-ary tree has internal vertices
$n = mi + 1$
here $m = 5$ $i = 100$
So $n = 5(100) + 1 = 501$ vertices

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Q.23 Prefix and Postfix notation.

a) i) $(x + xy) + (x / y)$

Prefix: light to left

$+ + x * x y \; / x y$

Postfix: light ~~to~~ left

$x \; x y * + \; x y / +$

b) i) Prefix expression.

$4 - 2 = 2$

$6 / 2 = 3$

$+ - \uparrow 3 2 1 \uparrow 2 3 3$

$+ - \uparrow 3 2 3^2 3$

$+ - \uparrow 3 2 9 3$

$+ - 8 9 3$

$+ 9 - 8 3$

$+ 1 3$

$1 + 3 = 4$

ii) Postfix expression

$4 8 + 6 5 - * 3 2 - 2 2 + * /$

$4 + 8 \; 6 - 5 * 3 - 2 \; 2 + 2 * /$

$12 * 1 \; 1 4 * /$

$12 \; 1 * 4 /$

$12 \; 4 /$

$12 / 4$

3

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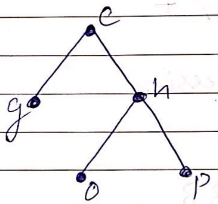
Q.24

i) Not a rooted full 3-ary tree as all the internal vertices do not have same children.

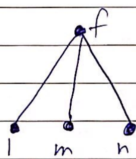
ii) Not a Balanced m-ary tree because its leaves are at 2, 3, 4 and 5.

iii) ~~st~~ subtrees at

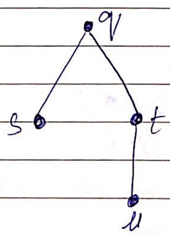
c.



f.



q



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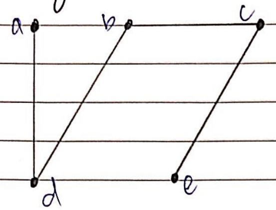
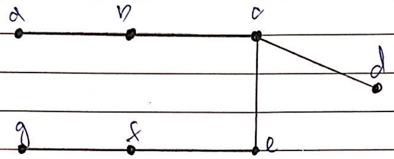
Q.25 Spanning Tree.

i)



ii)



iii)