$y = x^2 \longrightarrow f(x) = x^2$

$x = y^2 \longrightarrow f(y) = y^2$

Exercise 6.1

Area between the curve:

6.1.2 AREA FORMULA If f and g are continuous functions on the interval $[a, b]$, and if $f(x) \geq g(x)$ for all x in $[a, b]$, then the area of the region bounded above by $y = f(x)$, below by $y = g(x)$, on the left by the line $x = a$, and on the right by the line $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\,dx \qquad (1)$$

$\longrightarrow A = \int_a^b [f(y) - g(y)]\,dy$

▶ Example 1 Find the area of the region bounded above by $y = x + 6$, bounded below by $y = x^2$, and bounded on the sides by the lines $x = 0$ and $x = 2$.

$\longrightarrow 0 \rightarrow 2$

$$A = \int_0^2 [(x+6) - (x^2)]\,dx$$

$$= \int_0^2 x + 6 - x^2\,dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_0^2$$

$$= \left[\frac{2^2}{2} + 6 \times 2 - \frac{2^3}{3}\right] - [0]$$

$$= \frac{4}{2} + 12 - \frac{8}{3}$$

$$\boxed{A = 34/3 \text{ unit}^2}$$

▶ Example 2 Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

$$A = \int_a^b [f(x) - g(x)]\,dx$$

$$= \int_a^b [x^2 - x - 6]\,dx$$

$$= 3\int (x^2 - x - 6)\,dx$$

$y = x^2 \longrightarrow f(x) = x^2$

$y = x + 6 \longrightarrow g(x) = x + 6$

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$$= 3\int_{-2}^{3} (x^2 - x - 6)\, dx$$

$$x^2 - 3x + 2x - 6 = 0$$
$$x(x-3) + 2(x-3) = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3 \quad , x = -2$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x \right]_{-2}^{3}$$

$$= \left[\frac{3^3}{3} - \frac{3^2}{2} - 6(3) \right] - \left[\frac{(-2)^3}{3} - \frac{(-2)^2}{2} - 6(-2) \right]$$

$$= \left(\frac{27}{3} - \frac{9}{6} - 18 \right) - \left(\frac{-8}{3} - \frac{4}{2} + 12 \right)$$

$$A = \underbrace{-20.83}_{\text{neglect}} = 20.83 \text{ unit}^2$$

► **Example 4** Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.

$$x = y^2$$
$$y^2 = x$$
$$\cancel{y = \pm\sqrt{x}}$$
$$\boxed{\cancel{f(x) = \pm\sqrt{x}}}$$

$$\boxed{\cancel{g(x) = x - 2}}$$

$$A = \cancel{\int_a^b [f(x) - g(x)]\, dx}$$

$$=$$

$$\boxed{f(y) = y^2}$$

$$y = x - 2$$
$$x = y + 2$$
$$\boxed{g(y) = y + 2}$$

$$y^2 = y + 2$$
$$y^2 - y - 2 = 0$$
$$y^2 - 2y + y - 2 = 0$$

$$x = y^2 \qquad y = x - 2$$

~~$y^2 - 2y \dots$~~

$y=-1$
$y=2$

$x = y^2$ $\qquad$ $y = x - 2$

$x = y + 2$

$f(y) = y^2$ $\qquad$ $g(y) = y+2$

$$A = \int_{-1}^{2} \{f(y) - g(y)\}\, dy$$

$$A = \int_{-1}^{2} (y^2 - y - 2) = \underline{4.5 \text{ unit}^2}$$

7. $y = x^2,\ y = \sqrt{x},\ x = \frac{1}{4},\ x = 1$

8. $y = x^3 - 4x,\ y = 0,\ x = 0,\ x = 2$

9. $y = \cos 2x,\ y = 0,\ x = \pi/4,\ x = \pi/2$

$f(x) = \cos 2x$
$g(x) = 0$
$\int_{\pi/4}^{\pi/2} [\cos 2x - 0]\, dx$

Q. find Area b/w the Curve $y = x^2$ and $y = x^3$

$x^2 \rightarrow \int dx$

$f(x) = x^3$, $\quad g(x) = x^2$

$x^3 = x^2$
$x^3 - x^2 = 0$
$x^2(x-1) = 0$
$\boxed{x=0}$ $\quad$ $\boxed{x=1}$

$$A = \int_0^1 f(x) - g(x)\, dx$$

$$= \int_0^1 x^3 - x^2$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{4} - \frac{1}{3} = -0.255$$

neglect

-0.255 unit^2

$$= \frac{1}{4} - \frac{1}{3} = - -$$
$$= \underline{0.255}\ \text{unit}^2$$

$y = x^2$, $x = y^2$

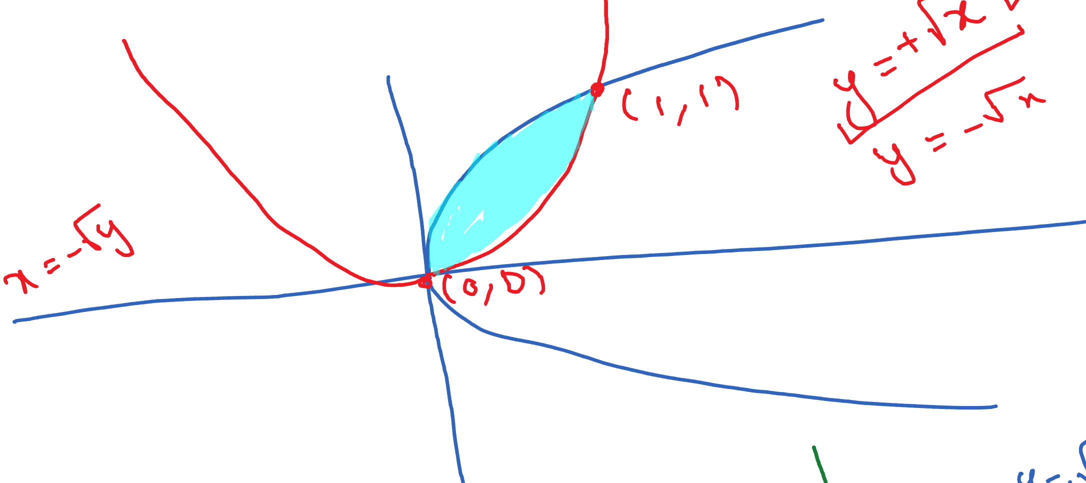
$x = \pm\sqrt{y}$

$x = -\sqrt{y}$

$x = +\sqrt{y}$

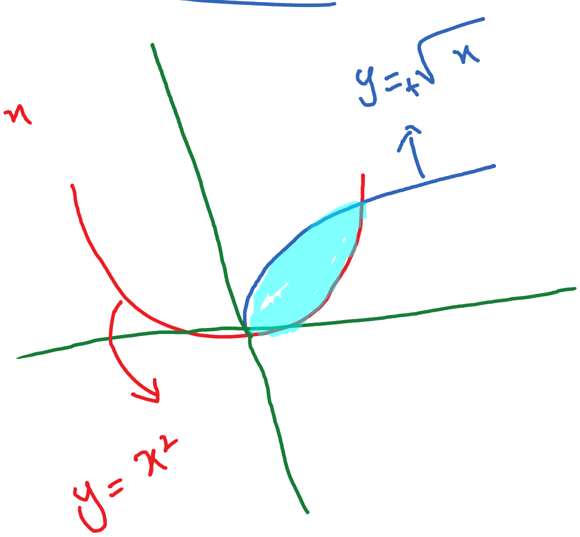
$y = +\sqrt{x}$

$y = -\sqrt{x}$



$$A = \int_a^b f(x) - g(x)\, dx$$

$$= \int_0^1 (x^2 - \sqrt{x})\, dx$$



$x = y + 2$

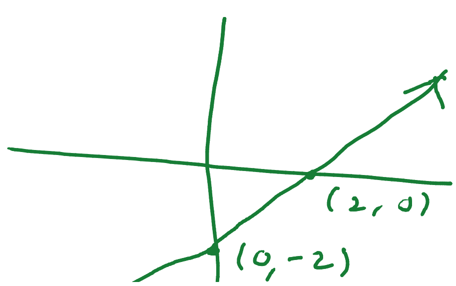
$0 = y + 2$

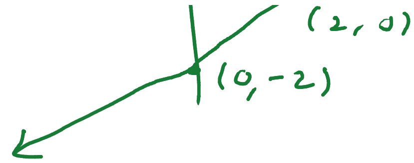
$\boxed{y = -2}$

$(0, -2)$

$x = 0 + 2$

$\boxed{x = 2} \longrightarrow (2, 0)$

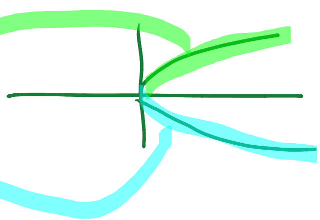




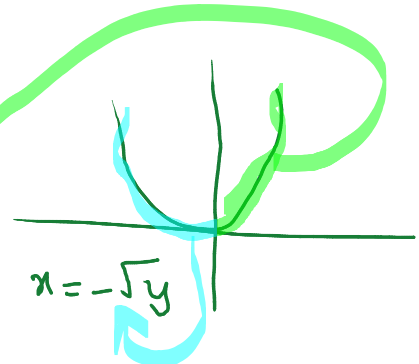
$x = y^2$

$y = \sqrt{x}$

$y = -\sqrt{x}$

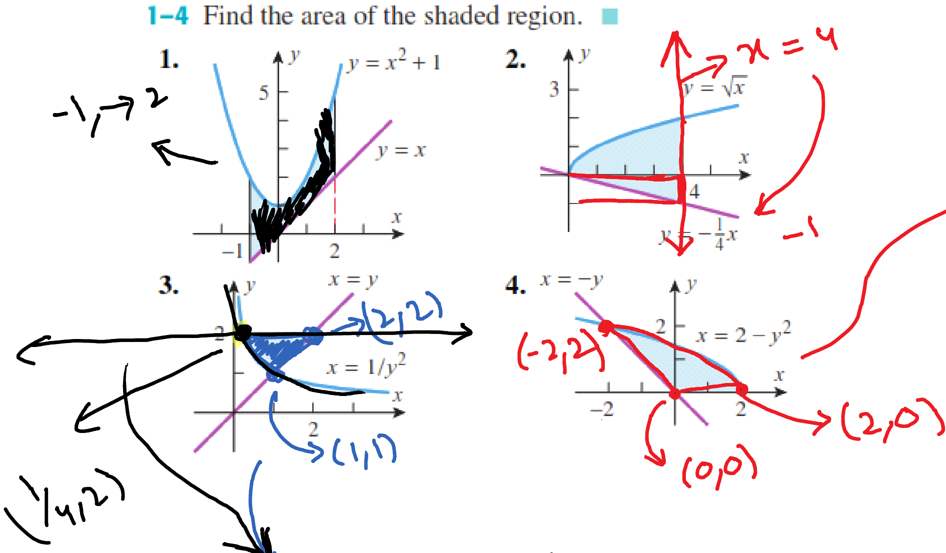


$y = x^2$



$x = +\sqrt{y}$ $\quad$ $x = -\sqrt{y}$

1–4 Find the area of the shaded region.



$f(y) = 2 - y^2$

$g(y) = -y$

$A = \int_0^2 f(y) - g(y)$

$A = \int_0^2 \left[(2-y^2) - (-y)\right] dy$

$\boxed{A = }$

$y = 2$ | $x = 1/y^2$

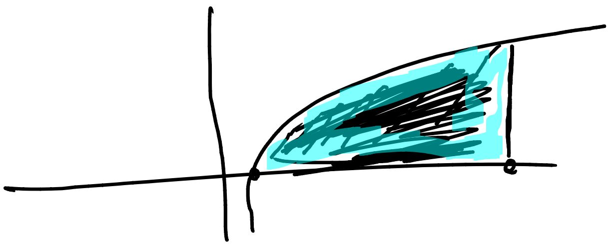
$x = 1/2^2 = 1/4$

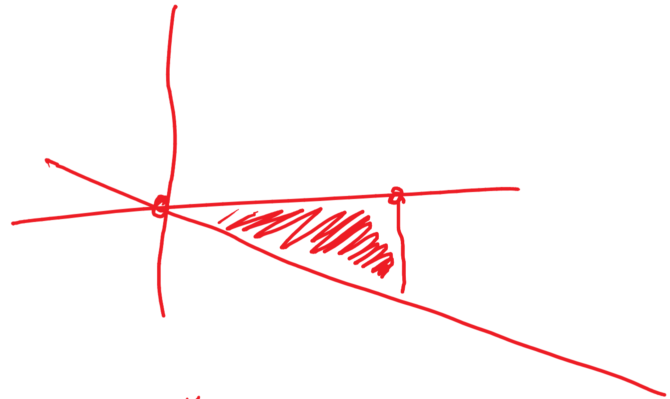
$y = 2$

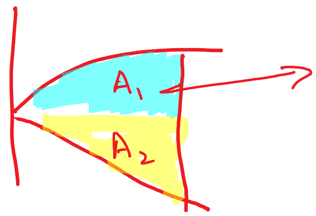
$x = y \longrightarrow f(y) = y$

$x = 1/y^2 \longrightarrow f(y) = 1/y^2$

$A = \int_1^2 \left[\frac{1}{y^2} - y\right] dy$







$A_1 = \int_0^4 \sqrt{x}\,dx = 5.3$

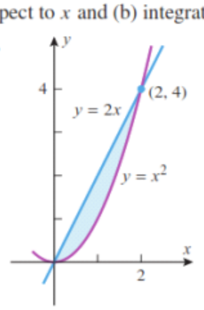
$A_2 = \int_0^4 \frac{1}{-4} x\,dx = -1/32$

$= 1/32$

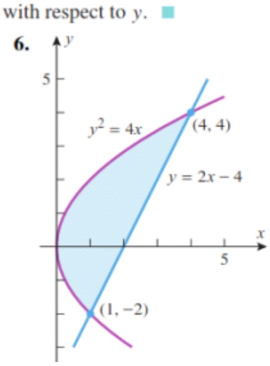
$-\frac{1}{4}\frac{x^2}{2} = -\frac{1}{8}\,16 = -2$

5–6 Find the area of the shaded region by (a) integrating with respect to x and (b) integrating with respect to y.

5.



6.



25. $y = xe^{x^2}, \quad y = 2|x|$

$y = 2x \quad | \; y = -2x$