Please enter data on page 477 in your calculator.

Males in L1

Females in L2

Section 9-1 Testing the Difference Between Two Means:

Objective: Test the difference between two large sample means, using the *z* test.

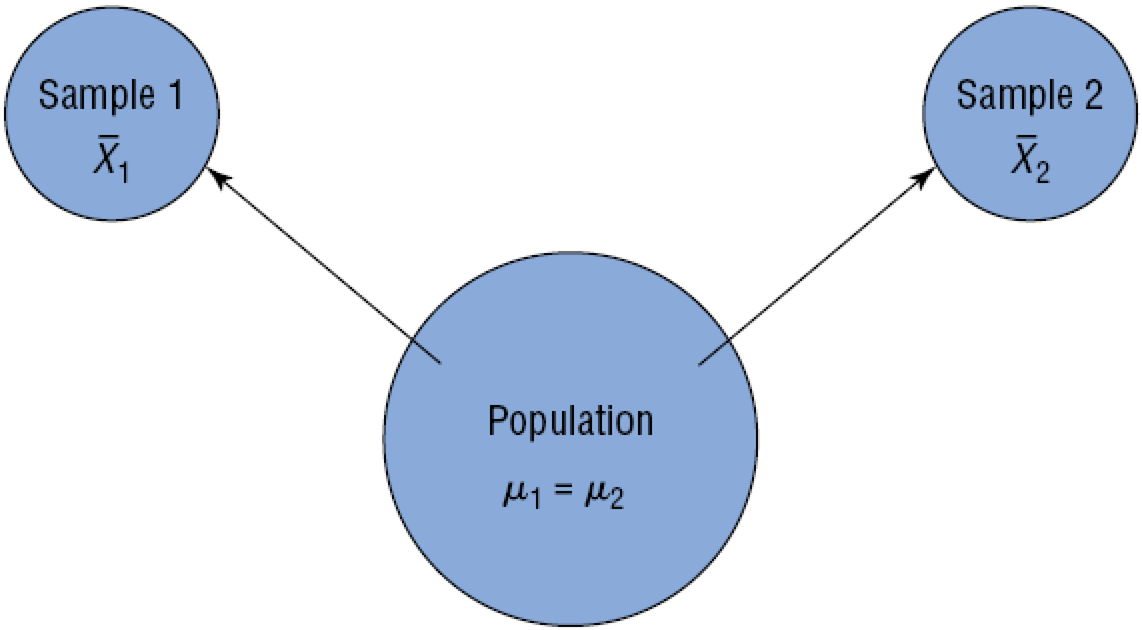
The Oscars

Comparing samples

- Compare period 1 and 3 test scores.
- Compare SAT scores of seniors in PSU stat class and seniors in Calc 2.
- Weight of runners who enrolled in a gym and those who did not.

Can you think of other comparison samples

Hypothesis Testing Situations in the Comparison of Means



(a) Difference is not significant

Do not reject H_0: $\mu_1 = \mu_2$ since $\bar{X}_1 - \bar{X}_2$ is not significant.

Hypothesis Testing Situations in the Comparison of Means

(b) Difference is significant

Reject H_0: $\mu_1 = \mu_2$ since $\bar{X}_1 - \bar{X}_2$ is significant.

9.1 Testing the Difference Between Two Means: Using the *z* Test

Assumptions:

1. The samples must be independent of each other. That is, there can be no relationship between the subjects in each sample.
2. The standard deviations of both populations must be known, and if the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

Two tailed

H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 \neq \mu_2$

OR

H_0: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 \neq 0$

Right tailed

H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 > \mu_2$

OR

H_0: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 > 0$

left tailed

H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 < \mu_2$

OR

H_0: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 < 0$

Formula for the z Test for Comparing Two Means from Independent Populations

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Example 9-1: Hotel Room Cost

A survey found that the average hotel room rate in New Orleans is $88.42 and the average room rate in Phoenix is $80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are $5.62 and $4.83, respectively. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the rates?

Hint: Make a table of information

Table of information

- Sample 1
- Use the context

$$\overline{x}_1 =$$

$$s_1 =$$

$$n_1 =$$

- Sample 2
- Use the context

$$\overline{x}_2 =$$

$$s_2 =$$

$$n_2 =$$

A survey found that the average hotel room rate in New Orleans is \$88.42 and the average room rate in Phoenix is \$80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are \$5.62 and \$4.83, respectively. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the rates?

Example 9-1: Hotel Room Cost

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Step 1: State the hypotheses and identify the claim.

H_0: $\mu_1 = \mu_2$ and H_1: $\mu_1 \neq \mu_2$ (claim)

Step 2: Find the critical value.

The critical value is $z = \pm 1.96$.

Example 9-1: Hotel Room Cost

A survey found that the average hotel room rate in New Orleans is $88.42 and the average room rate in Phoenix is $80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are $5.62 and $4.83, respectively. At α = 0.05, can it be concluded that there is a significant difference in the rates?

Step 3: Compute the test value.

$$z = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Example 9-1: Hotel Room Cost

A survey found that the average hotel room rate in New Orleans is \$88.42 and the average room rate in Phoenix is \$80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations of the populations are \$5.62 and \$4.83, respectively. At α = 0.05, can it be concluded that there is a significant difference in the rates?

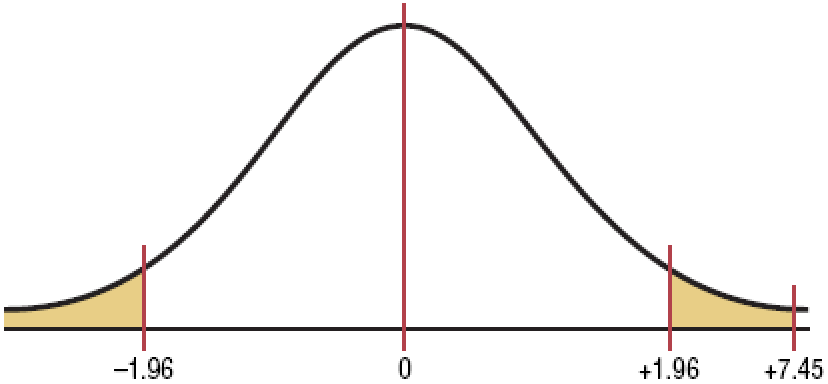
Step 3: Compute the test value.

$$z=\frac{\left(88.42-80.61\right)-\left(0\right)}{\sqrt{\frac{5.62^2}{50}+\frac{4.83^2}{50}}}=7.45$$

Example 9-1: Hotel Room Cost

Step 4: Make the decision.

Reject the null hypothesis at $\alpha = 0.05$, since $7.45 > 1.96$.



Step 5: Summarize the results.

There is enough evidence to support the claim that the means are not equal. Hence, there is a significant difference in the rates.

Data on page 477

Example 9-2

- A researcher hypothesis that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. At $\alpha = 0.10$, is there enough evidence to support the claim? Assume σ_1 and $\sigma_2 = 3.3$

Example 9-2: College Sports Offerings

A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. At $\alpha = 0.10$, is there enough evidence to support the claim? Assume σ_1 and $\sigma_2 = 3.3$.

Males					Females				
6	11	11	8	15	6	8	11	13	8
6	14	8	12	18	7	5	13	14	6
6	9	5	6	9	6	5	5	7	6
6	9	18	7	6	10	7	6	5	5
15	6	11	5	5	16	10	7	8	5
9	9	5	5	8	7	5	5	6	5
8	9	6	11	6	9	18	13	7	10
9	5	11	5	8	7	8	5	7	6
7	7	5	10	7	11	4	6	8	7
10	7	10	8	11	14	12	5	8	5

Example 9-2: College Sports Offerings

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_1 = \mu_2$ and H_1: $\mu_1 > \mu_2$ (claim)

Step 2: Compute the test value. Traditional method

Using a calculator, we find

For the males: $\bar{X}_1 = 8.6$ and $\sigma_1 = 3.3$

For the females: $\bar{X}_2 = 7.9$ and $\sigma_2 = 3.3$

Substitute in the formula.

$$z = \frac{\left(\bar{X}_1 - \bar{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(8.6 - 7.9) - (0)}{\sqrt{\frac{3.3^2}{50} + \frac{3.3^2}{50}}} = 1.06$$

Example 9-2: College Sports Offerings

Step 3: Find the *P-value*.

For $z = 0.939$

P-value = *0.174*

Step 4: Make the decision.

Do not reject the null hypothesis.

Step 5: Summarize the results.

There is not enough evidence to support the claim that colleges offer more sports for males than they do for females.

Example 9-2: College Sports Offerings

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_1 = \mu_2$ and H_1: $\mu_1 \neq \mu_2$ (claim)

Step 2: Compute the test value. **P-value method**

Using a calculator, **3: 2-SampZTest**

P=0.174

Example 9-2: College Sports Offerings

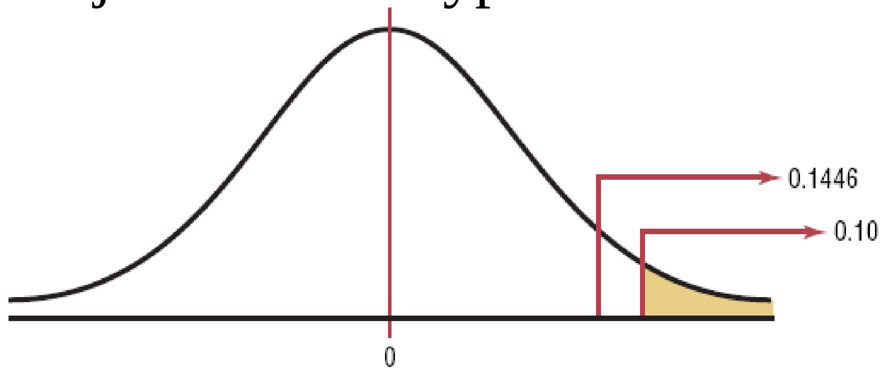
Step 3: compare the *P-value to* α.

$P > \alpha$

$0.1446 > \alpha$

Step 4: Make the decision.

Do not reject the null hypothesis.



Step 5: Summarize the results.

There is not enough evidence to support the claim that colleges offer more sports for males than they do for females.

Confidence Intervals for the Difference Between Two Means

Formula for the *z* confidence interval for the difference between two means from independent populations

$$\left(\bar{X}_1 - \bar{X}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \left(\mu_1 - \mu_2\right)$$

$$< \left(\bar{X}_1 - \bar{X}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Formula for Confidence Interval for Difference Between Two Means: Large Samples

$$(\overline{X}_1 - \overline{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^{\,2}}{n_1} + \frac{\sigma_2^{\,2}}{n_2}} < \mu_1 - \mu_2 < (\overline{X}_1 - \overline{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^{\,2}}{n_1} + \frac{\sigma_2^{\,2}}{n_2}}$$

- See page 478. Which test on your calculator do you think you should use?

Example 9-3

- Find the 95% confidence interval for the difference between the means for the data in Example 9-1.

- A survey found that the average hotel room rate in New Orleans is \$88.42 and the average room rate in Phoenix is \$80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations were \$5.62 and \$4.83, respectively. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the rates?

Example 9-3: Confidence Intervals

Find the 95% confidence interval for the difference between the means for the data in Example 9–1.

$$\left(\bar{X}_1 - \bar{X}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{X}_1 - \bar{X}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\left(88.42 - 80.61\right) - 1.96\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}} < \mu_1 - \mu_2 < \left(88.42 - 80.61\right) + 1.96\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}$$

$$7.81 - 2.05 < \mu_1 - \mu_2 < 7.81 + 2.05$$

$$5.76 < \mu_1 - \mu_2 < 9.86$$

- The interval does NOT contain the hypothesized difference between the means.
- In another words, zero is *not* within the interval, therefore, the decision is to REJECT the null hypothesis.
- If zero is within the interval the decision will be NOT to reject the null hypothesis.

On your own

- Study the examples in section 9.1

- Sec 9.1 page 479
- #7,13,16,19, 21