Date: _____

Discrete & continuous data

- => Mean
- => Median
- => Mode

} Central Tendency

- => Variance
- => Standard deviation
- => Quartile
- => Decile
- => Percentile
- => IQ Range

} Measure of dispersion

Date:

$$\frac{x_1 + x_2 + x_3 + \dots x_n}{n}$$

$$\bar{x} = \frac{\sum x}{n}$$

Median - Middlemost value in data
Mode - Most repeated value in data

} Measure of central tendency or location

Quartiles } Measure of non central tendency

Measure of location

Measure of central tendency
- Mean
- Median
- Mode

Mean = $\bar{X} = \frac{X_1 + X_2 \dots X_n}{n}$

$$\bar{x} = \frac{\sum_{i=1}^{n} x}{n}$$

Date: _____

d) Find mean (ie. $\bar{x}$) of the following data

1, 2, 2, 4, 5, 10

Ans $\frac{1+2+2+4+5+10}{6} = \frac{\bar{x}}{n} = \frac{24}{6} = 4$

x	
1	
2	
2	
4	
5	
10	

e) Find mean of the data 1, 1, 1, 51

$\frac{\bar{x}}{n} = \frac{1+1+1+51}{4} = \frac{54}{4} = 13.5$

x	
1	
1	
1	
51	

$\Sigma x = 54$

Date:

Median

Q. Find the median of the following data (odd case)

1st 2nd 3rd 4th 5th 6th 7th

0, 2, 3, 5, 20, 99, 100

* Data should be in ascending order (ordered)

$$\text{Median} = \left[\frac{n+1}{2}\right]^{th} \text{position}$$

~~e.g.~~ Ans $\frac{7+1}{2} = \frac{8}{2} = 4^{th}$

∴ 5 is the answer.

Q. Find the median of the following data (even case)

10, 20, 30, 40, 50, 60

$$\text{Median} = \left[\frac{n+1}{2}\right]^{th} \text{position}$$

$$= \left[\frac{6+1}{2}\right]^{th} = \left[\frac{7}{2}\right]^{th} = 3.5^{th} \text{ position}$$

3^{RD} value 4^{th} value

$$\frac{3^{RD} \text{ value} + 4^{th} \text{ value}}{2} = \frac{30+40}{2} = 35$$

Date:

Mode

Mode is the name of data that occurs most frequently.

Q. Find mode of 1, 1, 1, 2, 3, 4, 5
Ans 1

Q. Find mode of 5, 5, 5, 6, 8, 10, 10, 10.
Ans Mode = 5 and 10

* Such a data set is called Bimodal

Quartiles

Measure of non location

Quartile
Decile
Percentile
} - commonly used quartiles

Quartile split ordered data into four quarters

Q_1 = first Quartile 25%.
Q_2 = Second Quartile 50%.
Q_3 = Third Quartile 75%.

Date:

Q. Find Q_1, Q_2 and Q_3 of the following data

210, 220, 225, 225, 225, 235, 240, 250, 270, 280

Ans

210
220
225 → Q_1
225
Q_2 → 225 — $n = 10$
235
240
250 → Q_3
270
280

$$Q_i = i\,\frac{n}{4}$$

$$Q_1 = 1\left(\frac{10}{4}\right) = 2.5^{th} \text{ value}$$

$$Q_2 = 2\left(\frac{10}{4}\right) = 5^{th} \text{ value}$$

$$Q_3 = 3\left(\frac{10}{4}\right) = 7.5^{th} \text{ value}$$

Date: _____

Deciles

$D_i = i\left(\frac{n+1}{10}\right)^{Th}$ value

where $i = 1, 2, 3 \ldots 9$

$D_5 = 5\left(\frac{n+1}{10}\right)^{Th}$ value - median

$D_3 = 3\left(\frac{n+1}{10}\right)^{Th}$ value

$D_7 = 7\left(\frac{n+1}{10}\right)^{Th}$ value

Percentiles

$P_i = i\left(\frac{n+1}{100}\right)^{Th}$ value

$P_1 = 1\left(\frac{n+1}{100}\right)^{Th}$ value

$P_{25} = 25\left(\frac{n+1}{100}\right)^{Th}$ value

$\vdots$

$P_{99} = 99\left(\frac{n+1}{100}\right)^{Th}$ value

Date:

① Reading level of n = 16 student in a class

5, 6, 6, 6, 5, 8, 7, 7, 7, 8, 10, 9, 9, 9, 9, 9

Compute mean, median, mode, Quartile (Q_1 and Q_3) D_7, P_{25} and P_{50}

Ans. Mean $= \frac{\bar{x}}{n} = \frac{120}{16} = 7.5$

Median $= \frac{16+1}{2} = \frac{17}{2} = 8.5^{Th}$ value

5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 9, 9, 9, 9, 9, 10

$\frac{7+8}{2} = \frac{15}{2} = 7.5$

Mode = 9

$Q_1 = 1 \times \frac{16}{4} = 4^{Th}$ value $= 6$

$Q_3 = 3 \times \frac{16}{4} = 12^{Th}$ value $= 9$

$D_7 = 7\left(\frac{n+1}{10}\right) = 7\left(\frac{17}{10}\right) = 11.9^{Th}$ value $= 9$

$P_{25} = 25\left(\frac{17}{100}\right) = 4.25^{Th}$ value $= 6$

$P_{50} = 50\left(\frac{17}{100}\right) = 8.5^{Th}$ value $= 7.5$

Date:

Alternate Method

d. Transform the above data to be analyzed as group data.

Reading (x)	(F) frequency	$f_2 x_2$
5	2	10
6	3	18
7	3	21
8	2	16
9	5	45
10	1	10
	$\Sigma F = 16$	$\Sigma fx = 120$

Median for grouped data

$$l + \frac{h}{f}\left(\frac{\Sigma f}{2} - L.F\right)$$

where

l = lower class boundary
h = width of class interval
f = maximum frequency
$c.f$ = Cumulative frequency

Date:

Mode for grouped data

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

where

l = lower class boundary
f_m = maximum frequency in a data
f_1 = Preceding frequency
f_2 = Proceeding frequency
h = width / size of class interval

Quantile (grouped data)

$$Q_1 = l + \frac{h}{f}\left(\frac{\Sigma f}{4} - c.f\right)$$

$$Q_2 = l + \frac{h}{f}\left(2\frac{\Sigma f}{4} - c.f\right)$$

$$Q_3 = l + \frac{h}{f}\left(3\frac{\Sigma f}{4} - c.f\right)$$

OR

$$Q_i = l + \frac{h}{f}\left(i\frac{\Sigma f}{4} - c.f\right) \quad i = 1, 2, 3$$

similarly

$$D_i = l + \frac{h}{f}\left(i\frac{\Sigma f}{10} - c.f\right) \quad i = 1, 2, 3 \ldots 9$$

$$P_i = l + \frac{h}{f}\left(i\frac{\Sigma f}{100} - c.f\right) \quad i = 1, 2, 3 \ldots 99$$

Measure of Dispersion

Date: _____

- Range ⟶ Max value − Min value
- Quartile range / Quartile deviation

$$IQR = Q_3 - Q_1$$

$$Q.D = \frac{Q_3 - Q_1}{2}$$

- variance
- Standard Deviation.
- Mean absolute deviation

$D_1 = 0, 1, 5, 7, 9$ $\quad \bar{x} = 4.4$

$D_2 = 0, 1, 15, 17, 19$ $\quad \bar{x} = 10.4$

$D_3 = 0, 10, 20, 25, 35$ $\quad \bar{x} = 17$

Range

$R_1 = 9 - 0 = 9$

$R_2 = 19 - 0 = 19$

$R_3 = 35 - 0 = 35$

Mean absolute deviation

$$MAD_{\bar{x}} = \frac{\Sigma |x - \bar{x}|}{n} = \frac{15.6}{5} = 3.12$$

x	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$
0	$0 - 4.4 = -4.4$	4.4
1	$1 - 4.4 = -3.4$	3.4
5	$5 - 4.4 = 0.6$	0.6
7	$7 - 4.4 = 2.6$	2.6
9	$9 - 4.4 = 4.6$	4.6
	$\Sigma (x - \bar{x}) = 0$	⑨ 15.6

Date:

Median absolute deviation

$$MAD_{\tilde{x}} = \frac{\Sigma |x - \tilde{x}|}{n}$$

Mode absolute deviation

$$MAD_{\hat{x}} = \frac{\Sigma |x - \hat{x}|}{n}$$

If mode does not exist, then MAD does not exist

Variance

$$\sigma^2 = \frac{(|x - \bar{x}|)^2}{n}$$

* Continued from the previous table *

$(|x - \bar{x}|)^2$

19.36
11.56
0.36
6.76
21.16

$\sigma^2 = \frac{59.2}{5} = 11.8$

Date:

Standard Deviation

$$\sqrt{\sigma^2} = \sqrt{V(x)}$$

$$\sigma = \sqrt{11.8}$$
$$\sigma = 3.44$$

$$MAD_{\hat{x}} > MAD_{\tilde{x}} > MAD_{\bar{x}}$$

Coefficient of Variation

$$C.O.R = \frac{x_m - x_0}{x_3 + x_0} \times 100$$

$$C.O.D = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

$$C.O.M.A.D = \frac{MAD}{\bar{x}} \times 100$$

$$C.O.V = \frac{\sigma}{\mu} \times 100 = \frac{S.D}{Mean} \times 100$$

Important tools of Probability

Date:

Counting techniques

- Multiplicative rule $n_1 . n_2 . n_3 \ldots n_k$
- Additive rule $n_1 + n_2 + n_3 + \ldots n_k$

fundamental rules of counting:

i) Permutations
ii) Combinations

Q) There are four buses running b/w two cities. In how many ways can a man go from one city to the other and come back by a different bus?

Ans

Karachi —— Hyderabad

4 x 3 = 12

Q) How many results are possible, when a pair of die are thrown

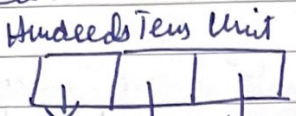
Ans. 6 faces on a die

$6 \times 6 = 36$

Q) How many lunches are possible consisting of soup and sandwich and a soft drink, If one can select from four soups, three sandwiches and two soft drinks

Date: _____

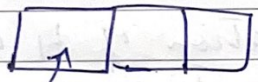
Q. How many three digit numbers can be formed from the digit 2, 4, 6 and 8. If repetitions are allowed.



4 4 4

$4 \times 4 \times 4$

Q. How many three digit numbers can be formed from the digits 0, 2, 4, 5 and 9 when each digit is allowed only once.



0 cannot be placed here hence, $4 \times 4 \times 3 = 48$ choices

Q. How many new arrangements can be made from the letters of word FAVOUR so that vowel will occupy even place

3 3 2 2 1 1

$3 \times 3 \times 2 \times 2 \times 1 \times 1 = 36$

Date:

Q) In how many ways can the letters of the word CAT be arranged.

CAT $\quad {}^3P_3$

Q) In how many ways can five people be lined up to get a bus.

$${}^5P_5 = \frac{5!}{(5-4)!} = \frac{120}{1!} = 120$$

Permutation of n different objects

Q) F find the number of permutation of 4 objects A, B, C, D taken 2 at a time.

Ans. $\quad {}^4P_2 = 12$

Q) Seven players of Pakistan's hockey team can play in any of the five forward line position. In how many ways can these positions be filled.

$${}^7P_5 = 2520$$

Date: _____

Permutation when repetition is allowed.

The number of permutation when repetition is allowed is given by

Q. In how many different ways can a 3-question true-false examination be answered.

Ans. $s = 3$

$n = 2$

No. of possible answers $(2)^3 = 8$

Q. In how many license plates of two letters followed by three digits can be made if the letters and digits can be repeated

Ans $(26)^2 \cdot (10)^5 = 676000$

Q. How many 2 digit numbers can be formed from the digits 1, 3, 7, 9 when the digits are

a) Not repeated

b) Repeated

Ans. a) 12

b) 16

Date:

Permutation of n objects when they are not all different.

$$\frac{n!}{n_1! n_2! n_3!}$$

Q.
In how many ways can the letters of the word STATISTICS be arranged.

Ans.

S = n_1 = 3 Times
T = n_2 = 3 Times
A = n_3 = 1 Time
I = n_4 = 2 Times
C = n_5 = 1 Time

$$\frac{10!}{3! \times 3! \times 1 \times 2! \times 1} = 50,400$$

Q. Find the number of permutations of 8 balls, taken 8 at a time, when 3 are red, 4 are green, and 1 is black and if one cannot distinguish b/w the same colour of balls.

$$\frac{8!}{3! \times 4! \times 1} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1}$$

$$= 280$$

Q. In how many ways can a committee of 3 students be selected from 4 students. (A, B, C, D)

n = 4 r = 3 ^nC_r

Date: _____

Combinations

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

Q. From a group of 10 boys and 6 girls, a committee of 3 boys and 2 girls is to be selected. How many ways can it be done?

Ans

$$^{10}C_3 \times {}^{6}C_2$$

$$= \frac{10!}{7! \times 3!} \times \frac{6!}{4! \times 2!}$$

$$= 120 \times 15 = 1800$$

Q. In how many ways can the city football team of 11 players be selected from 16 players.

$$^{16}C_{11}$$

Q. In how many ways can a cricket eleven be chosen out of 14 players? How many of them will
i) include a particular player
ii) exclude a particular

Ans $^{14}C_{10} = 364$ ways

Date:

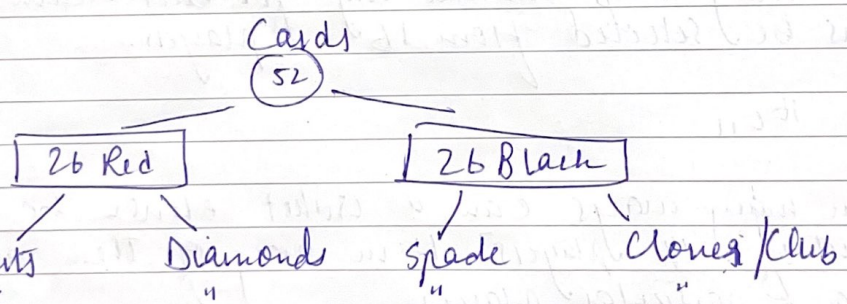
i) $^{13}C_{10}$ = 286 ways
ii) $^{13}C_{11}$ = 78 ways

Q. From a class containing 5 boys and 6 girls, a group of 5 students is to be selected. There are how many combinations
i) 3 boys and 2 girls $^5C_3 * {}^6C_2 = 150$
ii) 2 boys and 3 girls $^5C_2 \times {}^6C_3 = 200$
iii) 5 boys $^5C_5 * {}^6C_0 = 1$

Q. In how many ways, a hand of 4 cards can be selected from a deck of 52 cards.

$^{52}C_4 = 270725$

Sample space



Probab

The deg
is calle

P(A)

Q. Find
coin

Q. If
the pr

Probability

Date:

The degree of belief about a statement or event is called probability of the event.

$$P(A) = \frac{\text{No. of favourable outcomes}}{\text{total outcomes}}$$

Q. Find the probability of getting a head when a coin is tossed

$S = \{H, t\}$ $\quad P(A) = P(\text{head}) = \frac{1}{2}$

Q. If a die is rolled once, then what will be the probability of getting an even number

$S = \{1, 2, 3, 4, 5, 6\}$

$P(1) = \frac{1}{6}$

$P(\text{even}) = \left(\frac{3}{6}\right) = \frac{1}{2}$

Date:

Q. If two dice are rolled once, what will be the points of getting

i) Sum of ~~getting~~ dot is less than 5
ii) same dot
iii) Sum of dot is $\leq$ or $\geq$ 5

Ans. i)

ii) $\frac{1}{6}$

iii) $\frac{10}{36}$

Axioms of Probability

Date: _____

- If S is the sample space of an experiment and A & B are any two events then

(i) $P(A) \geq 0$

(ii) $P(S) = 1$

(iii) If A & B are the two disjoint mutually exclusive events then

$$P(A \cup B) = P(A) + P(B)$$

Addition law of Probability

If A & B are any two events then

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A & B are two mutually exclusive events i.e. $A \cap B = \emptyset$

$$P(A \cap B) = P(\emptyset) = 0$$

$$P(A \text{ or } B) = P(A) + P(B)$$

Date:

Q. If two dies are rolled one, find the
(i) probability of getting different numbers.

$P(\text{getting different numbers}) = \frac{30}{36} = \frac{5}{6}$

ii) probability of getting different numbers

Let B be the event of getting different numbers

$B = S/A \qquad S - A = A^c$

$P(B) = P(A^c) = 1 - P(A)$ } Derivation of Formula

$P(B) = 1 - P(A)$

$P(B) \downarrow$

different $= 1 - P(\text{same dot})$

$= 1 - \frac{6}{36} = \frac{30}{36}$

Q. A card is drawn from a set of 52 cards. What
(i) is the probability that the card is club or heart.

Ans. Let A be the event that card is club : $P(A) = 13/52$
Let B be the event that card is heart : $P(B) = 13/52$