MGT317
Operations Research (OR)

Linear Programming
&
Graphic Solution

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In this Lecture

- This topic concentrates on model formulation and computations in linear programming (LP).
- To illustrate the use of LP, real world applications in different areas will be formulated and solved.

TWO-VARIABLE MODEL

- We will study the graphical solution of two-variables LP.
- The two-variable s problem hardly exist in practice.
- The study will provide an introduction to the general simples algorithm in chapter 3

Example 2.1-1: The Reddy Mikks Company

- Reddy Mikks produces interior and exterior paints
- from two raw materials, Ml and M2:

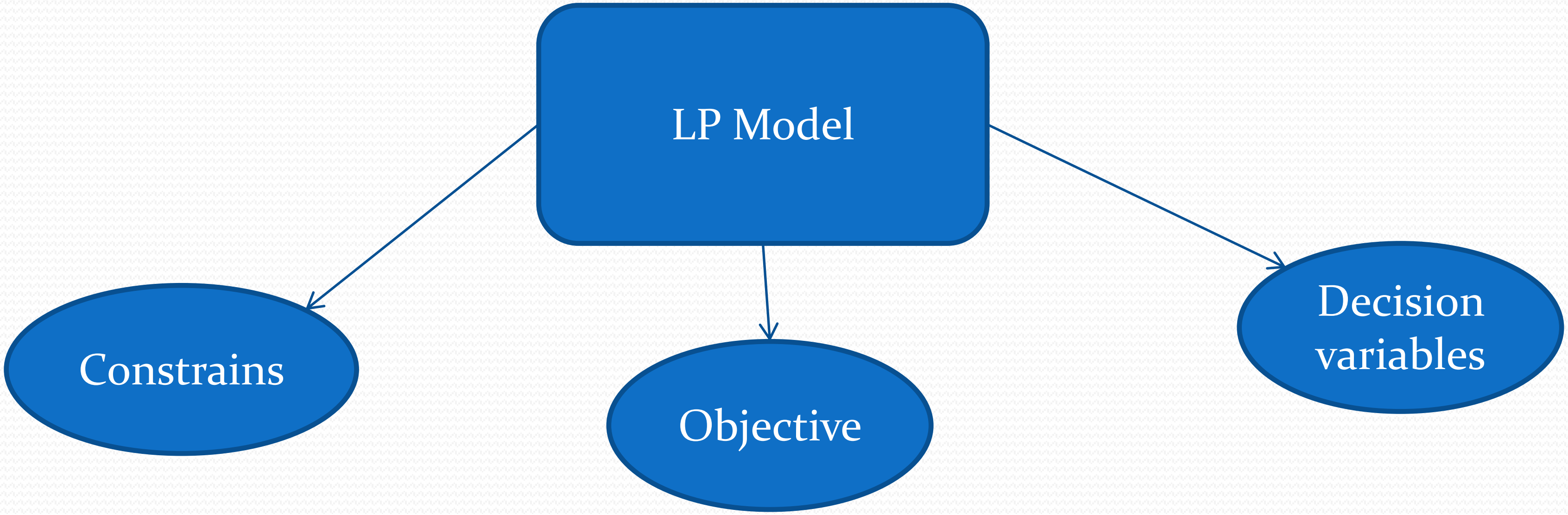
	Tons of raw material per ton of		Maximum daily availability (tons)
	Exterior paint	Interior paint	
M1	6	4	24
M2	1	2	6
Profit per ton (SR 1,000)	5	4	

- The daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton.
- The maximum daily demand for interior paint is 2 tons.

The Reddy Mikks Company

- Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit.

The LP models basic components



- **Decision variables** : that we seek to determine.
- **Objective** : goal that we need to optimize (maximize or minimize).
- **Constraints: restrictions** that the solution must satisfy.

Solution: The Reddy Mikks Company

- The first step in the development of the model is the definition of the decision variables.
- Since we need to determine the daily amounts to be produced of exterior and interior paints, we can define the variables as:

x_1=*Tons produced daily of exterior paint*

x_2=*Tons produced daily of interior paint*

The objective function

- To construct the objective function, note that the company wants to maximize (i.e., increase as much as possible) the total daily profit of both paints.
- Given that the profits per ton of exterior and interior paints are 5 and 4 (thousand) , respectively:

  - *Total profit from exterior paint*$=5x_1$
  - *Total profit from interior paint*$=4x_2$

Objective

- Maximize the total profit z:

$$Maximize \quad z = 5x_1 + 4x_2$$

- Next, we construct the constraints that restrict raw material usage and product demand.

Constraints

- The raw material restrictions are expressed verbally as

$$\textit{usage of raw materials} \leq \textit{maximum raw material available}$$

- The usage of raw material M1 by both paints is:

  $6x_1 + 4x_2$ *tons/day*

- The usage of M2 by both is:

  $1x_1 + 2x_2$ *tons/day*

- The daily availabilities of raw materials Ml and M2 are limited, hence we have to have:

$$6x_1 + 4x_2 \leq 24$$

$$1x_1 + 2x_2 \leq 6$$

Cont.

Constraints

- The excess of the daily production of interior over exterior paint should not exceed 1 ton, hence

$$x_2 - x_1 \leq 1$$

- The maximum daily demand of interior paint is limited to 2 tons, hence

$$x_2 \leq 2$$

- An implicit restriction is that variables cannot assume negative values:

$$x_1 , x_2 \geq 1$$

The complete Reddy Mikks model

Maximize	$z = 5x_1 + 4x_2$
subject to	$6x_1 + 4x_2 \leq 24$
	$x_1 + 2x_2 \leq 6$
	$-x_1 + x_2 \leq 1$
	$x_2 \leq 2$
	$x_1, x_2 \geq 0$

The complete Reddy Mikks model

- Any values of the variables that satisfy all five contraints constitue a feasible solution.
- The goal of the problem is to find the best feasible solution, or the optimum, that maximizes the total profit.

What is missing?

- We need to know how many feasible solution for Reddy Mikks model.
- Using values make this impossible because we have a huge number of possibilities we need to try.
- Graphical solution may leads us to the best solution.

Properties of LP model

- Proportional
- Additively
- Certainty
-

Graphical solution

- The graphical procedure includes two steps:
  - Determination of the feasible solution space.
  - Determination of the optimum solution from among all the feasible points in the solution space.

- We will see how to maximize and minimize the feasible solution.

The solution of the Reddy Mikks model

- The solution of the Reddy Mikks model requires two steps:
  1. Determination of the Feasible Solution Space:
     - The nonnegativity of the variables restricts the solution space area to the first quadrant of the x-y plane that lies above the x_1-axis and to the right of the x_2-axis.

$$x_1 , x_2 \geq 0$$

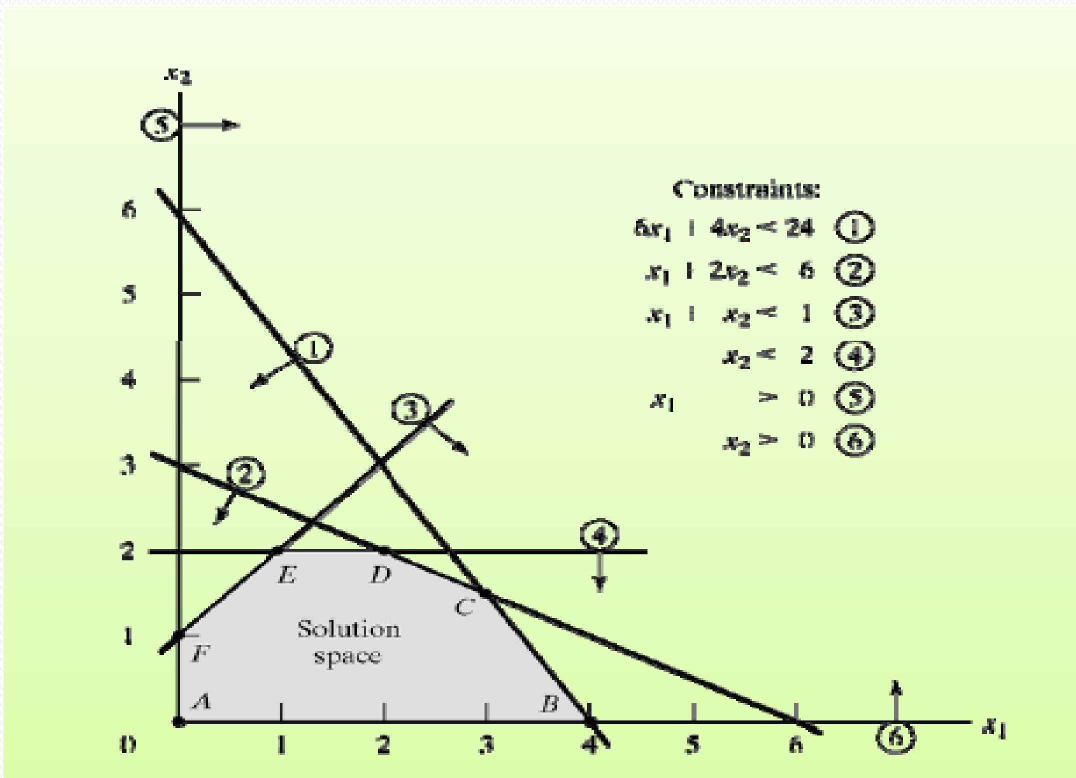
     - Replace each inequality with an equation and then graph the resulting straight line by locating two distinct points on it.

Replace $6x_1 + 4x_2 \leq 24$ with $6x_1 + 4x_2 = 24$

$1x_1 + 2x_2 \leq 6$ with $1x_1 + 2x_2 = 6$

Cont…

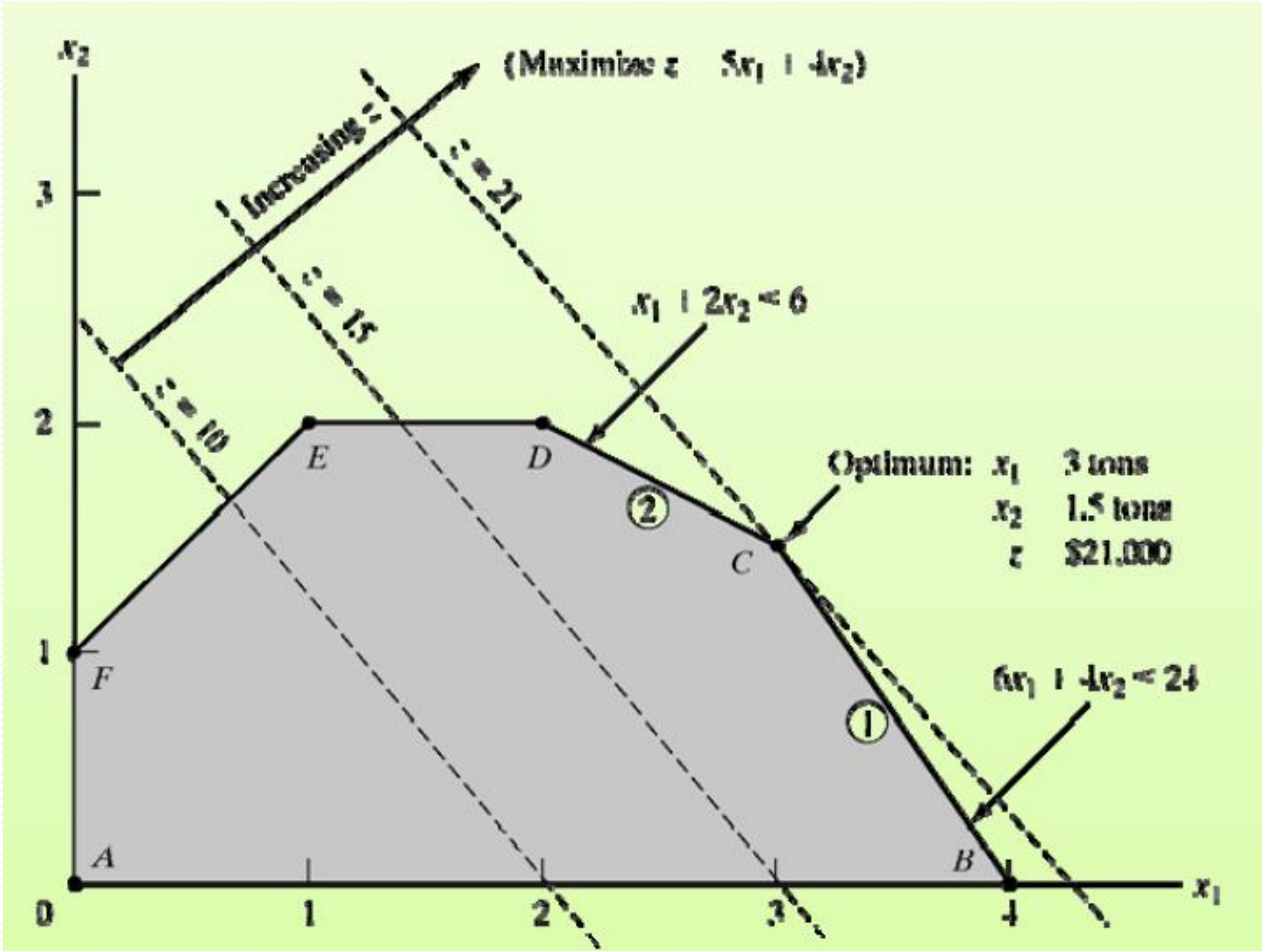
- Determine the feasible side by choosing (0 0) point as a reference point.
- Draw a line for each equation by taking two values for x_1 and x_2.



Second step

2. Determination of the Optimum Solution:

- Identify the direction in which the profit function $z=5x_1 + 4x_2$ increases by assigning arbitrary increasing values to z.
- Graph the resulting lines.



- The optimum solution is always associated with a corner point of the solution space.

- The solution space ABCDEF with infinite numbers of solution replaced with finite number of solution points.
- This is the key for the development of general algebric algorithm called the *Simples method*.

- *Will study it in ch3*

Home work

- Problem set 2.2A: question 1 & 2, page 19

Solution of a Minimization Model

- A farm uses at least 800 kg of feed daily, which is a mixture of corn and soybean meal with the following composition:

Feedstuff	Kg per kg of feedstuff		Cost (SR/kg)
	Protein	Fiber	
Corn	0.09	0.02	0.30
soybean	0.60	0.06	0.90

• The dietary requirements of the feed are at least 30% protein and at most 5% fiber.

What is required?

- The farm wants to determine the daily minimum cost feed mix.

Decision variables & objectives

- Decision variables

$$X_1 = kg\ of\ corn\ in\ the\ mix$$
$$X_2 = kg\ of\ soybean\ in\ the\ mix$$

- The objective criterion is the minimize the total cost:

$$Minimize\quad z = 0.3\,X_1 + 0.9\,X_2$$

Restrictions

- The farm needs 800 kg of feed a day:

$$X_1 + X_2 \geq 800$$

- The other restrictions on the mix are:

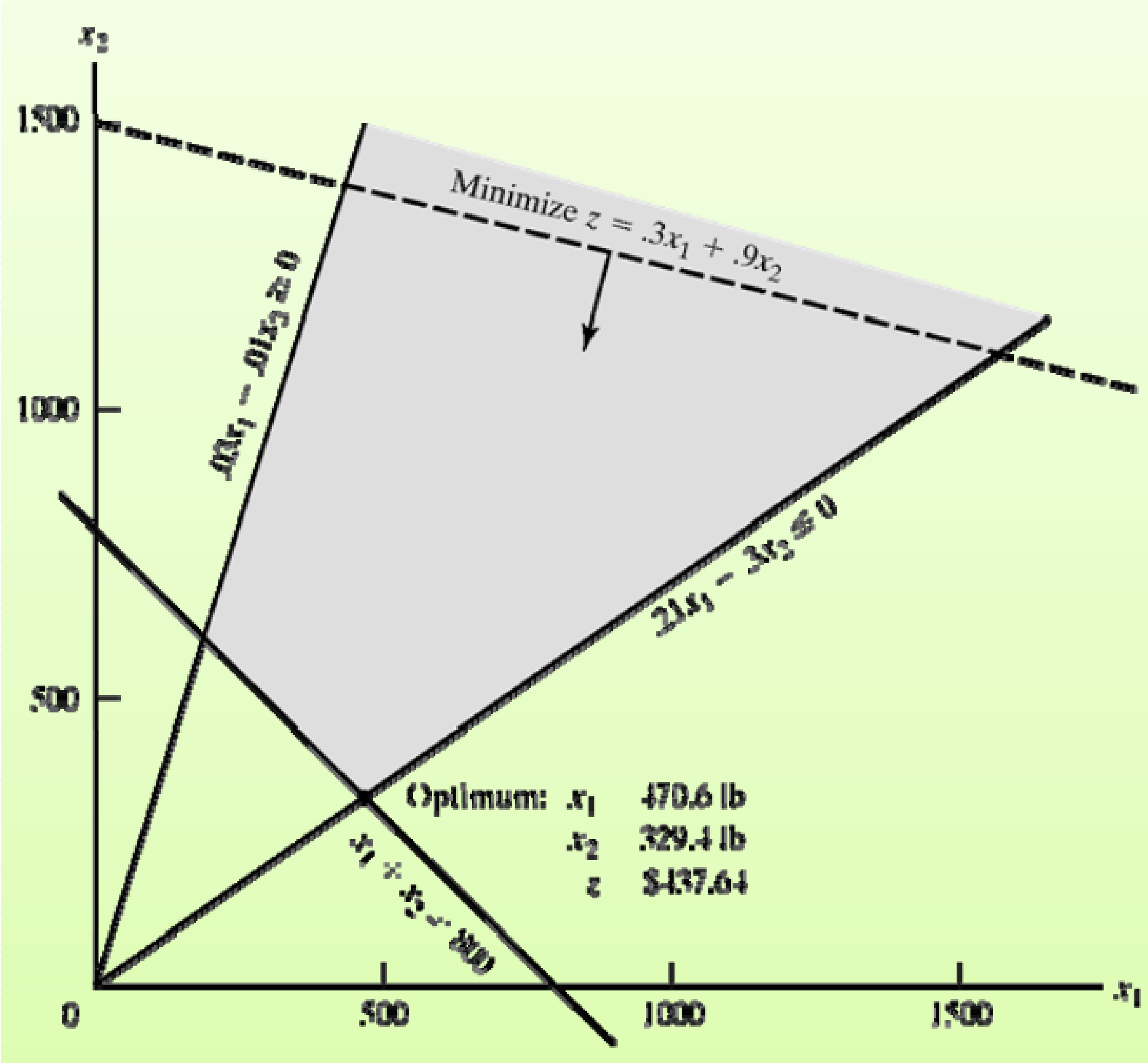
$$0.09X_1 + 0.6\,X_2 \geq 0.3(X_1 + X_2)$$
$$0.02X_1 + 0.06\,X_2 \leq 0.05(X_1 + X_2)$$

The model

- Minimize $z = 0.3\ X_1 + 0.9\ X_2$
- Subject

$$x_1 + x_2 \geq 800$$
$$0.21x_1 - 0.30\ x_2 \leq 0$$
$$0.03x_1 - 0.01\ x_2 \geq 0$$
$$x_1\ ,\ x_2 \geq 0$$

Graphical solution



Notes:

- The second and the third constrains pass through the origin point (0,0), e.g. (x_1 *=200)*so cannot be used as a reference point.
- That cannot help, because we need the area above the lines and under *z*-line to minimize *z*

Home work

- Problem set 2.2B, question 1, page 26

Any Questions about the Unit's Organisation