Applied Physics Assignment-1

Vectors

1. A displacement vector in the xy plane is 7.3 m long and directed at angle of 30° in Fig.1. Determine (a) the x component and (b) the y component of the vector.

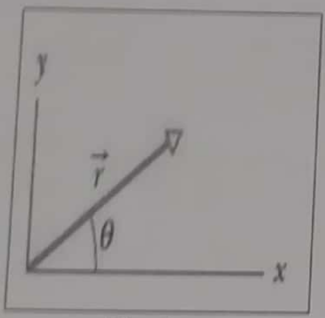


Fig-1

$r_x = r\cos\theta = 7.3 \times \cos 30$

(a) $\boxed{r_x = 6.321\ m}$

$r_y = r\sin\theta = 7.3 \times \sin 30$

(b) $\boxed{r_y = 3.65\ m}$

2. The two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in Fig-2 have equal magnitudes of 10m and the angles are $\theta_1 = 30^0$ and $\theta_2 = 105^0$. Find the (a) x and y components of their vector sum $\boldsymbol{r}$ (b) the magnitude of $\boldsymbol{r}$ and (c) the angle $\boldsymbol{r}$ makes with the positive direction of the x axis

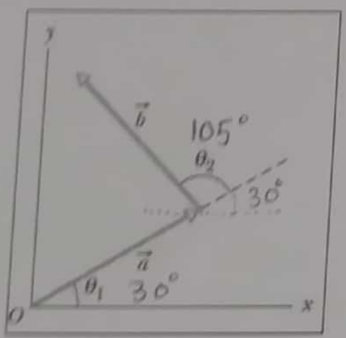


Fig-2

(a) $r_x = a_x + b_x = a\cos\theta_1 + b\cos\theta_2 = 10\cos 30° + 10\cos(30° + 105°)$

$r_x = 10\cos 30 + 10\cos 135° = 8.66 + (-7.071) = 1.59$

$r_y = a\sin\theta_1 + b\sin\theta_2 = 10\sin 30° + 10\sin(135°) = 10\times 0.5 + 10\times(0.707)$

$r_y = 12.07$

(b) $r = \sqrt{r_x^2 + r_y^2} = \sqrt{145.68 + 2.5281}$

$\boxed{r = \sqrt{148.2} = 12.17}$

$r = r_x\hat{i} + r_y\hat{j}$

$\boxed{r = 1.59\hat{i} + 12.07\hat{j}}$

(c) $\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{+12.07}{+1.59}\right)$

$\boxed{\theta = 82.49°}$

3. For the vectors in Fig. 3, with a = 4, b = 3, and c = 5, what are (a) the magnitude and the direction of $\boldsymbol{a} \times \boldsymbol{b}$, (b) the magnitude and) the direction of $\boldsymbol{a} \times \boldsymbol{c}$, and (c) the magnitude and the direction of $\boldsymbol{b} \times \boldsymbol{c}$?

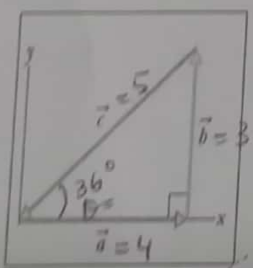


fig-3

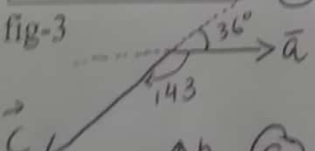
(a) $a = 4 \quad b = 3 \quad \theta = 90°$

$|a \times b| = ab\sin 90° = 4 \times 3(1) = \boxed{12 = |a \times b|}$

a is along x-axis & b is along y-axis, Hence a x b is along z-axis.

(b) $\theta = \tan^{-1}\left(\frac{3}{4}\right) \Rightarrow \theta = 36°$ Angle b/w a & c is $180 - 36 = 143°$

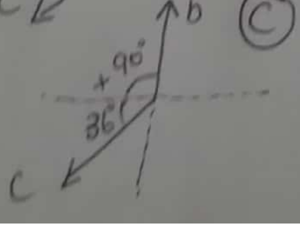
$a \times c = ac\sin 143 = 4 \times 5 \times \sin 143 = 12.03$ (-z-axis. b/c change in direction)



(c) $\theta = 126°$

$b \times c = bc\sin 126 = 3 \times 5\sin 126$

$\boxed{b \times c = 12}$ +z-axis



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4. By Considering the above problem -2 find the (a) ***a . b*** (b) ***a x b*** (c) angle between a and b

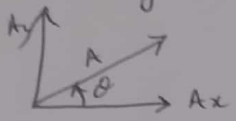
(a) $a \cdot b = ab\cos\theta = ab\cos 90 = 0$

(b) $a \times b = ab\sin\theta = 12$

(c) Angle b/w a & b is 90°

5. The *x* component of vector *A* is 25.0 m and the *y* component is 40.0 m. (a) What is the magnitude of *A* (b) What is the angle between the direction of and the positive direction of *x*?

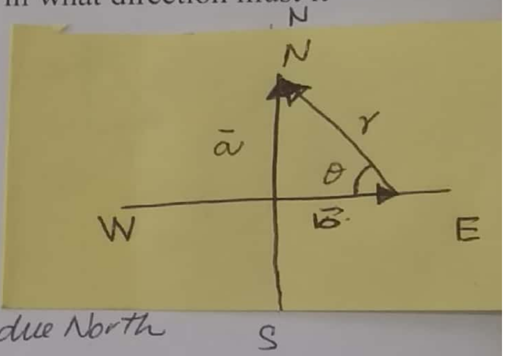
$A_x = 25$
$A_y = 40$



$|A| = \sqrt{(25)^2 + (40)^2}$
$= \sqrt{625 + 1600}$
$= \sqrt{2225}$
$\boxed{|A| = 47.16\ m}$

$\theta = \tan^{-1}\left(\frac{40}{25}\right)$
$= \tan^{-1}(1.6)$
$\boxed{\theta = 57^\circ}$

6. A ship sets out to sail to a point 120 km due north. An unexpected storm blows the ship to a point 100 km due east of its starting point. (a) How far and (b) in what direction must it now sail to reach its original destination?



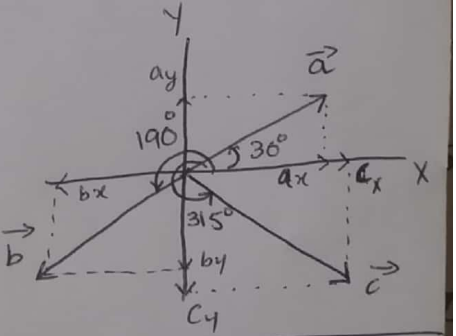
$a = 120km$
$b = 100km$

$|\bar{r}| = \sqrt{a^2 + b^2} = \sqrt{(120)^2 + (100)^2}$

$|\vec{r}| = 156.2\ km$

$\theta = \tan^{-1}\left(\frac{100}{120}\right) = \boxed{\theta = 39.79}$ west of due North

7. Three vectors ***a*** , ***b*** and ***c*** each have a magnitude of 50 m and lie in an xy plane. Their directions relative to the positive direction of the x axis are 30°, 195°, and 315°, respectively. What are (i) the magnitude and the angle of the vector ***a+b+c*** , and (ii) the magnitude and the angle of ***a-b+c***? What are the (iii) magnitude and angle of a fourth vector ***d*** such that ***(a+b) – (c+d) = 0*** ?



(a) $R = a + b + c$

$R_x = a_x + b_x + C_x = 50\cos 30 + 50\cos 190 + 50\cos 315$
$= 43.3 - 48.282 + 35.35 = \boxed{30.35 = R_x}$

$R_y = a_y + b_y + C_y = 50\sin 30 + 50\sin 190 + 50\sin 315$
$= 25 - 12.94 - 35.35 = \boxed{-23.29 = R_y}$

$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{(30.35)^2 + (-23.29)^2} = \sqrt{921 + 542.42} = \sqrt{1463.42} = \boxed{38.25 = |R|}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-23.29}{30.35}\right) \Rightarrow \boxed{\theta = -37.48^\circ}$

(b) $R_x = a_x - b_x + C_x = 43.3 + 48.28 + 35.35 = \boxed{126.93 = R_x}$

$R_y = a_y - b_y + C_y = 25 + 12.94 - 35.35 = \boxed{2.59 = R_y}$

$|R| = \sqrt{(16103) + (6.78)}$

$\boxed{|R| = 126.9}$

$\theta = \tan^{-1}\left(\frac{2.59}{126.93}\right) = \boxed{1.1^\circ = \theta}$

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8. Find the angle betwenn the vector A= 2i -3j+5k and the x, y,and z axes, respectively.

$A \cdot B = AB \cos\theta$

$A \cdot x = 2 \quad |A| = \sqrt{4+9+25} = 6.164$

$A \cdot y = -3 \quad \theta_x = \cos^{-1}\left(\frac{A \cdot x}{A}\right) \Rightarrow \boxed{\theta_x = 71.06^\circ}$

$A \cdot z = 5$

$\theta_y = \cos^{-1}\left(\frac{A \cdot y}{A}\right) = \boxed{\theta_y = 119^\circ}$

$\theta_z = \cos^{-1}\left(\frac{A \cdot z}{A}\right) = \boxed{\theta_z = 35.79^\circ}$

9. Calculate the angle between "r" and the positive z-axis. (c) Find the angle between "a" and "b". where a = 5i +4j -6k , b= -2i +2j+3k and c = 4i+3j+2k , r = a+b+c.

$r = 7\hat{i} + 9\hat{j} - \hat{k} \quad \boxed{|r| = 11.445}, \boxed{|a| = 8.77}, \boxed{|b| = 4.123}$

$r \cdot z = (7\hat{i} + 9\hat{j} - \hat{k}) \cdot \hat{k}$

$\boxed{r \cdot z = -1}$

$r \cdot z = |r||z| \cos\theta$

$\theta = \cos^{-1}\left(\frac{r \cdot z}{r}\right)$

$= \cos^{-1}\left(\frac{-1}{11.44}\right)$

$\boxed{\theta = 95^\circ}$

$a \cdot b = -10 + 8 - 18 = -20$

$\theta = \cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right)$

$= \cos^{-1}\left(\frac{-20}{36.18}\right)$

$\boxed{\theta = 123^\circ}$

10. Vector A has a magnitude of 6 units, vector B has a magnitude of 7 units, and A.B has a value of 14. What is the angle between the direction of A and B?

A = 6 units

B = 7 units

A.B = 14

$\theta = ?$

$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right)$

$= \cos^{-1}\left(\frac{14}{6 \times 7}\right)$

$= \cos^{-1}(0.33)$

$\boxed{\theta = 70.54^\circ}$

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Motion

Q.1

$r = (2t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$

Sol)

$r\,(\text{at} \Rightarrow t = 2) = [2(2)^3 - 5(2)]\hat{i} + [6 - 7(2)^4]\hat{j}$

$\boxed{r = 6\hat{i} - 106\hat{j}}$

$v = \frac{dr}{dt} = (6t^2 - 5)\hat{i} + (0 - 28t^3)\hat{j} = [6(2)^2 - 5]\hat{i} + [28(2)^3]\hat{j}$

$\boxed{v = 19\hat{i} - 224\hat{j}}$

$a = \frac{dv}{dt} = (12t - 0)\hat{i} + (-84t^2)\hat{j} = 12(2)\hat{i} - [84(2)^2]\hat{j}$

$\boxed{a = 24\hat{i} - 336\hat{j}}$

Q.2

$v_i = 18\,m/s$

$v_f = -30\,m/s$

$t = 2.4\,sec$

Sol)

$a_{avg} = \frac{v_f - v_i}{\Delta t} = \frac{-30 - 18}{2.4} = \boxed{-20\,m/s^2 = a}$

Q.3

$a = 9.8\,m/s^2$

$v_i = 0$

$v_f = \frac{1}{10} v_c = 3 \times 10^7\,m/s$

Sol)

$v_f = v_i + at \Rightarrow 3 \times 10^7 = 0 + 9.8t$

$\boxed{t = 3.1 \times 10^6\,sec}$

$S = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(9.8)(3.1 \times 10^6)^2$

$\boxed{S = 4.7 \times 10^{13}\,m}$

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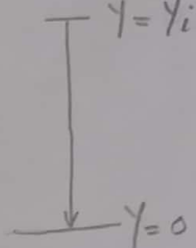
Q.4

$v_f = 24\,m/s$
$h = ?$
$t = ?$
$v_i = 0$

Sol



$v_f = v_i - gt \Rightarrow \quad 24 = 0 - (9.8)t$

$\boxed{t = 2.44\,sec}$

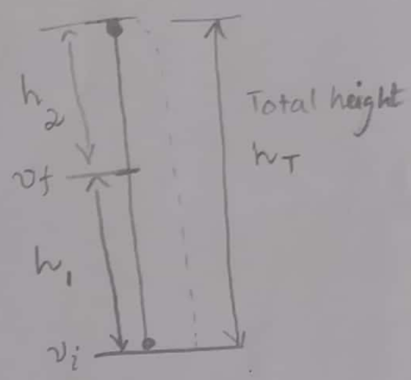
$Y = Y_i + v_i t - \frac{1}{2}gt^2$

$0 = Y + 0 - \frac{1}{2}9.8(2.44)^2$

$\boxed{Y = 29.4\,m}$

5

$t = 2.25\,sec$
$h_1 = 36.8\,m$
$v_i = ?$
$v_f = ?$
$h_2 =$



Sol

$Y = Y_i + v_i t - \frac{1}{2}gt^2$

$36.8 = 0 + v_i(2.25) - \frac{1}{2}(9.8)(2.25)^2$

$\boxed{v_i = 27.4\,m/s}$

$v_f = v_i - gt$
$= 27.4 - 9.8(2.25)$

$\boxed{v_f = 5.4\,m/s}$

For highest point $v_f = 0$ and time taken is

$v_f = v_i - gt$
$0 = 27.4 - (9.8)t$

$\boxed{t = 2.8\,sec}$

Total height $h_t \Rightarrow$ $Y = Y_i - v_i t - \frac{1}{2}gt^2$
$h_t = 0 - 24.7(2.8) - \frac{1}{2}(9.8)(2.8)^2$

$\boxed{h_t = 38.3\,m}$

So, $h_2 = h_t - h_1$
$= 38.3 - 36.8 = 1.5\,m$ $\quad \boxed{h_2 = 1.5\,m}$

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Q.6 $v_i = 40 km/h$

$v_f = 60 km/h$

$v_{avg} = ?$

Sol

Distance up = Distance down = D

So total Distance = 2D

$$v_{avg} = \frac{\text{Total distance}}{\text{Time Interval}} = \frac{2D}{t_{up} + t_{down}}$$

$$s = vt \Rightarrow t_{up} = \frac{s}{v} = \frac{D}{v_i}$$

$$t_{down} = \frac{D}{v_f}$$

So,

$$v_{avg} = \frac{2D}{\frac{D}{v_i} + \frac{D}{v_f}} = \frac{2\cancel{D}}{\frac{\cancel{D}(v_f + v_i)}{v_i v_f}} = \frac{2 v_i v_f}{v_f + v_i}$$

$$= \frac{2(40)(60)}{40+60} = 48 km/h$$

$$\boxed{v_{avg} = 48 km/h}$$

Q.7 $v_i = 12.4$

$h = 81.3$

$v_f = ?$

$v_i = 0$ ↓ For returning ∴ At highest point $V_i = 0$

Sol

$$v_f^2 = v_i^2 - 2gh = 0 - 2gh = -2(9.8)(81.3)$$

$$\boxed{v_f = \pm 41.8 m/s}$$

↑ Time for upward $\Rightarrow$ $v_f = v_i - gt \Rightarrow 0 = 12.4 - (9.8)t$

$t_u = 1.2653 sec$

↓ Time for downward $\Rightarrow$ $41.8 = 0 - (9.8)t \Rightarrow t_d = 4.265 sec$

Total $t = t_u + t_d = 1.265 + 4.265 = 5.53$

$$\boxed{t = 5.53 sec}$$

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Q.8

$v_f = 0$
$v_i = 360 km/h$
$S = 1.8km$
$a = ?$

Sol

$v_f^2 - v_i^2 = 2as$
$0 - (360)^2 = 2aS$

$$a = \frac{(360)^2}{2(1.8)} = 36$$

$$\boxed{a = 36 km/h}$$

Q.9

$a = -4.92 m/s^2$
$v_i = 24.6 m/s$
$v_f = 0$
$t = ?$
$S = ?$

Sol

$v_f = v_i + at \Rightarrow 0 = 24.6 + (-4.92)t$

$$\boxed{t = 5 sec}$$

$S = v_i t + \frac{1}{2}at^2 \Rightarrow (24.6)(5) + \frac{1}{2}(-4.92)(5)^2$
$= 123 - 61.5$

$$\boxed{S = 61.5m}$$

Q.10

$x = 50t + 10t^2$

Sol

$$v_{avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{x(t=3) - x(t=0)}{3 - 0} = \frac{240 - 0}{3} = 80$$

$$\boxed{v_{avg} = 80 m/s}$$

$v_{in} = \frac{dx}{dt} = 50 + 20t \Rightarrow 50 + 20(3) = 110 m/s$

$$\boxed{v_{in} = 110 m/s}$$

$a_{in} = \frac{d v_{in}}{dt} = 0 + 20 = 20$

$$\boxed{a_{in} = 20 m/s^2}$$

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