SM.

Date 2/2/2022.

~~reproduce.~~

↳ Simulation is to imitate the operation of a facility or process usually through a computer.

↳ Application Areas of Simulation :- (wsc).
  - ↳ Queuing System.
  - ↳ Manufacturing.
  - ↳ Construction Engineering.
  - ↳ Military Applications.
  - ↳ logistics.
  - ↳ Transportation.
  - ↳ Distributions.
  - ↳ Business Process Simulation.
  - ↳ Human System.
    - ↳ Simulation of AI.
    - ↳ Robots Simulation of Humans

↳ What Simulation does & when to use?
  - ↳ enables Study of internal interactions of complex systems.
  - ↳ informational, organizational & environmental changes can be simulated & the effect of those on the model behaviour can be observed.
  - ↳ the knowledge gained ~~from~~ in designing a Simulation model can suggest improvement in the system.
  - ↳ ~~An~~ animation shows a system in Simulated operation so that the plan can be visualized. (drone, plane crash, acoustic Engineering sound,

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Date 08/09/2022.

- When Simulation is not appropriate
  - Banks & Gapes paper - 1997.
    - when problem can be solved by (human sense) common sense
    - if problem can be solved ~~analyticatically~~ analytically.

- Terminologies used in simulation:-
  - System:
    - it is defined as a group of objects that are joined together in some regular interactions or interdependence towards the accomplishment of some purpose.
    - Components of system:-
      - Entity:
        - an object of interest in system.
      - Attributes:
        - it is a property of an entity

          quality of entity.
      - Activity:
        - Any crosses causing changes in the system.
      - Events:
        - it is an instantaneous occurrence that may change the state of the system.
      - State variables:
        - collection of variables necessary to describe system at any time relative to the object of study

          changes occur when we give input or output.

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Example 01:-

System:- Banking.

Entity:- Customers.

Attribute:- Balance.

Activity:- Making deposits.

Events:- Arrival, departure.

State variables:- No. of customers waiting in the queue, servers (service providers).

Example:-

System:- Production of Automobiles.

Entity:- Machince.

Attribute:- Speed, Efficiency.

Activity:- welding, Stamping.

Events:- Breakdown, failure of machine.

State variables:- busy or idle.

Server
- idle
- busy

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Numerical / $\boxed{\text{Simulation}}$ => error?
↳ approximation.

→ Ways to Study System :-

- Experiment with actual system.
- Experiment with model of the system.
  - Physical Model.
  - Mathematical Model.
    - Analytical Solution.
    - Simulation

→ Types Of Systems :-

↳ Discrete & continuous :

- Discrete
  - ↳ Queuing system
  - ↳ State variable change at particular time.
- continuous
  - ↳ velocity of airplane
  - ↳ State variable change continuously.

→ Classification of Simulation Model :-

↳ Static or Dynamic Model.

- Static
  - ↳ not change with time
  - ↳ building.
- Dynamic
  - ↳ change with time
  - ↳ velocity of airplane
  - ↳ queuing system.

↳ Deterministic or Stochastic
↓ ↓
↳ output predict. ↳ randomies.
D.E ~~↳ differential equation~~
↳ Discrete and Continuous.
↳ Discrete ~~Mod~~ & model continuous.
↳ no. of buses → discrete.
↳ Google map

↳ Discrete Event Simulation:-
It concerns the modelling of a system as it evolves overtime by a represeutation in which the state variable changes instantaneously at separate points of time.

track event's time → Simulation clock.

↳ ↳ Time Advance Mechanism:-
↳ Simulation clock:
↳ The variable in a simulation model that gives current value of a simulated time.
↳ give current time's value of a system.
↳ Track by 2 Mechanisms:
↳ Next Event Time Advance → Symbolic Tools use.
↳ fixed Increment Time Advance.

Server
idle busy

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↳ fixed Increment Time Advance.

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↳ Next Event Time Advance:

1) ↳ Set Simulation clock to zero.
2) ↳ The time of occurence of future events are determined.
3) The simulation clock is then advance to the time of occurence of the most imitent of these future event.
4) Process will be repeated till the stopping (defined) criteria is reached.

Example:-

Consider a service facility with a single server (MM1) for which we would like to estimate the (expected) average delay in queue of arriving customers.

↳ State Variables:-

- ↳ State of server => (idle or busy)
- ↳ No. of customers waiting in queue.
- ↳ Times of arrival of each person waiting in queue.

↳ Events:

- ↳ Arrival
- ↳ Departure.

↳ Next Event Time Advance Approach:-

t_i = time of arrival of the ith customer ($t_0 = 0$)

A_i = consecutive customers diff $= t_i - t_{i-1}$ (inter arrival time)

S_i = Time that server ~~actu~~ actually spends serving ith customer.

D_i = Delay in time of the ith customer

(departure time) C_i = Total time that ith customer completes service time $= t_i + D_i + S_i$.

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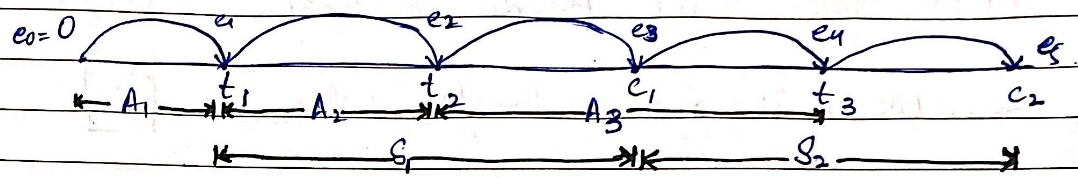
e_i = Time of occurrence of the i^{th} event of any type.



Read topic : 1.3 & 1.3.1

1.7.

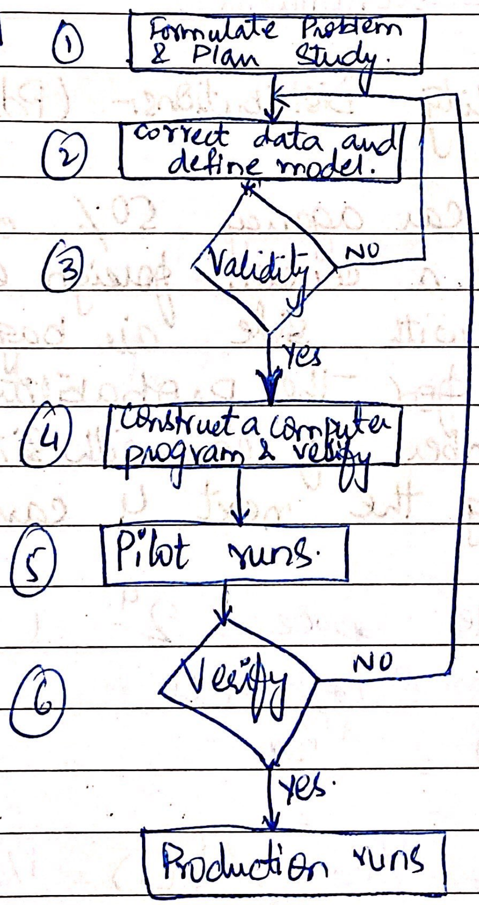
10 | 02 | 2022.

Check exact representation of Scenario → Validity

Performance Model → Service time (banking

Steps In a Sound Simulation Study :-

Fig:1.46
Pg #67



+ve correlation — both inc.

-ve correlation — 1 inc. & 2 dec

0 correlation — NO change

Analy

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→ Review Of Probability:-

Sample Space → all possible outcomes. $\{H, T\}$

Sample unit → One Single outcome: $\boxed{T}$

random variable → assign

$\{T, H\}$; Y = no. of heads.

$\boxed{y = 0}$

$\boxed{y = 1}$

→ Types
- → discrete
- → continuous.

↳ Discrete Probability Distributions:- (Pdf).

Example:-

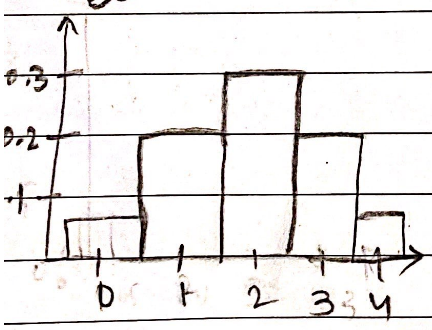
If a car agency 50% of its inventory of a certain foreign car equipped with side air bags. Find a formula for the probability distribution of the number of cars with side air bags among the next 4 cars sold.

Bell shaped. Sol:-

Sample Space $= 2^4$ (50%. Yes / No).

$x = 0, 1, 2, 3, 4$

$$f(x) = \frac{{}^4C_x}{2^4}$$



$f(0) = 1/16$ 0.0625 $f(1) = 1/4 = 0.25$

$f(2) = 3/8$ 0.3; $f(3) = 1/4$

$f(4) = 1/16$.

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↳ Cumulative Distribution Function :-

$$F(x) = P(X \leq x).$$

$$F(0) = f(0).$$

$$F(1) = f(0)+f(1) = F(0)+f(1).$$

$$F(2) = f(0)+f(1)+f(2) = F(1)+f(2).$$

$\vdots$

$$F(4) = 1.$$

↳ Continuous Probability Distribution :- single pts - zero.

↳ find atleast 2 pts - intervals - area under the curve

↳ $P(a < X \leq b) = P(a < X < b) + P(X = b) = P(a < X < b).$

↳ $\int_{-\infty}^{\infty} f(x)\, dx = 1.$

Examples:-

3.11. Sol :-

$$= \int_{-\infty}^{\infty} f(x)\, dx$$

$$= \int_{-\infty}^{-1} f(x)\, dx + \int_{-1}^{2} f(x)\, dx + \int_{2}^{\infty} f(x)\, dx.$$

$$= 0 + \int_{-1}^{2} \frac{x^2}{3}\, dx + 0$$

$$= \frac{1}{3} \left| \frac{x^3}{3} \right|_{-1}^{2} = \frac{1}{3}\left[\frac{2^3}{3} + \frac{1}{3}\right] = \frac{8+1}{3 \times 3}$$

$$= \frac{\cancel{9}}{\cancel{9}} = \boxed{9}$$

$$= P(0 < X \leq 1) = \int_{-\infty}^{\infty} f(x)\, dx$$

$$= \int_{-\infty}^{0} f(x)\, dx + \int_{0}^{1} f(x)\, dx + \int_{1}^{\infty} f(x)\, dx = \int_{0}^{1} f(x)\, dx$$

$$= \left| \frac{x^3}{3^{?}} \right|_{0}^{1} = \boxed{\frac{1}{3}}$$

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3.12.

$$F(x) = P(X \le x) = \int_0^x f(x)\,dx$$

$$= \int_x^{-1} 0 + \int_{-1}^x f(x)\,dx$$

$$= 0 + \int_{-1}^x (x^2/3)\,dx$$

$$= \left| x^3/9 \right|_{-1}^x = \boxed{(x^3+1)/9}$$

$$P(0 < X < 1) = F(1) - F(0) = \int_0^1 f(x)\,dx$$

Marginal skip, Conditional skip

Single $\int$ - area.
double $\int$ = volume.

Joint Probability Distribution more than 1 random vars.

3.14. Random → whose val. changed — 3b

X = no. of blue pens selected. — 2r

Y = no. of red pens selected. — 3g.

Sol:-

$$f(x,y) = \frac{{}^3C_x \, {}^2C_y \, {}^3C_{(x-y-2)}}{{}^8C_2}$$

		X		
	f(x,y)	0	1	2
Y	0			
	1			
	2			

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Date ____ 20__ $= 0 - 1 + \frac{3}{2}$ $= \frac{0 - x^2 + 3}{2} = \frac{1}{3}$

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$0 < x < \frac{1}{2}$

$\frac{1}{4} < y < \frac{1}{2}$.

3.15.

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{2}{5}(2x+3y)\,dx\,dy = 1.$$

$$= \int_0^1\int_0^1 \frac{2}{5}(2x+3y)\,dx\,dy = 1.$$

$$= \frac{2}{5}\int_0^1 \left| \frac{2x^2}{2} + \frac{3y^2 x}{2} \right|_0^1 dy = 1.$$

$$= \frac{2}{5}\int_0^1 \left[(0-1) + \frac{3y}{2} \right] dy$$

$$= \frac{2}{5}\int_0^{1} \left[\frac{10}{3} y + 1 + 3y \right] dy.$$

$$= \frac{2}{5}\left| y + \frac{3y^2}{2} \right|_0^1 = \frac{2}{5}\left[\left(1+\frac{3}{2}\right) - (0+0) \right]$$

$$= \frac{2}{5}\left(\frac{5}{2}\right) = \boxed{1}$$

Proved.

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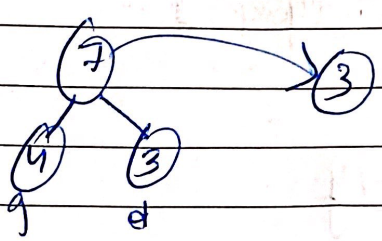
* Expected Mean:-

↳ $E(X) = \sum_x x f(x)$ | $E(X^2) = \sum x^2 f(x)$.

↳ $E(X) = \int_{-\infty}^{\infty} x f(x)\, d(x)$. | $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

Examples:-

4.1.



$$f(x) = \frac{{}^4C_x \; {}^3C_{3-x}}{{}^7C_3} \quad ; \forall \; x = 0, 1, 2, 3.$$

$$E(X) = \mu = \sum_x x f(x)$$
$$= (0) f(0) + (1) f(1) + (2) f(2) + (3) f(3).$$

4.3.

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx.$$

$$= \int_{-\infty}^{100} 0 + \int_{100}^{\infty} (20000 x / x^3)\, dx$$

$$= 0 + \left| \frac{-20000 x^{-1}}{} \right|_{100}^{\infty}$$

$$= \left| \frac{-20000}{x} \right|_{100}^{\infty} = \frac{-20000}{\infty} + \frac{20000}{100}$$

$$\boxed{= 200}$$

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↳ Function's ~~Pro~~ Expected Mean:-

↳ $E(g(x)) = \sum g(x) f(x) = \int_{-\infty}^{\infty} g(x) f(x)\, dx$

Example:-

4.4

$E(g(x)) = \sum g(x) f(x)$

$= \sum (2x-1) f(x)$

$= (2(4)-1) f(4) + (2(5)-1) f(5) + \dots$

↳ Joint Probability distribution :-

$E(g(X,Y)) = \sum_x \sum_y g(x,y) f(x,y) = \int \int g(x,y) f(x,y)$

$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y)\, dx\, dy$

↳ Variance :-

$\sigma^2 = E((X - \mu^2)) = \sum (x-\mu)^2 f(x) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

↳ $\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

↳ Covariance:-

↳ units same — magnitude measure

↳ +ve → both inc. / dec. dir same.

↳ −ve → one inc, one dec dir op.

↳ zero → non-linear.

↳ tells info about linear only.

↳ ~~ρ_{XY}~~ Correlation.

$$\boxed{\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}}$$

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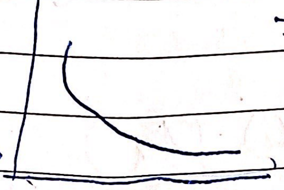
→ Exponential :- $f(x) = e.$

arrival time
↳ random var →



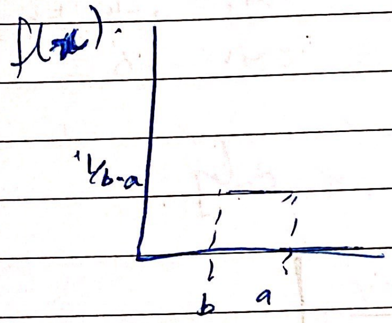
distribution parameters.
↳ normal → μ, σ^2
↳ exponential → mean = 1.
↳ mean - 0.5

$F(x) = 1 - e^{x/\beta}$; $F(x)^{-1} = \beta \ln(1 - u)$ → random var generate.
$= \beta \ln u.$
↳ mean of exponential.

Sample > 100.

→ Uniform :-

$U(0,1)$



CLT :-

Sample ↑ follow normal distribution.

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→ The Strong law of large Numbers

Let, $x_1, x_2, \dots, x_n$ be IID (Independent & Ideal Identically Distributed) random variables with finite means (μ), then theorem states;

$$\bar{x}(u) \to u \text{ as } u \to \alpha.$$ $\mu = E(x)$.

α ——— α ——— α.

Single Server Queing System :- (Single Channel Queue).

Example :

Q.

A small grocery store has only one checkout counter customers arrives at this checkout counter at random from 1 - 8 mins appart. Each poss. val. of interarrival times has the same prob. of occurrence. The service time vary from 1-6 mins with a given probs. The problem is to analyze the sys. by simulating the arrival & service of 10 customers.

Sol.

Inter arrival times distribution.

Interarrival times.	Probability	C.F	Assignment of Random nos.
1	0.125	0.125	0-125 (1-125).
2	0.125	0.250	126-250
3	0.125	0.375	251-375
4	0.125	0.500	376-500
5	0.125	0.625	501-625
6	0.125	0.750	626-750
7	0.125	0.875	751-875
8	0.125	1.00	876-1000

D·D = Delay Duration = Delay time.

A < D = Delay.

A = D = no delay

A > D = sys. idle time inc

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Service Time.

Service Times	Probability	C.F	Assignment of Random #s.
1	0.10	0.1	0-10
2	0.20	0.3	11-30
3	0.30	0.6	31-60
4	0.25	0.85	61-85
5	0.1	0.95	86-95
6	0.05	1.00 ~~0.100~~	96-100

Customers	Rand # of I.A	I.A times	Rand # of S.T	Service Time	C.F of I.A Arrival Time	D.T - A.T (prev)(curr) Delay Duration	S.T + A.T + D.D Departure time	S.T + D.D Time spend in sys.
1	256	3		2	3	0	5	2
2	75	1		3	4	1	8	4
3	400	4		4	8	0	12	4
4	215	3		2	11	1	~~23~~ 14	3
5	900	8		5	19	0	24	5
6	700	6		3	25	0	28	3
7	75	1		3	26	2	31	5
8	715	6		4	32	0	36	4
9	150	2		4	34	2	40	6
10	400	4		3	38	2	$\boxed{43}$	5
				33		8		

run-time of Simulation = 43

for 5 customers. idle time fraction 8/24

S.T = 16

~~idle~~

Server idle time = 24 − 16 in fraction 8/24.

busy = 16/24

Server
↳ idle → busy
↳ server idle time = C.T (prev) − C.T (curr)

0 = Idle
1 = busy.

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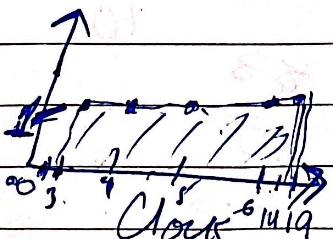
Mechanism → Next Event Time Advance.

Clock Time	Type of Event (Initialized)	Server Status	Idle duration of Server	Customer in queue.	
0	(Initial) I.	0		0	0
3	(Arr) A1	1	3-0=0	0	0
4	A2	1		0	1
5	(Deft) D1	1		1 0	0
8	A3/D2.	1		0	0
11	A4	1		0 1	1
12	D3	1		1 0	0
14	D4	0		0	0
19	A5	1	19 - 14 = 5	0	0
24	D5	0		0	0
25	A6	1	25 - 24 = 1	0	
26	A7	1		1	
28	D6	1		0	
31	D7	0		0	
32	A8	1	32 - 31 = 1	0	
34	A9	1		1 0	
36	D8	1		0	
38.	A10.	1		1/5	

how many ~~ppl~~ ppl are in queue. → Statistical Counters
State of Server.

40
43.

QT-Graph
↓
Server idle / busy at a particular time.



diagram

$\frac{\text{D.D}}{\text{Total no. of cust.}}$ = avg waiting time.

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(1) Average waiting time for the customers:-

$$\text{Avg. W.T for cust.} = \frac{\text{total time custs wait in q.}}{\text{total nos of custs.}}$$

$$= \frac{8}{10}$$

~~Avg. W.T for customer = 0.8.~~

Avg. W.T for customers = 0.8 (per customers)

(2) Probability that customers has to wait:-

$$= \frac{\text{customers who faced delay.}}{\text{Total customers.}}$$

$$= \frac{5}{10}$$

$$= 0.5$$

(3) Probability of idle servers:- (S.T). (fraction of idle time)

$$= \frac{\text{idle time (total)}}{\text{total simulation time}}$$

$$= \frac{10}{43}$$

$$= 0.232.$$

(4) Avg. Service Time:-

$$= \frac{\text{Sum of S.T}}{10}$$

$$= \frac{33}{10}$$

$$= 3.3.$$

43/10

close to

E(S).

no. of customers / rand vars in C.

~~diff will decr~~

if

$= 1 \times P(1) + 2P(2)$

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~~5~~

$$E(S) = \sum_x x \cdot f(x)$$
$$= (1)(0.1) + (2)(0.2) + (3)(0.3) + (4)(0.25) + (5)(0.1) + (6)(0.05)$$
$$E(S) = 3.2 \text{ mins.}$$

(5) The avg. time b/w ~~A~~ arrivals.

avg. of I.A.T. , Expected val.

(6) The avg. W.T a customer spends in sys

Q.	Initial		Chang	
I.A.T.	1-8	→	1-6	= 1/6 → Prob. Chan
S.T	1-6	→	1-8	

S.T	
1	0.1
2	0.2
3	0.3
4	0.05
5	0.25
6	0.025
7	0.025
8	0.05

Mid 1. → Theory
Prob.
Simulation.

Page #

S.T of all customers = busy time of server.
~~total - by~~ busy time of server = idle-time of server.

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- I.A ⇒ mean = 1.8 - 0.25 ... ppl ... average.

Quiz On Thursday. seedo & check

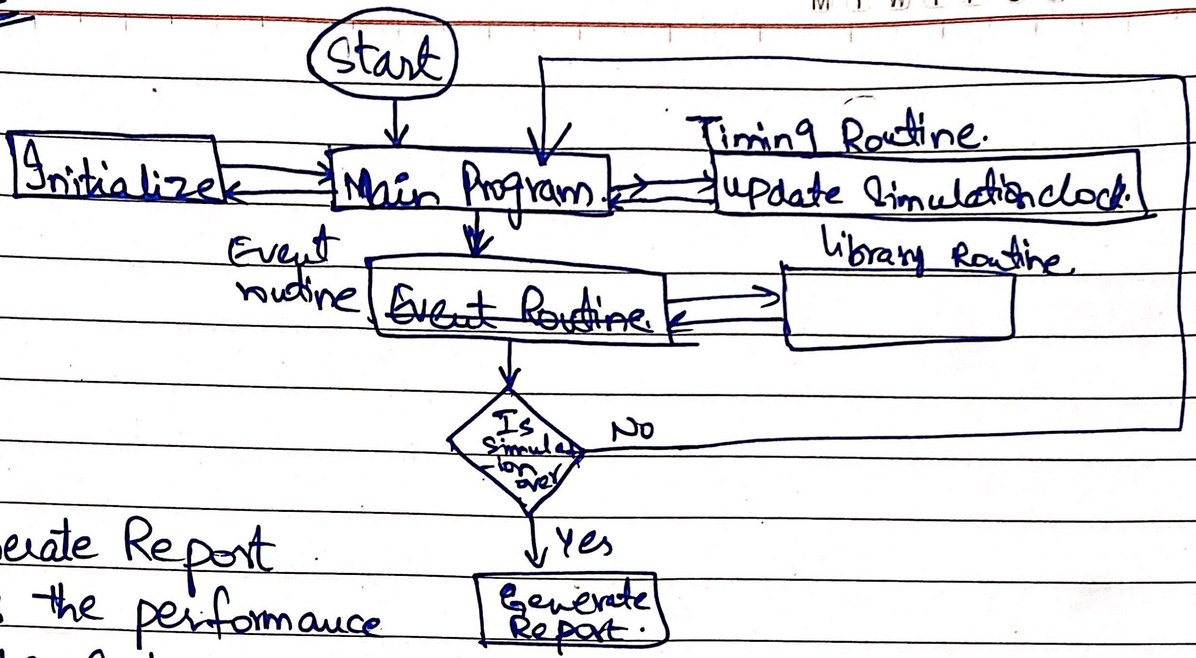
10 events means clock is updating 10 times.

This ~~event~~ topic is not included in Mid I.

Components & Organizational of a Discrete-Event Simulation Model:-

1. System State.
2. Simulation Clock. (we use Event wise Simulation clock)
3. Event list
4. Statistical counters.
5. Initialization Routine.
   ↳ FIRST ARRIVAL'S S.T = ∞.
6. Timing Routine.
   ↳ A Sub program that determines the next event from the event list & then advances the simulation clock to the time when that event is to occur.
7. Event Routine ⇒ A sub program that update the sys. state when a particular type of event occurs.
   decides arrival/departures & update the clock. same as time but for event perspective.
8. Library Routines.
   ↳ random #s.
9. Report generator.
10. Main Program.

Pg#10.



Generate Report tells the performance of the system.

Read pg # 9, 10, 18–27.

α ______ α ______ α.

A_1 = System initialize – arrival time of C_1.

Double Server.