day / date: 26-2-21

CS211 - Discrete Structures
Assignment #1, Spring 2021
S.M. Hassan Ali - K20-1052

Q.1.

a) A proposition that is True.
b) A proposition that is False, since Tallahassee is the capital of Florida.
c) A proposition that is True. $(2+3=5)$
d) A proposition that is False. $(5+7=10)$
e) It is not a proposition since value of x is not defined. $(x+2=11)$.
f) It is not a proposition because it is an imperative statement.

Q.2.

a) Smartphone B has the most RAM of 288 MB so that is true $(P \wedge Q \wedge R = T)$
b) Smartphone B has 288 Ram and 64 GB Rom (T) and resolution of 4Mp (T). $(T \wedge T = T)$
c) Smartphone B has not more resolution than A so $(T \wedge F = F)$.
d) Again smartphone B has greater Ram and Rom but lesser resolution. $(p \rightarrow q, T \rightarrow F = F)$
e) First statement of smartphone A is False and other statement is True. $(p \leftrightarrow q, F \leftrightarrow T = F)$
(In implication both proposition should have same values in order for T).

day / date:

P, Q, R, S, T, U

Q.3

a) P, a proposition is not True.

b) $R^S \wedge U$, ~~T ∨ F = T~~ ~~T ∨~~ $T \wedge T = T$

c) $Q \vee U$, $F \vee T = T$

d) $U \rightarrow P$, $F \rightarrow T = T$

e) $S \leftrightarrow P$, $T \leftrightarrow T = T$

P: Annual revenue of Acme Computer, 138 B

Q: Acme computer net profit 8 b.

R: A.R of Nadir Software, 87 b

S: N.S net profit 5 b.

T: A.R of Quixote Media 111 B.

U: Q.M net profit 13 B.

Q.4.

p: you have the flu. q: You miss the final examination.

r: You pass the course.

a) $p \rightarrow q$.

If you have the flu, then you will miss the final exams.

b) $\neg q \leftrightarrow r$

You will not miss the final exam if and only if you have pass the course.

c) $q \rightarrow \neg r$

If you miss the final exam, then you will not pass this course.

d) $p \vee q \vee r$

You have the flu OR you miss the final exam OR you pass the course.

day / date:

e) $(p \to \neg r) \vee (q \to \neg r)$

If you have the flu, the you will not pass the couse OR if you miss the final exam, you will not pass the course.

f) $(p \wedge q) \vee (\neg q \wedge r)$

You have the flu And you miss the final exam, OR You will not ~~pass the course~~ miss the final exam And you pass the course.

Q.5

a) $R \wedge \neg Q$
b) $P \wedge Q \wedge R$
c) $P \to R$ (R necessary for P)
d) $P \wedge \neg Q \wedge R$
e) $(P \wedge Q) \to R$ (P and Q is sufficient for R)
f) $R \leftrightarrow (P \vee Q)$

Q.6

a) • p only if q.
If you send me an e-mail, then I will remember to send you the address

b) • q is sufficient for ~~q~~ p
If you were born in US, then you citizen of this country

c) • if p, q
If you keep you text book, then it will be useful reference in your future course.

day / date:

d) • ~~if p, q~~ q if p
If the Red Wings's goalie plays well, then they will win the Stanley Cup.

e) • p implies q
If you get the job, then you had the best credentials.

f) • q whenever p.
If there is a storm, then the beach erodes.

g) • q is necessary for p.
If you log on to the server, then you have a valid password

h) • q unless ¬p (q unless negation of p)
If you do not ~~being~~ begin your climb too late, then you will reach the summit.

Q.7.
a) Ways to write conditional statement are:
- → if p, q
- → q if p
- → q when p
- → p implies q
- → p only if q
- → q whenever p
- → q is necessary for p
- → p is sufficient for q
- → q unless ¬p

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b) converse.

converse swaps the statement

$p \rightarrow q$ to $q \rightarrow p$

if p then q to if q then p

inverse.

it makes the negation of both statements.

$p \rightarrow q$ inverse is $\neg p \rightarrow \neg q$

if negation of p, then negation of q.

contrapositive.

swaps the statements as well as make them $\neg$

$p \rightarrow q$ contrapositive is $\neg q \rightarrow \neg p$

c) "If it is sunny tomorrow, then I will go for a walk in the woods.

converse:

If I go for a walk in the woods, then it is sunny tomorrow.

inverse:

If it is not sunny tomorrow, then I will not go for a walk in the woods.

contrapositive.

If I donot go for a walk in the woods, then it is not sunny tomorrow.

inverse of inverse:

If it is sunny tomorrow, then I will go for a walk in the woods.

inverse of its converse:
If I donot go for the walk in the woods, then it is not sunny tomorrow.

inverse of its contrapositive:
If I go for the walk in the woods, the it is sunny tomorrow.

Q.8.

$$\left.\begin{aligned} \neg(P \wedge Q) &= \neg P \vee \neg Q \\ \neg(P \vee Q) &= \neg P \wedge \neg Q. \end{aligned}\right\} \text{Demorgan's law.}$$

a) Jan is not rich OR unhappy.

b) Carlos will not cycle AND does not run tomorrow.

c) The fan is fast AND it is very cold.

d) Akram is fit OR Saleem is not injured.

Q.9.

a) OR exclusive since only one thing could be done to have True. ($F \oplus T = T$).

b) OR inclusive since both conditions can be true for a true. ($T \wedge T = T$, $T \vee F = T$)

c) OR inclusive since both the statements need to be true in order to be true. It contradicts the exclusive case.

day / date:

d) OR inclusive since both statements can become true for the output true.

Q.10.

a) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

$(p \wedge (p \wedge \neg q)) \vee (p \wedge q)$ De Morgan

$((p \wedge p) \wedge \neg q) \vee (p \wedge q)$ associative law

$(p \wedge \neg q) \vee (p \wedge q)$ idempotent law

$p \wedge (\neg q \vee q)$ negation law

$p \wedge T \equiv p$ identity law

b) $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$

$\neg(p \leftrightarrow q)$ Bi-implication law

$\neg((p \rightarrow q) \wedge (q \rightarrow p))$

$\neg(\neg p \vee q) \vee \neg(\neg q \vee p)$ implication law

$(p \wedge \neg q) \vee (q \wedge \neg p)$ Demorgan

$p \leftrightarrow \neg q$

day / date:

c) $\neg p \leftrightarrow \neg q \equiv p \leftrightarrow \neg q$

$(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$ Bi-implication.

$(\neg\neg p \vee q) \wedge (\neg q \vee \neg p)$ implication.

$(p \vee q) \wedge (\neg q \vee \neg p)$ double negation.

$(q \vee p) \wedge (\neg p \vee \neg q)$ commutative law

$(\neg q \rightarrow p) \wedge (p \rightarrow \neg q)$ rearrange & implication

$(p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$ Bi-implication again.

$p \leftrightarrow \neg q$

day / date:

d) $(p \wedge q) \rightarrow (p \rightarrow q) \equiv T$

$\neg(p \wedge q) \vee (\neg p \vee q)$ — implication law

$(\neg p \vee \neg q) \vee (\neg p \vee q)$ — De Morgan.

$(\neg p \vee \neg p) \vee (\neg q \vee q)$ — idempotent law.

$\neg p \cancel{\neg} \vee T$ — negation law

T — universal bound law

e) $\neg(p \vee \neg(p \wedge q)) \equiv F$

$\neg p \wedge (p \wedge q)$ — De Morgan.

$q \wedge (\neg p \wedge p)$ — associative law.

$q \wedge F$ — negation law

F — universal law.

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Q. 11

a) $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$

P	Q	R	$P \to R$	$Q \to R$	$(P \to R) \wedge (Q \to R)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

P	Q	R	$P \vee Q$	$(P \vee Q) \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	~~F~~ T

logically equivalent

$(p \to R) \wedge (Q \to R) \equiv (P \vee Q) \to r$.

b) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

P	Q	R	$P \to Q$	$P \to R$	$(P \to Q) \vee (P \to R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

P	Q	R	$(Q \vee R)$	$P \to (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$

logically equivalent.

day / date:

c) $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

P	Q	R	S	$(P \to Q)$	$(R \to S)$	$(P \to Q) \to (R \to S)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	T	F	F	F	T
T	F	F	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	~~F~~ T	T	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	T	T
F	F	F	F	T	T	T

day / date:

P	Q	R	S	(P→R)	(Q→S)	(P→R)→(Q→S)
T	T	T	T	T	T	T
T	T	T	F	T	T	T
T	T	F	T	F	F	T
T	T	F	F	F	F	T
T	F	T	T	F	T	T
T	F	T	F	~~F~~ T	T	T
T	F	F	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T
F	T	F	F	T	F	F
F	F	T	T	T	T	T
F	F	T	F	T	T	T
F	F	F	T	T	T	T
F	F	F	F	T	T	T

$$(p\to q)\to(r\to s) \not\equiv (p\to r)\to(q\to s)$$

NOt logically equivalent.

day / date:

Q.12 $P(m,n)$ "m divides n" U = +ve integer.

a) $P(4,5)$

F, because 4 does not divide 5 into a whole no

b) $P(2,4)$

T, 4 divide by 2 is 2.

c) $\forall m \forall n P(m,n)$.

F, because not all value of m can divide the all values of n.

d) $\exists m \forall n P(m,n)$

T, because one value exist that is 1 that divides each value of n.

e) $\exists n \forall m P(m,n)$

F, because some value of n is not divisible by all values of m as it increases.

f) $\forall n P(1,n)$

T, because all values of n, 1 divides them all.

day / date:

Q.13 U = real no

a) $\exists x (x^2 = 2)$

$x = \pm\sqrt{2}$

T, because some values ($\pm\sqrt{2}$) exist that makes existential quantifier true.

b) $\exists x (x^2 = -1)$

F, the function contradicts the domain and values belong to imaginary no.

c) $\forall x (x^2 + 2 \geq 1)$

T, all values of x $x^2 + 2 = 1$

even zero satisfies the function.

d) $\exists x (x^2 = x)$

T, there is a value such that $x = 1$ exist that makes it true.

Q.14 $F(x, y)$ "x can fool y" U = all people.

a) $\forall x F(x, Bob)$

b) $\forall y F(Alice, y)$

c) $\forall x \forall y F(x, y)$

d) $\neg \exists x \forall y \cancel{F} F(x, y)$

e) $\forall y \exists x F(x, y)$

day / date:

Q.15 $P(x)$: "x can speak Russian
$Q(x)$: "x knows the computer language C++".
U = all students at your school.

a) $\exists x(P(x) \land Q(x))$

b) $\exists x(P(x) \land \neg Q(x))$

c) $\forall x(P(x) \lor Q(x))$

d) $\neg \exists x(P(x) \lor Q(x))$

Q.16 $Q(x,y)$ "x has sent an e-mail msg to y"
U = all the student in class.

a) $\exists x \exists y Q(x,y)$

There is at least a student x who has sent an e-mail to atleast a student y.

b) $\exists x \forall y Q(x,y)$

There is at least a student x who has sent an e-mail to all students (y).

c) $\forall x \exists y Q(x,y)$

All of the students (x) in the class has sent at least an e-mail to someone in the class.

day / date:

d) $\exists y \forall x Q(x,y)$

There is at least a student (y) in the class who has been sent an e-mail in the class.

e) $\forall y \exists x Q(x,y)$

Every student (y) has been sent an e-mail from at least a student in the class.

f) $\forall x \forall y Q(x,y)$

Every student in the class has sent a message to every student in the class.

Q.17 $P(x,y)$ "Student x has taken class y".
$U(x)$ = students in the class
$U(y)$ = computer science students.

a) $\exists x \exists y P(x,y)$

There is a student in the class who has taken computer science course.

b) $\exists x \forall y P(x,y)$

There is a student in the class that has taken all of computer science courses.

c) $\forall x \exists y P(x,y)$
All of the students in the class have taken a course in computer science.

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day / date:

d) $\exists y \forall x P(x,y)$

There is a computer science course that every student has taken.

e) $\forall y \exists x P(x,y)$

Every cs course has been taken by a student in the class.

f) $\forall x \forall y P(x,y)$

All the students in the class have taken all cs courses.

Q.18

a) P = Alice is a maths major
q = Alice is a comp science major

using the rules of inference: addition.

$$\frac{p}{\therefore p \vee q}$$

b) P = Jerry is a maths major
q = Jerry is a comp science major.

using rule of inference: simplification.

$$\frac{p \wedge q}{\therefore p} \qquad (\therefore q)$$

day / date:

c) p = It is raining
q = Pool is closed.

using antecedent to check the implication.
(Modus ponens)

$$\begin{array}{l} p \rightarrow q \\ \underline{\quad p \quad} \\ \therefore q \end{array}$$

(order does not matter unless the consequent q changes its position)

d) p = It snows.
q = University is closed.

using consequent to check the implication
(Modus tollens)

$$\begin{array}{l} p \rightarrow q \\ \underline{\neg q \quad\quad} \\ \therefore \neg p \end{array}$$

e) p = I go swimming
q = I stay in the sun too long
r = I will sunburn.

using the rule of inference hypothetical syllogism.

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore p \rightarrow r \end{array}$$

day / date:

Q.19

P = Today is Tuesday
Q = I have a test in Maths ~~or Eco.~~
~~R = My Eco Professor is sick~~
R = I have a test in Eco.
S = My Eco Professor is sick.

$P \to (Q \lor R)$
$S \to \neg R$
$\underline{P \land S}$
$\therefore Q$

$\cancel{P \to (Q \lor R)}$
$\cancel{P}$

$P \land S$
P using simplication law
$P \to (Q \lor R)$
$Q \lor R$ using modus ponens.
S using S.L
$S \to \neg R$
$\neg R$ using m.p
$R \lor Q$ using commutative law
Q using elimination law.

Argument is valid (Tautology)

day / date:

b) P = Ali is a lawyer.
Q = He is ambitious
R = He is an early riser
S = He likes chocolates.

$P \rightarrow Q$
$R \rightarrow \neg S$
$Q \rightarrow R$
$\therefore P \rightarrow \neg S$

$P \rightarrow Q$
$Q \rightarrow R$

$P \rightarrow R$ using Hypothetical syllogism

$R \rightarrow \neg S$

$P \rightarrow \neg S$ using H.S.

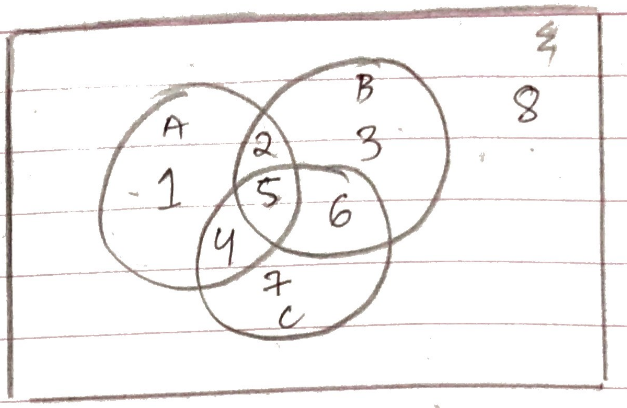
Argument is Valid (tautology).

Q.20 $U = \{1,2,3,4,5,6,7,8\}$

$A = \{1,2,4,5\}$

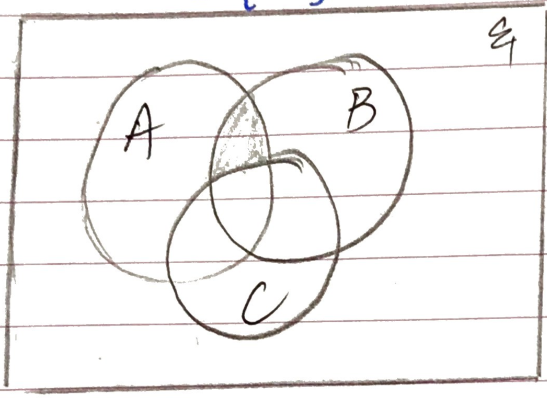
$B = \{2,3,5,6\}$

$C = \{4,5,6,7\}$



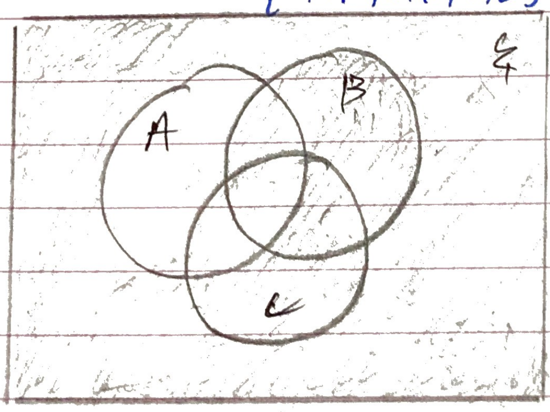
a) $(A \cap B) \cap C'$

$= \{2\}$



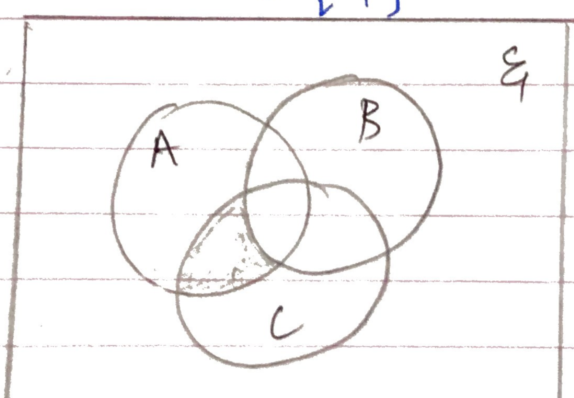
b) $A' \cup (B \cup C)$

$= \{2,3,4,5,6,7,8\}$

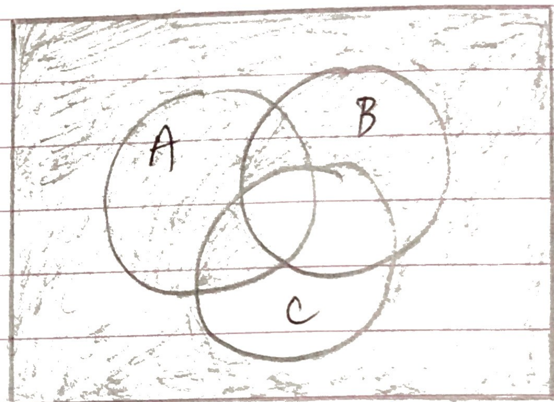


c) $(A-B) \cap C$

$= \{4\}$

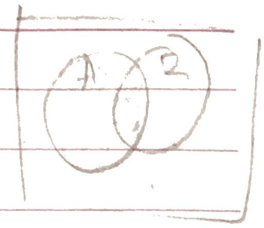


d) $(A \cap B') \cup C'$



$= \{1,2,3,4,8\}$

day / date:



Q.21

a) $(A-(A\cap B))\cap(B-(A\cap B))=\emptyset$

$(A\cap(A\cap B)')\cap(B\cap(A\cap B)')$ $(A-B)=(A\cap B')$

$(A\cap(A'\cup B'))\cap(B\cap(A'\cup B'))$

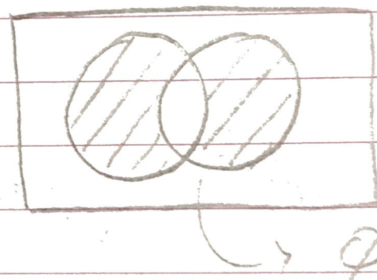
a b c a b c

$((A\cap A')\cup(A\cap B'))\cap((B\cap A')\cup(B\cap B'))$ Distributive Law

$(\emptyset\cup(A\cap B'))\cap((B\cap A')\cup\emptyset)$ complement Laws

$(A\cap B')\cap(B\cap A')$

$(A'\cap A)\cap(B'\cap B)$



$\emptyset A\cap\emptyset$

$\emptyset$

day / date:

b) $(A-B) \cup (A \cap B) = A$

$(A-B) = (A \cap B')$

$(A \cap B') \cup (A \cap B)$

a b a c

$A \cap (B \cup B')$

Distributive law

$A \cap U$

Identity law

A

c) $(A-B)-C = (A-C)-B$

a b

$A \cap B'$

$((A-B) \cap C')$

$(A-B) = (A \cap B')$

$((A \cap B') \cap C')$

Assosiative law

$((A \cap C') \cap B')$

a b

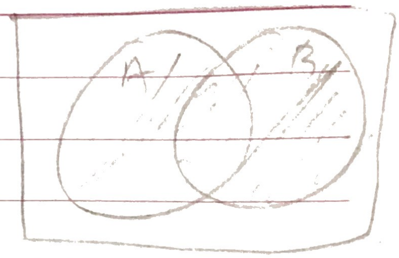
$(A \cap C') - B$

a b

$(A-C)-B$

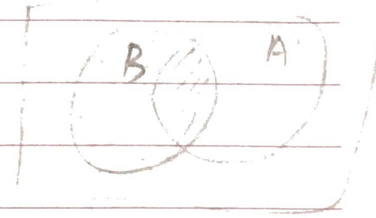
day / date:

d) $(B' \cup (\underset{a}{B'} - \underset{b}{A}))' = B$



$(B' \cup (B' \cap A'))'$ $\qquad (A-B) = A \cap B'$

$(B \cap (B' \cap A')')$ $\qquad$ De Morgan Law

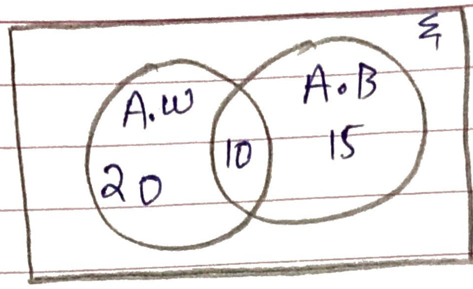


$(B \cap (B \cup A))$ $\qquad$ Absorption law

B

Q.22

a)



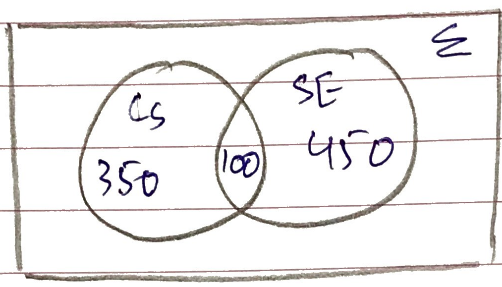
U
$\xi = 100$

$(A.W \cup A.B)' = ?$

$|A \cup B| = |A| + |B| - |A \cap B|$
$= 20 + 15 - 10$
$= 25$

$100 - 25 = 75$ (those apples can be sold).

b)

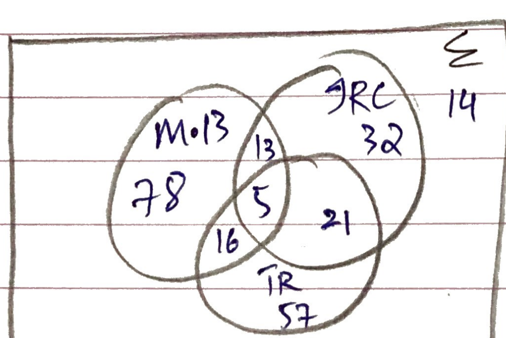


$U = 1000$

$|A \cup B| = 350 + 450 - 100$
$= 700$ (either of CS or SE)

$(A \cup B)' = 1000 - 700 = 300$ (neither of CS or SE).

c)



$U = ?$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$
$= 78 + 32 + 57 - 13 - 16 - 21 + 5$
$= 122 + 14$
$= 136$

d) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Set builder notation

$= \{(a,b) \mid a \in A \wedge b \in B \cap C\}$

$= \{(a,b) \mid a \in A \wedge (b \in B \wedge b \in C)\}$

$= \{(a,b) \mid (a \in A \wedge a \in A) \wedge (b \in B \wedge b \in C)\}$ Idempotent Law

$= \{(a,b) \mid a \in A \wedge (a \in A \wedge (b \in B \wedge b \in C))\}$ Associative Law

$= \{(a,b) \mid a \in A \wedge ((a \in A \wedge b \in B) \wedge b \in C)\}$ Associative Law

$= \{(a,b) \mid a \in A \wedge ((b \in B \wedge a \in A) \wedge b \in C)\}$ Commutative Law

$= \{(a,b) \mid a \in A \wedge (b \in B \wedge (a \in A \wedge b \in C))\}$ Associative Law

$= \{(a,b) \mid (a \in A \wedge b \in B) \wedge (a \in A \wedge b \in C)\}$ Associative Law

$= \{(a,b) \mid (a,b) \in A \times B \wedge (a,b) \in A \times C\}$ Cartesian product

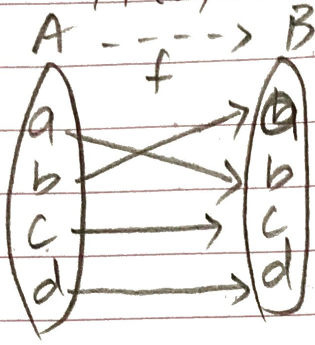
$= \{(a,b) \mid (a,b) \in (A \times B) \cap (A \times C)\}$ intersection

$= (A \times B) \cap (A \times C)$

Q.23

$A = \{a, b, c, d\}$ $B = \{a, b, c, d\}$

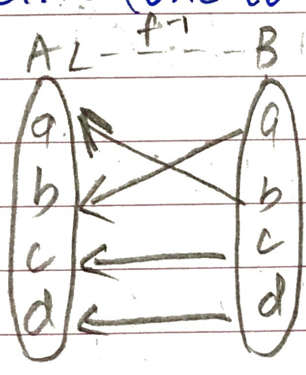
a) $f(a) = b, f(b) = a, f(c) = c, f(d) = d$



i) $f(A)$ (domain) $= \{a, b, c, d\}$
$f(B)$ (codomain) $: \{a, b, c, d\}$
Range $= \{a, b, c, d\}$ all mapped elements

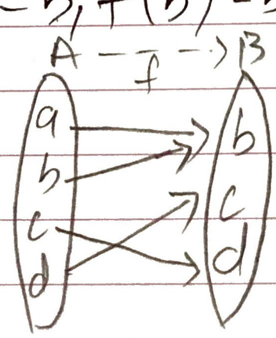
ii) Bijective because it has the property of both injective (one-to-one) and surjective (onto).

iii)



Inverse exist

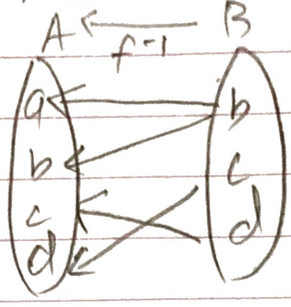
b) $f(a) = b, f(b) = b, f(c) = d, f(d) = c$



i) Domain $= \{a, b, c, d\}$ Co-domain $= \{b, c, d\}$
Range $= \{b, c, d\}$

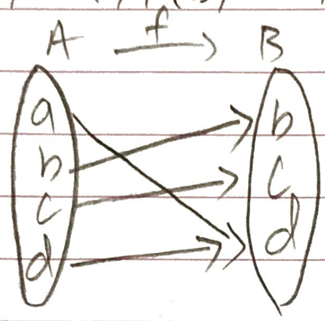
ii) Function but it is neither one-to-one nor onto since a,b has both image of b.

iii)



Inverse does not exist since a,b both are the images of b under f which is invalid for a function.

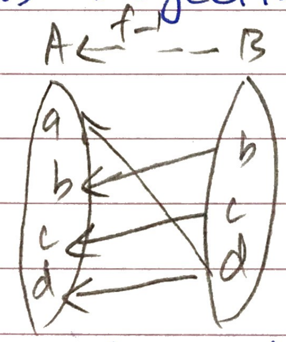
c) $f(a)=d, f(b)=b, f(c)=c, f(d)=d$



i) Domain $= \{a,b,c,d\}$ co-domain $= \{b,c,d\}$
Range $= \{b,c,d\}$

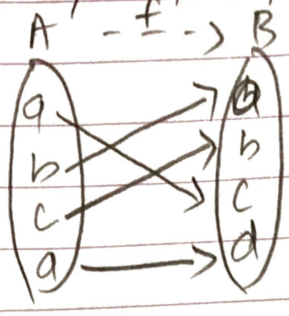
ii) It is surjective cause a,~~b~~d both have a image of d.

iii)



Inverse does not exist since a,d are both the images of d under f which is invalid for a function.

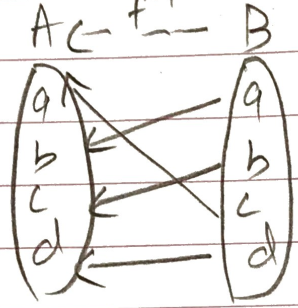
d) $f(a) = c, f(b) = a, f(c) = b, f(d) = d$



i) Domain $= \{a, b, c, d\}$ Co-domain $= \{a, b, c, d\}$
Range $= \{a, b, c, d\}$

ii) Bijective because both co-domain and range is same.

iii)



Inverse exist since each value of domain has a unique image and no domain is being left alone.

Q.24 $f(x) = \left\lfloor \frac{x^2}{3} \right\rfloor$ $f(s) = ?$

i) $S = \{-2, -1, 0, 1, 2, 3\}$

Domain whole no so range in whole no.

$f(s)$ = Image of all the domain of S

$f(-2) = \left\lfloor \frac{(-2)^2}{3} \right\rfloor = \left\lfloor \frac{4}{3} \right\rfloor = \cancel{\frac{4}{3}}\ 1$

$f(s) = \{0, 1, 3\}$

$f(-1) = \lfloor 1/3 \rfloor = 0$ $f(2) = \lfloor 4/3 \rfloor = 1$

$f(0) = \lfloor 0 \rfloor = 0$ $f(3) = \lfloor 3 \rfloor = 3$

$f(1) = \lfloor 1/3 \rfloor = 0$