Strategy to computationally model and resolve radioactive decay chain in engineering education by using the Runge-Kutta numerical method

F. T. C. S. Balbina[1], F. J. H. Moraes[1], E. Munin[1] and L. P. Alves[1]

[1] Biomedical Engineering Center, Anhembi Morumbi University (UAM), São José dos Campos, São Paulo, Brazil.
Center for Innovation, Technology and Education (CITE), São José dos Campos, São Paulo, Brazil

Abstract—
The objective of the present work is to offer an educational strategy targeting biomedical engineering/physics students focusing on the calculation of isotope concentrations and activities in radioactive decay chains, which is capable of demonstrating the behavior of these isotopes over time, by using an iterative process and basic mathematical operations. The computational modeling of the radioactive decay problem by solving ordinary differential equation systems using the Runge-Kutta Fourth Order numerical method is treated. The adopted physical and mathematical models are shown, as well as their computational routine.

Keywords— **Biomedical Engineering; Physics Education; Computational Modeling; Radioactive Chain Decay; Runge-Kutta.**

I. INTRODUCTION

Education is the main tool for people to evolve and be prepared to develop activities necessary to ensure their survival and progress [1]. The attributes of naturally decaying atoms, known as radioisotopes, give such atoms several applications on many fields of biomedical engineering: diagnosis (radio-pharmaceuticals or radio-tracers which emit gamma rays from within the body) [2], therapy (radioactive iodine-131 is used in small amounts to treat cancer and other conditions affecting the thyroid gland.) [3, 4, 5] and sterilization (gamma rays from a cobalt-60 source to sterilize plastic medical products and supplies such as syringes, gloves, clothing, and instruments that would be damaged by heat or high-pressure sterilization) [6].

The learn of radioactive decay involves the understanding of physical concepts that appear as confusing for students [7]. There is no secret that physics learning pathways have been tied to an image of difficulty, binding conceptual understanding to mathematical complexity [8]. It's not uncommon to have students leaving physics-based programs due to poor performance or lack of interest [9].

Data analysis, the study of cognitive mechanisms and change of mindset, from teaching-focused to a research-focused university, associated to programs that aid introductory students of physics and also science, technology, engineering, and mathematics at the initial phase, has been successfully implemented, addressing problems previously cited [10, 11, 12, 13, 14, 15, 16]. Even social interactions, which are directly linked to the creation of motivational environments that promote learning and development, have been accounted for [17, 18]. However, another way to approach this problem is to make physics equations to look "easier", and phenomena to be "visual" for basic engineering students.

The radioactive decay chain problem has an analytical solution for two elements, when the concentration of the second element equals to zero at an initial time. The solution starts to become more complex for a broader analysis including additional chain components over the time, with different concentrations at the beginning of the process. Relevant derivations of Baterman's reference method [19], *e.g.* matrix exponential approach [20, 21], still need an appreciable mathematical background for its use. Modern science offers a different way to deal with it: computational models associated with numerical methods, which provide approximate solutions.

Recent advances in technology allows for the adoption of computational models in engineering education. Back in 1990, reference books in Computational Physics already stated about the uses of computational languages like MATLAB or numerical methods like Runge-Kutta algorithms, on the solution of differential equations, and the advantages of their uses in physics [22]. More important, computational models can help students understand and absorb the concept of a set of equations as physical relationships among quantities [23].

Runge-Kutta methods solve a system of l ordinary differential equations (ODE) of the type $y'(t) = f(t, y(t))$ where $y : \mathbb{R} \to \mathbb{R}^l$ and $f : \mathbb{R} \times \mathbb{R}^l \to \mathbb{R}^l$, subjected to the initial condition $y(t_0) = y_0 \in \mathbb{R}^l$ with $t_0 \in \mathbb{R}$ [24] and has been used for numerical solution of ordinary systems, as well as fuzzy equations [25, 26]. It is a one-step method with multiple stages, where the number of stages determines the order of the method [26].

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