```
> restart
> with(HyperTypeSeq)
```

$$[AlgebraHolonomicSeq, HTS, HolonomicRE, REtoHTS] \quad (1)$$

HolonomiRE

```
> HolonomicRE(n!+1/n!,a(n))
```

$$(n+3)\,(n+1)^2\,a(n) - (n^2+3n+1)\,(n^2+3n+3)\,a(n+1) + n\,(n^2+4n+4)\,a(n+2) = 0 \quad (2)$$

```
> HolonomicRE(sin(n*Pi/4)^2,a(n))
```

$$-a(n) + a(n+1) - a(n+2) + a(n+3) = 0 \quad (3)$$

```
> HolonomicRE(sin(cos(n*Pi)*Pi/6)*sin(n*Pi/4),a(n))
```

$$a(n) + \sqrt{2}\,a(n+1) + a(n+2) = 0 \quad (4)$$

```
> HolonomicRE(sin(cos(n*Pi)*Pi/6)*sin(n*Pi/4),a(n),reshift=2)
```

$$a(n) + a(n+4) = 0 \quad (5)$$

New (March 2024): HolonomicRE can now compute recurrence equations from normal forms

```
> s:=n!*chi[{modp*(n,3)=1}]
```

$$s := n!\,\chi_{\{modp(n,3)=1\}} \quad (6)$$

```
> HolonomicRE(s,a(n))
```

$$-(n+3)\,(n+2)\,(n+1)\,a(n) + a(n+3) = 0 \quad (7)$$

```
> s:=n^2*chi[{modp*(n,2)=1}]+1/n!*chi[{modp*(n,4)=3}]
```

$$s := n^2\,\chi_{\{modp(n,2)=1\}} + \frac{\chi_{\{modp(n,4)=3\}}}{n!} \quad (8)$$

```
> HolonomicRE(s,a(n))
```

$$(n^6+30n^5+371n^4+2418n^3+8747n^2+16628n+12956)\,a(n) + (-n^6-18n^5-131n^4-490n^3-983n^2-992n-384)\,a(n+2) - (n+4)\,(n+3)\,(n^6+30n^5+371n^4+2418n^3+8747n^2+16628n+12956)\,(n+1)\,(n+2)\,a(n+4) + (n+4)\,(n+3)\,(n+5)\,(n+6)\,(n^6+18n^5+131n^4+490n^3+983n^2+992n+384)\,a(n+6) = 0 \quad (9)$$

REtoHTS

The sequence A212579

```
> RE:= a(n) = a(n-1)+2*a(n-2)-a(n-3)-2*a(n-4)-a(n-5)+2*a(n-6)+a
```

```
(n-7)-a(n-8)
```

$$RE := a(n) = a(n-1) + 2\,a(n-2) - a(n-3) - 2\,a(n-4) - a(n-5) + 2\,a(n-6) + a(n-7) - a(n-8) \quad \textbf{(10)}$$

```
> REtoHTS(RE,a(n),[0, 1, 8, 31, 80, 171, 308, 509, 780, 1137, 1584,
  2143, 2812])
```

$$\frac{4}{9} + \frac{31}{12}\,n - 3\,n^2 + \frac{67}{36}\,n^3 - \frac{1}{4}\,n\,\chi_{\{modp(n,\,2)=0\}} - \frac{4}{9}\,\chi_{\{modp(n,\,3)=0\}} - \frac{8}{9}\,\chi_{\{modp(n,\,3)=1\}} \quad \textbf{(11)}$$

```
> U:=proc(n) U(n):=subs([n=n-1,a=U],RE) end proc:
  U(0):=0:  U(1):=1:U(2):=8:U(3):=31:U(4):=80:U(5):=171:U(6):=308:U
  (7):=509:U(8):=780:U(9):=1137:U(10):=1584:U(11):=2143:U(12):=
  2812:
> REtoHTS(RE,a(n),U)
```

$$\frac{4}{9} + \frac{31}{12}\,n - 3\,n^2 + \frac{67}{36}\,n^3 - \frac{1}{4}\,n\,\chi_{\{modp(n,\,2)=0\}} - \frac{4}{9}\,\chi_{\{modp(n,\,3)=0\}} - \frac{8}{9}\,\chi_{\{modp(n,\,3)=1\}} \quad \textbf{(12)}$$

The sequence A033481
Requirement: FPS

```
> f:=(21 + 64*z + 32*z^2 - 5*z^3 - 56*z^4 - 28*z^5 - 14*z^6 - 7*
  z^7) / ((1 - z)*(1 + z + z^2))
```

$$f := \frac{-7\,z^7 - 14\,z^6 - 28\,z^5 - 56\,z^4 - 5\,z^3 + 32\,z^2 + 64\,z + 21}{(1-z)\,(z^2+z+1)} \quad \textbf{(13)}$$

```
> FPS(f,z,n)
```

$$7\,z^4 + 14\,z^3 + 28\,z^2 + 63\,z + 19 + \left(\sum_{n=0}^{\infty} 4\,z^n\right) + \left(\sum_{n=0}^{\infty} -2\,z^{3\,n}\right) + \left(\sum_{n=0}^{\infty} -3\,z^{3\,n+1}\right) \quad \textbf{(14)}$$

```
> RE:=FPS:-FindRE(f-(7*z^4 + 14*z^3 + 28*z^2 + 63*z + 19),z,u(n))
```

$$RE := (-n+1)\,u(n) + (4\,n-12)\,u(n-4) + (n-1)\,u(n-3) + (2\,n+2)\,u(n-2) + (-4\,n+12)\,u(n-1) + (-2\,n-2)\,u(n+1) = 0 \quad \textbf{(15)}$$

```
> U:=proc(n) U(n):=subs([n=n-1,u=U],solve(RE,u(n+1))) end proc:
  U(0):=2:U(1):=1:U(2):=4:U(3):=2:U(4):=1:U(5):=4:
> REtoHTS(RE,u(n),U)
```

$$4 - 2\,\chi_{\{modp(n,\,3)=0\}} - 3\,\chi_{\{modp(n,\,3)=1\}} \quad \textbf{(16)}$$

```
> REtoHTS(RE,u(n),[2,1,4,2,1,4])
```

$$4 - 2\,\chi_{\{modp(n,\,3)=0\}} - 3\,\chi_{\{modp(n,\,3)=1\}} \quad \textbf{(17)}$$

```
> REtoHTS(a(n+2)=a(n+1)+a(n),a(n),[0,1,1])
```

$$-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^n}{5} + \frac{\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^n}{5} \quad \textbf{(18)}$$

```
> U:=  proc(n) U(n):=2*(2*n-1)*U(n-1)/(n+1) end proc: U(0):=1: U(1)
  :=1:
> REtoHTS((n+2)*C(n+1)=2*(2*n+1)*C(n),C(n),U)
```

(19)

$$\frac{(2n)!\,(n+1)}{(n+1)!^2} \tag{19}$$

HTS

```
> HTS(sin(n*Pi/4)^2,n)
```

$$\frac{1}{2} - \frac{(-1)^{\frac{n}{2}} \chi_{\{modp(n, 2) = 0\}}}{2} \tag{20}$$

```
> HTS(sin(Pi*cos(n*Pi)/6)*cos(n*Pi/4),n)
```

$$\frac{(-1)^{\frac{n}{4}} \chi_{\{modp(n, 4) = 0\}}}{2} - \frac{\sqrt{2}\,(-1)^{\frac{n}{4}-\frac{1}{4}} \chi_{\{modp(n, 4) = 1\}}}{4} + \frac{\sqrt{2}\,(-1)^{\frac{n}{4}-\frac{3}{4}} \chi_{\{modp(n, 4) = 3\}}}{4} \tag{21}$$

```
> HTS(sin(cos(n*Pi/3)*Pi),n)
```

$$(-1)^{\frac{n}{3}-\frac{1}{3}} \chi_{\{modp(n, 3) = 1\}} - (-1)^{\frac{n}{3}-\frac{2}{3}} \chi_{\{modp(n, 3) = 2\}} \tag{22}$$

```
> HTS(tan(n*Pi/4),n)
```

$$\chi_{\{modp(n, 4) = 1\}} + \left(\lim_{n \to 2} \tan\left(\frac{n\pi}{4}\right)\right) \chi_{\{modp(n, 4) = 2\}} - \chi_{\{modp(n, 4) = 3\}} \tag{23}$$

```
> HTS(tan(n*Pi/3),n)
```

$$\sqrt{3}\,\chi_{\{modp(n, 3) = 1\}} - \sqrt{3}\,\chi_{\{modp(n, 3) = 2\}} \tag{24}$$

```
> HTS(cos(n*arccos(x)),n)
```

$$\frac{\left(x - \sqrt{x^2-1}\right)^n}{2} + \frac{\left(x + \sqrt{x^2-1}\right)^n}{2} \tag{25}$$

```
> HTS(sin(n*Pi/6)*cos(n*Pi/3)-sin(n*Pi/2),n)
```

$$-\frac{\mathrm{I}\left(\frac{\sqrt{3}}{2} - \frac{\mathrm{I}}{2}\right)^n}{4} + \frac{\mathrm{I}\left(\frac{\sqrt{3}}{2} + \frac{\mathrm{I}}{2}\right)^n}{4} - \frac{(-1)^{\frac{n}{2}-\frac{1}{2}} \chi_{\{modp(n, 2) = 1\}}}{2} \tag{26}$$

```
> HTS(sin(n*Pi/4)^2*cos(n*Pi/6)^2,n)
```

$$\frac{1}{4} - \frac{(-\mathrm{I})^n}{8} + \frac{\left(\frac{1}{2} - \frac{\mathrm{I}\sqrt{3}}{2}\right)^n}{8} + \frac{\left(\frac{1}{2} + \frac{\mathrm{I}\sqrt{3}}{2}\right)^n}{8} - \frac{\mathrm{I}\,(-1)^{\frac{n}{2}-\frac{1}{2}} \chi_{\{modp(n, 2) = 1\}}}{8} - \frac{3\,(-\mathrm{I})^{\frac{n}{3}} \chi_{\{modp(n, 3) = 0\}}}{8} - \frac{3\,\mathrm{I}\,(-1)^{\frac{n}{6}-\frac{1}{2}} \chi_{\{modp(n, 6) = 3\}}}{8} \tag{27}$$

```
> HTS(sin(n*Pi/4)^2*cos(n*Pi/6)^4,n,maxreorder=12)
```

$$
\frac{9}{32}+\frac{\left(4\,\mathrm{I}\,(-1)^{\frac{n}{2}}\sqrt{3}+8\left(-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)^{\frac{n}{2}}+3\,(-1)^{\frac{n}{2}}\right)\chi_{\{modp(n,\,2)\,=\,0\}}}{32}
-\frac{9\,\chi_{\{modp(n,\,3)\,=\,0\}}}{32}+\frac{\mathrm{I}\sqrt{3}\,(-\mathrm{I})^{\frac{n}{3}-\frac{2}{3}}\,\chi_{\{modp(n,\,3)\,=\,2\}}}{4}
+\frac{\left(-\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)^{\frac{n}{4}}\chi_{\{modp(n,\,4)\,=\,0\}}}{2}-\frac{\left(4\,\mathrm{I}\sqrt{3}+27\right)(-1)^{\frac{n}{6}}\,\chi_{\{modp(n,\,6)\,=\,0\}}}{32}
-\frac{\sqrt{3}\,(-1)^{\frac{n}{6}-\frac{5}{6}}\,\chi_{\{modp(n,\,6)\,=\,5\}}}{4}
\tag{28}
$$