```
> restart
> with(HyperTypeSeq)
```

$$[AlgebraHolonomicSeq, HTS, HTSproduct, HolonomicRE, REtoHTS] \quad (1)$$

HolonomiRE

```
> HolonomicRE(n!+1/n!,a(n))
```

$$(n+3)(n+1)^2 a(n) - (n^2+3n+1)(n^2+3n+3)\,a(n+1) + n(n^2+4n+4)\,a(n+2) = 0 \quad (2)$$

```
> HolonomicRE(sin(n*Pi/4)^2,a(n))
```

$$-a(n) + a(n+1) - a(n+2) + a(n+3) = 0 \quad (3)$$

```
> HolonomicRE(sin(cos(n*Pi)*Pi/6)*sin(n*Pi/4),a(n))
```

$$a(n) + \sqrt{2}\,a(n+1) + a(n+2) = 0 \quad (4)$$

```
> HolonomicRE(sin(cos(n*Pi)*Pi/6)*sin(n*Pi/4),a(n),reshift=2)
```

$$a(n) + a(n+4) = 0 \quad (5)$$

New (March 2024): HolonomicRE can now compute recurrence equations from normal forms

```
> s:=n!*chi[{modp*(n,3)=1}]
```

$$s := n!\,\chi_{\{modp(n,3)=1\}} \quad (6)$$

```
> HolonomicRE(s,a(n))
```

$$-(n+3)(n+2)(n+1)\,a(n) + a(n+3) = 0 \quad (7)$$

```
> s:=n^2*chi[{modp*(n,2)=1}]+1/n!*chi[{modp*(n,4)=3}]
```

$$s := n^2\,\chi_{\{modp(n,2)=1\}} + \frac{\chi_{\{modp(n,4)=3\}}}{n!} \quad (8)$$

```
> HolonomicRE(s,a(n))
```

$$(n^6+30n^5+371n^4+2418n^3+8747n^2+16628n+12956)\,a(n) + (-n^6-18n^5-131n^4-490n^3-983n^2-992n-384)\,a(n+2) - (n+4)(n+3)(n^6+30n^5+371n^4+2418n^3+8747n^2+16628n+12956)(n+1)(n+2)\,a(n+4) + (n+4)(n+3)(n+5)(n+6)(n^6+18n^5+131n^4+490n^3+983n^2+992n+384)\,a(n+6) = 0 \quad (9)$$

REtoHTS

The sequence A212579

```
> RE:= a(n) = a(n-1)+2*a(n-2)-a(n-3)-2*a(n-4)-a(n-5)+2*a(n-6)+a
```

```
(n-7)-a(n-8)
```

$$RE := a(n) = a(n-1) + 2\,a(n-2) - a(n-3) - 2\,a(n-4) - a(n-5) + 2\,a(n-6) + a(n-7) - a(n-8) \tag{10}$$

```
> REtoHTS(RE,a(n),[0, 1, 8, 31, 80, 171, 308, 509, 780, 1137, 1584, 2143, 2812])
```

$$\frac{4}{9} + \frac{31}{12}\,n - 3\,n^2 + \frac{67}{36}\,n^3 - \frac{1}{4}\,n\,\chi_{\{modp(n,\,2)=0\}} - \frac{4}{9}\,\chi_{\{modp(n,\,3)=0\}} - \frac{8}{9}\,\chi_{\{modp(n,\,3)=1\}} \tag{11}$$

```
> U:=proc(n) U(n):=subs([n=n-1,a=U],RE) end proc:
U(0):=0:  U(1):=1:U(2):=8:U(3):=31:U(4):=80:U(5):=171:U(6):=308:U(7):=509:U(8):=780:U(9):=1137:U(10):=1584:U(11):=2143:U(12):=2812:
> REtoHTS(RE,a(n),U)
```

$$\frac{4}{9} + \frac{31}{12}\,n - 3\,n^2 + \frac{67}{36}\,n^3 - \frac{1}{4}\,n\,\chi_{\{modp(n,\,2)=0\}} - \frac{4}{9}\,\chi_{\{modp(n,\,3)=0\}} - \frac{8}{9}\,\chi_{\{modp(n,\,3)=1\}} \tag{12}$$

The sequence A033481
Requirement: FPS

```
> f:=(21 + 64*z + 32*z^2 - 5*z^3 - 56*z^4 - 28*z^5 - 14*z^6 - 7*z^7) / ((1 - z)*(1 + z + z^2))
```

$$f := \frac{-7\,z^7 - 14\,z^6 - 28\,z^5 - 56\,z^4 - 5\,z^3 + 32\,z^2 + 64\,z + 21}{(1-z)\,(z^2+z+1)} \tag{13}$$

```
> FPS(f,z,n)
```

$$7\,z^4 + 14\,z^3 + 28\,z^2 + 63\,z + 19 + \left(\sum_{n=0}^{\infty} 4\,z^n\right) + \left(\sum_{n=0}^{\infty} -2\,z^{3\,n}\right) + \left(\sum_{n=0}^{\infty} -3\,z^{3\,n+1}\right) \tag{14}$$

```
> RE:=FPS:-FindRE(f-(7*z^4 + 14*z^3 + 28*z^2 + 63*z + 19),z,u(n))
```

$$RE := (-n+1)\,u(n) + (4\,n-12)\,u(n-4) + (n-1)\,u(n-3) + (2\,n+2)\,u(n-2) + (-4\,n+12)\,u(n-1) + (-2\,n-2)\,u(n+1) = 0 \tag{15}$$

```
> U:=proc(n) U(n):=subs([n=n-1,u=U],solve(RE,u(n+1))) end proc:
U(0):=2:U(1):=1:U(2):=4:U(3):=2:U(4):=1:U(5):=4:
> REtoHTS(RE,u(n),U)
```

$$4 - 2\,\chi_{\{modp(n,\,3)=0\}} - 3\,\chi_{\{modp(n,\,3)=1\}} \tag{16}$$

```
> REtoHTS(RE,u(n),[2,1,4,2,1,4])
```

$$4 - 2\,\chi_{\{modp(n,\,3)=0\}} - 3\,\chi_{\{modp(n,\,3)=1\}} \tag{17}$$

```
> REtoHTS(a(n+2)=a(n+1)+a(n),a(n),[0,1,1])
```

$$-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2} + \frac{1}{2}\right)^n}{5} + \frac{\sqrt{5}\left(\frac{\sqrt{5}}{2} + \frac{1}{2}\right)^n}{5} \tag{18}$$

```
> U:=  proc(n) U(n):=2*(2*n-1)*U(n-1)/(n+1) end proc: U(0):=1: U(1):=1:
> REtoHTS((n+2)*C(n+1)=2*(2*n+1)*C(n),C(n),U)
```

(19)

$$\frac{(2n)!\,(n+1)}{(n+1)!^2} \tag{19}$$

HTS

```
> HTS(sin(n*Pi/4)^2,n)
```

$$\frac{1}{2} - \frac{(-1)^{\frac{n}{2}}\,\chi_{\{modp(n,2)=0\}}}{2} \tag{20}$$

```
> HTS(sin(Pi*cos(n*Pi)/6)*cos(n*Pi/4),n)
```

$$\frac{(-1)^{\frac{n}{4}}\,\chi_{\{modp(n,4)=0\}}}{2} - \frac{\sqrt{2}\,(-1)^{\frac{n}{4}-\frac{1}{4}}\,\chi_{\{modp(n,4)=1\}}}{4} + \frac{\sqrt{2}\,(-1)^{\frac{n}{4}-\frac{3}{4}}\,\chi_{\{modp(n,4)=3\}}}{4} \tag{21}$$

```
> HTS(sin(cos(n*Pi/3)*Pi),n)
```

$$(-1)^{\frac{n}{3}-\frac{1}{3}}\,\chi_{\{modp(n,3)=1\}} - (-1)^{\frac{n}{3}-\frac{2}{3}}\,\chi_{\{modp(n,3)=2\}} \tag{22}$$

```
> HTS(tan(n*Pi/4),n)
```

$$\chi_{\{modp(n,4)=1\}} + \left(\lim_{n\to 2}\tan\left(\frac{n\pi}{4}\right)\right)\chi_{\{modp(n,4)=2\}} - \chi_{\{modp(n,4)=3\}} \tag{23}$$

```
> HTS(tan(n*Pi/3),n)
```

$$\sqrt{3}\,\chi_{\{modp(n,3)=1\}} - \sqrt{3}\,\chi_{\{modp(n,3)=2\}} \tag{24}$$

```
> HTS(cos(n*arccos(x)),n)
```

$$\frac{\left(x-\sqrt{x^2-1}\right)^n}{2} + \frac{\left(x+\sqrt{x^2-1}\right)^n}{2} \tag{25}$$

```
> HTS(sin(n*Pi/6)*cos(n*Pi/3)-sin(n*Pi/2),n)
```

$$-\frac{\mathrm{I}\left(\frac{\sqrt{3}}{2}-\frac{\mathrm{I}}{2}\right)^n}{4} + \frac{\mathrm{I}\left(\frac{\sqrt{3}}{2}+\frac{\mathrm{I}}{2}\right)^n}{4} - \frac{(-1)^{\frac{n}{2}-\frac{1}{2}}\,\chi_{\{modp(n,2)=1\}}}{2} \tag{26}$$

```
> HTS(sin(n*Pi/4)^2*cos(n*Pi/6)^2,n)
```

$$\frac{1}{4} - \frac{(-\mathrm{I})^n}{8} + \frac{\left(\frac{1}{2}-\frac{\mathrm{I}\sqrt{3}}{2}\right)^n}{8} + \frac{\left(\frac{1}{2}+\frac{\mathrm{I}\sqrt{3}}{2}\right)^n}{8} - \frac{\mathrm{I}\,(-1)^{\frac{n}{2}-\frac{1}{2}}\,\chi_{\{modp(n,2)=1\}}}{8} - \frac{3\,(-\mathrm{I})^{\frac{n}{3}}\,\chi_{\{modp(n,3)=0\}}}{8} - \frac{3\,\mathrm{I}\,(-1)^{\frac{n}{6}-\frac{1}{2}}\,\chi_{\{modp(n,6)=3\}}}{8} \tag{27}$$

```
> HTS(sin(n*Pi/4)^2*cos(n*Pi/6)^4,n,maxreorder=12)
```

$$\frac{9}{32} + \frac{\left(4\,\mathrm{I}\,(-1)^{\frac{n}{2}}\sqrt{3} + 8\left(-\frac{1}{2} - \frac{\mathrm{I}\sqrt{3}}{2}\right)^{\frac{n}{2}} + 3\,(-1)^{\frac{n}{2}}\right)\chi_{\{modp(n,\,2)=0\}}}{32} \tag{28}$$
$$- \frac{9\,\chi_{\{modp(n,\,3)=0\}}}{32} + \frac{\mathrm{I}\sqrt{3}\,(-\mathrm{I})^{\frac{n}{3}-\frac{2}{3}}\chi_{\{modp(n,\,3)=2\}}}{4}$$
$$+ \frac{\left(-\frac{1}{2} - \frac{\mathrm{I}\sqrt{3}}{2}\right)^{\frac{n}{4}}\chi_{\{modp(n,\,4)=0\}}}{2} - \frac{\left(4\,\mathrm{I}\sqrt{3}+27\right)(-1)^{\frac{n}{6}}\chi_{\{modp(n,\,6)=0\}}}{32}$$
$$- \frac{\sqrt{3}\,(-1)^{\frac{n}{6}-\frac{5}{6}}\chi_{\{modp(n,\,6)=5\}}}{4}$$

HTSproduct

New (March 2024): computing product of hypergeometric type terms

```
> s1:=HTS(sin(n*Pi/4)^2,n)
```

$$s1 := \frac{1}{2} - \frac{(-1)^{\frac{n}{2}}\chi_{\{modp(n,\,2)=0\}}}{2} \tag{29}$$

```
> s2:=HTS(cos(n*Pi/3)^2,n)
```

$$s2 := \chi_{\{modp(n,\,3)=0\}} + \frac{\chi_{\{modp(n,\,3)=1\}}}{4} + \frac{\chi_{\{modp(n,\,3)=2\}}}{4} \tag{30}$$

```
> s:=HTSproduct(s1,s2,n)
```

$$s := \frac{\chi_{\{modp(n,\,3)=0\}}}{2} + \frac{\chi_{\{modp(n,\,3)=1\}}}{8} + \frac{\chi_{\{modp(n,\,3)=2\}}}{8} - \frac{(-1)^{\frac{n}{2}}\chi_{\{modp(n,\,6)=0\}}}{2} \tag{31}$$
$$- \frac{(-1)^{\frac{n}{2}}\chi_{\{modp(n,\,6)=4\}}}{8} - \frac{(-1)^{\frac{n}{2}}\chi_{\{modp(n,\,6)=2\}}}{8}$$

*And we can now have fun with finding equivalent representations by combining **HTS** and **HTSproduct***

```
> HTS((sin(n*Pi/4)*cos(n*Pi/3))^2,n)
```

$$\frac{1}{8} - \frac{(-1)^{\frac{n}{2}} \chi_{\{modp(n,2)=0\}}}{8} + \frac{3\chi_{\{modp(n,3)=0\}}}{8} - \frac{3(-1)^{\frac{n}{6}} \chi_{\{modp(n,6)=0\}}}{8} \tag{32}$$

What recurrence equation for

```
> HolonomicRE(s,a(n))
```

$$-a(n) + a(n+3) - a(n+6) + a(n+9) = 0 \tag{33}$$

```
>
>
```