> *restart*

>

Gröbner bases elimination for **Proposition 1** and **Theorem 2** in the paper:

On Rational Recursion for Holonomic Sequences

Bertrand Teguia Tabuguia and James Worrell
Department of Computer Science, University of Oxford

Throughout $s(n+l+i) \rightarrow s_{l+i}$, $i=1, 2$

Degree 1 case

>

> $d := 1$

$$d := 1 \tag{1}$$

Case l+1:

> $p0 := add(\mathrm{alpha}[0, k] \cdot n^k, k = 0..d)$

$$p0 := \alpha_{0,1}\, n + \alpha_{0,0} \tag{2}$$

> $p1 := collect(add(add(\mathrm{binomial}(d-k+j, j) \cdot (\mathrm{alpha}[1, d-j] + \mathrm{beta}[l, d-j] \cdot s[l+1]), j = 0..k) \cdot n^{d-k}, k = 0..d), n, distributed)$

$$p1 := \left(\beta_{l,1}\, s_{l+1} + \alpha_{1,1}\right) n + \beta_{l,0}\, s_{l+1} + \beta_{l,1}\, s_{l+1} + \alpha_{1,0} + \alpha_{1,1} \tag{3}$$

>

> $L := [p0, normal(p1)]$

$$L := \left[\alpha_{0,1}\, n + \alpha_{0,0}, n\, \beta_{l,1}\, s_{l+1} + n\, \alpha_{1,1} + \beta_{l,0}\, s_{l+1} + \beta_{l,1}\, s_{l+1} + \alpha_{1,0} + \alpha_{1,1}\right] \tag{4}$$

> $vars := remove(has, indets([p1, p0]), \mathrm{beta})$ **minus** $\{n\}$

$$vars := \left\{\alpha_{0,0}, \alpha_{0,1}, \alpha_{1,0}, \alpha_{1,1}, s_{l+1}\right\} \tag{5}$$

> $J := PolynomialIdeals{:-}PolynomialIdeal(L, parameters = select(has, indets(p1), \mathrm{beta}))$

$$J := \left\langle \alpha_{0,1}\, n + \alpha_{0,0}, n\, \beta_{l,1}\, s_{l+1} + n\, \alpha_{1,1} + \beta_{l,0}\, s_{l+1} + \beta_{l,1}\, s_{l+1} + \alpha_{1,0} + \alpha_{1,1} \right\rangle \tag{6}$$

> $J := PolynomialIdeals{:-}EliminationIdeal(J, vars)$:

> $J := select(type, convert(J, list), polynom)$:

> $numelems(J)$

(7)

$$1 \tag{7}$$

> degree(J[1], s[l+1])

$$1 \tag{8}$$

> collect(J[1], s[l+1])

$$(-\alpha_{0,0}\beta_{l,1} + \alpha_{0,1}\beta_{l,0} + \alpha_{0,1}\beta_{l,1})\, s_{l+1} - \alpha_{0,0}\alpha_{1,1} + \alpha_{0,1}\alpha_{1,0} + \alpha_{0,1}\alpha_{1,1} \tag{9}$$

>

Case l+2:

>

Degree $d \leq 3$ case

>

> $d := 2$

$$d := 2 \tag{10}$$

> $p0 := add(\text{alpha}[0,k]\cdot n^k, k=0..d)$

$$p0 := \alpha_{0,2}\, n^2 + \alpha_{0,1}\, n + \alpha_{0,0} \tag{11}$$

> $p1 := collect(add(add(\text{binomial}(d-k+j, j)\cdot\text{alpha}[1, d-j], j=0..k)\cdot n^{d-k}, k=0..d), n, distributed)$

$$p1 := \alpha_{1,2}\, n^2 + (\alpha_{1,2} + 2\,\alpha_{1,1})\, n + \alpha_{1,2} + \alpha_{1,1} + \alpha_{1,0} \tag{12}$$

> $p2 := collect(add(add(\text{binomial}(d-k+j, j)\cdot 2^j\cdot(\text{alpha}[2, d-j] + \text{beta}[l, d-j]\cdot s[l+2]), j=0..k)\cdot n^{d-k}, k=0..d), n, distributed)$

$$p2 := (\beta_{l,2}\, s_{l+2} + \alpha_{2,2})\, n^2 + (4\,\beta_{l,1}\, s_{l+2} + \beta_{l,2}\, s_{l+2} + 4\,\alpha_{2,1} + \alpha_{2,2})\, n + 4\,\beta_{l,0}\, s_{l+2} + 2\,\beta_{l,1}\, s_{l+2} + \beta_{l,2}\, s_{l+2} + 4\,\alpha_{2,0} + 2\,\alpha_{2,1} + \alpha_{2,2} \tag{13}$$

> $L := [p0, p1, normal(p2)]:$

> $vars := remove(has, indets([p2, p1, p0]), \text{beta})$ **minus** $\{n\}$

$$vars := \{\alpha_{0,0}, \alpha_{0,1}, \alpha_{0,2}, \alpha_{1,0}, \alpha_{1,1}, \alpha_{1,2}, \alpha_{2,0}, \alpha_{2,1}, \alpha_{2,2}, s_{l+2}\} \tag{14}$$

> $J := PolynomialIdeals:\text{-}PolynomialIdeal(L, parameters = select(has, indets(p2), \text{beta}))$

$$J := \langle \alpha_{1,2}\, n^2 + (\alpha_{1,2} + 2\,\alpha_{1,1})\, n + \alpha_{1,2} + \alpha_{1,1} + \alpha_{1,0},\ n^2\,\alpha_{0,2} + n\,\alpha_{0,1} + \alpha_{0,0},\ n^2\,\beta_{l,2}\, s_{l+2} + n^2\,\alpha_{2,2} + 4\,n\,\beta_{l,1}\, s_{l+2} + n\,\beta_{l,2}\, s_{l+2} + 4\,n\,\alpha_{2,1} + n\,\alpha_{2,2} + 4\,\beta_{l,0}\, s_{l+2} + 2\,\beta_{l,1}\, s_{l+2} + \beta_{l,2}\, s_{l+2} + 4\,\alpha_{2,0} + 2\,\alpha_{2,1} + \alpha_{2,2}\rangle \tag{15}$$

> $J := PolynomialIdeals:\text{-}EliminationIdeal(J, vars):$

> $J := select(type, convert(J, list), polynom):$

> numelems(J)

$$7 \tag{16}$$

> degree(J[2], s[l+2])

$$1 \tag{17}$$

> $collect(J[2], s[l+2])$

$$\left(-2\,\alpha_{0,0}\,\alpha_{1,1}\,\beta_{l,2}+4\,\alpha_{0,0}\,\alpha_{1,2}\,\beta_{l,1}+\alpha_{0,1}\,\alpha_{1,0}\,\beta_{l,2}+\alpha_{0,1}\,\alpha_{1,1}\,\beta_{l,2}-4\,\alpha_{0,1}\,\alpha_{1,2}\,\beta_{l,0}-2\,\alpha_{0,1}\,\alpha_{1,2}\,\beta_{l,1}-4\,\alpha_{0,2}\,\alpha_{1,0}\,\beta_{l,1}-\alpha_{0,2}\,\alpha_{1,0}\,\beta_{l,2}+8\,\alpha_{0,2}\,\alpha_{1,1}\,\beta_{l,0}+\alpha_{0,2}\,\alpha_{1,1}\,\beta_{l,2}+4\,\alpha_{0,2}\,\alpha_{1,2}\,\beta_{l,0}-2\,\alpha_{0,2}\,\alpha_{1,2}\,\beta_{l,1}\right)s_{l+2}-2\,\alpha_{0,0}\,\alpha_{1,1}\,\alpha_{2,2}+4\,\alpha_{0,0}\,\alpha_{1,2}\,\alpha_{2,1}+\alpha_{0,1}\,\alpha_{1,0}\,\alpha_{2,2}+\alpha_{0,1}\,\alpha_{1,1}\,\alpha_{2,2}-4\,\alpha_{0,1}\,\alpha_{1,2}\,\alpha_{2,0}-2\,\alpha_{1,2}\,\alpha_{0,1}\,\alpha_{2,1}-4\,\alpha_{0,2}\,\alpha_{1,0}\,\alpha_{2,1}-\alpha_{0,2}\,\alpha_{2,2}\,\alpha_{1,0}+8\,\alpha_{0,2}\,\alpha_{1,1}\,\alpha_{2,0}+\alpha_{0,2}\,\alpha_{1,1}\,\alpha_{2,2}+4\,\alpha_{0,2}\,\alpha_{1,2}\,\alpha_{2,0}-2\,\alpha_{0,2}\,\alpha_{1,2}\,\alpha_{2,1} \tag{18}$$

>

>

>

>

>

>