> $restart$

>

Gröbner bases elimination for **Proposition 1** and **Theorem 2** in the paper:

On Rational Recursion for Holonomic Sequences

Bertrand Teguia Tabuguia and James Worrell

Throughout $s(n+l+i) \to s_{l+i}$, $i=1,2$

Degree 1 case

>

> $d := 1$

$$d := 1 \tag{1}$$

Case l+1:

> $p0 := add\left(\mathrm{alpha}[0,k]\cdot n^k, k=0..d\right)$

$$p0 := \alpha_{0,1}\, n + \alpha_{0,0} \tag{2}$$

> $p1 := collect\left(add\left(add\left(\mathrm{binomial}(d-k+j,j)\cdot(\mathrm{alpha}[1,d-j]+\mathrm{beta}[l,d-j]\cdot s[l+1]), j=0..k\right)\cdot n^{d-k}, k=0..d\right), n, distributed\right)$

$$p1 := \left(\beta_{l,1}\, s_{l+1} + \alpha_{1,1}\right) n + \beta_{l,0}\, s_{l+1} + \beta_{l,1}\, s_{l+1} + \alpha_{1,0} + \alpha_{1,1} \tag{3}$$

>

> $L := [p0, normal(p1)]$

$$L := \left[\alpha_{0,1}\, n + \alpha_{0,0},\ n\,\beta_{l,1}\, s_{l+1} + n\,\alpha_{1,1} + \beta_{l,0}\, s_{l+1} + \beta_{l,1}\, s_{l+1} + \alpha_{1,0} + \alpha_{1,1}\right] \tag{4}$$

> $vars := remove(has, indets([p1, p0]), \mathrm{beta})$ **minus** $\{n\}$

$$vars := \left\{\alpha_{0,0}, \alpha_{0,1}, \alpha_{1,0}, \alpha_{1,1}, s_{l+1}\right\} \tag{5}$$

> $J := PolynomialIdeals{:-}PolynomialIdeal(L, parameters = select(has, indets(p1), \mathrm{beta}))$

$$J := \left\langle \alpha_{0,1}\, n + \alpha_{0,0},\ n\,\beta_{l,1}\, s_{l+1} + n\,\alpha_{1,1} + \beta_{l,0}\, s_{l+1} + \beta_{l,1}\, s_{l+1} + \alpha_{1,0} + \alpha_{1,1}\right\rangle \tag{6}$$

> $J := PolynomialIdeals{:-}EliminationIdeal(J, vars)$:

> $J := select(type, convert(J, list), polynom)$:

> $numelems(J)$

$$1 \tag{7}$$

> $degree(J[1], s[l+1])$

$$1 \tag{8}$$

> $collect(J[1], s[l+1])$

$$\left(-\alpha_{0,0}\,\beta_{l,1}+\alpha_{0,1}\,\beta_{l,0}+\alpha_{0,1}\,\beta_{l,1}\right)s_{l+1}-\alpha_{0,0}\,\alpha_{1,1}+\alpha_{0,1}\,\alpha_{1,0}+\alpha_{0,1}\,\alpha_{1,1} \tag{9}$$

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Case l+2:

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Degree $d \leq 3$ case

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> $d := 2$

$$d := 2 \tag{10}$$

> $p0 := add\left(\text{alpha}[0,k]\cdot n^k, k=0..d\right)$

$$p0 := \alpha_{0,2}\,n^2+\alpha_{0,1}\,n+\alpha_{0,0} \tag{11}$$

> $p1 := add\left(\text{alpha}[1,k]\cdot n^k, k=0..d\right)$

$$p1 := \alpha_{1,2}\,n^2+n\,\alpha_{1,1}+\alpha_{1,0} \tag{12}$$

> $p2 := collect\Big(add\Big(add\big(\text{binomial}(d-k+j,j)\cdot 2^j\cdot(\text{alpha}[2,d-j]+\text{beta}[l,d-j]\cdot s[l+2]), j=0..k\big)\cdot n^{d-k}, k=0..d\Big), n, distributed\Big)$

$$p2 := \left(\beta_{l,2}\,s_{l+2}+\alpha_{2,2}\right)n^2+\left(4\,\beta_{l,1}\,s_{l+2}+\beta_{l,2}\,s_{l+2}+4\,\alpha_{2,1}+\alpha_{2,2}\right)n+4\,\beta_{l,0}\,s_{l+2}+2\,\beta_{l,1}\,s_{l+2}+\beta_{l,2}\,s_{l+2}+4\,\alpha_{2,0}+2\,\alpha_{2,1}+\alpha_{2,2} \tag{13}$$

> $L := [p0, p1, normal(p2)]:$

> $vars := remove(has, indets([p2, p1, p0]), \text{beta})$ **minus** $\{n\}$

$$vars := \left\{\alpha_{0,0}, \alpha_{0,1}, \alpha_{0,2}, \alpha_{1,0}, \alpha_{1,1}, \alpha_{1,2}, \alpha_{2,0}, \alpha_{2,1}, \alpha_{2,2}, s_{l+2}\right\} \tag{14}$$

> $J := PolynomialIdeals{:-}PolynomialIdeal(L, parameters = select(has, indets(p2), \text{beta}))$

$$J := \left\langle n^2\,\alpha_{0,2}+\alpha_{0,1}\,n+\alpha_{0,0}, n^2\,\alpha_{1,2}+n\,\alpha_{1,1}+\alpha_{1,0}, n^2\,\beta_{l,2}\,s_{l+2}+n^2\,\alpha_{2,2}+4\,n\,\beta_{l,1}\,s_{l+2}+n\,\beta_{l,2}\,s_{l+2}+4\,n\,\alpha_{2,1}+n\,\alpha_{2,2}+4\,\beta_{l,0}\,s_{l+2}+2\,\beta_{l,1}\,s_{l+2}+\beta_{l,2}\,s_{l+2}+4\,\alpha_{2,0}+2\,\alpha_{2,1}+\alpha_{2,2}\right\rangle \tag{15}$$

> $J := PolynomialIdeals{:-}EliminationIdeal(J, vars):$

> $J := select(type, convert(J, list), polynom):$

> $numelems(J)$

$$7 \tag{16}$$

> $degree(J[2], s[l+2])$

$$1 \tag{17}$$

> $collect(J[2], s[l+2])$

$$\left(-\alpha_{0,0}\,\alpha_{1,1}\,\beta_{l,2}+4\,\alpha_{0,0}\,\alpha_{1,2}\,\beta_{l,1}+\alpha_{0,0}\,\alpha_{1,2}\,\beta_{l,2}+\alpha_{0,1}\,\alpha_{1,0}\,\beta_{l,2}-4\,\alpha_{0,1}\,\alpha_{1,2}\,\beta_{l,0}\right.$$
$$-2\,\alpha_{0,1}\,\alpha_{1,2}\,\beta_{l,1}-\alpha_{0,1}\,\alpha_{1,2}\,\beta_{l,2}-4\,\alpha_{0,2}\,\alpha_{1,0}\,\beta_{l,1}-\alpha_{0,2}\,\alpha_{1,0}\,\beta_{l,2}+4\,\alpha_{0,2}\,\alpha_{1,1}\,\beta_{l,0}$$
$$\left.+2\,\alpha_{0,2}\,\alpha_{1,1}\,\beta_{l,1}+\alpha_{0,2}\,\alpha_{1,1}\,\beta_{l,2}\right)s_{l+2}-\alpha_{0,0}\,\alpha_{1,1}\,\alpha_{2,2}+4\,\alpha_{0,0}\,\alpha_{1,2}\,\alpha_{2,1}+\alpha_{0,0}\,\alpha_{1,2}\,\alpha_{2,2}$$
$$+\alpha_{0,1}\,\alpha_{1,0}\,\alpha_{2,2}-4\,\alpha_{0,1}\,\alpha_{1,2}\,\alpha_{2,0}-2\,\alpha_{0,1}\,\alpha_{1,2}\,\alpha_{2,1}-\alpha_{0,1}\,\alpha_{2,2}\,\alpha_{1,2}-4\,\alpha_{0,2}\,\alpha_{1,0}\,\alpha_{2,1}$$
$$-\alpha_{0,2}\,\alpha_{1,0}\,\alpha_{2,2}+4\,\alpha_{0,2}\,\alpha_{1,1}\,\alpha_{2,0}+2\,\alpha_{0,2}\,\alpha_{1,1}\,\alpha_{2,1}+\alpha_{0,2}\,\alpha_{1,1}\,\alpha_{2,2} \tag{18}$$

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