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Computations of examples from the paper:

On Rational Recursion for Holonomic Sequences (Examples 3-4)

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The algorithm is implemented as a command of the **NLDE** package, available at https://github.com/T3gu1a/D-algebraic-functions

> with(NLDE, HoloToSimpleRatrec)

$$[HoloToSimpleRatrec] \quad (1)$$

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Example 1:

Catalan numbers $\left(\frac{1}{n+1}\binom{2n}{n}\right)_n$

$$\left(\frac{1}{n+1}\binom{2n}{n}\right)_n \quad (1.1)$$

> $p := (n+2)\cdot s(n+1) - (4\cdot n+2)\cdot s(n)$

$$p := (n+2)\, s(n+1) - (4\,n+2)\, s(n) \quad (1.2)$$

> $HoloToSimpleRatrec(p, s(n))$

$$s(n+2) = \frac{2\, s(n+1)\, (8\, s(n) + s(n+1))}{10\, s(n) - s(n+1)} \quad (1.3)$$

$((-1)^n + n)_n$

$$((-1)^n + n)_n \quad (1.4)$$

> $p := (-2\,n-3)\, s(n) - 2\, s(n+1) + (2\,n+1)\, s(n+2)$

$$p := (-2\,n-3)\, s(n) - 2\, s(n+1) + (2\,n+1)\, s(n+2) \quad (1.5)$$

> $HoloToSimpleRatrec(p, s(n))$

(1.6)

$$s(n+3) = s(n+1) - s(n) + s(n+2) \tag{1.6}$$

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Example 2

$(n!^2)_n$

$$(n!^2)_n \tag{2.1}$$

> $p := s(n+1) - (n+1)^2 \cdot s(n)$

$$p := s(n+1) - (n+1)^2\, s(n) \tag{2.2}$$

> $HoloToSimpleRatrec(p, s(n))$

$$s(n+3) = \frac{s(n+2)\,\left(2\,s(n)\,s(n+1) + 2\,s(n)\,s(n+2) - s(n+1)^2\right)}{s(n)\,s(n+1)} \tag{2.3}$$

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Example 3

$(n!^3)_n$

$$(n!^3)_n \tag{3.1}$$

> $p1 := s(n+1) - (n+1)^3 \cdot s(n)$

$$p1 := s(n+1) - (n+1)^3\, s(n) \tag{3.2}$$

> $HoloToSimpleRatrec(p1, s(n), method = GB)$

$$s(n+3) = \frac{s(n+2)\,\left(4\,s(n)\,s(n+1)^2 - 4\,s(n)\,s(n+2)^2 + s(n+1)^3 + s(n+1)^2\,s(n+2)\right)}{s(n+1)\,\left(s(n)\,s(n+1) - s(n)\,s(n+2) - 2\,s(n+1)^2\right)} \tag{3.3}$$

$\left(n^2 + \sin\left(\frac{n \cdot \pi}{4}\right)^2\right)_n$

$$\left(n^2 + \sin\left(\frac{n \cdot \pi}{4}\right)^2\right)_n \tag{3.4}$$

> $p2 := (-2 \cdot n\text{^}2 - 8 \cdot n - 11) \cdot s(n) + (2 \cdot n\text{^}2 + 4 \cdot n + 5) \cdot s(n+1) + (-2 \cdot n\text{^}2 - 8 \cdot n - 11) \cdot s(n+2) + (2 \cdot n\text{^}2 + 4 \cdot n + 5) \cdot s(n+3) = 0$

$$p2 := \left(-2\,n^2 - 8\,n - 11\right)\,s(n) + \left(2\,n^2 + 4\,n + 5\right)\,s(n+1) + \left(-2\,n^2 - 8\,n - 11\right)\,s(n \tag{3.5}$$

$+ 2) + (2 n^2 + 4 n + 5) s(n + 3) = 0$

> HoloToSimpleRatrec(p2, s(n), method = GB)

$$s(n+5) = s(n) - 3 s(n+1) + 4 s(n+2) - 4 s(n+3) + 3 s(n+4) \tag{3.6}$$

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Example 4

$$\left(\binom{2 \cdot n}{n} \cdot \binom{3 \cdot n}{n}\right)_n$$

$$\left(\binom{2 \cdot n}{n} \cdot \binom{3 \cdot n}{n}\right)_n \tag{4.1}$$

> p3 := s(n + 1)·(n + 1)^2 − 3·(3·n + 1)·(3·n + 2)·s(n)

$$p3 := s(n+1) (n+1)^2 - 3 (3 n + 1) (3 n + 2) s(n) \tag{4.2}$$

> HoloToSimpleRatrec(p3, s(n), method = LA)

$$s(n+3) = \left(3 s(n+2) \left(26244 s(n) s(n+1) - 702 s(n) s(n+2) - 378 s(n+1)^2 + 13 s(n+1) s(n+2)\right)\right) \Big/ \left(5508 s(n) s(n+1) - 201 s(n) s(n+2) - 84 s(n+1)^2 + 4 s(n+1) s(n+2)\right) \tag{4.3}$$

$$\left(\frac{n^4}{2^n} + 3^n\right)_n$$

$$\left(\frac{n^4}{2^n} + 3^n\right)_n \tag{4.4}$$

> p4 := (15 n^4 + 48 n^3 + 36 n^2 − 24 n − 30) s(n) − (7 n^2 + 4 n + 4) (5 n^2 − 4 n − 4) s(n + 1) + (10 n^4 − 8 n^3 − 12 n^2 − 8 n − 2) s(n + 2)

$$p4 := (15 n^4 + 48 n^3 + 36 n^2 - 24 n - 30) s(n) - (7 n^2 + 4 n + 4) (5 n^2 - 4 n - 4) s(n+1) + (10 n^4 - 8 n^3 - 12 n^2 - 8 n - 2) s(n+2) \tag{4.5}$$

> HoloToSimpleRatrec(p4, s(n), method = LA)

$$s(n+6) = -10 s(n+4) - \frac{3 s(n)}{32} + \frac{31 s(n+1)}{32} - \frac{65 s(n+2)}{16} + \frac{35 s(n+3)}{4} + \frac{11 s(n+5)}{2} \tag{4.6}$$

Random polynomial (not necessarily the same as in the paper).

> p5 := s(n) + randpoly(n, degree = 5)

(4.7)

$$p5 := s(n) + 16\,n^5 + 52\,n^4 - 20\,n^3 - 4\,n^2 - 89\,n - 77 \tag{4.7}$$

> *HoloToSimpleRatrec*(*p5*, *s*(*n*), *method* = *LA*)

$$s(n+5) = -1920 + 5\,s(n+4) - 10\,s(n+3) + 10\,s(n+2) + s(n) - 5\,s(n+1) \tag{4.8}$$

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Other examples

$(n!^4)_n$

$$(n!^4)_n \tag{5.1}$$

> $p := s(n+1) + (n+1)^4 \cdot s(n)$

$$p := s(n+1) + (n+1)^4\,s(n) \tag{5.2}$$

> *HoloToSimpleRatrec*(*p*, *s*(*n*)) #*LA method by default*

$$s(n+5) = -\frac{1}{s(n)\,s(n+1)\,s(n+2)\,s(n+3)}\big(s(n+4)\,\big(24\,s(n)\,s(n+1)\,s(n+2)\,s(n+3) - 4\,s(n)\,s(n+1)\,s(n+2)\,s(n+4) + 6\,s(n)\,s(n+3)^2\,s(n+1) - 4\,s(n)\,s(n+2)^2 s(n+3) + s(n+1)^2\,s(n+2)\,s(n+3)\big)\big) \tag{5.3}$$

> *HoloToSimpleRatrec*(*p*, *s*(*n*), *method* = *GB*)

$$s(n+3) = -\big(s(n+2)\,\big(5\,s(n)\,s(n+1)^2 - 22\,s(n)\,s(n+2)\,s(n+1) + 5\,s(n)\,s(n+2)^2 + s(n+1)^3 - s(n+1)^2\,s(n+2)\big)\big) \Big/ \big(s(n+1)\,\big(s(n)\,s(n+1) - s(n)\,s(n+2) - 3\,s(n+1)^2\big)\big) \tag{5.4}$$

We use the **HolonomicRE** command of **HyperTypeSeq** to find recurrence equations from general terms. The package is available at
https://github.com/T3gu1a/HyperTypeSeq

$(n!^2 + n!)_n$

$$(n!^2 + n!)_n \tag{5.5}$$

> $p :=$ *HyperTypeSeq*:-*HolonomicRE*$(n!^2 + n!, s(n))$

$$p := (n+2)\,(n+1)^3\,s(n) - (n+2)\,\big(n^2 + 3\,n + 1\big)\,s(n+1) + s(n+2)\,n = 0 \tag{5.6}$$

> *HoloToSimpleRatrec*(*p*, *s*(*n*)) #*LA method by default*

$$s(n+6) = -\big(-153\,s(n+1)^2\,s(n+4)^3 + 92\,s(n+2)^3\,s(n+4)^2 - 27\,s(n+2)^3\,s(n+5)^2 - 12\,s(n+2)^2\,s(n+4)^3 - 8448\,s(n)\,s(n+1)\,s(n+2)\,s(n+4)^2 - 440\,s(n)\,s(n+1)\,s(n+2)\,s(n+5)^2 + 13032\,s(n)\,s(n+1)\,s(n+3)^2\,s(n+4) - 6828\,s(n)\,s(n+1)\,s(n+3)^2\,s(n+5) - 3612\,s(n)\,s(n+1)\,s(n+3)\,s(n+4)^2 + 344\,s(n)\,s(n+1)\,s(n+3)\,s(n+5)^2 - 547\,s(n)\,s(n+1)\,s(n+4)^2\,s(n+5) - 47\,s(n)\,s(n \tag{5.7}$$

$$+ 1) s(n+4) s(n+5)^2 + 10272 s(n) s(n+2)^2 s(n+3) s(n+4)$$
$$- 7578 s(n) s(n+2)^2 s(n+3) s(n+5) + 3164 s(n) s(n+2)^2 s(n+4) s(n+5)$$
$$+ 15468 s(n) s(n+2) s(n+3)^2 s(n+4) - 9666 s(n) s(n+2) s(n+3)^2 s(n+5)$$
$$- 276 s(n) s(n+2) s(n+3) s(n+4)^2 + 110 s(n) s(n+2) s(n+3) s(n+5)^2$$
$$+ 98 s(n) s(n+2) s(n+4)^2 s(n+5) + 9 s(n) s(n+2) s(n+4) s(n+5)^2$$
$$- 211 s(n) s(n+3)^2 s(n+4) s(n+5) + 3 s(n) s(n+3) s(n+4)^2 s(n+5)$$
$$+ 5172 s(n+1)^2 s(n+2) s(n+3) s(n+4) - 3832 s(n+1)^2 s(n+2) s(n+3) s(n$$
$$+ 5) + 2220 s(n+1)^2 s(n+2) s(n+4) s(n+5) + 736 s(n+1)^2 s(n+3) s(n$$
$$+ 4) s(n+5) - 2402 s(n+1) s(n+2)^2 s(n+3) s(n+4) + 1436 s(n+1) s(n$$
$$+ 2)^2 s(n+3) s(n+5) + 185 s(n+1) s(n+2)^2 s(n+4) s(n+5) + 1074 s(n$$
$$+ 1) s(n+2) s(n+3)^2 s(n+4) - 812 s(n+1) s(n+2) s(n+3)^2 s(n+5)$$
$$- 856 s(n+1) s(n+2) s(n+3) s(n+4)^2 + 42 s(n+1) s(n+2) s(n+3) s(n$$
$$+ 5)^2 - 3 s(n+1) s(n+2) s(n+4)^2 s(n+5) - 24 s(n+1) s(n+3)^2 s(n$$
$$+ 4) s(n+5) + 30 s(n+2)^2 s(n+3) s(n+4) s(n+5) + 704 s(n+1) s(n$$
$$+ 3)^3 s(n+4) - 384 s(n+1) s(n+3)^3 s(n+5) + 163 s(n+1) s(n+3)^2 s(n$$
$$+ 4)^2 + 9 s(n+1) s(n+3) s(n+4)^3 - 1010 s(n+2)^3 s(n+3) s(n+4)$$
$$+ 699 s(n+2)^3 s(n+3) s(n+5) - 120 s(n+2)^3 s(n+4) s(n+5) - 337 s(n$$
$$+ 2)^2 s(n+3)^2 s(n+4) + 193 s(n+2)^2 s(n+3)^2 s(n+5) - 106 s(n+2)^2 s(n$$
$$+ 3) s(n+4)^2 + 13 s(n+2) s(n+3)^3 s(n+4) - 6 s(n+2) s(n+3)^3 s(n+5)$$
$$+ 3 s(n+2) s(n+3)^2 s(n+4)^2 - 540 s(n) s(n+1) s(n+4)^3 - 2848 s(n) s(n$$
$$+ 2)^2 s(n+4)^2 - 78 s(n) s(n+2)^2 s(n+5)^2 - 296 s(n) s(n+2) s(n+4)^3$$
$$+ 4080 s(n) s(n+3)^3 s(n+4) - 2208 s(n) s(n+3)^3 s(n+5) + 1108 s(n) s(n$$
$$+ 3)^2 s(n+4)^2 - 12 s(n) s(n+3)^2 s(n+5)^2 + 68 s(n) s(n+3) s(n+4)^3$$
$$- 1998 s(n+1)^2 s(n+2) s(n+4)^2 - 160 s(n+1)^2 s(n+2) s(n+5)^2$$
$$+ 2532 s(n+1)^2 s(n+3)^2 s(n+4) - 1312 s(n+1)^2 s(n+3)^2 s(n+5)$$
$$- 1206 s(n+1)^2 s(n+3) s(n+4)^2 + 96 s(n+1)^2 s(n+3) s(n+5)^2$$
$$- 141 s(n+1)^2 s(n+4)^2 s(n+5) - 12 s(n+1)^2 s(n+4) s(n+5)^2 - 366 s(n$$
$$+ 1) s(n+2)^2 s(n+4)^2 - 141 s(n+1) s(n+2)^2 s(n+5)^2 - 120 s(n+1) s(n$$
$$+ 2) s(n+4)^3 + 9658 s(n) s(n+1) s(n+2) s(n+4) s(n+5) + 2554 s(n) s(n$$
$$+ 1) s(n+3) s(n+4) s(n+5) + 1579 s(n) s(n+2) s(n+3) s(n+4) s(n+5)$$
$$+ 529 s(n+1) s(n+2) s(n+3) s(n+4) s(n+5) + 27648 s(n) s(n+1) s(n$$
$$+ 2) s(n+3) s(n+4) - 20028 s(n) s(n+1) s(n+2) s(n+3) s(n+5)) \Big/$$
$$\big(276 s(n) s(n+3)^2 s(n+1) + 558 s(n) s(n+2)^2 s(n+3) + 280 s(n+1)^2 s(n$$
$$+ 2) s(n+3) + 1440 s(n) s(n+1) s(n+2) s(n+3) - 826 s(n) s(n+1) s(n$$

$$+ 2)\, s(n+4) - 52\, s(n+2)^3\, s(n+3) + 24\, s(n+2)^3\, s(n+4) - 12\, s(n$$
$$+ 2)^2\, s(n+3)^2 + 99\, s(n)\, s(n+3)^3 + 52\, s(n+1)^2\, s(n+3)^2 + 18\, s(n$$
$$+ 1)^2\, s(n+4)^2 + 18\, s(n+1)\, s(n+3)^3 + 48\, s(n)\, s(n+1)\, s(n+2)\, s(n+5)$$
$$- 127\, s(n)\, s(n+1)\, s(n+3)\, s(n+4) - 23\, s(n)\, s(n+1)\, s(n+3)\, s(n+5)$$
$$- 174\, s(n)\, s(n+2)\, s(n+3)\, s(n+4) + 3\, s(n)\, s(n+2)\, s(n+3)\, s(n+5) - 45\, s(n$$
$$+ 1)\, s(n+2)\, s(n+3)\, s(n+4) + 70\, s(n)\, s(n+1)\, s(n+4)^2 - 276\, s(n)\, s(n$$
$$+ 2)^2\, s(n+4) + 10\, s(n)\, s(n+2)^2\, s(n+5) + 561\, s(n)\, s(n+2)\, s(n+3)^2$$
$$- 12\, s(n)\, s(n+2)\, s(n+4)^2 + 9\, s(n)\, s(n+3)^2\, s(n+4) - 171\, s(n+1)^2\, s(n$$
$$+ 2)\, s(n+4) + 13\, s(n+1)^2\, s(n+2)\, s(n+5) - 40\, s(n+1)^2\, s(n+3)\, s(n+4)$$
$$- 6\, s(n+1)^2\, s(n+3)\, s(n+5) - 87\, s(n+1)\, s(n+2)^2\, s(n+3) + 38\, s(n$$
$$+ 1)\, s(n+2)^2\, s(n+4) + 3\, s(n+1)\, s(n+2)^2\, s(n+5) + 59\, s(n+1)\, s(n$$
$$+ 2)\, s(n+3)^2)$$

> *HoloToSimpleRatrec*$(p, s(n), method = GB)$

$$s(n+6) = (\tag{5.8}$$
$$-6486923191185797164733082294296052743518364194845119601265638\, s(n)\, s(n$$
$$+ 2)\, s(n+5)^2$$
$$- 5371655126481772723594613911139129183271653304186539673945402 11\, s(n)$$
$$s(n+3)^2 s(n+5)$$
$$- 7295634143611993341981373358322395024209270096246746458560 2039\, s(n)\, s(n$$
$$+ 3)\, s(n+4)^2$$
$$- 1872803427745091177511002344703782531219307276062803193698940\, s(n)\, s(n$$
$$+ 3)\, s(n+5)^2$$
$$- 2532463202548028869037239095949960263404740233466013260753787 86\, s(n$$
$$+ 1)\, s(n+2)\, s(n+4)^2$$
$$+ 2295693667612789193162161777163233282947879081391510639410 3354\, s(n$$
$$+ 1)\, s(n+2)\, s(n+5)^2$$
$$+ 1001151696073414761645751550952304509206205829255284269564958903\, s(n$$
$$+ 1)\, s(n+3)^2\, s(n+4)$$
$$- 798121984167012144362005682869059969431285388158104876163905 87\, s(n$$
$$+ 1)\, s(n+3)^2\, s(n+5)$$
$$+ 1604905089000785496532422574671915224675101053803948444127129 79\, s(n$$
$$+ 1)\, s(n+3)\, s(n+4)^2$$
$$+ 2672719380676225405192175016210269811099860479044868793885259\, s(n$$
$$+ 1)\, s(n+3)\, s(n+5)^2$$

$- 65515875282490058719186180225530872119998893149394645668887740\, s(n$
$+ 1)\, s(n+4)^2\, s(n+5)$
$- 17229661759882478940747919146674709573371139992914818268071 4\, s(n$
$+ 1)\, s(n+4)\, s(n+5)^2$
$- 274441639320505326719804535620672083460987856193505179864486197\, s(n$
$+ 2)^2\, s(n+3)\, s(n+4)$
$- 508017681011438603179674047933449120959075006837018612761625 08\, s(n$
$+ 2)^2\, s(n+3)\, s(n+5)$
$- 489398174874685101955441904111927721158228832087800575480082 20\, s(n$
$+ 2)^2\, s(n+4)\, s(n+5)$
$- 405164615505446075979232391154401708834452714803600896032811 31\, s(n$
$+ 2)\, s(n+3)^2\, s(n+4)$
$- 299398760393033520637210420740737932068330359391292304432453 87\, s(n$
$+ 2)\, s(n+3)^2\, s(n+5)$
$- 333496018044048756678796477828097032903154776073444757754559 77\, s(n$
$+ 2)\, s(n+3)\, s(n+4)^2$
$+ 1525865706435833923120739950953207415171771936062475602698580\, s(n$
$+ 2)\, s(n+3)\, s(n+5)^2$
$- 2259618630845210246222215101442458313940000986407009274951804\, s(n$
$+ 2)\, s(n+4)^2\, s(n+5)$
$+ 942146309424926284869804606989083623448858420618158696420368\, s(n$
$+ 3)^2\, s(n+4)\, s(n+5)$
$+ 239654942877716238788972189971614499662682738485561784383124734\, s(n$
$+ 1)\, s(n+4)^3$
$- 18922682655468179377441249134847753649466060280670935155724698\, s(n$
$+ 2)^3\, s(n+5)$
$+ 192060129389455430774761688237697971910873234988581525164879354\, s(n$
$+ 2)^2\, s(n+4)^2$
$+ 8736043888190494221583658460301061399857128497817519839833548\, s(n$
$+ 2)^2\, s(n+5)^2$
$- 19547694354438368333609349661520454587351929232135405362815304\, s(n$
$+ 2)\, s(n+3)^3$
$+ 34357098701087040312877218407254729545862187392310727407029860\, s(n$
$+ 2)\, s(n+4)^3$

$$- 11378178974854322052728295733230058088672152535717130738895652\, s(n+3)^{3}\, s(n+4)$$

$$- 17747989036976007888929029734925956961380348666434418035852 01\, s(n+3)^{3}\, s(n+5)$$

$$- 11913994587305650910957287320483545956259686498767832556849921\, s(n+3)^{2}\, s(n+4)^{2}$$

$$- 3609676741672517236085026503308562894691797034447684148 6430\, s(n+3)\, s(n+4)^{3}$$

$$+ 51651107932284233647484694925280502528971114378588409555446 3028\, s(n)\, s(n+3)^{2} s(n+1)$$

$$+ 86787688042792721860515557508652873445535073681708104045009 4548\, s(n)\, s(n+2)^{2} s(n+3)$$

$$+ 73383387310700640971114378912349885993766361984644526717056 9716\, s(n+1)^{2}\, s(n+2)\, s(n+3)$$

$$- 57261207895944826365090448179405769091998117299558109586011 89\, s(n+3)^{4}$$

$$+ 44585585133783195400716815094946375560954518262884988853476 58764\, s(n)\, s(n+1)\, s(n+2)\, s(n+3)$$

$$- 48158818137167454916280816370610029333517546643927856435413 94428\, s(n)\, s(n+1)\, s(n+2)\, s(n+4)$$

$$- 11189663193728362661132850827520022423653149180878623956406 9648\, s(n+2)^{3}\, s(n+3)$$

$$+ 43238214587309939576070661524467735325336368047503244623128 676\, s(n+2)^{3}\, s(n+4)$$

$$- 46093332884842065661896969769314181499221176794969792680513 691\, s(n+2)^{2}\, s(n+3)^{2}$$

$$+ 60511966174226418286549164430499188515426756030140442878380 4076\, s(n)\, s(n+3)^{3}$$

$$- 12721676076365214231463837789568860770839751401501068984661 556\, s(n+1)^{2}\, s(n+3)^{2}$$

$$+ 16796830553946854959777847540315092966506241198528042724343 0460\, s(n+1)\, s(n+3)^{3}$$

$$+ 70627778950870878315606632861865227266173113985936251270122 1614\, s(n)\, s(n+1)\, s(n+2)\, s(n+5)$$

$$+ 58326292113183081323026165689965073354347804257475339806495 9832\, s(n)\, s(n$$

$+ 1)\, s(n+3)\, s(n+4)$

$- 42329727137858906887438291656317062543600358404610914464268330\, s(n)\, s(n$

$+ 1)\, s(n+3)\, s(n+5)$

$+ 370353880032623324156175936424086832921026159788656042409121 8212\, s(n)$

$s(n+2)\, s(n+3)\, s(n+4)$

$- 1430311916012557270135406454279666275086508945339222126678336894\, s(n)$

$s(n+2)\, s(n+3)\, s(n+5)$

$- 191699166080307805996251294031941750185751199962488384950238350\, s(n$

$+ 1)\, s(n+2)\, s(n+3)\, s(n+4)$

$+ 311047812871417083959741627094252227629409595480602426367133436\, s(n)$

$s(n+2)^2 s(n+4)$

$- 571936750647552232371121009085506610052407174002124681443933342\, s(n)$

$s(n+2)^2 s(n+5)$

$+ 2445290197986849428879788711040519944408419973575766679824107888\, s(n)$

$s(n+2)\, s(n+3)^2$

$- 3860929146851183273129650155735156665642407057595938475510479024\, s(n)$

$s(n+2)\, s(n+4)^2$

$+ 4711496142517602856722614845031864746210074550402368988152786881\, s(n)$

$s(n+3)^2 s(n+4)$

$- 768023992612816043456330007409782426619726220954042386806184300\, s(n$

$+ 1)^2 s(n+2)\, s(n+4)$

$+ 817376655407800915199059175401364681608568589477711108 76026218\, s(n$

$+ 1)^2 s(n+2)\, s(n+5)$

$+ 216375512298752770145409486137917750743631730491249719166588 00\, s(n$

$+ 1)^2 s(n+3)\, s(n+4)$

$+ 697221751509086332886883073702275449327202131332667347488497 14\, s(n$

$+ 1)^2 s(n+3)\, s(n+5)$

$- 5618994364943975524908053475836207386662734475764697293971848 12\, s(n$

$+ 1)\, s(n+2)^2 s(n+3)$

$+ 3058070903844698776027935886459790365830735640468202966341686 64\, s(n$

$+ 1)\, s(n+2)^2 s(n+4)$

$- 43740839717516989099377621291646674538226835004690216981194 8\, s(n$

$+ 1)\, s(n+2)^2 s(n+5)$

$+ 3731466306360882881645859249852333682093528163863591475064392 00\, s(n$

$$+ 1) s(n + 2) s(n + 3)^2$$
$$+ 645764547848328072731171913179884568421198182513577238011626772 s(n) s(n$$
$$+ 2) s(n + 4) s(n + 5)$$
$$+ 123693938752216808427517479427544585082481703161361003630093\,27 s(n) s(n$$
$$+ 3) s(n + 4) s(n + 5)$$
$$+ 564928898515334497292732133354812693742564937184549391804544\,96 s(n$$
$$+ 1) s(n + 2) s(n + 3) s(n + 5)$$
$$- 292131776090460080721000954686601471591446626809479421369645660 s(n$$
$$+ 1) s(n + 2) s(n + 4) s(n + 5)$$
$$- 156356860162089094213699441979942686062209206693802445823590304 s(n$$
$$+ 1) s(n + 3) s(n + 4) s(n + 5)$$
$$- 242748381734956516765603308423473397064522024386903988931294\,39 s(n$$
$$+ 2) s(n + 3) s(n + 4) s(n + 5)) \Big/$$
$$\Big(905811985229887835153481371204056255128364973369295151590\,60 s(n) s(n$$
$$+ 1) s(n + 3)$$
$$- 574443412510387282026316365730667386342756272020632258366419\,60 s(n) s(n$$
$$+ 2) s(n + 3)$$
$$+ 179411594204368128839719714283603566172939547349043351901449\,68 s(n) s(n$$
$$+ 2) s(n + 4)$$
$$- 330288115246358664053075464340343261236804142439085346785562 s(n) s(n$$
$$+ 2) s(n + 5)$$
$$- 288999687244783209523423419816242958322364260987198826160125\,6 s(n) s(n$$
$$+ 3) s(n + 4)$$
$$+ 431031575781269754218209699945445718150796568432194458608\,28 s(n) s(n$$
$$+ 3) s(n + 5)$$
$$- 130940007748032153799161397213377412098896465716703713378897\,52 s(n$$
$$+ 1) s(n + 2) s(n + 3)$$
$$- 192267336601047214302196428321643630926665032959459241620604\,34 s(n$$
$$+ 1) s(n + 2) s(n + 4)$$
$$+ 154323412392927960547424910418541305383997190474649742126924\,2 s(n$$
$$+ 1) s(n + 2) s(n + 5)$$
$$+ 262735601917128481328733506809437222429317280926963016793816\,7 s(n$$
$$+ 1) s(n + 3) s(n + 4)$$
$$- 544087889317105851340486805790298164366317681731363671001848 s(n$$
$$+ 1) s(n + 3) s(n + 5)$$

$$- 43226284678985371970833773755030127911588590732945675275262 4\, s(n+2)\, s(n+3)\, s(n+4)$$

$$+ 5564310757197865903120253291612412087269736625692520751314095\, s(n)\, s(n+3)^{2}$$

$$+ 3589807180308358992801872302090100694974986098872185246617532\, s(n+1)^{2}\, s(n+3)$$

$$+ 7211961420109568641490962635595198322600758861230339074812692\, s(n+1)\, s(n+2)^{2}$$

$$- 470716947131682691750006730215757915082689471410107409762077\, s(n+1)\, s(n+3)^{2}$$

$$+ 371791271718281061933007614323551068632606281802215488321134\, s(n+1)\, s(n+4)^{2}$$

$$- 3705745882542364589363216987878092828946162057165242588254711\, s(n+2)^{2}\, s(n+3)$$

$$- 3851402587481658403720244573549681690913379041855233806862926\, s(n+2)^{2}\, s(n+4)$$

$$+ 544087889317105851340486805790298164366317681731363671001848\, s(n+2)^{2}\, s(n+5)$$

$$+ 888352064008817700860752156164844026934517921127864992090055\, s(n+2)\, s(n+3)^{2}$$

$$- 6003741964822359322904423898429806292436096649163703283821094 6\, s(n)\, s(n+2)^{2}$$

$$+ 2122624420312141484184260331415431581590022447850166509887166\, s(n+1)^{2}\, s(n+2)$$

$$+ 2047275970799500193992868187116082114437859746794757379196478 6\, s(n)\, s(n+2)\, s(n+1)$$

$$+ 6954101424863829693117481116317591435360779065599374601182458\, s(n+2)^{3}$$

$$+ 604715750715726577753301232267502104832796255272412644314 90\, s(n+3)^{3}\Big)$$

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