

```
> restart
```

```
>
```

Implementation of the algorithm highlighted in the paper:

On Rational Recursion for Holonomic Sequences (Examples 3-4)

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The algorithm is implemented as a command of the NLDE package, available at
<https://github.com/T3gula/D-algebraic-functions>

```
> with(NLDE, HoloToSimpleRatrec)
[ HoloToSimpleRatrec ] (1)
```

```
> RE1:=s(n+1) - (n+1)^3*s(n) = 0
RE1 := s(n+1) - (n+1)^3 s(n) = 0 (2)
```

```
> HoloToSimpleRatrec(RE1,s(n),method=GB)
s(n+3) = (s(n+2) (4 s(n) s(n+1)^2 - 4 s(n) s(n+2)^2 + s(n+1)^3 + s(n+1)^2 s(n+2))) / (s(n+1) (s(n) s(n+1) - s(n) s(n+2) - 2 s(n+1)^2)) (3)
```

```
> RE2:=-n*s(n) + (n+1)*s(n+1) + s(n+3)*n
RE2 := -n s(n) + (n+1) s(n+1) + s(n+3) n (4)
```

```
> HoloToSimpleRatrec(RE2,s(n),method=GB)
s(n+4) = 1 / (s(n) - s(n+3)) (s(n) s(n+1) - 2 s(n) s(n+2) + s(n+1) s(n+2) - s(n+1) s(n+3) + 2 s(n+2) s(n+3)) (5)
```

```
> RE3:=add(randpoly(n,degree=1,coeffs=rand(-1..1))*s(n+j),j=0..4)
RE3 := (-n+1) s(n) + s(n+1) n - s(n+2) n + s(n+3) - s(n+4) (6)
```

```
> HoloToSimpleRatrec(RE3,s(n),method=GB)
s(n+5) = -1 / (s(n) - s(n+1) + s(n+2)) (s(n) s(n+1) - 2 s(n) s(n+2) + 2 s(n) s(n+3) - s(n) s(n+4) + s(n+1) s(n+2) - s(n+2)^2 + s(n+3)^2 - s(n+3) s(n+4)) (7)
```

```
> RE4:=add(randpoly(n,degree=2,coeffs=rand(-1..1))*s(n+j),j=0..4)
RE4 := (n^2 - n - 1) s(n) + s(n+1) n + s(n+2) (n+1) + (-n^2 - 1) s(n+3) - n^2 s(n+4) (8)
```

```
> HoloToSimpleRatrec(RE4,s(n),method=GB)
s(n+6) = (4 s(n+4)^3 + 9 s(n+1) s(n+4) s(n+5) + s(n+2) s(n+4) s(n+5) + 2 s(n (9)
```

$$\begin{aligned}
& + 3) s(n+4) s(n+5) + 4 s(n) s(n+1) s(n+2) + 4 s(n) s(n+1) s(n+3) \\
& + 6 s(n) s(n+1) s(n+4) - 12 s(n) s(n+1) s(n+5) - 5 s(n) s(n+2) s(n+3) \\
& + 8 s(n) s(n+2) s(n+4) + s(n) s(n+2) s(n+5) - 2 s(n) s(n+3) s(n+4) \\
& - 4 s(n) s(n+4) s(n+5) - 5 s(n+1) s(n+2) s(n+3) - 7 s(n+1) s(n+2) s(n \\
& + 4) + 2 s(n+1) s(n+2) s(n+5) - 6 s(n+1) s(n+3) s(n+4) + 12 s(n+1) s(n \\
& + 3) s(n+5) - 6 s(n) s(n+4)^2 + 8 s(n) s(n+5)^2 - 2 s(n+1)^2 s(n+3) - 3 s(n \\
& + 1)^2 s(n+4) + 6 s(n+1)^2 s(n+5) + s(n+1) s(n+2)^2 - 4 s(n+1) s(n+3)^2 - s(n \\
& + 1) s(n+4)^2 - 4 s(n+1) s(n+5)^2 + 4 s(n+2)^2 s(n+3) - s(n+2)^2 s(n+5) \\
& + 5 s(n+2) s(n+3)^2 - 4 s(n+2) s(n+4)^2 - 2 s(n+2) s(n+5)^2 + 2 s(n+3)^2 s(n \\
& + 4) + 6 s(n+3) s(n+4)^2 - 8 s(n+3) s(n+5)^2 + 3 s(n+4)^2 s(n+5) - 6 s(n \\
& + 4) s(n+5)^2 - 2 s(n) s(n+2)^2 - 2 s(n+1)^2 s(n+2) \big) / (10 s(n) s(n+1) \\
& - 5 s(n) s(n+3) - 5 s(n) s(n+5) - 5 s(n+1)^2 - s(n+1) s(n+2) - 9 s(n+1) s(n \\
& + 3) - 6 s(n+1) s(n+4) + 3 s(n+1) s(n+5) + 2 s(n+2) s(n+3) + s(n+2) s(n \\
& + 5) + 5 s(n+3)^2 + 6 s(n+3) s(n+4) + 6 s(n+3) s(n+5) + 4 s(n+4) s(n+5))
\end{aligned}$$

> HoloToSimpleRatrec (RE1, s (n))

$$s(n+4) = \frac{1}{s(n) s(n+1) s(n+2)} (s(n+3) (6 s(n) s(n+1) s(n+2) + 3 s(n) s(n+1) s(n+3) - 3 s(n) s(n+2)^2 + s(n+1)^2 s(n+2))) \quad (10)$$

> HoloToSimpleRatrec (RE2, s (n))

$$s(n+4) = \frac{1}{s(n) - s(n+3)} (s(n) s(n+1) - 2 s(n) s(n+2) + s(n+1) s(n+2) - s(n+1) s(n+3) + 2 s(n+2) s(n+3)) \quad (11)$$

> HoloToSimpleRatrec (RE3, s (n))

$$s(n+5) = -\frac{1}{s(n) - s(n+1) + s(n+2)} (s(n) s(n+1) - 2 s(n) s(n+2) + 2 s(n) s(n+3) - s(n) s(n+4) + s(n+1) s(n+2) - s(n+2)^2 + s(n+3)^2 - s(n+3) s(n+4)) \quad (12)$$

> HoloToSimpleRatrec (RE4, s (n))

$$\begin{aligned}
s(n+6) = & (4 s(n+4)^3 + 9 s(n+1) s(n+4) s(n+5) + s(n+2) s(n+4) s(n+5) \\
& + 2 s(n+3) s(n+4) s(n+5) + 4 s(n) s(n+1) s(n+2) + 4 s(n) s(n+1) s(n+3) \\
& + 6 s(n) s(n+1) s(n+4) - 12 s(n) s(n+1) s(n+5) - 5 s(n) s(n+2) s(n+3) \\
& + 8 s(n) s(n+2) s(n+4) + s(n) s(n+2) s(n+5) - 2 s(n) s(n+3) s(n+4) \\
& - 4 s(n) s(n+4) s(n+5) - 5 s(n+1) s(n+2) s(n+3) - 7 s(n+1) s(n+2) s(n \\
& + 4) + 2 s(n+1) s(n+2) s(n+5) - 6 s(n+1) s(n+3) s(n+4) + 12 s(n+1) s(n \\
& + 3) s(n+5) - 6 s(n) s(n+4)^2 + 8 s(n) s(n+5)^2 - 2 s(n+1)^2 s(n+3) - 3 s(n \\
& + 1)^2 s(n+4) + 6 s(n+1)^2 s(n+5) + s(n+1) s(n+2)^2 - 4 s(n+1) s(n+3)^2
\end{aligned} \quad (13)$$

$$\begin{aligned}
 & -s(n+1)s(n+4)^2 - 4s(n+1)s(n+5)^2 + 4s(n+2)^2s(n+3) - s(n+2)^2s(n+5) \\
 & + 5s(n+2)s(n+3)^2 - 4s(n+2)s(n+4)^2 - 2s(n+2)s(n+5)^2 + 2s(n+3)^2s(n+4) \\
 & + 6s(n+3)s(n+4)^2 - 8s(n+3)s(n+5)^2 + 3s(n+4)^2s(n+5) - 6s(n+4)s(n+5)^2 \\
 & - 2s(n)s(n+2)^2 - 2s(n+1)^2s(n+2)) / (10s(n)s(n+1) - 5s(n)s(n+3) - 5s(n)s(n+5) \\
 & - 5s(n+1)^2 - s(n+1)s(n+2) - 9s(n+1)s(n+3) - 6s(n+1)s(n+4) + 3s(n+1)s(n+5) \\
 & + 2s(n+2)s(n+3) + s(n+2)s(n+5) + 5s(n+3)^2 + 6s(n+3)s(n+4) + 6s(n+3)s(n+5) + 4s(n+4)s(n+5))
 \end{aligned}$$

$\left[\begin{array}{l} > \\ = \\ > \\ = \\ > \end{array} \right]$