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Computations of examples from the paper:

On Rational Recursion for Holonomic Sequences (Examples 1-4)

Bertrand Teguia Tabuguia and James Worrell
Department of Computer Science, University of Oxford

The algorithm is implemented as a command of the **NLDE** package, available at
https://github.com/T3gu1a/D-algebraic-functions

> $with(NLDE, HoloToSimpleRatrec)$

$$[HoloToSimpleRatrec] \tag{1}$$

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Example 1:

Catalan numbers $\left(\frac{1}{n+1}\binom{2n}{n}\right)_n$

$$\left(\frac{1}{n+1}\binom{2n}{n}\right)_n \tag{1.1}$$

> $p := (n+2)\cdot s(n+1) - (4\cdot n+2)\cdot s(n)$

$$p := (n+2)\, s(n+1) - (4\,n+2)\, s(n) \tag{1.2}$$

> $HoloToSimpleRatrec(p, s(n))$

$$s(n+2) = \frac{2\, s(n+1)\, (8\, s(n) + s(n+1))}{10\, s(n) - s(n+1)} \tag{1.3}$$

$((-1)^n + n)_n$

$$((-1)^n + n)_n \tag{1.4}$$

> $p := (-2\,n-3)\, s(n) - 2\, s(n+1) + (2\,n+1)\, s(n+2)$

$$p := (-2\,n-3)\, s(n) - 2\, s(n+1) + (2\,n+1)\, s(n+2) \tag{1.5}$$

> $HoloToSimpleRatrec(p, s(n))$

(1.6)

$$s(n+3) = s(n+1) - s(n) + s(n+2) \tag{1.6}$$

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Example 2

$(n!^2)_n$

$$(n!^2)_n \tag{2.1}$$

> $p := s(n+1) - (n+1)^2 \cdot s(n)$

$$p := s(n+1) - (n+1)^2\, s(n) \tag{2.2}$$

> $HoloToSimpleRatrec(p, s(n))$

$$s(n+3) = \frac{s(n+2)\,\left(2\, s(n)\, s(n+1) + 2\, s(n)\, s(n+2) - s(n+1)^2\right)}{s(n)\, s(n+1)} \tag{2.3}$$

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Example 3

$(n!^3)_n$

$$(n!^3)_n \tag{3.1}$$

> $p1 := s(n+1) - (n+1)^3 \cdot s(n)$

$$p1 := s(n+1) - (n+1)^3\, s(n) \tag{3.2}$$

> $HoloToSimpleRatrec(p1, s(n), method = GB)$

$$s(n+3) = \frac{s(n+2)\,\left(4\, s(n)\, s(n+1)^2 - 4\, s(n)\, s(n+2)^2 + s(n+1)^3 + s(n+1)^2\, s(n+2)\right)}{s(n+1)\,\left(s(n)\, s(n+1) - s(n)\, s(n+2) - 2\, s(n+1)^2\right)} \tag{3.3}$$

$\left(n^2 + \sin\left(\frac{n \cdot \pi}{4}\right)^2\right)_n$

$$\left(n^2 + \sin\left(\frac{n \cdot \pi}{4}\right)^2\right)_n \tag{3.4}$$

> $p2 := (-2 \cdot n\text{^}2 - 8 \cdot n - 11) \cdot s(n) + (2 \cdot n\text{^}2 + 4 \cdot n + 5) \cdot s(n+1) + (-2 \cdot n\text{^}2 - 8 \cdot n - 11) \cdot s(n+2) + (2 \cdot n\text{^}2 + 4 \cdot n + 5) \cdot s(n+3) = 0$

$$p2 := \left(-2\, n^2 - 8\, n - 11\right)\, s(n) + \left(2\, n^2 + 4\, n + 5\right)\, s(n+1) + \left(-2\, n^2 - 8\, n - 11\right)\, s(n \tag{3.5}$$

$+ 2) + (2 n^2 + 4 n + 5) s(n + 3) = 0$

> $HoloToSimpleRatrec(p2, s(n), method = GB)$

$$s(n + 5) = s(n) - 3 s(n + 1) + 4 s(n + 2) - 4 s(n + 3) + 3 s(n + 4) \tag{3.6}$$

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Example 4

$$\left(\binom{2 \cdot n}{n} \cdot \binom{3 \cdot n}{n}\right)_n$$

$$\left(\binom{2 \cdot n}{n} \cdot \binom{3 \cdot n}{n}\right)_n \tag{4.1}$$

> $p3 := s(n + 1) \cdot (n + 1)^{\wedge}2 - 3 \cdot (3 \cdot n + 1) \cdot (3 \cdot n + 2) \cdot s(n)$

$$p3 := s(n + 1) (n + 1)^2 - 3 (3 n + 1) (3 n + 2) s(n) \tag{4.2}$$

> $HoloToSimpleRatrec(p3, s(n), method = LA)$

$$s(n + 3) = \left(3 s(n + 2) \left(26244 s(n) s(n + 1) - 702 s(n) s(n + 2) - 378 s(n + 1)^2 + 13 s(n + 1) s(n + 2)\right)\right) \Big/ \left(5508 s(n) s(n + 1) - 201 s(n) s(n + 2) - 84 s(n + 1)^2 + 4 s(n + 1) s(n + 2)\right) \tag{4.3}$$

$$\left(\frac{n^4}{2^n} + 3^n\right)_n$$

$$\left(\frac{n^4}{2^n} + 3^n\right)_n \tag{4.4}$$

> $p4 := (15 n^4 + 48 n^3 + 36 n^2 - 24 n - 30) s(n) - (7 n^2 + 4 n + 4) (5 n^2 - 4 n - 4) s(n + 1) + (10 n^4 - 8 n^3 - 12 n^2 - 8 n - 2) s(n + 2)$

$$p4 := (15 n^4 + 48 n^3 + 36 n^2 - 24 n - 30) s(n) - (7 n^2 + 4 n + 4) (5 n^2 - 4 n - 4) s(n + 1) + (10 n^4 - 8 n^3 - 12 n^2 - 8 n - 2) s(n + 2) \tag{4.5}$$

> $HoloToSimpleRatrec(p4, s(n), method = LA)$

$$s(n + 6) = -10 s(n + 4) - \frac{3 s(n)}{32} + \frac{31 s(n + 1)}{32} - \frac{65 s(n + 2)}{16} + \frac{35 s(n + 3)}{4} + \frac{11 s(n + 5)}{2} \tag{4.6}$$

Random polynomial (not necessarily the same as in the paper).

> $p5 := s(n) + randpoly(n, degree = 5)$

(4.7)

$$p5 := s(n) + 16\,n^5 + 52\,n^4 - 20\,n^3 - 4\,n^2 - 89\,n - 77 \tag{4.7}$$

> *HoloToSimpleRatrec*(*p5*, *s*(*n*), *method* = *LA*)

$$s(n+5) = -1920 + 5\,s(n+4) - 10\,s(n+3) + 10\,s(n+2) + s(n) - 5\,s(n+1) \tag{4.8}$$

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Other examples

$(n!^4)_n$

$$(n!^4)_n \tag{5.1}$$

> $p := s(n+1) + (n+1)^4 \cdot s(n)$

$$p := s(n+1) + (n+1)^4\,s(n) \tag{5.2}$$

> *HoloToSimpleRatrec*(*p*, *s*(*n*)) *#LA method by default*

$$s(n+5) = -\frac{1}{s(n)\,s(n+1)\,s(n+2)\,s(n+3)}\big(s(n+4)\,\big(24\,s(n)\,s(n+1)\,s(n+2)\,s(n+3) - 4\,s(n)\,s(n+1)\,s(n+2)\,s(n+4) + 6\,s(n)\,s(n+3)^2\,s(n+1) - 4\,s(n)\,s(n+2)^2 s(n+3) + s(n+1)^2\,s(n+2)\,s(n+3)\big)\big) \tag{5.3}$$

> *HoloToSimpleRatrec*(*p*, *s*(*n*), *method* = *GB*)

$$s(n+3) = -\big(s(n+2)\,\big(5\,s(n)\,s(n+1)^2 - 22\,s(n)\,s(n+2)\,s(n+1) + 5\,s(n)\,s(n+2)^2 + s(n+1)^3 - s(n+1)^2\,s(n+2)\big)\big) \Big/ \big(s(n+1)\,\big(s(n)\,s(n+1) - s(n)\,s(n+2) - 3\,s(n+1)^2\big)\big) \tag{5.4}$$

We use the **HolonomicRE** command of **HyperTypeSeq** to find recurrence equations from general terms. The package is available at
https://github.com/T3gu1a/HyperTypeSeq

$(n!^2 + n!)_n$

$$(n!^2 + n!)_n \tag{5.5}$$

> $p :=$ *HyperTypeSeq:-HolonomicRE*$\big(n!^2 + n!, s(n)\big)$

$$p := (n+2)\,(n+1)^3\,s(n) - (n+2)\,\big(n^2 + 3\,n + 1\big)\,s(n+1) + s(n+2)\,n = 0 \tag{5.6}$$

> *HoloToSimpleRatrec*(*p*, *s*(*n*)) *#LA method by default*

$$s(n+6) = -\big(-153\,s(n+1)^2\,s(n+4)^3 + 92\,s(n+2)^3\,s(n+4)^2 - 27\,s(n+2)^3\,s(n+5)^2 - 12\,s(n+2)^2\,s(n+4)^3 - 3\,s(n+1)\,s(n+2)\,s(n+4)^2\,s(n+5) - 24\,s(n+1)\,s(n+3)^2\,s(n+4)\,s(n+5) + 30\,s(n+2)^2\,s(n+3)\,s(n+4)\,s(n+5) - 8448\,s(n)\,s(n+1)\,s(n+2)\,s(n+4)^2 - 440\,s(n)\,s(n+1)\,s(n+2)\,s(n+5)^2 + 13032\,s(n)\,s(n+1)\,s(n+3)^2\,s(n+4) - 6828\,s(n)\,s(n+1)\,s(n+3)^2\,s(n+5) \tag{5.7}$$

$$- 3612 s(n) s(n+1) s(n+3) s(n+4)^{2} + 344 s(n) s(n+1) s(n+3) s(n+5)^{2}$$
$$- 547 s(n) s(n+1) s(n+4)^{2} s(n+5) - 47 s(n) s(n+1) s(n+4) s(n+5)^{2}$$
$$+ 10272 s(n) s(n+2)^{2} s(n+3) s(n+4) - 7578 s(n) s(n+2)^{2} s(n+3) s(n+5)$$
$$+ 3164 s(n) s(n+2)^{2} s(n+4) s(n+5) + 15468 s(n) s(n+2) s(n+3)^{2} s(n+4)$$
$$- 9666 s(n) s(n+2) s(n+3)^{2} s(n+5) - 276 s(n) s(n+2) s(n+3) s(n+4)^{2}$$
$$+ 110 s(n) s(n+2) s(n+3) s(n+5)^{2} + 98 s(n) s(n+2) s(n+4)^{2} s(n+5)$$
$$+ 9 s(n) s(n+2) s(n+4) s(n+5)^{2} - 211 s(n) s(n+3)^{2} s(n+4) s(n+5)$$
$$+ 3 s(n) s(n+3) s(n+4)^{2} s(n+5) + 5172 s(n+1)^{2} s(n+2) s(n+3) s(n+4)$$
$$- 3832 s(n+1)^{2} s(n+2) s(n+3) s(n+5) + 2220 s(n+1)^{2} s(n+2) s(n+4) s(n$$
$$+ 5) + 736 s(n+1)^{2} s(n+3) s(n+4) s(n+5) - 2402 s(n+1) s(n+2)^{2} s(n$$
$$+ 3) s(n+4) + 1436 s(n+1) s(n+2)^{2} s(n+3) s(n+5) + 185 s(n+1) s(n$$
$$+ 2)^{2} s(n+4) s(n+5) + 1074 s(n+1) s(n+2) s(n+3)^{2} s(n+4) - 812 s(n$$
$$+ 1) s(n+2) s(n+3)^{2} s(n+5) - 856 s(n+1) s(n+2) s(n+3) s(n+4)^{2}$$
$$+ 42 s(n+1) s(n+2) s(n+3) s(n+5)^{2} - 1312 s(n+1)^{2} s(n+3)^{2} s(n+5)$$
$$- 1206 s(n+1)^{2} s(n+3) s(n+4)^{2} + 96 s(n+1)^{2} s(n+3) s(n+5)^{2}$$
$$- 141 s(n+1)^{2} s(n+4)^{2} s(n+5) - 12 s(n+1)^{2} s(n+4) s(n+5)^{2} - 366 s(n$$
$$+ 1) s(n+2)^{2} s(n+4)^{2} - 141 s(n+1) s(n+2)^{2} s(n+5)^{2} - 120 s(n+1) s(n$$
$$+ 2) s(n+4)^{3} + 704 s(n+1) s(n+3)^{3} s(n+4) - 384 s(n+1) s(n+3)^{3} s(n+5)$$
$$+ 163 s(n+1) s(n+3)^{2} s(n+4)^{2} + 9 s(n+1) s(n+3) s(n+4)^{3} - 1010 s(n$$
$$+ 2)^{3} s(n+3) s(n+4) + 699 s(n+2)^{3} s(n+3) s(n+5) - 120 s(n+2)^{3} s(n$$
$$+ 4) s(n+5) - 337 s(n+2)^{2} s(n+3)^{2} s(n+4) + 193 s(n+2)^{2} s(n+3)^{2} s(n$$
$$+ 5) - 106 s(n+2)^{2} s(n+3) s(n+4)^{2} + 13 s(n+2) s(n+3)^{3} s(n+4) - 6 s(n$$
$$+ 2) s(n+3)^{3} s(n+5) + 3 s(n+2) s(n+3)^{2} s(n+4)^{2} - 540 s(n) s(n$$
$$+ 1) s(n+4)^{3} - 2848 s(n) s(n+2)^{2} s(n+4)^{2} - 78 s(n) s(n+2)^{2} s(n+5)^{2}$$
$$- 296 s(n) s(n+2) s(n+4)^{3} + 4080 s(n) s(n+3)^{3} s(n+4) - 2208 s(n) s(n$$
$$+ 3)^{3} s(n+5) + 1108 s(n) s(n+3)^{2} s(n+4)^{2} - 12 s(n) s(n+3)^{2} s(n+5)^{2}$$
$$+ 68 s(n) s(n+3) s(n+4)^{3} - 1998 s(n+1)^{2} s(n+2) s(n+4)^{2} - 160 s(n$$
$$+ 1)^{2} s(n+2) s(n+5)^{2} + 2532 s(n+1)^{2} s(n+3)^{2} s(n+4) + 2554 s(n) s(n$$
$$+ 1) s(n+3) s(n+4) s(n+5) + 1579 s(n) s(n+2) s(n+3) s(n+4) s(n+5)$$
$$+ 529 s(n+1) s(n+2) s(n+3) s(n+4) s(n+5) + 27648 s(n) s(n+1) s(n$$
$$+ 2) s(n+3) s(n+4) - 20028 s(n) s(n+1) s(n+2) s(n+3) s(n+5)$$
$$+ 9658 s(n) s(n+1) s(n+2) s(n+4) s(n+5)) \Big/ (276 s(n) s(n+3)^{2} s(n+1)$$
$$+ 558 s(n) s(n+2)^{2} s(n+3) + 280 s(n+1)^{2} s(n+2) s(n+3) + 1440 s(n) s(n$$
$$+ 1) s(n+2) s(n+3) - 826 s(n) s(n+1) s(n+2) s(n+4) + 99 s(n) s(n+3)^{3}$$

$$
\begin{aligned}
&+ 52\, s(n+1)^2 s(n+3)^2 + 18\, s(n+1)^2 s(n+4)^2 + 18\, s(n+1)\, s(n+3)^3 \\
&- 52\, s(n+2)^3 s(n+3) + 24\, s(n+2)^3 s(n+4) - 12\, s(n+2)^2 s(n+3)^2 \\
&+ 48\, s(n)\, s(n+1)\, s(n+2)\, s(n+5) - 127\, s(n)\, s(n+1)\, s(n+3)\, s(n+4) \\
&- 23\, s(n)\, s(n+1)\, s(n+3)\, s(n+5) - 174\, s(n)\, s(n+2)\, s(n+3)\, s(n+4) \\
&+ 3\, s(n)\, s(n+2)\, s(n+3)\, s(n+5) - 45\, s(n+1)\, s(n+2)\, s(n+3)\, s(n+4) \\
&+ 70\, s(n)\, s(n+1)\, s(n+4)^2 - 276\, s(n)\, s(n+2)^2 s(n+4) + 10\, s(n)\, s(n+2)^2 s(n \\
&+ 5) + 561\, s(n)\, s(n+2)\, s(n+3)^2 - 12\, s(n)\, s(n+2)\, s(n+4)^2 + 9\, s(n)\, s(n \\
&+ 3)^2 s(n+4) - 171\, s(n+1)^2 s(n+2)\, s(n+4) + 13\, s(n+1)^2 s(n+2)\, s(n+5) \\
&- 40\, s(n+1)^2 s(n+3)\, s(n+4) - 6\, s(n+1)^2 s(n+3)\, s(n+5) - 87\, s(n \\
&+ 1)\, s(n+2)^2 s(n+3) + 38\, s(n+1)\, s(n+2)^2 s(n+4) + 3\, s(n+1)\, s(n \\
&+ 2)^2 s(n+5) + 59\, s(n+1)\, s(n+2)\, s(n+3)^2 \big)
\end{aligned}
$$

> $HoloToSimpleRatrec(p, s(n), method = GB)$

$$
\begin{aligned}
s(n+6) = \big(\qquad & \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \textbf{(5.8)} \\
&- 6486923191185797164733082294296052743518364194845119601265638\, s(n)\, s(n \\
&+ 2)\, s(n+5)^2 \\
&- 5371655126481772723594613911139129183271653304186539673945402 11\, s(n) \\
&s(n+3)^2 s(n+5) \\
&- 7295634143611993341981373358322395024209270096246746458560 2039\, s(n)\, s(n \\
&+ 3)\, s(n+4)^2 \\
&- 1872803427745091177511002344703782531219307276062803193698940\, s(n)\, s(n \\
&+ 3)\, s(n+5)^2 \\
&- 2532463202548028869037239095949960263404740233466013260753787 86\, s(n \\
&+ 1)\, s(n+2)\, s(n+4)^2 \\
&+ 2295693667612789193162161777163233282947879081391510639410 3354\, s(n \\
&+ 1)\, s(n+2)\, s(n+5)^2 \\
&+ 1001151696073414761645751550952304509206205829255284269564958903\, s(n \\
&+ 1)\, s(n+3)^2 s(n+4) \\
&- 798121984167012144362005682869059969431285388158104876163905 87\, s(n \\
&+ 1)\, s(n+3)^2 s(n+5) \\
&+ 1604905089000785496532422574671915224675101053803948444127129 79\, s(n \\
&+ 1)\, s(n+3)\, s(n+4)^2 \\
&+ 2672719380676225405192175016210269811099860479044868793885259\, s(n \\
&+ 1)\, s(n+3)\, s(n+5)^2 \\
&- 6551587528249005871918618022553087211999889314939464566888740\, s(n
\end{aligned}
$$

$+ 1) s(n+4)^2 s(n+5)$

$- 1722966175988247894074791914667470957337113999291481826807 14\, s(n$

$+ 1) s(n+4) s(n+5)^2$

$- 2744416393205053267198045356206720834609878561935051798644 86197\, s(n$

$+ 2)^2 s(n+3) s(n+4)$

$- 5080176810114386031796740479334491209590750068370186127616 2508\, s(n$

$+ 2)^2 s(n+3) s(n+5)$

$- 4893981748746851019554419041119277211582288320878005754800 8220\, s(n$

$+ 2)^2 s(n+4) s(n+5)$

$- 4051646155054460759792323911544017088344527148036008960328 1131\, s(n$

$+ 2) s(n+3)^2 s(n+4)$

$- 2993987603930335206372104207407379320683303593912923044324 5387\, s(n$

$+ 2) s(n+3)^2 s(n+5)$

$- 3334960180440487566787964778280970329031547760734447577545 5977\, s(n$

$+ 2) s(n+3) s(n+4)^2$

$+ 1525865706435833923120739950953207415171771936062475602698 580\, s(n$

$+ 2) s(n+3) s(n+5)^2$

$- 2259618630845210246222215101442458313940000986407009274951 804\, s(n$

$+ 2) s(n+4)^2 s(n+5)$

$+ 9421463094249262848698046069890836234488584206181586964203 68\, s(n$

$+ 3)^2 s(n+4) s(n+5)$

$- 5726120789594482636509044817940576909199811729955810958601 189\, s(n+3)^4$

$+ 6457645478483280727311719131798845684211981825135772380116 26772\, s(n) s(n$

$+ 2) s(n+4) s(n+5)$

$+ 1236939387522168084275174794275445850824817031613610036300 9327\, s(n) s(n$

$+ 3) s(n+4) s(n+5)$

$+ 5649288985153344972927321333548126937425649371845493918045 4496\, s(n$

$+ 1) s(n+2) s(n+3) s(n+5)$

$- 2921317760904600807210009546866014715914466268094794213696 45660\, s(n$

$+ 1) s(n+2) s(n+4) s(n+5)$

$- 1563568601620890942136994419799426860622092066938024458235 90304\, s(n$

$+ 1) s(n+3) s(n+4) s(n+5)$

$- 2427483817349565167656033084234733970645220243869039889312 9439\, s(n$

$+ 2) s(n+3) s(n+4) s(n+5)$

$$+ 239654942877716238788972189971614499662682738485561784383124734\, s(n+1)\, s(n+4)^{3}$$

$$- 189226826554681793774412491348477536494660602806709351557246 98\, s(n+2)^{3}\, s(n+5)$$

$$+ 192060129389455430774761688237697971910873234988581525164879354\, s(n+2)^{2}\, s(n+4)^{2}$$

$$+ 87360438881904942215836584603010613998571284978175198398335 48\, s(n+2)^{2}\, s(n+5)^{2}$$

$$- 195476943544383683336093496615204545873519292321354053628153 04\, s(n+2)\, s(n+3)^{3}$$

$$+ 343570987010870403128772184072547295458621873923107274070298 60\, s(n+2)\, s(n+4)^{3}$$

$$- 113781789748543220527282957332300580886721525357171307388956 52\, s(n+3)^{3}\, s(n+4)$$

$$- 177479890369760078889290297349259569613803486664344180358520 1\, s(n+3)^{3}\, s(n+5)$$

$$- 119139945873056509109572873204835459562596864987678325568499 21\, s(n+3)^{2}\, s(n+4)^{2}$$

$$- 360967674167251723608502650330856289469179703444768414864 30\, s(n+3)\, s(n+4)^{3}$$

$$+ 516511079322842336474846949252805025289711143785884095554463028\, s(n)\, s(n+3)^{2} s(n+1)$$

$$+ 867876880427927218605155575086528734455350736817081040450094548\, s(n)\, s(n+2)^{2} s(n+3)$$

$$+ 733833873107006409711143789123498859937663619846445267170569716\, s(n+1)^{2}\, s(n+2)\, s(n+3)$$

$$+ 4458558513378319540071681509494637556095451826288498885347658764\, s(n)\, s(n+1)\, s(n+2)\, s(n+3)$$

$$- 4815881813716745491628081637061002933351754664392785643541394428\, s(n)\, s(n+1)\, s(n+2)\, s(n+4)$$

$$+ 605119661742264182865491644304991885154267560301404428783804076\, s(n)\, s(n+3)^{3}$$

$$- 127216760763652142314638377895688607708397514015010689846615 56\, s(n+1)^{2}\, s(n+3)^{2}$$

$+ 16796830553946854959777847540315092966506241198528042724343046 0\, s(n$

$+ 1)\, s(n+3)^{3}$

$- 11189663193728362661132850827520022423653149180878623956406964 8\, s(n$

$+ 2)^{3}\, s(n+3)$

$+ 4323821458730993957607066152446773532533636804750324462312867 6\, s(n$

$+ 2)^{3}\, s(n+4)$

$- 4609333288484206566189696976931418149922117679496979268051369 1\, s(n$

$+ 2)^{2}\, s(n+3)^{2}$

$+ 70627778950870878315606632861865227266173113985936251270122161 4\, s(n)\, s(n$

$+ 1)\, s(n+2)\, s(n+5)$

$+ 58326292113183081323026165689965073354347804257475339806495983 2\, s(n)\, s(n$

$+ 1)\, s(n+3)\, s(n+4)$

$- 4232972713785890688743829165631706254360035840461091446426833 0\, s(n)\, s(n$

$+ 1)\, s(n+3)\, s(n+5)$

$+ 370353880032623324156175936424086832921026159788656042409121821 2\, s(n)$

$s(n+2)\, s(n+3)\, s(n+4)$

$- 143031191601255727013540645427966627508650894533922212667833689 4\, s(n)$

$s(n+2)\, s(n+3)\, s(n+5)$

$- 19169916608030780599625129403194175018575119996248838495023835 0\, s(n$

$+ 1)\, s(n+2)\, s(n+3)\, s(n+4)$

$+ 31104781287141708395974162709425222762940959548060242636713343 6\, s(n)$

$s(n+2)^{2} s(n+4)$

$- 57193675064755223237112100908550661005240717400212468144393334 2\, s(n)$

$s(n+2)^{2} s(n+5)$

$+ 244529019798684942887978871104051994440841997357576667982410788 8\, s(n)$

$s(n+2)\, s(n+3)^{2}$

$- 386092914685118327312965015573515666564240705759593847551047902 4\, s(n)$

$s(n+2)\, s(n+4)^{2}$

$+ 471149614251760285672261484503186474621007455040236898815278688 1\, s(n)$

$s(n+3)^{2}\, s(n+4)$

$- 76802399261281604345633000740978242661972622095404238680618430 0\, s(n$

$+ 1)^{2}\, s(n+2)\, s(n+4)$

$+ 8173766554078009151990591754013646816085685894777111087602621 8\, s(n$

$+ 1)^{2}\, s(n+2)\, s(n+5)$

$$+ 21637551229875277014540948613791775074363173049124971916658800\, s(n+1)^2\, s(n+3)\, s(n+4)$$

$$+ 69722175150908633288688307370227544932720213133266734748849714\, s(n+1)^2\, s(n+3)\, s(n+5)$$

$$- 561899436494397552490805347583620738666273447576469729397184812\, s(n+1)\, s(n+2)^2\, s(n+3)$$

$$+ 305807090384469877602793588645979036583073564046820296634168664\, s(n+1)\, s(n+2)^2\, s(n+4)$$

$$- 437408397175169890993776212916466745382268350046902169811948\, s(n+1)\, s(n+2)^2\, s(n+5)$$

$$+ 373146630636088288164585924985233368209352816386359147506439200\, s(n+1)\, s(n+2)\, s(n+3)^2\Big) \Big/$$

$$\Big(90581198522988783515348137120405625512836497336929515159060\, s(n)\, s(n+1)\, s(n+3)$$

$$- 57444341251038728202631636573066738634275627202063225836641960\, s(n)\, s(n+2)\, s(n+3)$$

$$+ 17941159420436812883971971428360356617293954734904335190144968\, s(n)\, s(n+2)\, s(n+4)$$

$$- 330288115246358664053075464340343261236804142439085346785562\, s(n)\, s(n+2)\, s(n+5)$$

$$- 2889996872447832095234234198162429583223642609871988261601256\, s(n)\, s(n+3)\, s(n+4)$$

$$+ 43103157578126975421820969994544571815079656843219445860828\, s(n)\, s(n+3)\, s(n+5)$$

$$- 13094000774803215379916139721337741209889646571670371337889752\, s(n+1)\, s(n+2)\, s(n+3)$$

$$- 19226733660104721430219642832164363092666503295945924162060434\, s(n+1)\, s(n+2)\, s(n+4)$$

$$+ 1543234123929279605474249104185413053839971904746497421269242\, s(n+1)\, s(n+2)\, s(n+5)$$

$$+ 2627356019171284813287335068094372224293172809269630167938167\, s(n+1)\, s(n+3)\, s(n+4)$$

$$- 544087889317105851340486805790298164366317681731363671001848\, s(n+1)\, s(n+3)\, s(n+5)$$

$- 432262846789853719708337737550301279115885907329456752752624\, s(n$

$+ 2)\, s(n + 3)\, s(n + 4)$

$+ 6954101424863829693117481116317591435360779065599374601182458\, s(n + 2)^{3}$

$+ 60471575071572657775330123226750210483279625527241264431490\, s(n + 3)^{3}$

$- 60037419648223593229044238984298062924360966491637032838210946\, s(n)$

$s(n + 2)^{2}$

$+ 2122624420312141484184260331415431581590022447850166509887166\, s(n$

$+ 1)^{2}\, s(n + 2)$

$+ 5564310757197865903120253291612412087269736625692520751314095\, s(n)$

$s(n + 3)^{2}$

$+ 3589807180308358992801872302090100694974986098872185246617532\, s(n$

$+ 1)^{2}\, s(n + 3)$

$+ 7211961420109568641490962635595198322600758861230339074812692\, s(n$

$+ 1)\, s(n + 2)^{2}$

$- 470716947131682691750006730215757915082689471410107409762077\, s(n$

$+ 1)\, s(n + 3)^{2}$

$+ 371791271718281061933007614323551068632606281802215488321134\, s(n$

$+ 1)\, s(n + 4)^{2}$

$- 3705745882542364589363216987878092828946162057165242588254711\, s(n$

$+ 2)^{2}\, s(n + 3)$

$- 3851402587481658403720244573549681690913379041855233806862926\, s(n$

$+ 2)^{2}\, s(n + 4)$

$+ 544087889317105851340486805790298164366317681731363671001848\, s(n$

$+ 2)^{2}\, s(n + 5)$

$+ 888352064008817700860752156164844026934517921127864992090055\, s(n$

$+ 2)\, s(n + 3)^{2}$

$+ 20472759707995001939928681871160821144378597467947573791964786\, s(n)\, s(n$

$+ 2)\, s(n + 1))$