```
> restart
>
```

Implementation of the algorithm highlighted in the paper:

On Rational Recursion for Holonomic Sequences (Example 3)

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The algorithm is implemented as a command of the **NLDE** package, available at https://github.com/T3gu1a/D-algebraic-functions

```
> with(NLDE, HoloToSimpleRatrec)
```

$$[HoloToSimpleRatrec] \tag{1}$$

```
> RE:=s(n + 1) - (n + 1)^3*s(n) = 0
```

$$RE := s(n+1) - (n+1)^3 s(n) = 0 \tag{2}$$

```
> HoloToSimpleRatrec(RE,s(n))
```

$$s(n+3) = \frac{s(n+2)\left(4\,s(n)\,s(n+1)^2 - 4\,s(n)\,s(n+2)^2 + s(n+1)^3 + s(n+1)^2\,s(n+2)\right)}{s(n+1)\left(s(n)\,s(n+1) - s(n)\,s(n+2) - 2\,s(n+1)^2\right)} \tag{3}$$

```
> RE:=-n*s(n) + (n + 1)*s(n + 1) + s(n + 3)*n
```

$$RE := -n\,s(n) + (n+1)\,s(n+1) + s(n+3)\,n \tag{4}$$

```
> HoloToSimpleRatrec(RE,s(n))
```

$$s(n+4) = \frac{1}{s(n) - s(n+3)}\left(s(n)\,s(n+1) - 2\,s(n)\,s(n+2) + s(n+1)\,s(n+2) - s(n+1)\,s(n+3) + 2\,s(n+2)\,s(n+3)\right) \tag{5}$$

```
> RE:=add(randpoly(n,degree=1,coeffs=rand(-1..1))*s(n+j),j=0..4)
```

$$RE := -s(n) + (-n+1)\,s(n+1) - s(n+2)\,n + (-n-1)\,s(n+3) \tag{6}$$

```
> HoloToSimpleRatrec(RE,s(n))
```

$$s(n+4) = -\frac{1}{s(n) - 3\,s(n+1) - 2\,s(n+2) - s(n+3)}\left(s(n)\,s(n+2) + s(n)\,s(n+3) - s(n+1)^2 - 2\,s(n+1)\,s(n+2) - 3\,s(n+1)\,s(n+3)\right) \tag{7}$$

```
> RE:=add(randpoly(n,degree=2,coeffs=rand(-1..1))*s(n+j),j=0..4)
```

$$RE := \left(-n^2 + n + 1\right) s(n) + \left(-n^2 - n - 1\right) s(n+1) + (-n-1)\,s(n+2) + \left(n^2 - n - 1\right) s(n+3) + \left(-n^2 - 1\right) s(n+4) \tag{8}$$

```
> HoloToSimpleRatrec(RE,s(n))
```

$$
\begin{aligned}
s(n+6) = -\big(&4\,s(n)\,s(n+1)\,s(n+2) + 16\,s(n)\,s(n+1)\,s(n+3) + 2\,s(n)\,s(n+2)\,s(n+3) + 4\,s(n)\,s(n+2)\,s(n+4) + 2\,s(n)\,s(n+2)\,s(n+5) - 15\,s(n)\,s(n+3)\,s(n+4) \\
&+ 12\,s(n)\,s(n+3)\,s(n+5) - 8\,s(n+1)\,s(n+2)\,s(n+3) + 4\,s(n+1)\,s(n+2)\,s(n+4) + 2\,s(n+1)\,s(n+2)\,s(n+5) + 3\,s(n+1)\,s(n+3)\,s(n+4) + 10\,s(n+1)\,s(n+3)\,s(n+5) \\
&+ 2\,s(n+2)\,s(n+3)\,s(n+4) - 8\,s(n)\,s(n+2)^2 + 4\,s(n+1)^2\,s(n+2) + 2\,s(n+2)^3 + 4\,s(n+3)^3 - 4\,s(n)\,s(n+1)\,s(n+5) + 10\,s(n)\,s(n+4)\,s(n+5) \\
&- 5\,s(n+2)\,s(n+3)\,s(n+5) + s(n+2)\,s(n+4)\,s(n+5) + 3\,s(n+3)\,s(n+4)\,s(n+5) + 6\,s(n)\,s(n+1)\,s(n+4) - 4\,s(n)\,s(n+3)^2 + 8\,s(n+1)^2\,s(n+3) \\
&- 4\,s(n+1)\,s(n+2)^2 - 18\,s(n+1)\,s(n+3)^2 + 3\,s(n+1)\,s(n+4)^2 + 7\,s(n+2)^2\,s(n+3) - 8\,s(n+2)^2\,s(n+4) - s(n+2)\,s(n+3)^2 + 3\,s(n+2)\,s(n+4)^2 - 6\,s(n)\,s(n+4)^2 \\
&+ 2\,s(n+1)^2\,s(n+4) - 9\,s(n+3)\,s(n+4)^2 + 11\,s(n+3)^2\,s(n+4) - 4\,s(n+1)^2\,s(n+5) - 12\,s(n+3)^2\,s(n+5) + 11\,s(n+4)^2\,s(n+5) - 5\,s(n+4)^3 + 6\,s(n)\,s(n+5)^2 \\
&+ 2\,s(n+1)\,s(n+5)^2 - 2\,s(n+2)\,s(n+5)^2 - 6\,s(n+3)\,s(n+5)^2 + 3\,s(n+4)\,s(n+5)^2\big) \Big/ \big(12\,s(n)\,s(n+1) + 4\,s(n)\,s(n+2) - 4\,s(n)\,s(n+3) - 12\,s(n)\,s(n+4) \\
&+ 3\,s(n)\,s(n+5) + 6\,s(n+1)^2 - s(n+1)\,s(n+2) - 14\,s(n+1)\,s(n+3) + 6\,s(n+1)\,s(n+4) + s(n+1)\,s(n+5) - s(n+2)^2 - 3\,s(n+2)\,s(n+3) + 7\,s(n+2)\,s(n+4) \\
&- s(n+2)\,s(n+5) + 4\,s(n+3)^2 + 8\,s(n+3)\,s(n+4) - 3\,s(n+3)\,s(n+5) - 12\,s(n+4)^2 + 4\,s(n+4)\,s(n+5)\big)
\end{aligned}
\tag{9}
$$