



Scientific and Technical Computing

Hardware and Code Optimization

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UT Austin, 10/1/19 & ...

Our Computer: CPU, Cache, Memory, 'Connection'

CPU

1. Pipelined operation

System designed to get 1 opc

Memory

1. Data streams

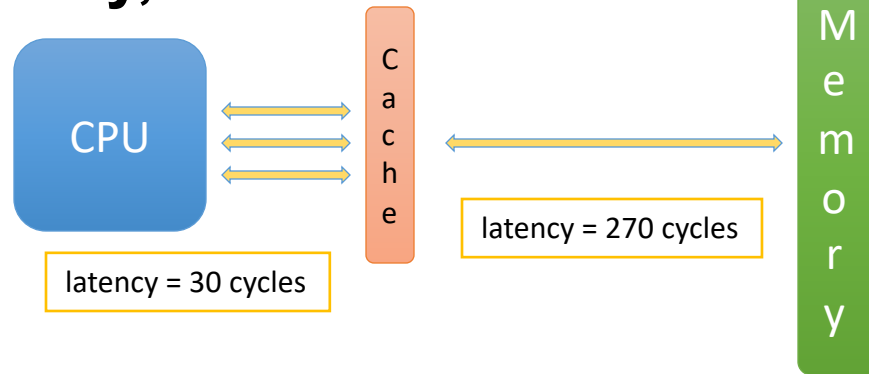
System designed to support 1 wpc (for one row)

Caches

1. Managed by run-time
2. Cache size (for stencil update)

System designed for 'enough' bandwidth to support 2 rows

Size: at least $3 \times n$ words



Our computer has been somewhat 'hypothetical' so far
We have designed the specs so that we get 'optimal'
performance for a stencil update

Concurrency!

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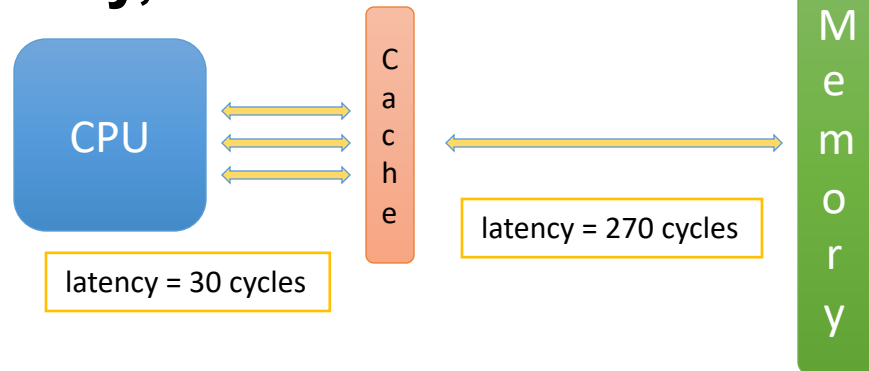
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Requirement: Size of the cache = $3 \times n$

n could be any number, any large number

Size of cache in hardware certainly not adjustable
Also differences between chip generations

Outline

CPU & Memory, latency, bandwidth, wpc,
opc ...

Data streaming, pipelining, caches (part 1)

Caches: software (short)

Caches (working principles)

There are at least 4 ‘working principles’
that we have to cover

My ‘big’ plan

Cover many hardware fundamentals as they guide code design

- loosely in decreasing order of importance

For each hardware feature:

Add details as necessary to describe a simplified, yet functional
‘working model’

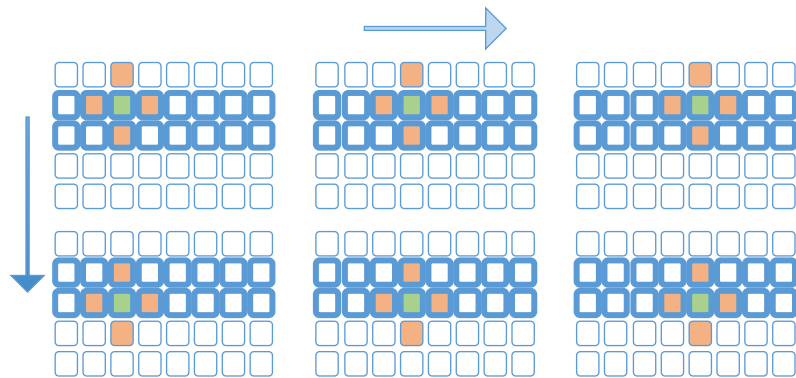
Discussion: Working around the size limitation

Example:

$n=500$, cache size=300 words

At what iteration ' i ' do we (approximately) start to replace data in the cache?

- ' i ' is inner loop



```
do j=1, n
  do i=1, n
    y(i,j) = 0.25 * (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))
  enddo
enddo
```

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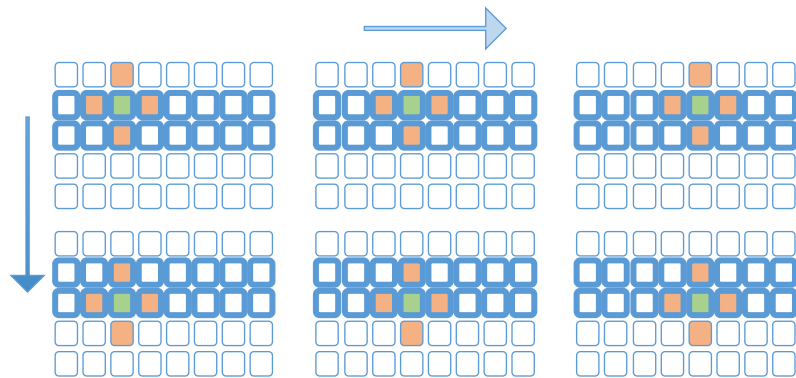
Example:

$n=500$, cache size=300 words

At what iteration ' i ' do we (approximately) start to replace data in the cache? $i \sim 100$

So what do we do when we reach ' $i=100$ '

- Hint: going further to the right is a 'dead end'



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do j=1, n
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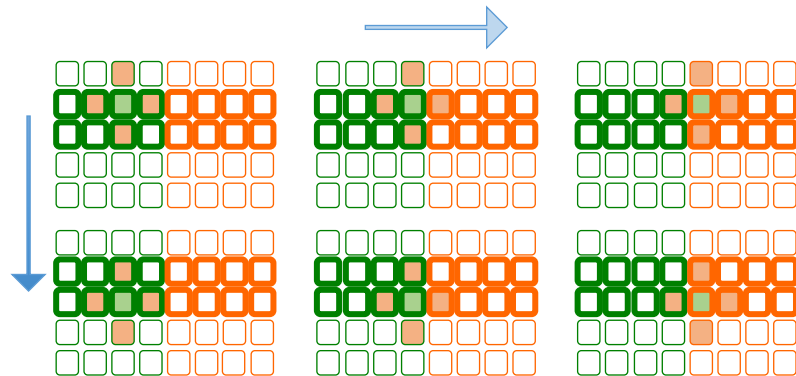
$n=500$, cache size=300 words

At what iteration 'i' do we (approximately) start to replace data in the cache? $i \sim 100$

So what do we do when we reach 'i=100'

- Hint: going further to the right is a 'dead end'
- So we go one row down
- The green area first, then the orange area

So how do we do this in code?



```
do j=1, n
  do i=1, n
    y(i,j) = 0.25 * (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))
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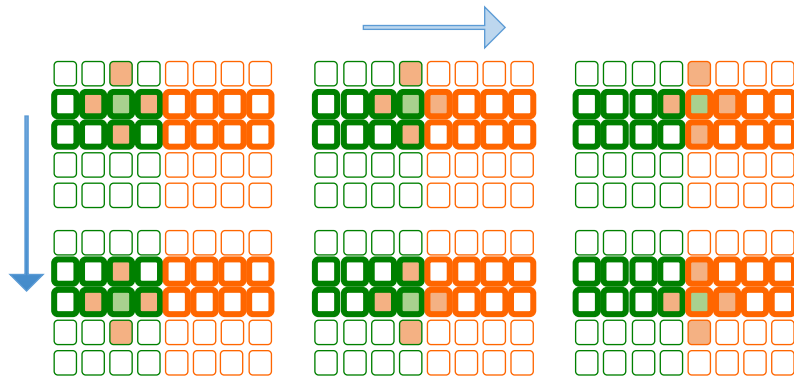
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n=500, cache size=300 words

So how do we do this in code?

- What is the width of a strip?
- How many strips?
- How many loops in the code?



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do j=1, n
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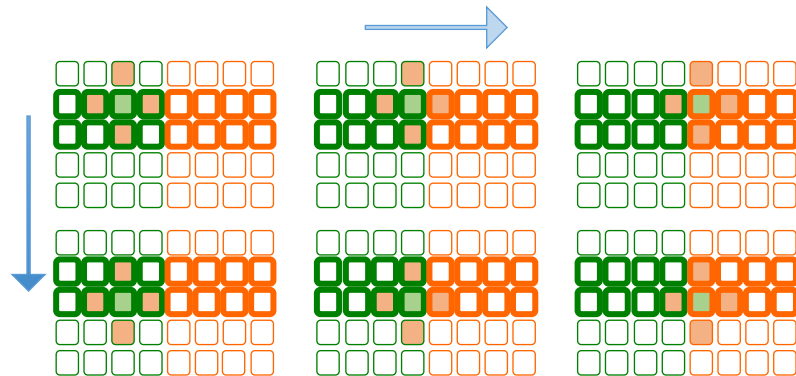

Discussion: Working around the size limitation

Example:

$n=500$, cache size=300 words

So how do we do this in code?

- What is the width of a strip? 100
- How many strips? 5
- How many loops in the code? 3 (up from 2)



```
n = 500; ns = 100
do iout=1, ...
  do j=1, n
    is = ...
    ie = ...
    do i=is, ie
      y(i,j) = 0.25 * (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))
    enddo
  enddo
enddo
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Discussion: Working around the size limitation

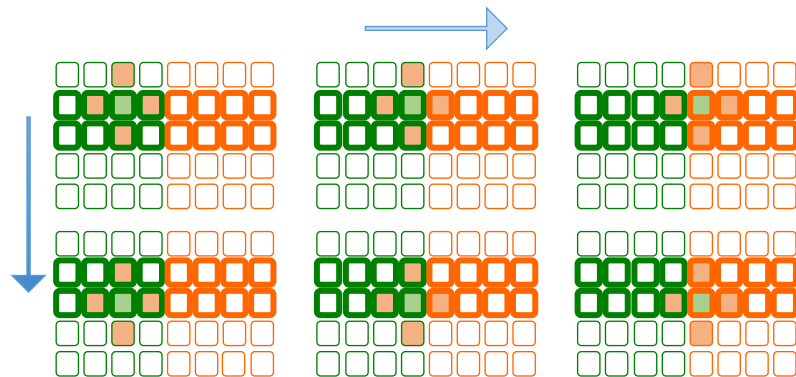
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  do j=1, n
    is = ...
    ie = ...
    do i=is, ie
      y(i,j) = 0.25 *
    enddo
  enddo
enddo
```



We go left to right first (**x**-direction)

We go in **y**-direction second

Hence left to right is the inner loop. Loop index is '**i**'

The 'fast' loop, i.e. the inner loop 'exceeds' to size of the cache

We split up the inner loop in 2 loops. Indexes '**i**' and '**iout**'

The loop '**j**' that re-uses the data is in the middle

is and **ie** are set so that the pair of loops (**i** and **iout**) covers the whole computational domain, left to right, exactly once

Cache blocking

Re-use data before it is evicted

Breaking a loop into 2 (or more) parts

(There can be cache blocking for multiple loops)

Note:

In our example we have been overly optimistic

Width of the strip stretched to the max

Real application: other data is also stored in cache
(there are also other processes)

Let's fill in the blanks

```
n = 500; ns = 100
do iout=1, ...
  do j=1, n
    is = ...
    ie = ...
    do i=is, ie
      y(i,j) = 0.25 * (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))
    enddo
  enddo
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```

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Be aware that for arbitrary pairs (n,ns) the code will be more complicated

Consider:

N=495; ns=100

```
n = 495; ns = 100
do iout=1, ...
  do j=1, n
    is = ...
    ie = ...
    do i=is, ie
      y(i,j) = 0.25 * (x(i-1,j) + x(i+1,j) + x(i,j-1) + x(i,j+1))
    enddo
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Cache blocking

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Breaking a loop into 2 (or more) parts

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1. Be aware that for arbitrary pairs (n, ns) the code will be more complicated

2. Cache size=300 → $ns=100$
 ns is way(!) too large (by 2×)
Why?

Note:

In our example we have been overly optimistic

Width of the strip stretched to the max

Real application: other data is also stored in cache
(there are also other processes)

In our example, why should the width of the strip (ns) be smaller than 50?

Usually numerical tests (trials) are used to determine a suitable size for the cache blocking

Tests are repeated, if:

- Architecture changes (different machine)
- Implementation changes (more/less data in loop kernel)

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n = 500; ns = 100
do iout=1, ...
  do j=1, n
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Cache blocking

Re-use data before it is evicted

Breaking a loop into 2 (or more) parts

There can be cache blocking for multiple loops

2. Cache size=300 → ns=100
ns is way(!) too large (by 2×)
Why?

Everything moving between
memory and CPU is cached:
Not just 'x' but also 'y'

Note:

In our example we have been overly optimistic

Width of the strip stretched to the max

Real application: other data is also stored in cache
(there are also other processes)

In our example, why should the width of the strip (ns) be
smaller than 50?

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n = 500; ns = 50
do iout=1, ...
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Limitations

Conflicting goals

- Purpose of cache: fast --- low latency/high bandwidth
- Organization of cache: FIFO (First In - First Out) or LRU

Match or no-match?

How do we find a match?

How do we find an element to evict?

4	6	12	11
5	1	14	10
9	8	2	3
7	15	13	16

Problems to tackle

1. Speed
2. FIFO
3. Storage efficiency

For illustration purposes
Cache is drawn as a 2d 'array'
You can imagine that the cache is organized as a 1d array

Basic principle

The cache is small, therefore it can hold data only for a (very) limited time (time in cycles)

How long (# of cycles) and how often data is re-used depends on the code (implementation of the algorithm)

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- Purpose of cache: fast --- low latency/high bandwidth
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Keep in this in mind when thinking about how a cache works:

- The cache stores where the data came from, i.e. the original address
- The cache stores the actual value

These are the addresses, not the actual data

Assume that in this example we encounter the addresses

Consecutive and in order.

So address '1', and its content, was stored first,
i.e. is the oldest

Address '2', and its content, is the second oldest

Address '16' was stored last

Problems to tackle

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You can imagine that the cache is organized as a (long) 1d array

Example:

Loading element #3: This element is stored in the cache. How do we find element #3

Loading element #17: This is not a match; #17 loaded from memory and will replace oldest element in cache

Limitations: The Cache can be quite large

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Note:

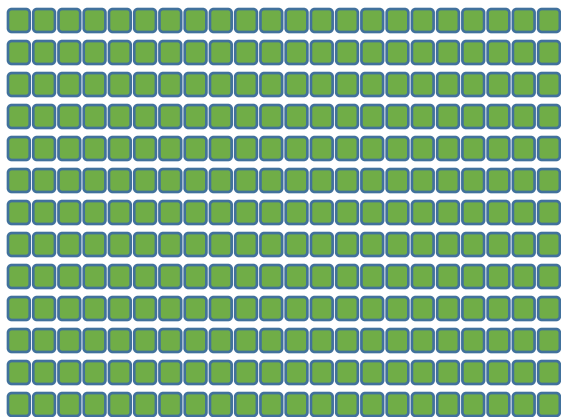
For every cell in the cache the original address has to be stored

Address = 1 word

Therefore half the cache stores addresses

Half the cache stores data

... and then some ordering is needed



OK, for a cache holding 16 words we may be able to compare all addresses and also make FIFO work

Cache size may exceed 1Mw

More than a million entries

(1Mw = 4 or 8 Mb)

So how can we devise a fast strategy?

Let's tackle 'fast access' first,
and then add some FIFO
and then storage efficiency