The MathletFactory Tutorial

Markus Gronau, Tim Paehler

January 11th 2005

Contents

1 General informations **7**

1.1 What is the MathletFactory? 7
1.2 About this document 7
1.3 Available resources 8
1.4 Setting up the environment 8

2 Creating mathlets **9**

2.1 First steps 9
2.1.1 The choice of a mathlet type 9
2.1.2 Creating the applet class 10
2.1.3 Setting the title 10
2.1.4 Implementing a `main`-method 10
2.2 Using a Canvas 11
2.3 Using a ControlPanel 12
2.4 Using advanced BaseApplet functionality 13
2.4.1 Structure of a mathlet 13
2.4.2 Using Tabs 13
2.4.3 Adding a reset button 14
2.4.4 Adding a screenshot button 14
2.4.5 Extending directly the `BaseApplet` 14
2.5 Using the MumieTheme 15
2.5.1 Property Names 15
2.5.2 Fonts 15
2.5.3 Colors 15
2.6 Writing Help Files 16
2.6.1 Default Help Files 16
2.6.2 Individual Help Files 16
2.6.3 Displaying Help Files 16
2.7 Example: A triangle and its altitudes 17

3 MMObjects **23**

3.1 Introduction 23
3.2 Displaying MMObjects 23

3.2.1 Representing content in Containers 23
3.2.2 Representing content in Canvases 24
3.2.3 Display Properties 24
3.3 Rendering cycle 25
3.4 Number class 25
3.5 Adding new MMObjects 26
3.5.1 Introduction 26
3.5.2 Extending the `MMDefaultCanvasObject` 26
3.5.3 Implementing the Transformer 27
3.5.4 Using and Referencing Drawables in `CanvasTransformers` 28
3.5.5 Registration of the MMObject-Transformer Mapping . . . 29

4 Interactivity 31
4.1 Dependencies 31
4.2 Updaters 32
4.3 Animations 32
4.3.1 Creating steps 33
4.3.2 Using animation-dependencies 33
4.3.3 Displaying the controls 33

5 MMPanels 35
5.1 Common Functionality of *MMPanels* 35
5.1.1 Initialisation 35
5.1.2 Detecting User Changes 35
5.1.3 Text visibility 35
5.1.4 Panel size 36
5.2 Overview of *MMPanels* 36
5.2.1 `MMNumberPanel` 36
5.2.2 `MMNumberMatrixPanel` 36
5.2.3 `MMFunctionPanel` 37
5.2.4 `MMDoubleSliderPanel` 37

6 Internationalization 39
6.1 Storing messages in a file 39
6.2 Using messages in a mathlet 39

A Overview of Implemented MMObjects 41
A.1 Affine Geometric Objects 41
A.1.1 Affine Points 41
A.1.2 Affine Lines 42
A.1.3 Ellipses 42
A.1.4 Coordinate System 42
A.2 Analysis Objects 42

A.2.1 Monovariate Functions 42
A.2.2 Multivariate Functions 43
A.2.3 Series 43
A.2.4 Sequences 43
A.2.5 Vector Fields 43
A.3 Numbers 43
A.4 Intervals and Sets 44
A.5 Linear Algebra Objects 44
A.5.1 Vector spaces and Vectors 44
A.5.2 Matrices 44
A.5.3 Endomorphisms 44
A.5.4 Polynomials 45
A.5.5 Equations and Relations 45

B Review of Function Visualisation **47**
B.1 Mathematical Approach 47
B.2 Implemented Model 48
B.3 Review of implemented function types 49
B.4 Fig: Function Structure I 51
B.5 Fig: Function Structure II 52
B.6 Fig: Implementation of Splines 53

C The MathletFactory from a System Developer's Perspective **55**
C.1 MVC Architecture of the MathletFactory 55
C.1.1 Requirements 55
C.1.2 Fundamental Concepts 55
C.1.3 Model 57
C.1.4 View 57
C.1.5 Controller 57
C.2 Arithmetic and Geometric Model 57
C.2.1 Number Types 57
C.2.2 Vectors, Vector spaces and Matrices 58
C.2.3 Affine and Projective Geometry 58
C.2.4 Numerical Computing 59
C.2.5 Compound Example 59

D Algebraic Object Model and Formal Languages **61**
D.0.6 Lexical, Syntactic and Semantic Analysis 61
D.0.7 Introduction to Formal Languages 62
D.0.8 Types of Grammars 63
D.0.9 From Syntactic to Semantic Analysis 64
D.0.10 Formal Languages Used by the MathletFactory 64
D.0.11 Tree Architecture 65

D.0.12 Basic Tree Model 66
D.0.13 Object Model of Operations 67
D.0.14 Object Model of Relations 70
D.1 Applications 71
D.2 Arithmetic and Geometric Symbolic View Architecture 75
D.3 Algebraic Symbolic View Architecture 76
D.3.1 View Architecture of Operations 76
D.3.2 View Architecture of Relations 77
D.3.3 Metrics of View Components 77
D.4 Graphical View and Controller Architecture 78

E Miscellaneous Topics **83**
E.1 Commands for embedding applets in websites 83
E.2 Table: DisplayProperties in MMObjects and Drawables 84
E.3 Classes and their locations/packages 85
E.4 Code: TriangleAltitude-Applet 89

Chapter 1

General informations

1.1 What is the MathletFactory?

The *MathletFactory* is a reusable component system for creating web-based interactive mathematical applets (mathlets) which allows the rapid development of such programs for instructional web systems.
The *MathletFactory* is part of the project MuMIE (http://www.mumie.net).

Following the mathematical paradigm to separate logic from its representation, it consists of both a mathematical library and a presentation and interaction framework with an extensible event system.
On the mathematical side it contains a large and still growing collection of mathematical objects (up to now: number classes, matrices, polynomials, vectorspaces, functions etc.) which give it a power almost compared to that of a CAS. On the interactive side it allows users to place the mathematical objects in different contexts and interactively change their state. By defining dependencies between objects and update chains it allows learners to experience mathematics at work simply by dragging objects (e.g. a vector point) or by setting values from hand (e.g. values of a matrix) and watching the applet dynamically displaying the new result.

1.2 About this document

This document adresses to mathlet developers who wants to get familiar with the creation of mathlets using the MathletFactory's library. It does not aim to describe each single class but to illustrate the necessary techniques and philosophies for the construction of a mathlet. This technical stuff will be accompanied by comprehensive examples and hints. A complete overview of all classes can be found in the APIDOC. The use of the APIDOC will be assumed.

1.3 Available resources

The MathletFactory library as well as the APIDOC, examples, sources and other documentation can be downloaded from the internet site:
http://www.mathletfactory.de

1.4 Setting up the environment

In order to compile and run java-applets you will need the Java Software Development Kit (JDK/SDK) of (at least) version 1.4 as well as the Java3D-classes for 3D-Mathlets and the JavaMediaFramework (JMF) for screen capture support.
For viewing applets in your internet browser, you will need a Java-plugin.
All files can be downloaded for free from http://java.sun.com.
The Java APIDOC and Java tutorial are also available there.

You will need to put the MathletFactory library (JAR-file) into the java-classpath where the compiler and the runtime will search for the classes (see your java-documentation for informations about setting the classpath).

The use of a professional Integrated Development Editor (IDE) instead of a simple text editor is recommended. There you can include the APIDOC and the sources into the editor for an easier programming.

If you want to publish your own mathlets in the internet, you will need to create a HTML-file where you can embed the applet (see Appendix) and to have a copy of the MathletFactory-library on your server where the browser plugin can find it.

Chapter 2

Creating mathlets

2.1 First steps

2.1.1 The choice of a mathlet type

The mathlet-class needs to extend one of the template classes below. There are mathlet template types for the most common needs. The main distinction of theses templates is made by the number, the arrangement and the dimension of their canvases.
The following template types are available:

- **"No-Canvas"** – none canvas and 1 ControlPanel

$$NoCanvasApplet^1$$

- **"Single"** – 1 single canvas and below 1 ControlPanel

$$2D : SingleG2DCanvasApplet^2, 3D : SingleJ3DCanvasApplet^3$$

- **"Side-By-Side"** – 2 canvases arranged horizontally and below 1 Control-Panel

$$2D : SideBySideG2DCanvasApplet^2, 3D : SideBySideJ3DCanvasApplet^3$$

- **"Upper-Lower"** – 2 canvases arranged vertically and below 1 Control-Panel

$$2D : UpperLowerG2DCanvasApplet^2$$

[1]net.mumie.mathletfactory.appletskeleton
[2]net.mumie.mathletfactory.appletskeleton.g2d
[3]net.mumie.mathletfactory.appletskeleton.j3d

- **"Upper-Middle-Lower"** – 3 canvases arranged vertically and below 1 ControlPanel

$$2D : UpperMiddleLowerG2DCanvasApplet^2$$

The ControlPanel resides below the canvases, except for the `NoCanvasApplet`: there it covers the whole mathlet's space.

2.1.2 Creating the applet class

The "entry point" of an applet is the `init()` method which will be called by the web-browser to load and initialize the applet.
An empty mathlet can look like this (e.g. for a "Single-Canvas"-applet):

```
import net.mumie.mathletfactory.appletskeleton.g2d.*;

public class MyApplet extends SingleG2DCanvasApplet {
...
   public void init() {
      super.init() // needed to initialize the template
      ...  // your entry point
   }
}
```

Note that the call `super.init()` is needed to initialize the super class.

The browser plugin (or the appletviewer) needs a HTML page to start the applet. There the location to the compiled applet, its size and other parameters are defined. See *Appendix: Commands for embedding applets in websites*.

2.1.3 Setting the title

The title is shown at top of the mathlet, above the canvases or the control panel. It can be set with

```
setTitle(String)
```

defined in the `BaseApplet` class.

2.1.4 Implementing a `main`-method

In order to start the mathlet as an application (i.e. outside the browser from the command line) you have to write:

```
import ...
import net.mumie.mathletfactory.util.*;
```

```
public class MyApplet extends SingleG2DCanvasApplet {
   ...
   public static void main(String[] args) {
      MyApplet applet = new MyApplet();
      applet.init();
      applet.start();
      BasicApplicationFrame frame =
               new BasicApplicationFrame(applet, 400, 300);
      frame.pack();
      frame.setVisible(true);
   }
}
```

2.2 Using a Canvas

A canvas is a paint board where *MMObjects* , images and text can be displayed. It can hold either 2- or 3-dimensional objects, their class is respectively `MMG2DCanvas` or `MMJ3DCanvas`, subclasses of `MM2DCanvas` and `MM3DCanvas`.[4] The common class of these two different canvas types is `MMCanvas`, where the biggest part of functionality is implemented.
Objects can be added with

`addObject(MMCanvasObjectIF`[5]`)`

defined in `MMCanvas`[6].

[4] The distinction of two 2D and 3D canvas classes was made to allow in future releases the usage of other graphics implementations than the Java-proper ones (*Graphics2D* and *Java3D*).

[5] net.mumie.mathletfactory.mmobject

[6] net.mumie.mathletfactory.display

2.3 Using a ControlPanel

The `ControlPanel`[7] represents a container for GUI(Graphical User Interface)-components, providing a flexible text editor-like layout manager. It allows the programmer to add components and to lay out them in the mathlet as easy as writing a text: the ControlPanel's height will be divided upon all lines where each of them can have a different horizontal alignment (left, right and center). Their height is determined by the components's preferred sizes.
At the begin (means no object has been added yet) a "line pointer" starts at the first line and adds all following components to it. A line will be terminated by a line break. The pointer will then jump to the second line.
Note: No further adding and alignment setting will be possible for this line!
It is possible to use tabstops and other space holders for exact positioning.

The ControlPanel is a base functionality of the BaseApplet and all of its extending classes. In most cases it is positioned under the canvases (except for the NoCanvasApplet: there it fills the whole vertical space). It can be accessed through `BaseApplet.getControlPanel()` or used indirectly by one of the delegate methods defined in the BaseApplet. These delegaters are named after their implementations in the ControlPanel, except the `add()` method: its corresponding delegate method is `addControl()`.[8]
It is possible to create a new ControlPanel-instance (e.g. in a standalone applet or in a new tab) but it is not necessary to do this in the predefined mathlet classes.

Method summary:

- `add(JComponent)` – adds a component to the actual line's end
- `insertLineBreak()` – jumps to the next line
- `insertLineBreak(int)` – jumps `int` lines downwards
- `insertTab()` – inserts a single tabstop
- `insertTab(int)` – inserts `int` tabstops
- `insertHSpace(int), insertVSpace(int)` – inserts place holders with `int` pixels width/height
- sets the alignment of the actual and the following lines
  - `setLeftAlignment()`
  - `setCenterAlignment()`
  - `setRightAlignment()`

[7] net.mumie.mathletfactory.appletskeleton.util
[8] This distinction was made to show the difference between the add-method defined in the Applet-class and this one.

2.4 Using advanced BaseApplet functionality

2.4.1 Structure of a mathlet

The BaseApplet defines a common design for all mathlets where the components are placed into predefined panels/panes (containers able to hold GUI-components). The 3 main panels of a mathlet are (read from top to bottom):

- title pane containing a label with the mathlet's title
- center pane containing the canvases and/or a control-panel
- button pane containing the help, reset, screenshot and animation buttons

The center pane itself is divided into (from top to bottom)

- canvas pane containing a single canvas or an arrangement of several canvases
- control-panel pane containing a ControlPanel

2.4.2 Using Tabs

It is possible to open a new tab in the control-panel-, canvas- and center-pane. This is done by calling `add(String, Component)` on the following getters:

- getCanvasTabbedPane()
- getCenterTabbedPane()
- getControlTabbedPane()

These methods return a customized instance of a JTabbedPane[9] (an instance of `TabbedPanel` more precisly) which will show the real tab (with the tab's title) only when more than one tab has been added.
Note: all of the BaseApplet's tabbed panes have allready a tab (containg the canvases, etc.). But the tab's titles are not visible because they are the only tabs!

The add-method of these tabbed panes request a string for the tab's title and an instance of Component[10] which can be e.g. a new ControlPanel, Canvas or JPanel (they all extends the class `Component`).
The title of the first (default) tab can be set through the `TabbedPanel`-proper method `setTitleAt(int, String)`. The instance of this underlying SWING-class can be accessed through `getTabbedPane()` in the class `TabbedPanel`.

[9] javax.swing.JTabbedPane
[10] java.awt.Component

2.4.3 Adding a reset button

The button itself can be added by simply calling `addResetButton()` inside an extended BaseApplet-class. By pressing the button, the method `public void reset()` will be called in the mathlet. To react on this event you will have to overwrite this method in the applet-class.

2.4.4 Adding a screenshot button

The button will be visible when calling `addScreenShotButton()` inside an extended BaseApplet-class.

2.4.5 Extending directly the `BaseApplet`

Each `BaseApplet` consists of several panels which are not yet added to the visible GUI (the *center*, *canvas* and *control pane*). Extending directly the `BaseApplet` class means that this work has to been done explicitly.
The biggest part of a `BaseApplet` is the *center pane* which takes all the place between the title and the bottom bars. In a case of a so called "no-canvas-applet" the *center pane* has been added only a *ControlPanel*. Note that each of theses panels in `BaseApplet` is an extended version of `JPanel` and `JTabbedPanel`, allowing to show register cards only if more than 1 component has been added. Theses panels haves the additional identifier *tabbed* [11]. In `NoCanvasApplet` we have to write:

```
getCenterPane().add(getControlTabbedPanel());
```

In `SingleG2DCanvasApplet` we have to write:

```
getCanvasTabbedPanel().add(getCanvasPane());
getCanvasPane().setLayout(new BorderLayout());
m_canvas = new MMG2DCanvas();
getCanvasPane().add(m_canvas, BorderLayout.CENTER);
getCanvasPane().add(getControlTabbedPanel(), BorderLayout.SOUTH);
getCenterPane().add(getCanvasTabbedPanel());
```

[11] In the case of the *center pane*, this and the *center tabbed pane* are exactly the same.

2.5 Using the MumieTheme

The MathletFactory uses an extended version of the Java SWING Metal-theme. It is used to retrieve default values for fonts and colors from a property-file.
The class must be used in a static way, i.e. calling `MumieTheme.DEFAULT_THEME` gets the actual single working instance of that class. Values can be retrieved through the various `get`-methods.
The default property files are located in the `resource` folder of the distribution. It is possible to write its own files or to change the existing ones to manage its own default values.
Due to the architecture of SWING, all properties should be (re-)loaded in the `updateUI()` method of the corresponding SWING-class because after an update of the user interface, all dependant classes will reload their default properties from SWING.

2.5.1 Property Names

By default, each SWING component listens to various properties defining its font, color or border. Every property starts with the name of the component by omitting the "SWING-J" (i.e. "Button" instead of "JButton"), followed by a dot and the property type: "font", "background", etc.
There also exist special properties for several *MathletFactory* classes (e.g. Canvas.background).

2.5.2 Fonts

A legal fonts declaration for the MumieTheme consists of 3 comma separated values: font name, style and size.
The font style is coded into the following strings: PLAIN, BOLD, ITALIC, BOLD-ITALIC.
Example:

```
Button.font=Dialog,BOLD-ITALIC,12
```

2.5.3 Colors

A legal fonts declaration for the MumieTheme consists of three 3 comma separated color components for red, green and blue (RGB).
Example:

```
Button.background=230,230,230
```

2.6 Writing Help Files

2.6.1 Default Help Files

By default, each mathlet comes along with a set of help files describing the base common functionality of each mathlet. The type of the mathlet (g2d, no-canvas, j3d) will be questioned to decide which default help files will be needed.
The available files are located under `resource/html`.

2.6.2 Individual Help Files

Each mathlet can have its own individual help file which must be a valid HTML file. If the applet parameter `helpURL` was not set, the mathlet will try to search for its help file with the following assumptions:

- the help file is located in the same directory as the mathlet or
- in the subdirectory `help` of the mathlet's directory
- the help file is named after the mathlet with the suffixes `_info.html` or `_info_<locale>.html`

2.6.3 Displaying Help Files

By default, each mathlet contains at the bottom left corner a *help* button opening the help for this mathlet.
When the mathlet was started as an applet inside a web browser, it will try to open the help file inside a new browser window. If it was started as an application, no applet context is available so the mathlet will show the help file in an own window. Note that in this case the Java class `JEditorPane`, responsible for displaying the file in the mathlet window, can only parse HTML 3.2. In case of a browser environment, the HTML version of the help file must be readable by the web browser.

2.7 Example: A triangle and its altitudes

We want to create an applet which draws a triangle with three moveable points and computes the altitudes.

The canvas

Generally for any visualization we need a so called "canvas" (or several of them) to draw our objects of visualization. The *MathletFactory* for this purpose provides different predefined canvases, such as a single or a side-by-side canvas for both Graphics2D or Java3D applets. Any applet therefore shall extend one of the abstract classes

- `SingleG2DCanvasApplet`,
- `SideBySideG2DCanvasApplet`,
- `SingleJ3DCanvasApplet`, or
- `SideBySideJ3DCanvasApplet`.

There is also an an applet skeleton for pure symbolic displaying with no canvas, the `NoCanvasApplet`.

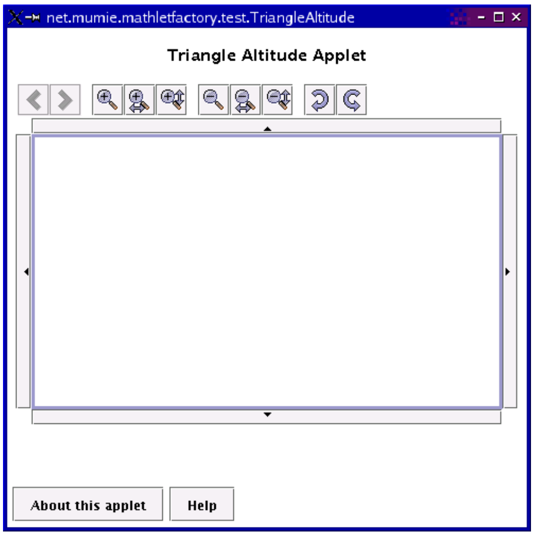
The methods init() and initializeObjects()

The method `init()` is called from the applet context (i.e. the browser or appletviewer) or by a wrapper-main method (don't forget, your applet won't work otherwise!). It calls at least the method **`initializeObjects(),`** and adds all created objects to the canvas.

With an empty implementation, we can already have a look at our efforts (see below) until now. As can be seen, some control icons are automatically added as well as a standard "help"-button.

```
import net.mumie.mathletfactory.appletskeleton.g2d.SingleG2DCanvasApplet;
import net.mumie.mathletfactory.util.BasicApplicationFrame;

public class TriangleAltitude extends SingleG2DCanvasApplet{

  public void init() {
    super.init();
    setTitle("Triangle Altitude Applet");
    initializeObjects();
  }

  protected void initializeObjects(){
  }

  public static void main(String[] args){
    TriangleAltitude myApplet = new TriangleAltitude();
    myApplet.init();
```

```
    BasicApplicationFrame f = new BasicApplicationFrame(myApplet,500);
    f.pack();
    f.setVisible(true);
  }
}
```



A single canvas applet.

Number classes

Almost all objects are based on a so called number class like `MDouble`, `MRational`, `MInteger`, or `MComplex`. They represent the real (with IEEE `double` precision), rational, natural, or complex numbers. All computations are done within this class.

Affine2DPoints

For the triangle we need three points A, B, C. They are realized by a so called `MMAffine2DPoint` and defined by the representing number class and the coordinates:

```
A = new MMAffine2DPoint(MDouble.class, -0.3, 0.3);
B = new MMAffine2DPoint(MDouble.class, 0.25, 0.25);
C = new MMAffine2DPoint(MDouble.class, -0.25, -0.25).
```

MouseTranslateHandler and KeyboardTranslateHandler

Since we want the points to be moveable by mouse and keyboard we create a so called `Affine2DMouseTranslateHandler` and an `Affine2DKeyboardTranslateHandler` and add them to the points:

```
amth = new Affine2DMouseTranslateHandler(getCanvas());
akth = new Affine2DKeyboardTranslateHandler(getCanvas());
A.addHandler(akth);     A.addHandler(amth);
B.addHandler(akth);     B.addHandler(amth);
C.addHandler(akth);     C.addHandler(amth);
```

Affine2DLineSegments

Now we need some line segments to connect the points of the triangle:

```
AB = new MMAffine2DLineSegment(A,B);
BC = new MMAffine2DLineSegment(B,C);
CA = new MMAffine2DLineSegment(C,A).
```

We also need line segments which represent the altitudes:

```
altitude_AB = new MMAffine2DLineSegment(C,getPerpendicularFoot(A,B,C));
altitude_BC = new MMAffine2DLineSegment(A,getPerpendicularFoot(B,C,A));
altitude_CA = new MMAffine2DLineSegment(B,getPerpendicularFoot(C,A,B)).
```

The method `getPerpendicularFoot()` returns a `MMAffine2DPoint` representing the footpoint of the altitude. Some mathematical computation is done within this method which is not of interest for us now.

Maybe the footpoint of an altitude does not lie on an edge of the triangle. So we extend the edges by another line segment:

```
aFootC = new MMAffine2DLineSegment(A,getPerpendicularFoot(A,B,C));
bFootC = new MMAffine2DLineSegment(B,getPerpendicularFoot(A,B,C));
bFootA = new MMAffine2DLineSegment(B,getPerpendicularFoot(B,C,A));
cFootA = new MMAffine2DLineSegment(C,getPerpendicularFoot(B,C,A));
cFootB = new MMAffine2DLineSegment(C,getPerpendicularFoot(C,A,B));
aFootB = new MMAffine2DLineSegment(A,getPerpendicularFoot(C,A,B)).
```

Add objects to the canvas

Now all objects are created we have to add them to the canvas:

```
getCanvas().addObject(A);
getCanvas().addObject(B);
getCanvas().addObject(C);

getCanvas().addObject(altitude_AB);
getCanvas().addObject(altitude_BC);
getCanvas().addObject(altitude_CA);
```

```
getCanvas().addObject(AB);
getCanvas().addObject(BC);
getCanvas().addObject(CA);

getCanvas().addObject(aFootC);
getCanvas().addObject(bFootC);
getCanvas().addObject(cFootA);
getCanvas().addObject(cFootB);
getCanvas().addObject(aFootB);
getCanvas().addObject(bFootA);
```

Display Properties

As can be seen in figure ??? all the lines and points have the standard color black. If we want to give them another color we have to define `PointDisplayProperties` and `LineDisplayProperties` and set them for the points and lines:

```
private PointDisplayProperties pp = new PointDisplayProperties();
private LineDisplayProperties ll = new LineDisplayProperties();
private LineDisplayProperties mm = new LineDisplayProperties();
private LineDisplayProperties kk = new LineDisplayProperties();
```

```
pp.setObjectColor(Color.blue);
ll.setObjectColor(Color.red);
mm.setObjectColor(Color.red);
mm.setFilled(false);
kk.setObjectColor(Color.yellow);

A.setDisplayProperties(pp);
B.setDisplayProperties(pp);
C.setDisplayProperties(pp);

altitude_AB.setDisplayProperties(kk);
altitude_BC.setDisplayProperties(kk);
altitude_CA.setDisplayProperties(kk);

AB.setDisplayProperties(ll);
BC.setDisplayProperties(ll);
CA.setDisplayProperties(ll);

aFootC.setDisplayProperties(mm);
bFootC.setDisplayProperties(mm);
bFootA.setDisplayProperties(mm);
cFootA.setDisplayProperties(mm);
cFootB.setDisplayProperties(mm);
aFootB.setDisplayProperties(mm);
```

The result can be seen in figure ??.

Dependency

Now our points are moveable but the lines do not move with them. Obviously the position of the lines depend on the position of the points. This is described by a so called `DependencyAdapter` and the method `dependsOn()`. For example for the edges of the triangle we have

```
DependencyAdapter DPA = new DependencyAdapter() {
  public void doUpdate(MMObjectIF dependant, MMObjectIF[] free) {
    MMAffine2DLineSegment line = (MMAffine2DLineSegment) dependant;
    line.setInitialPoint((MMAffine2DPoint)free[0]);
    line.setEndPoint((MMAffine2DPoint)free[1]);
  }
};
AB.dependsOn(new MMObjectIF[]{A,B},DPA);
BC.dependsOn(new MMObjectIF[]{B,C},DPA);
CA.dependsOn(new MMObjectIF[]{C,A},DPA);
```

Hereby the first argument `new MMObjectIF[]{A,B}` of the method `dependsOn()` is an array of objects on which the object `AB` depends. This array is passed to the method `doUpdate` in the `DependencyAdapter DPA` as parameter `free`. `AB` is passed to the method `doUpdate` in the `DependencyAdapter` as parameter `dependent`. `doUpdate` describes the action to perform when an object of the array `free` is changed.

Reset, Screenshot, About this applet button

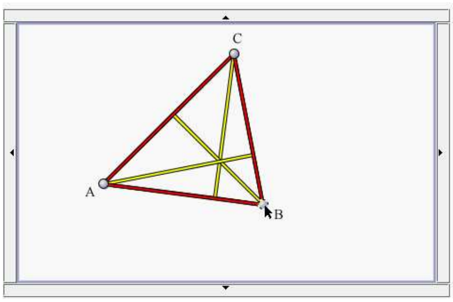
With the commands

```
addResetButton();
addScreenShotButton();
```

we can add a reset and a screenshot button. The functionality of the reset button is defined in the method `reset()`, which calls the method `initializeObjects()` and repaints the canvas:

```
public void reset(){
  initializeObjects();
  getCanvas().renderScene();
  getCanvas().repaint();
}.
```

A HTML-description of the functionality of the applet can be saved in a file with the name `<class_name>_info.html`. This desciption is opend in another window by clicking the about-this-applet button.



The completed applet

Now our first applet is ready. The full program can be found in the Appendix (TriangleAltitude).

Summary: Building Mathlets

These examples should illuminate the process of rapid applet development. The linear process model we used can be generalized to the following steps:

1. Choose the display type and numbers of displays to be used and extend the corresponding applet skeleton.
2. Add the chosen mmobjects and their iconic or symbolic representations
3. Add necessary handlers
4. Create the update graph by adding updaters and creating dependencies

Chapter 3

MMObjects

3.1 Introduction

MMObjects (Multimedial Mathematical Objects) are the primary entities with which an application programmer of the MathletFactory has to deal with. The MathletFactory tutorial gives an overview of the framework that provides the simple use of *MMObjects* . This can be summarized in one sentence: *MMObjects* encapsulate the mathematical and interactivity state and are mapped onto graphic primitives (so called *drawables*) and panel representation (so called *MMPanels*) by the use of specialised *transformers*.
An *MMObject* is either a super class of a mathematical class (located in `net.mumie.mathletfactory.math`) implementing the `MMObjectIF` interface or an extension of the class `MMDefaultCanvasObject` or `DefaultMMObject`, implementing itself this interface and its necessary methods.
Note:

- – every *MMObject* 's name starts with the two capital letters "MM"
- – every *MMObject* is located in the tree branch `net.mumie.mathletfactory.mmobject`

3.2 Displaying MMObjects

A single *MMObject* can be displayed in multiple instances of container- and canvas drawables. (a "Container" means a box where GUI-elements such as buttons, labels and textfields can be added and displayed)(e.g. the `ControlPanel` or a simple `JPanel`). Thereby it is possible to display the *MMObject* 's content for different transform types in separate drawables and even in multiple instances for the same type.

3.2.1 Representing content in Containers

Each time one of the methods

- getAsContainerContent()
- getAsContainerContent(int)

is called, a new container drawable instance is created and returned. The first method returns the default container drawable, the second the drawable designated by the transform type. It is necessary to store it in a variable because these methods will never return this same instance again. The returned component can then by casted to the drawables runtime class to gain access to class-specific functionality.

3.2.2 Representing content in Canvases

Each time one of the methods

- getAsCanvasContent()
- getAsCanvasContent(int)

is called, a new container drawable instance is created and returned. The first method returns the default container drawable, the second the drawable designated by the transform type. It is necessary to store it in a variable because these methods will never return this same instance again. The returned component can then by casted to the drawables runtime class to gain access to class-specific functionality.

3.2.3 Display Properties

Each *MMObject* has its own display properties which define appearance related settings such as colors, fonts or transparency. They can be returned and set with

- `getDisplayProperties()`
- `setDisplayProperties(DisplayProperties)`

Both methods are defined in the interface `MMObjectIF`.
Moreover some MM-classes have extended properties where special settings are possible. By casting to the `DisplayProperties`-runtime class they offer additional methods to change other useful settings such as the line width, point radius or the basic shape of a drawable:

```
MMAffine2DPoint p1;
// ... initializing the point ...
PointDisplayProperties pdp = (PointDisplayProperties)p1.getDisplayProperties();
pdp.setPointRadius(10);
```

Sometimes we want to have 2 identical display properties, except 1 setting. Avoid such code:

```
DisplayProperties dp1 = new DisplayProperties();
DisplayProperties dp2 = dp1; (*)
dp1.setTransparency(0.25);
dp2.setTransparency(0.75); // dp1 will be changed!
```

It is possible to call `clone()` on an instance of `DisplayProperties` to copy the settings into a new independant instance. The line (*) must be:
`DisplayProperties dp2 = dp1.clone();`

See *Appendix: Usage of DisplayProperties in* MMObjects *and Drawables* for a list of available properties-classes and their implementing *MMObjects* and the drawables using them.

3.3 Rendering cycle

Each rendering cycle is started either by a canvas, by the mathlet author (calling a manual repaint on the *MMObject* with the `render()` method) or by a handler (calling itself the `render()` method).
The *MMObject* forwards the repaint request to all of its working transformers which will set the internal data of the *MMObject* 's drawables accordingly to the actual mathematical content.
Calling `render()` on a *MMObject* is therefore equivalent to calling `render()` on every transformer instance hold by the *MMObject* .

Example: 2D point
The `MMAffine2DTransformer` gets the coordinates of its "master" (here: `Affine2DPoint`) and passes them to its drawable (here: `G2DPointDrawable`) which will draw a point at the new coordinates in the canvas.
When dragging the point with the mouse, an instance of the class `Affine2DMouseTranslateHandler` will set the new coordinates in the master (a `MMAffine2DPoint`) and the rendering cycle will continue as described above.

3.4 Number class

Most *MMObjects* (or maybe their underlying mathematical classes) are dependant of a number type, means calculations are made through the number-class-proper arithmetic operations/methods.
All MM-classes need this number class as parameter for their constructors for initializing their internal fields with numbers of this class. A change of the class outside the constructor (i.e. after initializing) is generally not possible.
The available number classes are (located in `net.mumie.mathletfactory.math.number`):
`MDouble, MComplex, MRational, MComplexRational, MBigRational, MInteger, MNatural, MRealNumber, Zmod5.`

3.5 Adding new MMObjects

This document gives a brief overview of how to add user defined *MMObjects* to the MathletFactory. This is done in three steps: Extending the `MMDefaultCanvasObject`, implementing the associated transformer(s) and registering the *MMObject* -transformer mapping in the specific transformer.properties file(s).

3.5.1 Introduction

MMObjects are the primary entities with which an application programmer of the MathletFactory has to deal with. The MathletFactory tutorial gives an overview of the framework that provides the simple use of *MMObjects* . This can be summarized in one sentence: *MMObjects* encapsulate the mathematical and interactivity state and are mapped onto graphic primitives (so called *drawables*) and panel representation (so called *MMPanels*) by the use of specialised *transformers*.

3.5.2 Extending the `MMDefaultCanvasObject`

Writing a *MMObject* usually starts by implementing a subclass of the `MMDefaultCanvasObject` class[1]. This class provides already the base functionality for handling the interactivity state (handlers, updaters, dependencies), therefore only the mathematical state has to be implemented. An alternative approach would be to extend one of the mathematical classes by implementing the `MMCanvasObjectIF` interface. This was, for example, done in the implementation of some *MMObjects* representing affine geometric entities (points, lines, etc.).
The abstract methods to be implemented when extending `MMDefaultCanvasObject` are the two following:

`getDefaultTransformType()` Should return one of the transform types specified as constant in the class `GeneralTransformer`. As may be guessed, this value is important for determining the correct transformer for the *MMObject* when invoking the methods `MMDefaultCanvasObject.getAsContainerContent()` and `MMDefaultCanvasObject.getAsCanvasContent()`.

`getNumberClass()` This method implements the number class used by the *MMObject* . This must be one of the subclasses of `MNumber` and specifies, whether the mathematical entity bases on integer, rational, real, etc. numbers. The number class is needed for initializing all *MMObjects* , a change after initialization is usually not possible.

[1] *MMObjects* with no graphical representation can use instead the class `MMDefaultObject` which represents a *MMObject* with only a symbolic representation

3.5.3 Implementing the Transformer

The transformer offers functionality responsible for the rendering, i.e. the mapping of *MMObjects* onto graphical primitives (drawables) or panel representations (MMPanels). Note that it does not implement the actual graphical representation itself but keeps it as a reference and updates its configuration according to the *MMObject* 's state. This allows it to reuse graphical primitives for a wide range of *MMObjects* .
For allowing sophisticated geometry rendering, the MathletFactory offers a two level rendering approach using Math coordinates, world coordinates and screen coordinates with the appropriate transformation functions `math2World` and `math2Screen`. However, for simple (usually affine) rendering, only one of these transformations has to be specified other than the identity.
Corresponding to the different rendering subsystems, there are three different types of transformers. The G2D transformers (located in the subpackage `transformer.g2d`) perform rendering for 2D representations using the Java2D api, the J3D transformers (located in the subpackage `transformer.j3d`) perform rendering for 3D representations using the Java3D api and the NOC (=No Canvas) transformers (located in the subpackage `transformer.noc`) perform rendering onto Components.
Provided the drawable or MMPanel exists, implementing a transformer is quite straightforward and can be done by copying an existing transformer and editing a few methods. These methods differ with the type of the transformer and are listed in the following subsections:

G2D Transformers

For G2D Transformers, extending the class `Affine2DDefaultTransformer` offers a good starting point. Besides the constructor, where the drawables are specified (see below), the following methods need to be implemented:

`synchronizeMath2Screen()` This method is used for rendering the mathematical state directly onto the screen.

`synchronizeWorld2Screen()` This method is used for rendering the world coordinates (already calculated from the mathematical state) onto the screen. This method is usually invoked by `synchronizeMath2Screen()` after `math2World` transformation has been performed.

J3D Transformers

For J3D Transformers, extending the class `Affine3DDefaultTransformer` offers a good starting point. Besides the constructor, where the drawables are specified (see below), the following methods need to be implemented:

`synchronizeMath2Screen()` This method is used for rendering the mathematical state. As the actual representation on the screen depends on the viewers position, this method should only implement the transformation from mathematical state to resulting world coordinates, leaving the projection on the screen to the 3D rendering system.

`getWorldPickPointFromMaster()` This method should return a 3D point, that might act as a 'center of gravity' when determining the medium distance of the drawable from the viewer.

NOC Transformers

For NOC transformers, the class `ContainerObjectTransformer` should be extended, representations using matrices may be derived from `TransformerUsingMatrixPanel`. The following methods need to be implemented:

`initialize(MMObjectIF master)` This method is used for creating the MMPanel, handing it over the master *MMObject* needed for call-backs (e.g. editing of entries).

`render()` This method is used for rendering the mathematical state directly onto the screen. Note that unlike in the canvas transformers, where rendering does not include the actual drawing of the object (this is done by the canvas), this method should contain a `repaint()` call for the MMPanel after transferring the mathematical state.

3.5.4 Using and Referencing Drawables in `CanvasTransformers`

Since an *MMObject* transformed by a single transformer may have multiple drawables (e.g. a cuboid would be displayed by using multiple rectangles), some of which may be active or inactive (e.g. a line would be either rendered by a line drawable or - in the degenerated case - by a point drawable), the developer has to specify the type of drawables needed by the Transformer. The `CanvasTransformers` class therefore offers support for multiple types of drawables, the following is an excerpt from its source code:

```
/**
 * this array holds all possible {@link ...CanvasDrawable}s
 * necessary to visualize the mathematics
 */
protected CanvasDrawable[] m_allDrawables;

/**
 * this array holds additional <code>CanvasDrawables</code> that might be
 * required by the &quot;real&quot; mathematics. Here we think of extra
 * presentation of boundary values for functions defined on borel sets
 * (these might be displayed as point objects, whereas the actual function
 * graph is displayed as a polygon) etc.
 */
protected CanvasDrawable[] m_additionalDrawables;

/** If a drawable is contained in this set, it neither rendered, nor drawn. */
protected Set m_invisibleDrawables = new HashSet();

/**
```

```
   * this will be the instance of the current active (valid)
   * <code>CanvasDrawable</code> and always points to one of the drawables
   * stored in {@link #m_allDrawables}.
   */
  protected CanvasDrawable m_activeDrawable;
```

Since the fields are used in various methods, they should be initialized in the constructor.

3.5.5 Registration of the MMObject-Transformer Mapping

After the transformer has been constructed, it needs to be registered for a chosen transform type and in a specific display. This is done by adding an entry to the `transformer.properties.g2d`, `transformer.properties.j3d` or `transformer.properties.noc` file (depending on the type of the transformer), located in the transformer package. This entry is of the following form:
`<transform type>#<screen type>#<mmobject class>=<transformer class>`
where `transform type` is either the default transform type of the *MMObject* or the argument specified in the `getAsCanvasContent(int transformType)` or `setCanvasTransformer(int transformType, int screenType)` call creating the transformer and `screen type` is `ST_GRAPHICS2D`, `ST_J3D` or `ST_NO_CANVAS`. The possible values for both of these variables are constants defined in the `GeneralTransformer` class.

Chapter 4

Interactivity

4.1 Dependencies

Sometimes a MM-object needs for its calculations the values of other objects. This implies that it must be recalculate its value whenever its "parameters" (i.e. the other objects) change to be up-to-date.
It is possible to define relations between MM-objects so that one of them is dependant of the others. When one of the later is changed, the depending object will be updated. This can be done be implementing all 3 methods defined in the `DependencyIF`-interface

- `doUpdate()`
- `doUpdate(MMObjectIF dependant, MMObjectIF free)`
- `doUpdate(MMObjectIF dependant, MMObjectIF[] free)`

or by overwriting one of the methods of the `DependencyAdapter`-class (which implements the interface with empty methods). The later possibility is recommended because it is sufficient to use only 1 of them: all 3 methods will be called one after the other during one update cycle. Using the interface will cause to implement 2 methods with an empty body.
After the *doUpdate*-method has been called, the *dependant* object will be rendered.

Say we have 3 MM-objects named *dependant, free1, free2* and we want the *dependant* to calculate its value if one of the free-objects changes its value:

```
MMObjectIF dependant, free1, free2;
... //initialize the fields above
dependant.dependsOn(new MMObjectIF[] {free1, free2},
    new DependencyAdapter() {
      public void doUpdate() {
        ... // calculate
      }
```

```
    }
  );
```

If the MM-objects involved in this update are NOT known in the *doUpdate*-method (i.e. the adapter-class is used for many dependencies) the passed MM-objects must be casted to their original class to use the full functionality of the MM-class.

Example: line segment between 2 points
We want to create a line between 2 points that listens to changes of its start- and endpoint. The *dependant* wourld be a `MMAffine2DLineSegment` and the 2 *free*-objects instances of `MMAffine2DPoint`:

```
import net.mumie.mathletfactory.mmobject.geom.affine.*;
import net.mumie.mathletfactory.action.updater.DependencyAdapter;
...
MMAffine2DLineSegment line1, line2;
MMAffine2DPoint start1, start2, end1, end2;
...
DependencyAdapter lineAdapter = new DependecyAdapter() {
  public void doUpdate(MMObjectIF dependant, MMObjectIF[] free) {
    MMAffine2DLineSegment line = (MMAffine2DLineSegment) dependant;
    MMAffine2DPoint p1 = (MMAffine2DPoint)free[0];
    MMAffine2DPoint p2 = (MMAffine2DPoint)free[1];
    line.setInitialPoint(p1);
    line.setEndPoint(p2);
  }
};
line1.dependsOn(new MMObjectIF[]{start1, end1}, lineAdapter);
line2.dependsOn(new MMObjectIF[]{start2, end2}, lineAdapter);
...
```

4.2 Updaters

4.3 Animations

Animations allow a succession of actions during a defined duration. They are defined by one or many steps defining themselves the "real" actions. The steps's order is determined by the order the steps have been added to the animation (FIFO principle). Successive actions should be grouped into separate steps where a better control of initializations between two successive actions can be reached. Actions which are dependant of the step's progress (e.g. dragging an object from one position to another) must be made to process a number between zero (begin)(0

4.3.1 Creating steps

An animation step is created with a *long*-value as duration or with a second parameter, the call count as an *int*-number.

The animating calcuation part is done by the *proceed(Progress)*-method which must be overwritten by

4.3.2 Using animation-dependencies

Animation-dependencies work similiarly to the "normal" depedencies except that the *doUpdate*-methods have a third argument: the progress.

4.3.3 Displaying the controls

The components available to control an animation are:

- the *button panel* with the main control buttons: `getButtonPanel()`
- the *description label* with the currently displayed message: `getDescriptionLabel()`
- the *options panel* for changing settings: `getOptionsPanel()`

These 3 components are grouped into the *animation panel*: `getAnimationPanel()`

It is possible to place the *button panel* into the bottom bar of the mathlet, between the *help- and reset-button* by using inside a *BaseApplet*-extending class `setAnimationPanel(Animation)`.

Chapter 5

MMPanels

This chapter deals with the panels used to render *MMObjects* in a container. These so called *MMPanels* have a set of common functionality and properties along with a specialisation for their *MMObjects* , their "masters".

Every *MMPanel* derives from the homonymous class from the package *net.mumie.mathletfactory.display.noc* or from a subclass of it, called `MMEditablePanel`.

5.1 Common Functionality of *MMPanels*

5.1.1 Initialisation

Each *MMPanel* has a constructor of at least 2 parameters where the first is the *Master-MMObject* and the second is the transformer used to update this container drawable. *MMPanels* are intended to work *for* a *MMObject* and not to be instantiated by the **new** operator. The only way to create a new instance of a *MMPanel* is to call *getAs-ContainerContent()* on the *MMObject* to be displayed.

5.1.2 Detecting User Changes

Every *MMPanel* contains a flag to indicate that the user has changed or entered data in the panel:

`isEdited()` Returns if this *MMPanel* was edited manually by the user.

`setEdited(boolean)` Sets the `edited` flag. My be used to reset the flag.

5.1.3 Text visibility

There are 2 ways to hide the content in a *MMPanel* without hiding the entire component.
The first is an absolute mechanism that hides the text until other specified.
The second is mechanism hides the text as long as the user has not edited the panel's

value. It makes use of the `edited` flag.

The following methods overwrite the foreground with the background color to hide the text. Note that only the "real content" can be hidden, i.e. the layout and style elements are not infected (e.g. the braces of a matrix or of an intervall).

`boolean isTextVisible()` Returns if the content of this *MMPanel* is visible. Default is true.

`void setTextVisible(boolean)` Sets the content's visibility to the given boolean value.

`boolean isTextVisibleBeforeEdited()` Returns if the panel's content will only be visible when the user enters data. Default is true.

`void setTextVisibleBeforeEdited(boolean visible)` Sets the `textVisibleBeforeEdited`-flag.

5.1.4 Panel size

By default, each *MMPanel* takes as much place as it needs but will never exceed its *preferred size*. But sometimes this dynamic resizing is not desired and, if the panel's best dimension is known, can be turned off be setting his own preferred size by invoking these two methods:

`void setWidth(int)` Sets the preferred width of this *MMPanel* . Setting -1 will restore the default (automatic) size.

`void setHeight(int)` Sets the preferred height of this *MMPanel* . Setting -1 will restore the default (automatic) size.

5.2 Overview of *MMPanels*

5.2.1 `MMNumberPanel`

This *MMPanel* is used to render all numbers in the *MathletFactory* . It is derived from the `OperationPanel` which is basically used to render symbolic function expressions. It is used in many other *MMPanels* where numbers can be edited.

5.2.2 `MMNumberMatrixPanel`

This *MMPanel* is used to render (number) matrices, linear equation systems and vectors/tuples. It makes use of the *MMPanels* of its number components to render them. The matrix border can be changed to determinant, brackets and braces type. It extends the *MMPanel* with the possibility to set or get all attributes from its subcomponents in a single call.

`boolean isCompletelyEdited()` Returns if all subcomponents were edited. Uses the `edited` flag of its numbers.

`MMNumberPanel getEntryPanel(int row, int col)` Returns the `MMNumberPanel` of the matrix entry with the given indices.

5.2.3 MMFunctionPanel

This *MMPanel* is used to render symbolic function expressions for all function objects. It is derived from the `OperationPanel`.

5.2.4 MMDoubleSliderPanel

This *MMPanel* is used to render double values as a slider. By this way the user can drag the slider to change the number value.

Chapter 6

Internationalization

Internationalization of mathlets can be done by using localizable messages (identified by keys) for e.g. the title or descriptions inside the mathlet. The mathlet developer is encouraged to use localizable messages instead of static strings in order to free the code from language dependant expressions and therefore to provide a more flexible language integration.

At start-up, the mathlet tries to load the correct messages-file for the executing system, else it tries to load it in english.

6.1 Storing messages in a file

Each language has its own abbreviation, e.g. "en" for english, "de" for german or "fr" for french. The filename of the message-file is determined by the prefix "`Messages_`" followed by the language abbreviation and the file ending "`.properties`" (e.g. `Messages_en.properties` for english). This file must reside in the mathlet's directory and the locale of the executing system must correspond to that of the filename. A message in the "en"-file could be:

myApplet.title = This is the title in english!

whereas in the "de"-file:

myApplet.title = Dies ist der Titel auf deutsch!

If no such message file is found, the default file ("`Messages.properties`") will be taken.

6.2 Using messages in a mathlet

Message strings stored in a language file can be read through the `getString(String key)` -method defined in the `BaseApplet`-class. Since every mathlet template extends this class, this method is available in every mathlet.

Appendix A

Overview of Implemented MMObjects

Note:
Every *MMObject* must have a copy constructor with its own entity as single parameter. These constructors will not be listed below.

Note:
Every constructor which is not a copy constructor has a `Class` parameter as first (or even single) argument. This field must be one of the number classes providing calculations on a specific number field for that *MMObject* . In the further, a constructor with no arguments ("empty constructor") means with no arguments except the `Class` parameter.

A.1 Affine Geometric Objects

These objects exist whether in 2D or 3D space and are catacterized by their affine coordinates.
Location: net.mumie.mathletfactory.mmobject.geom.affine

A.1.1 Affine Points

They can be constructed with their coordinates or by an empty constructor (in this case they get the coordinates of the origin). These can be changed by various `get`- and `set`-methods.
Implementations:
`MMAffine2DPoint` - represents an affine 2D point
`MMAffine3DPoint` - represents a point in the affine 3D space

A.1.2 Affine Lines

These objects are caracterized by 2 points they are running through. They can be infinitely long or a segment between these points.
Implementations:
`MMAffine2DLine` - represents an infinitely long line in 2D
`MMAffine2DLineSegment` - represents a line segment in 2D
`MMAffine3DLine` - represents an infinitely long line in 3D
`MMAffine3DLineSegment` - represents a line segment in 3D

A.1.3 Ellipses

Ellipses are internally represented by a symmetric 3x3 `net.mumie.mathletfactory.util.math.NumberM` They can be constructor either by this matrix or by the center, by the radian between the semi axes and the canonic coordinate system and by the length of the semi axes or by the two focal points of the ellipse and by the sum of the distances between an arbitrary point on the ellipse and the two focal points.
Implementations:
`MMAffine2DEllipse` - represents an ellipse in 2D space
`MMAffine3DEllipse` - represents an ellipse in 3D space

A.1.4 Coordinate System

A 2D coordinate system can be constructed either with default display options and the origin as center or with custom settings for center, axes and grid line display.
Implementations:
`MMCoordinateSystem` - represents a configurable 2D coordinate system

A.2 Analysis Objects

A.2.1 Monovariate Functions

Monovariate real valued functions are constructed either by implementing one of the `evaluate` methods defined in the interface `FunctionOverRIF` or by using a symbolic string representation of the function using an *Operation* .
Implementations:
`MMFunctionDefByOp` - represents a function defined by an *Operation*
`MMFunctionDefinedByExpression` - represents a function defined by an implementation of `FunctionOverRIF`
`MMFunctionDefinedBySamples` - represents a function defined by a discrete set of points
`MMPiecewiseFunction` - represents a piecewise function defined by an *Operation* for each interval

A.2.2 Multivariate Functions

Multivariate functions are defined by a single expression or by distinct expressions for each dimension (parametric functions).
Implementations:
`MMFunctionOverR2` - represents a function in the R^2 defined by an *Operation* for x and y
`MMOneChainInR2` - represents a monovariate piecewise continuous function in the R^2
`MMParametricFunctionInR2` - represents a parametric function in the R^2
`MMParametricFunctionInR3` - represents a parametric function in the R^3

A.2.3 Series

All implemented series work with *Operations* but also implement the interface `FunctionOverRIF`.
Implementations:
`MMSeriesDefByOp` - represents a series defined by an *Operation*
`MMFourierSeriesDefByOp` - represents a Fourier series of a periodic function
`MMFunctionSeriesDefByOp` - represents a monovariate real valued function series
`MMPowerSeriesDefByOp` - represents a power series
`MMTaylorSeriesDefByOp` - represents a Taylor series

A.2.4 Sequences

Implementations:
`MMSequence` - represents a sequence defined by an implementation of `SequenceAdapter`
`MMSequenceDefByOp` - represents a sequence defined by an *Operation*
`MMFunctionSequenceDefByOp` - represents a monovariate real valued function sequence defined by an *Operation*
`MMRecursiveSequenceDefByOp` - represents a recursive sequence defined by an *Operation*

A.2.5 Vector Fields

Implementations:
`MMVectorField2DOverR2DefByExpression` - represents a vector field defined by an implementation of `VectorField2DOverR2IF`
`MMVectorField2DOverR2DefByComponents` - represents a vector field defined by 2 *Operations*

A.3 Numbers

Location: net.mumie.mathletfactory.mmobject.number

A.4 Intervals and Sets

Location: net.mumie.mathletfactory.mmobject.set **Implementations:**
`MMInterval` - represents a simple set with a starting and an ending value
`MMNumberSet` - represents a number set defined by an implementation of `NumberSetIF`
`MMSetDefByRel` - represents a set defined by a *Relation*

A.5 Linear Algebra Objects

Location: net.mumie.mathletfactory.mmobject.algebra.linalg

A.5.1 Vector spaces and Vectors

In the Mumie MathletFactory, vector spaces and vectors are tightly coupled. This results from the design principle, that vector spaces have a basis that can be different than the canonical default basis (and which is expressed in coordinates with respect to that default basis). For vectors this means, that the basis of the vector space they belong to also determines the coordinates of the vector with respect to the canonical basis.
Because of this, there is no possible way to create a vector without a vectorspace, the only methods to create one are the `getNewMMVectorFromDefaultCoordinates()` and `getNewVectorFromDefaultCoordinates()` methods of the associated vector space.
Implementations:
`MMDefaultR2` - represents the vector space R^2
`MMDefaultR2Vector` - represents a vector in the R^2
`MMDefaultR3` - represents the vector space R^3
`MMDefaultR3Vector` - represents a vector in the R^3

A.5.2 Matrices

Matrices are internally stored row-wise in a one-dimensional array. Their indexing begins with 1 (as opposed to Java array indexing) for row and column indexes.
Implementations:
`MMNumberMatrix` - represents a (mxn) number matrix
`MMNumberTuple` - represents a column number vector/tuple
`MMOpMatrix` - represents a (mxn) operation matrix
`net.mumie.mathletfactory.mmobject.util.MMStringMatrix` - represents a (mxn) string matrix

A.5.3 Endomorphisms

An endomorphism E represent a linear map from a vector space into itself. It therefore requires either a vector space as argument (which sets E to the identity) or a basis of that vector space $b_1, b_2, ...; b_n$ in form of a vector array and its image under the endomorphism $Eb_1, Eb_2, ..., Eb_n$.

The Matrix of the endomorphism can be set and queried by the methods `setDefaultMatrixRepresentation`
and `getDefaultMatrixRepresentation()`.
Implementations:
`MMDefaultR2Endomorphism` - represents an endomorphism in R^2
`MMDefaultR3Endomorphism` - represents an endomorphism in R^3

A.5.4 Polynomials

Implementations:
`MMPolynomial` - represents a polynomial both as algebraic entity and as function over R
`MMBezierPolynomial` - represents a bezier polynomial

A.5.5 Equations and Relations

Implementations:
`MMRelation` - represents an arbitrary complex algebraic relation
`MMEquationSystem` - represents a system of 1 or more equations

Appendix B

Review of Function Visualisation

B.1 Mathematical Approach

This section deals with the visualisation of real valued functions by their (two dimensional) graph. For a given function $f : I \mapsto \mathbb{R}$ we want to display the set $\{(t, f(t)) |\ t \in I\} \subset \mathbb{R}^2$ (at least, this will be the default visualisation type).
All java function classes within the *MathletFactory* have to implement the interface `FunctionOverRIF`. This is a very simple interface only declaring the instruction how to get the "y" from the "x":

- `double evaluate(double x)`
- `void evaluate(MMNumber x, MMNumber y)`
- `void evaluate(double[] x, double[] y)`

The first method using primitive java doubles is the method used for visualisation. In fact, the second method will not make sense for all types of functions. But think of a polynomial having only rational valued coefficients: this method might then be used to picture the fact that rational arguments are mapped to rational result values. The third method will evaluate for an array of input values the output by using the array y.
During the code development and conception it turned out that for this rendering purpose it is comfortable to generalize the visualisation to the so called *1-chain* in $\mathbb{R}$ over $\mathbb{Z}$.
This *1-Chain* in $\mathbb{R}$ is defined as a formal expression[1]

$$\sum_{i=0}^{N-1} \alpha_i f_i \qquad (\alpha_i \in \mathbb{Z},\ f_i : I_i \mapsto \mathbb{R},\ N \in \mathbb{N}).$$

The f_i are supposed to be continous real valued functions defined on closed Intervals $I_i \subset \mathbb{R}$.

[1]To be conform with the java indexing in arrays, we let the summation index run from 0 to $N-1$.

B.2 Implemented Model

In the *MathletFactory* we have the class `OneChainInRIF` modeling these mathematical objects. This modeling is slightly different from the expression above. Here we have

$$\sum_{i=0}^{N-1} f_i \qquad (f_i : B_i \mapsto \mathbb{R},\ N \in \mathbb{N}).$$

That is, all the coefficients α_i are equal to 1. The B_i are elementary sets in $\mathbb{R}$ (i.e. finite union of disjoint intervals that may be of any type) that are modeled by the class `FiniteBorelSet`. Furthermore the f_i need not to be continous, they are only required to be java classes that implement the `FunctionOverRIF`. The class `OneChainInRIF` so far mainly declares the following two methods

- `FunctionOverRIF getEvaluateExpressionInComponent(int indexOfComponent)` returns the i-th evaluation expression (corresponding to the element f_i above) as a `FunctionOverRIF`.
- `FiniteBorelSet getBorelSetInComponent(int indexOfComponent)` returns the i-th borel set.

The common interface for all the "MMFunction objects" that shall be rendered by their graph is `MMOneChainInRIF` that extends `OneChainInRIF` and `Discretizable1DIF` by adding the methods

- `setVerticesCount(int i)`
- `int getVerticesCount()`

These both methods are essential for the discretisation of the function graph (but are of no meaning for the mathematical content and so are a "pure" *MM-feature*).

Because all 1D-function classes do implement this interface, it is possible to use a single transformer type that does all the rendering stuff, the `OneChainInRTransformer`.
This transformer will render each set

$$S_i := \{(t, f_i(t))|\ t \in B_i\} \qquad (0 \leq i \leq N-1)$$

as a 2d-polygon. Observe that for a given function defined on an elementary set (a `FiniteBorelSet` in *MathletFactory* terminology) we have $N = 1$ and there is only a single set S_0 to be displayed.
Let n_i be the number of intervals in the ith elementary set, then according to the decomposition

$$B_i = I_{i,0} \cup I_{i,1} \cup \ldots \cup I_{i,n_i-1}$$

the transformer will discretise the curve on each $I_{i,j}$ corresponding to the value returned by `getVerticesCount()`. The `OneChainInRTransformer` is smart enough not to stupidly discretize each interval and then evaluate the suitable function expression – it does perform a check which parts of the function will really be visible on the screen.
A further improving but not yet implemented feature would be an adaptable discretisation algorithm due to the (local) change rate (i.e. the derivative) of the function to display.

B.3 Review of implemented function types

- `MMFunctionDefinedBySamples`
  This function is determined by an array `Affine2DPoint[] p` of N defining sample points. The domain is equal to $\bigcup_{i=0}^{N-1}\{p_x[i]\}$ and the evaluate method will simply return the corresponding $p_y[i]$ value. This class is also the base class for various types of spline classes which are also treated as "sample point defined" functions.

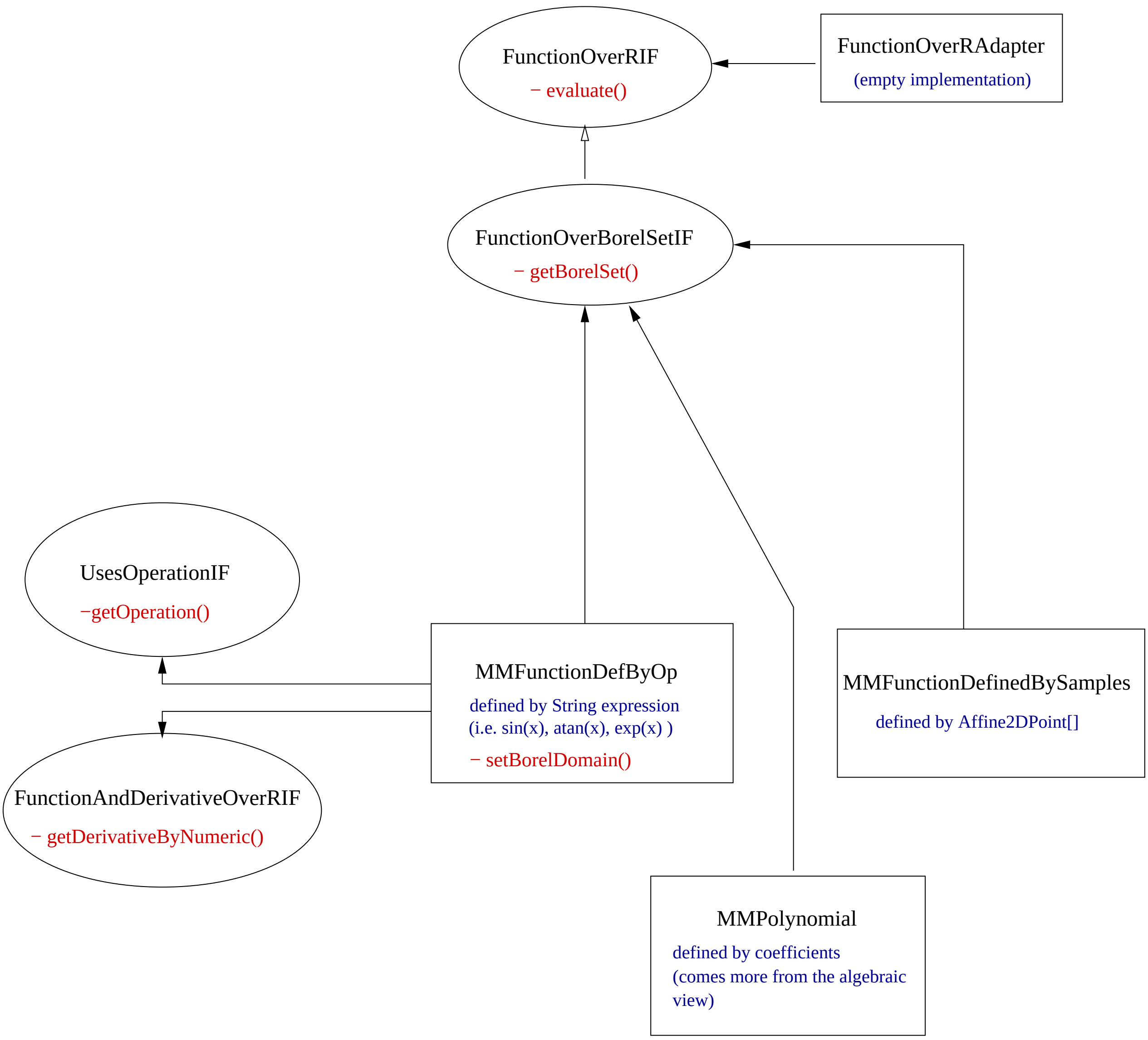
- `MMFunctionDefByOp`
  This type explicitly holds an instance of `FiniteBorelSet` as domain and a `String` expression (i.e. sin(x), exp(x), ...). A parsing mechanism ensures a fast realisation for the method `double evaluate(double x)`, which is always used for rendering.

- `MMFunctionDefinedByExpression`
  This class also holds explicitly its domain, but additionally holds an instance of `FunctionOverRIF`. The latter is responsible for the "real evaluating" and offers a flexible approach for defining more sophisticated functions. By using the method `void setFunctionExpression(FunctionOverRIF f)`, there is a fast approach to define arbitrary functions.

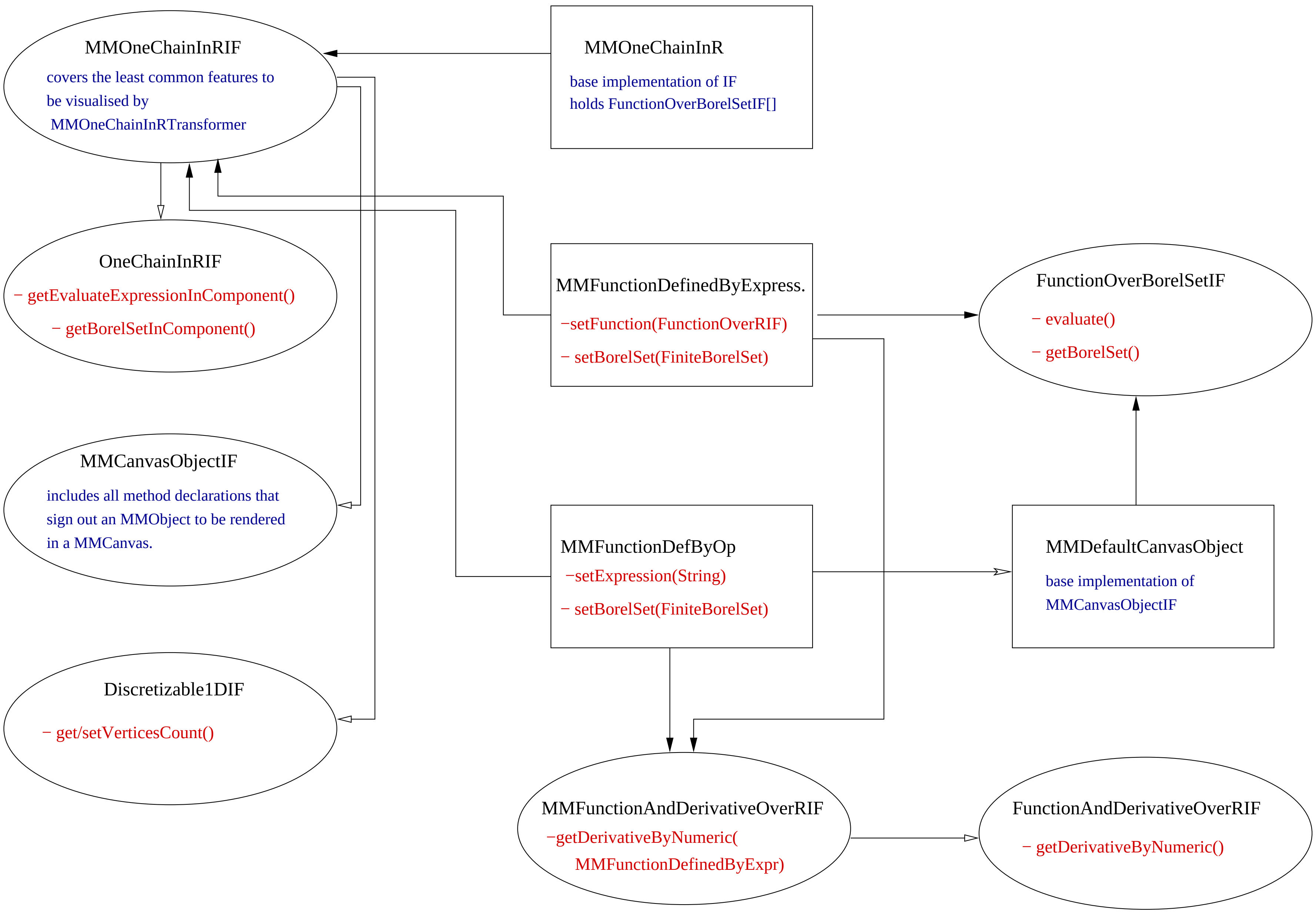
- `MMPolynomial`
  Essentially defined on it's coefficients, we treat this class as a real valued function for standard rendering.

Remark:
Observe that all these functions have a very similar implementation of the `MMOneChainInRIF`. All of these classes define a 1-chain consisting of a single evaluation expression and because all of them do implement the interface `FunctionOverRIF` the method `FunctionOverRIF getEvaluateExpression(int i)` can simply be coded by returning the class itsself.

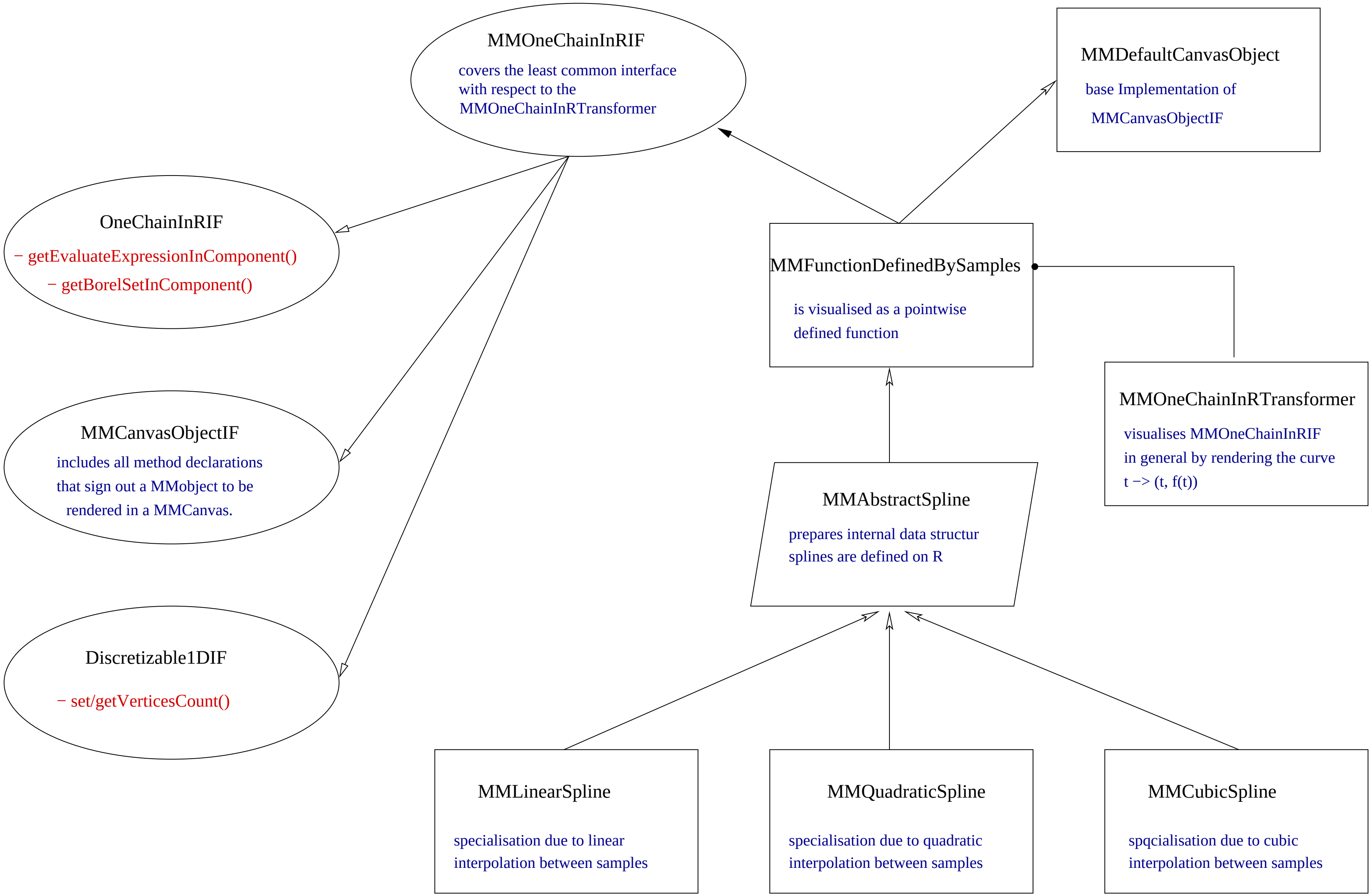
B.4 Fig: Function Structure I



B.5 Fig: Function Structure II



B.6 Fig: Implementation of Splines



Appendix C

The MathletFactory from a System Developer's Perspective

This appendix contains an overview of the MathletFactory from a system developer's perspective. This perspective is necessary for developing new MMObjects or display components that extend the set of mathematical entities represented by the MathletFactory.
In the following sections we omit the details, which can be found at the API documentation but give a structural overview that follows the Model-View-Controller architecture.

C.1 MVC Architecture of the MathletFactory

C.1.1 Requirements

Following the didactic model of Bruner, mathematics can be regarded as a system with three complementing representations: The *enactive* representation of a mathematical entity is determined by what you can *do* with it, the *iconic* representation is an image or sketch that *visualises* one or more of its properties and the *symbolic* representation *denotes* it in a formal language system. One of the main conclusions of Bruner's Theory is, that although professional mathematicians almost exclusively use symbolic representations, the other types of representations play a vital role in learning mathematics. If we use this didactic principle as a requirement for the architecture of a mathematics learning framework, it makes sense to allow different representations for mathematical objects. This can be done best by using the Model-View-Controller Pattern[1], an architectural pattern that separates the data of an entity from its presentation and application logic.

C.1.2 Fundamental Concepts

A good starting point when describing the MathletFactory is to describe what happens, when a student uses an applet created with the MathletFactory.

[1] [Bu96]

If, for example, a student drags the graphical representation of a three dimensional vector on a canvas with the mouse, the canvas generates an event that is sent to the `CanvasController`. This instance checks all objects contained in the canvas if they are meant, by using the mouse coordinates and the canvas' internal projection parameters. If an object has been picked and it can handle the type of event (by owning an appropriate handler), the event is delivered to it by calling `MMObjectIF.doAction()`. The `MMObject` then delegates the event to the specified handler, which processes the event (e.g. translating the end-point of the vector in a plane parallel to the viewport by transforming the new mouse coordinates to world coordinates) and modifies the state of the `MMObject` (e.g. setting its coordinates to new values). After this it is checked, if any other objects depend on the vector (for example, the vector could be designated as normal for a plane). For this it is checked, if any `Updater`s are associated to the object and if so, their `update()` method is called, changing the state of any dependent objects (which in turn might also have other updaters and so on). After the update graph traversal has been finished, The views of the objects that were changed, are redrawn (both the graphical and the symbolic representation), by invoking `render()` and `draw()` in all transformers of the `MMObject`. By this the student is informed of the result of his action and may proceed with further actions.

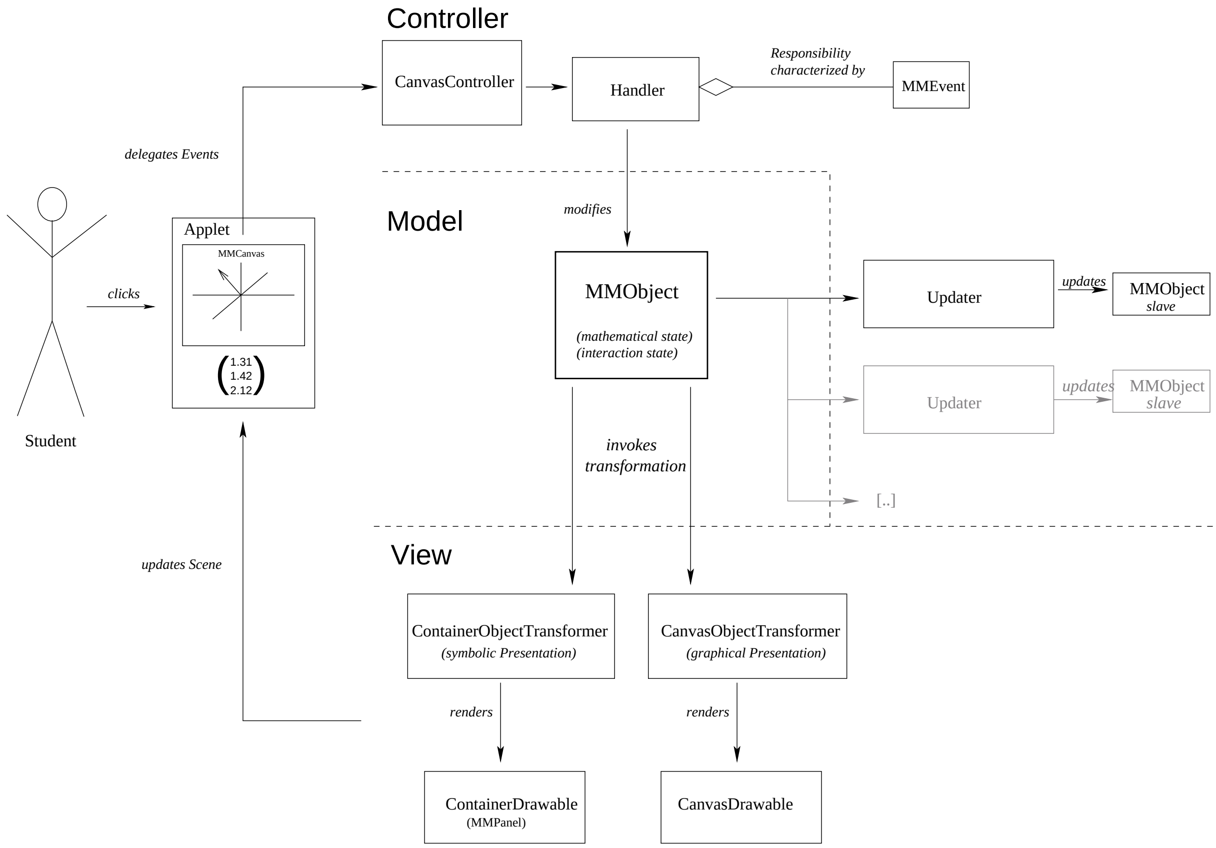


Fig. 1: An action cycle in the MVC architecture

C.1.3 Model

The core model used in the MathletFactory is the *MMObject*, represented by a class that implements the `MMObjectIF` interface. An instance of this class contains on the one hand the mathematical information of the object, on the other hand it also owns references to the handlers that allow its manipulation and to updaters that connects the object with other MMObjects in the update graph. It also contains the link to the view components.

C.1.4 View

The view component consists basically of two objects: The *transformer* (represented by a subclass of `GeneralTransformer`) the *drawable* (represented by the interface `Drawable`). While the drawable is the actual displaying unit, the transformer establishes a link between model and drawable and knows how to repaint it, when the MMObject has changed.

C.1.5 Controller

In the controller section we differentiate between objects that directly manipulate MMObjects and those that are responsible for constructing the update graph. The directly manipulating objects are represented by two different classes: Handlers (subclasses of `MMHandler`) for manipulating iconic representations in a canvas (e.g. selecting and dragging vectors with the mouse) and panels (subclasses of `MMPanel`) for editing symbolic representations. Note that the panel is also a drawable into which the controller-code (which consists of only a few lines) has been integrated into.

C.2 Arithmetic and Geometric Model

In order to compute a wide variety of mathematical problems, the Mumie MathletFactory offers a flexible and economic model for representing the geometric and arithmetic properties of mathematical entities and allows an accessible interface for easy manipulation. In the following, we briefly describe the model architecture of the basic entities.

C.2.1 Number Types

Starting with the specification that all mathematical objects – directly or indirectly – use numbers, we assign to each MMObject a specific number type that can be changed upon construction or sometimes even at run time. By this it is, for example, possible to use the same sequence object for displaying both real valued sequences and complex valued sequences. In the first case the object is instantiated by invoking the constructor with the argument `MComplex.class`, the second with the argument `MDouble.class`. The implementation is done using an abstract base class `MNumber` of which all number types are subclasses. Currently, the following number types exist:

Number Type	Set of numbers modelled	Internal Java type used
`MNatural`	$\mathbb{N}$	`BigInteger`
`MInteger`	$\mathbb{Z}$	`double`
`MRational`	$\mathbb{Q}$	`long`
`MBigRational`	$\mathbb{Q}$	`BigInteger`
`MDouble`	$\mathbb{R}$	`double`
`MComplex`	$\mathbb{C}$	`double`
`MZmod5`	$\mathbb{Z}/5\mathbb{Z}$	`int`

The last is an experimental type that demonstrates the use of finite fields in the MathletFactory and will soon be replaced by a generic class modelling $\mathbb{Z}/p\mathbb{Z}$.

C.2.2 Vectors, Vector spaces and Matrices

While the subclasses of `MNumber` provide the base for one-dimensional number computing, the class `NumberMatrix` – representing an $m \times n$-matrix – is fundamental for the calculation in linear spaces of higher dimension. As MMObjects it has also exclusively uses the generic number interface allowing each instance of a `NumberMatrix` to represent a matrix of different number type.
The `NumberMatrix` is extended by `NumberTuple`, a class that represents $m \times 1$-matrices and which is basically used as coordinates for vectors or matrix columns/rows. It therefore offers additional functionality like the norm, the dot product, etc.
Vectors in turn are modelled by subclasses of the abstract `NumberVector`. It is a specific trait, that each vector of the MathletFactory 'knows' the vector space in which it exists and is represented by coordinates that are relative to its associated basis. By this it is possible to transform all vectors of a chosen vector space by changing the space's basis. The `NumberVectorSpace` class therefore provides a wide range of methods to manipulate its basis.

C.2.3 Affine and Projective Geometry

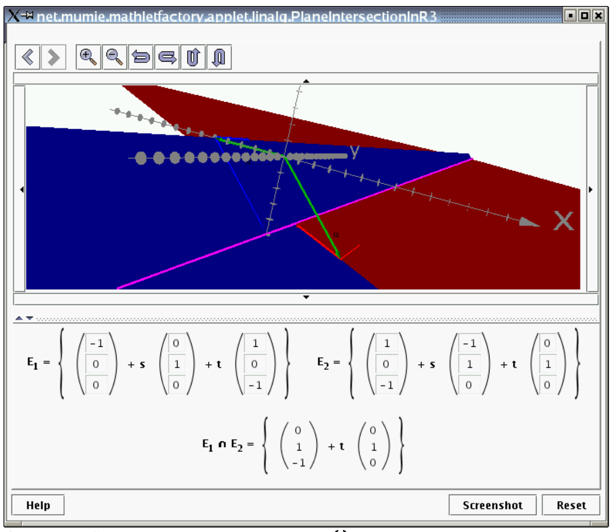
The vector space model is also used in the geometric classes. Like most CAG (Computer Aided Geometry) Modelling software. All internal data is stored in homogeneous (i.e. projective geometry) coordinates. This allows the easy transformation of mathematical entities also for affine geometry. For example, when calculating the intersection of two planes in the three dimensional space $\mathbb{R}$ (i.e. the intersection of 2D affine subspaces within a 3D affine space) simply the projective hyperplanes have to be intersected, reducing the problem to finding the null space of the matrix that contains the projective coordinates of the subspaces' basis (see extended description of this example below).

C.2.4 Numerical Computing

Numerically, almost all affine and projective geometric operations – like the example above – base on the Gauss algorithm implemented in the class `EchelonForm`. This ensures a high reusage and offers an easy optimisation opportunity: If the Gauss algorithm is made faster, a wide range of computations will be executed faster.

C.2.5 Compound Example

To demonstrate, how all the concepts described above work together, we give a 'real life' example: In the mathlet depicted below, the intersection of two planes in three dimensional is computed and displayed (For details refer to the documented source code):




```
AffineSpace.intersected()
          |
AffineSpace.intersect()
          |
ProjectiveSpace.intersected()
          |
NumberVectorSpace.intersected()
          |
SolveHomogen.intersection()
          |
SolveHomogen.nullSpace()
          |
EchelonForm.getREFMatrix()
          |
EchelonForm.toReducedRowEchelonForm()
```

Fig. 2: The intersection of two planes in three-dimensional space and its stack trace to the Gauss algorithm

This starts in by invoking the method `AffineSpace.intersected(AffineSpace with)` in one of the planes with the other as argument (the plane class descends from the affine space class). This method does nothing but the invocation of its projective representation to intersect with the projective representation of the other plane. This is done by giving their two vector bases to the method `SolveHomogen.intersection(NumberTuple[] span1, NumberTuple[] span2)` which in turn calls the `nullSpace(NumberMatrix matrix)` method with a (3×6) matrix as argument that contains all the base vectors as columns. This method in turn uses the Gauss algorithm implemented in `EchelonForm.getREFMatrix(NumberMatri matrix)` to transform the matrix to reduced echelon form to determine its null space basis. The null space basis is then used as parameters for constructing the basis of the intersecting space, which is returned by the `intersected()` method.

Appendix D

Algebraic Object Model and Formal Languages

Since often it is not only needed to let the application developer perform computations with the MathletFactory but also the student (or teacher/tutor) himself, the arithmetic and geometric model is complemented by an algebraic model that allows the input of expressions in a formal language. By this, the flexibility of the mathlets is increased and allows them to be used as tools in open learning scenarios.

D.0.6 Lexical, Syntactic and Semantic Analysis

In order to analyse and interpret a formal language, we need structures that operate on three different levels of language: the first is the lexical analysis which analyses the alphabet of symbols being used, the second is the syntactic analysis that applies the rules for building words and sentences. On the third level we have the semantic analysis that analyses the meaning of the words.

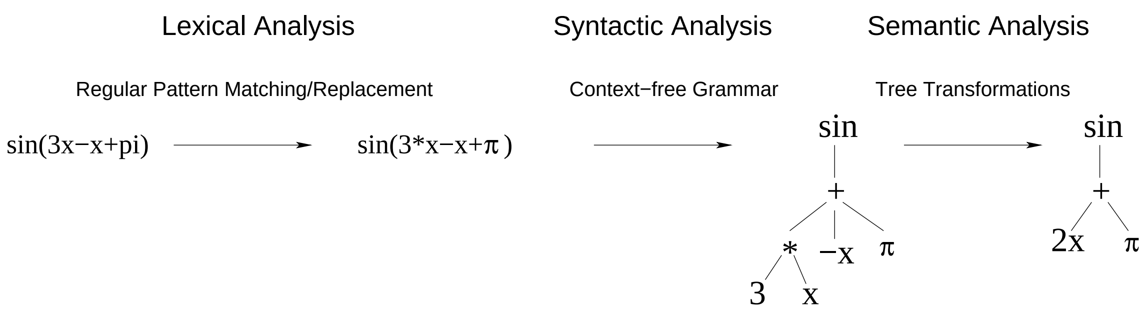


Fig. 3: The different stages in mathematical expressions analysis performed by the MathletFactory

In the MathletFactory, each stage of language processing is performed by a specific unit: The lexical analysis is performed by a set of regular expressions (which also do some replacements that allow an increased robustness like `2x` $\to$ `2*x`) and a small

scanner unit. The syntactic analysis is done by a parser that implements a context-free grammar (see below) and the semantic analysis is left to a rule-based tree automaton that operates on the operation tree generated by the parser.
Note that it is quite easy to reproduce a string out of the tree representation by doing a depth-first order traversal, thus closing the sequence to a loop. This is very important for interactive work with mathematics, where the user enters an expression, watches the response of the system and may want to re-edit his input for a receiving a different result.

D.0.7 Introduction to Formal Languages

The main idea of the MathletFactory's algebraic object model takes advantage of the formal languages used in mathematics. Computer science has developed a rich set of methods to interpret these formal languages of which we can only give a short introduction, see [HU79] for details.
A formal language can be defined as a concatenation of symbols from an alphabet. These concatenations are called the *words*. The formal language $\mathcal{L}(\Sigma)$ over an alphabet Σ that consists of all possible words can thus inductively be defined as:

1. $\sigma \in \mathcal{L}$ for all $\sigma \in \Sigma$.
2. $w \in \mathcal{L}$ for $w = u.\sigma$ with $u \in \mathcal{L}, \sigma \in \Sigma$.

with . being the concatenation operator. The language that consists of all words over Σ is also called Σ^*, where the *-operator means, that the resulting set contains all finite concatenations $\sigma_1.\sigma_2 \ldots \sigma_n, n \in \mathbb{N}$ of symbols $\sigma_i \in \Sigma$, and the empty word ε.

Of course we are more interested in languages that allow only certain words. For example, we may want `(x+1)` to be a word of our language, but not `+)1)x`. We thus need a higher structure that tells us, which words belong to the language, a *grammar*. A grammar is defined by the accepted alphabet (the *terminals*) and by a set of explicit rules how a word can be decomposed into these. For example, one could state the following rules for simple arithmetic expressions of numbers represented by `NUM`:

1. `+`, `-`, `*`, `/` $\in \mathcal{L}$, `NUM` $\subset \mathcal{L}$,*
2. $w \in \mathcal{L}$ for $w = u.$`*`$.v$ or $w = u.$`/`$.v$ with $u, v \in \mathcal{L}$,
3. $w \in \mathcal{L}$ for $w = u.$`+`$.v$ or $w = u.$`+`$.v$ with $u, v \in \mathcal{L}$.

We have already differentiated between products and sums of rational numbers, because for the semantic analysis (i.e. evaluating the arithmetic expressions) it is necessary to consider the precedence of operations. One could define grammars like we

*Note, that for avoiding trivial rules it is better to regard a number like `1234` to be represented by a single symbol, not as a concatenation of symbols as one would presume by the fact that it is a concatenation of digits in our common writing system. In the following we will also regard function identifier like `cos` as a single symbol in order to keep our grammar compact.

did in the example above, but for handling more complex grammars it is useful to state a grammar in the Backus-Naur form (BNF), which regards grammars as a tuple (T, N, s, R), where $T = \Sigma$ is the set of terminals, N is the set of nonterminal symbols (i.e. variables that may contain concatenations of terminals), s is the name of the starting variable (i.e. the variable whose content is tested to be a valid word of the specified language) and $R \subset (N \cup T)^*.N.(N \cup T)^* \times (N \cup T)^*$ is the set of rules that need to apply for a word of the specified language:

```
G(T,N,s,R)

 T:     NUM,'+','-','*','/'
 N:     expr, term, fac
 s:     expr
 R:
 (1)    expr      ->      term { '+' term } | term { '-' term } .
 (2)    term      ->      fact { '*' fact } | fact { '/' fact } .
 (3)    fact      ->      NUM.
```

We see, that the operator precedence is ensured by using different variables for summands and for factors. The expression `3*4+1` could thus be tested by the grammar as follows (we enclose the symbol with its type in parentheses):

$$\begin{aligned}
\texttt{(expr 3*4+1)} &\overset{(1)}{\rightarrow} \texttt{(term 3*4).+.(term 1)} \\
&\overset{(2)}{\rightarrow} \texttt{(fact 3).*.(fact 4).+.(fact 1)} \\
&\overset{(3)}{\rightarrow} \texttt{(NUM 3).*.(NUM 4).+.(NUM 1).}
\end{aligned}$$

After this none of the rules can be applied to the expression anymore which means that `3*4+1` is a word specified by G. This testing method also gives an idea, how a parser that accepts words of $\mathcal{L}(G)$ could be implemented: By a recursive reduction algorithm, where each rule is modelled by an according method.[1]

D.0.8 Types of Grammars

Noam Chomsky has provided a hierarchy of grammars where each type produces a language that is a subset of a language produced by a lower type. The type of a grammar depends completely on the specified rules R:

Type	Name	Constraints for R
1	context sensitive	$\lvert u\rvert \leq \lvert v\rvert$ for all $R : u \mapsto_G v$
2	context free	$R \subseteq N \times (N \cup T)^*$
3	regular	$R \subseteq N \times (\{\varepsilon\} \cup T \cup N.V)^*$

[1]This is called the Top-Down approach, which is mainly used in functional or rule-based language implementations, imperative language implementations often also use a Bottom-Up approach, where for each step only as much symbols are read in as needed for applying the next rule.

This means for example, that regular grammars produce only a subset of context free grammars, which in turn produce a subset of context sensitive languages. Which type suits our needs? There are some features for which we need at least a context free grammar. For example, an expression containing parentheses is only valid in mathematics, when every opening parenthesis has its closing counterpart. On the other hand we want to use parentheses on all levels. This means we need a rule $r \in R$ that has the form `prim` $\rightarrow$ `'(' expr ')'`, which is not possible for regular grammars, which allow no terminals on both sides of a non-terminal. Context sensitive grammars in turn would allow rules with non-terminals and terminals mixed on the left side thus allowing a higher semantic analysis already in the syntax phase, one could, for example, add something like with the following rule:
`NUM '*' '(' VAR '+' VAR ')'` $\rightarrow$ `'(' NUM '*' VAR '+' NUM '*' VAR ')'`,
thus implementing the distributive law for words containing two variables `VAR` multiplied with a number `NUM`. But as the reader might guess, a program that parses context sensitive languages is hard to implement, so we do things like this in a separate semantic analysis step (see D.0.13).

D.0.9 From Syntactic to Semantic Analysis

After the parser has accepted the mathematical expression, we need to transform it into a data structure that allows easy semantic analysis. In our case it is suitable to represent it as a syntax tree or as a word of a regular tree language [Co03]. For example, sin(2x+π) is interpreted as `(sin (+ (* 2 x) pi))`. The expression can then be symbolically transformed by using tree automata (see below) or numerically evaluated, which is done by a recursive procedure that evaluates the tree from the leaves (numbers or variables and parameters with assigned values) up to the root node.

D.0.10 Formal Languages Used by the MathletFactory

The MathletFactory uses two formal languages, which are both context free and read by a recursive descent parser [ASU86]: The operation language *Op* and the relation language *Rel*.

The Operation Language *Op* The language *Op* is used for modelling algebraic operations as they occur in functions, equations, etc. It can be used by any mathematical object that can be characterised by a numerically evaluable symbolic expression. The grammar of *Op* is as follows:

```
G(T,N,s,R)

 T:      NUM, VAR, SIN, COS, SINH, COSH, EXP, ASIN, ACOS, LN, SQRT, '+', '-', '*', '/', '^',
 N:      expr, term, fac, pot, prim
 s:      expr
 R:
 (1) expr      ->      term { '+' term } | term { '-' term } .
 (2) term      ->      fact { '*' fact } | fact { '/' fact } .
```

```
(3) fact      ->      SIN pot | COS pot | SINH pot | COSH pot | EXP pot | LN pot | ABS pot
                      | ASIN pot | ACOS pot | TAN pot | ATAN pot | SQRT pot | FLOOR pot | pot.
(4) pot       ->      prim {'^' prim}.
(5) prim      ->      VAR | NUM | '(' expr ')' | '|' expr '|' .
```

The Relation Language *Rel* The language *Rel* is used for modelling algebraic relations as they occur in set definitions, propositions, etc. As one might already guess from the fact that relations consist of operations, words of *Op* are part of the letters of *Rel*. More precisely, the *Rel* terminal `SIMP` is a simple relation that contains a left and right hand side $\texttt{expr}_i \in Op$, which are separated by a relation sign ($=,\neq,\geq,\leq,>$ or $<$). The grammar of *Rel* is as follows:

```
G(T,N,s,R)

T:       SIMP, NOT, AND, OR, NOT
N:       rel, cla, sub, prim
s:       rel
R:
(1) rel       ->      sub { OR sub }.
(2) sub       ->      cla { AND cla }.
(3) cla       ->      NOT prim | prim.
(4) prim      ->      SIMP | ALL | NULL | '[' rel ']'.
```

Note that in *Rel* we use square brackets for overriding precedence, whereas in *Op* we use parentheses, allowing to parse a complete relation containing operations in a single pass.

D.0.11 Tree Architecture

After the syntactic analysis (i.e. the construction of the operation/relation tree from a user/application provided string), the semantic analysis (i.e. transformation or evaluation of the tree) takes place, again initiated by user or application action. We demonstrate this in a short example:

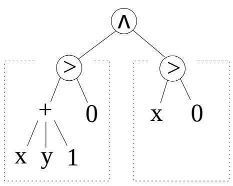


Fig. 4

In the figure above the relation $x + y + 1 > 0 \wedge x > 0$ is displayed in its tree representation (e.g. as a parsed result of the string '`x+y+1>0 AND x>0`'). This tree has two levels: a relation level (the encircled nodes) and an operation level (the nodes below the relation nodes). The relation consists of $\wedge$-Conjunction as root node with two *simple* relations ($x+y+1 > 0$ and $x > 0$, the dashed boxes in the figure) as leaves. The relation leaves contain the operations $x + y + 1$, 0 and x, 0. After binding x and y to certain numeric values, the relation can be evaluated, returning either `true` or `false`, depending on the evaluation results of the operations.

The approach of using tree representations for mathematical expressions is adopted by all modern Computer Algebra Systems (e.g. [Wo91], [Wa91], [Mo93]), but their almost purely functional model allows neither typing (e.g. no type distinction between operations and relations)[2] nor the integration in an object oriented graphical user interface.
For our requirements, we will adopt the concept of a functional representation, but merge it with an object oriented model. This means for the architectural design that we use the different types `Operation` (any symbolic expression that can be numerically evaluated) and `Relation` (any symbolic expression that can be either true or false). We choose these entities because they are closed under the most transformations we will apply onto them (for example, opposed to the set of equations – the set of relations is closed under equivalence transformations, the set of analytic operations is closed under derivation), so we can still use a functional model when transforming trees of each type.

Tree Representation vs. Flat Representation Apart from being a construct of formal languages over which human-computer interaction is possible, a tree architecture also grants a greater flexibility than flat structures. For example, if we want to model a finite representation of a Borel σ-algebra in $\mathbb{R}$ this could be implemented by a single class, owning a list of intervals, upon which the operations `intersect`, `join`, `complement` etc. are resolved. This can be quite costly if we want to compute the join or intersection of two Borel sets that are widely 'scattered', though the user will usually test inclusion in the result only for some points or a single interval displayed on the screen. Also, we could not model the special case of infinite intervals like $\mathbb{Z}$, $\mathbb{R}\backslash 2\pi\mathbb{Z}$, etc. which we need when displaying periodic behaviours.
The solution to this problem is to implement the Borel set as a tree structure that has intervals and 'periodic intervals' as leaves and operations upon Borel sets as inner nodes. When the user wants the set to be displayed on the screen, the observed interval and an ε for precision (e.g. pixel width) is given to the Borel set and a distribution of points and lines satisfying these parameters is computed.
As this example suggests, constructing trees of mathematical entities allows a higher degree of generalisation without losing flexibility and simplicity. We will also see in section D.1 that trees are easily transformable, adding a lot of functionality to tree-structured mathematical entities.

D.0.12 Basic Tree Model

All tree-organised implementations of mathematical entities share a common base class, the `AbstractTreeNode`, which offers abstract tree functionality regardless of type, like adding or removing children, searching and replacement of descendants, etc. From these, the basic nodes for mathematical entities are derived (at this time `OpNode`, `RelNode` and `SetNode`), the subclasses of which are the concrete implementations of

[2]For example, in Maple variables can hold relations as well as boolean or numerical values, making expressions like `y = (y=1);` or `plot(sin(true),x=0..Pi);` computable.

mathematical operations and operands. The tree of these nodes is referenced by the model of the mathematical entity (`Operation`, `Relation`, `BorelSet`), which serves as static container and fixed reference when performing tree transformations:

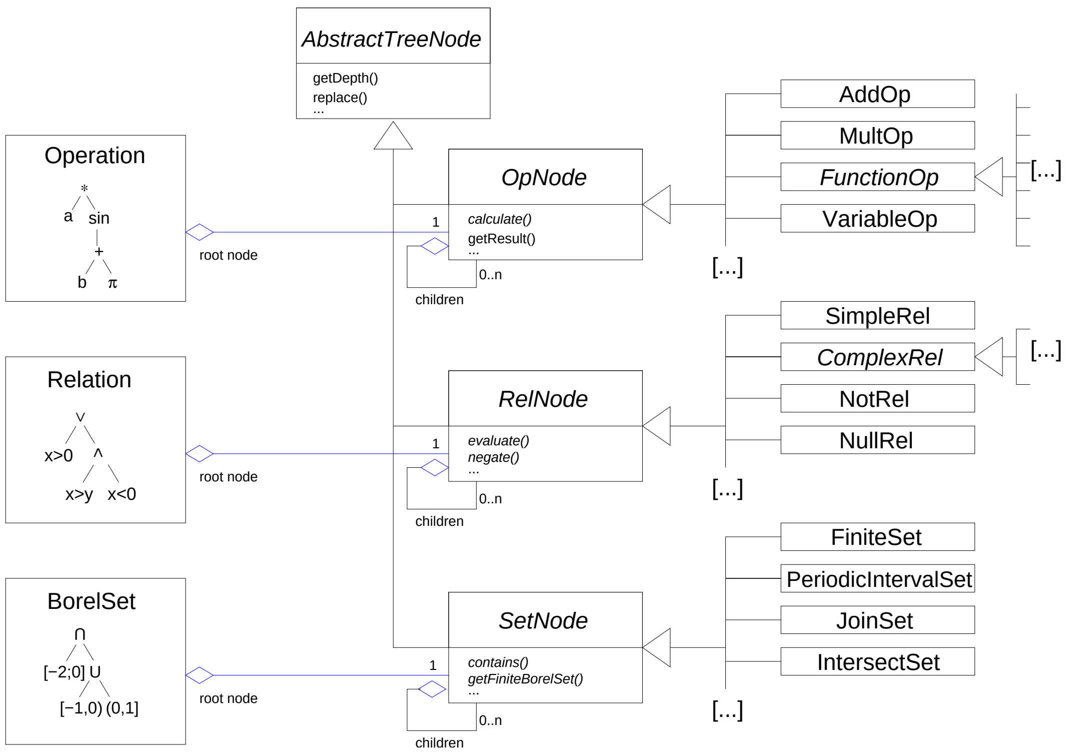


Fig. 5: Inheritance and aggregation model of tree nodes

D.0.13 Object Model of Operations

Generation and Structure of Operations The Language *Op* is syntactically analysed by an recursive descent parser. When parsing an expression it creates a corresponding operation tree that is contained in an `Operation` object. The operation tree consists of nodes that are of type `OpNode` and whose inner nodes are functions and operations while the leaves are basically variables and numbers. For example, parsing the expression "sin(2x+π)" generates the operation tree `(sin (+ (* 2 x) pi)`:

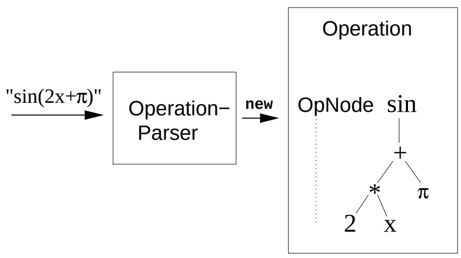


Fig. 6

The `Operation` class bundles the functionality for any operation that can be found in a function or on the left or right hand side of an equation, etc. It contains a reference to the operation tree and keeps track of state information like the variables used and normalisation status (see below).
The operation tree is made of instances of the class `OpNode`, which is an abstract class that provides the common functionality for all operation nodes. It models an elementary operation with a factor and an exponent so the above example could also have the form `(sin (+ 2x pi))`. From the functional view the factor and the exponent are not essentially necessary, since there are also nodes for power and multiplication operations, but with addition and multiplication being the most common operations, this reduces the depth of most trees. This again decreases the costs of analysis and synthesis of larger trees and allows a higher amount of order within the trees (for example, children of a multiplication node can be ordered by power, making fraction handling easier). Adding factor and exponent fields to an operation node is also a common technique used in CAS [Bau02].
Below, a part of the definition of `algebra.op.OpNode` is shown:

```
...

public abstract class OpNode implements Cloneable, Comparable, NumberTypeDependentIF {
  ...
  /** The base value, which is calculated in {@link #calculate}. */
  protected MMNumber m_base;

  /**
   *  A numerical factor is stored for each operation, to reduce the tree complexity
   *  and allow group actions.
   */
  protected MMNumber m_factor;

  /**
   *  The exponent is stored for each operation, to reduce the tree complexity
   *  and allow group actions.
   */
  protected int m_exponent = 1;

  /**
   *  The children of the node, may be null (leaves), one node (e.g.\ functions) or
   *  an arbitrary number of nodes (e.g.\ multiplication).
   */
  protected OpNode[] m_children;

  /** The direct ancestor of this node in the Operation Tree. */
  protected OpNode m_parent;

  /** The number class being used. */
  protected Class m_numberClass;

  ...
```

Operation Transformations and Normal Form As mathematics often consists of transforming expressions, operations can be transformed in multiple ways. For example, the addition of the number 3 to an existing operation can be done by creating a new addition node that has the old operation tree and a '3'-node as children and taking the addition node as the root node for the new operation. There are of course many other possible transformations like substitution, separation of variables, factorisation, derivation, etc. To implement these in an object model, a two level approach is used: Transformations that create a new operation tree with specific rules for each node (like calculating the derivative or the inverse) are handled by the node object itself, whereas transformations that alter the existing tree structure (expansion, simplification, etc.) are handled by a transformer facility called `OpTransform`. This two-level approach allows to combine the power of recursive algorithms with the structural advantage of object-orientation.

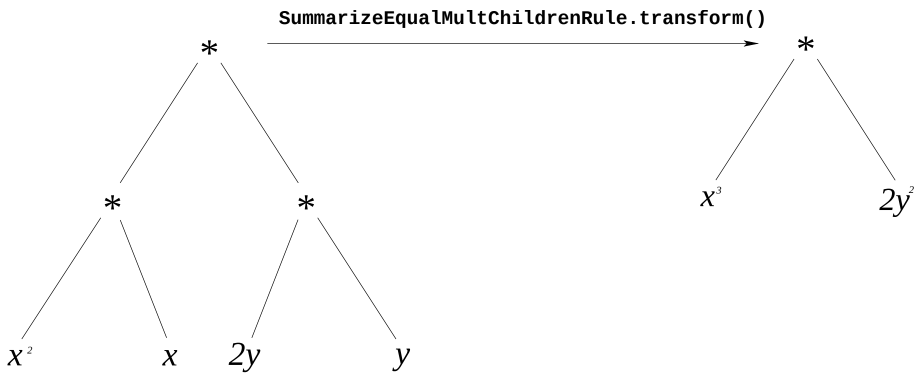


Fig. 7: Transformation of a subtree by a rule

The `OpTransform` object is a tree automaton that transforms trees by using a set of rules (located in the subpackage `algebra.op.rule`), which specify a certain subtree pattern and a transformation that is performed if the tree matches the pattern.

For example, in the tree (* (* x^2 x) (* 2y y)) (see figure) the condition for a multiplication node containing two or more equal children (without regarding their factors or exponents) applies to both '*' children of the root node. The rule object therefore raises the power of the first child by the number of the other children, transforming the tree to (* x^3 $2y^2$). This would also apply to the root node if a previous application of `CollapseEqualOpsRule` had flattened the tree to (* x^2 x 2y y).

In order to apply a rule like in the example above, a certain structure of the tree has to be assumed. For example, the tree `(* (* (^ x 2)) x)` is mathematically equivalent to (* x^2 x), but it is harder to analyse and transform. Because the rules should be kept as simple as possible (since there are many of them and their set should be easily expandable), it is reasonable to specify a normal form for operation trees.
The normalisation rules are located in `algebra.op.rule.normalize`. Here are some examples:

Name	Application Example
`NormalizeMultRule`	`(* 4x 6y)` $\rightarrow$ `24(* x y)`
`NormalizeExponentsRule`	`(*` x^4 y^6`)` $\rightarrow$ `(*` x^2 y^3`)`^{2}
`CollapsePowerRule`	`(^ (^ x y) z)` $\rightarrow$ `(^ x (* y z))`
`HandleFunctionSymmetryRule`	`(sin -x)` $\rightarrow$ `-(sin x)`

This set can be easily expanded, most of the rules consist of less than 20 lines of code. All the rules are applied in a deterministic order and since they always produce defined results, it can be ensured that after none of the rules can be applied to any node in the tree anymore, the operation has a unique defined form. This form is defined as the normal form of the operation (for the specific rule-set). Trees having the same normal form are considered to be mathematically equal by the system.

D.0.14 Object Model of Relations

Generation and Structure of Relations Analogous to operations, relations are usually generated by parsing a word of *Rel* (see the grammar in D.0.10) and by constructing a relation tree composed of `RelNodes`. The inner nodes of the relation tree are the logical operations $\vee$, $\wedge$ and $\neg$, while the leaves are either simple relations (equations or inequations) or special nodes like `AllRel` and `NullRel`, which either relate everything (making interpretation of the used domain necessary) or nothing.

Transformation of Relations, Normal Form Like operations, relations can be transformed in multiple ways, the most used transformations are the logic transformations that negate, conjunct or disjunct subtrees. But there are also complex transformations that occur when a simple relation is transformed. For example, if the inequation $x^2 - x > 0$ is divided by x this leads to the equivalent complex relation $x - 1 > 0 \wedge \frac{1}{x} > 0 \vee x - 1 < 0 \wedge \frac{1}{x} < 0$ (see figure).

The functionality for handling relation transformations and its special cases are implemented in the class `RelTransform`.

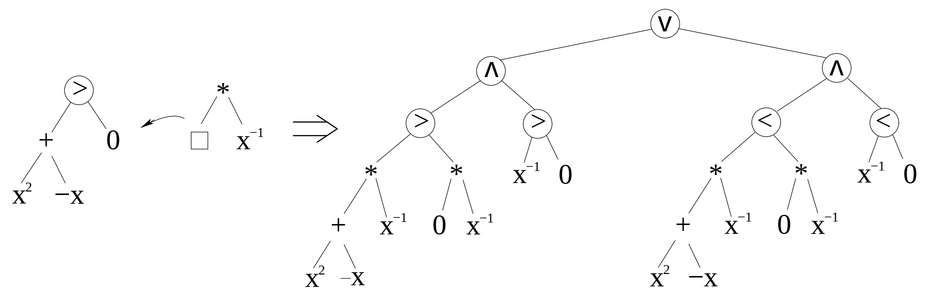


Fig. 8: Transformation of a relation

For comparation and simplification of relations, again, a higher order is needed for the trees to facilitate the development of tree analysis and transformation code.

For example, the relation $x = y \wedge y = x$ should be recognised as redundant by the system and be simplified to $x = y$. As for operation trees, this is done by defining a normal form for relations and implementing rules for normalisation.
The normalisation rules are located in `algebra.rel.rule.normalize`. Here are some examples:

Name	Application Example
`MoveOrUpwardsRule`	`[[x=0 OR y=0] AND z=0]` → `[x=0 AND z=0] OR [y=0 AND z=0]`
`CollapseComplexRule`	`[[x=0 AND y=0] AND [z=0]` → `[x=0 AND y=0 AND z=0]`
`RemoveNotRelRule`	`[NOT [x>=y]]` → `[x < y]`

D.1 Applications

To demonstrate the practical use of the algebraic object model, we give two application examples.

Symbolic Derivation The symbolic derivative of a differentiable function can be computed automatically by simply applying the well known rules for elementary functions and chaining them together for compound functions, using the derivation rules for sums, products and compositions.

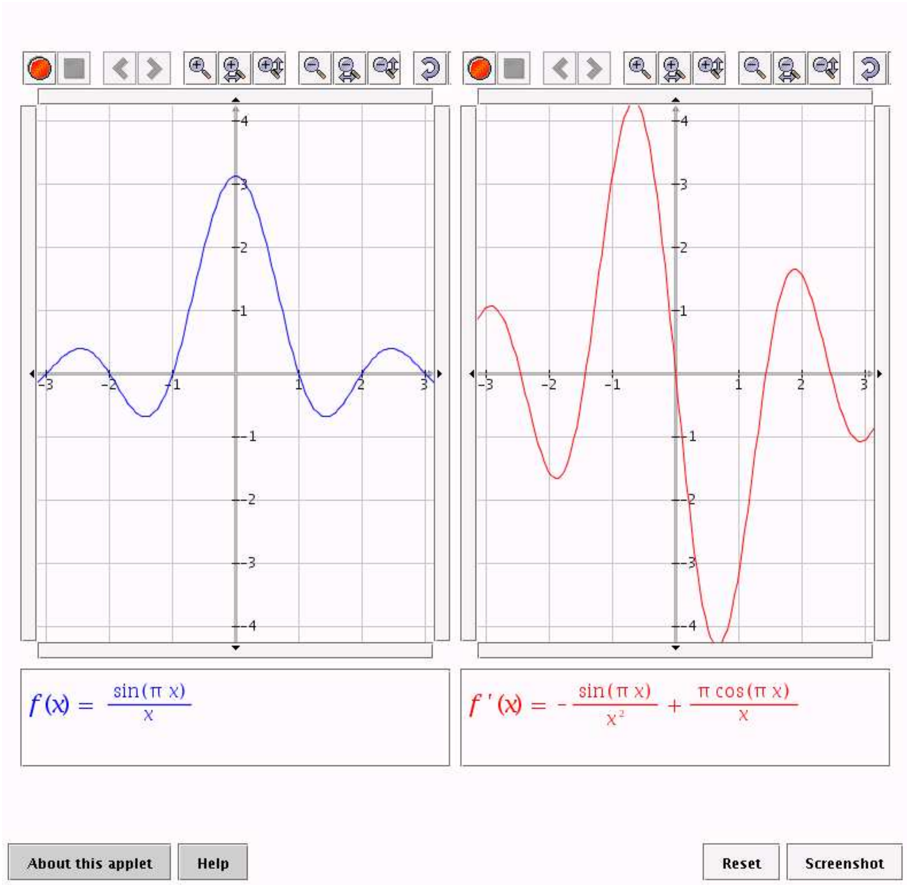


Fig. 9: The function and derivative plotter

The application presented to the user is an applet that plots an arbitrary function typed in by the user and additionally computes and displays the derivative (if any exists).

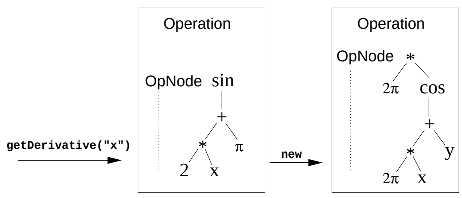


Fig. 10: Object model for deriving operations

From a technical perspective this is done by using an `MMFunctionDefinedByOperation` object from which another instance is created with an `Operation` that is returned by the `getDerivative()` method of the first operation.
Deriving an operation can be implemented completely on a per-node basis, therefore

the method `Operation.getDerivative()` simply delegates the calculation to the root node of the operation tree. The derivation code of an operation node has two parts: a generic part that does the derivation for the internal factor and exponent and a specific part that varies on the node type. Below is a code snippet containing the node specific derivation code in the class `SinOp`:

```
/**
 *  Implements <i> (sin(f(x)))' = cos(f(x)) * f`(x) </i>.
 */
public OpNode getDerivative(String variable){

  if(getMaxPowerOfVariable(variable) == 0)
    return new NumberOp(m_numberClass, 0);

  // cos(f(x))
  OpNode cosOp = new CosOp(m_numberClass);
  cosOp.setChildren(new OpNode[]{(OpNode)(m_children[0].clone())});

  // f'(x)
  OpNode derivedChild = m_children[0].getDerivative(variable);

  // cos(f(x)) * f'(x)
  MultOp derivedCosOp = new MultOp(m_numberClass);
  derivedCosOp.setChildren(new OpNode[]{cosOp, derivedChild});
  return deriveNode(derivedCosOp);
}
```

The generic part implemented in the abstract class `OpNode` is as follows

```
/**
 *  Implements <i>(m*a(x)^n)' = (n*a(x) ^ (n-1) * m*a'(x)</i>.
 *  @param derivedNode a'(x)
 */
protected OpNode deriveNode(OpNode derivedNode){
  if(m_exponent == 1){
    derivedNode.setFactor(m_factor.copy());
    return derivedNode;
  }
  MultOp derivedPower = new MultOp(m_numberClass);
  OpNode newPower = (OpNode)clone();
  newPower.m_factor.mult(NumberFactory.newInstance(m_numberClass, m_exponent));
  newPower.m_exponent--;
  derivedPower.setChildren(new OpNode[]{newPower, derivedNode});
  return derivedPower;
}
```

Definition Range of Operations Another useful application of the MathletFactory algebraic object model is the ability to compute the definition range of an operation. We have used this functionality to add an extra warning when displaying mathematical entities that use operations which are only partially defined.
When determining definition gaps, it poses a serious implementation problem to numerically calculate the complete definition range and display it graphically for any function (as the example $f(x) = \frac{1}{\sin(\frac{1}{x})}$ in the figure below shows).
The MathletFactory addresses this problem by displaying the definition range of a function as a symbolic expression. Though this expression often has an implicit form

(showing only a relation of the used variables that must be satisfied) the user is warned of existing definition gaps, and he is principally able to find out where they are.[3]

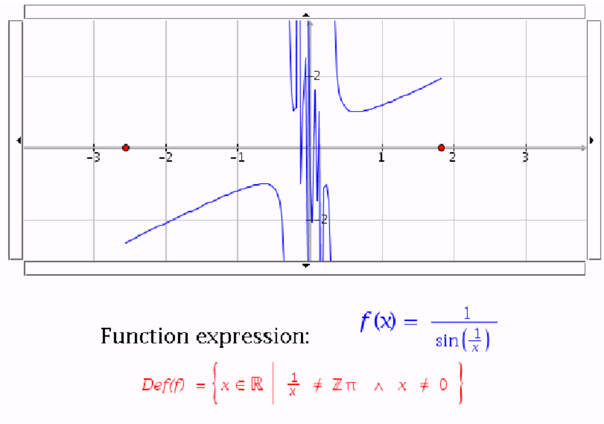


Fig. 11: The function plotter with definition range detector

From the technical perspective the computation of the definition range for an arbitrary operation is also solved on a per-node basis. By calling the method `getDefinedRel()` the `OpNode` object creates a `RelNode` object which represents a relation that a member of the definition range for the operation must satisfy. The `RelNode`s of the children of an `OpNode` are connected by an `AndRel` conjunction, forming a relation tree that is at last anchored in a `Relation` object by the enclosing `Operation.getDefinedRelation()` method. For the `OpNode`s there is – like the computation of the derivative – a generic and a node-specific implementation. The generic part, implemented in `OpNode` is as follows:

[3]Also, by using the `Relation.toContentMathML()` export method, an application programmer is able to link it with a computer algebra system or to implement a numerical solution of his own.

```
/**
 * Returns the relation for which the operation represented by this node is defined.
 * @see #getDefinedRel(OpNode operand)
 */
public RelNode getDefinedRel(){

  // by default the operation is defined totally for any variable
  RelNode definedRel = new AllRel(m_numberClass);

  // retrieve the definition range of this node with the child as argument
  // (non unary operations overload this method)
  if(m_children != null)
    definedRel = getNodeDefinedRel(m_children[0]);

  // for nodes with a negative exponent the base may not become zero
  if(getExponent() < 0){
    OpNode nodeWithoutExponent = (OpNode)clone();
    nodeWithoutExponent.setExponent(1);
    definedRel = new AndRel(definedRel, new NotRel(nodeWithoutExponent.getZeroRel()));
  }

  // intersect the defintion range of this node with the definion range of the children
  if(m_children == null)
    return definedRel;
  else
    return new AndRel(definedRel, getChildrenDefinedRel());
}

/**
 * Returns the relation subtree for which this operation with
 * <code>operand</code> as child is defined. It does not consider the
 * exponent of this node, which is be checked in {@link #getDefinedRel}.
 */
public abstract RelNode getNodeDefinedRel(OpNode operand);
```

The specific part is implemented by the subclasses of `OpNode` in `getNodeDefinedRel()`. Here is an example for `TanOp` returning a relation that says that the cosine of its operand may not be zero:

```
public RelNode getNodeDefinedRel(OpNode operand){
  return new NotRel(new CosOp(m_numberClass).getZeroRel((OpNode)operand.clone()));
}
```

D.2 Arithmetic and Geometric Symbolic View Architecture

One pattern that has been consequently used in the development of the symbolic view architecture is, that every mathematical inclusion is transformed to a ‘containedness’ relation on the view component level. This allows an easy development and support for a multitude of symbolic representations. We illustrate this with the example of constructing the parametric view for an affine plane in $\mathbb{R}^3$:

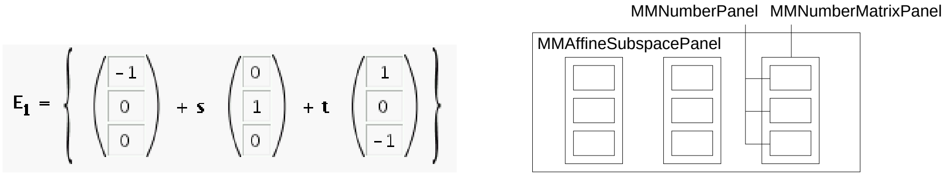


Fig. 12: The symbolic view of a plane and its structure

On Java level, the view is simply a subclass of `JPanel`, namely an `MMAffineSubspacePanel`. This class acts not only as a view for a plane in $\mathbb{R}^3$, but also for a line or point in $\mathbb{R}^3$ or in $\mathbb{R}^2$, thus making it possible to use it as a view of a dynamically generated affine subspace (like the intersection or join of other affine spaces). But let us first consider, that it displays a regular (non-degenerated) plane. In this case, we need to display three vector displays: one displaying the origin vector and two for the direction vectors. This is simply done by adding three `MMNumberMatrixPanel`s, which is the standard display component for a vector of any dimension, but also of course for any number matrix of arbitrary form. These panels in turn contain all entries as `MMNumberPanel`, the symbolic representation for numbers of any type.[4] A simple update mechanism using the Java property support ensures, that changes performed by the user are recorded by the master MMObject. On the other hand, if the mathematical state of the MMObject changes (e.g. by updating or user interactions on the graphical level), the changes are immediately displayed, allowing the user to continuously watch the symbolic perspective of his actions.
This applies not only to changing the vectors of the affine subspace, but also to changing its dimension: If the plane was the result of an intersection of two other planes (which were geometrically identical) and one of these planes changed, making the intersection a line or an empty space, the symbolic view adapts to these cases without complaints.

D.3 Algebraic Symbolic View Architecture

In the following, we will also sketch the view architecture used by the MathletFactory's algebraic object model, for details the commented source code and API documentation should be consulted.
The architecture for symbolically displaying algebraic entities is closely related to the structure of the algebraic object model: A tree of operation nodes is mapped on a tree of view nodes that recursively draw the expression on a panel, whereas a tree of relation nodes is mapped on a tree of panels with each parent containing its children.

D.3.1 View Architecture of Operations

The implementation of the symbolic view for an operation is the `OperationPanel`. This is a GUI component that draws the expression string on its screen area when

[4]Note, that the inclusion does not end there, for the number panel itself contains an operation panel (see below) to allow the display of constants like $\frac{3}{2}pi$, etc.

asked to repaint. This is done by view nodes (an analogon to the TEX noads[5]), each of which corresponds to an operation node of the `Operation`. Apart from the reference to its operation node, a view node keeps track of its metrics which is determined by the metrics of its children (if any) and the font of the panel. For example, the view node for a square root must know the width and height of its child expression in order to fully enclose the radicand (see figure).
So when a repaint event is sent to the operation panel, the panel asks its root view node to draw itself on the panel. If the root view node has children, it asks them to calculate their metrics (which in turn may depend on their children´s metrics, etc.) and then draws the expression it represents accordingly.

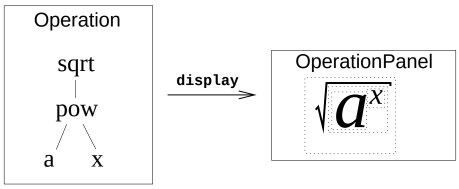


Fig. 13: Displaying operations, the dashed lines mark the `OpViewNodes` that draw on the `OperationPanel`.

D.3.2 View Architecture of Relations

The inclusion of operations in relations can also be transferred to the view architecture: The view component for a simple relation, a `SimpleRelationPanel` is merely a panel containing two `OperationPanels` with a relation sign label between them. Complex relations are displayed by instances of `RelationContainer`: Panels that contain either `SimpleRelationPanels` or other `RelationContainers`. The root node itself is contained in a component called `RelationPanel`.

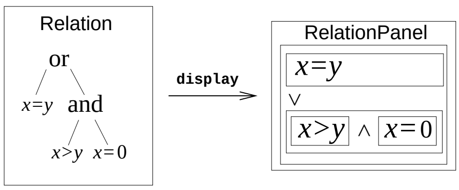


Fig. 14: Relation trees are rendered in a container hierarchy rooted by an `RelationPanel`.

D.3.3 Metrics of View Components

The MathletFactory uses the standard metrics model of typography[6]: The metrics of each glyph is characterised by its width, ascent and descent; for the rendering of fractions, sub- and superscripts the baseline must also be recorded. Operation view nodes

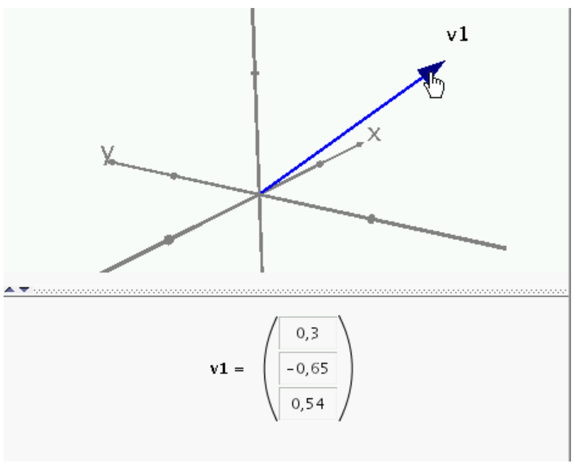
[5] [Kn82]
[6] [He93]

keep their metrics parameters in an object called `ViewNodeMetrics`. On the panel level (anything that uses `OperationPanels` or container trees of these) this is done by implementing the interface `Alignable`, which declares methods for retrieving the metrics parameters. By using this interface it is ensured that two operations can be horizontally aligned (i.e. having the same baseline), even if they have different heights or one of them has a border (e.g. for marking it as editable).

Fig. 15: The baseline, ascent and descent of a font.

D.4 Graphical View and Controller Architecture

At the time of writing, implementations for the Java2D system library and the Java3D API exist, but previous prototype implementations have also been tested with the third party graphics library JavaView[7]. The architecture works as follows: For each different display type there exists a specific canvas (a subclass of `MMCanvas`) that can be added to an applet like any other GUI component. If an application programmer wants to display a certain MMObject, he simply calls `addObject(MMObjectIF object)` in the canvas with the MMObject as argument and the systems automatically assigns the appropriate (or a previously chosen) visual representation.



[7] [JV04]

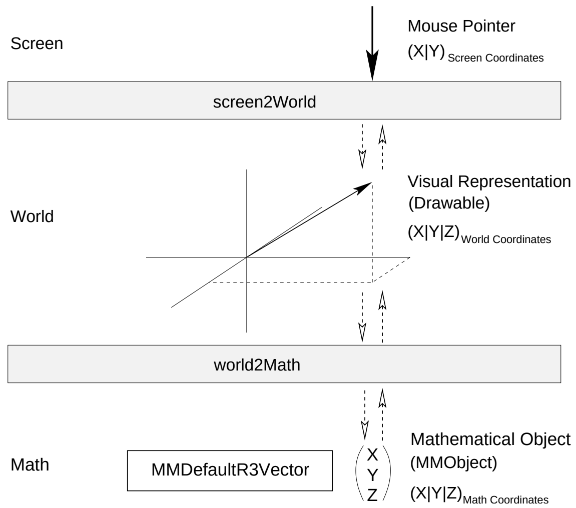


Fig. 16: Dragging a 3D vector: How the display and interaction system works

For example, when a user drags the end point of a 3D vector with the mouse, the mouse coordinates (measured in pixels) are transformed by a matrix `screen2World` into world coordinates. These describe a virtual space, where the visual representations of mathematical objects 'live'. The mathematical objects themselves reside in a separate coordinate space called the math space. This allows them to be independent of the worlds geometry and dimension, thus allowing, for example, projections from a spherical geometry into the euclidian world coordinate space. For euclidian math spaces the transformation is simply done by another matrix called `world2Math`. So when the user drags the vector, the mouse coordinates are transformed into world and math coordinates. The object's coordinates are changed and the graphical view updates accordingly, allowing further interaction. For 2D (or 1D) vectors the mechanism is the same.

We have given only a short overview of a complex system; yet, an application developer needs not know about all this, because there is a large set of prefabricated *handlers* that implement almost all desired functionality for manipulating MMObjects. So if an application developer wants to construct a mathlet, where the user may drag a vector (or any other MMObject), he only has to add the appropriate handler to it.

Bibliography

[ASU86] A: Aho, R. Sethi, J. Ullman. Compilers - Principles, Techniques and Tools. Addison Wesley. 1996.

[Bau02] C. Bauer, A. Frink, R. Kreckel.
Introduction to the GiNaC Framework for Symbolic Computation within the C++ Programming Language.
Journal of Symbolic Computation Volume 33, Number 1, 2002.

[Bu96] F. Buschmann et al. Pattern-Oriented Software Architecture, Volume 1: A System of Patterns. Addison-Wesley 1996.

[Co03] H. Comon et al. Tree Automata Techniques and Applications. Preprint, 2003.
http://www.grappa.univ-lille3.fr/tata

[He93] R. Hersch (Ed.). Visual and Technical Aspects of Type. Cambridge University Press. 1993.

[HU79] J.E. Hopcroft and J.D. Ullman. Introduction to Automata Theory, Languages and Computation. Addison Wesley. 1979.

[JV04] JavaView homepage. 2004.
http://www.javaview.de

[Kn82] D. Knuth. Documented TEX source code. 1982.
http://www.ctan.org/tex-archive/systems/knuth/tex/tex.web.

[Mo93] K. Morisse. Datenstrukturen und Speicherverwaltung in MuPAD. In: mathPAD Vol. 3 No. 2. 1993.
http://www.mupad.de/mathpad.shtml

[Wa91] Maple Language Reference Manual. Waterloo Maple Publishing. 1991.

[Wo91] S. Wolfram. The Mathematica Book. Cambridge. 1991.

Appendix E

Miscellaneous Topics

E.1 Commands for embedding applets in websites

It is possible to embed applets in websites via the traditional (and even deprecated) `<applet>`-tag or via the recommended `<object>`-tag.
Below is a template for an `<object>`-tag:

```
<object classid="java:MyApplet.class" codetyte="application/java"
      codebase="./" archive="MyApplet.jar" width="400" height="300">
  <param name="parameter1" value="...value...">
  <param name="parameter2" value="...value...">
</object>
```

Overview of attributes:

- `classid` specifies the name of the applet with prefix "java:" and suffix ".class". Note that no path must be specified!

- `codebase` specifies the path to the class-file relative to the HTML page's location

- `archive` specifies a comma-separated list of needed archive files (e.g. zipped libraries where also the applet class can be located)

E.2 Table: DisplayProperties in MMObjects and Drawables

Properties-Class	implementing MM-Class	using Drawable

`LineDisplayProperties`	`MMAffine2DLine`
	`MMAffine2DLineSegment`
	`MMAffine2DRay`
	`MMAffine3DLine`
	`MMAffine3DLineSegment`
	`MMCoordinateSystem`
	`MMDefaultRNVector`
	`MMVectorField2DOverR2-DefByComponents`
	`MMVectorField2DOverR2-DefByExpression`
		`G2DLineDrawable`
		`J3DLineSegmentDrawable`
		`J3DPolyLineDrawable`

`PointDisplayProperties`	`MMAffine2DPoint`
	`MMAffine3DPoint`
	`MMDefaultRN`
	`MMBezierPolynomialAdvanced`
		`G2DPointDrawable`
		`J3DPointDrawable`

`PolygonDisplayProperties`	`MMAffine2DPolygon`
	`MMFunctionDefByOp`
	`MMFunctionDefinedByExpression`
	`MMFunctionDefinedBySamples`
	`MMPiecewiseFunction`
	`MMStepFunction`
	`MMBezierPolynomialAdvanced`
	`MMPolynomial`
	`MMParametricFunctionInR2`
	`MMOneChainInR2`
		`G2DPolygonDrawable`

`SurfaceDisplayProperties`	`MMFunctionOverR2`
	`MMParametricFunctionInR3`
	`MMAffine3DPlane`
	`MMAffine3DSubspace`
		(no explicit drawable)

E.3 Classes and their locations/packages

This section lists the most used classes with their location inside the library. All following package descriptions must have the explicit prefix `net.mumie.mathletfactory`.

Class	Package
ActionManager	action
Affine2DKeyboardGridTranslateHandler	action.handler
Affine2DKeyboardTranslateHandler	action.handler
Affine2DMouseGridTranslateHandler	action.handler
Affine2DMouseTranslateHandler	action.handler
Affine3DKeyboardTranslateHandler	action.handler
Affine3DMouseTranslateHandler	action.handler
AffineLineSegmentBetweenPointsUpdater	action.updater
Animation	uitl.animation
AnimationDependencyAdapter	uitl.animation
AnimationDependencyUpdater	uitl.animation
BaseApplet	appletskeleton
BasicApplicationFrame	util
CanvasControllerIF	action
CanvasImage	util
CanvasMessage	util
Canvas2DObjectTransformer	transformer
Canvas3DObjectTransformer	transformer
ContainerObjectTransformer	transformer
ControlPanel	appletskeleton.util
DefaultCanvasController	action
DependencyAdapter	action.updater
DependencyUpdater	action.updater
DisplayProperties	display
FunctionAndDerivativeOverRIF	math.analysis.function
FunctionOverBorelSetIF	math.analysis.function
FunctionOverRIF	math.analysis.function
GeneralTransformer	transformer
LinearMap	math.algebra.linalg
LinearMapDefByVectorsUpdater	action.updater
LineDisplayProperties	display

Class	Package
MathUtilLib	math.util
MatrixLayout	appletskeleton.util
MMAffine2DEllipse	mmobject.geom.affine
MMAffine3DEllipse	mmobject.geom.affine
MMAffine2DLine	mmobject.geom.affine
MMAffine3DLine	mmobject.geom.affine
MMAffine2DLineSegment	mmobject.geom.affine
MMAffine3DLineSegment	mmobject.geom.affine
MMAffine2DPoint	mmobject.geom.affine
MMAffine3DPoint	mmobject.geom.affine
MMComplex	mmobject.number
MMComplexRational	mmobject.number
MMCoordinateSystem	mmobject.geom.affine
MMDefaultCanvasObject	mmobject
MMDefaultObject	mmobject
MMDouble	mmobject.number
MMEditablePanel	display.noc
MMEquationSystem	mmobject.algebra
MMFunctionDefByOp	mmobject.analysis.function
MMFunctionDefinedByExpression	mmobject.analysis.function
MMFunctionDefinedBySamples	mmobject.analysis.function
MMFunctionPanel	display.noc.function
MMG2DCanvas	display.g2d
MMInteger	mmobject.number
MMInterval	mmobject.set
MMJ3DCanvas	display.j3d
MMNumberMatrix	mmobject.algebra.linalg
MMNumberMatrixPanel	display.noc.matrix
MMNumberPanel	display.noc.number
MMNumberSet	mmobject.set
MMNumberTuple	mmobject.algebra.linalg
MMObjectIF	mmobject
MMOpMatrix	mmobject.algebra.linalg
MMPanel	display.noc
MMPiecewiseFunction	mmobject.analysis.function
MMPolynomial	mmobject.algebra.poly
MMRational	mmobject.number
MMRelation	mmobject.algebra
MMSetDefByRel	mmobject.set
MMString	mmobject.util
MMStringMatrix	mmobject.util
MumieTheme	appletskeleton.util
NoCanvasApplet	appletskeleton

Class	Package
NumberFactory	math.number
NumberMatrix	math.algebra.linalg
NumberTuple	math.algebra.linalg
NumberTypeDependentIF	math.number
OneChainInRIF	math.analysis.function
OneChainInRNIF	math.analysis.function.multivariate
Operation	math.algebra.op
OperationPanel	display.noc.op
OpMatrix	math.algebra.linalg
OpParser	math.algebra.op
OpTuple	math.algebra.linalg
PointDisplayProperties	display
PolygonDisplayProperties	display
Progress	util.animation
PropertyHandlerIF	mmobject
Relation	math.algebra.rel
RelParser	math.algebra.rel
SequenceAdapter	math.analysis.sequence
SequenceIF	math.analysis.sequence
SeriesIF	math.analysis.sequence
SideBySideG2DCanvasApplet	appletskeleton.g2d
SideBySideJ3DCanvasApplet	appletskeleton.j3d
SingleG2DCanvasApplet	appletskeleton.g2d
SingleJ3DCanvasApplet	appletskeleton.j3d
SpecialCaseEvent	action.message
SpecialCaseListener	action.message
Step	util.animation
SurfaceDisplayProperties	display
TabbedPanel	appletskeleton.util
TextLayout	appletskeleton.util
TextPanel	appletskeleton.util
UpperLowerG2DCanvasApplet	appletskeleton.g2d
UpperMiddleLowerG2DCanvasApplet	appletskeleton.g2d
UsesOpArrayIF	math.algebra.op
UsesOperationIF	math.algebra.op
UsesRelationIF	math.algebra.rel
VectorField2DOverR2IF	math.analysis.vectorfield
VectorFunctionOverBorelSetIF	math.analysis.function.multivariate

Class	Package
VectorFunctionOverDomainIF	math.analysis.function.multivariate
VectorFunctionOverRIF	math.analysis.function.multivariate

E.4 Code: TriangleAltitude-Applet

```
<...> // some imports omitted

public class TriangleAltitude extends SingleG2DCanvasApplet{

  private MMAffine2DPoint A, B, C;
  private MMAffine2DLineSegment AB, BC, CA;
  private MMAffine2DLineSegment aFootC, bFootA, cFootB, bFootC, cFootA, aFootB;
  private MMAffine2DLineSegment altitude_AB, altitude_BC, altitude_CA;

  private PointDisplayProperties pp;
  private LineDisplayProperties ll, kk, mm;

  private Affine2DKeyboardTranslateHandler akth;
  private Affine2DMouseTranslateHandler amth;

  public void init() {
    setTitle("Triangle Altitude Test");
    createObjects();
    initializeObjects();
    createDependencies();

    getCanvas().addObject(aFootC);
    getCanvas().addObject(bFootC);
    getCanvas().addObject(bFootA);
    getCanvas().addObject(cFootA);
    getCanvas().addObject(cFootB);
    getCanvas().addObject(aFootB);
    getCanvas().addObject(altitude_AB);
    getCanvas().addObject(altitude_BC);
    getCanvas().addObject(altitude_CA);
    getCanvas().addObject(AB);
    getCanvas().addObject(BC);
    getCanvas().addObject(CA);
    getCanvas().addObject(A);
    getCanvas().addObject(B);
    getCanvas().addObject(C);

    addResetButton();
    addScreenShotButton();
  }

  private void createObjects() {
    amth = new Affine2DMouseTranslateHandler(getCanvas());
    akth = new Affine2DKeyboardTranslateHandler(getCanvas());
    pp = new PointDisplayProperties();
    ll = new LineDisplayProperties();
    mm = new LineDisplayProperties();
    kk = new LineDisplayProperties();
```

```
  A = new MMAffine2DPoint(MDouble.class, -0.3, 0.3);
  A.addHandler(akth);
  A.addHandler(amth);
  B = new MMAffine2DPoint(MDouble.class, 0.25, 0.25);
  B.addHandler(akth);
  B.addHandler(amth);
  C = new MMAffine2DPoint(MDouble.class, -0.25, -0.25);
  C.addHandler(akth);
  C.addHandler(amth);

  AB = new MMAffine2DLineSegment(A, B);
  BC = new MMAffine2DLineSegment(B, C);
  CA = new MMAffine2DLineSegment(C, A);

  aFootC = new MMAffine2DLineSegment(A, getPerpendicularFoot(A, B, C));
  bFootC = new MMAffine2DLineSegment(B, getPerpendicularFoot(A, B, C));
  bFootA = new MMAffine2DLineSegment(B, getPerpendicularFoot(B,C, A));
  cFootA = new MMAffine2DLineSegment(C, getPerpendicularFoot(B,C, A));
  cFootB = new MMAffine2DLineSegment(C, getPerpendicularFoot(C,A, B));
  aFootB = new MMAffine2DLineSegment(A, getPerpendicularFoot(C,A, B));

  altitude_AB = new MMAffine2DLineSegment(C, getPerpendicularFoot(A,B, C));
  altitude_BC = new MMAffine2DLineSegment(A, getPerpendicularFoot(B,C, A));
  altitude_CA = new MMAffine2DLineSegment(B, getPerpendicularFoot(C,A, B));
}

protected void initializeObjects(){
  amth.setDrawDuringAction(true);
  amth.setUpdateDuringAction(true);

  pp.setObjectColor(Color.blue);
  ll.setObjectColor(Color.red);
  mm.setObjectColor(Color.red);
  mm.setFilled(false);
  kk.setObjectColor(Color.yellow);

  A.setDisplayProperties(pp);
  B.setDisplayProperties(pp);
  C.setDisplayProperties(pp);

  AB.setDisplayProperties(ll);
  BC.setDisplayProperties(ll);
  CA.setDisplayProperties(ll);

  aFootC.setDisplayProperties(mm);
  bFootC.setDisplayProperties(mm);
  bFootA.setDisplayProperties(mm);
  cFootA.setDisplayProperties(mm);
  cFootB.setDisplayProperties(mm);
  aFootB.setDisplayProperties(mm);
```

```
    altitude_AB.setDisplayProperties(kk);
    altitude_BC.setDisplayProperties(kk);
    altitude_CA.setDisplayProperties(kk);

    A.setFromXY(-0.3, 0.3);
    B.setFromXY(0.25, 0.25);
    C.setFromXY(-0.25, -0.25);
  }

  public void createDependencies();
    DependencyAdapter DPA = new DependencyAdapter() {
      public void doUpdate(MMObjectIF dependant, MMObjectIF[] free) {
        MMAffine2DLineSegment line = (MMAffine2DLineSegment) dependant;
        line.setInitialPoint((MMAffine2DPoint)free[0]);
        line.setEndPoint((MMAffine2DPoint)free[1]);
      }
    };
    AB.dependsOn(new MMObjectIF[]{A, B}, DPA);
    BC.dependsOn(new MMObjectIF[]{B, C}, DPA);
    CA.dependsOn(new MMObjectIF[]{C, A}, DPA);

    DPA = new DependencyAdapter() {
      public void doUpdate(MMObjectIF dependant, MMObjectIF[] free) {
        MMAffine2DLineSegment line = (MMAffine2DLineSegment) dependant;
        line.setInitialPoint((MMAffine2DPoint)free[2]);
        line.setEndPoint(getPerpendicularFoot((MMAffine2DPoint)free[0],
                                              (MMAffine2DPoint)free[1],
                                              (MMAffine2DPoint)free[2]));
      }
    };
    altitude_AB.dependsOn(new MMObjectIF[]{A, B, C}, DPA);
    altitude_BC.dependsOn(new MMObjectIF[]{B, C, A}, DPA);
    altitude_CA.dependsOn(new MMObjectIF[]{C, A, B}, DPA);

    DPA = new DependencyAdapter() {
      public void doUpdate(MMObjectIF dependant, MMObjectIF[] free) {
        MMAffine2DLineSegment line = (MMAffine2DLineSegment) dependant;
        line.setInitialPoint((MMAffine2DPoint)free[0]);
        line.setEndPoint(getPerpendicularFoot((MMAffine2DPoint)free[1],
                                              (MMAffine2DPoint)free[2],
                                              (MMAffine2DPoint)free[3]));
      }
    };
    aFootC.dependsOn(new MMObjectIF[]{A, A, B, C}, DPA);
    bFootC.dependsOn(new MMObjectIF[]{B, A, B, C}, DPA);
    bFootA.dependsOn(new MMObjectIF[]{B, B, C, A}, DPA);
    cFootA.dependsOn(new MMObjectIF[]{C, B, C, A}, DPA);
    cFootB.dependsOn(new MMObjectIF[]{C, C, A, B}, DPA);
    aFootB.dependsOn(new MMObjectIF[]{A, C, A, B}, DPA);
  }
```

```
  public void reset(){
    initializeObjects();
    getCanvas().renderScene();
    getCanvas().repaint();
  }

  private MMAffine2DPoint getPerpendicularFoot(MMAffine2DPoint A,
                                               MMAffine2DPoint B,
                                               MMAffine2DPoint C){
  <...>
  }

  public static void main(String[] args){
    TriangleAltitude myApplet = new TriangleAltitude();
    myApplet.init();
    BasicApplicationFrame f = new BasicApplicationFrame(myApplet, 500);
    f.pack();
    f.setVisible(true);
  }
}
```