Scientific Programming and Dynamic Modelling in Julia

April 27, 2022

1 Exercise 1 - Solutions

1.1 Logistic Map

1.1.1 Code the Logistic Map

Write a function that returns a N steps long trajectory of a logistic map given an initial condition x_0 and parameter value r, where $1 < r < 4$.

[1]:
```
import Pkg
Pkg.activate(".")

using Plots

f(x; r=3) = r*x*(1-x) # thats the rhs

# that's a slower version of the code
function logistic_map_slow(x_0, r, N=50)
    x = [x_0]

    for i=2:N
        push!(x, f(x[i-1];r))
    end
    x
end

# that's a faster version that pre-allocates memory, T here stands for a␣
 ↪non-specified type
# it could be a Float64, the compiler automatically recognizes what is needed.
function logistic_map(x_0::T, r::T, N::Integer=50) where {T}
    x = zeros(T,N)
    x[1] = x_0

    for i=2:N
        x[i] = f(x[i-1]; r)
    end
    x
end
```

```
  Activating project at `~/Nextcloud/TUMLecture/dyn-
modelling`
```

```
logistic_map (generic function with 2 methods)
```

1.1.2 Plot trajectories

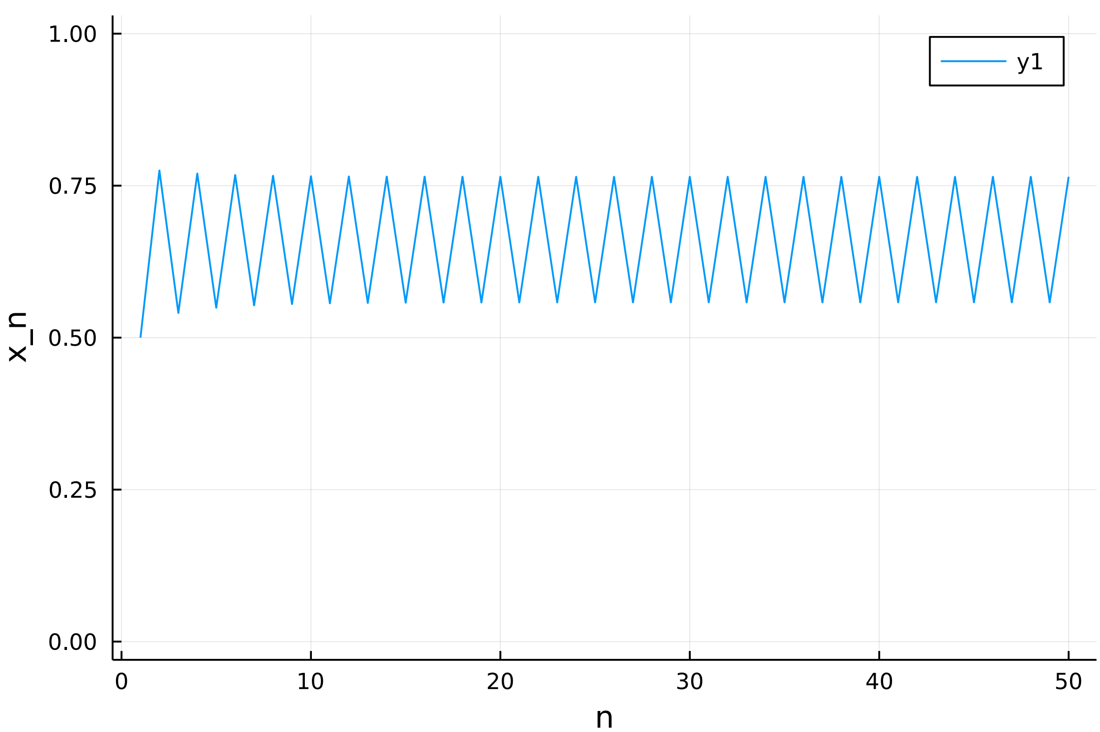
Julia has one major plotting library `Plots.jl` that can use different backends for plotting (like Python's matplotlib or plotly). After importing the library with `using Plots`, a basic plot is called by `plot(x,y)`. If you want to add to an exisiting plot, use `plot!(x,y)`. You can adjust the plot by adding keyword arguments. Some common keyword arguments are:

- `ylims=[lower_limit, upper_limit]`
- `xlims=[lower_limit, upper_limit]`
- `title`
- `xlabel`
- `ylabel`
- all further keyword arguments are listed there: https://docs.juliaplots.org/stable/attributes/

For those how are familiar with Python, you can also use `matplotlib.pyplot` directly, there is a Julia wrapper, called `PyPlot.jl`. The syntax is almost the same as in Python. See its documentation (https://github.com/JuliaPy/PyPlot.jl) for how exactly it translates.

Now, plot trajectories of the logistic map for different values of $1 < r < 4$, that are $N = 50$ steps long.

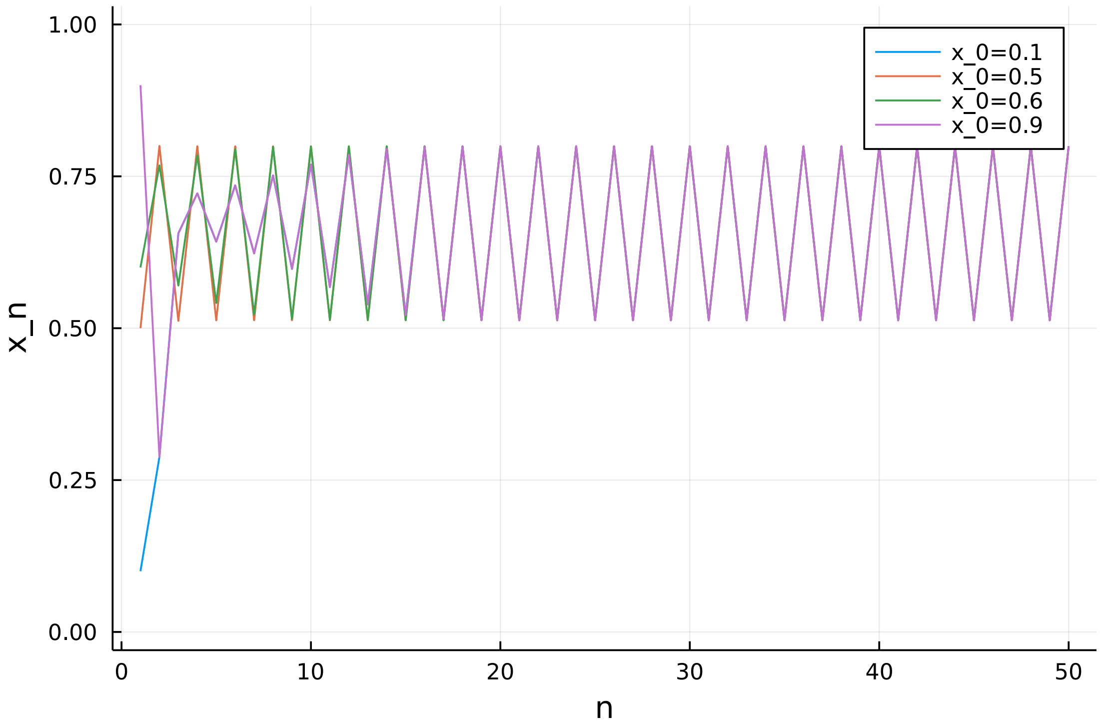
[2]:
```
plot(logistic_map(0.5, 3.1),ylims=[0,1],xlabel="n",ylabel="x_n")
```



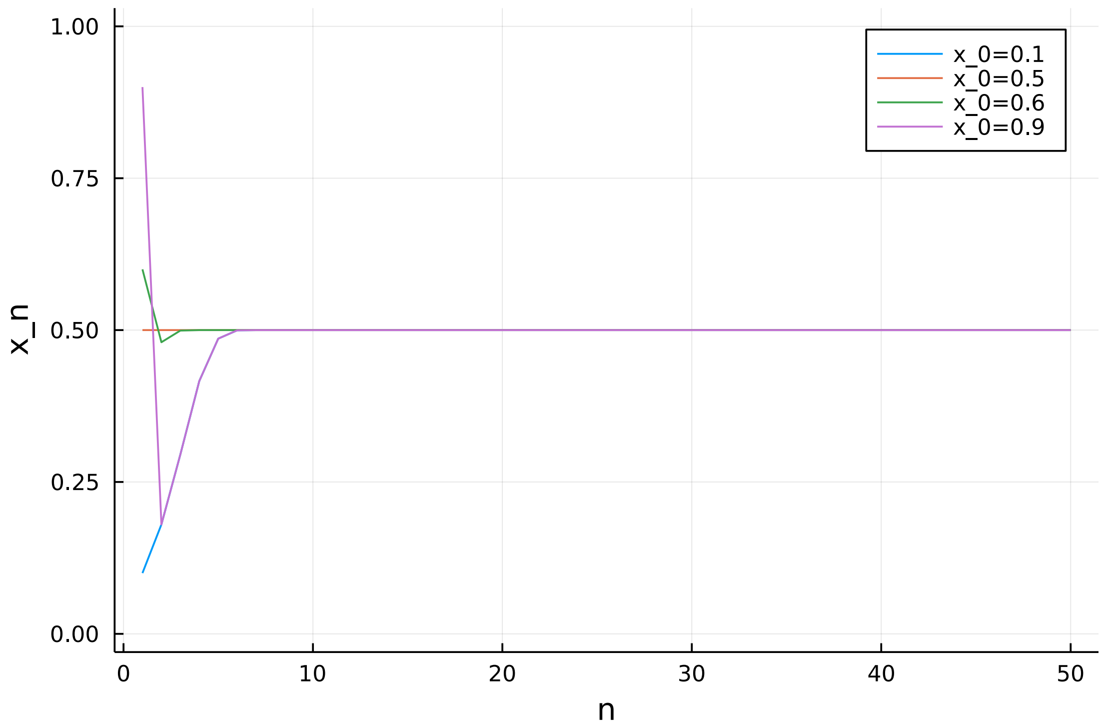
1.1.3 Explore the Logistic Map

If you keep the r constant, e.g. at $r = 2.9$ and $r = 3.2$, what are you observing for different initial conditions x_0?

[3]:
```
plot(logistic_map(0.1, 3.2),ylims=[0,1],xlabel="n",ylabel="x_n",label="x_0=0.1")
plot!(logistic_map(0.5, 3.2),ylims=[0,1],label="x_0=0.5")
plot!(logistic_map(0.6, 3.2),ylims=[0,1],label="x_0=0.6")
plot!(logistic_map(0.9, 3.2),ylims=[0,1],label="x_0=0.9")
```

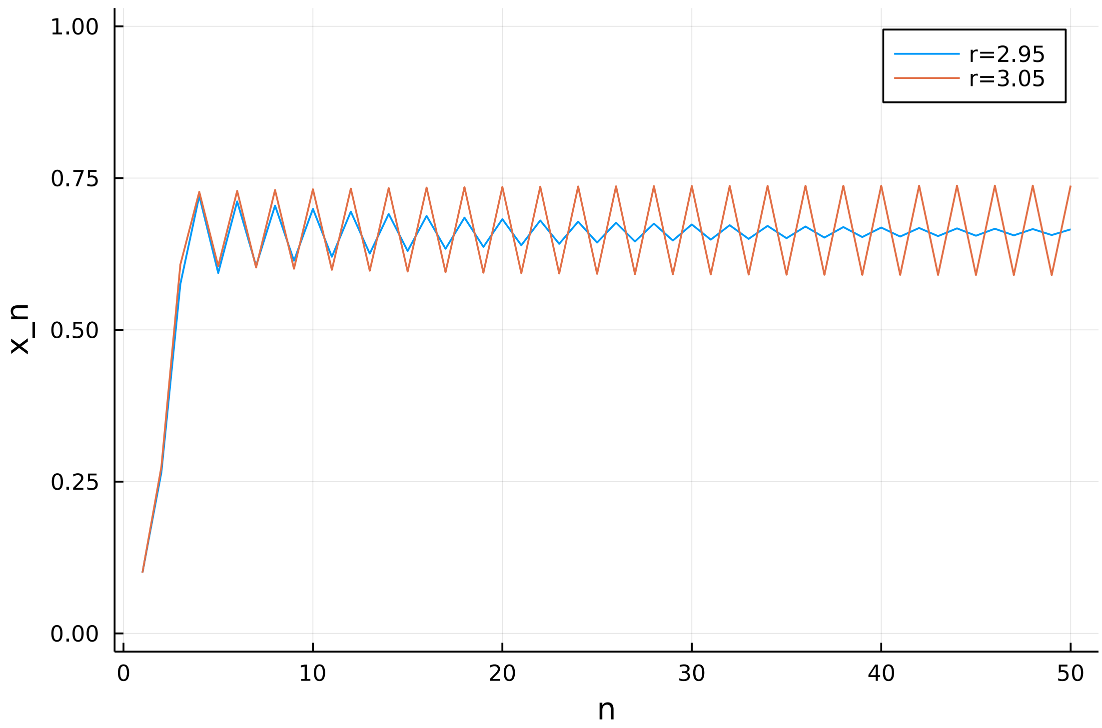


[4]:
```
plot(logistic_map(0.1, 2.),ylims=[0,1],xlabel="n",ylabel="x_n",label="x_0=0.1")
plot!(logistic_map(0.5, 2.),ylims=[0,1],label="x_0=0.5")
plot!(logistic_map(0.6, 2.),ylims=[0,1],label="x_0=0.6")
plot!(logistic_map(0.9, 2.),ylims=[0,1],label="x_0=0.9")
```

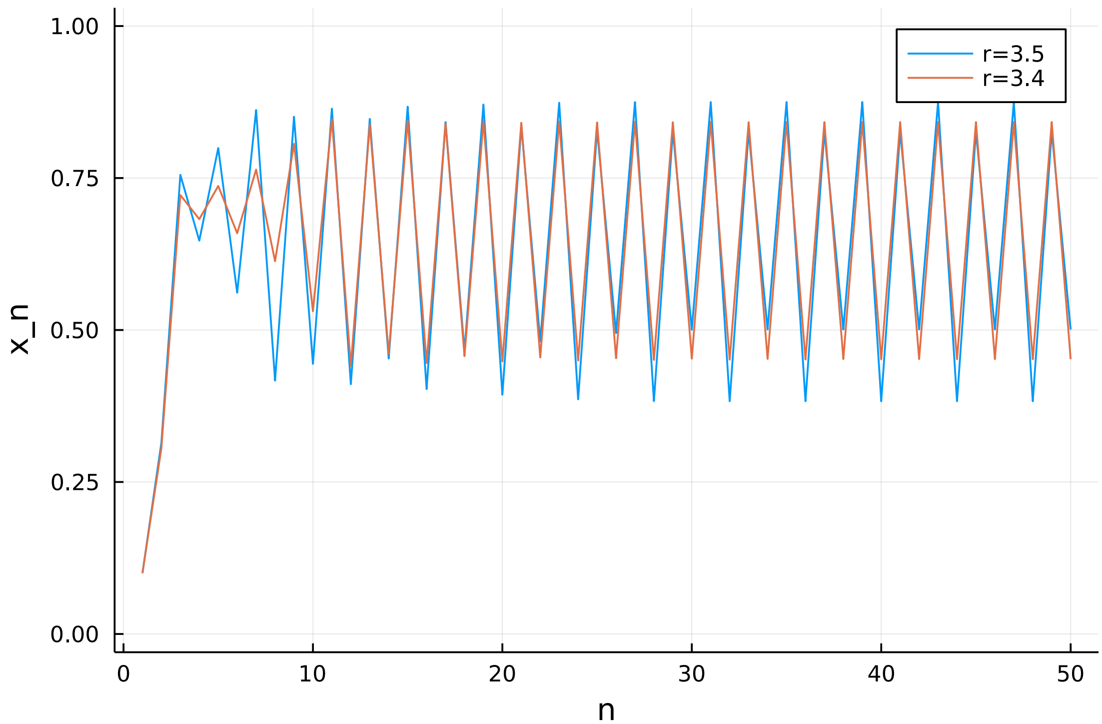


At which points $r_c \in [2.7; 3.6]$ does the trajectory change its behaviour and how?

[5]:
```
plot(logistic_map(0.1, 2.95),ylims=[0,1],label="r=2.95",xlabel="n",ylabel="x_n")
plot!(logistic_map(0.1, 3.05),ylims=[0,1],label="r=3.05")
# Oscillations don't damp out any more at r=3
```



[6]:
```
plot(logistic_map(0.1, 3.5),ylims=[0,1],label="r=3.5",xlabel="n",ylabel="x_n")
plot!(logistic_map(0.1, 3.4),ylims=[0,1],label="r=3.4")
```



The period of the oscillation doubles, at $r = 3.4$ the oscillation has a period 2 and at $r = 3.5$ the period is 4. That's the first period doubling of the logistic map.

The rest, we will do a bit more systematic now

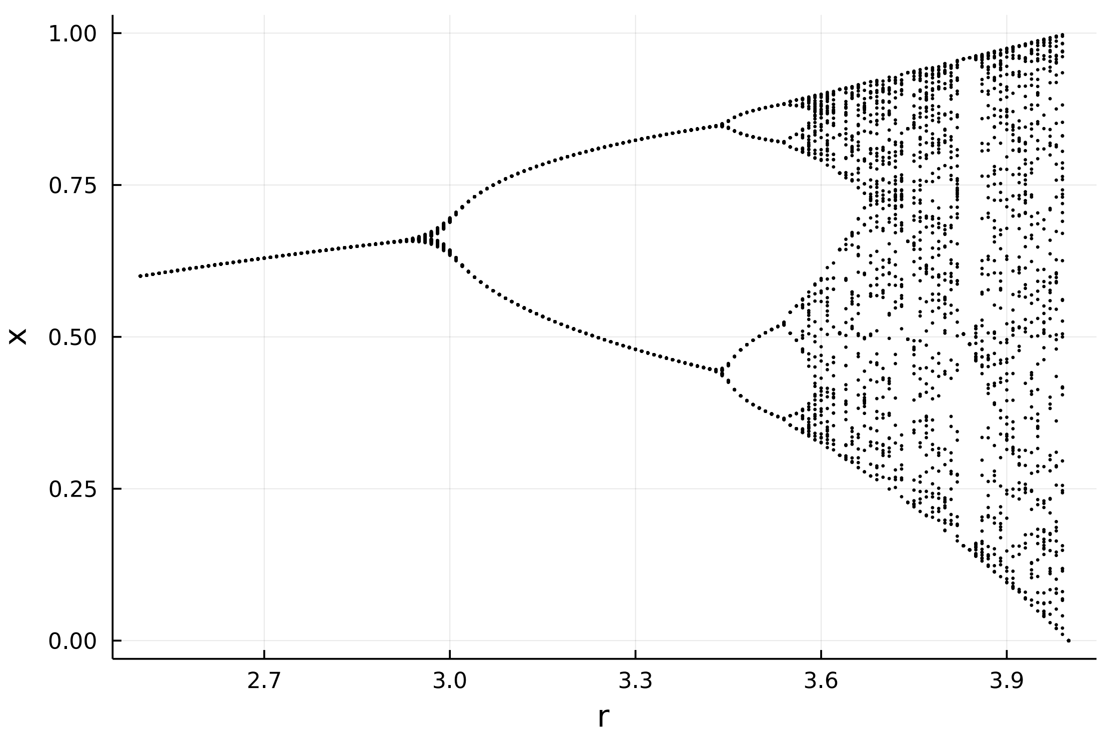
1.1.4 Plotting a famous diagram

Plot a diagram with r on the x axis and the last 50 points of 100 steps long trajectories of a logistic map on the y axis for $2.5 < r < 4$. Use the same initial conditions x_0 for every trajectory.

Tips

- use **scatter!** For the plots. The keyword argument **markersize** determines the size of the scatter points, it should be < 1 here
- If you use any plot inside of a loop, use the **show=true** argument so that your editor really shows the plot

[7]:
```
r_vals = 2.5:0.01:4
plot(0,0) # initialize an empty plot
for r   r_vals
    y = logistic_map(0.5, r, 100)
    scatter!(ones(50)*r, y[51:end], xlims=[r_vals[1], r_vals[end]],ylims=[0,1], show=true, markersize = 0.8,legend=nothing)
end
plot!(0,0, xlabel="r",ylabel="x")
```



What you are seeing is a simple version of a orbit diagram or bifurcation diagram. We will talk about what happens there in the next lectures.