In [ ]:

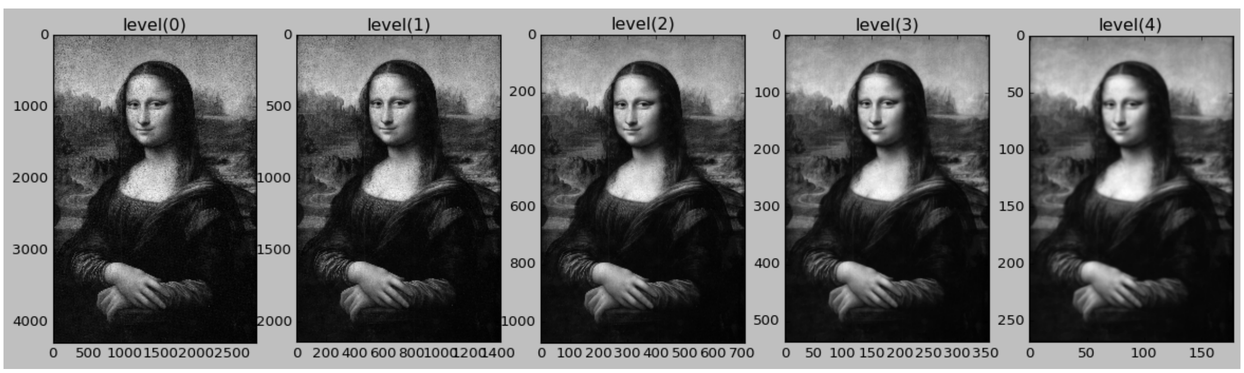
```python
import cv2
import matplotlib.pyplot as plt
import numpy as np
from functions import *
plt.style.use(plt.style.available[5])
```

5.1. Pyramid

5.1.1. For the “Mona Lisa” image, build a 5 level Gaussian pyramid and display it in a format. Also, implement and display a Laplacian (difference of Gaussian (DoG)) pyramid.

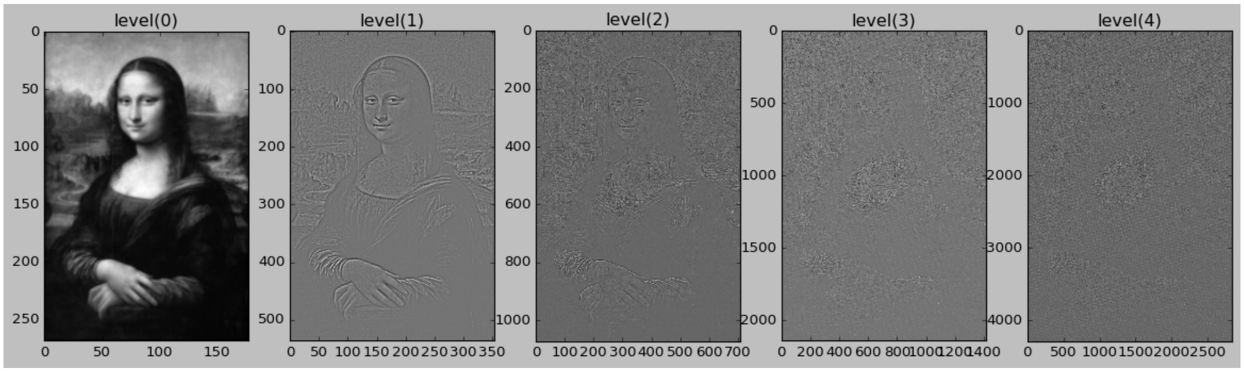
In [ ]:

```python
mona = cv2.imread('mona lisa.jpg',cv2.IMREAD_GRAYSCALE)

# gaussian pyramid
gaussian_pyr = gaussian_pyramid(mona,4)

figure = plt.figure(figsize=(18,18))
n = len(gaussian_pyr)

for i in range(n):
    figure.add_subplot(1,n,i+1)
    plt.imshow(gaussian_pyr[i],cmap='gray')
    plt.title(f"level({i})",color='black')
```



In [ ]:

```python
# laplacian pyramid
l_pyramid = laplacian_pyramid(gaussian_pyr)

figure = plt.figure(figsize=(18,18))
n = len(l_pyramid)

for i in range(n):
    figure.add_subplot(1,n,i+1)
    plt.imshow(l_pyramid[i],cmap='gray')
    plt.title(f"level({i})",color='black')
```



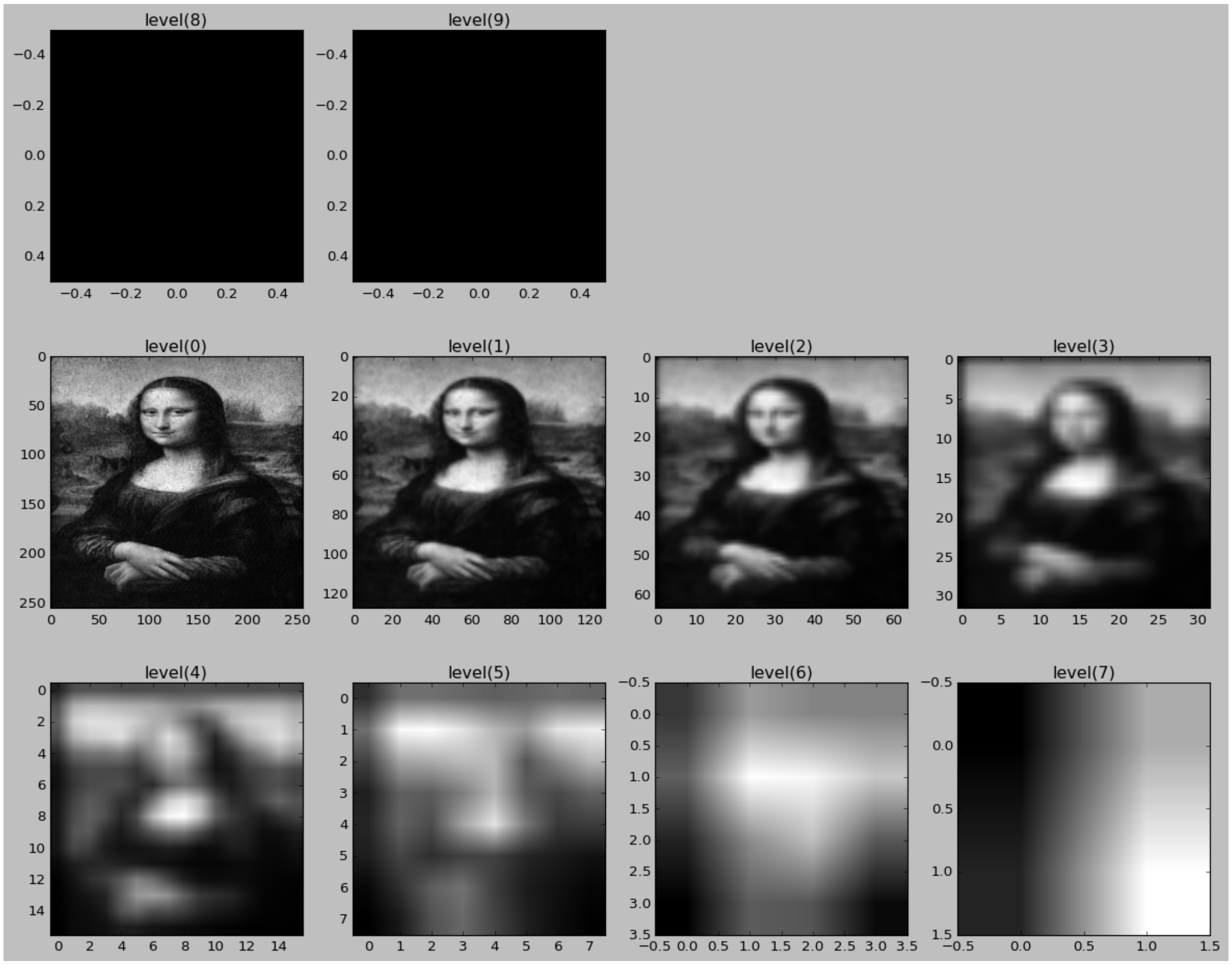
5.1.2. Describe how separability and cascading can help to speed up Gaussian smoothing and design a fast algorithm for computing a 3-step gaussian pyramid (filtered with σ, √2σ, 2σ) of a 2D image using pseudo-code.

5.1.3. Given an image of size $N \times N$, where $N = 2 J$, what is the maximum number of levels you can have in an approximation pyramid representation? (The maximum level is reached when the coarsest level has only 1 pixel). What is the total number of pixels in the pyramid (i.e. including pixels at all pyramid levels)? How does this number compare with the original number of pixels in the image? Since this number is larger than the original pixel number, what are some of the benefits of using the approximation pyramid? (give some examples). Repeat the step for the prediction residual pyramid. Display and discuss the results.

In [ ]:

```
mona = cv2.imread('mona lisa.jpg',cv2.IMREAD_GRAYSCALE)
# resize 'mona lisa' to make it divisible by 2
mona_r = cv2.resize(mona, (256, 256)).copy()

gaussian_pyr = gaussian_pyramid(mona_r,9)

figure = plt.figure(figsize=(18,18))
n = len(gaussian_pyr)
print(n)
j=1
for i in range(n):
    x = i%8
    if x == 4:
        j +=1
    figure.add_subplot(j,4,x+1)

    plt.imshow(gaussian_pyr[i],cmap='gray')
    plt.title(f"level({i})",color='black')
```

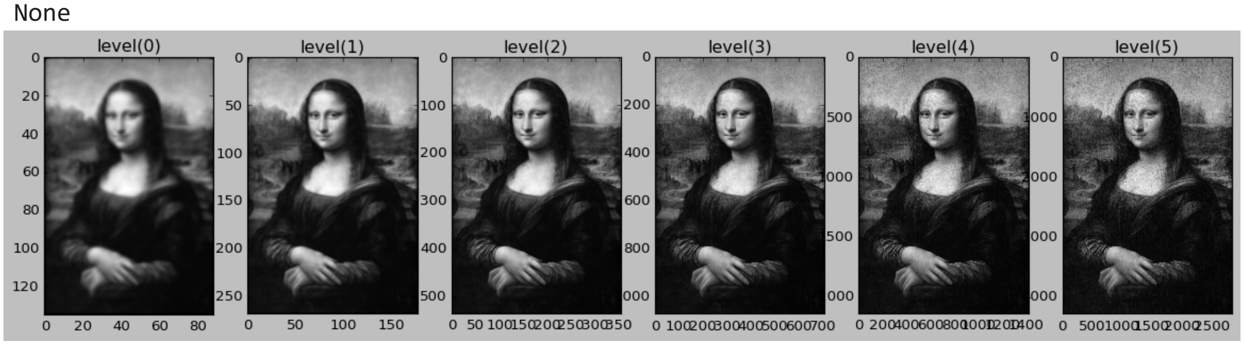
```
10
```



In [ ]:

```
mona = cv2.imread('mona lisa.jpg',cv2.IMREAD_GRAYSCALE)

gaussian_pyr = gaussian_pyramid(mona,5)

laplacian_pyr,reconstruct_pyr = pyramid_reconstruct(gaussian_pyr)
n = len(reconstruct_pyr)

figure = plt.figure(figsize=(18,18))
for i in range(n):
    figure.add_subplot(1,n,i+1)
    plt.imshow(reconstruct_pyr[i],cmap='gray')
    plt.title(f"level({i})",color='black')

print(print(f'MSE: {mean_square_error(mona,reconstruct_pyr[-1])}'))
```

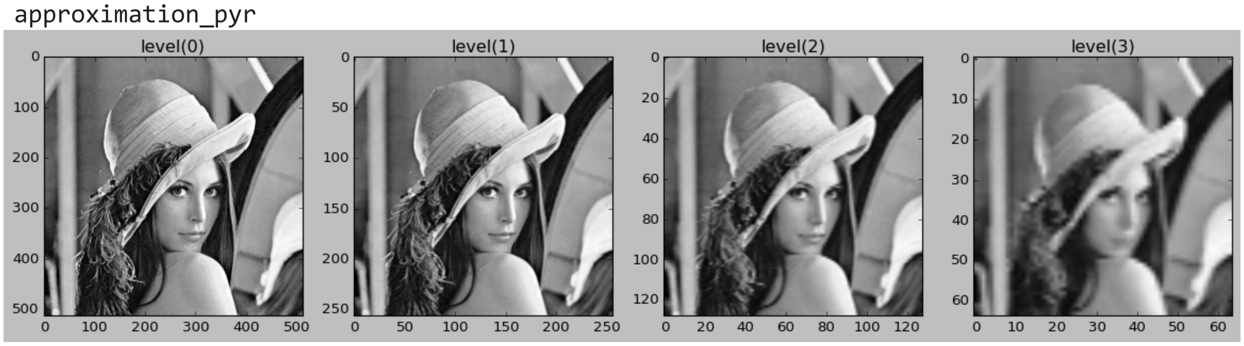
```
MSE: 0.000000
None
```



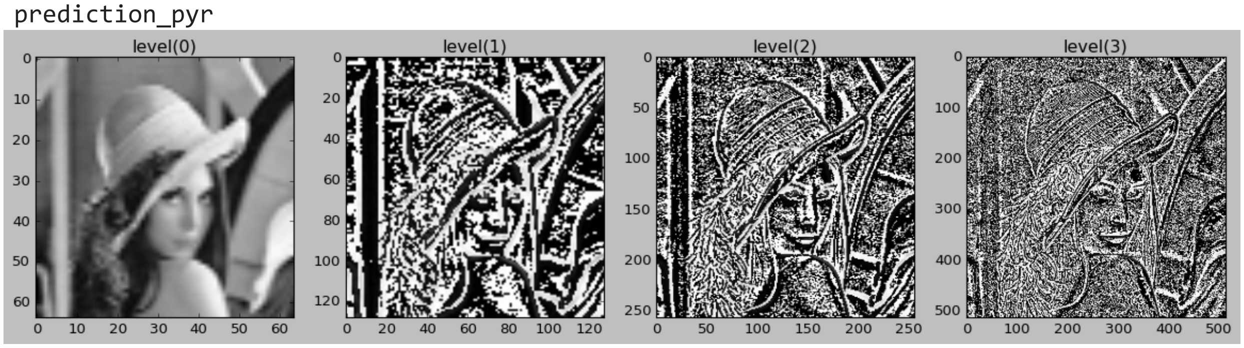
5.1.4. For the grayscale Lena image, manually compute a 3-level approximation pyramid and corresponding prediction residual pyramid. Use 2x2 averaging for the approximation and use pixel replication for the interpolation filters.

In [ ]:
```
lena = cv2.imread('Lena.bmp',cv2.IMREAD_GRAYSCALE)

# the approximation_pyramid function uses 2*2 box filter for filtering kernel
approximation_pyr = approximation_pyramid(lena,3)

# the pyramid_reconstruct function,use a approximation pyramid (either with gaussian k
# to rebulid the original image.
prediction_pyr , reconstruct_pyr = pyramid_reconstruct(approximation_pyr)
n=len(approximation_pyr)

print("approximation_pyr")
figure = plt.figure(figsize=(18,18))
for i in range(n):
    figure.add_subplot(1,n,i+1)
    plt.imshow(approximation_pyr[i],cmap='gray')
    plt.title(f"level({i})",color='black')
plt.show()

print('prediction_pyr')
figure2 = plt.figure(figsize=(18,18))
for i in range(n):
    figure2.add_subplot(1,n,i+1)
    plt.imshow(prediction_pyr[i],cmap='gray')
    plt.title(f"level({i})",color='black')
plt.show()

print('reconstruct_pyr')
figure3 = plt.figure(figsize=(18,18))
for i in range(n):
    figure3.add_subplot(1,n,i+1)
    plt.imshow(reconstruct_pyr[i],cmap='gray')
    plt.title(f"level({i})",color='black')
plt.show()
```

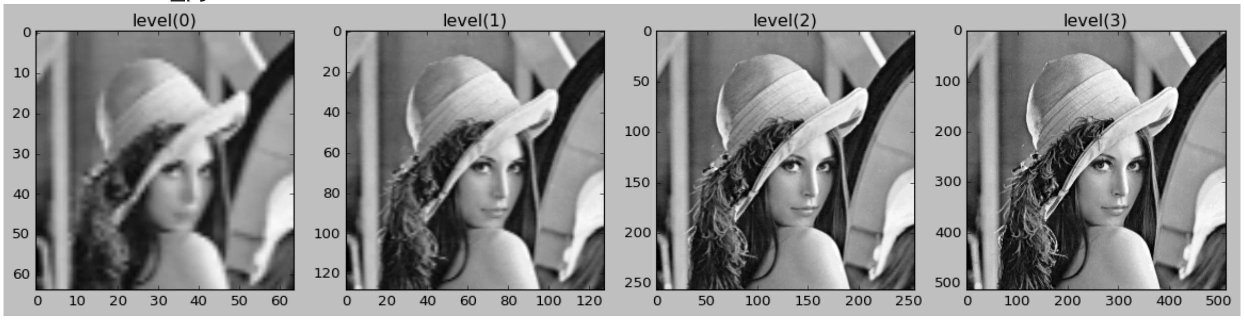
approximation_pyr



prediction_pyr



reconstruct_pyr



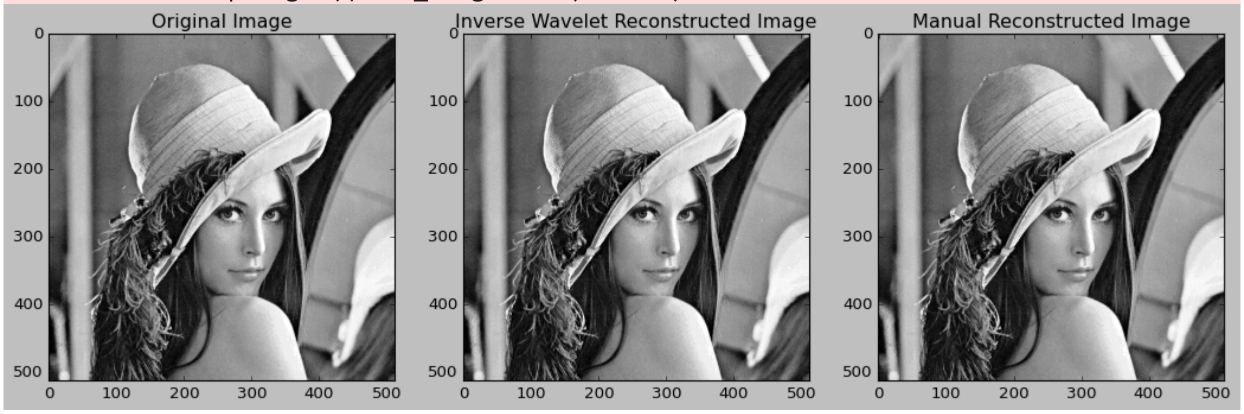
5.1.5. For the grayscale Lena Image, compute the wavelet transform (with 3-level) using the Haar analysis filters. Comment on the differences between the pyramids generated in Prob. 5.1.2 with the ones generated here.

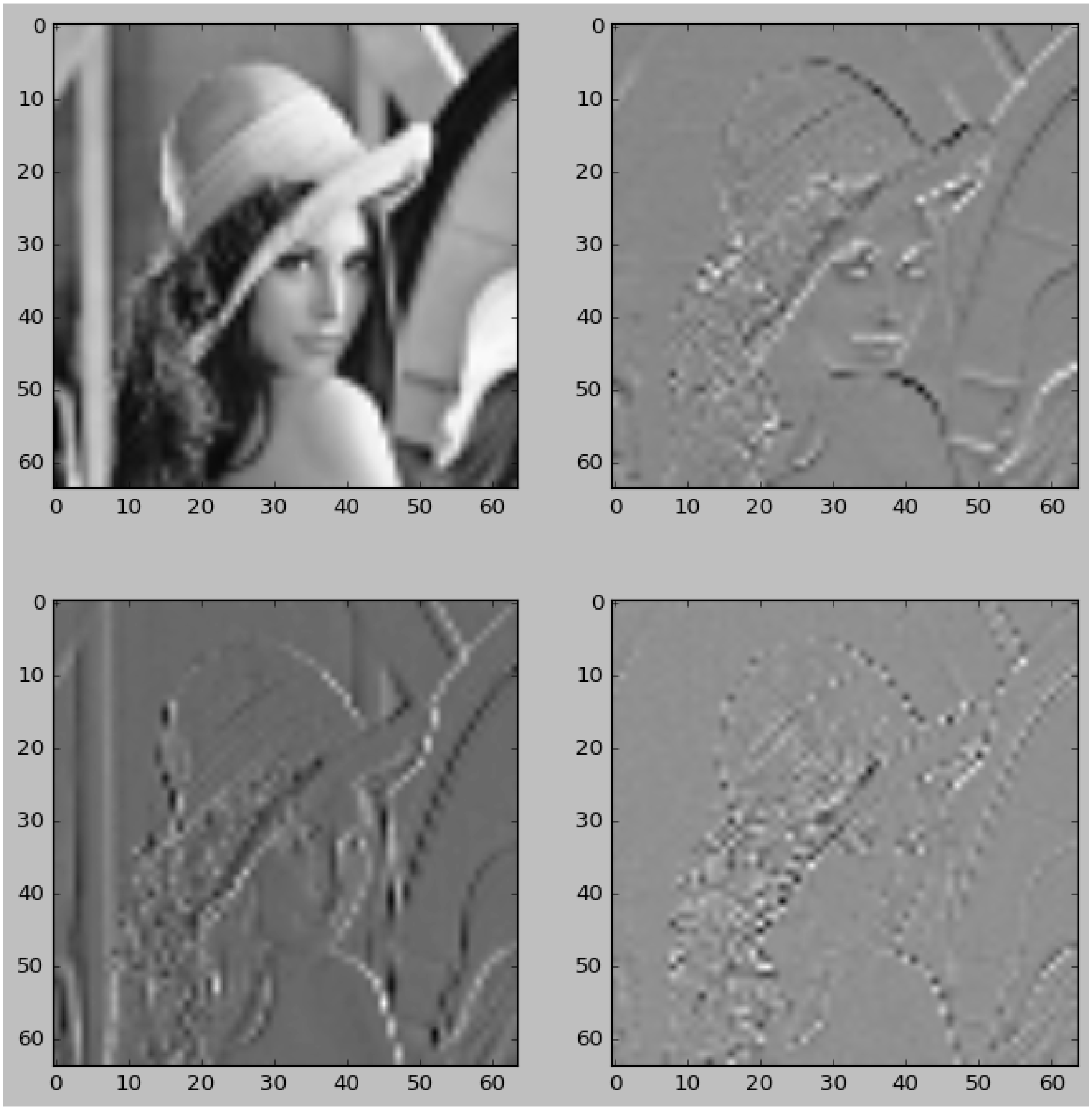
In [ ]:
```
import pywt

lena = cv2.imread('Lena.bmp',cv2.IMREAD_GRAYSCALE)
level = 3

# wavelet transform. The function returns coefficients of transform
c_matrix ,coefficients = wavelet_payramid(lena,level)

# coefficients of last level
cA = coefficients[0]
(cH,cV,cD) = coefficients[(-1*level)]

# inverse wavelet transform
# using previous coefficients, we rebuild the image
lena_i = pywt.waverec2(coefficients, 'haar', mode='periodization')
lena_i = lena_i.astype('uint8')

figure = plt.figure(figsize=(16,16))
figure.add_subplot(1,3,1)
plt.imshow(lena,cmap='gray')
plt.title('Original Image')

figure.add_subplot(1,3,2)
plt.imshow(lena_i,cmap='gray')
plt.title('Inverse Wavelet Reconstructed Image')

figure.add_subplot(1,3,3)
plt.imshow(reconstruct_pyr[-1],cmap='gray')
plt.title('Manual Reconstructed Image')

print(f'Wavelet MSE: {mean_square_error(lena,lena_i)}')
print(f'Wavelet PSNR: {PSNR(lena,lena_i)}')
print(f'Manual MSE: {mean_square_error(lena,reconstruct_pyr[-1])}')
print(f'Manual PSNR: {PSNR(lena,reconstruct_pyr[-1])}')


figure2 = plt.figure(figsize=(10, 10))
figure2.add_subplot(2, 2, 1)
plt.imshow(cA, cmap='gray')

figure2.add_subplot(2, 2, 2)
plt.imshow(cH, cmap='gray')
```

```python
figure2.add_subplot(2, 2, 3)
plt.imshow(cV, cmap='gray')

figure2.add_subplot(2, 2, 4)
plt.imshow(cD, cmap='gray')
plt.show()

# the whole wavelet pyramid
plt.imshow(c_matrix,cmap='gray')
```

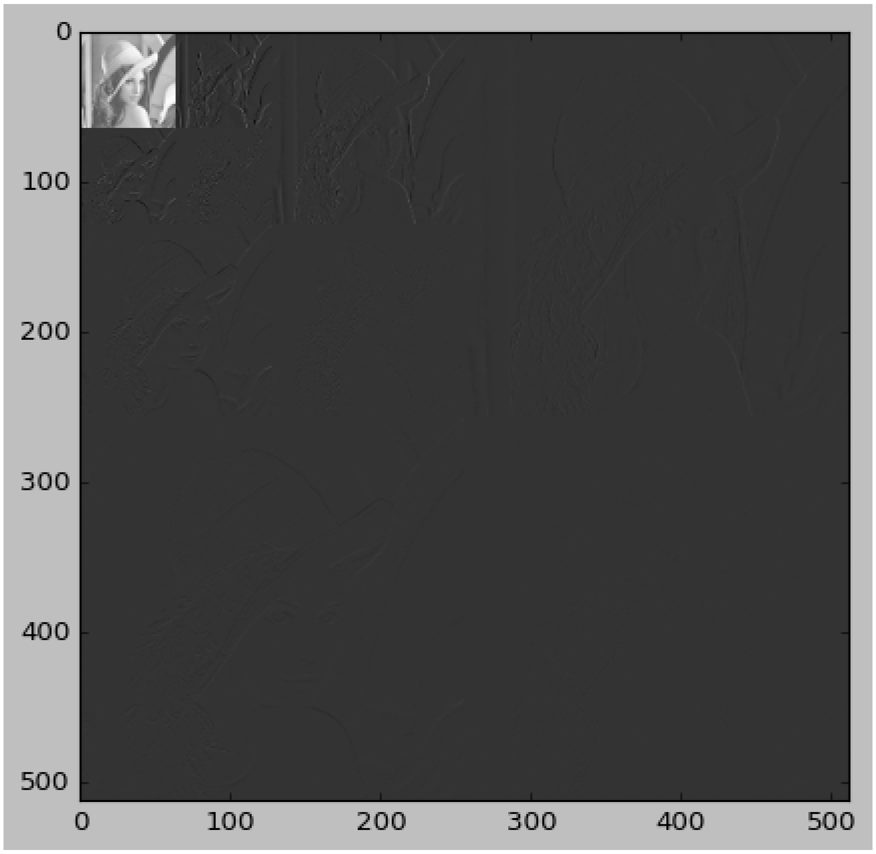
```
Wavelet MSE: 0.000084
Wavelet PSNR: 88.89197601997365
Manual MSE: 0.000000
Manual PSNR: inf
```

```
c:\Users\siedt\AppData\Local\Programs\Python\Python310\lib\site-packages\skimage\metr
ics\simple_metrics.py:163: RuntimeWarning: divide by zero encountered in scalar divid
e
  return 10 * np.log10((data_range ** 2) / err)
```





Out[ ]: <matplotlib.image.AxesImage at 0x244166090f0>



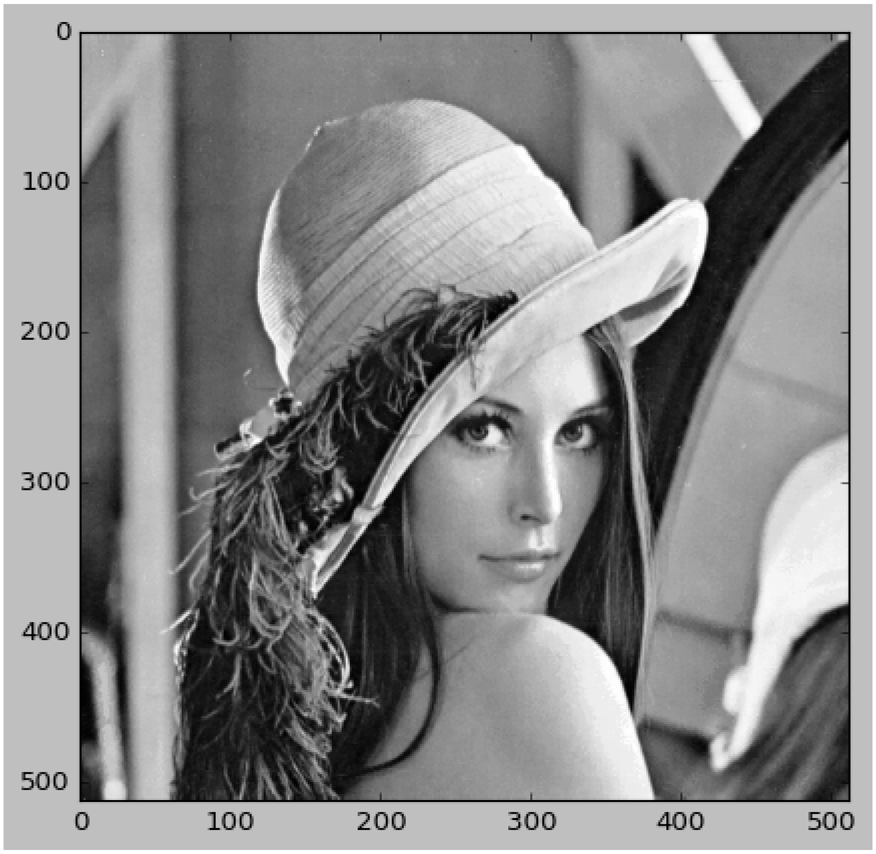
5.1.6. Quantize all the wavelet coefficients (whole sub-bands) created in Prob. 5.1.4 by a step size of $\gamma = 2$. Then reconstruct the image from the quantized wavelet coefficients using Haar synthesis filter. Report PSNR values and discuss the results. $c'(u, v) = \gamma \times sgn[c(u, v)] \times floor\ [\,|c(u, v)|\ \gamma\,]$, c represents the wavelet coefficient Note: you can use dwt2, idwt2, and psnr functions for problems 5.

In [ ]:

```
# extract sub-bands coefficients
cA = coefficients[0]
(cH1,cV1,cD1) = coefficients[-1]
(cH2,cV2,cD2) = coefficients[-2]
(cH3,cV3,cD3) = coefficients[-3]

# quantize coefficients
cA_new = coefficientQuantizer(cA)
cH1_new = coefficientQuantizer(cH1)
cV1_new = coefficientQuantizer(cV1)
cD1_new = coefficientQuantizer(cD1)
cH2_new = coefficientQuantizer(cH2)
cV2_new = coefficientQuantizer(cV2)
cD2_new = coefficientQuantizer(cD2)
cH3_new = coefficientQuantizer(cH3)
cV3_new = coefficientQuantizer(cV3)
cD3_new = coefficientQuantizer(cD3)

# making coefficients list
c = [cA_new,(cH3_new, cV3_new, cD3_new) ,(cH2_new, cV2_new, cD2_new),(cH1_new, cV1_ne

# inverse wavelet transform
lena_i = pywt.waverec2(c, 'haar', mode='periodization')
lena_i = lena_i.astype('uint8')
```

```python
plt.imshow(lena_i,cmap='gray')

print(f'MSE: {mean_square_error(lena,lena_i)}')
print(f'PSNR: {PSNR(lena,lena_i)}')
```

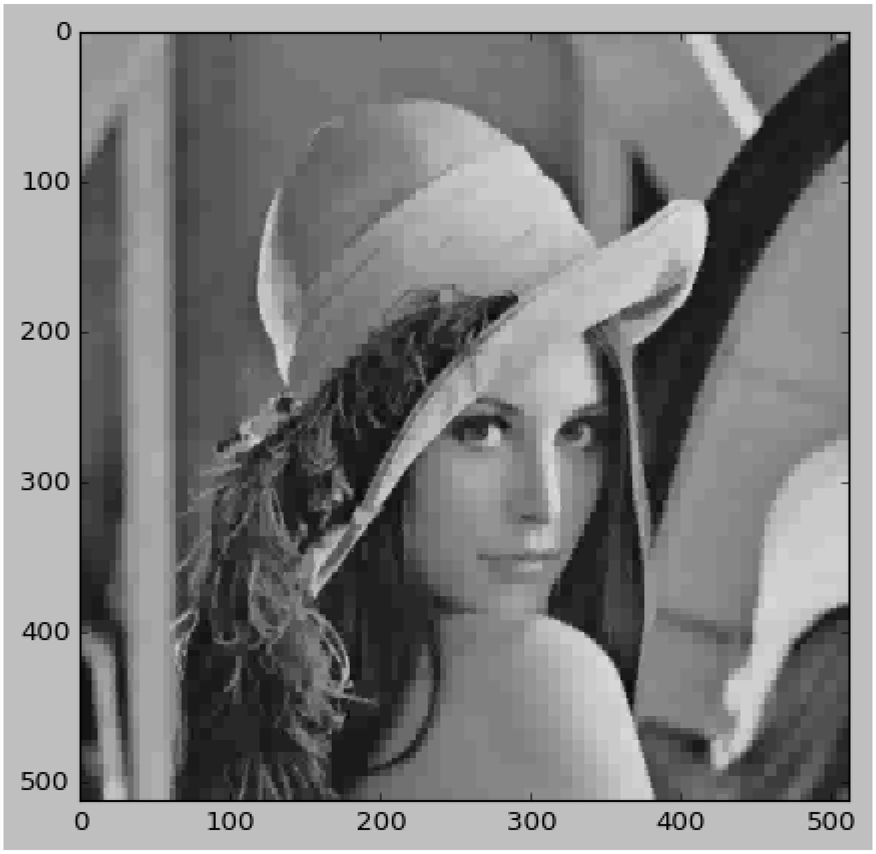
```
MSE: 1.367565
PSNR: 46.77132334238592
```



In [ ]:

```python
# quantize the coefficients, step = 50
cA_new = coefficientQuantizer(cA,50)
cH1_new = coefficientQuantizer(cH1,50)
cV1_new = coefficientQuantizer(cV1,50)
cD1_new = coefficientQuantizer(cD1,50)
cH2_new = coefficientQuantizer(cH2,50)
cV2_new = coefficientQuantizer(cV2,50)
cD2_new = coefficientQuantizer(cD2,50)
cH3_new = coefficientQuantizer(cH3,50)
cV3_new = coefficientQuantizer(cV3,50)
cD3_new = coefficientQuantizer(cD3,50)

# making coefficients list
c50 = [cA_new,(cH3_new, cV3_new, cD3_new) ,(cH2_new, cV2_new, cD2_new),(cH1_new, cV1_r

# inverse wavelet transform
lena_i = pywt.waverec2(c50, 'haar', mode='periodization')
lena_i = lena_i.astype('uint8')

plt.imshow(lena_i,cmap='gray')

print(f'MSE: {mean_square_error(lena,lena_i)}')
print(f'PSNR: {PSNR(lena,lena_i)}')
```

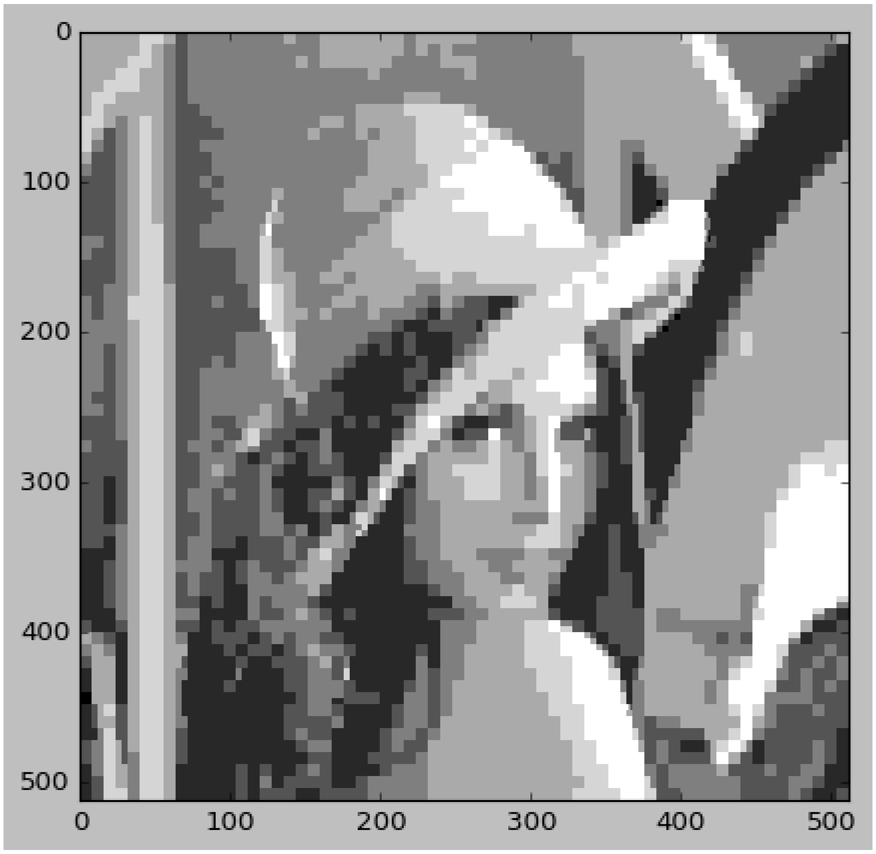
```
MSE: 102.903538
PSNR: 28.006500551142388
```



In [ ]:

```
# quantize the coefficients, step = 255
cA_new = coefficientQuantizer(cA,255)
cH1_new = coefficientQuantizer(cH1,255)
cV1_new = coefficientQuantizer(cV1,255)
cD1_new = coefficientQuantizer(cD1,255)
cH2_new = coefficientQuantizer(cH2,255)
cV2_new = coefficientQuantizer(cV2,255)
cD2_new = coefficientQuantizer(cD2,255)
cH3_new = coefficientQuantizer(cH3,255)
cV3_new = coefficientQuantizer(cV3,255)
cD3_new = coefficientQuantizer(cD3,255)

# making coefficients list
c255 = [cA_new,(cH3_new, cV3_new, cD3_new) ,(cH2_new, cV2_new, cD2_new),(cH1_new, cV1_

# inverse wavelet transform
lena_i = pywt.waverec2(c255, 'haar', mode='periodization')
lena_i = lena_i.astype('uint8')

plt.imshow(lena_i,cmap='gray')

print(f'MSE: {mean_square_error(lena,lena_i)}')
print(f'PSNR: {PSNR(lena,lena_i)}')
```

```
MSE: 619.742523
PSNR: 20.208690648882424
```


0
100
200
300
400
500
0
100
200
300
400
500