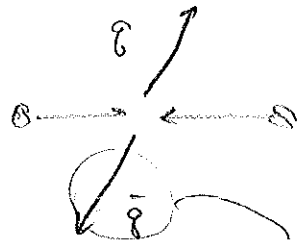
BAILES: see Ellenberg p. 188: GOD OR SIMULATION?

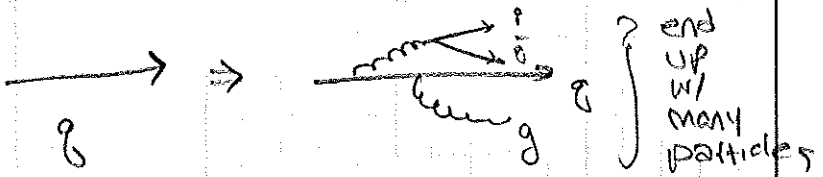
FOLLOWING P.6 of Lec 25 (Sierra Boxing slac stats '11)

Some physics: @ proton colliders, one can produce high energy quarks

quarks: stuff protons & neutrons are "made of"



BUT THESE RADIATE OFF GLUONS (strong force) LIKE CRAZY

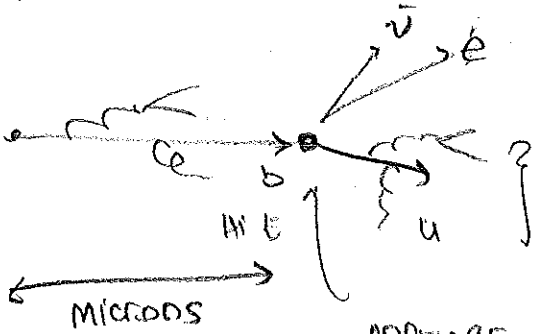


call this a jet.

THERE ARE DIFFERENT KINDS OF QUARKS.

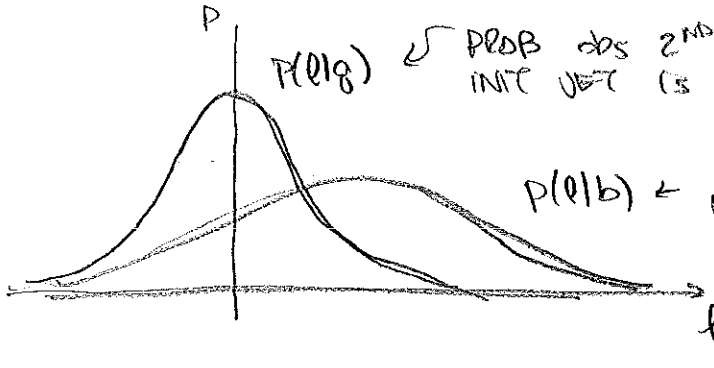
one of them is called the bottom (b) quark & has a special property @ colliders: ↙ rel to $\sqrt{s}$: 8 TeV

they explode after travelling a few microns



APPEARS AS A SECOND VERTEX

BUT: other things in the proton-proton collision can give displaced vertices ... so how do we tag b-jets?



$P(\ell|q)$ ← PROB dist 2ND vertex @ ℓ IF YOU KNOW INIT JET IS LIGHT QUARK

$P(\ell|b)$ ← prob 2ND vert @ ℓ if you know INIT JET IS b quark

so what do you do?
└ think about this.

Hypothesis tests

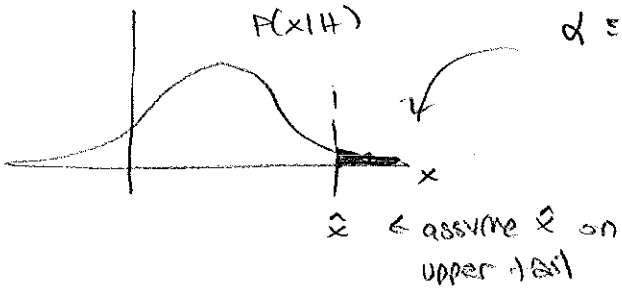
given <u>data</u> $\hat{x}$, & <u>theory</u> (hypothesis) H.
($\hat{x}$: set of measurements; H: set of parameters)

HOW COMPATIBLE IS H w/ $\hat{x}$? def <u>Likelihood</u>.

$L(H) = p(x|H)$ ← prob. that you'd measure x ASSUMING H is true.

↑ easy enough to calculate. BUT THIS IS STILL <u>NOT</u> $p(H|x)$

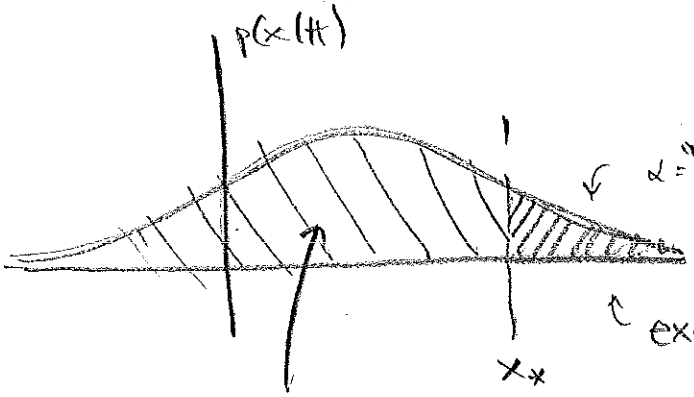
ONE WAY TO USE THIS:



$$\alpha = \int_{\hat{x}}^{\infty} dx\ p(x|H)$$

α IS: PROB THAT ANOTHER MEAS OF x WOULD BE AT LEAST AS CRAZY AS $\hat{x}$.

if $\alpha = 1\%$, say H excluded @ 1% confidence



a criterion for exclusion.

↑ exclude H if $\hat{x}$ is here

$(1-\alpha)$: confidence level

When $p(x|H)$ is gaussian, can relate p-values to standard deviations

eg $1\sigma \rightarrow \alpha = 16\%$
$2\sigma \rightarrow \alpha = 2.3\%$
$3\sigma \rightarrow \alpha = 0.14\%$.

but these numbers mean something even if $p(x|H)$ is not gaussian

unlike these, which are defined to be properties of a gaussian.

p-hacking: eg XKCD #882 "Significant"

- almost $\alpha = 5\%$ threshold
- Do JELLYBEANS CAUSE ACNE? NO, w/ $\alpha = 5\%$.
  - ↳ test each of the 20 individual colors. LIKELY THAT ONE WILL HAVE $\alpha \geq 5\%$.

That's what α means.

then → "Scientists discover that green jellybeans cause acne!!"

→ ACTUALLY AN EXAMPLE OF LOOK ELSEWHERE EFFECT, TRIALS FACTOR.

↳ eg this matters in big data fields like HEP if you test 1000s of hypotheses, you're going to get some 3σ results!

Deep question: what is $p(H|\hat{x})$?

NOW YOU HAVE TO PICK A TEAM:

(F) FREQUENTIST: impossible to answer
can give p-value / α ... but
this does not measure $p(H|\hat{x})$

(B) BAYESIAN: obviously:

$$p(H|\hat{x}) = \frac{P(\hat{x}|H)\,P(H)}{P(\hat{x})}$$

$p(H|\hat{x})$ ↑ POSTERIOR

$P(H)$ ← PRIOR

$P(\hat{x})$ ← $=1$ we measure $\hat{x}$

PRIOR: prob. prior to measurement that H is true.

↳ SUBJECTIVE AS HELL
(OR BASED ON INCOMPLETE KNOWLEDGE)

eg: P(LHC will destroy earth | pre-LHC data)

$P(\hat{x}|H) \sim 50\%$

BUT $P(H) \ll 1$.

L. LYONS: (F) USE IMPECCABLE LOGIC TO ANSWER A QUESTION THAT NOBODY CARES ABOUT

(B) ANSWERS THE Q'S EVERYONE WANTS, BUT BASED ON ASSUMPTIONS THAT NOBODY BELIEVES.

if you have a family of Hypotheses : $H = H(a)$

parameter, eg Higgs mass

prior: no idea what a is; $p(a) = \text{const.}$

posterior: $p(a) \sim p(\hat{x}|a)$

FLAT DIST. IS A CONST ? CANCELS.

$$p(a|\hat{x}) = \frac{p(\hat{x}|a)}{\int da'\, p(\hat{x}|a')} \cdot \frac{p(a)}{p(a)}$$

↑ normalize

Why this is subtle:

we thought that $p(a) = \text{const}$ is conservative (reflects total agnosticism)

but what if we used an equally valid parameter, $b = a^2$?

↳ eg m_H or m_H^2 ?
SAME INFO, BUT m_H^2 IS THE THING THAT SHOWS UP IN INVARIANTS.

if $p(a)$ is FLAT, $p(b)$ is NOT

if $a = a(b)$

eg. if $a = m_H$, $b = m_H^2$

$$p_a(a)\,da = p_a(a(b)) \left(\frac{da}{db}\right) db$$

$$\frac{da}{db} = \frac{d\sqrt{b}}{db} = \frac{1}{2}\frac{1}{\sqrt{b}}$$

$$p_b(b)$$

So even choice of parameterization is subjective!

↳ related to MEASURE PROBLEM of multiverse theories

HOW PEOPLE TEND TO BE IN MY FIELDS

hep-ph : BAYESIAN

↳ I want to ask: what is prob. that $m_h \in [124, 126]$ GeV ?

(frequentist: 0 or 1)

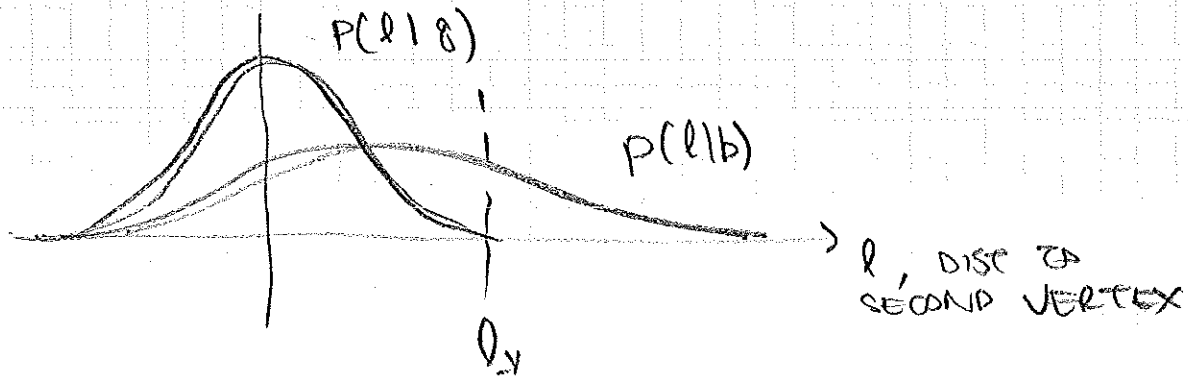
astro-ph : so many uncertainties — need priors to make conclusions

BAYESIAN

hep-ex : FREQUENTIST

↳ given O(1000) collaborators, will never agree to a prior. ☹

OUR ORIGINAL QUESTION: b-tagging



↳ AT SOME CUT VALUE:

if $\hat{\ell} > \ell_x$, THEN CALL IT A b-jet.

HAVE TO PICK l_* TO BALANCE TWO THINGS

① EFFICIENCY: $\varepsilon_b = \int_{l_*}^{\infty} dl \; p(l|b)$

(want ≈ 1

given an actual b-jet, we want to identify it as one.

② PURITY $\varepsilon_g = \int_{l_*}^{\infty} dl \; p(l|g)$

↑ want $\ll 1$

given a normal jet, we want to avoid calling it a b-jet

(Type I & Type II errors)

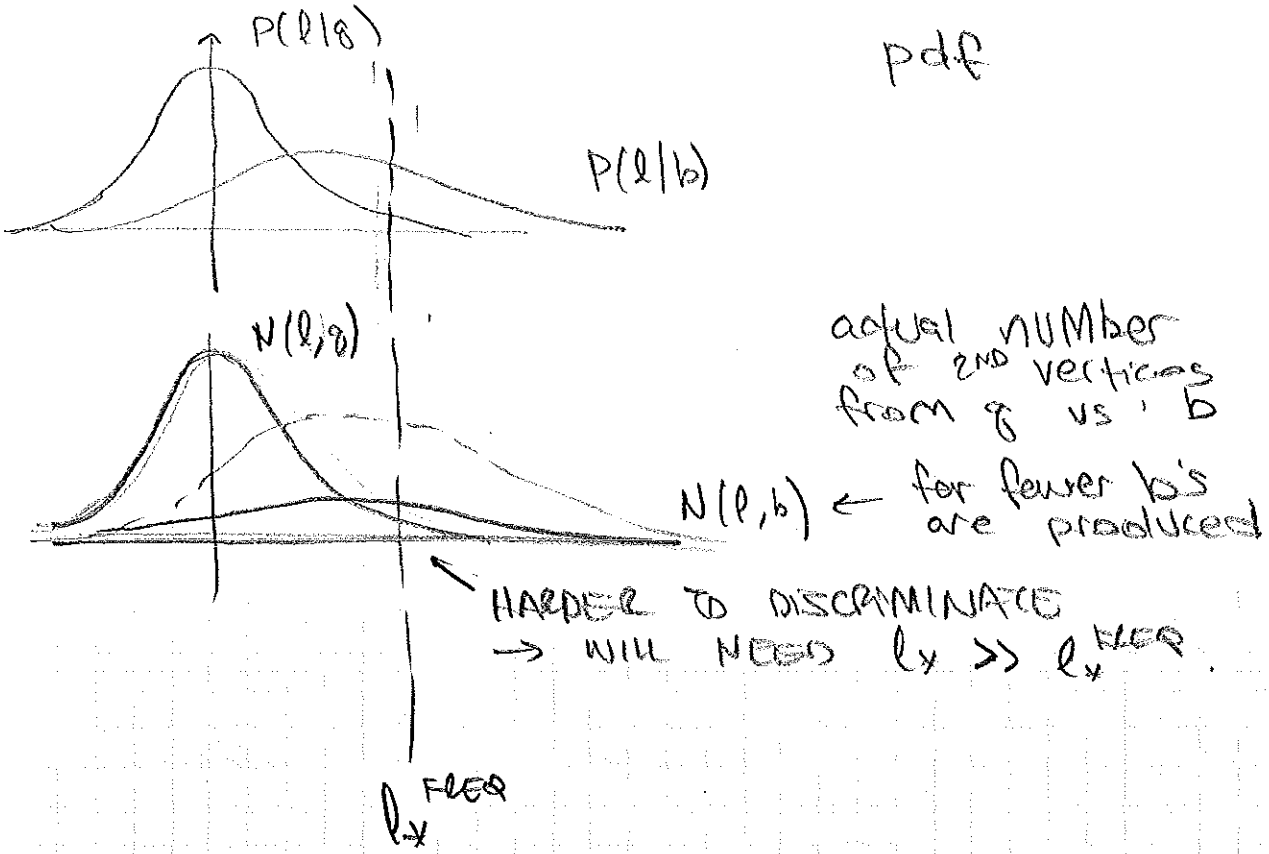
FREQUENTIST choose l_* to balance ε_b & ε_g

BAYESIAN: one step further: we have PRIOR INFO!

$p(g) \gg p(b)$ ← pp collisions @ 7 TeV will tend to produce more normal jets.

Bez PROB: $\frac{p(l|b)}{p(l|g)} \frac{p(b)}{p(g)}$ ← $\frac{p(b)}{p(g)} \ll 1$

⇒ WANT TO PICK l_* LARGER than FREQUENTIST.



lesson: be careful of tails!