Homework 1: **Dimensional Analysis**

Course: Physics 231, *Methods of Theoretical Physics* (2017)
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Due by: Friday, October 6

1 Warm Up

As a warm up, write out the dimensions of the following quantities in the form $[Q] = L^\alpha, M^\beta T^\gamma$, that is: write out the length, mass, and time dimensions.

(a) Electric charge, e. (In lecture we wrote out the answer; derive it.)

(b) Action ($S = \int dtL$, where L is the Lagrangian)

(c) Magnetic field, B

(d) Energy.

2 Natural Units

In high energy physics one typically works in **natural units** where we work only in mass dimension[1], $[Q] = M^\beta$. High energy physicists are lazy, so they prefer to just write $[Q] = \beta$. Thus the mass of the electron has dimension $[m_e] = 1$.

This 'gets rid' of the length and time dimensions. In order to do this *without* throwing away information, one uses 'universal constants' to convert between length/time and mass[2]. Fortunately, nature provides us with such constants: the speed of light, c, and Planck's constant, $\hbar = h/2\pi$. When using natural units, experts often just say that we impose the following curious conditions:

$$c = 1 \qquad\qquad \hbar = 1 \ . \tag{.1}$$

This looks completely wrong from dimensional analysis. What gives?

(a) First, get a sense of the numbers involved. Write out the numerical values of c and $\hbar$ in cgs (centimeters, grams, seconds) units. Don't use more than 2 significant figures, that's just showing off.

(b) Suppose I have some quantity Q with dimension $[Q] = L^\alpha M^\beta T^\gamma$ for some numbers α, β, γ. What natural units is really saying is that we want to work with a different set of dimensions: energy E, c, and $\hbar$:

$$[Q] = E^x c^y \hbar^z.$$

Find the values of x, y, and z as a function of α, β, γ.

[1] Actually, we work in dimensions of energy. You already know how energy and mass are related from problem 1b.

[2] In lecture we considered the meaning of 'three apples.' If apples have some standardized monetary value on the world apple market, then that value is a conversion between the unit of 'apple' to a unit of currency, for example.

(c) When a high energy physicist says that $[Q]$ has dimension x in natural units, what they really mean is that $[Q] = E^x c^y \hbar^z$ for some y and z. For a such a quantity, you already know what the 'ordinary' dimensions are so that you can multiply by the appropriate powers of c and $\hbar$ to get the number in any other units. For example, the mass of the proton m_p is 1 GeV, where GeV is a unit of energy. We say that $[m_p] = 1$ in natural units. What are the values of x and y? Convert 1 GeV into grams.

(d) Repeat problem 1 in natural units.

(e) Convert the electron mass $m_e = 0.5$ MeV into a length. What does this length correspond to? (Think about what the possibilities are from quantum mechanics, check your answer.)

(f) Explain the following poetic statement: "high-energy colliders are microscopes."

3 Energy from a sudden explosion

Suppose a large amount of energy is released in a small volume. This explosion produces a spherical shock wave where there is a sharp increase in air pressure. How does the characteristic size of the shock wave depend on time? Start by identifying the three relevant parameters for the problem and giving their dimensions in $L^\alpha M^\beta T^\gamma$ form. If you're stuck, see "The Formation of a Blast Wave by a Very Intense Explosion." by Geoffrey Taylor[3].

4 Errors in High School Physics

In lecture we examined the application of dimensional analysis to estimate the error on a simple high school physics calculation: the time it takes for an object to reach the ground after being dropped from rest at height h. Here we're assuming everything is human scale and on the surface of the Earth so that the zeroth order solution is $t_0 = \sqrt{2h/g}$. Estimate the size of the correction from special relativity. If you need a hint, this problem came from "Dimensional analysis, falling bodies, and the fine art of not solving differential equations" by Craig Bohren[4].

Extra Credit

These problems are not graded and are for your edification. You are strongly encouraged to explore and discuss these topics, especially if they are in a field of interest to you.

1 Renormalization Group as Dimensional Analysis

Read the paper "Dimensional Analysis in Field Theory: An Elementary Introduction to Broken Scale Invariance and the Renormalization Group Equations" by Paul Stevenson[5]. This paper describes the phenomenon of dimensional transmutation in quantum/statistical field theory in

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[3] http://www.jstor.org/stable/98395

[4] *Am. J. Phys.* **72** 4 , April 2004 http://dx.doi.org/10.1119/1.1574042

[5] Annals Phys. **132** (1981) 383, http://dx.doi.org/10.1016/0003-4916(81)90072-5

a way that strips it of the mysticism that tends to appear when you first learn field theory. In particular, it explains why dimensionless 'coupling constants' are not really constant and are scale dependent. Understand the dimensional analysis theorem in the paper and then understand how that theorem is evaded in actual physics. Theorists should take time to understand this paper carefully.

2 Allometry

These two problems come from *Mathematical Methods in Classical Mechanics* by the eminent mathematician V.I. Arnold.

(a) A desert animal has to cover great distance between sources of water. How does the maximal time the animal can run depend on the size L of the animal?

(b) How does the height of an animal's jump depend on its size? Use the fact that the force applied by muscles is proportional to the strength of bones, which is itself is proportional to their cross section.