GREEN'S FUNCTION CONVENTIONS → APPEL

$$\partial^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial \underline{r}^2}$$

$$\delta^4(x) = \delta(t)\,\delta^3(\underline{r}) \quad \Leftarrow \text{ not } \delta(ct)\,\delta^3(\underline{r})$$

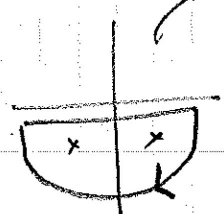
$$d^4K = dE\, d^3\underline{k}\, (2\pi)^{-4} \quad \Leftarrow \text{ not } (dE/c)\, d^3\underline{k}$$

$$\int_{-\infty}^{\infty} \frac{1}{k^2 - (E/c)^2} e^{-iEu}\, dE = \int_{-\infty}^{\infty} \frac{-1}{\mathcal{E}^2 - k^2} e^{-i\mathcal{E}cu}\, c\, d\mathcal{E}$$

$$\mathcal{E} \equiv E/c$$

$$\xrightarrow[\text{PUSH}]{\text{POLE}} \int_{-\infty}^{\infty} \frac{-e^{-i\mathcal{E}cu}}{(\mathcal{E}-(k-i\epsilon))(\mathcal{E}+(k+i\epsilon))}\, c\, d\mathcal{E}$$

$$= \oint_{\bar{C}_-} \frac{-e^{-i\mathcal{E}cu}}{[\mathcal{E}-(k-i\epsilon)][\mathcal{E}-(-k-i\epsilon)]}\, c\, d\mathcal{E}$$



-ve ORIENTATION

$$\mathcal{E} = k \qquad \mathcal{E} = -k$$

$$= -2\pi i \sum \text{Res}$$

$$= -2\pi i \cdot \left[\frac{-c e^{-ikcu}}{+2k} + \frac{-c e^{+ikcu}}{-2k} \right]$$

$$= \frac{-i\pi c}{k}\left(e^{ikcu} - e^{-ikcu} \right)$$

thanks to Wei-Xiang Feng for pointing this out.

WHAT IF WE STUCK W/ natural-like units ?

then: $d^4K = \frac{dE}{c}\, d^3\underline{k}$

$$\delta^{(4)}(x) = \delta(ct)\,\delta^{(3)}(\underline{x})$$

$$\partial^2 = \frac{\partial^2}{(ct)^2} - \frac{\partial^2}{\partial \underline{x}^2}$$

$$\partial^2 G = \partial^2 \int d^4K\, e^{-iK\cdot x}\, \tilde{G}(k) = \delta^{(4)}(x)$$

$$= \partial^2 \int \frac{dE}{c}\, d^3\underline{k}\, e^{-i\frac{E}{c}\cdot ct}\, e^{i\underline{k}\cdot\underline{x}}\, \tilde{G} = \int \frac{dE}{c}\, d^3\underline{k}\, e^{-i\frac{E}{c}ct}\, e^{i\underline{k}\cdot\underline{x}}$$

THESE FACTORS OF c CANCEL
SO CALCULATION OF $\tilde{G}$ PROCEEDS
AS IN PREVIOUS CASE.

BUT: $G(x-x') = \int d^4K\, \tilde{G} = \int \frac{dE}{c}\, d^3\underline{k}\, \tilde{G}$

EXTRA FACTOR OF c^{-1}

SAME AS BEFORE

So $G(x-x')$ is DIFFERENT !
THAT'S OKAY: WHEN WE SOLVE $\partial^2 A_\mu(x) = j_\mu(x)$

$$A_\mu(x) = \int d^4x'\, j_\mu(x')\, G(x-x')$$

HAS EXTRA c^{-1}

IN NATURAL-LIKE UNITS,
$d^4x' = d(ct)\, d^3\underline{x}$

EXTRA FACTOR OF c

CANCEL!