Distributed finite-time regulation of a class of networked heterogeneous multi-agent systems

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Distributed consensus of a class of networked heterogeneous multi-agent systems

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Abstract

In this paper, we consider the consensus problem of a class of heterogeneous multi-agent systems composed of the linear first-order and second-order integrator agents together with the nonlinear Euler–Lagrange (EL) agents. First, we propose a distributed consensus protocol under the assumption that the parameters of heterogeneous system are exactly known. Sufficient conditions for consensus are presented and the consensus protocol accounting for actuator saturation is developed. Then, by combining adaptive controller and PD controller together, we design a protocol for the heterogeneous system with unknown parameters (in the nonlinear EL dynamics). Based on graph theory, Lyapunov theory and Barbalat′s Lemma, the stability of the controllers is proved. Simulation results are also provided to illustrate the effectiveness of the obtained results.

1. Introduction

Recently, there has been a growing interest in multi-agent consensus due to its broad applications such as cooperative control of unmanned aerial vehicles, wireless sensor network, and spacecraft formation. There are two main approaches for controlling multi-agent systems: centralized and distributed. The distributed architecture is extensively studied due to its tremendous merits such as low energy consumption, low communication burden and relatively high reliability [1]. Many efforts have been devoted to the distributed consensus problem and three typical systems have received great attention by virtue of their broad practical applications,

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i.e. linear first-order integrator system, second-order integrator system and the nonlinear EL system (see Refs. [2–15], to name just a few).

Most of the existing results deal with homogeneous multi-agent dynamical systems, i.e. it is assumed that each subsystem has the same dynamics. Actually, the dynamics of the agents coupled with each other may be different when different kinds of agents share common goals in some practical applications [20]. For example, due to restrictions and external uncertainty, networked multiple robot systems with different shapes and abilities modeled by heterogeneous dynamics are more applicable than the homogeneous systems in real world [20,21]. Therefore, it is of great interest and importance to study the consensus problem about the heterogeneous systems. In [16], a sufficient condition for consensus of high-order integrator heterogeneous systems is proposed. Based on Lyapunov theory, the consensus problems of heterogeneous systems consisting of first-order and second order integrator agents are considered with and without velocity measurements in [17] and [18], respectively, and saturated consensus protocol is also taken into consideration. Reference [19] studies the output consensus for a class of heterogeneous uncertain linear multi-agent system by embedding the internal models into the controller. Based on the properties of nonnegative matrices, consensus for discrete-time heterogeneous multi-agent systems composed of first-order and second-order integrator agents is studied in [20]. By applying a power integrator method and Lyapunov theory, [21] proposes two kinds of finite-time consensus protocols for heterogeneous multi-agent systems composed of first-order and second-order integrator agents. In addition, in [22], consensus solution to heterogeneous multi-agent systems with unknown communication delays is studied by frequency-domain methods.

Note that the aforementioned results mainly deal with linear heterogeneous multi-agent systems, yet little attention has been paid on the consensus problem of complex heterogeneous systems composed of both linear and nonlinear agents, and the extension of previous work to this kind of heterogeneous system is nontrivial. It is well known that EL systems are commonly used to formulate a class of mechanical systems such as robotic manipulators, attitudes of spacecraft and walking robots [24]. However, the consensus results for first- and second-order dynamics cannot be directly applied to Lagrangian systems due to their inherent nonlinearity, especially when there exist parametric uncertainties [12]. For example, in the multi-robot systems, due to common goal and dynamic environments, some robots should be modeled by linear second-order integrator equation and others should be modeled by EL equation, then new coordination protocols need to be developed. Motivated by this, this paper extends the consensus results to the heterogeneous systems composed of linear first-order, second-order integrator agents and nonlinear EL dynamical agents. In particular, we first propose a consensus protocol for systems with completely known parameters, then adaptive consensus protocol is also developed for unknown parametric systems. To the best of our knowledge, there is still no result which comprehensively considers the heterogeneous consensus problem with nonlinear EL dynamics and unknown parameters.

This paper is organized as follows: In Section 2, the systems′ model, graph theory and control objective are presented. The consensus problems of heterogeneous multi-agent systems with exactly known parameters and unknown parameters are studied in Section 3. In Section 4, the simulation results are given to illustrate the effectiveness of the controllers. Finally, we draw our conclusions in Section 5.

Notation. $\mathbb{R}:=(-\infty,\infty)$, $\mathbb{R}^+:=(0,\infty)$. $\|A\|$ is the matrix 2-norm of matrix A. $|x|$ stands for the stand Euclidean norm for the of vector $x\in\mathbb{R}^n$. For any function $f:\mathbb{R}_{\geq 0}\to\mathbb{R}^n$, the $\mathbb{L}_\infty-$norm is defined as $\|f\|_\infty=\sup_{t\geq 0}|f(t)|$, and the $\mathbb{L}_2-$norm as $\|f\|_2^2=\int_0^\infty|f(t)|^2\,dt$. The $\mathbb{L}_\infty$ and $\mathbb{L}_2$ spaces are defined as the sets $\{f:\mathbb{R}_{\geq 0}\to\mathbb{R}^n:\|f\|_\infty<\infty\}$ and $\{f:\mathbb{R}_{\geq 0}\to\mathbb{R}^n:\|f\|_2<\infty\}$, respectively.

2. Problem formulation and background

2.1. Heterogeneous multi-agent systems

Consider a heterogeneous system composed of $l(0<l<m)$ first-order integrator agents, $m-l$ second-order agents and $n-m(m<n)$ EL agents. The well-known first-order integrator agents are given by

$$\dot{x}_i(t)=\tau_i(t), \quad i\in\mathcal{I}_l \tag{1}$$

where $x_i(t)\in\mathbb{R}^p$ and $\tau_i(t)\in\mathbb{R}^p$ are the position and the control input, respectively. $\mathcal{I}_l\triangleq\{1,\ldots,l\}$. In addition, the dynamics of the second-order integrator agents is given by

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ \dot{v}_i(t)=\tau_i(t), \quad i\in\mathcal{I}_l/\mathcal{I}_m\end{cases} \tag{2}$$

where $x_i(t)\in\mathbb{R}^p$, $v_i(t)\in\mathbb{R}^p$ and $\tau_i(t)\in\mathbb{R}^p$ are the position, the velocity and the control input, respectively. $\mathcal{I}_m\triangleq\{1,\ldots,m\}$. Furthermore, EL system dynamics is formulated by

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ M_i(x_i)\dot{v}_i+C_i(x_i,v_i)v_i=\tau_i(t), \quad i\in\mathcal{I}_m/\mathcal{I}_n\end{cases} \tag{3}$$

where $\mathcal{I}_n\triangleq\{1,\ldots n\}$, where $x_i(t)\in\mathbb{R}^p$, $v_i(t)\in\mathbb{R}^p$ and $\tau_i(t)\in\mathbb{R}^p$ are the position, the velocity and the control input, respectively. $M_i(x_i)\in\mathbb{R}^{p\times p}$ is the general inertia matrix, and $C_i(x_i,\dot{x}_i)\in\mathbb{R}^{p\times p}$ is the matrix of Coriolis and centrifugal forces. Before proceeding, we assume that the EL system satisfies the following properties [25]:

Property 1. *The inertial matrix $M_i(x_i)$ is lower and upper bounded, i.e.*

$$0<\lambda_m\{M_i(x_i)\}I\le M_i(x_i)\le\lambda_M\{M_i(x_i)\}I<\infty \tag{4}$$

Property 2. *The dynamic parameters of EL system is linearly parameterizable, i.e.*

$$M_i(x_i)\ddot{x}_i+C_i(x_i,\dot{x}_i)\dot{x}_i=Y_i(x_i,\dot{x}_i,\ddot{x}_i)\theta_i \tag{5}$$

where θ_i is a constant r-dimensional vector of parameters whose elements include the moments of inertial and external disturbances, and $Y(\cdot)\in\mathbb{R}^{p\times r}$ is the matrix of known functions of the generalized coordinates and their higher derivatives.

Property 3. *The matrix $\dot{M}_i(x_i)-2C_i(x_i,\dot{x}_i)$ is skew-symmetric, i.e. for a given vector $r\in\mathbb{R}^p$, we have*

$$r^T(\dot{M}_i(x_i)-2C_i(x_i,\dot{x}_i))r=0 \tag{6}$$

Property 4. *The generalized centripetal-Coriolis matrix $C_i(x_i,\dot{x}_i)$ is bounded with x_i. For all vectors $x_i\in\mathbb{R}^p$, $C_i(x_i,\dot{x}_i)$ satisfies*

$$\|C_i(x_i,\dot{x}_i)\|\le k_C\|\dot{x}_i\| \tag{7}$$

for some constant $k_C>0$. This property can also be found in [9] and [12].

2.2. Graph theory

For the heterogeneous multi-agent systems, assume that each agent is a node and the information flow of the agents is represented by an undirected graph $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$, where $\mathcal{V}(\mathcal{G})=\{v_1,v_2,\ldots,v_n\}$ is the set of nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A}=\{a_{ij}\}$ is the weighted adjacency matrix with nonnegative adjacency elements a_{ij}. The node indexes belong to a finite index set $\mathcal{I}_n = \{1, \ldots, n\}$. For a undirected graph, $\forall i, j \in \mathcal{I}_n$, if $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$, then $(v_j, v_i) \in \mathcal{E}(\mathcal{G})$. An edge of $\mathcal{G}$ is denoted by (v_i, v_j) and it is said to be incoming with respect to v_j and outgoing with respect to v_i. A path is the sequence of edges of the form $(v_1, v_2), (v_2, v_3), \ldots$. An undirected graph is said to be connected if there exists a path between any two distinct vertices of the graph. If $\mathcal{G}$ is an undirected graph, the adjacency matrix $\mathcal{A}$ is symmetric.

2.3. Control objective

In this paper, we require the networked heterogeneous systems to reach state consensus. The heterogeneous systems is said to reach consensus if

$$\begin{aligned} &\lim_{t\to\infty} \| x_j - x_i \| = 0, \quad i, j \in \mathcal{I}_n \\ &\lim_{t\to\infty} \| v_j - v_i \| = 0, \quad i, j \in \mathcal{I}_l / \mathcal{I}_n \end{aligned} \tag{8}$$

3. Main results

3.1. Consensus with completely known parameters

In this section, we assume that the parameters of the heterogeneous are exactly known. System (1)–(3) can also be written as

$$\begin{cases} \dot{x}_i = \tau_i, & i \in \mathcal{I}_l \\ \dot{x}_i = v_i, & i \in \mathcal{I}_l / \mathcal{I}_m \\ \dot{v}_i = \tau_i, & i \in \mathcal{I}_l / \mathcal{I}_m \\ \dot{x}_i = v_i, & i \in \mathcal{I}_m / \mathcal{I}_n \\ M_i(x_i)\dot{v}_i + C_i(x_i, v_i)v_i = \tau_i, & i \in \mathcal{I}_m / \mathcal{I}_n \end{cases} \tag{9}$$

We present the following consensus protocol for system (9) as follows:

$$\tau_i = \begin{cases} \sum_{j=1}^{n} a_{ij}(x_j - x_i), & i \in \mathcal{I}_l, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i) - \Lambda_i v_i, & i \in \mathcal{I}_l / \mathcal{I}_m, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i) - \Phi_i v_i, & i \in \mathcal{I}_m / \mathcal{I}_n, j \in \mathcal{I}_n \end{cases} \tag{10}$$

where Λ_i and Φ_i are $p \times p$ positive-definite diagonal matrices, and $\mathcal{A} = [a_{ij}]$ is the weighted adjacency matrix.

Theorem 1. *Consider the heterogeneous system (9). If the undirected graph $\mathcal{G}$ is connected and τ_i is designed as Eq. (10), then the states of all the agents reach a consensus in the sense of Eq. (8) for all the initial conditions.*

Proof. Construct the following Lyapunov function

$$V(t)=\sum_{i=m+1}^{n} v_i^T M_i(x_i)v_i+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(x_j-x_i)^T\left(x_j-x_i\right)+\sum_{i=l+1}^{m} v_i^T v_i$$

Taking the derivative of $V(t)$ along (9) and (10) yields

$$\begin{aligned}
\dot V(t)=&\sum_{i=m+1}^{n} v_i^T \dot M_i(x_i)v_i+2\sum_{i=m+1}^{n} v_i^T M_i(x_i)\dot v_i+\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(x_j-x_i)^T(\dot x_j-\dot x_i)\\
&+2\sum_{i=l+1}^{m} v_i^T\dot v_i=\sum_{i=m+1}^{n} v_i^T \dot M_i(x_i)v_i+2\sum_{i=m+1}^{n} v_i^T\left(\sum_{j=1}^{n} a_{ij}(x_j-x_i)-\Phi_i v_i-C_i(x_i,v_i)v_i\right)\\
&+\sum_{i=1}^{l}\sum_{j=1}^{l} a_{ij}(x_j-x_i)^T(\dot x_j-\dot x_i)+\sum_{i=l+1}^{n}\sum_{j=1}^{l} a_{ij}(x_j-x_i)^T(\dot x_j-v_i)\\
&+\sum_{i=1}^{l}\sum_{j=l+1}^{n} a_{ij}(x_j-x_i)^T(v_j-\dot x_i)+\sum_{i=l+1}^{n}\sum_{j=l+1}^{n} a_{ij}(x_j-x_i)^T(v_j-v_i)\\
&+2\sum_{i=l+1}^{m} v_i^T\left(\sum_{j=1}^{n} a_{ij}(x_j-x_i)-\Lambda_i v_i\right)\\
=&\sum_{i=m+1}^{n} v_i^T(\dot M_i(x_i)-2C_i(x_i,v_i))v_i+2\sum_{i=m+1}^{n}\sum_{j=1}^{n} a_{ij}v_i^T(x_j-x_i)\\
&-2\sum_{i=m+1}^{n} v_i^T\Phi_i v_i+\sum_{i=1}^{n}\sum_{j=1}^{l} a_{ij}(x_j-x_i)^T\dot x_j-\sum_{i=1}^{l}\sum_{j=1}^{n} a_{ij}(x_j-x_i)^T\dot x_i\\
&+\sum_{i=1}^{n}\sum_{j=l+1}^{n} a_{ij}(x_j-x_i)^T v_j-\sum_{i=l+1}^{n}\sum_{j=1}^{n} a_{ij}(x_j-x_i)^T v_i\\
&+2\sum_{i=l+1}^{m}\sum_{j=1}^{n} a_{ij}v_i^T(x_j-x_i)-2\sum_{i=l+1}^{m} v_i^T\Lambda_i v_i
\end{aligned}$$

As graph $\mathcal{G}$ is undirected, $a_{ij}=a_{ji}$. Thus we have

$$\sum_{i=1}^{n}\sum_{j=1}^{l} a_{ij}(x_j-x_i)^T\dot x_j=-\sum_{i=1}^{l}\sum_{j=1}^{n} a_{ij}(x_j-x_i)^T\dot x_i$$

$$\sum_{i=1}^{n}\sum_{j=l+1}^{n} a_{ij}(x_j-x_i)^T v_j=-\sum_{i=l+1}^{n}\sum_{j=1}^{n} a_{ij}(x_j-x_i)^T v_i$$

And using Property 3, then we obtain

$$\begin{aligned}
\dot V(t)=&2\sum_{i=m+1}^{n}\sum_{j=1}^{n} a_{ij}v_i^T(x_j-x_i)-2\sum_{i=m+1}^{n} v_i^T\Phi_i v_i\\
&-2\sum_{i=1}^{l}\sum_{j=1}^{n} a_{ij}(x_j-x_i)^T\dot x_i-2\sum_{i=l+1}^{n}\sum_{j=1}^{n} a_{ij}(x_j-x_i)^T v_i\\
&+2\sum_{i=l+1}^{m}\sum_{j=1}^{n} a_{ij}v_i^T(x_j-x_i)-2\sum_{i=l+1}^{m} v_i^T\Lambda_i v_i
\end{aligned}$$

$$= -2\sum_{i=m+1}^{n} v_i^T \Phi_i v_i - 2\sum_{i=1}^{l} \dot{x}_i^T \sum_{j=1}^{n} a_{ij}(x_j - x_i) - 2\sum_{i=l+1}^{m} v_i^T \Lambda_i v_i$$
$$= -2\sum_{i=m+1}^{n} v_i^T \Phi_i v_i - 2\sum_{i=1}^{l} \left(\sum_{j=1}^{n} a_{ij}(x_j - x_i)\right)^T \left(\sum_{j=1}^{n} a_{ij}(x_j - x_i)\right) - 2\sum_{i=l+1}^{m} v_i^T \Lambda_i v_i \le 0$$

Since $V \ge 0$ and $\dot{V} \le 0$, we can get $v_i(i \in \mathcal{I}_l/\mathcal{I}_n)$ and $\sum_{j=1}^{n} a_{ij}(x_j - x_i)(i \in \mathcal{I}_n)$ is bounded. For the second-order agents, as $\dot{v}_i = \sum_{j=1}^{n} a_{ij}(x_j - x_i) - \Lambda_i v_i$, then $\dot{v}_i(i \in \mathcal{I}_l/\mathcal{I}_m)$ is also bounded. By Properties 1 and 4, $M_i(x_i)$ and $C_i(x_i, \dot{x}_i)$ are bounded, then according to $M_i(x_i)\dot{v}_i + C_i(x_i, v_i)v_i = \sum_{j=1}^{n} a_{ij}(x_j - x_i) - \Phi_i v_i$, we conclude that $\dot{v}_i(i \in \mathcal{I}_m/\mathcal{I}_n)$ is also bounded. By differentiating $\dot{V}(t)$, we can see that $\ddot{V}(t)$ is bounded. Combining with the fact that $\dot{V}(t)$ is uniformly continuous in time, we can conclude that $\dot{V}(t) \to 0$ as $t \to 0$ according to Barbalat′s Lemma, which implies $v_i = 0(i \in \mathcal{I}_l/\mathcal{I}_n)$ and $\sum_{i=1}^{n} a_{ij}(x_j - x_i) = 0$ as $t \to \infty$. Then we obtain

$$\sum_{j=1}^{n} x_i^T \sum_{j=1}^{n} a_{ij}(x_j - x_i) = 0$$

Because the communication graph $\mathcal{G}$ is undirected and connected, it follows that

$$\sum_{j=1}^{n} \sum_{j=1}^{n} a_{ij}(x_j - x_i)^T (x_j - x_i) = 0$$

which implies that $x_i = x_j(i, j \in \mathcal{I}_n)$ as $t \to \infty$. This completes the proof. □

Remark 1. We consider a heterogeneous system composed of linear first-order, second-order and nonlinear EL dynamic agents in a unified framework. In protocol (10), each agent communicates with its neighbors of different dynamics to reach consensus. Note that in [17], consensus problem of heterogeneous system consisting of linear first-order and second-order agents is studied. If $m = n$, system (9) is the same as the one in [17], we therefore extend the results to more complicated heterogeneous systems coupled with nonlinear EL systems.

Remark 2. The protocol can be also given with relative velocity information as follows:

$$\tau_i = \begin{cases} \sum_{j=1}^{n} a_{ij}(x_j - x_i), & i \in \mathcal{I}_l, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i) - \Lambda_i \sum_{j=1}^{n} a_{ij}(v_j - v_i), & i \in \mathcal{I}_l/\mathcal{I}_m, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i) - \Phi_i \sum_{j=1}^{n} a_{ij}(v_j - v_i), & i \in \mathcal{I}_m/\mathcal{I}_n, j \in \mathcal{I}_n \end{cases} \tag{11}$$

To prove the stability of Eq. (11), we can also use the Lyapunov function in the proof of Theorem 1. Differentiating $V(t)$ gives that $V(t) = -2\sum_{i=m+1}^{n} a_{ij}(v_j - v_i)^T \Phi_i (v_j - v_i) - 2\sum_{i=1}^{l} \dot{x}_i^T \dot{x}_i - 2 \sum a_{ij}(v_j - v_i)^T \Lambda_i (v_j - v_i) \le 0$. Similar to the proof of Theorem 1, we can prove that by using Eq. (11), the control objective equation (8) can be achieved. The proof is omitted here due to space limitation.

Remark 3. In many applications, the states of heterogeneous are synchronized with the desired relative difference. This problem resembles a vehicle formation problem in [23]. Assume that the desired state difference is given as a constant $x_{ij}^d \in \mathbb{R}^p$, for each (v_i, v_j), $i,j \in \mathcal{I}_n$, $x_{ij}^d = -x_{ji}^d$. Then the control objective is shown as

$$\begin{aligned}&\lim_{t\to\infty} \| x_j - x_i - x_{ij}^d \| = 0, \quad i,j \in \mathcal{I}_n \\ &\lim_{t\to\infty} \| v_i \| 0, \quad i \in \mathcal{I}_l / \mathcal{I}_n\end{aligned} \tag{12}$$

the distributed controller can be designed as

$$\tau_i = \begin{cases} \sum_{j=1}^n a_{ij}(x_j - x_i - x_{ij}^d), & i \in \mathcal{I}_l, j \in \mathcal{I}_n \\ \sum_{j=1}^n a_{ij}(x_j - x_i - x_{ij}^d) - \Lambda_i v_i, & i \in \mathcal{I}_l / \mathcal{I}_m, j \in \mathcal{I}_n \\ \sum_{j=1}^n a_{ij}(x_j - x_i - x_{ij}^d) - \Phi_i v_i, & i \in \mathcal{I}_m / \mathcal{I}_n, j \in \mathcal{I}_n \end{cases} \tag{13}$$

Using the Lyapunov function as

$$V(t) = \sum_{i=m+1}^n v_i^T M_i(x_i) v_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij}(x_j - x_i - x_{ij}^d)^T \left(x_j - x_i - x_{ij}^d \right) + \sum_{i=l+1}^m v_i^T v_i$$

we can prove that Eq. (12) can be achieved. The proof procedures are similar to Theorem 1, and it is omitted here.

Similar to [4] and [17], we can also extend the results to the scenarios of actuator saturation. The consensus algorithm is designed as

$$\tau_i = \begin{cases} \sum_{j=1}^n a_{ij} \tanh(x_j - x_i), & i \in \mathcal{I}_l, j \in \mathcal{I}_n \\ \sum_{j=1}^n a_{ij} \tanh(x_j - x_i) - \Lambda_i \tanh(v_i), & i \in \mathcal{I}_l / \mathcal{I}_m, j \in \mathcal{I}_n \\ \sum_{j=1}^n a_{ij} \tanh(x_j - x_i) - \Phi_i \tanh(v_i), & i \in \mathcal{I}_m / \mathcal{I}_n, j \in \mathcal{I}_n \end{cases} \tag{14}$$

where $\tanh(\cdot)$ is defined component-wise for a vector. In contrast to Eq. (10), bounded inputs are introduced in Eq. (14) to account for actuator saturation. It follows that $\| \tau_i \|_\infty \le \max\{ \sum_{j=1}^n a_{ij} + \| \Lambda_i \|_\infty, \sum_{j=1}^n a_{ij} + \| \Phi_i \|_\infty \}$. Then we have

Corollary 2. *Consider the heterogeneous system (9). If the undirected graph $\mathcal{G}$ is connected and τ_i is designed as Eq. (14), then the states of all the agents reach a consensus in the sense of Eq. (8) for all the initial conditions.*

Proof. Consider the following Lyapunov function

$$V(t) = \sum_{i=m+1}^n v_i^T M_i(x_i) v_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij} \mathbf{1}^T \ln\left[\cosh\left(x_j - x_i \right) \right] + \sum_{i=l+1}^m v_i^T v_i$$

The derivative of $V(t)$ is given by

$$\dot{V}(t) = -2\sum_{i=1}^{l}\left(\sum_{j=1}^{n} a_{ij}\tanh(x_j - x_i)\right)^T\left(\sum_{j=1}^{n} a_{ij}\tanh(x_j - x_i)\right)$$
$$-2\sum_{i=m+1}^{n} v_i^T \Phi_i \tanh(v_i) - 2\sum_{i=l+1}^{m} v_i^T \Lambda_i \tanh(v_i) \le 0$$

Similar to the proof of Theorem 1, it follows from Barbalat′s Lemma that

$$\lim_{t\to\infty} \|x_j - x_i\| = 0, \quad i,j \in \mathcal{I}_n$$
$$\lim_{t\to\infty} \|v_i\| = 0, \quad i \in \mathcal{I}_l/\mathcal{I}_n$$

This completes the proof. □

Next, we extend the results to the leader-following network, i.e. regulation problem. Assume that there exists a static leader denoted as x_0 and only partial agents can communicate with the leader. If the ith agent can communicate with the leader, then $b_i > 0$, otherwise $b_i = 0$, $i \in \mathcal{I}_n$. Then the protocol can be formulated as

$$\tau_i = \begin{cases} \sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i), & i \in \mathcal{I}_l, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i) - \Lambda_i v_i, & i \in \mathcal{I}_l/\mathcal{I}_m, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i) - \Phi_i v_i, & i \in \mathcal{I}_m/\mathcal{I}_n, j \in \mathcal{I}_n \end{cases} \tag{15}$$

We have the following corollary:

Corollary 3. *Consider the heterogeneous system (9). If there exists a static leader and at least one agent can communicate with it, and the undirected graph $\mathcal{G}$ is connected and τ_i is designed as Eq. (15), then the states of all the agents reach to the state of the leader for all the initial conditions.*

Proof. Let $y_i = x_i - x_0$, $\dot{y}_i = \dot{x}_i$, then the closed-loop system can be written as

$$\begin{cases} \dot{y}_i = \sum_{j=1}^{n} a_{ij}(y_j - y_i) - b_i y_i, & i \in \mathcal{I}_l \\ \dot{y}_i = v_i, & i \in \mathcal{I}_l/\mathcal{I}_m \\ \dot{v}_i = \sum_{j=1}^{n} a_{ij}(y_j - y_i) - b_i y_i - \Lambda_i v_i, & i \in \mathcal{I}_l/\mathcal{I}_m \\ \dot{y}_i = v_i, & i \in \mathcal{I}_m/\mathcal{I}_n \\ M_i(y_i + x_0)\dot{v}_i + C_i(y_i + x_0, v_i)v_i = \sum_{j=1}^{n} a_{ij}(y_j - y_i) - b_i y_i - \Phi_i v_i, & i \in \mathcal{I}_m/\mathcal{I}_n \end{cases} \tag{16}$$

Take a Lyapunov function as

$$V(t) = \sum_{i=m+1}^{n} v_i^T M_i v_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(y_j - y_i)^T\left(y_j - y_i\right) + \sum_{i=l+1}^{m} v_i^T v_i + \sum_{i=1}^{n} b_i y_i^T y_i$$

The following proof is quite similar to Theorem 1, we can prove that $x_i = x_j = x_0 (i, j \in \mathcal{I}_n)$ and $\dot{x}_i = 0$ as $t \to \infty$. The detailed proof is omitted here due to space limitation. □

3.2. Consensus with parametric uncertainties

In this section, we assume that there exist uncertain parameters in the heterogeneous system, i.e. $M_i(x_i)$ and $C_i(x_i, v_i)$ in EL system (3) are not precisely known. To this end, we use the adaptive control technology to solve the problem. Inspired by [12], the protocol of the ith agent is designed as

$$\tau_i = \begin{cases} \sum_{j=1}^{n} a_{ij}(x_j - x_i), & i \in \mathcal{I}_l, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i) - \Xi_i v_i, & i \in \mathcal{I}_l/\mathcal{I}_m, j \in \mathcal{I}_n \\ Y_i\hat{\theta}_i - \lambda\varepsilon_i, & i \in \mathcal{I}_m/\mathcal{I}_n, j \in \mathcal{I}_n \end{cases} \tag{17}$$

where

$$\varepsilon_i = v_i - \kappa\sum_{j=1}^{n} a_{ij}(x_j - x_i) \tag{18}$$

and

$$Y_i\hat{\theta}_i = \kappa\hat{M}_i(x_i)\sum_{j=1}^{n} a_{ij}(\dot{x}_j - \dot{x}_i) + \kappa\hat{C}_i(x_i, v_i)\sum_{j=1}^{n} a_{ij}(x_j - x_i) \tag{19}$$

with $\kappa > 0$, $\lambda > 0$, $\Xi_i \in \mathbb{R}^{p\times p}$ is a positive-definite diagonal matrix. Y_i is a known function of the generalized coordinates. $\hat{\theta}_i$ is the time varying estimates of the constant p-dimensional parameters given by θ_i. $\hat{M}_i(x_i)$ and $\hat{C}_i(x_i, v_i)$ are the estimates of $M_i(x_i)$ and $C_i(x_i, v_i)$, respectively. Substituting Eq. (17) into Eq. (9) yields

$$\begin{cases} \dot{x}_i = \sum_{j=1}^{n} a_{ij}(x_j - x_i), & i \in \mathcal{I}_l \\ \dot{x}_i = v_i, & i \in \mathcal{I}_l/\mathcal{I}_m \\ \dot{v}_i = \sum_{j=1}^{n} a_{ij}(x_j - x_i) - \Xi_i v_i, & i \in \mathcal{I}_l/\mathcal{I}_m \\ \dot{x}_i = v_i, & i \in \mathcal{I}_m/\mathcal{I}_n \\ M_i(x_i)\dot{\varepsilon}_i + C_i(x_i, v_i)\varepsilon_i = Y_i\tilde{\theta}_i - \lambda\varepsilon_i, & i \in \mathcal{I}_m/\mathcal{I}_n \end{cases} \tag{20}$$

where $\tilde{\theta}_i(t) = \theta_i - \hat{\theta}_i(t)$ is the estimation errors of θ_i, and $\hat{\theta}_i(t)$ evolves as

$$\dot{\hat{\theta}}_i(t) = \frac{\Gamma_i Y_i^T \varepsilon_i}{\lambda} \tag{21}$$

where Γ_i is a constant positive definite matrix. Then the following result is in order:

Theorem 4. *Consider the heterogeneous system (9) with uncertain parameters. If the undirected graph $\mathcal{G}$ is connected and τ_i is designed as Eq. (17), then the states of all the agents reach a consensus in the sense of Eq. (8) for all the initial conditions.*

Proof. Consider the following Lyapunov candidate function

$$V(t)=\frac{1}{2\lambda}\sum_{i=m+1}^{n}\varepsilon_i^T M_i(x_i)\varepsilon_i+\frac{1}{2}\sum_{i=m+1}^{n}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i$$
$$+\frac{\kappa}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(x_j-x_i)^T\left(x_j-x_i\right)+\kappa\sum_{i=l+1}^{m}v_i^Tv_i$$

The derivative of $V(t)$ along the trajectory of Eq. (20) is given by

$$\begin{aligned}
\dot{V}(t)&=\frac{1}{2\lambda}\sum_{i=m+1}^{n}\varepsilon_i^T\dot{M}_i(x_i)\varepsilon_i+\frac{1}{\lambda}\sum_{i=m+1}^{n}\varepsilon_i^TM_i(x_i)\dot{\varepsilon}_i+\sum_{i=m+1}^{n}\tilde{\theta}_i^T\Gamma_i^{-1}\dot{\tilde{\theta}}_i\\
&\quad+\kappa\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(x_j-x_i)^T\left(\dot{x}_j-\dot{x}_i\right)+2\kappa\sum_{i=l+1}^{m}v_i^T\dot{v}_i\\
&=\frac{1}{2\lambda}\sum_{i=m+1}^{n}\varepsilon_i^T\dot{M}_i(x_i)\varepsilon_i+\frac{1}{\lambda}\sum_{i=m+1}^{n}\varepsilon_i^T\left(Y_i\tilde{\theta}_i-\lambda\varepsilon_i-C_i(x_i,v_i)\varepsilon_i-Y_i\tilde{\theta}_i\right)\\
&\quad+\kappa\sum_{i=1}^{l}\sum_{j=1}^{l}a_{ij}(x_j-x_i)^T(\dot{x}_j-\dot{x}_i)+\kappa\sum_{i=l+1}^{n}\sum_{j=1}^{l}a_{ij}(x_j-x_i)^T(\dot{x}_j-v_i)\\
&\quad+\kappa\sum_{i=1}^{l}\sum_{j=l+1}^{n}a_{ij}(x_j-x_i)^T(v_j-\dot{x}_i)+\kappa\sum_{i=l+1}^{n}\sum_{j=l+1}^{n}a_{ij}(x_i-x_j)^T(v_j-v_i)\\
&\quad+2\kappa\sum_{i=l+1}^{m}v_i^T\left(\sum_{j=1}^{n}a_{ij}(x_j-x_i)-\Xi_iv_i\right)\\
&=-\sum_{i=m+1}^{n}\left(v_i-\kappa\sum_{j=1}^{n}a_{ij}(x_j-x_i)\right)^T\left(v_i-\kappa\sum_{j=1}^{n}a_{ij}(x_j-x_i)\right)\\
&\quad-2\kappa\sum_{i=1}^{l}\sum_{j=1}^{n}a_{ij}(x_j-x_i)^T\dot{x}_i-2\kappa\sum_{i=l+1}^{n}\sum_{j=1}^{n}a_{ij}(x_j-x_i)^Tv_i\\
&\quad+2\kappa\sum_{i=l+1}^{m}\sum_{j=1}^{n}a_{ij}v_i^T(x_j-x_i)-2\kappa\sum_{i=l+1}^{m}v_i^T\Xi_iv_i\\
&=-\sum_{i=m+1}^{n}v_i^Tv_i-\kappa^2\sum_{i=m+1}^{n}\left(\sum_{j=1}^{n}a_{ij}(x_j-x_i)\right)^T\left(\sum_{j=1}^{n}a_{ij}(x_j-x_i)\right)\\
&\quad-2\kappa\sum_{i=1}^{l}\left(\sum_{j=1}^{n}a_{ij}(x_j-x_i)\right)^T\left(\sum_{j=1}^{n}a_{ij}(x_j-x_i)\right)-2\kappa\sum_{i=l+1}^{m}v_i^T\Xi_iv_i
\end{aligned}$$

Since κ and β are positive constants, it follows that $\dot{V}(t)\le 0$. Similar to the proof of Theorem 1, we obtain that $v_i(i\in\mathcal{I}_l/\mathcal{I}_m)$, $\sum_{j=1}^{n}a_{ij}(x_j-x_i)(i\in\mathcal{I}_n)$ and $\dot{v}_i(i\in\mathcal{I}_l/\mathcal{I}_m)$ are bounded. It is known that $\tilde{\theta}_i$ and ε_i are also bounded. Next we show $\dot{v}_i(i\in\mathcal{I}_m/\mathcal{I}_n)$ is bounded, and firstly, we need to show the boundeness of ε_i. Because of the boundedness $M_i(x_i)$, $C_i(x_i,v_i)$, $\hat{M}_i(x_i)$, $\hat{C}_i(x_i,v_i)$, ε_i, $\tilde{\theta}_i$ and $\hat{\theta}_i$, we can get that Y_i is bounded from Eq. (19). Note that in

$M_i(x_i)\dot{\varepsilon}_i + C_i(x_i, v_i)\varepsilon_i = Y_i\tilde{\theta}_i - \lambda\varepsilon_i$, $M_i(x_i)$, $C_i(x_i, v_i)$ and ε_i are all bounded, then it follows that $\dot{\varepsilon}_i$ is bounded. Hence, by differentiating Eq. (18), we can get that $\dot{v}_i(i \in \mathcal{I}_m/\mathcal{I}_n)$ is bounded. Therefore, $\ddot{V}(t)$ is bounded. By invoking Barbalat′s Lemma, we conclude that $\dot{V}(t) \to 0$ as $t \to \infty$. Finally, it can be concluded that $v_i = 0(i \in \mathcal{I}_l/\mathcal{I}_n)$ and $\sum_{i=1}^{n} a_{ij}(x_j - x_i) = 0(i,j \in \mathcal{I}_n)$ as $t \to \infty$. The following proof is the same as proof of Theorem 1. It follows that

$$\lim_{t\to\infty} \| x_j - x_i \| = 0, \quad i,j \in \mathcal{I}_n$$

$$\lim_{t\to\infty} \| v_i \| = 0, \quad i \in \mathcal{I}_l/\mathcal{I}_n$$

This concludes the proof. □

Remark 4. From the proof of Theorem 1, we can see that the consensus is derived without using the *persistently exciting* (PE) condition [26]. Therefore, the coordination of the heterogeneous systems will still be reachable in the absence of the PE condition. In this case, we certainly would not expect the parameter convergence. In fact, when consensus is reached, $\dot{\hat{\theta}}_i = 0$ with $\hat{\theta}_i$ being some constant vector. Therefore, no conclusion can be drawn about the behavior of the estimation error $\hat{\theta}_i$ in the absence of the PE condition, except that it converges to a constant vector.

Remark 5. Similar to Remark 2, we can also extend the protocol (17) to the case of desired differences. The algorithm is given by

$$\tau_i = \begin{cases} \sum_{j=1}^{n} a_{ij}(x_j - x_i - x_{ij}^d), & i \in \mathcal{I}_l, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i - x_{ij}^d) - \Xi_i v_i, & i \in \mathcal{I}_l/\mathcal{I}_m, j \in \mathcal{I}_n \\ Y_i\hat{\theta}_i - \lambda\varepsilon_i, & i \in \mathcal{I}_m/\mathcal{I}_n, j \in \mathcal{I}_n \end{cases} \tag{22}$$

with

$$\begin{cases} \varepsilon_i = v_i - \kappa \sum_{j=1}^{n} a_{ij}(x_j - x_i - x_{ij}^d) \\ \dot{\hat{\theta}}_i(t) = \frac{\Gamma_i Y_i^T \varepsilon_i}{\lambda} Y_i\hat{\theta}_i = \kappa\hat{M}_i(x_i)\sum_{j=1}^{n} a_{ij}(\dot{x}_j - \dot{x}_i) + \kappa\hat{C}_i(x_i, v_i)\sum_{j=1}^{n} a_{ij}(x_j - x_i - x_{ij}^d) \end{cases}$$

Remark 6. Like Corollary 3, the following algorithm can deal with the regulation problem of heterogeneous systems with uncertain EL agents:

$$\tau_i = \begin{cases} \sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i), & i \in \mathcal{I}_l, j \in \mathcal{I}_n \\ \sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i) - \Xi_i v_i, & i \in \mathcal{I}_l/\mathcal{I}_m, j \in \mathcal{I}_n \\ Y_i\hat{\theta}_i - \lambda\varepsilon_i, & i \in \mathcal{I}_m/\mathcal{I}_n, j \in \mathcal{I}_n \end{cases} \tag{23}$$

with

$$\begin{cases} \varepsilon_i = v_i - \kappa\left(\sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i)\right) \\ \dot{\hat{\theta}}_i(t) = \frac{\Gamma_i Y_i^T \varepsilon_i}{\lambda} Y_i \hat{\theta}_i = \kappa \hat{M}_i(x_i)\left(\sum_{j=1}^{n} a_{ij}(\dot{x}_j - \dot{x}_i) - b_i \dot{x}_i\right) + \kappa \hat{C}_i(x_i, v_i)\left(\sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i)\right) \end{cases}$$

where $i \in \mathcal{I}_m / \mathcal{I}_n, j \in \mathcal{I}_n$. We can prove that algorithm (23) solves the regulation problem. The proof is similar to the one of Corollary 3, and it is omitted here.

4. Numerical simulation

In this section, we present numerical simulation results to illustrate the effectiveness of our protocols. Consider a heterogeneous systems composed of two first-order integrator agents, two second-order integrator agents and two EL agents, and the model is shown as

$$\begin{cases} \dot{x}_i(t) = \tau_i(t), & i \in \{1,2\} \\ \ddot{x}_i(t) = \tau_i(t), & i \in \{3,4\} \\ M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i = \tau_i(t), & i \in \{5,6\} \end{cases} \tag{24}$$

where

$$M_i = \begin{bmatrix} a_1 + 2a_3 \cos x_{i(2)} + 2a_4 \sin x_{i(2)} & a_2 + a_3 \cos x_{i(2)} + a_4 \sin x_{i(2)} \\ a_2 + a_3 \cos x_{i(2)} + a_4 \sin x_{i(2)} & a_2 \end{bmatrix}$$

$$C_i = \begin{bmatrix} a_4 \cos x_{i(2)} - a_3 \sin x_{i(2)} & (a_4 \cos x_{i(2)} - a_3 \sin x_{i(2)})(\dot{x}_{i(1)} + \dot{x}_{i(2)}) \\ a_3 \sin x_{i(2)} - a_4 \cos x_{i(2)} & 0 \end{bmatrix}$$

with $a_1 = I_1 + m_l l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2$, $a_2 = I_e + m_e l_{ce}^2$, $a_3 = m_e l_1 l_{ce} \cos \delta_e$, $a_4 = m_e l_1 l_{ce} \sin \delta_e$ and $x_i = [x_{i(1)}, x_{i(2)}]^T$, $\tau_i = [\tau_{i(1)}, \tau_{i(2)}]^T$. In this simulation, the values $m_1 = 1.2$, $l_1 = 1.2$, $m_e = 2.5$ $\delta_e = 30^\circ$, $I_1 = 0.15$, $l_{c1} = 0.5$, $I_e = 0.25$, $l_{ce} = 0.6$ are taken.

Let vertices 1 and 2 denote single integrator agents, vertices 3 and 4 denote double integrator agents and vertices 5 and 6 represent EL agents. The communication topology is presented as Fig. 1. The initial state is chosen as $x_1(0) = [-1.5, -0.4]^T$, $x_2(0) = [1.9, 1.5]^T$, $x_3(0) = [-1.8, -1.5]^T$, $x_4(0) = [0.1, 0.6]^T$, $x_5(0) = [1.4, 1.9]^T$, $x_6(0) = [-0.5, -1.8]^T$, $v_2(0) = [-0.2, 0.1]^T$, $v_3(0) = [-0.8, 1.9]^T$, $v_4(0) = [0.1, -0.3]^T$, $v_5(0) = [0.6, 0.8]^T$.

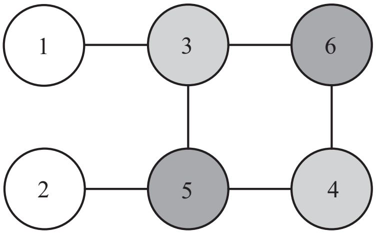

Fig. 1. Communication and sensing graph.

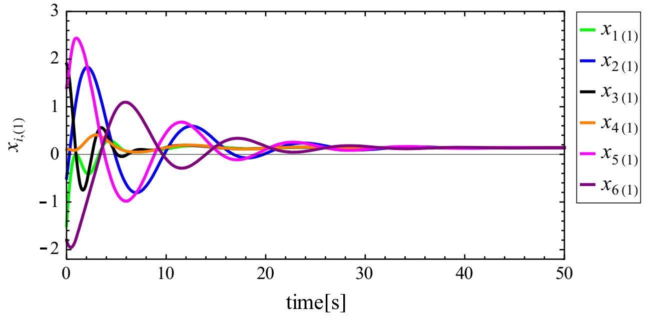

Fig. 2. State of $x_{i(1)}$.

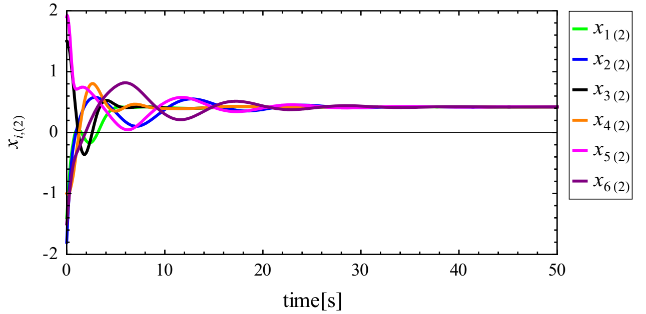

Fig. 3. State of $x_{i(2)}$.

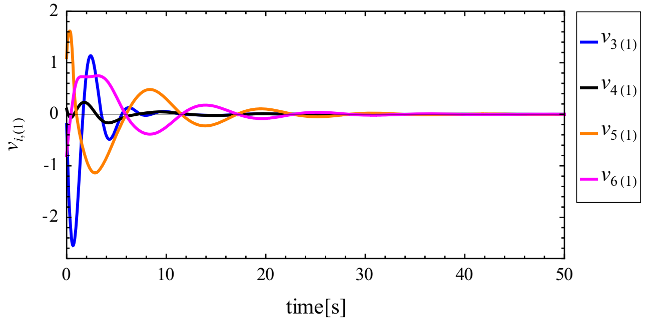

Fig. 4. State of $v_{i(1)}$.

By applying algorithm (15), we get the simulation results as shown in Fig. 2–5. From these figures, we know that protocol (15) can solve the regulation problem of heterogeneous system (24).

As to the heterogeneous system with unknown parameters, we use protocol (23) to finish the simulation. Assume that the initial condition is same as which in Eq. (24). Denote $\varepsilon_i = (\varepsilon_{i(1)}, \varepsilon_{i(2)})^T = \sum_{j=1}^{n} a_{ij}(x_j - x_i) + b_i(x_0 - x_i)$. In Eq. (23), $\theta = [a_1, a_2, a_3, a_4]^T$, and all of these parameters are set to be 10–80% of accuracy of their real values. Accordingly, $Y_i \in \mathbb{R}^{2\times 4}$ is

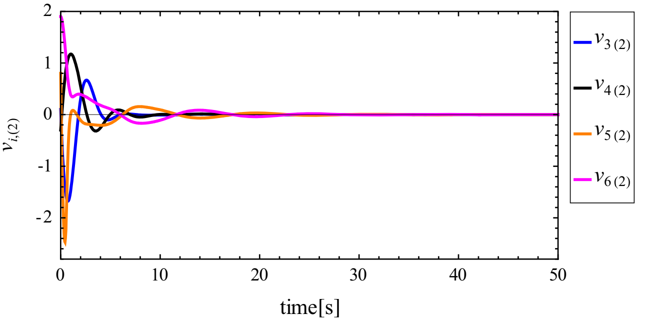

Fig. 5. State of $v_{i(2)}$.

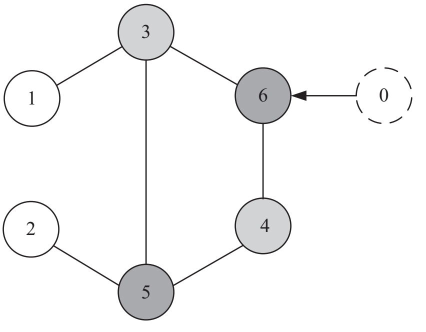

Fig. 6. Communication topology.

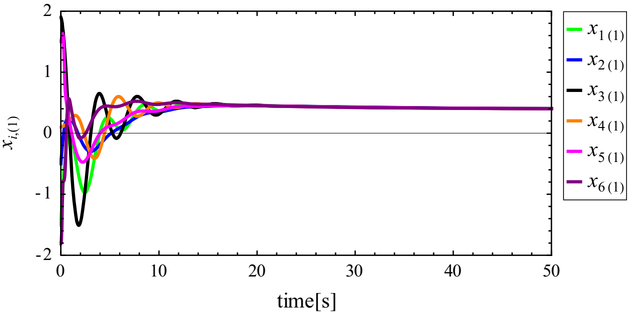

Fig. 7. State of $x_{i(1)}$ with protocol (23).

presented as

$$Y_i = \begin{bmatrix} \dot{\varepsilon}_{i(1)} & \dot{\varepsilon}_{i(2)} & y_{13} & y_{14} \\ 0 & \dot{\varepsilon}_{i(1)} + \dot{\varepsilon}_{i(2)} & y_{23} & y_{24} \end{bmatrix}$$

where

$$y_{13} = (2\dot{\varepsilon}_{i(1)} + \dot{\varepsilon}_{i(2)}) \cos\ q_{i(2)} - (\varepsilon_{i(1)}\dot{q}_{i(2)} + \varepsilon_{i(2)}\dot{q}_{i(1)} + \varepsilon_{i(2)}\dot{q}_{i(2)})\ \sin\ q_{i(2)}$$

$$y_{14} = (2\dot{\varepsilon}_{i(1)} + \dot{\varepsilon}_{i(2)}) \sin q_{i(2)} + (\varepsilon_{i(1)}\dot{q}_{i(2)} + \varepsilon_{i(2)}\dot{q}_{i(1)} + \varepsilon_{i(2)}\dot{q}_{i(2)}) \cos q_{i(2)}$$
$$y_{23} = \dot{\varepsilon}_{i(1)} \cos q_{i(2)} + \varepsilon_{i(1)}\dot{q}_{i(1)} \sin q_{i(2)}$$
$$y_{24} = -\varepsilon_{i(1)}\dot{q}_{i(1)} \cos q_{i(2)} + \dot{\varepsilon}_{i(1)} \sin q_{i(2)}$$

We assume agent 0 to be the static leader with the state $x_0 = [0.4, -0.8]^T$ and the communication topology is shown in Fig. 6. The simulation results are given in Figs. 7–10, and as these figures show, all the states reach to the state of agent 0, i.e. $x_0 = [0.4, -0.8]^T$. We can conclude that protocol (23) solves the regulation problem.

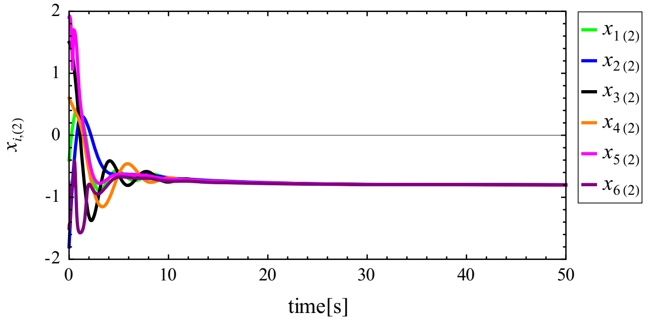

Fig. 8. State of $x_{i(2)}$ with protocol (23).

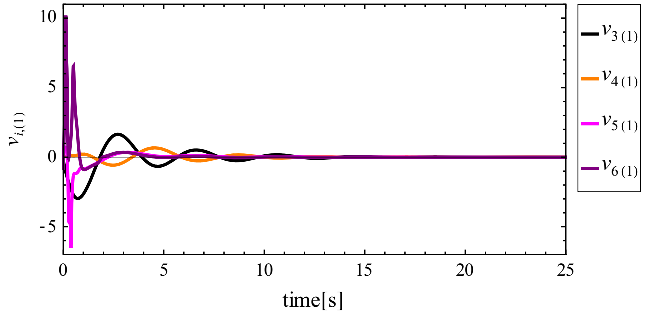

Fig. 9. State of $v_{i(1)}$ with protocol (23).

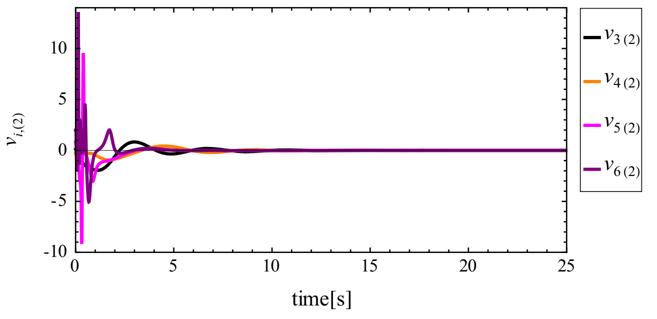

Fig. 10. State of $v_{i(2)}$ with protocol (23).

5. Conclusions

In this paper, we have investigated a class of heterogenous multi-agent system composed of the linear first-order, second-order integrator agents and the nonlinear EL agents. Both heterogeneous multi-agent systems with exactly known parameters and unknown parameters are taken into consideration. Sufficient conditions for the consensus of heterogeneous multi-agent system are obtained, and distributed protocols are presented to solve the consensus problem. Especially in the case where there exist parametric uncertainties in the EL agents, we combine PD controller and adaptive controller together to deal with the consensus problem. Our future work will focus on the consensus problem of heterogeneous system with more complex communication topologies, for example, switching topologies or stochastic topologies.

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