Fixed-Time Consensus Tracking of Heterogeneous Multi-agent Systems*

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***Abstract*—Fixed-time consensus tracking problem of the heterogeneous multi-agent systems comprised of first-order agents and second-order agents with uncertain disturbances is studied in this paper. To solve consensus tracking problem of heterogeneous multi-agent systems, we design a novel distributed fixed-time observer for each follower to estimate leader's states, and the decentralized fixed-time sliding mode controllers which can suppress disturbances to make each follower track the leader's states within fixed-time. By using graph theory and Lyapunov stability theory, fixed-time consensus tracking conditions are presented for our proposed algorithms. Simulation examples show the effectiveness of theoretical results.**

***Index Terms*—Fixed-time consensus tracking, distributed fixed-time observer, sliding mode control, heterogeneous multi-agent systems**

I. Introduction

With the development of network communication technology in recent years, cooperative control in multi-agent systems has been applied in many research fields, such as multi-robot formation control [1], wireless sensor networks [2], and multi-vehicle cooperative control [3], etc. Consensus tracking problem, which is regarded as one of the most fundamental problems in multi-agent cooperative control, has received scholars' research interest [4]-[6]. In the leader-following consensus tracking problem [7], all followers are required to track leader's states via locally exchanging information.

For multi-agent systems, many researchers used nonlinear control methods to solve the consensus problem. Sliding mode control, as an excellent nonlinear robust control method [8], has been used extensively to achieve finite-time or fixed-time consensus tracking for multi-agent systems and there are many achievements in this aspect. Zuo [9] proposed a sliding mode surface and a distributed fixed-time controller for the second-order multi-agent systems to achieve fixed-time consensus convergence. Jin [10] proposed a nonsingular terminal sliding mode controller which is based on the sliding mode observer to achieve finite-time consensus. With the help of the Artstein's transformation, Ni [11] achieved fixed-time consensus tracking by designing observers and sliding-mode controllers for second-order multi-agent systems with input delay. Tian [12] presented a sliding mode manifold and designed a distributed controller to achieve fixed-time consensus tracking for high-order leader-following multi-agent systems. But each agent's control law in [12] needs the neighbors' control input. Yu [14] designed a full-order sliding mode manifold, which avoided singularity problems and weakened chattering [13], and a distributed sliding mode controller to achieve second-order consensus, but its settling-time estimation cannot be explicitly provided in [14].

In the above research results, the dynamic models of all agents in the multi-agent systems are similar, but many multi-agent systems are always heterogeneous in reality. Therefore, the heterogeneous multi-agent systems has attracted more attention and many researchers have achieved a lot of results in this field. Zheng [15, 16] investigated the heterogeneous multi-agent systems with and without velocity measurements, and constructed the consensus algorithms to achieve consensus. Moreover, Zhu [17] designed a nonlinear consensus protocol to solve finite-time consensus problem of the heterogeneous multi-agent systems with first-order leader or second-order leader. Liu [18] investigated consensus seeking problem for heterogeneous multi-agent systems with identical communication delay under a connected and symmetric topology, and obtained consensus conditions according to Nyquist and generalized Nyquist stability criteria. Zheng [19] investigated average-consensus tracking problem of the heterogeneous multi-agent systems without and with communication delay, and obtained the consensus conditions based on Nyquist stability criterion. Yuan [20] considered the heterogeneous multi-agent systems with a leader and distinct-order followers, and designed a two-layer control scheme composed of the upper-layer cooperative estimator estimating the state of leader and the lower-layer distributed output regulator realizing the exact output regulation performance.

Motivated by the above discussions, this paper investigates the fixed-time consensus tracking problem of the heterogeneous multi-agent systems formed by first-order and second-order agents. The main contributions of this paper are listed as follows. Firstly, considering that only a part of followers can obtain the states of leader, the distributed fixed-time observer makes each follower estimate the position and speed of leader within fixed time. Compared with [11], besides, it's easy to design the upper bound of observer's settling time. Secondly, compared with [15–17], we propose a distributed observer and decentralized control method to achieve fixed-time consensus

This work is supported by National Natural Science Foundation of China under Grant 61973139 and Six Talent Peaks Project in Jiangsu Province under Grant 2015-DZXX-011.

tracking for heterogeneous multi-agent systems. Moreover, the algorithms suppresses disturbances and provides an explicit settling-time estimation.

II. Preliminaries

A. Graph Theory

Considering a multi-agent system composed of a leader and N followers, the topology between N followers is described as $G \in \{V, E, A\}$ with a set of vertices $V = \{1, 2, \cdots, N\}$, a set of edges $E \subseteq V \times V$ and a weighted adjacency matrix $A = [a_{ij}] \in R^{N \times N}$ with $a_{ij} \geq 0$. In the graph G, the elements of A is nonnegative and $e_{ij} = (i, j) \in E$ denotes a directed edge from vertex i to j. The set of i's neighbors is denoted by $N_i = \{j \in V : (i, j) \in E\}$ and the series of different vertices $V = \{i, \cdots, j\}$ is a path from vertex i to vertex j if two vertices of the series of vertices are neighbors. Moreover, we assume $a_{ij} > 0$ if $e_{ij} = (i, j) \in E$, $a_{ij} = 0$ if $e_{ij} = (i, j) \notin E$, and $a_{ii} = 0$ for all $i \in V$. G is an undirected graph if $a_{ij} = a_{ji}$, and the undirected graph is connected if there exists a path between any two different vertices. The Laplacian matrix of the graph G is denoted as $L = D - A = [l_{ij}] \in R^{N \times N}$, where the degree matrix D is denoted as $D = \text{diag}\{\Sigma_{j=1}^{N} a_{ij}, i \in V\}$. Define $H = L + B$ with $B = \text{diag}\{b_1, b_2, \cdots, b_N\}$, where $b_i > 0$ if the follower i has direct link with the leader and $b_i = 0$ otherwise. Herein, the properties of H will be given in Lemma 1.

Lemma 1 [21]: The Laplacian matrix L is semi-positive definite and the matrix H is positive definite, if the graph G is undirected and connected and each follower has a path to the leader.

B. Problem Description

Consider the heterogeneous multi-agent systems comprised of a leader, M first-order followers and $N - M$ second-order followers where N, M are positive integers and $M < N$. The dynamics of N followers are described as

$$\begin{aligned} &\dot{x}_k(t) = u_k(t) + d_k(t), \quad k = 1, 2, \cdots, M, \\ &\dot{x}_l(t) = v_l(t), \\ &\dot{v}_l(t) = u_l(t) + d_l(t), \quad l = M+1, M+2, \cdots, N, \end{aligned} \tag{1}$$

where $x_k(t) \in R$, $u_k(t) \in R$ and $d_k(t) \in R$ are the position, the control input and the external disturbance of agent k, respectively. $x_l(t) \in R$, $v_l(t) \in R$, $u_l(t) \in R$ and $d_l(t)$ are the position, the velocity, the control input and the external disturbance of agent l, respectively.

The dynamics of the leader is given by

$$\begin{aligned} &\dot{x}_0(t) = v_0(t), \\ &\dot{v}_0(t) = u_0(t), \end{aligned} \tag{2}$$

where $x_0(t) \in R$, $v_0(t) \in R$ and $u_0(t) \in R$ are the position, the velocity and the control input of the leader, respectively.

Assumption 1: $|d_k(t)| \leq \rho_k$ and $|d_l(t)| \leq \rho_l$, where ρ_k, ρ_l are positive.

Assumption 2: In the multi-agent systems, only a subset of the followers can directly obtain the position and velocity of leader, but the leader's control input $u_0(t)$ is known to all followers.

Assumption 3: The topology G formed by N followers is undirected and each follower has a path to the leader.

Definition 1: The multi-agent systems (1) and (2) achieves globally fixed-time consensus tracking, if there exists a bounded time constant $T \in [0, T_{max}]$, so that the states of each follower satisfies $\lim_{x \to T_{max}} \parallel x_i(t) - x_0(t) \parallel = 0$.

C. Some Useful Lemmas

Consider the following system

$$\dot{x}(t) = f(x, t), \quad x(0) = x_0, \tag{3}$$

where $x \in R^P$ is the state and $f : R^+ \times R^P \rightarrow R^P$ is a nonlinear function.

Lemma 2 [22]: For the system (3), if there is a continuous positive definite function $V(x) : R^P \rightarrow R^+ \cup \{0\}$ satisfying

$$\dot{V} \leq -\alpha V^{p/q} - \beta V^{m/n},$$

where $\alpha > 0, \beta > 0$ and p, q, m, n are positive odd integers satisfying $m/n > 1$ and $0 < q/p < 1$. Then, the zero equilibrium state of system (3) is globally fixed-time stable where the settling time satisfies

$$T \leq \frac{1}{\alpha} \frac{q}{q-p} + \frac{1}{\beta} \frac{n}{m-n}, \quad \forall x_0 \in R. \tag{4}$$

Lemma 3 [23]: For the system (3), there is a continuous positive definite function $V(x) : R^P \rightarrow R^+ \cup \{0\}$ satisfying

$$\dot{V} \leq -\mu V^{\sigma}, \tag{5}$$

where $\mu > 0, \sigma > 0$, and the zero equilibrium state of system (3) is asymptotically stable if $\sigma = 1$. If $0 < \sigma = p/q < 1$ and p, q are positive odd integers, the zero equilibrium state of system (3) is finite-time stable, where the settling-time satisfies

$$T \leq \frac{|V_0|^{(1-\sigma)}}{\mu(1-\sigma)}. \tag{6}$$

If σ is odd integer and $\sigma > 1$, for every $\varepsilon > 0$, $V(x) < \varepsilon$ is achieved within bounded time, where the settling-time satisfies

$$T \leq \frac{1}{\mu(\sigma - 1)\varepsilon^{(\sigma-1)}}. \tag{7}$$

Lemma 4 [24]: For the system (3), there is a continuous positive definite function $V(x) : R^P \rightarrow R^+ \cup \{0\}$ satisfying

$$\dot{V} \leq -\frac{p}{qT^c} \Big(1 + (V^{q/p})^2\Big)^{3/2} V^{(1-q/p)}, \tag{8}$$

where p, q are positive odd integers satisfying $0 < p/q < 1$ and $T^c > 0$. Then, the zero equilibrium state of system (3) is globally fixed-time stable and the settling time is bounded by

$$T \leq T^c, \quad \forall x_0 \in R. \tag{9}$$

Lemma 5 [25]: If $\xi_1, \xi_2, \cdots, \xi_n \geq 0$ and $0 < \delta \leq 1$, then $\sum_{i=1}^{n} \xi_i^{\delta} \geq \Big(\sum_{i=1}^{n} \xi_i\Big)^{\delta}$. If $\xi_1, \xi_2, \cdots, \xi_n \geq 0$ and $\delta > 1$, then $\sum_{i=1}^{n} \xi_i^{\delta} \geq n^{1-\delta}\Big(\sum_{i=1}^{n} \xi_i\Big)^{\delta}$.

III. Main Results

This section is mainly divided into two parts. In the first part, we designs the distributed fixed-time observers estimating leader's states for each follower and proves its effectiveness theoretically. In the second part, the decentralized fixed-time sliding-mode controllers are designed for first-order and second-order followers respectively, and the effectiveness is also proved theoretically.

A. Distributed Fixed-time Observers

In the multi-agent systems, the followers which don't have direct communication link to leader cannot get the position and velocity of leader. The distributed fixed-time observer is designed as

$$\begin{aligned}\dot{\theta}_i =&\omega_i - c\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\theta_i-\theta_j)+b_i(\theta_i-x_0)\Big)^{(1-p/q)}\\&- g\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\theta_i-\theta_j)+b_i(\theta_i-x_0)\Big)^{(1+2p/q)},\\ \dot{\omega}_i =&u_0 - c\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\omega_i-\omega_j)+b_i(\omega_i-v_0)\Big)^{(1-p/q)}\\&- g\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\omega_i-\omega_j)+b_i(\omega_i-v_0)\Big)^{(1+2p/q)},\end{aligned} \tag{10}$$

where $\theta_i \in R$ and $\omega_i \in R$ are the estimations of leader's position and velocity respectively for ith follower through the observer, p, q are positive odd integers satisfying $q > p > 0$, $sig(\cdot)^\alpha =| \cdot |^\alpha \, sign(\cdot)$ and c, g satisfy:

$$\begin{aligned}c &= \frac{\sqrt{2}q}{\lambda_1(H)pT_1},\\ g &= \frac{\sqrt{2}qN^{p/q}}{\lambda_1(H)pT_1},\end{aligned} \tag{11}$$

where $T_1 > 0$.

Theorem 1: Assume that the dynamics of leader is described by (2) and *Assumption 2*, *Assumption 3* hold. The distributed observer (10) estimates the leader's position x_0 and velocity v_0 within fixed time globally and the settling-time function t_1 satisfies

$$t_1 \le 2T_1. \tag{12}$$

Proof: The estimated errors are defined as $\tilde{\theta}_i = \theta_i - x_0$ and $\tilde{\omega}_i = \omega_i - v_0$. Combined with (10), we have

$$\begin{aligned}\dot{\tilde{\theta}}_i =&\tilde{\omega}_i - c\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\tilde{\theta}_i-\tilde{\theta}_j)+b_i\tilde{\theta}_i\Big)^{(1-p/q)}\\&- g\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\tilde{\theta}_i-\tilde{\theta}_j)+b_i\tilde{\theta}_i\Big)^{(1+2p/q)},\\ \dot{\tilde{\omega}}_i =&- c\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\tilde{\omega}_i-\tilde{\omega}_j)+b_i\tilde{\omega}_i\Big)^{(1-p/q)}\\&- g\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\tilde{\omega}_i-\tilde{\omega}_j)+b_i\tilde{\omega}_i\Big)^{(1+2p/q)}.\end{aligned} \tag{13}$$

Let $\tilde{\omega} = [\tilde{\omega}_1, \tilde{\omega}_2, \cdots, \tilde{\omega}_n]^T$, and we design the Lyapunov function as $V_1 = \frac{1}{2}\tilde{\omega}^T H\tilde{\omega}$. Then, we have

$$\begin{aligned}\dot{V}_1 =& \sum_{i\in N_i}\frac{dV_1}{d\tilde{\omega}_i}\cdot\dot{\tilde{\omega}}_i\\ =&-c\sum_{i\in N_i}\left|\sum_{j\in N_i} a_{ij}(\tilde{\omega}_i-\tilde{\omega}_j)+b_i\tilde{\omega}_i\right|^{(2-p/q)}\\ &-g\sum_{i\in N_i}\left|\sum_{j\in N_i} a_{ij}(\tilde{\omega}_i-\tilde{\omega}_j)+b_i\tilde{\omega}_i\right|^{(2+2p/q)}.\end{aligned} \tag{14}$$

Based on *Lemma 5*, we get

$$\begin{aligned}\dot{V}_1 \le& -c\left(\sum_{i\in N_i}\left(\sum_{j\in N_i} a_{ij}(\tilde{\omega}_i-\tilde{\omega}_j)+b_i\tilde{\omega}_i\right)^2\right)^{(2-p/q)/2}\\ &-gN^{(-p/q)}\left(\sum_{i\in N_i}\left(\sum_{j\in N_i} a_{ij}(\tilde{\omega}_i-\tilde{\omega}_j)+b_i\tilde{\omega}_i\right)^2\right)^{(1+p/q)}\\ \le& -c(2\lambda_1(H)V_1)^{\frac{2-p/q}{2}} - gN^{(-p/q)}(2\lambda_1(H)V_1)^{(1+p/q)}.\end{aligned} \tag{15}$$

Defining $\eta = \sqrt{2\lambda_1(H)V_1}$, (15) can be rewritten as

$$\begin{aligned}\dot{\eta} \le& -c\lambda_1(H)\eta^{(1-p/q)} - gN^{(-p/q)}\lambda_1(H)\eta^{(1+2p/q)}\\ \le& -\frac{\sqrt{2}q}{pT_1}\left(\eta^{(1-p/q)} + \left(\left(\eta^{p/q}\right)^2\right)^{3/2}\eta^{(1-p/q)}\right)\\ \le& -\frac{q}{pT_1}\left(\eta^{2(1-p/q)/3} + \left(\eta^{p/q}\right)^2\eta^{2/3(1-p/q)}\right)^{3/2}\\ =& -\frac{q}{pT_1}\left(1+\left(\eta^{p/q}\right)^2\right)^{3/2}\eta^{(1-p/q)}.\end{aligned} \tag{16}$$

Based on *Lemma 4*, v_0 can be estimated within fixed time and the settling-time t_1 meets

$$t_1 \le T_1. \tag{17}$$

When $t \ge T_1$, $\tilde{\omega}_i = \omega_i - v_0 = 0$. Combined with (13), we have

$$\begin{aligned}\dot{\tilde{\theta}}_i =& - c\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\tilde{\theta}_i-\tilde{\theta}_j)+b_i\tilde{\theta}_i\Big)^{(1-p/q)}\\&- g\cdot sig\Big(\sum_{j\in N_i} a_{ij}(\tilde{\theta}_i-\tilde{\theta}_j)+b_i\tilde{\theta}_i\Big)^{(1+2p/q)}.\end{aligned} \tag{18}$$

Define $\tilde{\theta} = [\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_n]^T$, and the Lyapunov function is designed as $V = \frac{1}{2}\tilde{\theta}^T H\tilde{\theta}$. T_1 is also the convergence time of $\tilde{\theta}$ according to the same process as above. Consequently, the distributed observer (10) estimates the leader's states within fixed time and the settling-time function satisfies $t_1 \le 2T_1$. Theorem 1 is proved. □

Remark 1: The settling-time function (17) has an upper bound which only depends on T_1, and T_1 includes no other parameters and is easy to be designed.

B. Decentralized Fixed-time Sliding-mode Controller

For the heterogeneous agents (1) and (2), the tracking errors are defined as

$$\begin{aligned} e_k &= x_k - \theta_k, \\ e_l &= x_l - \theta_k, \end{aligned} \tag{19}$$

and $\theta_k = x_0$, $\omega_k = v_0$ when $t \geq t_1$.

For first-order followers, the terminal sliding-mode surface are constructed as

$$s_k = e_k + \alpha_k \int_0^t sig^{r_k}(e_k)\, dt + \beta_k \int_0^t sig^{\kappa_k}(e_k)\, dt, \tag{20}$$

where $\alpha_k > 0, \beta_k > 0, 0 < r_k = p_k/q_k < 1$, $\kappa_k > 1$. The control inputs are designed by

$$\begin{aligned} u_k =& \omega_0 - \alpha_k sig^{r_k}(e_k) - \beta_k sig^{\kappa_k}(e_k) - \alpha_k sig^{r_k}(s_k) \\ & - \beta_k sig^{\kappa_k}(s_k) - D_k sign(s_k), k = 1, 2, \cdots, M, \end{aligned} \tag{21}$$

where $D_k \geq \rho_k$.

For second-order followers, the nonsingular terminal sliding-mode surface are constructed as

$$s_l = \begin{cases} \dot{e}_l + \beta_l {e_l}^{\kappa_l}, & |e_l(t)| > \varepsilon_l, \\ \dot{e}_l + \alpha_l sign(e_l), & |e_l(t)| \leq \varepsilon_l, \end{cases} \tag{22}$$

where $\alpha_l > 0, \beta_l > 0, 0 < r_l = p_l/q_l < 1$, $\kappa_l > 1$. The control inputs are designed by

$$\begin{aligned} u_l \;\; = \;\; & \begin{cases} \dot{\omega}_0 - \beta_l \kappa_l {e_l}^{\kappa_l - 1} \dot{e}_l - \alpha_l sig^{r_l}(s_l) \\ -\beta_l sig^{\kappa_l}(s_l) - D_l sign(s_l), \qquad |e_l(t)| > \varepsilon_l \\ \dot{\omega}_0 - \alpha_l sig^{r_l}(s_l) - \beta_l sig^{\kappa_l}(s_l) - D_l sign(s_l), \\ \qquad\qquad\qquad\qquad |e_l(t)| \leq \varepsilon_l, \end{cases} \\ & l = M+1, M+2, \cdots, N, \end{aligned} \tag{23}$$

where $D_l \geq \rho_l$.

Theorem 2: Under *Assumption 1*, the states of the heterogeneous multi-agent systems (1) and (2) reach the sliding mode surface (20) and (22) in fixed-time, if the decentralized control inputs are designed as (21) and (23), respectively. The heterogeneous multi-agent system (1) achieves consensus tracking in fixed-time along the sliding mode surface (20) and (22).

Proof: We divide the proof into two parts.

Part 1: The proof of the first-order followers tracking the leader in fixed time.

The Lyapunov function is chosen as $V_2 = s^2$. Then we have

$$\begin{aligned} \dot{V}_2 &= 2 s_k \dot{s}_k \\ &= 2 s_k (\dot{e}_k + \alpha_k sig^{r_k}(e_k) + \beta_k sig^{\kappa_k}(e_k)) \\ &= 2 s_k (-\alpha_k sig^{r_k}(s_k) - \beta_k sig^{\kappa_k}(s_k) + d_k - D_k sign(s_k)) \\ &= -2\alpha_k |s_k|^{r_k+1} - 2\beta_k |s_k|^{\kappa_k+1} - d_k s_k - D_k |s_k| \\ &\leq -2\alpha_k {V_2}^{(r_k+1)/2} - 2\beta_k {V_2}^{(\kappa_k+1)/2}. \end{aligned} \tag{24}$$

Based on *Lemma 2*, the first-order followers reach the corresponding sliding-mode surface and the settling-time T_{2k} satisfies

$$T_{2k} \leq \frac{1}{\alpha_k(1-r_k)} + \frac{1}{\beta_k(\kappa_k - 1)}. \tag{25}$$

After the first-order followers reach the corresponding sliding-mode surface, we obtain $s_k = 0$. With (20), we have

$$\dot{e}_k = -\alpha_k sig^{r_k}(e_k) - \beta_k sig^{\kappa_k}(e_k). \tag{26}$$

The Lyapunov function is chosen as $V_3 = {e_k}^2$. Then, $\dot{V}_3$ satisfies

$$\begin{aligned} \dot{V}_3 &= 2 e_k \dot{e}_k \\ &= 2 e_k (-\alpha_k sig^{r_k}(e_k) - \beta_k sig^{\kappa_k}(e_k)) \\ &= -2\alpha_k {V_3}^{(r_k+1)/2} - 2\beta_k {V_3}^{(\kappa_k+1)/2}. \end{aligned} \tag{27}$$

Based on *Lemma 2*, after the first-order followers reach the corresponding sliding-mode surface, the first-order followers track the leader in fixed-time and the settling time T_{3k} satisfies

$$T_{3k} \leq \frac{1}{\alpha_k(1-r_k)} + \frac{1}{\beta_k(\kappa_k - 1)}. \tag{28}$$

Defining $T_2^{max} = max(T_{21} + T_{31}, \cdots, T_{2k} + T_{3k})$, all the first-order followers track the leader in fixed-time and the settling time t_2 satisfies $t_2 \leq T_2^{max}$.

Part 2: The proof of the second-order followers tracking the leader in fixed time.

When $|e_l(t)| > \varepsilon_l$, the second-order followers reach the corresponding sliding-mode surface and the settling time T_{4l} satisfies

$$T_{4l} \leq \frac{1}{\alpha_l(1-r_l)} + \frac{1}{\beta_l(\kappa_l - 1)}. \tag{29}$$

The proof is similar to that of first-order followers.

After the second-order followers reach the corresponding sliding mode surface, we obtain $s_l = 0$. With (22), we have

$$\dot{e}_l = -\beta_l {e_l}^{\kappa_l}. \tag{30}$$

Based on Lemma 3, if $s_l = 0$, the second-order followers converge to the set $M = \{e_l : e_l \leq \varepsilon_l\}$ in finite-time and the settling time T_{5l} satisfies

$$T_{5l} \leq \frac{1}{\beta_l(\kappa_l - 1)\varepsilon_l^{(\kappa_l - 1)}}. \tag{31}$$

When $|e_l(t)| \leq \varepsilon_l$, the symbol function $sign(\cdot)$ cannot be differentiated, so we consider two cases. Before the second-order followers track the leader, if $e_l = 0$ and $\dot{e}_l \neq 0$, e_l will leave the origin immediately so that the state of $e_l = 0$ can be negligible. Then, we have $\dot{s}_l = u_l - \dot{v}_0$. After the second-order followers track the leader, $sign(e_l) = 0$. Therefore, $\dot{s}_l = u_l - \dot{\omega}_0$. Then, the second-order followers reach the corresponding sliding-mode surface and the settling time T_{6l} satisfies

$$T_{6l} \leq \frac{1}{\alpha_l(1-r_l)} + \frac{1}{\beta_l(\kappa_l - 1)}. \tag{32}$$

The proof is similar to that of first-order followers.

With $s_l = 0$ and (22), we have

$$\dot{e}_l = -\alpha_l sign(e_l). \tag{33}$$

The Lyapunov function is designed as $V_4 = {e_l}^2$. Then, $\dot{V}_4$ satisfies

$$\begin{aligned}\dot{V}_4 &= 2e_l\dot{e}_l \\ &= 2e_l(-\alpha_l sign(e_l)) \\ &= -2\alpha_l\,|e_l| \\ &= -2\alpha_l V_4^{1/2}.\end{aligned} \tag{34}$$

Based on *Lemma 3*, after $s_l = 0$, the second-order followers track the leader in fixed-time and the settling time T_{7l} satisfies

$$T_{7l} \leq \frac{\varepsilon_l}{\alpha_l}. \tag{35}$$

Defining $T_3^{max} = max(T_{41} + T_{51} + T_{61} + T_{71}, \cdots, T_{4l} + T_{5l} + T_{6l} + T_{7l})$, all the second-order followers track the leader in fixed-time and the settling time t_3 satisfies $t_3 \leq T_3^{max}$. In summary, all followers track leader in fixed-time and the settling-time $t_4 = max(t_2, t_3)$. Then, the heterogeneous agents (1) and (2) achieve consensus tracking in fixed-time and the settling-time satisfying $t_5 \leq t_1 + t_4$. Theorem 2 is proved. □

IV. Simulation Example

Consider the heterogeneous multi-agent systems comprised of a leader and six followers with the communication topology shown in Fig.1, where $1, 2, 3, 4$ are first-order followers and $5, 6$ are second-order followers. The leader's input u_0, initial position x_0 and velocity v_0 are considered as $u_0(t) = -\sin(t)$, $x_0(0) = 0$ and $v_0(0) = 1$. The followers' initial position are considered as $[x_1(0), x_2(0), x_3(0), x_4(0), x_5(0), x_6(0)] = [1, 2, 3, 4, 5, 6]$. The observer parameters are considered as $p = 1, q = 3, T_1 = 2, \lambda_1(H) = 0.236$. The first-order followers' controller parameters are considered as $\alpha_k = 3, \beta_k = 5, D_k = 0.5, \kappa_k = 5/3, r_k = 1/3$ and the disturbance is considered as $d_k = 0.5\sin(t)$. The second-order followers' controller parameters are considered as $\alpha_l = 3, \beta_l = 5, D_l = 0.5, \kappa_l = 5/3, r_l = 1/3, \varepsilon_l = 2$, and the disturbance is considered as $d_l = 0.5\sin(t)$.

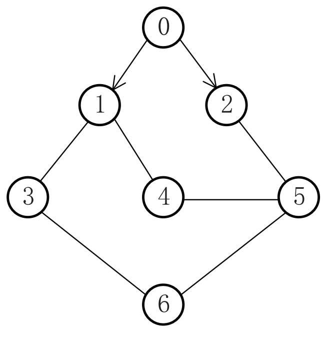

Fig. 1. The communication topology of agents.

The simulation of proposed algorithms are illustrated in Figs.2-4. Figs.2-3 show that the distributed observer (10) estimates the leader's state in fixed-time, where the settling-time satisfies $t_1 \leq 4s$. It is shown in Fig.4 that all first-order followers track leader' states in fixed-time, where the settling-time satisfies $t_2 \leq 1.6s$. All second-order followers track the leader's states in fixed-time, where the settling-time satisfies $t_3 \leq 2.46s$. Therefore, all followers track the leader' states in fixed-time, where the settling-time satisfies $t_4 \leq 2.46s$. Hence, the heterogeneous multi-agent systems (1) and (2) achieves consensus tracking in fixed time, and the settling-time satisfies $t_5 \leq 6.46s$.

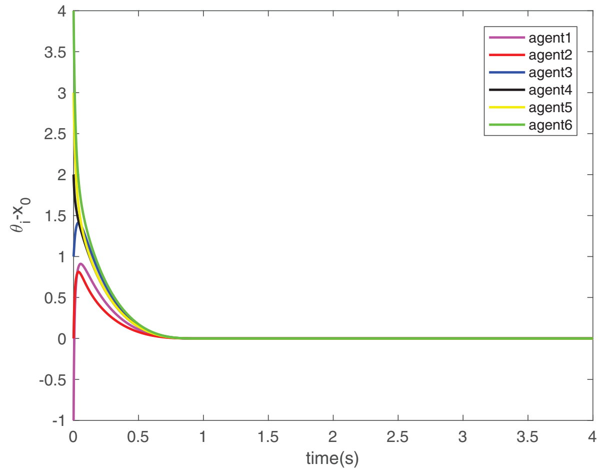

Fig. 2. The estimated error $\theta_i - x_0$ with the fixed-time observer in (10).

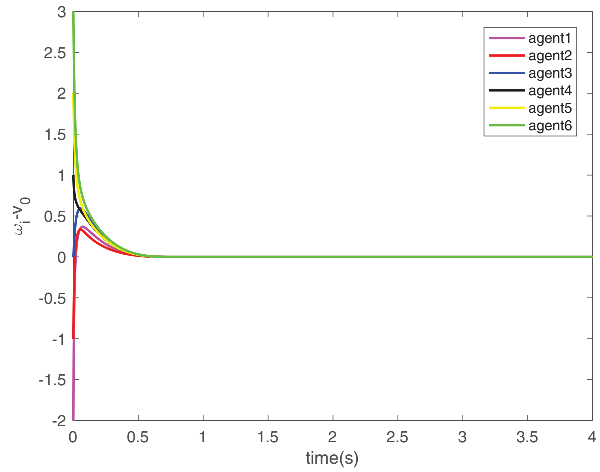

Fig. 3. The estimated error $\omega_i - v_0$ with the fixed-time observer in (10).

V. Conclusion

This paper investigated the fixed-time consensus tracking problem of heterogeneous multi-agent systems with uncertain disturbances. The control algorithm is divided into two part comprised of the state observer and sliding mode controllers. With the distributed fixed-time observer, each follower estimates states information of leader in fixed-time. The decentralized sliding mode controllers are designed for followers to track the leader by using the estimated leader's state.

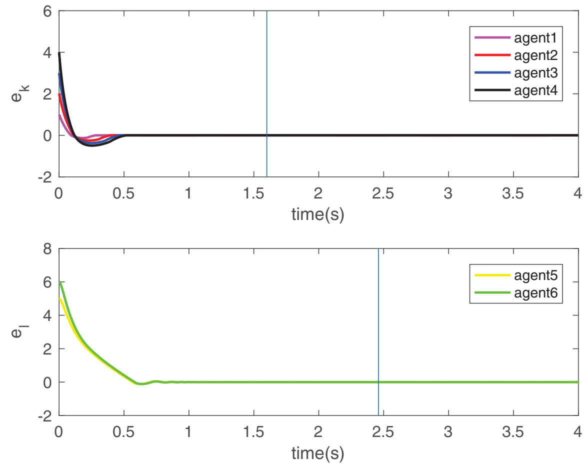

Fig. 4. The tracking error e_k, e_l of followers.

The effectiveness of the algorithm is proved by Lyapunov's theorem and illustrative examples.

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