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Fixed-time Consensus Algorithm for Second-order Multi-agent Systems with Bounded Disturbances

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***Abstract*—This paper addresses the problem of fixed-time consensus algorithm for the second-order multi-agent systems with bounded disturbances. The proposed fixed-time protocol can ensure a fixed -time consensus. First, the control law is designed to make the true speed track the virtual speed in a fixed time based on the fixed-time control method. Then, based on the backstepping design method, the virtual speed is designed, which makes the consensus be achieved in a fixed-time. Compared with the finite-time consensus results, the fixed-time consensus results can ensure the convergence settling time regardless of the initial states of the agents. Finally, a example is presented to show the effectiveness of the fixed-time consensus algorithm.**

Keywords*—*fixed-time; consensus; second-order; multi-agent systems; disturbances

I. INTRODUCTION

Cooperative control of the multi-agent systems is the main research direction of current international context and leads to lots of significant results such as formation control [1], [2], flocking [3], [4], data fusion [5] and so on. As a basic problem of cooperative control, Consensus is a typical collective behavior that requires the agents to converge to a common value or a agreement by communicating with their neighbours [6].

Noticeably, most of the existing works about the consensus problem for multi-agent systems are asymptotic consensus results which means that the consensus can only be achieved as time approaches infinity [7]–[9]. In [7], the distributed consensus algorithm was proposed for linear multi-agent systems with delays and noises in transmission channels. The consensus problems with the fixed and switching topologies were researched in [8], and two consensus protocols for the networks with and without time-delays were introduced. For the the external disturbances, input delays and nonlinear uncertainties of the system model, the consensus control algorithm for multi-agent systems has been researched in [9]. However, for the consensus of multi-agent systems, convergence speed is a very important performance property. This naturally leads to the analysis and construction of convergence speed control protocols for multi-agent systems.

Compared with the convergence speed of the asymptotic results, the finite-time consensus results are much better in practical aspplications. Motivated by these advantages of finite-time control protocols, for instance, faster convergence speed, higher accuracy and more robustness to nonlinear uncertainties [10]. Finite-time consensus control of multi-agent systems has attracted much attention in recent years [10]–[12]. The finite-time consensus control problems for first-order multi-agent systems were researched in [11]. By adding a power integrator method, the authors proposed continuous finite-time consensus protocols for leaderless and leader-follower second-order multi-agent systems [12]. [11] and [12] were extended to the high-order multi-agent systems in [13], where two finite-time consensus protocols were proposed with state feedback and output feedback for the normal form high-order systems. However, in these researches, the finite-time consensus is based on the initial conditions of the agents, in practical applications, the initial states of the agents may be unavailable in advance.

To deal with these constraints, some new works based on the fixed-time stability [14] have been studied, which can ensure the convergence settling time in spite of the initial states of the agents. In [15], the continuous fixed-time consensus algorithm was proposed for first-order multi-agent systems, and the fixed-time pinning control problem also be taken into consideration. The fixed-time leader-following problem was researched for first-order multi-agent systems with nonlinear uncertainties in [16]. In [17], the fixed-time consensus algorithms with linear and nonlinear state measurements ware proposed for first-order multi-agent systems. Due to the nonlinear property of the fixed-time convergence controller, it is difficult to extend the first-order results to the multi-agent systems with more complex dynamics. The first attempt is studied in [18] based on the terminal sliding mode control method, where the fixed-time consensus tracking control algorithms for second-order multi-agent systems was proposed. Note that, unknown dynamics and disturbances were not considered in these linear multi-agent systems.

Motivated by the existing works, in this brief, fixed-time consensus algorithm for second-order multi-agent systems was proposed. Compared with the previous works related to this brief, the results shown in this paper have the following features. First, the fixed-time consensus algorithm can avoid the convergence time depends on the initial conditions of the agents. Second, the multi-agent systems was modeled as

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second-order systems, in practice, a large class of mechanical systems are required to be second-order dynamic model. Third, the multi-agent systems with external disturbances are considered.

The rest of this paper is organized as follows: In Sect. II, preparation, problem description and the dynamical model of the second-order multi-agent systems are proposed. In Sect. III, the fixed-time consensus algorithm are proposed for the second-order multi-agent systems with bounded disturbances. In Sect. IV, a example is given to illustrate the fixed-time consensus algorithm. The conclusions are proposed in Sect. V.

II. Preparation and Problem Description

A. Graph Theory

In this section, firstly, some relevant graph concepts will be introduced. secondly, some lemmas are given which will be used later. Last, the problem formulation is presented.

For an undirected graph G with N agents is expressed as $G=(V,E,A)$ and the N agents can be considered as N nodes. The $V=\{1,2,\ldots,n\}$ means a set of nodes and $E\subseteq V\times V$ is the set of edges. An edge$(j,i)\in E$ means that note i can obtain information from note j and note j can also obtain information from note i. Adjacency matrix $A=[a_{ij}]\in R^{n\times n}$ is a $N*N$ matrix, whose elements can defined as that: $a_{ij}=\begin{cases}>0,(j,i)\in E\\ 0, otherwise\end{cases}$. Define the degree matrix as $D=diag\left[d_1,\ldots,d_N\right]$ with $d_i=\sum\limits_{j=1,j\neq i}^{N}a_{ij}$. The Laplacian matrix $L=[l_{ij}]\in R^{n\times n}$ of G is defined as $L=D-A$. In addition, if exists a path between any two agents, the graph G is connected.

Remark 1: For an undirected graph G , both A and L are symmetric. For a connected graph, the Laplacian has a single zero eigenvalue and the the vector of ones is the corresponding eigenvector [19].

B. Some Lemmas

Lemma 1 [20]: For a connected undirected graph G , the matrix L of G has these properties. $x^TLx=\frac{1}{2}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}a_{ij}(x_i-x_j)^2$, for any $x=[x_1,x_2,\ldots x_n]^T$, and the L is positive semi-definite and has n positive real eigenvalues. Assume that the eigenvalues are given by $0,\lambda_2,\cdots,\lambda_n$ satisfying $0\le\lambda_2\le\cdots\le\lambda_n$. The λ_2 is the second smallest eigenvalue. Furthermore, if $1^Tx=0$, then $x^TLx\ge\lambda_2x^Tx$.

Lemma 2 [14]: If there have a continuous radially unbounded function $V:R^n\to R_+\cup\{0\}$ such that

(1) $V(x)=0\Leftrightarrow x=0$;

(2) the solution of $x(t)$ satisfied the inequality $D*V(x(t))\le-\alpha V^p(x(t))-\beta V^q(x(t))$ for the $\alpha,\beta>0, p=1-\frac{1}{2\gamma}, q=1+\frac{1}{2\gamma}$ and $\gamma>1$;

then, we can get the globally fixed time stable and the convergence time T satisfies that $T\le T_{\max}:=\frac{\pi\gamma}{\sqrt{\alpha\beta}}$.

Lemma 3 [21]: Let $\varepsilon_1,\varepsilon_2,\ldots\varepsilon_M\ge 0$. Then

$$\sum_{i=1}^{M}\varepsilon_i^p\ge\left(\sum_{i=1}^{M}\varepsilon_i\right)^p, 0<p\le 1$$
$$\sum_{i=1}^{M}\varepsilon_i^p\ge M^{1-p}\left(\sum_{i=1}^{M}\varepsilon_i\right)^p, 1<p\le\infty$$

Lemma 4 [22]: If $|y|$ denotes the absolute value of real number y, we can get

$$\frac{d}{dy}|y|^{\alpha+1}=(\alpha+1)sig(y)^{\alpha},$$

$$\frac{d}{dy}sig(y)^{\alpha+1}=(\alpha+1)|y|^{\alpha}.$$

C. Problem Formulation

The system is composed of M agents, with x_i denoting the position of the agent i , v_i denoting the velocity of the agent i , u_i is the control input, and f_i is the bounded disturbance.

Let $sig(x)^{\chi}=sign(x)|x|^{\chi}$, where $\chi>0$, $x\in R$.

For the second-order multi-agent systems, the dynamics of the agents can be decribed as:

$$\begin{aligned}\dot{x}_i&=v_i,\\ \dot{v}_i&=u_i+f_i.\end{aligned}\tag{1}$$

The design of the consensus algorithm mainly divided into two steps:

(1) Based on the backstepping control method, the velocity v_i as a true velocity, the virtual control $v_i{}^*$ is designed to make the x_i reach agreement.

(2) Based on Fixed-time control method, control law u_i is designed to make the true velocity v_i can track the virtual velocity $v_i{}^*$ in the fixed time.

The virtual velocity $v_i{}^*$ is designed as

$$\begin{aligned}v_i{}^* = & -c_1 sig\left(\sum_{j=1}^{M}a_{ij}(x_i-x_j)\right)^{\alpha}\\ & -c_2 sig\left(\sum_{j=1}^{M}a_{ij}(x_i-x_j)\right)^{\beta},\end{aligned}\tag{2}$$

where c_1 and c_2 are positive constants, $\alpha\in(0,1)$, $\beta>1$.

Assumption 1: There exists a known and positive constant F_i such that $|f_i|\le F_i$.

III. Fixed -Time Consensus Algorithm

In this section, the Fixed-time consensus controllers for second-order multi-agent systems are presented as follows.

Based on the backstepping design method, define the error e_i

$$e_i=v_i-v_i^*,\tag{3}$$

From (2) and (3), one has

$$\begin{aligned}\dot{e}_i &= u_i+f_i\\&+c_1\alpha\left|\sum_{j=1}^{M}a_{ij}(x_i-x_j)\right|^{\alpha-1}\sum_{j=1}^{M}a_{ij}(v_i-v_j)\\&+c_2\beta\left|\sum_{j=1}^{M}a_{ij}(x_i-x_j)\right|^{\beta-1}\sum_{j=1}^{M}a_{ij}(v_i-v_j).\end{aligned}\tag{4}$$

The u_i control input is designed as

$$\begin{aligned}u_i &= -c_3sig(e_i)^p-c_4sig(e_i)^q\\&-c_1\alpha\left|\sum_{j=1}^{M}a_{ij}(x_i-x_j)\right|^{\alpha-1}\sum_{j=1}^{M}a_{ij}(v_i-v_j)\\&-c_2\beta\left|\sum_{j=1}^{M}a_{ij}(x_i-x_j)\right|^{\beta-1}\sum_{j=1}^{M}a_{ij}(v_i-v_j)\\&-F_isign(e_i),\end{aligned}\tag{5}$$

where c_3 and c_4 are positive constants, $p\in(0,1)$, $q>1$.

Theorem 1: Under Assumptions all above, the multi-agent systems, composed of the controlled plant (1) and controller (5), can achieve fixed-time consensus with the settling time T bounded as follows

$$T\le T_1+T_2\le T_{\max},$$

$$\begin{aligned}T_{\max}: &= T_{1max}+T_{2max}\\&=\frac{\pi\gamma_1}{2\sqrt{c_3c_4M^{\frac{1-q}{2}}}}\\&+\frac{\pi\gamma_2}{2\lambda_2\sqrt{c_1c_2M^{\frac{1-\beta}{2}}}},\end{aligned}\tag{6}$$

where $p=1-\frac{1}{\gamma_1}$, $q=1+\frac{1}{\gamma_1}$, $\gamma_1>1$, $\alpha=1-\frac{1}{\gamma_2}$, $\beta=1+\frac{1}{\gamma_2}$, $\gamma_2>1$.

Proof:

(1) From (4) and (5), $\dot{e}_i$ can be expressed as

$$\begin{aligned}\dot{e}_i &= f_i-c_3sig(e_i)^p\\&-c_3sig(e_i)^q-F_isign(e_i).\end{aligned}\tag{7}$$

Choose the Lyapunov candiate fuction as

$$V_1=\frac{1}{2}\sum_{i=1}^{M}e_i^2.\tag{8}$$

Differentiating (8)

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1}^{M}e_i\dot{e}_i\\&=\sum_{i=1}^{M}((e_if_i-F_i|e_i|)\\&\quad -c_3e_isig(e_i)^p-c_4e_isig(e_i)^q)\\&\le -c_3\sum_{i=1}^{M}|e_i|^{p+1}-c_4\sum_{i=1}^{M}|e_i|^{q+1}\\&= -c_3\sum_{i=1}^{M}\left(e_i^2\right)^{\frac{p+1}{2}}-c_4\sum_{i=1}^{M}\left(e_i^2\right)^{\frac{q+1}{2}}\\&\le -c_3(\sum_{i=1}^{M}e_i^2)^{\frac{p+1}{2}}-c_4M^{\frac{1-q}{2}}(\sum_{i=1}^{M}e_i^2)^{\frac{q+1}{2}}\\&\le -c_3(2V_1)^{\frac{p+1}{2}}-c_4M^{\frac{1-q}{2}}(2V_1)^{\frac{q+1}{2}}.\end{aligned}\tag{9}$$

Because of $\frac{p+1}{2}\in(0,1)$, $\frac{q+1}{2}\in(1,2)$, according to lemma 2, there is a fixed-time T_1 that make the e_i converge to 0 and the v_i converge to the v_i^*in a fixed-time. And the convergence time T_1 satisfied

$$T_1\le T_{1max}:=\frac{\pi\gamma_1}{\sqrt{\bar{\alpha}\bar{\beta}}},\tag{10}$$

where $p=1-\frac{1}{\gamma_1}$, $q=1+\frac{1}{\gamma_1}$, $\gamma_1>1$, $\bar{\alpha}=c_32^{\frac{p+1}{2}}$, $\bar{\beta}=c_4M^{\frac{1-q}{2}}2^{\frac{q+1}{2}}$.

So the convergence time satisfied

$$\begin{aligned}T_1\le T_{1max}: &= \frac{\pi\gamma_1}{\sqrt{c_32^{\frac{p+1}{2}}c_42^{\frac{q+1}{2}}M^{\frac{1-q}{2}}}}\\&=\frac{\pi\gamma_1}{2\sqrt{c_3c_4M^{\frac{1-q}{2}}}}.\end{aligned}\tag{11}$$

(2) Note that, when the error $e_i=0$, the (3) can be expressed as

$$\begin{aligned}v_i &= -c_1\left(\sum_{j=1}^{M}a_{ij}(x_i-x_j)\right)^{\alpha}\\&\quad -c_2\left(\sum_{j=1}^{M}a_{ij}(x_i-x_j)\right)^{\beta}.\end{aligned}\tag{12}$$

Consider the Lyapunov candiate fuction as

$$\begin{aligned}V_2 &= \frac{1}{2}x^TLx\\&=\frac{1}{4}\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i-x_j)^2,\end{aligned}\tag{13}$$

where $x = [x_1, x_2, \dots x_M]^T$, and set $s_i = \sum_{j=1}^{M} a_{ij}(x_i - x_j)$. Differentiating (13)

$$\begin{aligned}
\dot{V}_2 &= x^T L \dot{x} \\
&= \sum_{i=1}^{M}\sum_{j=1}^{M} a_{ij}(x_i - x_j)\dot{x}_i \\
&= \sum_{i=1}^{M}\sum_{j=1}^{M} a_{ij}(x_i - x_j) \\
&\quad \left(-c_1 sig\left(\sum_{j=1}^{M} a_{ij}(x_i - x_j)\right)^{\alpha}\right. \\
&\quad \left. -c_2 sig\left(\sum_{j=1}^{M} a_{ij}(x_i - x_j)\right)^{\beta}\right) \\
&= -c_1 \sum_{i=1}^{M} |s_i|^{\alpha+1} - c_2 \sum_{i=1}^{M} |s_i|^{\beta+1} \\
&= -c_1 \sum_{i=1}^{M} \left(s_i^2\right)^{\frac{\alpha+1}{2}} - c_2 \sum_{i=1}^{M} \left(s_i^2\right)^{\frac{\beta+1}{2}} \\
&\le -c_1 \left(\sum_{i=1}^{M} s_i^2\right)^{\frac{\alpha+1}{2}} - c_2 M^{\frac{1-\beta}{2}} \left(\sum_{i=1}^{M} s_i^2\right)^{\frac{\beta+1}{2}}. \quad (14)
\end{aligned}$$

Since $Lx = [s_1, s_2, \dots s_M]^T$, one has $\sum_{i=1}^{M} s_i^2 = (Lx)^T Lx = x^T L^2 x$. Let $L^{\frac{1}{2}} 1 = \omega = [\omega_1, \omega_2, \dots, \omega_M]^T$. Then, $\omega^T \omega = \left(L^{\frac{1}{2}} 1\right)^T L^{\frac{1}{2}} 1 = 1L1 = 0$, which means $\omega = 0$. Then, $1^T L^{\frac{1}{2}} x = \omega^T x = 0$.

So, one has

$$\sum_{i=1}^{M} s_i^2 = \left(L^{\frac{1}{2}} x\right) L \left(L^{\frac{1}{2}} x\right) \ge \lambda_2 x^T L x = 2\lambda_2 V_2, \quad (15)$$

where λ_2 is the second smallest eigenvalue of matrix L. Then $\dot{V}_2$ can be expressed as

$$\begin{aligned}
\dot{V}_2 &\le -c_1 (2\lambda_2 V)^{\frac{\alpha+1}{2}} - c_2 M^{\frac{1-\beta}{2}} (2\lambda_2 V)^{\frac{\beta+1}{2}} \\
&= -c_1 (2\lambda_2)^{\frac{\alpha+1}{2}} V_2^{\frac{\alpha+1}{2}} \\
&\quad -c_2 M^{\frac{1-\beta}{2}} (2\lambda_2)^{\frac{\beta+1}{2}} V_2^{\frac{\beta+1}{2}}. \quad (16)
\end{aligned}$$

Because of $\frac{\alpha+1}{2} \in (0,1)$, $\frac{\beta+1}{2} \in (1,2)$, according to lemma 2, there is a fixed-time T_2 that make the x_i achieve the consensus when the e_i has converged to 0. And the consensus time T_2 satisfied

$$T_2 \le T_{2max} := \frac{\pi\gamma_2}{\sqrt{\tilde{\alpha}\tilde{\beta}}}, \quad (17)$$

where $\alpha = 1 - \frac{1}{\gamma_2}$, $\beta = 1 + \frac{1}{\gamma_2}$, $\gamma_2 > 1$, $\tilde{\alpha} = c_1 (2\lambda_2)^{\frac{\alpha+1}{2}}$, $\tilde{\beta} = c_2 M^{\frac{1-\beta}{2}} (2\lambda_2)^{\frac{\beta+1}{2}}$.

So the convergence time satisfied

$$\begin{aligned}
T_2 \le T_{2max} : &= \frac{\pi\gamma_2}{\sqrt{c_1 (2\lambda_2)^{\frac{\alpha+1}{2}} c_2 (2\lambda_2)^{\frac{\beta+1}{2}} M^{\frac{1-\beta}{2}}}} \\
&= \frac{\pi\gamma_2}{2\lambda_2 \sqrt{c_1 c_2 M^{\frac{1-\beta}{2}}}}. \quad (18)
\end{aligned}$$

Thus, the consensus will be achieved within the period $T = T_1 + T_2 \le T_{\max} := T_{1max} + T_{2max} = \frac{\pi\gamma_1}{2\sqrt{c_3 c_4 M^{\frac{1-q}{2}}}} + \frac{\pi\gamma_2}{2\lambda_2\sqrt{c_1 c_2 M^{\frac{1-\beta}{2}}}}$. So the fixed-time consensus is achieved.

(3) Gloal stability analysis

First, when $t \in [0, T_1]$, from *Lemma 2* and (9), the error e_i is bounded. Define $e = [e_1, e_2, \dots e_M]^T$, $v^* = [v_1^*, x_2, \dots v_M^*]^T$. From (1) and (3), one has

$$x = e + v^* = -c_1 sig(Lx)^{\alpha} - c_2 sig(Lx)^{\beta} + e. \quad (19)$$

According to (19), and the error e_i is bounded, the x is bounded when $t \in [0, T_1]$. As similar as x, the v is also bounded when $t \in [0, T_1]$

Second, when $t \in (T_1, T_2]$, from *Lemma 2* and (14), the consensus will be achieved in a fixed-time. □

Remark 2: when $f_i = 0$, the multi-agent systems with bounded disturbances will degenerate into normal form second-order linear systems. We can choose the F_i arbitrarily, the fixed-time consensus can also be ensured.

Remark 3: From the prove, ones can find that the consensus protocol proposed in this paper has the ability to resist the disturbances.

Remark 4: when e_i has converged to 0 in a fixed-time, one has that the v_i converge to the v_i^*in a fixed-time. If the x_i can achieve the consensus in a fixed-time when the e_i has converged to 0, the fixed-time consensus results of the second-order multi-agent systems will be achieved.

Remark 5: when $c_1 \ne 0$, $c_2 = 0$, $c_3 \ne 0$ and $c_4 = 0$, the finite-time consensus will be achieved. So, the results for the second-order multi-agent systems in this brief are more general results.

IV. Simulation Results

In this section, we will list numerical examples and simulations to demonstrate the effectiveness of the method that we have proposed in this paper.

It is assumed that the multi-agent systems consist of 5 agents, and the relationship between its agents is represented by the following undirected graph which is shown in Fig. 1.

The dynamic models of the second-order multi-agent systems are described by

$$\begin{aligned}
\dot{x}_i &= v_i \\
\dot{v}_i &= u_i + 2sin(t), \quad (20)
\end{aligned}$$

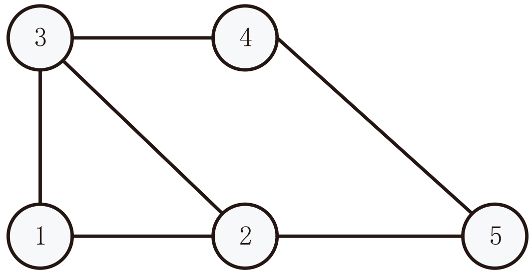

Fig. 1. The undirected network topology.

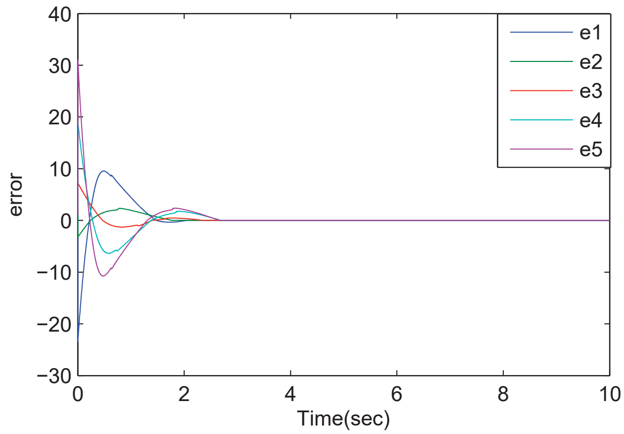

Fig. 2. The errors of the agents.

where f_i=2sin(t), it can be seen that the bounded disturbances f_i satisfies Assumption 1, and we can assume that $F_i = 2$.

From the graph, we can get the Laplacian matrix:

$$\mathrm{L} = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{bmatrix}.$$

The Laplacian matrix L satisfies *Lemma 1*, and the second smallest eigenvalue $\lambda_2 = 1.38$. The simulation is conducted by assuming that the initial position $x(0) = [-8 \ \ -2 \ \ \ 2 \ \ \ 6 \ \ \ 9]^T$, and the velocity $v(0) = [2 \ 1 \ 3 \ 2 \ 1]^T$. The control gains are chose as $c_1 = 2$, $c_2 = 2$, $\alpha = 0.5$, $\beta = 1.5$, and we can get $\gamma_2 = 2$, which satisfies $\alpha = 1 - \frac{1}{\gamma_2}$, $\beta = 1 + \frac{1}{\gamma_2}$. Then, choose $c_3 = 0.25$, $c_4 = 1$, $p = 1$, $q = 1.5$, and we can get $\gamma_1 = 2$, which satisfies $p = 1 - \frac{1}{\gamma_1}$, $q = 1 + \frac{1}{\gamma_1}$. According to the f_i, we can choose $F_i = 2$. From the analysis of Theorem 1, the time to converge T satisfies that $T \leq T_1 + T_2 \leq 12.77$, and $T_{1max} = 9.37$, $T_{2max} = 3.40$.

In this example, Fig. 2 shows that the errors converge to 0 in less than three seconds, which means the v_i converge to the v_i^* in less than three seconds. The convergence time of errors satisfies that $T_1 \leq T_{1max}$. The velocities of the M agents are plotted in Fig. 3. The convergence time T is about three seconds which satisfies that $T \leq T_1 + T_2 \leq 12.77$ are shown

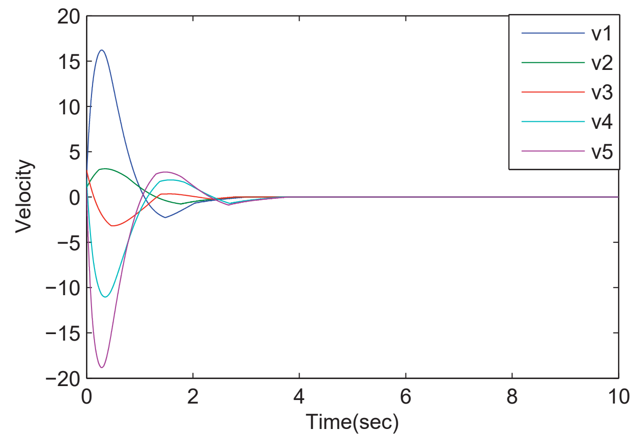

Fig. 3. The velocities of the agents.

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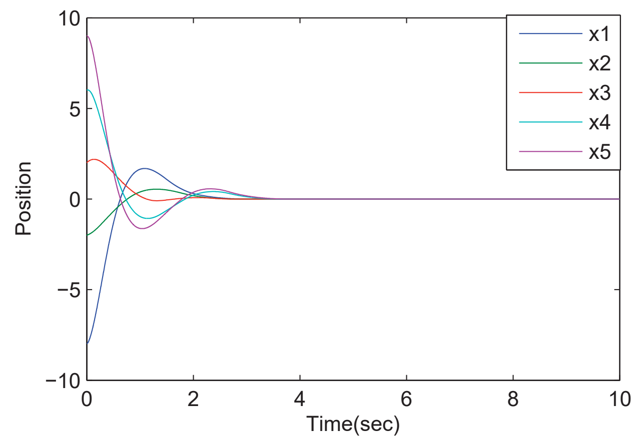

Fig. 4. The positions of the agents.

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in Fig. 4.

V. Conclusions

A fixed-time consensus control approach for second-order multi-agent systems with bounded disturbances proposed in this paper. Firstly, based on the backstepping design method, the virtual speed is designed, which makes the consensus be achieved in a fixed-time. Then, based on the fixed-time control method, the control law is designed to make the true speed track the virtual speed in a fixed time. The fixed-time consensus algorithm can avoid the convergence time depends on the initial conditions of the agents. Finally, a example for fixed-time consensus control problem has been presented to show the effectiveness of the control algorithm.

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