A Distributed Dynamic Event-Triggered Control Approach to Consensus of Linear Multiagent Systems With Directed Networks

Wenfeng Hu, *Member, IEEE*, Chunhua Yang, *Member, IEEE*, Tingwen Huang, *Senior Member, IEEE*, and Weihua Gui, *Member, IEEE*

***Abstract*—In this paper, we study the consensus problem for a class of linear multiagent systems, where the communication networks are directed. First, a dynamic event-triggering mechanism is introduced, including some existing static event-triggering mechanisms as its special cases. Second, based on the dynamic event-triggering mechanism, a distributed control protocol is developed, which ensures that all agents can reach consensus with an exponential convergence rate. Third, it is shown that, with the dynamic event-triggering mechanism, the minimum interevent time between any two consecutive triggering instants can be prolonged and no agent exhibits Zeno behavior. Finally, an algorithm is provided to avoid continuous communication when the dynamic event-triggering mechanism is implemented. The effectiveness of the results is confirmed through a numerical example.**

***Index Terms*—Directed communication networks, dynamic event-triggering mechanism, event-triggered consensus, linear dynamics.**

I. Introduction

As one of core cooperative control problems, consensus of large-scale interconnected systems has been paid great attention in the past decade. Typical results can be found in the pioneering works [1], [2] and the references therein. However, among these works, there is an assumption that each agent uses continuous signals from its neighbors to design suitable feedback control laws. Such an assumption implies that an ideal communication network with unlimited broad bandwidth is required, which is not reality in practice [3], [4]. Instead of continuous signals, sampled signals provide an alternative way that allows each agent to use sampled signals rather than continuous signals from its neighbors for the control design [5]. Usually, signals from agents are sampled with a fixed time period. When the time period is relatively small and/or the consecutive sampled signals are little fluctuating, a large number of *unnecessary* sampled signals will be transmitted among the agents through communication channels, leading to a large waste of network bandwidth and energy if some devices are battery-powered [6], [7].

As an aperiodic sampling approach [8], event-triggered sampling outperforms periodic sampling due to its capability of reducing the frequency of sampling. The comparison between event-triggered sampling and periodic sampling was made in detail for a certain isolated system in [9]. Under event-triggered sampling, several theoretical results on input-to-state stability were first developed in [10]. Recently, for the purpose of reducing communication burden among agents, some event-triggered strategies were incorporated in the consensus protocols for multiagent systems with various dynamics. For instance, with the communication times determined by some certain event-triggered schemes, the consensus problem was addressed for multiagent systems [11] with first-order dynamics [12], [13], or second-order dynamics [14], [15], or high order linear dynamics [16]–[19]. More specifically, the communication networks among all agents were assumed to be undirected in [16], while directed in [17]–[21]. As an abnormal phenomenon usually appears in a hybrid system, Zeno behavior means an unlimited accumulation of executions at some time. To exclude Zeno behavior, a constant is introduced in the triggering condition such that the minimum time elapsed between two consecutive triggering instants is strictly greater than zero [17]. The drawback is that the consensus is achieved with an error related to the introduced constant. In [19], a state-dependent triggering condition was proposed, while it is only proved that Zeno behavior will not appear for the closed-loop system. Recently, Girard [22] generalized the results in [10] by introducing a so-called dynamic triggering mechanism in the stability analysis of linear systems. Later on, the idea of dynamic triggering mechanism was incorporated in the event-triggered consensus protocols for the linear multiagent systems [23] and discrete time-delay complex dynamical networks [24], respectively. Significant extensions were further investigated in [25] and [26], where dynamic event-triggered transmission schemes were proposed for distributed set-membership estimation over wireless sensor networks and distributed formation control problem of a networked multiagent system, respectively. A novel distributed event-triggered sampled-data transmission strategy was proposed [26]–[28], with which the minimum interevent time can be guaranteed to be at least one sampling period. In this case, the Zeno-free property can be strictly guaranteed. As for the leader-following cases [29], Xu *et al.* [30] and Zhang *et al.* [31] introduced the event-triggered control into leader-following multiagent systems and T–S fuzzy systems, respectively. More results on this topic can be found in [32] and the references therein.

In this paper, the leaderless consensus problem for a class of multiagent systems is considered, where the communication networks are assumed to be directed. The main contributions of this paper include the following aspects.

1) A novel dynamic event-triggering mechanism is proposed, which includes the static triggering mechanisms proposed by Hu *et al.* [16] and Liu *et al.* [19] as its special cases. It is shown that the proposed dynamic event-triggering mechanism can ensure a larger interevent time than the static ones.
2) An event-triggered control protocol is developed based on the novel dynamic triggering mechanism. Based on this control

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W. Hu, C. Yang, and W. Gui are with the School of Information Science and Engineering, Central South University, Changsha 410083, China (e-mail: wenfenghu@csu.edu.cn; ychh@csu.edu.cn; gwh@csu.edu.cn).

T. Huang is with the Science Program, Texas A&M University at Qatar, Doha, Qatar (e-mail: tingwen.huang@qatar.tamu.edu).

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protocol, consensus can be reached with an exponential convergence rate, which is different from that in [17].

3) It is proven by contradiction that, with the proposed event-triggered control protocol, Zeno behavior can be excluded for any agent, while it is not involved in [26] and [27].
4) An implementation algorithm is presented using intermittent communication rather than continuous communication. An example is given to show the effectiveness of the proposed algorithm.

II. Problem Formulation

A. Preliminaries

For a real symmetric matrix M with appropriate dimensions, we define $\lambda_{\min}(M) = \min_i \lambda_i(M)$ and $\lambda_{\max}(M) = \max_i \lambda_i(M)$, where $\lambda_i(M)$ is any eigenvalue of M. If a real symmetric matrix M is positive (semi positive) definite, it is denoted as $M > 0$ $(M \geq 0)$. Denote $\mathrm{col}(x_1, \ldots, x_n) = [x_1^T, \ldots, x_N^T]^T$, where $x_i \in R^n (i = 1, \ldots, N)$. We use $\|x\|$ to represent the Euclidean norm of vector x, and $\|A\| = \sup_{x\neq 0}(\|Ax\|/\|x\|)$, where A is a matrix. If $A \in R^{m\times n}$ and $B \in R^{p\times q}$, then $A \otimes B \in R^{mp\times nq}$, where $\otimes$ denotes the Kronecker product. The set of natural numbers is denoted by $\mathbb{N}$.

In this paper, the interaction topology among all agents is directed, which is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The total number of the agents is N. Then N agents constitute the node set $\mathcal{V} = \{1, \ldots, N\}$, and all the communication channels between two agents constitute the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. In particular, if there exists a link starting from agent j and ending at agent i, or equivalently, if information can be directly transmitted from agent j to agent i, it is denoted by $(i, j) \in \mathcal{E}$. In this case, j is called an in-neighbor of i, and all the in-neighbors constitute agent i's in-neighbor set, defined as $\mathcal{N}_i = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}$. The out-neighbor set of agent i is denote by $\mathcal{M}_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. Define $\mathcal{A}(\mathcal{G}) = (a_{ij})_{N\times N}$, where $a_{ij} = 1$ if j is an in-neighbor of agent i, and $a_{ij} = 0$ otherwise, and $\mathcal{A}(\mathcal{G})$ is called the adjacency matrix. We let $L = (l_{ij})_{N\times N}$, where $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \neq j$, and L is called the Laplacian matrix of $\mathcal{G}$. For an undirected graph, it follows that $a_{ij} = a_{ji}, \forall i, j = 1, \ldots, N$, which is not necessarily true for a directed graph. Some ordered edges of form $(i_1, i_2), \ldots, (i_{k-1}, i_k)$ constitute a directed path from i_k to i_1 in the directed graph, and in this case it is said that agent i_1 can be reached by i_k. In a directed graph $\mathcal{G}$, if an agent can be reached by any other agents in the graph, $\mathcal{G}$ is called strongly connected.

B. Problem Formulation

We focus on a class of multiagent systems, in which the dynamics of agent i are given as follows:

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \ldots, N \tag{1}$$

where x_i is the n-dimensional state; u_i is the m-dimensional control input to be designed, and A and B are the system matrices with proper dimensions. Suppose that all agents communicate with each other via a directed graph $\mathcal{G}$.

This paper aims to develop a distributed control protocol u_i based on event-triggered communication, such that the so-called consensus of all agents in system (1) can be achieved by the control protocol, namely, $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0$.

To proceed with, we need the following assumptions, which are quite standard.

Assumption 1: (A, B) is stabilizable.

Assumption 2: The directed graph $\mathcal{G}$ is strongly connected.

Lemma 1 [33]: Under Assumption 1, the following algebraic Riccati equation (ARE) has a unique solution $P > 0$:

$$PA + A^T P - PBB^T P + I_n = 0. \tag{2}$$

Lemma 2 [34]: Under Assumption 2, there exists a vector $\xi = [\xi_1, \xi_2, \ldots, \xi_N]^T$ with all the elements to be positive such that $\xi^T L = 0$. Furthermore, let $\Xi = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_N)$. Then $\hat{L} = [(\Xi L + L^T \Xi)/2]$ is a symmetric matrix and $\sum_{j=1}^{N} \hat{L}_{ij} = \sum_{j=1}^{N} \hat{L}_{ji} = 0$ for all $i = 1, 2, \ldots, N$.

Lemma 3 [35]: Under Assumption 2, for a directed network with its corresponding Laplacian matrix L, the following holds:

$$a(L) = \min_{x^T\xi=0, x\neq 0} \frac{x^T \hat{L} x}{x^T \Xi x} > 0 \tag{3}$$

where $\hat{L} = [(\Xi L + L^T \Xi)/2]$, $\xi = [\xi_1, \xi_2, \ldots, \xi_N]^T$, and $\Xi = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_N)$, with $\xi_i > 0, i = 1, \ldots, N$, $\xi^T L = 0$, and $\sum_{i=1}^{N} \xi_i = 1$.

It is noted that $a(L)$ is called the general algebraic connectivity for a strongly connected graph. In particular, if the graph is undirected, then $a(L) = \lambda_2(L)$.

III. Main Results

A. Dynamic Event-Triggered Control Protocol

Inspired by [13], first, we define the following combined measurement variable:

$$q_i(t) = \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)).$$

Then, similar to [16], we propose a state feedback control protocol for agent i as

$$u_i(t) = Kq_i\left(t_k^i\right), \; t \in \left[t_k^i, t_{k+1}^i\right) \tag{4}$$

where K is the gain matrix to be designed. Note that agent i only needs the combined measurement $q_i(t)$ at some intermittent instants $t_0^i, t_1^i, \ldots$, which are called triggering times (or event times). At each triggering time instant t_k^i, agent i needs to calculate $q_i(t_k^i)$ via the communication with its neighbors. If the current triggering time instant is denoted by t_k^i, the next triggering time t_{k+1}^i can be determined by the following triggering mechanism:

$$\begin{cases} t_{k+1}^i = \inf\left\{t > t_k^i | h_i^d\left(e_i(t), \underset{j\in\mathcal{N}_i}{\cup} x_j(t), \eta_i(t)\right) \geq 0\right\} \\ \dot{\eta}_i(t) = g_i(e_i(t), q_i(t), \eta_i(t)) \end{cases} \tag{5}$$

where $\eta_i(t)$ is the internal dynamic state and $e_i(t)$ is the measurement error to be defined.

It is observed that the triggering function $h_i^d(\cdot)$ depends not only on the state of the controlled system but also on the internal dynamic state, i.e., $\eta_i(t)$. Such a triggering mechanism is called a dynamic event-triggering mechanism, which is first proposed for a single controlled plant in [22]. If the internal dynamic state $\eta_i(t)$ is not involved, then the dynamic event-triggering mechanism reduces to the so-called static event-triggering mechanism of form $h_i^s(e_i(t), \cup_{j\in\mathcal{N}_i} x_j(t)) \geq 0$. In the multiagent system, both the static event-triggering mechanism and the dynamic one are required to be distributed in the sense that the triggering mechanism can only use information from its neighboring agents.

Based on the definition of the combined measurement, we define the following measurement error:

$$e_i(t) = q_i\left(t_k^i\right) - q_i(t). \tag{6}$$

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Associated with the controller (4), the resultant closed-loop system is given by

$$\dot{x}_i = Ax_i + BK(e_i(t) + q_i(t)),\ \ i = 1, \ldots, N. \tag{7}$$

Put $q_1(t), \ldots, q_N(t)$ in a stack, i.e., $q(t) = \mathrm{col}(q_1(t), \ldots, q_N(t))$, and one has $q(t) = -(L \otimes I_n)x(t)$. Based on (7), it follows that:

$$\dot{q}(t) = (I_N \otimes A - L \otimes BK)q(t) - (L \otimes BK)e(t) \tag{8}$$

where $x(t)$ and $e(t)$ are also in stack forms.

Let $\xi_M = \max_i(\xi_1, \ldots, \xi_N)$ and $\xi_m = \min_i(\xi_1, \ldots, \xi_N)$. From Lemma 1, the following equation has a solution $P > 0$:

$$PA + A^T P - \alpha' PBB^T P + I_n = 0 \tag{9}$$

where $\alpha' = 2\mu a(L) - \mu^2 \xi_M \|L\|^2 > 0$ with $a(L)$ given in Lemma 3 and μ being a parameter to be determined.

The main result of this paper can be summarized as follows.

Theorem 1: Under Assumptions 1 and 2, let $K = \mu B^T P$, where $0 < \mu < (2a(L)/[\xi_M \|L\|^2])$ and $P > 0$ is the solution to (9). Then, all agents of the multiagent system (1) reach consensus under the control protocol (4) with t_{k+1}^i determined by

$$\begin{cases} t_{k+1}^i = \inf\Big\{t > t_k^i \,|\, \|e_i(t)\|^2 - \delta_i \|q_i(t)\|^2 - \pi_i \eta_i(t) \geq 0\Big\} \\ \dot{\eta}_i(t) = -\beta_i \eta_i(t) + \theta_i\Big(\delta_i \|q_i(t)\|^2 - \|e_i(t)\|^2\Big),\ \eta_i(0) > 0 \end{cases} \tag{10}$$

where $\beta_i > 0$, $\theta_i \geq \|PBB^T P\|$, $\pi_i > 0$, and $\delta_i = (\xi_i \sigma_i / \theta_i)$ with $0 < \sigma_i < 1$. Moreover, under the proposed event-triggered control protocol, Zeno behavior is excluded for any agent.

Proof: We consider the following Lyapunov function candidate:

$$W(t) = V(t) + \sum_{i=1}^{N} \eta_i(t) \tag{11}$$

where $V(t) = q^T(t)(\Xi \otimes P)q(t) \geq 0$. According to (10), one has $\|e_i(t)\|^2 - \delta_i \|q_i(t)\|^2 \leq \pi_i \eta_i(t)$, which implies that

$$\dot{\eta}_i \geq -\beta_i \eta_i - \theta_i \pi_i \eta_i. \tag{12}$$

Applying the comparison principle gives

$$\eta_i(t) \geq \eta_i(0) e^{-(\beta_i + \theta_i \pi_i)t} > 0$$

which leads to $W(t) > 0$.

Let $\hat{A} = [(PA + A^T P)/2]$ and $\hat{B} = PBB^T P$. It follows from (9) that:

$$\begin{aligned} 2\hat{A} - \Big(2\mu a(L) - \mu^2 \xi_i \|L\|^2\Big)\hat{B} \\ \leq PA + A^T P - \alpha' PBB^T P = -I_n. \end{aligned} \tag{13}$$

Notice that $q^T(t)(\xi \otimes I_n) = -x^T(t)(L^T \xi \otimes I_n) = 0$. From Lemma 3, we have

$$\begin{aligned} \dot{V}(t) &= q^T(t)\Big(\Xi \otimes 2\hat{A} - 2\hat{L} \otimes \mu\hat{B}\Big)q(t) \\ &\quad - 2q^T(t)\Big(\Xi L \otimes \mu\hat{B}\Big)e(t) \\ &\leq q^T(t)\Big(\Xi \otimes 2\hat{A} - 2\mu a(L)\Xi \otimes \hat{B}\Big)q(t) \\ &\quad + q^T(t)\Big(\Xi L L^T \Xi \otimes \mu^2 \hat{B}\Big)q(t) + e^T(t)\Big(I_N \otimes \hat{B}\Big)e(t) \\ &= \sum_{i=1}^{N} q_i^T(t)\Big[\xi_i\Big(2\hat{A} - \Big(2\mu a(L) - \mu^2 \xi_i \|L\|^2\Big)\hat{B}\Big)\Big]q_i(t) \\ &\quad + \|\hat{B}\| \sum_{i=1}^{N} \|e_i(t)\|^2 \\ &\leq -\sum_{i=1}^{N} \xi_i \|q_i(t)\|^2 + \|\hat{B}\| \sum_{i=1}^{N} \|e_i(t)\|^2. \end{aligned} \tag{14}$$

From (10) and (14), it is not difficult to verify that

$$\begin{aligned} \dot{W}(t) &= \dot{V}(t) + \sum_{i=1}^{N} \dot{\eta}_i(t) \\ &\leq -\sum_{i=1}^{N}(\xi_i - \theta_i \delta_i)\|q_i(t)\|^2 + \sum_{i=1}^{N}\Big(\|\hat{B}\| - \theta_i\Big)\|e_i(t)\|^2 \\ &\quad + \sum_{i=1}^{N}(-\beta_i \eta_i) \\ &\leq -\sum_{i=1}^{N} \xi_i(1 - \sigma_i)\|q_i(t)\|^2 - \sum_{i=1}^{N}(\beta_i \eta_i). \end{aligned} \tag{15}$$

On the other hand

$$V(t) = \sum_{i=1}^{N} \xi_i q_i^T(t) P q_i(t) \leq \lambda_{\max}(P) \sum_{i=1}^{N} \xi_i \|q_i(t)\|^2. \tag{16}$$

It follows from (15) and (16) that:

$$\begin{aligned} \dot{W}(t) &\leq -(1 - \sigma_M) \sum_{i=1}^{N} \xi_i \|q_i(t)\|^2 - \sum_{i=1}^{N}(\beta_i \eta_i) \\ &\leq -l_0 W(t) \end{aligned} \tag{17}$$

where $l_0 = \min([(1 - \sigma_M)/\lambda_{\max}(P)], \beta_m) > 0$ with $\sigma_M = \max_i\{\sigma_i\}$ and $\beta_m = \min_i\{\beta_i\}$. It follows that: $W(t) \leq W(0)e^{-l_0 t}$. Thus, $W(t)$ is decreased to 0 exponentially with a decay rate l_0. Since $\xi_m \lambda_{\min}(P)\|q(t)\|^2 \leq V(t) < W(t)$, $\|q(t)\|$ also decays to 0 exponentially. Therefore, under the assumption that the graph is strongly connected, we can conclude that all agents reach consensus exponentially.

In order to ensure that the triggering mechanism (10) is implementable, we need to prove that Zeno behavior does not occur for any agent. Suppose that there exists one agent, i.e., agent i, Zeno behavior appears at some time T_0. Then one has $\lim_{k \to \infty} t_k^i = T_0$. In light of the property of limit, it is concluded that for any $\varepsilon_0 > 0$, there exists $N(\varepsilon_0)$ such that $t_k^i \in (T_0 - \varepsilon_0, T_0 + \varepsilon_0)$ for $\forall k \geq N(\varepsilon_0)$, which implies that $t_{N(\varepsilon_0)+1}^i - t_{N(\varepsilon_0)}^i < 2\varepsilon_0$.

Note that $\sum_{i=1}^{N} \|q_i(t)\|^2 = \|q(t)\|^2 \leq (V(t)/[\xi_m \lambda_{\min}(P)])$ and $V(t) \leq W(t) \leq W(0)$. Then

$$\|q_i(t)\| \leq \|q(t)\| \leq \sqrt{\frac{W(0)}{\xi_m \lambda_{\min}(P)}} \doteq W_0. \tag{18}$$

Since $\|e_i(t)\|$ is piecewise continuously differentiable in the interval $[t_k^i, t_{k+1}^i)$, the Dini derivative of $\|e_i(t)\|$ can be calculated as follows:

$$\begin{aligned} D^+\|e_i(t)\| &\leq \frac{\|e_i^T\|}{\|e_i\|}\|\dot{e}_i\| = \left\| -\sum_{j=1}^{N} a_{ij}\big(\dot{x}_j(t) - \dot{x}_i(t)\big) \right\| \\ &= \left\| -Aq_i(t) + \mu BB^T P \sum_{j=1}^{N} a_{ij}\Big(q_i\Big(t_k^i\Big) - q_j\Big(t_{k_j'}^j\Big)\Big) \right\| \\ &\leq \|A\|\|q_i(t)\| + \mu\|BB^T P\| \left\| \sum_{j=1}^{N} a_{ij}\Big(\Big\|q_i\Big(t_k^i\Big)\Big\| + \Big\|q_j\Big(t_{k_j'}^j\Big)\Big\|\Big) \right\| \\ &\leq W_0\Big(\|A\| + \mu\Big\|BB^T P\Big\|(1 + |\mathcal{N}_i|)\Big) \\ &\doteq \hat{W}_0 \end{aligned} \tag{19}$$

where $k_j' = \arg\max_{k \in \mathbb{N}}\{t_k^j \mid t_k^j \leq t\}$, and $D^+\|e_i(t)\|$ denotes the right-hand derivative of $\|e_i(t)\|$ when $t = t_k^i$.

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Since an event is triggered only if the triggering condition in (10) is satisfied and $\|e_i(t)\|$ is reset to 0, one has $\|e_i(t)\| \geq \sqrt{\delta_i\|q_i(t)\|^2 + \pi_i\eta_i} \geq \sqrt{\pi_i\eta_i}$ at $t_k^{i-}, k = 1, 2, \ldots$ Define $f(t^-) = \lim_{s\to t^-} f(s)$. Then

$$\left\|e_i\left(t_k^{i-}\right)\right\| \geq \sqrt{\pi_i\eta_i\left(t_k^{i-}\right)} = \sqrt{\pi_i\eta_i(0)}e^{-\frac{\beta_i+\theta_i\pi_i}{2}t_k^{i-}}. \tag{20}$$

It follows from (19) and (20) that:

$$t^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)} \geq \frac{1}{\hat{W}_0}\sqrt{\pi_i\eta_i(0)}e^{-\frac{\beta_i+\theta_i\pi_i}{2}t^i_{N(\varepsilon_0)+1}}.$$

Let $\varepsilon_0 > 0$ be a solution of the following equation:

$$\frac{1}{\hat{W}_0}\sqrt{\pi_i\eta_i(0)}e^{-\frac{\beta_i+\theta_i\pi_i}{2}T_0} = 2\varepsilon_0 e^{\frac{\beta_i+\theta_i\pi_i}{2}\varepsilon_0}.$$

Then

$$\begin{aligned} t^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)} &\geq \frac{1}{\hat{W}_0}\sqrt{\pi_i\eta_i(0)}e^{-\frac{\beta_i+\theta_i\pi_i}{2}(T_0+\varepsilon_0)} \\ &= 2\varepsilon_0 \end{aligned} \tag{21}$$

which contracts the fact that $t^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)} < 2\varepsilon_0$. Thus, the above supposition does not hold, which implies that agent i does not exhibit Zeno behavior. The proof is completed. ■

Remark 1: In comparison with the triggering condition in [16], which has the form of $\|e_i(t)\| - \delta_i\|q_i(t)\| \leq 0$, the triggering condition in this paper introduces a dynamic variable $\eta_i(t)$, that is, $\|e_i(t)\| - \delta_i\|q_i(t)\| \leq \pi_i\eta_i(t)$. Clearly, if setting $\eta_i(t) \equiv 0$, the dynamic triggering mechanism reduces to the static one in [16]. Since $\pi_i\eta_i(t) > 0$, it is shown that the proposed dynamic triggering mechanism can produce a larger interevent time interval than the static triggering mechanism, definitely leading to an exclusion of the Zeno behavior.

Remark 2: Under a dynamic triggering mechanism, there are two challenging issues to be addressed: 1) how to design a dynamic event-triggered control law and 2) how to prove the Zeno-free property under the continuous-time framework. Recalling some results in [24]–[26], the design of suitable event-triggered control laws depends on solutions to a number of matrix inequalities, whose existence may be not easily guaranteed. However, in this paper, the desirable matrix gain can be obtained easily from a unique solution of a certain ARE. Moreover, from the proof of Theorem 1, one can see that the second issue is well addressed by contradiction. Nevertheless, in [24] and [25], the dynamics of each agent are considered in the discrete-time domain, which means that the issue of Zeno-free is not involved.

B. Implementation Algorithm Without Continuous Communication

In Theorem 1, the triggering mechanism is given in (10). When this mechanism is implemented, there are two problems to be solved. On the one hand, in a directed communication network, how could agent i calculate $q_i(t)$ at the triggering time t_k^i? This problem becomes difficult if the communication network is directed, because in this case agent i cannot inform its neighbors to send information at t_k^i decided by agent i itself. On the other hand, when checking the event-triggering condition, there is a need to continuously monitor $e_i(t)$ and $q_i(t)$, which means that continuous communication is still required. How to avoid such continuous communication is still challenging. In this paper, we propose a novel implementation algorithm only based on intermittent communication instead of continuous communication. The basic idea is to reproduce $q_i(t)$ (and thus $e_i(t)$) through calculation rather than communication. During the process, we only use information of them at the previous triggering time.

Algorithm 1 Implementation Algorithm

Initialization:
(1). Set $t_0^1 = \cdots = t_0^N = 0$, $e_i(0) = 0$, and $k_i = 0$ for $i = 1, \ldots, N$;
(2). Agent i receives $x_j(0),\ j \in \mathcal{N}_i$;
(3). Agent i sends $x_i(0)$ to agent $j,\ j \in \mathcal{M}_i$;
(4). Compute $q_i(0)$ and sends $q_i(0)$ to agent $j,\ j \in \mathcal{M}_i$.
while $t < T$, T is the desired lifespan of the system.
do
Compute $q_i(t)$ from (23) based on the updated $q_j(t^j_{k_j}),\ j \in \mathcal{N}_i$, compute $e_i(t)$ from (6), and get $\eta_i(t)$ from (10).
for $i = 1, \ldots, N$
if equation (25) is satisfied
(1). Update $k_i = k_i + 1$, $t^i_{k_i} = t$, $q_i(t^i_{k_i}) = q_i(t)$, and $e_i(t) = 0$;
(2). Sends $q_i(t^i_{k_i})$ to agent $j,\ j \in \mathcal{M}_i$;
(3). Update $u_i(t) = \mu B^T P q_i(t^i_{k_i})$.
end if
end for
end while

To reproduce $q_i(t)$, we consider its derivative with respect to t on the interval $[t^i_k, t^i_{k+1})$. From (1) and (4), one has

$$\begin{aligned} \dot{q}_i(t) &= Aq_i(t) + BK\left[\sum_{j=1}^N a_{ij}\left(q_j\left(t^j_{k'_j}\right) - q_i\left(t^i_k\right)\right)\right] \\ &= Aq_i(t) + \alpha_i(t) \end{aligned} \tag{22}$$

where $\alpha_i(t) = BK\left[\sum_{j=1}^N a_{ij}(q_j(t^j_{k'_j}) - q_i(t^i_k))\right]$.

It is noted that $\alpha_i(t)$ keeps constant if no in-neighbors send updated $q_j(t^j_{k'_j}), j \in \mathcal{N}_i$ to agent i and $q_i(t)$ is not updated either. Without loss of generality, during the time interval $[t^i_k, t^i_{k+1})$, suppose that $\alpha_i(t)$ needs to be updated at $t'_1, t'_2, \ldots, t'_l$, where $t'_0 = t^i_k < t'_1 < \cdots < t'_l < t^i_{k+1} = t'_{l+1}$. In other words, $\alpha_i(t)$ is updated for l times in $[t^i_k, t^i_{k+1})$. Then, for $t \in [t'_j, t'_{j+1}), j = 0, \ldots, l$, one has $\alpha_i(t) = \alpha_i(t'_j)$.

If no confusion is caused, we let $w_j = \alpha_i(t'_j)$ for $j = 0, \ldots, l$. If $A \neq 0$, it follows from (22) that:

$$\begin{aligned} q_i(t) &= e^{A\left(t-t^i_k\right)}q_i\left(t^i_k\right) \\ &\quad + \left(\int_{t'_0}^{t'_1} + \int_{t'_1}^{t'_2} + \cdots + \int_{t'_l}^{t}\right)e^{A(t-\tau)}\alpha_i(\tau)d\tau \\ &= e^{At}\left[W_l + R(t)\right] \end{aligned} \tag{23}$$

where $R(t) = \int_{t'_l}^t e^{A(t-\tau)}d\tau w_l$ and

$$W_l = \begin{cases} e^{-At^i_k}q_i\left(t^i_k\right), & l = 0 \\ e^{-At^i_k}q_i\left(t^i_k\right) + \sum_{h=0}^{l-1}\int_{t'_h}^{t'_{h+1}} e^{-A\tau}d\tau w_h, & l \geq 1. \end{cases} \tag{24}$$

In this case, $q_i(t)$ can be reproduced exactly based on the information at $t^i_k, t'_1, \ldots, t'_l$. The next triggering time, namely, t^i_{k+1} can be determined by the following equation:

$$\|q_i\left(t^i_k\right) - q_i(t)\|^2 - \delta_i\|q_i(t)\|^2 - \pi_i\eta_i(t) = 0. \tag{25}$$

Then, the implementation algorithm can be summarized as Algorithm 1.

Remark 3: The above implementation algorithm is partly inspired from the self-triggered strategy in [16], where $q_i(t)$ is only measured (through communication) at the triggering time instants for agent i. In contrast, in this paper, $q_i(t)$ is continuously reproduced based on

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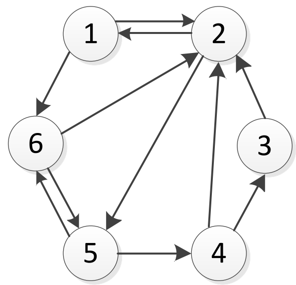

Fig. 1. Graph $\mathcal{G}$ in the example.

the intermittent updated $q_j(t^j_{k_j})$, $j \in \mathcal{N}_i$. Thus, continuous communication is avoided. Compared with the self-triggered strategies in [16] and some previous works, the developed implementation algorithm is less conservative. It should be mentioned that, by introducing a dynamic variable, the implementation of the dynamic event-triggering mechanism under directed networks becomes more complicated.

IV. Examples

To verify the theoretical results, this section considers a spacecraft formation flying problem, which was investigated in [16, Example 2]. In this example, each agent represents a spacecraft flying in the low Earth orbit, and there are six agents constituting the multiagent system. After some transformation, the spacecraft formation flying problem can be converted to the consensus problem of a linear multiagent system. In this case, each agent is with the general linear dynamics of form (1), where

$$A = \begin{bmatrix} 0 & I_3 \\ A_1 & A_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I_3 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3\omega_0^2 & 0 \\ 0 & 0 & -\omega_0^2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 2\omega_0 & 0 \\ -2\omega_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Specifically, the state can be written as $x_i = \mathrm{col}(\bar{x}_i, \bar{y}_i, \bar{z}_i, v_i^x, v_i^y, v_i^z)$, where $(\bar{x}_i, \bar{y}_i, \bar{z}_i)$ is the distance from the desired position in the X-Y-Z directions; (v_i^x, v_i^y, v_i^z) is the velocity in the three directions, and $\omega_0 = 0.001$ is the angular rate of the satellite. Suppose that the agents communicate with each other via a directed communication graph given in Fig. 1.

It can be verified that both Assumptions 1 and 2 hold in this case, and thus all the conditions of Theorem 1 are satisfied. Then, Theorem 1 can be applied to design the consensus protocol and the event-triggering mechanism for the concerned multiagent system. According to Lemma 2, it is found that

$$\xi = [0.2500 \ 0.1071 \ 0.1071 \ 0.2143 \ 0.1786 \ 0.1429]$$

which satisfies $\xi^T L = 0$ and $\sum_{i=1}^N \xi_i = 1$. The other parameters are given as

$$a(L) = 0.7939, \quad \mu = 0.0869 \le \frac{2a(L)}{\xi_M \|L\|^2} = 0.2895$$

$$K = \mu B^T P = \begin{bmatrix} 0.2795 & -0.0007 & 0 & 0.7623 & 0 & 0 \\ 0.0007 & 0.2795 & 0 & 0 & 0.7623 & 0 \\ 0 & 0 & 0.2795 & 0 & 0 & 0.7623 \end{bmatrix}$$

$$\delta = [2.8 \ 1.2 \ 1.2 \ 2.4 \ 2.0 \ 1.6] \times 10^{-3}$$

$$\theta_i = \left\| PBB^T P \right\| = 87.3971, \quad \beta_i = 0.004, \quad \pi_i = 0.002.$$

Under the arbitrarily selected initial conditions, we conduct some simulations in MATLAB. Define the position error on x-axis as $e_{ij} = \bar{x}_i - \bar{x}_j, \forall i \neq j$. The evolution curves of agents' position error e_{ij} and velocity v_i^x via the event-triggered consensus protocol are presented in Fig. 2. It is shown that all agents' positions and velocities on x-axis reach consensus eventually. Besides, the triggering time instants for six agents are recorded, as shown in Fig. 3, which also clearly show the interevent time interval for each agent. In the first 60 s, the triggering numbers for six agents are recorded as: 60, 133, 97, 79, 87, and 86. Moreover, taking agent 1 as an example,

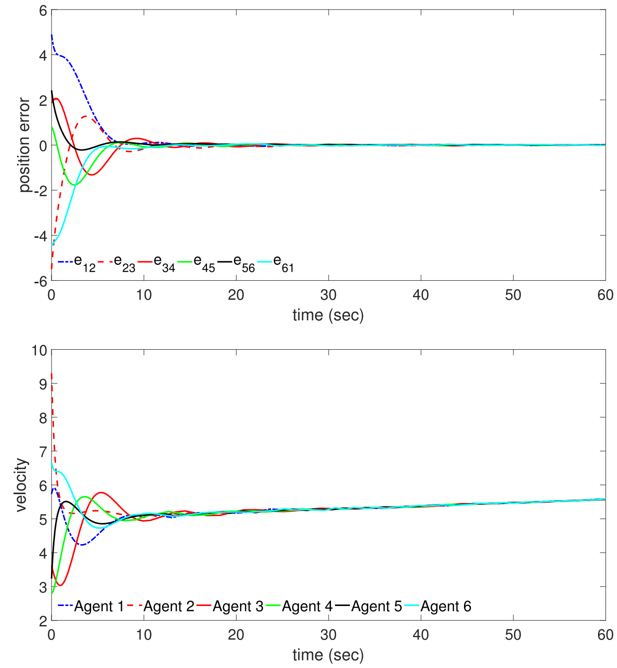

Fig. 2. Evolution of agents' position error and velocity via the event-triggered control protocol.

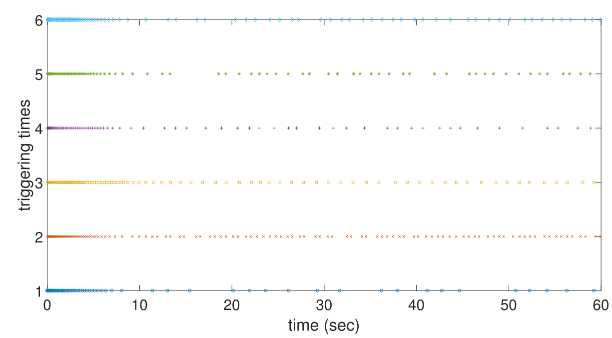

Fig. 3. Triggering time instants for the agents.

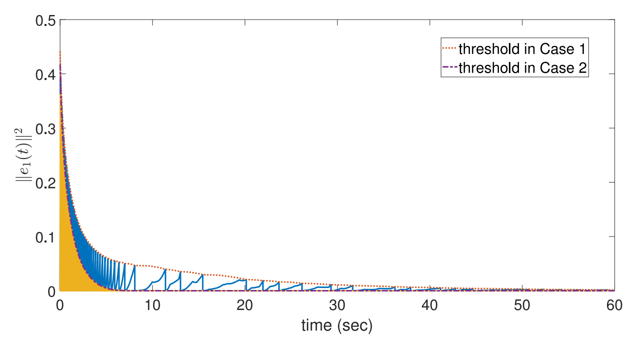

Fig. 4. Evolution of agent 1's error with respect to the threshold in two cases.

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TABLE I
PERFORMANCE COMPARISON WITH STATIC TRIGGERING MECHANISM

Control scheme	π_i	Triggering numbers for agents					
		1	2	3	4	5	6
Case 1	0.002	60	133	97	79	87	86
Case 2	0	587	770	997	685	640	783

the evolution curves of the combined measurement error $\|e_1(t)\|^2$ and its triggering threshold in the triggering condition (10) are also presented in Fig. 4 (case 1). It can be shown that all agents are triggered independently and synchronously, and the consensus problem in the example is solved.

To make a comparison, the simulations are also constructed with the static event-triggering mechanism. In doing so, the evolution of agent 1's error with respect to the threshold is also presented in Fig. 4 (case 2). The triggering numbers for the agents with the dynamic triggering mechanism and the static one are, respectively, recorded in Table I. It can be observed that the triggering numbers are significantly reduced under the dynamic triggering mechanism.

V. CONCLUSION

This paper has proposed a novel dynamic event-triggered control approach to address the consensus problem. The considered multiagent systems are with the general linear dynamics and the communication graph is directed. This paper represents a significant extension of this paper in [16]. It is shown that, all agents reach consensus exponentially with the proposed event-triggered control protocol, and Zeno behavior will not appear for any agent. Furthermore, an implementation algorithm requiring only intermittent communication has been presented. Future researches will focus on the event-triggered control of multiagent systems where the communication networks are time-varying.

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