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Distributed dynamic event-triggered consensus control for multi-agent systems under fixed and switching topologies[☆]

Yifei Li[a], Xiangdong Liu[a], Haikuo Liu[b], Changkun Du[b], Pingli Lu[a,*]

[a] *School of Automation, Beijing Institute of Technology, Beijing 100081, China*

[b] *School of Mechatronical Engineering, Beijing Institute of Technology, Beijing 100081, China*

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Abstract

This paper is devoted to the dynamic event-triggered consensus problem of general linear multi-agent systems under fixed and switching directed topologies. Two distributed dynamic event-triggered strategies, where internal dynamic variables are involved, are introduced for each agent to achieve consensus asymptotically. Compared with the existing static triggering strategies, the purposed dynamic triggering strategies result in larger inter-execution times and less communication energy among agents. In addition, neither controller updates nor triggering threshold detections require continuous communication in the purposed control strategies. It is also proven that the Zeno behavior is strictly ruled out under fixed and switching directed topologies. Finally, the effectiveness of the theoretical analysis is demonstrated by numerical simulations.

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* Corresponding author.

E-mail address: pinglilu@bit.edu.cn (P. Lu).

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1. Introduction

Cooperative control of multi-agent systems (MASs), which initially stems from the collective behavior of biological groups in nature such as school of fishes and flocking of birds, has received increasing attention due to its wide applications in diverse fields, e.g., distributed optimization [1,2], drone/satellite formation flight [3,4], wireless sensor networks [5], tracking control [6,7] and so on. Thus, the consensus problem, as a basic issue of cooperative control of MASs, which aims at designing a suitable control strategy to make the states of all agents reach a common quantity related to certain control performance, has attracted many scholars to do a lot of research work in this field in highly interconnected present world (see [8–12] and the references therein).

As the essential foundation for consensus control of MASs, communication is often assumed to be continuous in traditional control strategies, which require all agents to be equipped with sufficient communication resources and communication bandwidth. Obviously, it is difficult to maintain such an environment to meet the needs of the systems' long-term operation in practical applications. To avoid this disadvantage, distributed controllers with intermittent communication have been studied recently. A commonly used strategy present in [13] is periodic sampling. Even through the periodic sampling strategy enables intermittent communication among agents, the controller still updates periodically even after the control target has been achieved, which leads to a waste of energy. In this sense, significant research efforts, aiming to alleviate the requirements of continuous/periodic communication and reduce communication cost, have led to the emergence of the event-triggered control scheme. Under this scheme, information transmits only when some specific events are triggered, for instance, a measurement error exceeds a pre-designed threshold. Since then, plenty of scholars have conducted research on event-triggered consensus problems, see [14–25]. For MASs with single- or double-integrator dynamics, many results can be found in [14,15]. For MASs of general linear models over undirected [16–18] and directed graphs [19], event-triggered consensus control algorithms were presented, respectively. However, the event-triggered functions still need to continuously access to the neighbors' state information in these works, which brings about a paradox to the original purpose of saving communication energy by introducing the event-triggered strategy. To solve this problem, considering general linear MASs under directed graphs, the authors of [20,21] put forward a strict time-dependent and a state-dependent event-triggered consensus control protocol, respectively. Furthermore, the authors achieved finite-time consensus via a model-based event-triggered scheme in [22]. An event-triggered based latest study for nonlinear networked systems can be found in [23–26], which focus on the adaptive model predictive control problem, the output feedback tracking control problem, the adaptive control problem with unknown external disturbance and the constrained optimal control problem under an identifier-critic network framework, respectively.

It is worth noting that all the results aforementioned are obtained under the framework of static event-triggered control strategies. However, a new class of dynamic event-triggered control strategies including internal dynamic variables was presented in [27] with several merits including significantly larger average inter-event times. Therefore, dynamic event-triggered strategies have been used to solve the consensus problem of MASs in recent years (see e.g., [28–33]). The authors of [28] investigated the dynamic event-triggered average consensus problem of single-integrator MASs. Consensus of second-order MASs was studied in [29] with a centralized dynamic triggering condition. However, these works restricted their scope in agents with integrator-type dynamics. For general linear MASs, the idea of the dynamic triggering mechanism was applied for solving the distributed formation problem in [30] and the adaptive consensus control problem in [31,32], respectively. Although [30–32] was extended to general linear MASs to dispose of the dynamic consensus problem, the above-mentioned works are all based on undirected graphs. In fact, the undirected topology is bidirectional, which means that the communication and energy consumption will be double than the unidirectional link. The directed interaction topology is more realistic in application, which includes the undirected graph as its special cases. In addition, for general linear MASs with directed graphs, Hu et al. [33] also solved the consensus problem under dynamic triggering frame. However, the downside is that continuous communication is needed for each agents' event detection, which goes against the original intention of saving energy. To this end, these partly motivate our work to investigate dynamic event-triggered consensus of general linear MASs under directed graphs without continuous communication both for the controller updating and triggering detection, which is more meaningful and practical.

In actual applications, the communication topologies of MASs may inevitably change due to many factors, such as link failure, external obstacles or new creations. Therefore, the event-triggered consensus control for MASs under switching topologies has constituted a very active field of current study. The authors of [34] addressed the consensus problem for a class of any order MASs under switching topologies, which included kinds of inconsistent topologies. Later on, a decentralized event-triggered controller using an open-loop estimate of the neighbors state was designed under fixed and switching topologies in [35]. The authors of [36] introduced a mode-dependent dwell time approach to solve the event-triggered consensus problem of general linear MASs under switching topologies. Then, the bounded average consensus of MASs with switching topologies was achieved by proposing a predictor-like consensus protocol based on the persistent dwell time in [37]. Nevertheless, all the foregoing works on linear MASs under switching topologies are attended in the static event-triggered strategies. To the best of our knowledge, the design of distributed dynamic event-triggered consensus strategies for general linear MASs under switching topologies is still open and awaits a breakthrough. Besides, it is nontrivial to choose an appropriate switching law and guarantee the solvability of dynamic event-triggered problems under the switching case.

Motivated by the aforementioned observations, this paper will discuss the distributed consensus control for general linear MASs under fixed and switching directed graphs via the novel distributed dynamic event-triggered strategy. More challenges are posed in the consensus stability analysis and the Zeno behavior exclusion due to the more general agents' models and more complex communication topologies. The principal contributions are summarized as follows:

- Firstly, the issue that continuous access to neighbors' states are still required in the design of the triggering mechanism is ignored in many existing works on both static and dynamic event-triggered schemes (see [14,16,19,27]). In this endeavor, the proposed event-triggered control schemes avoid the continuous communication in not only the controller update but also the triggering detection.
- Secondly, compared to the static event-triggered mechanism (e.g., [19–21]), the dynamic event-triggered functions with internal dynamic variables proposed in this paper yield larger triggering intervals, which is beneficial to avoid Zeno behavior in practical applications. Moreover, the static triggering mechanisms proposed by Dimarogonas et al. [14] and Liu et al. [21] are special cases of ours.

- Thirdly, different from most of the existing works on dynamic event-triggered mechanisms which mainly focus on integrator-type dynamics or undirected graphs [27–32], the proposed results in this paper will show the consensus for general linear MASs under fixed and switching directed topologies, respectively.

The rest of this paper is organized as follows. Some preliminaries including useful knowledge and the dynamics are introduced in Section 2. Sections 3 and 4 present the main results. Section 5 illustrates the results through simulation examples. Section 6 concludes the paper.

Notation: Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}^{m\times n}$ be the set of $m\times n$ real matrices, respectively. $\mathcal{I}$ is a set of positive integers and $\mathcal{I}_N=\{1,2,\ldots,N\}$. $\mathbf{0}_N$ and $\mathbf{1}_N$ mean the $N\times 1$ column vector of all zeros and ones, respectively. For a vector $x\in\mathbb{R}^n$, $x>0$ means that every entry $x_i>0$. For a symmetric matrix P, $P>0$ means that P is positive definite and $\lambda_{\max}(P)$ ($\lambda_{\min}(P)$) means the maximum (minimum) eigenvalues of P. Denote $\|\cdot\|$ as the Euclidean norm for vectors and the induced 2-norm for matrices. The superscript T and the symbol $\otimes$ represent the transpose for matrices and the Kronecker product, respectively. Consider matrices $A=[a_{ij}]\in\mathbb{R}^{m\times n}$ and $B=[b_{ij}]\in\mathbb{R}^{p\times q}$, there exists $A\otimes B=[a_{ij}B]\in\mathbb{R}^{p\times q}$. Meanwhile, if A and B are positive definite (semidefinite) matrix, $A\otimes B$ is positive definite (semidefinite) matrix.

2. Preliminaries

In this section, we introduce some definitions in algebraic graph theory and the considered MASs briefly.

2.1. Graph theory

Consider a group of MASs with N agents. A directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ consists of a nonempty finite node set $\mathcal{V}=\{v_1,\ldots,v_N\}$, an edge set $\mathcal{E}\in(\mathcal{V}\times\mathcal{V})$ and a weighted adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$. The edge $(v_i,v_j)\in\mathcal{E}$ indicates that the node v_j can receive information from the node v_i or the node v_i can broadcast information to the node v_j. The neighbor set of node i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}|(i,j)\in\mathcal{E}\}$. The adjacency matrix $\mathcal{A}$ of a directed graph is given by $a_{ii}=0$, $a_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$, and $a_{ij}=0$ otherwise. The Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{N\times N}$, where $l_{ii}=\sum_{j=1}^{N}a_{ij}$, $l_{ij}=-a_{ij}$ with $i\neq j$. A directed path that links v_j with v_i is a sequence of edges in a directed graph of the form $(v_i,v_{i1}),(v_{i1},v_{i2}),\ldots,(v_{il},v_j)$ with distinct nodes $v_{ik}(k=1,2,\ldots,l)$. A directed graph $\mathcal{G}$ is strongly connected if there is a directed path from every node to every other node.

2.2. Problem formulation

Consider a linear MAS consisting of N identical agents, indexed by $1,\ldots,N$. The dynamics of the ith agent is described by

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\quad i\in\mathcal{I}_N, \tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ is the agent state, $u_i(t)\in\mathbb{R}^p$ is the control input. $A\in\mathbb{R}^{n\times n}$ and $B\in\mathbb{R}^{n\times p}$.

The objective of this paper is to design distributed dynamic event-triggered consensus strategies for each agent such that the states of all the agents achieve consensus. For this purpose, we need the following assumptions and lemmas.

Assumption 1. The matrix pair (A, B) in Eq. (1) is stabilizable.

Lemma 1 [38]**.** *For a strongly connected directed graph $\mathcal{G}$, zero is a simple eigenvalue of $\mathcal{L}$ with the corresponding right eigenvector $\mathbf{1}_N$, that is $\mathcal{L}\mathbf{1}_N = 0$, and there exists a positive vector $\mathbf{r} = (r_1, \dots, r_N)^T$ satisfying $\mathbf{r}^T\mathbf{1}_N = 1$ such that $\mathbf{r}^T\mathcal{L} = \mathbf{0}_N$.*

Lemma 2 [39]**.** *The general algebraic connectivity of a strongly connected graph $\mathcal{G}$ associated with the Laplacian matrix $\mathcal{L}$ is defined by*

$$\alpha(\mathcal{L}) = \min_{\mathbf{r}^T x=0, x\neq 0} \frac{x^T\tilde{\mathcal{L}}x}{x^T R x},$$

where $\tilde{\mathcal{L}} = \frac{1}{2}(R\mathcal{L} + \mathcal{L}R^T), R = \mathrm{diag}(r_1, \dots, r_N)$, and $\mathbf{r} = (r_1, \dots, r_N)^T$ satisfying $\mathbf{r}^T\mathcal{L} = \mathbf{0}_N$ and $\sum_{i=1}^N r_i = 1$.

Lemma 3 [39]**.** *Given any $x, y \in \mathbb{R}^N$, Young's inequality states that for any $\phi > 0$, $x^T y \leq \frac{x^T x}{2\phi} + \frac{\phi y^T y}{2}$.*

Lemma 4 [40]**.** *For any positive definite matrix $P_1 \in \mathbb{R}^{n\times n}$ and symmetric matrix $P_2 \in \mathbb{R}^{n\times n}$, the following inequality holds that*

$$x^T(t)P_2x(t) \leq \lambda_{\max}(P_1^{-1}P_2)x^T(t)P_1x(t).$$

3. Dynamic event-triggered consensus control under fixed topology

In this section, a dynamic event-triggered control strategy will be proposed to deal with the consensus problem for the linear MAS (1) under fixed directed graph without the Zeno behavior. The communication topology throughout this section is assumed as the following assumption such that it has the related property described in Lemma 1.

Assumption 2. The directed graph $\mathcal{G}$ among agents is strongly connected.

To start with, define the estimate state as $\bar{x}_i(t) = e^{A(t-t^i_{k_i})}x_i(t^i_{k_i})$, where $t^i_{k_i}$ and $x_i(t^i_{k_i})$ are the latest event-triggered time and the latest broadcast state of agent i, respectively. So the measurement error $e_i(t)$ is defined as

$$e_i(t) = \bar{x}_i(t) - x_i(t), \quad i \in \mathcal{I}_N.$$

A dynamic event-triggered consensus control protocol is proposed for each agent as follows:

$$u_i(t) = cK\sum_{j=1}^N a_{ij}(\bar{x}_i(t) - \bar{x}_j(t)), \tag{2}$$

where $c > 0$, a_{ij} is the ijth entry of the adjacency matrix $\mathcal{A}$ and the feedback gain matrix $K \in \mathbb{R}^{p\times n}$ is chosen by $K = -B^T P$ with a positive matrix P to be decided.

Now, we introduce an internal dynamic variable satisfying

$$\dot{\eta}_i(t) = -\chi_i\eta_i(t) + \xi_i\Big(\frac{\epsilon}{\alpha^*}\|z_i(t)\|^2 - \|e_i(t)\|^2\Big), \tag{3}$$

with $z_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}(\bar{x}_i(t) - \bar{x}_j(t))$, $\eta_i(0) > 0$, $\chi_i > 0$, $\epsilon > 0$, $\alpha^* > \gamma\|\mathcal{L}\|^2$, $\xi_i \in [0, \gamma]$ and $\gamma = (1 - \frac{1}{\alpha})\|M\|^2 + \frac{c}{\beta}\lambda_{\max}(\mathcal{L}^T R^2\mathcal{L} \otimes PBB^T P) + 2c\|M^T R\mathcal{L} \otimes PBB^T P\|$, where $i \in \mathcal{I}_N$, α, β are Young inequality parameters satisfying $0 < \alpha < 1$ and $\beta > 0$.

Inspired by Yi et al. [28], we assume that the first triggering time $t_1^i = 0$, so the triggering times $\{t_k^i\}_{k=2}^{\infty}$ is determined by

$$t_{k_i+1}^i = \max_{r \geq t_{k_i}^i} \left\{ (\|e_i(t)\|^2 \leq \frac{\epsilon}{\alpha^*}\|z_i(t)\|^2 + \frac{\eta_i(t)}{\theta_i}, \forall t \in [t_{k_i}^i, r] \right\}, \tag{4}$$

where $\theta_i > \frac{\gamma - \xi_i}{\chi_i}$.

Remark 1. The proposed mechanism (4) is called the dynamic event-triggered mechanism since it involves an internal variable $\eta_i(t)$. Besides, the following static event-triggered mechanism,

$$t_{k_i+1}^i = \max_{r \geq t_{k_i}^i} \left\{ \|e_i(t)\| \leq \sqrt{\frac{\epsilon}{\alpha^*}}\|z_i(t)\|, \forall t \in [t_{k_i}^i, r] \right\}. \tag{5}$$

can be seen as a limit case of the dynamic event-triggered mechanism (4) when the parameter θ_i goes larger enough. In other words, the static triggering law proposed in [14] and [21] can be seen as a special case of the proposed dynamic scheme (4). In addition, with the internal variable $\eta_i(t)$ being always nonnegative, the measurement error $\|e_i(t)\|$ of the static mechanism will reach its threshold earlier than the dynamic mechanism under the premise of selecting the same parameters, which means the dynamic event-triggered scheme here has larger triggered intervals.

Remark 2. Under the proposed dynamic triggering strategy, the continuous communication is never needed for not only the controller update but also the triggering detection. We use the zero-input estimate state $\bar{x}_i(t)$ rather than directly using the discrete static state $x_i(t_{k_i}^i)$ (such as the controllers in [32] and [33]), which can reduce the accumulation of measurement error $e_i(t)$ and save communication resource consumptions. What's more, different from the static event-triggered scheme, the dynamic scheme here involving an internal dynamic variable $\eta_i(t)$ plays an essential role in excluding Zeno behavior.

Then, we define $\delta_i(t) = x_i(t) - \sum_{j=1}^N r_j x_j(t)$ as a disagreement vector for each agent, where r_j is the jth row of the vector $\mathbf{r}$ defined in Lemma 2. It follows from $\delta(t) = [\delta_1^T(t), \ldots, \delta_N^T(t)]^T$ that the compact form of the disagreement vector is

$$\delta(t) = x(t) - (\mathbf{1}_N \mathbf{r}^T \otimes I_n)x(t) = (M \otimes I_n)x(t).$$

with $M = (I_N - \mathbf{1}_N \mathbf{r}^T)$. Meanwhile, we also define $\bar{\delta}(t) = (M \otimes I_n)\bar{x}(t)$.

With defining the stack vectors $x(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$, $e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$, the closed-loop form of the system (1) can be expressed as

$$\dot{x}(t) = (I_N \otimes A + c\mathcal{L} \otimes BK)x(t) + (c\mathcal{L} \otimes BK)e(t). \tag{6}$$

Therefore, in view of Eq. (6), it can be easily seen that the disagreement vector $\delta(t)$ satisfies

$$\dot{\delta}(t) = (I_N \otimes A + c\mathcal{L} \otimes BK)\delta(t) + (c\mathcal{L} \otimes BK)e(t), \tag{7}$$

where we have used the fact that $M\mathcal{L} = (I_N - \mathbf{1}_N \mathbf{r}^T)\mathcal{L} = \mathcal{L}(I_N - \mathbf{1}_N \mathbf{r}^T) = \mathcal{L}M = \mathcal{L}$.

In what follows, we present the following theorem to cope with the dynamic consensus problem under fixed graph.

Theorem 1. *Consider the linear MAS (1) and suppose that Assumptions 1–2 hold. Under the proposed distributed dynamic event-triggered consensus protocol composed of controller (2) and the dynamic triggering mechanism (4), the event-triggered consensus problem can be solved for any initial states if the parameters* c, ϵ, α, β *are selected such that* $\frac{c\beta}{1-\alpha}\lambda_{\max}(PBB^TP)+\frac{\epsilon}{1-\alpha}<-\lambda_{\max}(R\otimes Q)$, *where* $Q=PA+A^TP-2c\alpha(\mathcal{L})PBB^TP<0$. *In addition, the Zeno behavior can be excluded.*

Proof. According to Eqs. (3) and (4), we have $\dot{\eta}_i(t)\geq -(\chi_i+\frac{\xi_i}{\theta_i})\eta_i(t)$. So it is easy to get

$$\eta_i(t)>\eta_i(0)e^{-(\chi_i+\frac{\xi_i}{\theta_i})t}>0. \tag{8}$$

Therefore, considering the dynamic event-triggered function, we choose the following Lyapunov function:

$$W=\delta^T(R\otimes P)\delta+\sum_{i=1}^{N}\eta_i(t). \tag{9}$$

The time derivative of W along the closed-loop system is given by

$$\begin{aligned}\dot{W}&=2\delta^T(R\otimes P)\dot{\delta}+\sum_{i=1}^{N}\dot{\eta}_i(t)\\&=\delta^T(R\otimes(PA+A^TP)-c(R\mathcal{L}+\mathcal{L}^TR)\otimes PBB^TP)\delta-2\delta^T(cR\mathcal{L}\otimes PBB^TP)e+\sum_{i=1}^{N}\dot{\eta}_i(t).\end{aligned} \tag{10}$$

It follows from Lemma 2 that

$$-\delta^Tc((R\mathcal{L}+\mathcal{L}^TR)\otimes PBB^TP)\delta\leq-2c\alpha(\mathcal{L})\delta^T(R\otimes PBB^TP)\delta.$$

Thus, we can get that

$$\begin{aligned}\dot{W}&\leq\delta^T(R\otimes PA+A^TP-2c\alpha(\mathcal{L})PBB^TP)\delta-2\delta^T(cR\mathcal{L}\otimes PBB^TP)e+\sum_{i=1}^{N}\dot{\eta}_i(t)\\&\leq\lambda_{\max}(R\otimes Q)[\bar{\delta}-(M\otimes I_n)e]^T[\bar{\delta}-(M\otimes I_n)e]\\&\quad-2[\bar{\delta}-(M\otimes I_n)e]^T(cR\mathcal{L}\otimes PBB^TP)e+\sum_{i=1}^{N}\dot{\eta}_i(t),\end{aligned} \tag{11}$$

where we have used the fact that $\bar{\delta}=\delta+(M\otimes I_n)e$.

The first term in Eq. (11) yields

$$\begin{aligned}&\lambda_{\max}(R\otimes Q)[\bar{\delta}-(M\otimes I_n)e]^T[\bar{\delta}-(M\otimes I_n)e]\\&=\lambda_{\max}(R\otimes Q)[\bar{\delta}^T\bar{\delta}-2\bar{\delta}^T(M\otimes I_n)e+e^T(M^TM\otimes I_n)e]\\&\leq\lambda_{\max}(R\otimes Q)[(1-\alpha)\bar{\delta}^T\bar{\delta}+\left(1-\frac{1}{\alpha}\right)e^T(M^TM\otimes I_n)e],\end{aligned} \tag{12}$$

where we have used the Youngs inequality $\bar{\delta}^T(M\otimes I_n)e\leq\frac{\alpha}{2}\bar{\delta}^T\bar{\delta}+\frac{1}{2\alpha}e^T(M^TM\otimes I_n)e$ and $0<\alpha<1$.

Given $\beta > 0$, the second term in Eq. (11) can be handled according to Lemma 3

$$
\begin{aligned}
&-2\left[\bar{\delta} - (M \otimes I_n)e\right]^T (cR\mathcal{L} \otimes PBB^T P)e \\
&\le c\beta\lambda_{\max}(PBB^T P)\bar{\delta}^T\bar{\delta} + \left[\frac{c}{\beta}\lambda_{\max}(\mathcal{L}^T R^2\mathcal{L} \otimes PBB^T P) + 2c\left\|M^T R\mathcal{L} \otimes PBB^T P\right\|\right]\|e\|^2. \qquad (13)
\end{aligned}
$$

Thus, substituting Eqs. (3), (12) and (13) into Eq. (11) yields that

$$
\begin{aligned}
\dot{W} \le& \lambda_{\max}(R \otimes Q)\left[(1-\alpha)\bar{\delta}^T\bar{\delta} + \left(1 - \frac{1}{\alpha}\right)\|M\|^2\|e\|^2\right] + c\beta\lambda_{\max}(PBB^T P)\bar{\delta}^T\bar{\delta} \\
&+ \left[\frac{c}{\beta}\lambda_{\max}(\mathcal{L}^T R^2\mathcal{L} \otimes PBB^T P) + 2c\left\|M^T R\mathcal{L} \otimes PBB^T P\right\|\right]\|e\|^2 \\
&- \sum_{i=1}^{N}\chi_i\eta_i(t) + \sum_{i=1}^{N}\xi_i\left(\frac{\epsilon}{\alpha^*}\|z_i(t)\|^2 - \|e_i(t)\|^2\right) \\
\le& [\lambda_{\max}(R \otimes Q)(1-\alpha) + c\beta\lambda_{\max}(PBB^T P)]\left\|\bar{\delta}\right\|^2 + \sum_{i=1}^{N}(\gamma - \xi_i)\|e_i(t)\|^2 \\
&+ \sum_{i=1}^{N}\xi_i\frac{\epsilon}{\alpha^*}\|z_i(t)\|^2 - \sum_{i=1}^{N}\chi_i\eta_i(t). \qquad (14)
\end{aligned}
$$

Since the fact that $z = (\mathcal{L} \otimes I_n)\bar{x} = (\mathcal{L} \otimes I_n)\bar{\delta}$, we have $\|z\|^2 \le \|\mathcal{L}\|^2\left\|\bar{\delta}\right\|^2$. Then, according to the event-triggered function (4), (14) can be rewritten as

$$
\dot{W} \le \mathcal{P}\left\|\bar{\delta}\right\|^2 - \sum_{i=1}^{N}\left(\chi_i - \frac{\gamma - \xi_i}{\theta_i}\right)\eta_i(t) < 0 \qquad (15)
$$

with $\mathcal{P} = \lambda_{\max}(R \otimes Q)(1-\alpha) + c\beta\lambda_{\max}(PBB^T P) + \epsilon < 0$.

Therefore, we can conclude that the disagreement vector $\delta \to \mathbf{0}$ as $t \to \infty$, which means the MAS (1) can achieve the asymptotical consensus.

Now, we prove that Zeno behavior is strictly ruled out for each agent. Firstly, we suppose that Zeno behavior is existed, which implies that there exists an agent i, such that $\lim_{k\to+\infty} t^i_{k_i} = T_0$, where T_0 is a positive constant.

Let $\varepsilon_0 = \frac{1}{2\|A\|}\ln(\frac{1}{\varpi}\sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\chi_i+\frac{\xi_i}{\theta_i})T_0} + 1) > 0$, where $\varpi = \frac{\|A\|}{c\rho\|BK\|}$. Then according to the property of limits, there exists a positive integer $N(\varepsilon_0)$ such that

$$
t^i_{k_i} \in [T_0 - \varepsilon_0, T_0], \forall k \ge N(\varepsilon_0). \qquad (16)
$$

Thus, it implies that $t^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)} < 2\varepsilon_0$. Noting that Eq. (8) holds, we can conclude that one sufficient condition to guarantee that the inequality in Eq. (4) holds is

$$
\|e_i(t)\| \le \sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\chi_i+\frac{\xi_i}{\theta_i})t}. \qquad (17)
$$

Based on the fact that the interval between any two consecutive triggering events is bounded, it is obvious that $e^{A(t-t^i_{k_i})}$ is bounded for $\forall t \in [t^i_{k_i}, t^i_{k_i+1})$. In light of Eq. (6), it is easy to verify that $(\mathbf{r}^T \otimes e^{-At})x$ is an invariant quantity. Therefore, deriving from $x_i = \delta_i + \sum_{j=1}^{N} r_i x_j$, we obtain that $x(t)$ is finite for any finite t. Thus, we can get that for $\forall t \in [t^i_{k_i}, t^i_{k_i+1})$, the triggering error $e_i(t) = \bar{x}_i(t) - x_i(t) = e^{A(t-t^i_{k_i})}x_i(t^i_{k_i}) - x_i(t)$ is also bounded. It follows from the fact $z = (\mathcal{L} \otimes I_n)(x+e) = (\mathcal{L} \otimes I_n)(\delta + e)$ that z is bounded. Thus, we use ρ to denote the upper bound of $\|z_i(t)\|$. According to Eq. (1), it can be obtained that

$$\dot{e}_i(t) = Ae_i(t) - cBKz_i(t).$$

Next, based on the fact that once an event is triggered for agent i, the measurement error will be reseted to zero. So the solution of the above equation follows that

$$\|e_i(t)\| \le c\|BK\| \int_{t^i_{k_i}}^{t} \left\|e^{A(t-s)}z_i(s)\right\| ds \le \frac{c\|BK\|}{\|A\|}\rho(e^{\|A\|(t-t^i_{k_i})} - 1).$$

Thus, for any triggering instant $t^i_{k_i}$, it can be concluded that one sufficient condition to guarantee the above inequality is

$$\frac{c\|BK\|}{\|A\|}\rho(e^{\|A\|(t-t^i_{k_i})} - 1) \le \sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\chi_i + \frac{\xi_i}{\theta_i})t}. \tag{18}$$

Then one gets

$$\begin{aligned} t^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)} &\ge \frac{1}{\|A\|}\ln\left(\frac{1}{\varpi}\sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\chi_i + \frac{\xi_i}{\theta_i})t^i_{N(\varepsilon_0)+1}} + 1\right) \\ &\ge \frac{1}{\|A\|}\ln\left(\frac{1}{\varpi}\sqrt{\frac{\eta_i(0)}{\theta_i}}e^{-\frac{1}{2}(\chi_i + \frac{\xi_i}{\theta_i})T_0} + 1\right) = 2\varepsilon_0, \end{aligned} \tag{19}$$

which contradicts to Eq. (16). Therefore, Zeno behavior is excluded for each agent.

Thus, the proof is accomplished. □

4. Dynamic event-triggered consensus control under switching topologies

In this section, the proposed dynamic event-triggered control strategy in Section 3 is extended to the MASs under switching topologies. To address our main result, we need to introduce the switching law of communication topologies among agents.

The time-varying interaction topologies for the MASs are modeled by switched directed graph $\mathcal{G}_{\sigma(t)}$, where $\sigma(t)$ is a piece-wise constant switching signal with $\sigma(t) : [0, \infty) \to \mathbb{P}$, which indicates a graph may be switched within a limited set $\mathbb{P}$ at time t such that $\{\mathcal{G}_\sigma : \sigma \in \mathbb{P}\}$ contains all possible graphs in $\mathcal{G}$ among N agents. Thus, it can be obtained that $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}_{\sigma(t)})$, where $\mathcal{V} = \{v_1, \dots, v_N\}$ and $|\mathcal{V}| = N$. Besides, we define the Laplacian matrix of $\mathcal{G}_{\sigma(t)}$ as $\mathcal{L}_{\sigma(t)}$, the left eigenvector corresponding to the zero eigenvalue of $\mathcal{L}_{\sigma(t)}$ as $\mathbf{r}_{\sigma(t)} = [r_{\sigma(t),1}, \dots, r_{\sigma(t),N}]^T$, and $R_{\sigma(t)} = \mathrm{diag}(\mathbf{r}_{\sigma(\mathrm{t})})$.

Consider a set of infinite time intervals $[t_k, t_{k+1})$ with $t_1 = 0$ and each time interval is nonempty, bounded and contiguous. Suppose that within each interval $[t_k, t_{k+1})$, there exists a set of nonoverlapping subinterval sequences

$$[t_{k,0}, t_{k,1}), \dots, [t_{k,r}, t_{k,r+1}), \dots, [t_{k,\omega_k-1}, t_{k,\omega_k}), t_k = t_{k,0}, t_{k+1} = t_{k,\omega_k},$$

with $0 < \mu_1 \le t_{k,r+1} - t_{k,r} \le \mu_2, (r = 0, 1, \dots, \omega_k - 1)$. In addition, the communication topology $\mathcal{G}_{\sigma(t)}$ switches at $t_{k,r}$ and is fixed within the interval $[t_{k,r}, t_{k,r+1})$. Next, the following assumption about the communication topologies is given to provide support for better analysis and proof of the conclusion in this section.

Assumption 3. Within each internal $[t_{k,r}, t_{k,r+1})$, the graph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}_{\sigma(t)})$ contains a strongly connected subgraph $\widehat{\mathcal{G}}_{\sigma(t)} = (\widehat{\mathcal{V}}_{\sigma(t)}, \widehat{\mathcal{E}}_{\sigma(t)}, \widehat{\mathcal{A}}_{\sigma(t)})$, where $\widehat{\mathcal{V}}_{\sigma(t)} \subseteq \mathcal{V}$ and $|\widehat{\mathcal{V}}_{\sigma(t)}| = \widehat{N}_{\sigma(t)} \leq N$. On the basis of these, there exists $\bigcap_{t\in[t_k,t_{k+1})} \widehat{\mathcal{V}}_{\sigma(t)} \neq \emptyset$ and $\bigcup_{t\in[t_k,t_{k+1})} \mathcal{V}_{\sigma(t)} = \mathcal{V}$.

Under Assumption 3 and Lemma 1, it can be concluded that zero is a simple eigenvalue of $\widehat{\mathcal{L}}_{\sigma(t)}$ with the corresponding right eigenvector $\mathbf{1}_{\widehat{N}_{\sigma(t)}}$, namely, $\widehat{\mathcal{L}}_{\sigma(t)}\mathbf{1}_{\widehat{N}_{\sigma(t)}} = 0$, and there exists a positive vector $\hat{\mathbf{r}}_{\sigma(t)} = [\hat{r}_1, \ldots, \hat{r}_{\widehat{N}_{\sigma(t)}}]^T$ satisfying $\hat{\mathbf{r}}^T_{\sigma(t)}\mathbf{1}_{\widehat{N}_{\sigma(t)}} = 1$ such that $\hat{\mathbf{r}}^T_{\sigma(t)}\widehat{\mathcal{L}}_{\sigma(t)} = \mathbf{0}_{\widehat{N}_{\sigma(t)}}$. Then, one has

$$\widehat{M}_{\sigma(t)}\widehat{\mathcal{L}}_{\sigma(t)} = \widehat{\mathcal{L}}_{\sigma(t)}\widehat{M}_{\sigma(t)} = \widehat{\mathcal{L}}_{\sigma(t)}$$

with $\widehat{M}_{\sigma(t)} = I_{\widehat{N}_{\sigma(t)}} - \mathbf{1}_{\widehat{N}_{\sigma(t)}}\mathbf{r}^T_{\sigma(t)}$.

Next, based on the controller under fixed directed topology in Section 3, we design a dynamic event-triggered consensus control protocol for the MAS (1) in the case of switching topologies as follows:

$$u_i(t) = cK\sum_{j\in\mathcal{N}_i} a^{\sigma(t)}_{ij}(\bar{x}_i(t) - \bar{x}_j(t)), \tag{20}$$

where $a^{\sigma(t)}_{ij}$ is the ijth entry adjacency matrix $\mathcal{A}_{\sigma(t)}$ associated to the graph $\mathcal{G}_{\sigma(t)}$ and other parameters are defined same as these in Eq. (2). In addition, it is worth mentioning that the definitions of the measurement error $e_i(t)$ and the disagreement vector $\delta_i(t)$ are same as the case of fixed topology.

Hence, the closed-loop form of Eq. (1) can be expressed as

$$\begin{aligned}\dot{x}(t) &= (I_N \otimes A + c\mathcal{L}_{\sigma(t)} \otimes BK)x(t) + (c\mathcal{L}_{\sigma(t)} \otimes BK)e(t),\\ \dot{\delta}(t) &= (I_N \otimes A + c\mathcal{L}_{\sigma(t)} \otimes BK)\delta(t) + (c\mathcal{L}_{\sigma(t)} \otimes BK)e(t),\end{aligned} \tag{21}$$

with $x(t) = [x^T_1(t), \ldots, x^T_N(t)]^T$, $\delta(t) = [\delta^T_1(t), \ldots, \delta^T_N(t)]^T$, $e(t) = [e^T_1(t), \ldots, e^T_N(t)]^T$.

In the following, we reorder N agents such that the state variables of $\widehat{N}_{\sigma(t)}$ nodes contained in the subgraph $\widehat{\mathcal{G}}_{\sigma(t)}$ at time t are in the first $\widehat{N}_{\sigma(t)}$ dimensions of the state vector of the entire system, that is, we choose a permutation matrix $T_{\sigma(t)} \in \mathbb{R}^N$ such that

$$\tilde{x}(t) = (T^T_{\sigma(t)} \otimes I_n)x(t) = [\tilde{x}^T_1(t), \ldots, \tilde{x}^T_{\widehat{N}_{\sigma(t)}}(t), \tilde{x}^T_{\widehat{N}_{\sigma(t)}+1}(t), \ldots, \tilde{x}^T_N(t)]^T,$$

where $T_{\sigma(t)}$ satisfies $T^T_{\sigma(t)}T_{\sigma(t)} = I_N$. Then, it can be obtained that

$$\begin{aligned}\widetilde{\mathcal{L}}_{\sigma(t)} &= T^T_{\sigma(t)}\mathcal{L}_{\sigma(t)}T_{\sigma(t)} = \begin{bmatrix} \widehat{\mathcal{L}}_{\sigma(t)} & \mathbf{0} \\ \mathbf{0} & * \end{bmatrix},\\ \widetilde{R}_{\sigma(t)} &= T^T_{\sigma(t)}R_{\sigma(t)}T_{\sigma(t)} = \begin{bmatrix} \widehat{R}_{\sigma(t)} & \mathbf{0} \\ \mathbf{0} & * \end{bmatrix},\\ \tilde{\bar{x}}(t) &= (T^T_{\sigma(t)} \otimes I_n)\bar{x}(t) = [\tilde{\bar{x}}^T_1(t), \ldots, \tilde{\bar{x}}^T_{\widehat{N}_{\sigma(t)}}(t), \tilde{\bar{x}}_{\widehat{N}_{\sigma(t)}+1}(t), \ldots, \tilde{\bar{x}}^T_N(t)]^T,\\ \tilde{\delta}(t) &= (T^T_{\sigma(t)} \otimes I_n)\delta(t) = [\tilde{\delta}^T_1(t), \ldots, \tilde{\delta}^T_{\widehat{N}_{\sigma(t)}}(t), \tilde{\delta}^T_{\widehat{N}_{\sigma(t)}+1}(t), \ldots, \tilde{\delta}^T_N(t)]^T.\end{aligned} \tag{22}$$

For the agents contained in the directed strongly connected subgraph $\widehat{\mathcal{G}}_{\sigma(t)}$, $\tilde{x}^c_{\sigma(t)} = [\tilde{x}^T_1(t), \ldots, \tilde{x}^T_{\widehat{N}_{\sigma(t)}}(t)]^T$, $\tilde{e}^c_{\sigma(t)} = [\tilde{e}^T_1(t), \ldots, \tilde{e}^T_{\widehat{N}_{\sigma(t)}}(t)]^T$, $\tilde{\delta}^c(t) = [\tilde{\delta}^T_1(t), \ldots, \tilde{\delta}^T_{\widehat{N}_{\sigma(t)}}(t)]^T$ are defined as the corresponding state vector, measurement error and disagreement vector, respectively.

For the rest agents not included in $\widehat{\mathcal{G}}_{\sigma(t)}$, we also define $\tilde{x}^r(t) = [\tilde{x}^T_{\widehat{N}_{\sigma(t)}+1}(t), \ldots, \tilde{x}^T_N(t)]^T$, $\tilde{e}^r(t) = [\tilde{e}^T_{\widehat{N}_{\sigma(t)}+1}(t), \ldots, \tilde{e}^T_N(t)]^T$, $\tilde{\delta}^r(t) = [\tilde{\delta}^T_{\widehat{N}_{\sigma(t)}+1}(t), \ldots, \tilde{\delta}^T_N(t)]^T$ as the corresponding state vector, measurement error and disagreement vector, respectively.

Now, we are ready to analyze the consensus under switching topologies with the following assumption.

Assumption 4. The lower bound of the topologies switching interval μ_1 satisfies that

$$\mu_1 > \frac{\ln\theta}{\varphi_1}, \tag{23}$$

where $\theta = \max_{i,j\in\mathbb{P},i\neq j}\{\lambda_{\max}(\Xi_i^{-1}\Xi_j)\}$, $\varphi_1 = \max_{\sigma(t)\in\mathbb{P}}\{\frac{\mathcal{P}_{\sigma(t)}}{\lambda_{\max}(\widehat{R}_{\sigma(t)}\otimes P)}\}$ with $\Xi_{\sigma(t)} = \begin{bmatrix}\widehat{R}_{\sigma(t)} & 0\\ 0 & I_{N-\widehat{N}_{\sigma(t)}}\end{bmatrix}$.

Remark 3. The dwell time technique provided in Assumption 4 is employed to guarantee the dynamic event-triggered consensus under switching topologies. Because a small dwell time, that is a small $\mu_1 > 0$, may lead to an excessive switching frequency of topologies. Finally, a high frequency jump may occur to the system, which will affect the consensus. Therefore, in order to avoid the above situation, it's necessary to give a certain restriction to the lower bound of the switching interval μ_1, as shown in Eq. (23).

Thus, similarly to Section 3, an internal dynamic variable is proposed as

$$\dot{\tilde{\eta}}_i(t) = -\chi_i\tilde{\eta}_i(t) + \xi_i\Big(\frac{\epsilon}{\alpha^*}\|\tilde{z}_i(t)\|^2 - \|\tilde{e}_i(t)\|^2\Big), \tag{24}$$

where $\tilde{z}_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}(\bar{\tilde{x}}_i(t) - \bar{\tilde{x}}_j(t))$, $\tilde{\eta}_i(0) > 0$ and the parameters satisfy $\alpha^* > \max_{\sigma(t)\in\mathbb{P}}\{\gamma_{\sigma(t)}\left\|\widehat{\mathcal{L}}_{\sigma(t)}\right\|^2\}$ and $0 \le \xi_i \le \min_{\sigma(t)\in\mathbb{P}}\{\gamma_{\sigma(t)}\}$ with $\gamma_{\sigma(t)} = (1-\frac{1}{\alpha})\left\|\widehat{M}_{\sigma(t)}\right\|^2 + \frac{c}{\beta}\lambda_{\max}(\widehat{\mathcal{L}}^T_{\sigma(t)}\widehat{R}^2_{\sigma(t)}\widehat{\mathcal{L}}_{\sigma(t)}\otimes PBB^TP) + 2c\left\|\widehat{M}^T_{\sigma(t)}\widehat{R}_{\sigma(t)}\widehat{\mathcal{L}}_{\sigma(t)}\otimes PBB^TP\right\|$. The definition of parameters ϵ, χ_i, α, β are same as the case of fixed topology.

We only consider the agents contained in the directed and strongly connected subgraph $\widehat{\mathcal{G}}_{\sigma(t)}$ in each interval $[t_{k,r}, t_{k,r+1})$, because there is no interaction among agents not included in $\widehat{\mathcal{G}}_{\sigma(t)}$. Thus, without loss of generality, we assume there is only one such subgraph $\widehat{\mathcal{G}}_{\sigma(t)}$ with $\widehat{N}_{\sigma(t)}$ nodes $(0 < \widehat{N}_{\sigma(t)} < N)$ in the communication topology $\mathcal{G}_{\sigma(t)}$. Based on these, we establish a dynamic event-triggered scheme. Assume the first triggering time as $t_1^i = 0$, so the triggering times $\{t_k^i\}_{k=2}^{\infty}$ is determined by

$$t^i_{k_i+1} = \max_{r\ge t^i_{k_i}}\{(\|\tilde{e}_i(t)\|^2 \le \frac{\epsilon}{\alpha^*}\|\tilde{z}_i(t)\|^2 + \frac{\tilde{\eta}_i(t)}{\theta_i}, \forall t\in[t^i_{k_i}, r]\}, \tag{25}$$

where $\theta_i > \max\limits_{\sigma(t)\in\mathbb{P}}\{\frac{\widehat{N}_{\sigma(t)}(\gamma_{\sigma(t)}-\xi_i)}{\widehat{N}_{\sigma(t)}\chi_i-\varphi_1}\}$.

Remark 4. Since the controller (20) may change along with the switch of graphs, which will cause the jump of the system, the main difficulty here is how to design an appropriate switching law to guarantee the consistency and stability of the system. In this paper, we introduce the dwell-time for the topologies to guarantee the consensus of the MASs under the switching topologies.

Next, the main conclusion about the MAS (1) under switching topologies is put forward as follows. For the sake of simplicity, the time variable t in $\sigma(t)$ will be omitted during the proof, namely, using σ instead of $\sigma(t)$.

Theorem 2. *Consider the MAS (1) with switching topologies satisfying Assumptions 3–4. Under the proposed distributed dynamic event-triggered consensus protocol (20) and the triggering function (25), the event-triggered consensus can be achieved for any initial states if the parameters are selected such that*

$$c < \min_t \left\{ \frac{(\alpha-1)\lambda_{\max}(\widehat{R}_{\sigma(t)} \otimes Q_{\sigma(t)}) - \epsilon}{\beta\lambda_{\max}(PBB^TP)}, \frac{\varphi_1\lambda_{\max}(P)}{2\zeta\lambda_{\max}(PBB^TP)} \right\},$$

where $Q_{\sigma(t)} = PA + A^TP - 2c\zeta PBB^TP < 0$ with $\zeta = \min_{\sigma(t)\in\mathbb{P}}\{\alpha(\widehat{\mathcal{L}}_{\sigma(t)})\}$. Moreover, the Zeno behavior can be excluded.

Proof. According to Eqs. (24) and (25), we have $\dot{\tilde{\eta}}_i(t) \geq -(\chi_i + \frac{\xi_i}{\theta_i})\tilde{\eta}_i(t)$. So it is easy to get

$$\tilde{\eta}_i(t) > \tilde{\eta}_i(0)e^{-(\chi_i+\frac{\xi_i}{\theta_i})t} > 0. \tag{26}$$

Consider the piecewise Lyapunov function

$$V(t) = \tilde{\delta}(t)^T(\Xi_\sigma \otimes P)\tilde{\delta}(t) + \sum_{i=1}^{N}\tilde{\eta}_i(t),$$

where Ξ_σ is a positive matrix defined in Assumption 4.

Let

$$V(t) = V_1(t) + V_2(t), \tag{27}$$

where we denote

$$V_1(t) = \tilde{\delta}^{c\,T}(t)(\widehat{R}_\sigma \otimes P)\tilde{\delta}^c(t) + \sum_{i=1}^{\widehat{N}_\sigma}\tilde{\eta}_i^c(t) + \tilde{\delta}^{r\,T}(t)(I_{N-\widehat{N}_\sigma} \otimes P)\tilde{\delta}^r(t),$$

$$V_2(t) = \sum_{i=\widehat{N}_\sigma+1}^{N}\tilde{\eta}_i^r(t).$$

Firstly, we analyze the first part $V_1(t)$. The first two parts of $V_1(t)$ is the candidate Lyapunov function with respect to the agents contained in the topology $\widehat{\mathcal{G}}_\sigma$. However, in the third part of $V_1(t)$, there is no interaction among agents not included in the subgraph $\widehat{\mathcal{G}}_\sigma$, so the closed-loop system (21) here can be rewritten as

$$\dot{\tilde{\delta}}^r(t) = (I_{N-\widehat{N}_\sigma} \otimes A)\tilde{\delta}^r(t).$$

Thus, by combining these three pieces, for $t \in [t_{k,r}, t_{k,r+1})$, the derivative of $V_1(t)$ satisfies

$$\begin{aligned}
\dot{V}_1(t) &= 2\tilde{\delta}^{c\,T}(t)(\widehat{R}_\sigma \otimes P)\dot{\tilde{\delta}}^c(t) + \sum_{i=1}^{\widehat{N}_\sigma}\dot{\tilde{\eta}}_i^c(t) + 2\tilde{\delta}^{r\,T}(t)(I_{N-\widehat{N}_\sigma} \otimes P)\dot{\tilde{\delta}}^r(t) \\
&= \tilde{\delta}^{c\,T}(t)\Big[\widehat{R}_\sigma \otimes (PA + A^TP) - c(\widehat{R}_\sigma\widehat{\mathcal{L}}_\sigma + \widehat{\mathcal{L}}_\sigma^T\widehat{R}_\sigma) \otimes PBB^TP\Big]\tilde{\delta}^c(t) \\
&\quad - 2\tilde{\delta}^{c\,T}(t)(c\widehat{R}_\sigma\widehat{\mathcal{L}}_\sigma \otimes PBB^TP)\tilde{e}^c(t) + \sum_{i=1}^{\widehat{N}_\sigma}\dot{\tilde{\eta}}_i^c(t) + \tilde{\delta}^{r\,T}(t)\Big[I_{N-\widehat{N}_\sigma} \otimes (PA + A^TP)\Big]\tilde{\delta}^r(t).
\end{aligned}$$

According to Assumption 3, during $[t_{k,r}, t_{k,r+1})$, $\widehat{\mathcal{G}}_\sigma$ is a fixed strongly connected directed graph. Then, it follows from Lemma 2 that

$$\begin{aligned}-\tilde{\delta}^{c\,T}(t)[c(\widehat{R}_\sigma\widehat{\mathcal{L}}_\sigma + \widehat{\mathcal{L}}_\sigma^T\widehat{R}_\sigma)\otimes PBB^TP]\tilde{\delta}^c(t) &\le -2c\alpha(\widehat{\mathcal{L}}_\sigma)\delta^T(t)(\widehat{R}_\sigma\otimes PBB^TP)\tilde{\delta}^c(t)\\ &\le -2c\zeta\tilde{\delta}^{c\,T}(t)(\widehat{R}_\sigma\otimes PBB^TP)\tilde{\delta}^c(t)\end{aligned}$$

with $\zeta = \min_{\sigma\in\mathbb{P}}\{\alpha(\widehat{\mathcal{L}}_\sigma)\}$.

Owing to $PA + A^TP - 2c\zeta PBB^TP < 0$, it yields

$$\tilde{\delta}^{r\,T}(t)\Big[I_{N-\widehat{N}_\sigma}\otimes(PA + A^TP)\Big]\tilde{\delta}^r(t) < 2c\zeta\lambda_{\max}(PBB^TP)\left\|\tilde{\delta}^r(t)\right\|^2.$$

Similarly to the proof of Theorem 1, one gets

$$\begin{aligned}\dot{V}_1(t) &\le [\lambda_{\max}(\widehat{R}_\sigma\otimes Q_\sigma)(1-\alpha) + c\beta\lambda_{\max}(PBB^TP)]\left\|\bar{\tilde{\delta}}^c(t)\right\|^2 + \gamma_\sigma\|\tilde{e}^c(t)\|^2\\ &+\sum_{i=1}^{\widehat{N}_\sigma}\dot{\tilde{\eta}}_i^c(t) + 2c\zeta\lambda_{\max}(PBB^TP)\left\|\tilde{\delta}^r(t)\right\|^2.\end{aligned}$$

Combining the triggering function (25) with (24) leads to

$$\dot{V}_1(t) \le \mathcal{P}_\sigma\left\|\bar{\tilde{\delta}}^c(t)\right\|^2 - \sum_{i=1}^{\widehat{N}_\sigma}\left(\chi_i - \frac{\gamma_\sigma - \xi_i}{\theta_i}\right)\tilde{\eta}_i^c(t) + 2c\zeta\lambda_{\max}(PBB^TP)\left\|\tilde{\delta}^r(t)\right\|^2, \tag{28}$$

where $\mathcal{P}_\sigma = \lambda_{\max}(\widehat{R}_\sigma\otimes Q_\sigma)(1-\alpha) + c\beta\lambda_{\max}(PBB^TP) + \epsilon < 0$.

Next, we analyze the second part $V_2(t)$, where the agents are not contained in the topology $\widehat{\mathcal{G}}_\sigma$. Thus, taking the time derivative of $V_2(t)$ gives

$$\dot{V}_2(t) = \sum_{i=\widehat{N}_\sigma+1}^{N}\dot{\tilde{\eta}}_i^r(t) = -\sum_{i=\widehat{N}_\sigma+1}^{N}\chi_i\tilde{\eta}_i^r(t) < 0 \tag{29}$$

with $\chi_i > 0$ and $\tilde{\eta}_i^r(t) > 0$.

Then, add Eqs. (28) and (29) to get

$$\begin{aligned}\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t)\\ &\le \mathcal{P}_\sigma\left\|\bar{\tilde{\delta}}^c(t)\right\|^2 + 2c\zeta\lambda_{\max}(PBB^TP)\left\|\tilde{\delta}^r(t)\right\|^2 - \sum_{i=1}^{\widehat{N}_\sigma}\left(\chi_i - \frac{\gamma_\sigma - \xi_i}{\theta_i}\right)\tilde{\eta}_i^c(t).\end{aligned} \tag{30}$$

In light of the form of $V_1(t)$ in Eq. (27), it is not difficult to deduce that

$$V_1(t^*) \le \lambda_{\max}(\widehat{R}_\sigma\otimes P)\left\|\tilde{\delta}^c(t^*)\right\|^2 + \lambda_{\max}(P)\left\|\tilde{\delta}^r(t^*)\right\|^2 + \sum_{i=1}^{\widehat{N}_\sigma}\tilde{\eta}_i^c(t^*),$$

where t^* is the latest triggering instant, that is, $t^* = \max_k\{t_{k_i}^i \le t, i\in\mathcal{I}_N\}$.

Then, one can calculate that

$$\left\|\tilde{\delta}^c(t^*)\right\|^2 \ge \frac{V_1(t^*) - \lambda_{\max}(P)\left\|\tilde{\delta}^r(t^*)\right\|^2 - \sum_{i=1}^{\widehat{N}_\sigma}\tilde{\eta}_i^c(t^*)}{\lambda_{\max}(\widehat{R}_\sigma\otimes P)}. \tag{31}$$

Substituting Eq. (31) into Eq. (30) yields

$$\dot{V}(t) < -\varphi_1 V_1(t^*) - \sum_{i=1}^{\widehat{N}_\sigma}\left[\chi_i - \frac{\gamma_\sigma - \xi_i}{\theta_i} - \frac{\varphi_1}{\widehat{N}_\sigma}\right]\tilde{\eta}_i^c(t) + \left[2c\zeta\lambda_{\max}(PBB^T P) + \varphi_1\lambda_{\max}(P)\right]\left\|\tilde{\delta}^r(t)\right\|^2, \tag{32}$$

where $\varphi_1 = \max_{\sigma\in\mathbb{P}}\{-\frac{\mathcal{P}_\sigma}{\lambda_{\max}(\widehat{R}_\sigma\otimes P)}\} > 0$.

Since the parameters θ_i and c are defined above, it follows that $\sum_{i=1}^{\widehat{N}_\sigma}\left[\chi_i - \frac{\gamma_\sigma - \xi_i}{\theta_i} - \frac{\varphi_1}{\widehat{N}_\sigma}\right] > 0$ and $2c\zeta\lambda_{\max}(PBB^T P) + \varphi_1\lambda_{\max}(P) < 0$.

Thus, it can be obtained that

$$\dot{V}(t) \le -\varphi_1 V_1(t^*) < 0. \tag{33}$$

Furthermore, $V_1(t)$ is a monotonically decreasing function, and for $\forall t \in [t_{k,r}, t_{k,r+1})$, there is always $t > t^*$, so it enforces

$$V_1(t) \le V_1(t^*),$$

Taking the above function into Eq. (33), one deduces

$$\dot{V}(t) \le -\varphi_1 V_1(t) < 0. \tag{34}$$

Therefore, for $\forall t \in [t_{k,r}, t_{k,r+1})$, one derives

$$V(t) \le e^{-\varphi_1(t-t_{k,r})}V_1(t_{k,r}) \le e^{-\varphi_1(t-t_{k,r})}V(t_{k,r})$$

with $V_1(t) < V(t)$.

Due to the above function, the following inequality can be obtained at the switching instant $t_{k,r+1}$.

$$V(t_{k,r+1}^-) \le e^{-\varphi_1(t_{k,r+1}-t_{k,r})}V(t_{k,r}) \le e^{-\varphi_1\mu_1}V(t_{k,r}). \tag{35}$$

Moreover, it follows from Lemma 4 that

$$V(t_{k,r+1}) \le \theta V(t_{k,r+1}^-), \tag{36}$$

where θ has been defined in Assumption 4.

By combining Eq. (35) with Eq. (36) and picking a suitable μ_1 satisfied Eq. (23), it holds that

$$V(t_{k,r+1}) \le \theta e^{-\varphi_1\mu_1}V(t_{k,r}) \le e^{-\vartheta\mu_1}V(t_{k,r}),$$

where $\vartheta = \varphi_1 - \frac{\ln\theta}{\mu_1} > 0$.

We obtain by iteration that

$$V(t_{k,r+1}) \le e^{-(r+1)\vartheta\mu_1}V(t_{k,0}) = e^{-(r+1)\vartheta\mu_1}V(t_k)$$

and

$$V(t_{k+1}) = V(t_{k,\omega_k}) < e^{-\omega_k\vartheta\mu_1}V(t_{k,0}) = e^{-\omega_k\vartheta\mu_1}V(t_k).$$

Then, the properties of the Lyapunov function $V(t)$ over the entire time axis are analyzed as follows.

Firstly, without loss of generality, for arbitrarily given a non-switching instant $t \in (t_{k,r}, t_{k,r+1})$, it leads to

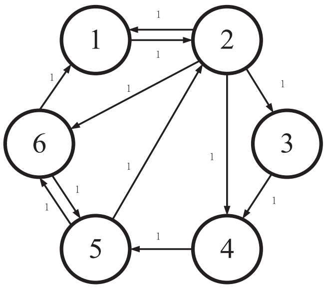


Fig. 1. The communication graph among agents.

$$V(t) \le e^{-\varphi_1(t-t_{k,r})}V(t_{k,r}) \le e^{-[\varphi_1(t-t_{k,r})+r\vartheta\mu_1]}V(t_k) \le e^{-[\varphi_1(t-t_{k,r})+(\sum_{i=1}^{k-1}\omega_i+r)\vartheta\mu_1]}V(t_1)$$
$$\le e^{-[\varphi_1(t-t_{k,r})+\omega^*\vartheta\mu_1]}V(0) \le e^{-\omega^*\vartheta\mu_1}V(0)$$

with $t_1 = 0$ and $\omega^* = (\sum_{i=1}^{k-1}\omega_i + r)$.

In view of $t \le (\omega^* + 1)\mu_2$ and $\omega^* + 1 \ge 2$, one gets

$$V(t) \le e^{-\omega^*\vartheta\mu_1}V(0) \le e^{-\frac{\omega^*\vartheta\mu_1}{(\omega^*+1)\mu_2}t}V(0) \le e^{-\frac{\vartheta\mu_1}{2\mu_2}t}V(0).$$

Secondly, for a switching instant $t = t_{k,r+1}$, one obtains

$$V(t) \le e^{-\omega^*\vartheta\mu_1}V(0) \le e^{-\frac{\vartheta\mu_1}{\mu_2}t}V(0) \le e^{-\frac{\vartheta\mu_1}{2\mu_2}t}V(0).$$

To sum up, for $\forall t \in [0, +\infty)$, it follows that

$$0 < V(t) \le e^{-\frac{\vartheta\mu_1}{2\mu_2}t}V(0). \tag{37}$$

Besides, Eq. (37) enforces that

$$\lim_{t\to+\infty}\tilde{\delta}(t) = \lim_{t\to+\infty}\delta(t) = \mathbf{0}.$$

Then, it can be easily concluded that

$$\lim_{t\to+\infty} x_i(t) - x_j(t) = \mathbf{0},$$

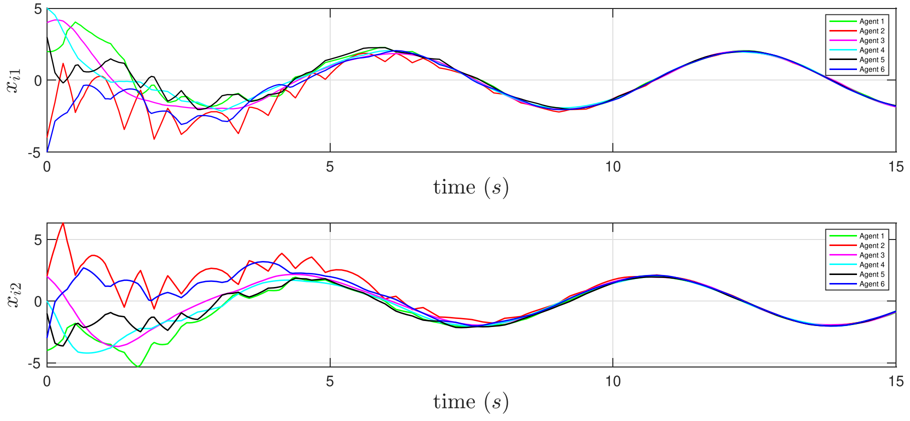
which implies each agent achieves consensus asymptotically.

In the following, we will prove under the dynamic event-triggered strategy (25), there does not exhibit the Zeno behavior.

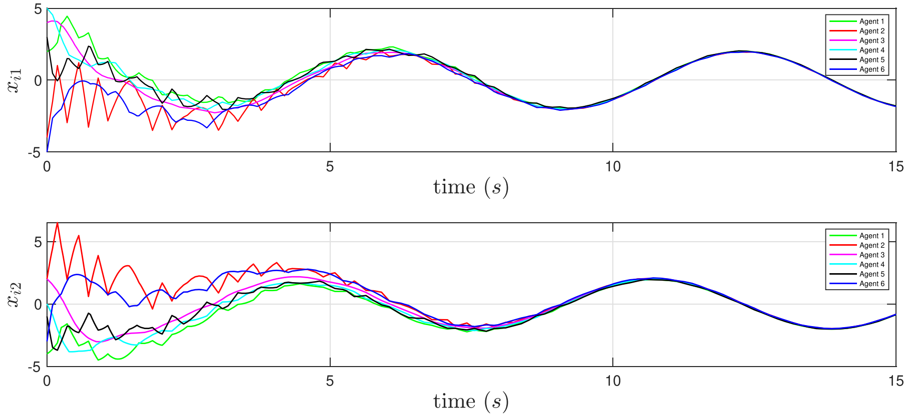
In light of the facts that the interaction among agents changes over time and the topology switching directly affects the event triggering of agents, the Zeno behavior can be analyzed in the following three cases.

Case 1: In the first case, we assume that no topology switches during the time interval between two consecutive triggered events. Thus, the topology is fixed between two triggered events. It can be obtained from Assumption 3 that only the agents contained in the subgraph $\widehat{\mathcal{G}}_\sigma$ will communicate, so we only consider the agents in the set $\widehat{\mathcal{V}}_\sigma$. Similarly to the proof of Theorem 1, no Zeno behavior occurs since it can be proven that infinite triggering is impossible in any finite interval by contradiction under Eqs. (21) and (25). The details are omitted here for brevity.

Case 2: In the second case, only one topology switches during the time interval between two consecutive triggered events. Firstly, consider the event-triggered interval $[t^i_{k_i}, t^i_{k_i+1})$ with $t^i_{k_i}$



(a) States evolution of MAS under dynamic event-triggered strategy (4)



(b) States evolution of MAS under static event-triggered strategy (5)

Fig. 2. States evolution of MAS under different event-triggered strategies with fixed topology.

being the topology switching instant. Since the interaction topology is fixed during $[t^i_{k_i}, t^i_{k_i+1})$, we can analyze as same as the method of case 1. Next, consider the triggering does not occur when the topology switches. Denote t_s be the topology switching instant, that is $t^i_{k_i} < t_s < t^i_{k_i+1}$. Therefore, there must exist a positive constant ε such that $t^i_{k_i+1} - t^i_{k_i} > \varepsilon > 0$.

Case 3: In the third case, there are at least two topologies switching during the time interval between two consecutive triggered events. Without loss of generality, suppose two topologies have been switched in the event-triggered interval $[t^i_{k_i}, t^i_{k_i+1})$. t_s and t_{s+1} are denoted as the two switching instants. Then it follows from Assumption 4 that $t_{s+1} - t_s > \mu_1 > \frac{\ln\theta}{\varphi_1}$, that is,

$$t^i_{k_i+1} - t^i_{k_i} > t_{s+1} - t_s > \mu_1 > \frac{\ln\theta}{\varphi_1} > 0.$$

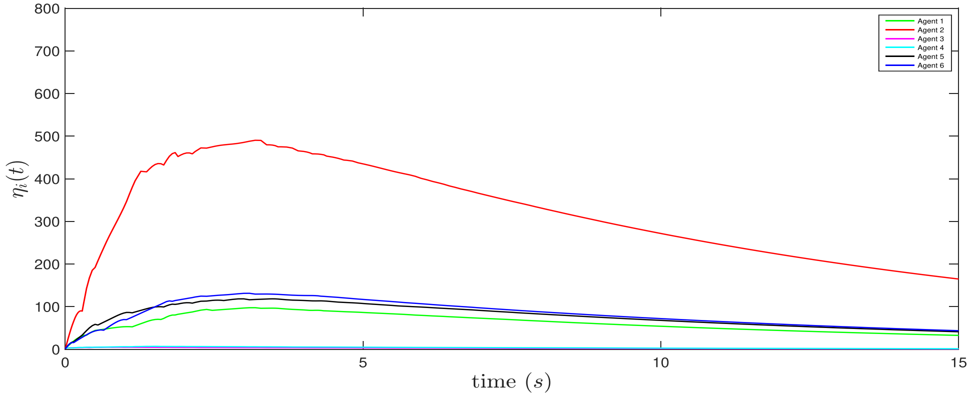


Fig. 3. The dynamic variable $\eta_i(t)$ in Eq. (3).

In summary, in any finite interval, only finite triggerings can happen, which means that the Zeno behavior is excluded for all the agents.

This completes the proof. □

In what follows, we give an algorithm for the proposed dynamic event-triggered consensus protocol under switching topologies.

Algorithm 1. In view of Assumption 1 holding, the dynamic event-triggered consensus protocol composed of controller (20) and dynamic triggering mechanism (25) can be constructed via the following steps.

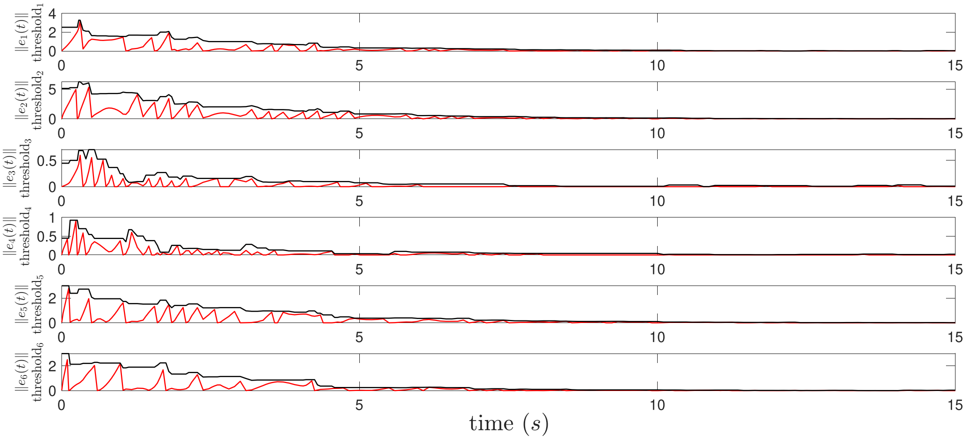
Step 1. solve the following linear matrix inequality $AP^{-1} + P^{-1}A^T - 2BB^T < 0$ to get a symmetric positive definite solution $P > 0$.

Step 2. Choose the feedback matrix $K = -B^T P$ and select the coupling gain $c > \frac{1}{\zeta}$ with $\zeta = \min_{\sigma(t)\in\mathbb{P}}\{\alpha(\widehat{\mathcal{L}}_{\sigma(t)})\}$ such that $PA + A^T P - 2c\zeta PBB^T P < 0$ holds.

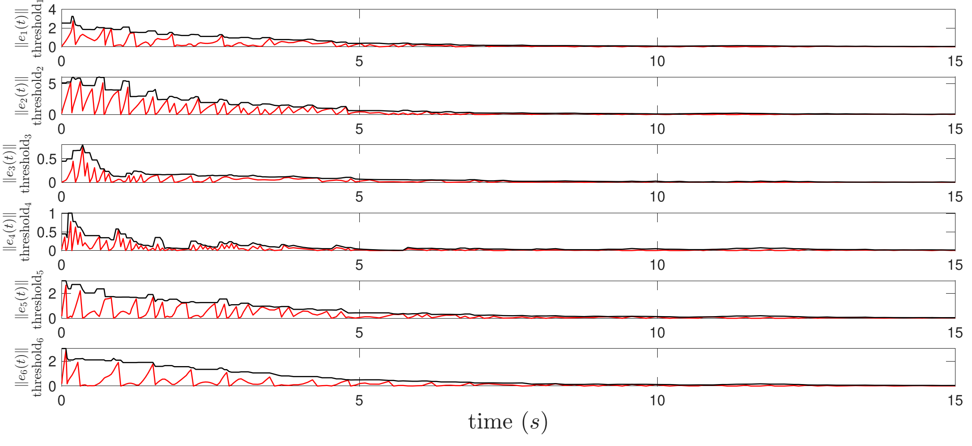
Step 3. Select the appropriate α, β, ϵ, α^*, χ_i, ξ_i, θ_i to satisfy the constraint condition in Theorem 2.

Step 4. Choose the remaining parameters in Eq. (25).

Remark 5. For arbitrary (A, B) satisfying Assumption 1, one can always find a positive definite matrix P such that $Q_{\sigma(t)} < 0$ holds. In addition, there exists uniformly bounds of the parameters for all possible graphs in the finite set $\{\widehat{\mathcal{G}}_\sigma : \sigma \in \mathbb{P}\}$. In practice, the parameters can be selected more conservatively if only they satisfy their bounds. For instance, we can choose the parameters off-line catering using the worst interaction topology before running Algorithm 1. Besides, the proposed Algorithm 1 for the chosen control parameters can also be used for the fixed topology as a particular case, so it is omitted here. Thus, it is easy to know that we can always find appropriate parameters, which means that the proposed dynamic triggering approach is implementable under directed fixed and switching topologies.



(a) The triggering errors and thresholds for each agent under dynamic event-triggered strategy (4)



(b) The triggering errors and thresholds for each agent under static event-triggered strategy (5)

Fig. 4. The triggering errors and thresholds for each agent under different event-triggered strategies.

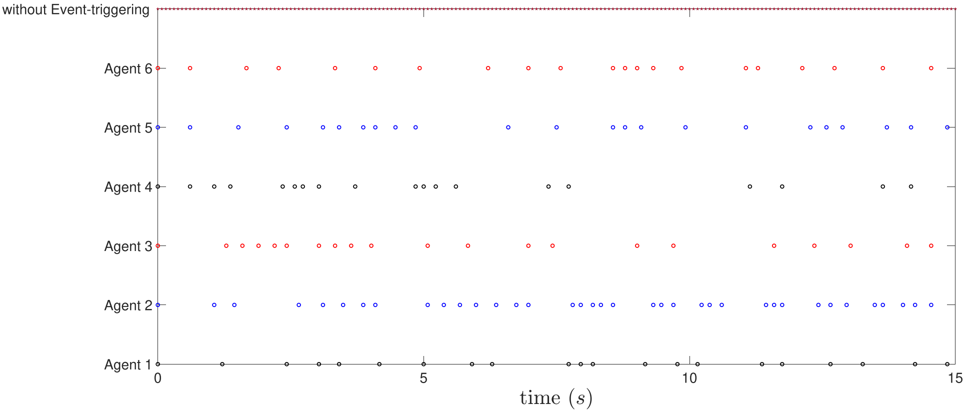
5. Simulation example

In this section, we demonstrate the theoretical results by two numerical examples and make the comparison between the dynamic event-triggered control strategy and traditional static one.

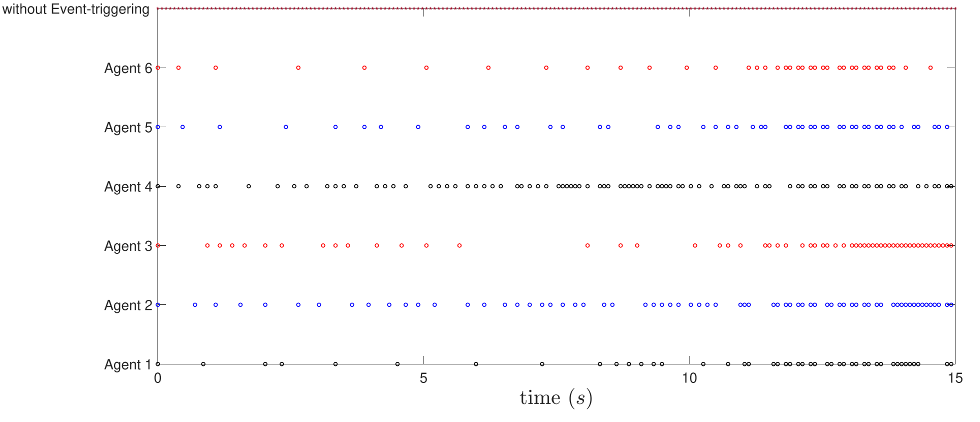
5.1. Example of fixed topology

Consider a group of 6 agents with general linear dynamics (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$



(a) The triggering times for each agent under dynamic event-triggered strategy (4)



(b) The triggering times for each agent under static event-triggered strategy (5)

Fig. 5. The triggering times for each agent under different event-triggered strategies and the comparison with the case without event-triggered strategy.

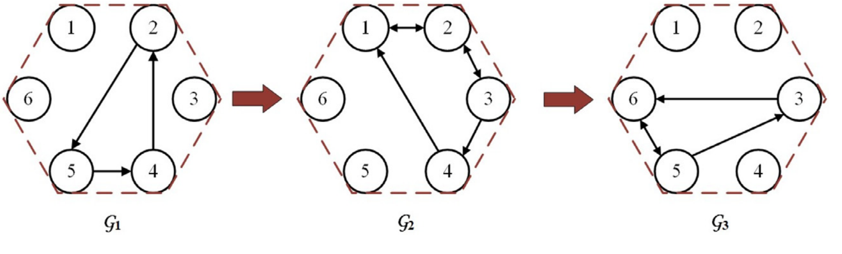
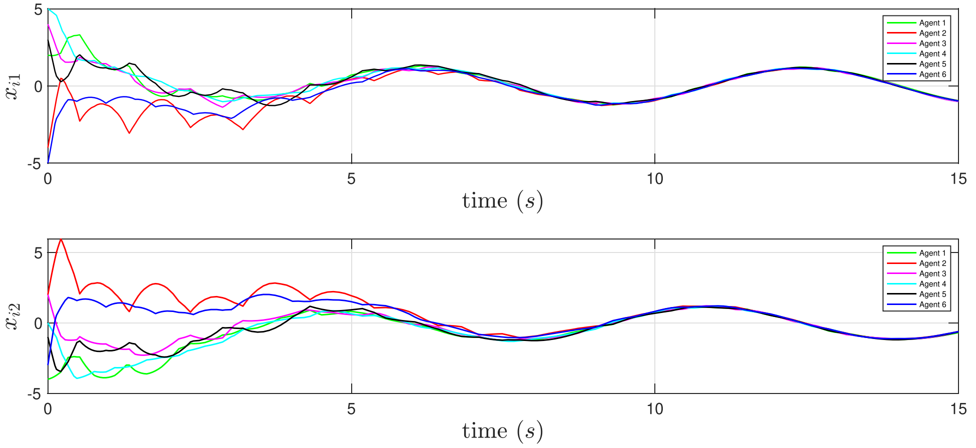
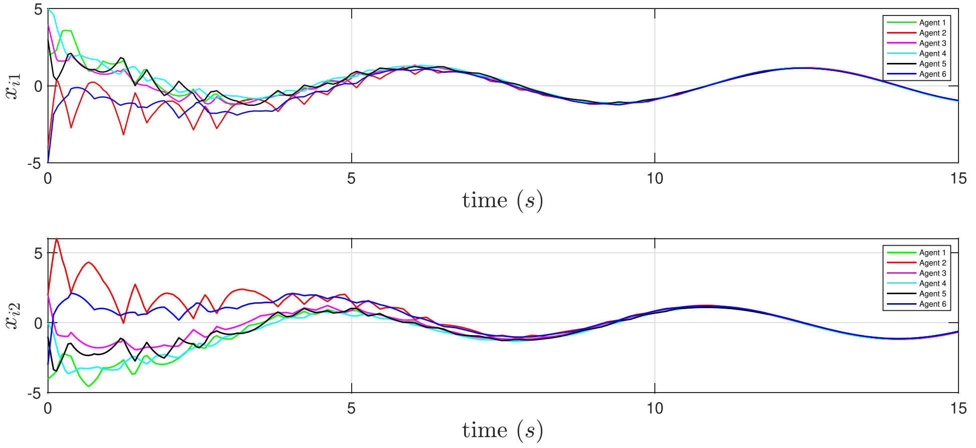


Fig. 6. The switching communication topologies among agents.



(a) States evolution of MAS under dynamic event-triggered strategy (25)



(b) States evolution of MAS under static event-triggered strategy

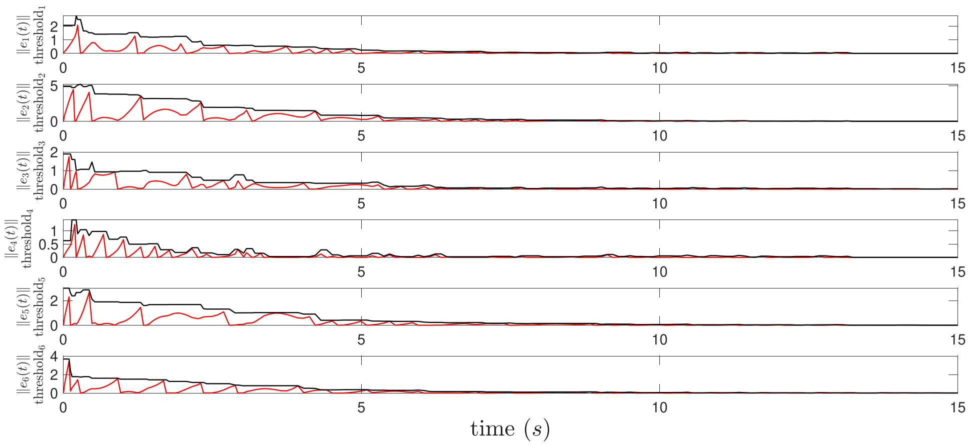
Fig. 7. States evolution of MAS under different event-triggered strategy with switching topologies.

It is easy to detect that Assumption 1 holds. The communication graph is shown in Fig. 1 satisfying Assumption 2 evidently.

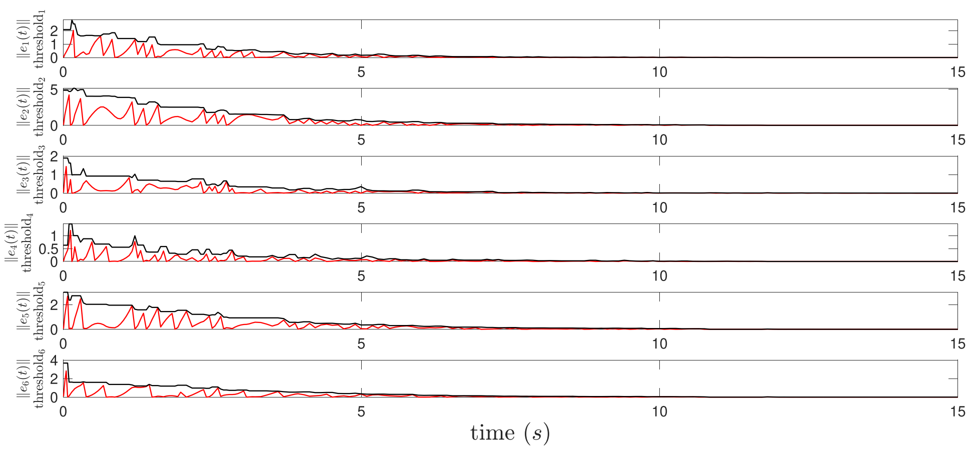
By steps of Algorithm 1, we can calculate that

$$P = \begin{bmatrix} 0.4654 & 0 \\ 0 & 0.2668 \end{bmatrix}$$

and the feedback gain matrix $K = [-0.4654 \ -0.2668]$. Then the parameters are selected as $c = 3.6,\ \alpha^* = 100,\ \epsilon = 1$ and $\chi_i = 0.1,\ \xi_i = 20,\ \theta_i = 30,$ where $i = 1, 2, \dots, 6$. Besides, the initial states are given by $x_1(0) = [2,\ -4]^T, x_2(0) = [-4,\ 2]^T, x_3(0) = [4,\ 2]^T, x_4(t) = [5,\ 0]^T, x_5(t) = [3,\ -1]^T, x_6(t) = [-5, -3]^T$. According to the dynamic triggering law (4), we choose $\eta_1(0) = 2,\ \eta_2(0) = 0.5,\ \eta_3(0) = \eta_4(0) = 3,\ \eta_5(0) = \eta_6(0) = 1.$



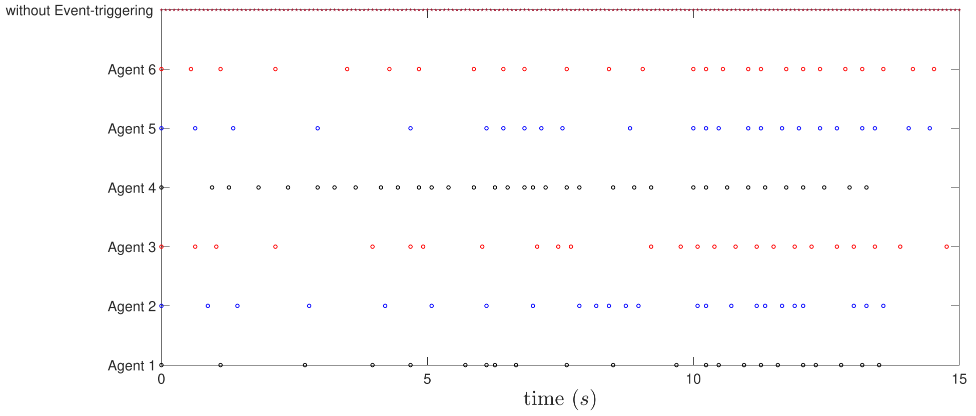
(a) The triggering errors and thresholds for each agent under dynamic event-triggered strategy (25)



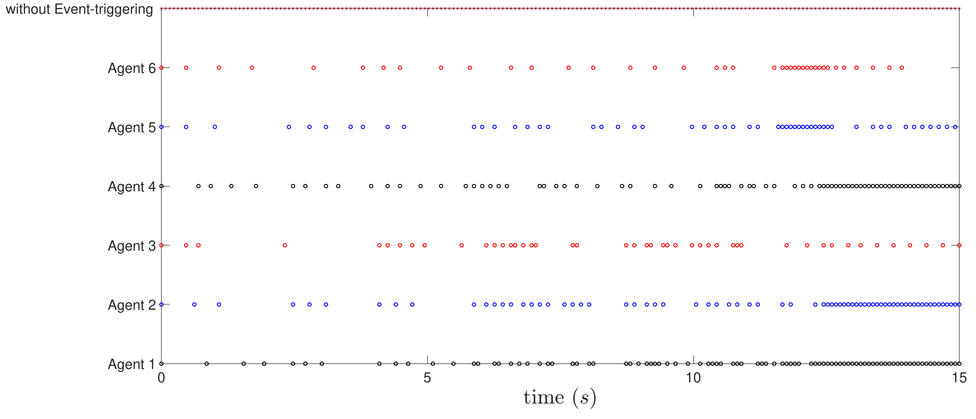
(b) The triggering errors and thresholds for each agent under static event-triggered strategy

Fig. 8. The triggering errors and thresholds for each agent under different event-triggered strategy with switching topologies.

The simulation results are shown in Figs. 2–5. The state evolutions of each agent under dynamic and static triggering strategies are shown in Fig. 2, which shows consensus is indeed achieved. Besides, the tendency of trajectories in these two cases evolves roughly same due to the same system dynamics and initial values. Fig. 3 depicts the trajectories of dynamic variable $\eta_i(t)$ in Eq. (3). Fig. 4 presents the thresholds and the triggering errors under different triggering laws for each agent. We can see that the dynamic variable, tracking errors and thresholds all converge to zero eventually. The corresponding triggering instants under dynamic and static triggering laws are shown in Fig. 5. For a clearer comparison, we record the triggering numbers for each agent with the dynamic and static triggering schemes under the directed fixed topology in Table 1, from which it can be intuitively observed that



(a) The triggering times for each agent under dynamic event-triggered strategy (25)



(b) The triggering times for each agent under static event-triggered strategy

Fig. 9. The triggering times for each agent under different event-triggered strategies and the comparison with the case without event-triggered strategy under switching topologies.

Table 1
Triggering numbers on each agent under different triggering schemes with fixed and switching topologies.

Agent index i		1	2	3	4	5	6
The fixed topology	Dynamic triggering scheme	21	37	21	19	23	21
	Static triggering scheme	45	69	57	90	50	37
The switching topologies	Dynamic triggering scheme	22	24	25	34	24	26
	Static triggering scheme	90	71	46	78	54	39

the triggering numbers are greatly reduced under the proposed dynamic triggering scheme. Consequently, based on completing the consensus task for MAS, the proposed dynamic approach has a satisfactory control performance with reducing communication and computation resource.

5.2. Example of switching topology

For the sake of making it easier to compare the performance of fixed and switching topologies, we consider the same general linear dynamics with a group of 6 agents as the case of the fixed topology in Section 5.1. Meanwhile, the system matrices A, B, the positive matrix P and the gain matrix K take the same value as these in the Section 5.1. Likewise, we choose identical values for all parameters and the initial states of each agent as the case of fixed topology. The possible interaction topologies are depicted in Fig. 6 satisfying Assumption 3. The process of switching topologies is $\mathcal{G}_1 \to \mathcal{G}_2 \to \mathcal{G}_3$.

Figs. 7 –9 describe the simulation results under switching topologies. The states evolved under dynamic and static triggering schemes are presented in Fig. 7, which demonstrates consensus of MAS can be achieved. Fig. 8 shows the triggering errors and thresholds under different triggering schemes for each agent. It can be seen that the threshold is piecewise continuous and goes to zero with an overall decreasing tendency. Fig. 9 depicts the corresponding triggering instants under dynamic and static triggering laws, respectively, which also implies the Zeno behavior is excluded. Under this case of switching topologies, we also record the triggering numbers with the different schemes in Table 1 for a clearer contradistinction. The triggering numbers under the dynamic triggering scheme are greatly reduced. In general, the consensus problem can be solved via the proposed dynamic event-triggered strategy of linear MASs with switching topologies.

6. Conclusion

This paper has investigated the consensus problem for general linear MASs under fixed and switching directed graphs. For fixed topology, a distributed dynamic event-triggered control protocol along with a dynamic triggering function has been addressed, where both controller updates and triggering condition detections utilize the discrete information of neighbors. Then, our dynamic event-triggered protocol has been extended to the MASs under switching topologies. The proposed results have shown that the consensus can be reached with any initial conditions and the Zeno behavior does not exist under these two types of topologies. Extending the results in this paper to a network of heterogeneous agents or jointly connected switching topologies, and designing a novel strategy when MASs contain some uncertainties or external disturbances are interesting directions for future study.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Yifei Li: Conceptualization, Methodology, Investigation, Software, Writing - original draft. **Xiangdong Liu:** Supervision. **Haikuo Liu:** Investigation, Writing - review & editing. **Changkun Du:** Investigation, Writing - review & editing. **Pingli Lu:** Supervision, Writing - review & editing.

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