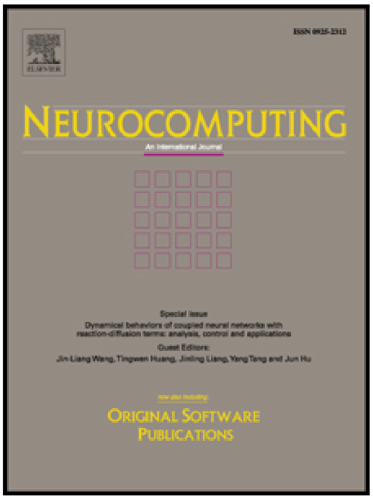
Communicated by Dr Hongyi Li

Accepted Manuscript



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PII: S0925-2312(17)30831-7
DOI: 10.1016/j.neucom.2017.04.061
Reference: NEUCOM 18428

To appear in: *Neurocomputing*

Received date: 15 November 2016
Revised date: 22 February 2017
Accepted date: 17 April 2017

Please cite this article as: Jian Liu, Yao Yu, Qing Wang, Changyin Sun, Fixed-time event-triggered consensus control for multi-agent systems with nonlinear uncertainties, *Neurocomputing* (2017), doi: 10.1016/j.neucom.2017.04.061

Fixed-time event-triggered consensus control for multi-agent systems with nonlinear uncertainties

Jian Liu[a], Yao Yu[a,*], Qing Wang[a], Changyin Sun[b,a]

[a]School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing 100083, China
[b]School of Automation, Southeast University, Nanjing, 210096, China

Abstract

This paper investigates the fixed-time event-triggered consensus control problem for multi-agent systems with nonlinear uncertainties. The fixed-time consensus protocols are presented based on event-triggered strategies which can significantly reduce energy consumption and the frequency of the controller updates. Both the centralized and the distributed consensus control strategies are considered. It is proved that under the proposed event-triggered consensus control strategies the Zeno behavior is avoided. Compared with the finite-time consensus, the fixed-time consensus can be achieved within a fixed settling time with arbitrary initial states of the agents. Finally, two examples are presented to show the effectiveness of the fixed-time event-triggered consensus protocols.

Keywords: fixed-time, event-triggered, consensus, multi-agent systems, nonlinear uncertainties

1. Introduction

Recent years, cooperative control of multi-agent systems received considerable attention and led to lots of significant results such as formation control [1, 2], flocking [3, 4], data fusion [5] and so on. As a basic problem of cooperative control, consensus is a typical collective behavior which requires all the agents to converge to a common value or an agreement by communicating with their neighbours [6].

Noticeably, most of the existing works about consensus problem of the multi-agent systems were asymptotic consensus results which meant that the consensus can only be achieved within infinite time[7, 8, 9, 10, 11]. In [7], the distributed consensus algorithm was considered for linear multi-agent systems with noise and delays. The consensus problems for the networks topologies with fixed and switching topologies were researched in [8], and the random interconnection failure was considered in [9]. For the system model with external disturbances and input delays, the consensus control algorithm for multi-agent systems had been researched in [10, 11]. However, convergence rate is an important property for the consensus of multi-agent systems. This naturally leads to the analysis and construction of the control protocols for consensus convergence rate.

Compared with the convergence rate of asymptotic results, the finite-time consensus results have better dynamic property. Motivated by the advantages of finite-time control protocols, for instance, higher accuracy and faster convergence rate [12], finite-time consensus control of multi-agent systems has attracted much attention in recent years [12, 13, 14]. The distributed finite-time consensus control protocols by continuous state feedback were proposed for networks of dynamic agents in [13]. By adding a power integrator method, the continuous finite-time consensus protoclols were proposed for leaderless and leader-follower multi-agent systems [14]. In these researches, the settling time of finite-time consensus is depend on the initial states of all the agents. Hence, the settling time can be sufficiently large if the initial states are very large.

To deal with these constraints, new works based on the notion of fixed-time stability [15] have been studied, which can ensure the settling time regardless of the initial states of the agents. In [16], the fixed-time consensus protocols

*Corresponding author
Email address: bkliujian@163.com(Jian Liu); yuyao@ustb.edu.cn(Yao Yu); sunnyqing1020@163.com(Qing Wang); cysun@seu.edu.cn (Changyin Sun)

with linear and nonlinear state measurements were proposed for linear multi-agent systems. In networked agents with directed and intermittent communications, if the sum of time intervals is larger than an analytically bounded value, the fixed-time consensus with be achieved with a larger convergence time were researched in [17]. Unfortunately, many practical systems are nonlinear and some essentially nonlinear phenomena can take place only in the presence of nonlinearity. This naturally brings about the analysis and construction of consensus control protocols for nonlinear multi-agent systems. In [18], a classical continuous fixed-time consensus protocol was proposed for multi-agent systems with the nonlinear disturbance, and the fixed-time pinning control problem also be taken into consideration. The fixed-time leader-following problem was researched for multi-agent systems with unknown nonlinear dynamics in [19].

In the above works, the continuous control strategy was considered, which may lead to large communication consumption and high frequency of controller's update. The event-triggered control is an effective way to overcome these disadvantages, and some instructive results have been achieved recently [20, 21, 22, 23, 24, 25]. In addition, event-triggered control is also widely applied to the discrete-time cases and complex networks[26, 27, 28]. Centralized event-triggered consensus control for multi-agent systems was first proposed in [20, 21]. The whole multi-agent systems share one event-triggered function depending on the states of all the agents and update all the control inputs at the same time. Compared with the centralized event-triggered function which requires the global state information, the distributed event-triggered function for each agent is based only on neighbours' information. In [23] a distributed event-triggered control algorithm was provided to investigate linear first-order system and extended to a linear second-order system. The distributed event-triggered consensus control for multi-agent systems with general linear dynamics were investigated in [24], and the consensus can be reached asymptotically. In [25], the event-triggered consensus of nonlinear multi-agent systems with nonlinear dynamics was considered. Taking into account of the convergence rate problems, the finite-time event-triggered consensus algorithms have been presented in [29, 30]. Both the leaderless case and the leader-following case were considered in [29]. Two sufficient conditions were proposed to reach finite-time consensus for multi-agent systems with fixed and switching network topologies in [30]. However, these algoithms hadn't take fixed-time event-triggered consensus or nonlinear uncertainties into consideration.

Motivated by the existing works, in this brief, fixed-time event-triggered consensus algorithms for multi-agent systems with nonlinear uncertainties are proposed. Compared with the previous works related to this brief, the results shown in this paper have the following features. Firstly, both the centralized and the distributed event-triggered consensus control strategies are developed. Secondly, the fixed-time consensus algorithms can ensure the settling time regardless of the initial conditions of the agents. Finally, the multi-agent systems with nonlinear uncertainties are considered. To the author's knowledge, few results about fixed-time event-triggered consensus for multi-agent systems with nonlinear uncertainties are available till now.

The rest of this paper is organized as follows: In Sect.2, preparation, problem description and the dynamical model of the multi-agent systems are given. In Sect.3, the fixed -time centralized event-triggered consensus algorithm is proposed for the multi-agent systems with nonlinear uncertainties. The fixed-time distributed event-triggered consensus algorithm is discussed in Sect.4. In Sect.5, two examples are provided to illustrate the fixed-time event-triggered consensus algorithms. The conclusions are drawn in Sect.6.

2. Preparation and problem description

In this section, firstly, the basic graph theory will be introduced. secondly, some lemmas are given which will be used later. Thirdly, the problem formulation is presented.

2.1. Graph theory

For an undirected graph G with N agents can be expressed as $G = (V_g, E, A)$ and the N agents can be considered as N nodes. The $V_g = \{1, 2, \ldots, n\}$ means a set of nodes and $E \subseteq V_g \times V_g$ is the set of edges. An edge$(j, i) \in E$ means that note i can obtain information from note j and note j can also obtain information from note i. Agent i and j can communicate with each other, then they are called neighbors. Adjacency matrix $A = [a_{ij}] \in R^{n\times n}$ is a $N * N$ matrix, whose elements can defined as that: $a_{ij} = \begin{cases} > 0, (j, i) \in E \\ 0, otherwise \end{cases}$. Define the degree matrix as $D = diag\,[d_1, \ldots, d_N]$ with

$d_i = \sum\limits_{j=1, j\neq i}^{N} a_{ij}$. The Laplacian matrix $L = [l_{ij}] \in R^{n\times n}$ of G is defined as $L = D - A$. In addition, if there exists a path between any two agents, the graph G is connected.

Remark 1. For an undirected graph G, both A and L are symmetric. For a connected graph, the Laplacian has a single zero eigenvalue, and $\mathbf{1}$ is the corresponding eigenvector [31].

2.2. Some lemmas

Lemma 1[32]. For a connected undirected graph G , the Laplacian matrix L of G has the following properties. $x^T Lx = \frac{1}{2}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} a_{ij}(x_i - x_j)^2$, for any $x = [x_1, x_2, \ldots x_n]^T$, and the L is positive semi-definite and has n positive real eigenvalues . 0 is a simple eigenvalue of L and $\mathbf{1}$ is the associated eigenvector. Assume that the eigenvalues of L are denoted by $0, \lambda_2, \cdots, \lambda_n$ satisfying $0 \leq \lambda_2 \leq \cdots \leq \lambda_n$. Then the second smallest eigenvalue $\lambda_2 \geq 0$. Furthermore, if $1^T x = 0$, then $x^T Lx \geq \lambda_2 x^T x$.

Lemma 2[15]. If there have a continuous radially unbounded function $V : R^n \to R_+ \cup \{0\}$ such that

(1) $V(x) = 0 \Leftrightarrow x = 0$;

(2) the solution $x(t)$ satisfied the inequality $D * V(x(t)) \leq -(\tilde{\alpha}V^{\tilde{p}}(x(t)) - \tilde{\beta}V^{\tilde{q}}(x(t)))^k$ for some $\tilde{\alpha}, \tilde{\beta}, \tilde{p}, \tilde{q}, k > 0, \tilde{p}k < 1$ and $\tilde{q}k > 1$;

then, the globally fixed-time stable can be achieved and the settling time T satisfies that $T(x_0) \leq \frac{1}{\tilde{\alpha}^k(1-\tilde{p}k)} + \frac{1}{\tilde{\beta}^k(\tilde{q}k-1)}$, $\forall x_0 \in R^n$. If $k = 1$, the globally fixed-time stable with settling time T bounded by

$$T \leq T_{\max} := \frac{1}{\tilde{\alpha}\,(1 - \tilde{p})} + \frac{1}{\tilde{\beta}\,(\tilde{q} - 1)}$$

where $\tilde{\alpha}, \tilde{\beta} > 0$, $p \in (0, 1)$ and $q \in (1, \infty)$.

Remark 2. According to Lemma 2, we can find that the settling time T bounded by $T \leq T_{\max}$. The $T_{\max}$ is only dependent on $\tilde{\alpha}, \tilde{\beta}, \tilde{p}$ and $\tilde{q}$. These parameters are dependent of the controllers' parameters, the number of the agents and the second smallest eigenvalue of matrix L regardless of the initial states.

Lemma 3[33]. Let $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_M \geq 0$. Then

$$\sum_{i=1}^{M} \varepsilon_i^p \geq \left(\sum_{i=1}^{M} \varepsilon_i\right)^p, 0 < p \leq 1$$
$$\sum_{i=1}^{M} \varepsilon_i^p \geq M^{1-p}\left(\sum_{i=1}^{M} \varepsilon_i\right)^p, 1 < p \leq \infty$$

2.3. Problem formulation and fixed-time consensus

The multi-agent systems with nonlinear uncertainties are composed of M agents, the communication topology of the M agents is a connected undirected graph which will be introduced later, and the dynamics of the agents can be described as:

$$\dot{x}_i(t) = u_i(t) + f(x_i(t), t) \tag{1}$$

where $x_i(t)$ is the state, $u_i(t)$ is the control input and $f(x_i(t), t)$ is a nonlinear function.

In this paper, according to the notion of fixed-time stable, the fixed-time consensus problem can be defined as follows.

Definition 1. The fixed-time consensus of the multi-agent systems (1) is that there exists a fixed-time T such that $\lim\limits_{t\to T}\left|x_i(t) - x_j(t)\right| = 0$, and $x_i(t) = x_j(t)$ when $t \geq T$ $(i, j = 1, 2, \ldots, M)$. The settling time T is bounded, i.e., $\exists T_{\max} > 0$, such that $T \leq T_{\max}$ with arbitrary initial states.

Define $x(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T \in R^M$ and $u(t) = [u_1(t), u_2(t), \ldots, u_M(t)]^T \in R^M$.

Let $y^{\chi} = \left[y_1^{\chi}, y_2^{\chi}, \ldots, y_M^{\chi}\right]^T$, where $\chi > 0$, $y = [y_1, y_2, \ldots, y_M]^T \in R^M$.

For the multi-agent systems with nonlinear uncertainties, the nonlinear term $f(x_i(t), t)$ should satisfy the following conditions.

Assumption 1. There is a known and positive constant γ satisfying

$$\left|f(x_i(t), t) - f(x_j(t), t)\right| \leq \gamma \left|x_i(t) - x_j(t)\right| \tag{2}$$

Remark 3. If the nonlinear term $f(x_i(t), t) = 0$, it will become the linear multi-agent systems studied in [29, 30], and in these cases they solved the finite-time consensus problems.

Remark 4. Compared with the convergence rate of asymptotic results, the finite-time consensus results have better dynamic property, for instance, higher accuracy and faster convergence rate. However, the settling time of the finite-time consensus depends on the initial states of all the agents. Hence, the settling time can be sufficiently large if the initial states are very large. In this paper, the fixed-time consensus can ensure the settling time regardless of the initial states of the agents.

3. Fixed-time centralized event-triggered consensus algorithm

In this section, the fixed-time centralized event-triggered consensus algorithm for multi-agent systems with nonlinear uncertainties is presented as follows.

For centralized event-triggered consensus algorithm, the multi-agent systems only have one global event triggered function. At the event time, all the agents will update the control law simultaneously while update control inputs once.

With the centralized event-triggered strategy, the control input for agent i is designed as

$$u_i(t) = -c_1\left(\sum_{j=1}^{M} a_{ij}(x_i(t_k) - x_j(t_k))\right)^p - c_2\left(\sum_{j=1}^{M} a_{ij}(x_i(t_k) - x_j(t_k))\right)^q - c_3\left(\sum_{j=1}^{M} a_{ij}(x_i(t_k) - x_j(t_k))\right) \tag{3}$$

where c_1, c_2 and c_3 are positive constants, t_k is the triggered time, $x_i(t_k)$ is the state of agent i at the k th event instants. $p \in (0, 1)$ and $q \in (1, \infty)$ are the ratios of positive odd numbers.

Define

$$y_i(t) = \sum_{j=1}^{M} a_{ij}(x_i(t) - x_j(t)) \tag{4}$$

So one has $y(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T = Lx \in R^M$.

Then (3) can be expressed as

$$\begin{aligned} u(t) &= -c_1 y^p(t_k) - c_2 y^q(t_k) - c_3 y(t_k) \\ &= -(e(t) + c_1 y^p(t) + c_2 y^q(t) + c_3 y(t)) \end{aligned} \tag{5}$$

where $e(t) = c_1 y^p(t_k) + c_2 y^q(t_k) + c_3 y(t_k) - c_1 y^p(t) - c_2 y^q(t) - c_3 y(t)$.

So the measurement error can be defined as follows

$$e_i(t) = c_1 y_i^p(t_k) + c_2 y_i^q(t_k) + c_3 y_i(t_k) - c_1 y_i^p(t) - c_2 y_i^q(t) - c_3 y_i(t) \tag{6}$$

Theorem 1. Under Assumptions all above, consider the multi-agent systems, which composed of the controlled plant (1) and controller (5), the event-triggered function for all the agents has the following form

$$\|e(t)\| \leq \frac{1}{2} c_3 \|y(t)\| \tag{7}$$

Thus, the events are triggered when $\|e(t)\| > \frac{1}{2}c_3\|y(t)\|$. At any event instant, all the agents will update the control law simultaneously and the measurement error will be set to zero. Furthermore, the multi-agent systems with nonlinear uncertainties can achieve fixed-time consensus if the following condition is satisfied

$$2\gamma \le c_3\lambda_2 \tag{8}$$

where λ_2 is the second smallest eigenvalue of matrix L, and the fixed time T bounded as follows

$$T \le T_{\max} := \frac{1}{c_1 2^{\frac{p-1}{2}}\lambda_2^{\frac{p+1}{2}}(1-p)} + \frac{1}{c_2 2^{\frac{q-1}{2}}\lambda_2^{\frac{q+1}{2}} M^{\frac{1-q}{2}}(q-1)} \tag{9}$$

Proof.

Consider the Lyapunov candiate fuction as

$$\begin{aligned} V &= \frac{1}{2}x^T(t)Lx(t) \\ &= \frac{1}{4}\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))^2 \end{aligned} \tag{10}$$

From the Lemma 1, it can be obtained that $\sum_{i=1}^{M} y_i^2(t) = \left(L^{\frac{1}{2}}x(t)\right)L\left(L^{\frac{1}{2}}x(t)\right) \ge \lambda_2 x^T(t)Lx(t)$, and $\sum_{i=1}^{M} y_i^2(t) = \left(L^{\frac{1}{2}}x(t)\right)L\left(L^{\frac{1}{2}}x(t)\right) \le \lambda_n x^T(t)Lx(t) \le 2\lambda_n V(t) \le 2\lambda_n V(0)$. Then one has $|y_i(t)| < \|y(t)\| \le \sqrt{2\lambda_n V(0)}$.

Differentiating (10)

$$\begin{aligned}
\dot{V} &= x^T(t)L\dot{x}(t) \\
&= -y^T(t)\left(e(t) + c_1 y^p(t) + c_2 y^q(t) + c_3 y(t) - f(t)\right) \\
&\le \|y(t)\|\,\|e(t)\| - \frac{1}{2}c_3\|y(t)\|^2 - \frac{1}{2}c_3\sum_{i=1}^{M}y_i^2(t) + \sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))f_i(t) - c_1\sum_{i=1}^{M}y_i^{p+1}(t) - c_2\sum_{i=1}^{M}y_i^{q+1}(t) \\
&\le \|y(t)\|\,\|e(t)\| - \frac{1}{2}c_3\|y(t)\|^2 - \frac{1}{2}c_3\sum_{i=1}^{M}y_i^2(t) + \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))\left(f_i(t)-f_j(t)\right) \\
&\quad - c_1\sum_{i=1}^{M}\left(y_i^2(t)\right)^{\frac{p+1}{2}} - c_2\sum_{i=1}^{M}\left(y_i^2(t)\right)^{\frac{q+1}{2}} \\
&\le -c_1\left(\sum_{i=1}^{M}y_i^2(t)\right)^{\frac{p+1}{2}} - c_2 M^{\frac{1-q}{2}}\left(\sum_{i=1}^{M}y_i^2(t)\right)^{\frac{q+1}{2}} + \frac{1}{2}\gamma\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))^2 - \frac{1}{2}c_3\sum_{i=1}^{M}y_i^2(t) \\
&\le -c_1(2\lambda_2 V)^{\frac{p+1}{2}} - c_2 M^{\frac{1-q}{2}}(2\lambda_2 V)^{\frac{q+1}{2}} + 2\gamma V - c_3\lambda_2 V \\
&= (2\gamma - c_3\lambda_2)V - c_1(2\lambda_2)^{\frac{p+1}{2}}V^{\frac{p+1}{2}} - c_2 M^{\frac{1-q}{2}}(2\lambda_2)^{\frac{q+1}{2}}V^{\frac{q+1}{2}}
\end{aligned} \tag{11}$$

If inequation (8) is founded, one has $\dot{V} \le -c_1(2\lambda_2)^{\frac{p+1}{2}}V^{\frac{p+1}{2}} - c_2 M^{\frac{1-q}{2}}(2\lambda_2)^{\frac{q+1}{2}}V^{\frac{q+1}{2}}$. According to lemma 2, one has $\tilde{\alpha}=c_1(2\lambda_2)^{\frac{p+1}{2}}$, $\tilde{\beta}=c_2M^{\frac{1-q}{2}}(2\lambda_2)^{\frac{q+1}{2}}$, $\tilde{p} = \frac{p+1}{2}$, $\tilde{q} = \frac{q+1}{2}$, and the fixed-time consensus can be obtained, and the consensus time T satisfies

$$T \le T_{\max} := \frac{1}{\tilde{\alpha}(1-\tilde{p})} + \frac{1}{\tilde{\beta}(\tilde{q}-1)} = \frac{1}{c_1 2^{\frac{p-1}{2}}\lambda_2^{\frac{p+1}{2}}(1-p)} + \frac{1}{c_2 2^{\frac{q-1}{2}}\lambda_2^{\frac{q+1}{2}}M^{\frac{1-q}{2}}(q-1)} \tag{12}$$

Under the proposed strategy, the inter-event interval is lower bounded, so the fixed-time centralized event-triggered consensus protocol has no Zeno behaviour. This can be proven in the following theorem:

Theorem 2. Consider the multi-agent system (1) with nonlinear uncertainties, for any initial condition, the inter-event interval $\{t_{k+1} - t_k\}$ is lower bounded by the strictly positive τ as follows

$$\tau = \frac{c_3}{2(\eta_1+\eta_2)^2 + c_3(\eta_1+\eta_2)} \tag{13}$$

where $\eta_1 = c_1 p(\lambda_n V(0))^{(p-1)/2} + c_2 q(\lambda_n V(0))^{(q-1)/2} + c_3$, $\eta_2 = \|L\|\left(c_1(\lambda_n V(0))^{(p-1)/2} + c_2(\lambda_n V(0))^{(q-1)/2} + c_3\right)$, $\gamma = \max\{\gamma_1, \gamma_1, \ldots, \gamma_M\}$.

Proof.

Define

$$\varphi(t)=\|e(t)\|/\|y(t)\| \tag{14}$$

From (3), (4) and (6), one has

$$\begin{aligned}
\dot{\varphi}(t) &= \frac{\|e(t)\|' \, \|y(t)\| - \|e(t)\| \, \|y(t)\|'}{\|y(t)\|^2} \\
&\leq \frac{\left(c_1 p\|y(t)\|^{p-1} + c_2 q\|y(t)\|^{q-1} + c_3\right)\|\dot{y}(t)\|}{\|y(t)\|} + \frac{\|e(t)\| \, \|\dot{y}(t)\|}{\|y(t)\| \, \|y(t)\|} \\
&\leq \left(\eta_1 + \frac{\|e(t)\|}{\|y(t)\|}\right)\frac{\|L\dot{x}\|}{\|y(t)\|} \\
&\leq (\eta_1 + \varphi(t))\frac{\|L(u(t) + f(t))\|}{\|y(t)\|} \\
&\leq (\eta_1 + \varphi(t))(\eta_2 + \varphi(t)) \\
&\leq (\eta_1 + \eta_2 + \varphi(t))^2
\end{aligned}$$

The $\varphi(t)$ satisfies the bound $\varphi(t) \leq \phi(t, \phi_0)$, where $\phi(t, \phi_0)$ is the solution of $\dot{\phi} \leq (\eta_1 + \eta_2 + \varphi(t))^2$, $\phi(0, \phi_0) = \phi_0$. The solution of the above differential equation is $\phi(\tau, 0) = \frac{\tau(\eta_1+\eta_2)^2}{1-\tau(\eta_1+\eta_2)}$. Because the inter-event times are bounded from below by the time τ that satisfies $\phi(\tau, 0) = \frac{1}{2}c_3$, one has $\tau = \frac{c_3}{2(\eta_1+\eta_2)^2+c_3(\eta_1+\eta_2)}$, and the proof is completed.

Remark 5. According to the theoretical proof, the convergence rate of consensus can be chosen arbitrarily. In practical applications, this character can well satisfy the strict settling time requirement.

Remark 6. From (7) and (11), it's easy to find that the controllers update the control inputs at the same time and the consensus of the multi-agent system with nonlinear uncertainties can be achieved in a fixed-time.

4. Fixed -time distributed event-triggered consensus algorithm

In this section, the fixed-time distributed event-triggered consensus problem for multi-agent systems with nonlinear uncertainties is investigated. Compared with the centralized event-triggered function which requires the global state information, the distributed event-triggered function for each agent is only based on neighbours' information, so the distributed event-triggered strategy can relieve the occupation of network abundantly and have better system properties.

For distributed event-triggered consensus algorithm, each agent has its own sampling time sequence, which is decided by its respective distributed event-triggered function.

By using the distributed event-triggered strategy, the control input for agent i is designed as

$$u_i(t) = -c_4\left(\sum_{j=1}^{M} a_{ij}(x_i(t_c^i) - x_j(t_c^i))\right)^{\alpha} - c_5\left(\sum_{j=1}^{M} a_{ij}(x_i(t_c^i) - x_j(t_c^i))\right)^{\beta} - c_6\left(\sum_{j=1}^{M} a_{ij}(x_i(t_c^i) - x_j(t_c^i))\right) \tag{15}$$

where c_4, c_5 and c_6 are positive constants, $\alpha \in (0, 1)$ and $\beta \in (1, \infty)$ are the ratios of positive odd numbers. The control law for the agent i is updated at its own event times $t_0^i, t_1^i, \ldots,$.

Remark 7. In this paper, the control law is composed of $\sum_{j=1}^{M} a_{ij}(x_i(t_c^i) - x_j(t_c^i))$ which is similar to [24] rather than $\sum_{j=1}^{M} a_{ij}(x_i(t_c^i) - x_j(t_{c'}^j))$, where $c' \triangleq \arg\min_{l \in N_+ : t \geq t_l^j}\left(t - t_l^j\right)$, as in [25, 30]. In these works, the control law for the agent i is updated both at its own event times $t_0^i, t_1^i, \ldots,$ as well as at the event times of its neighbors $t_0^j, t_1^j, \ldots,$ j is the neighbor of i. In our scheme, the agent i is updated at only its own event times, and the number of triggered events of the whole system will be reduced.

Remark 8. Compared with the centralized event-triggered consensus algorithm that triggers all the agents at the same time, for distributed event-triggered consensus algorithm, the controller (15) only depends on the states at time t_c^i, so each agent is triggered at its own event time.

Define

$$y_i(t) = \sum_{j=1}^{M} a_{ij}(x_i(t) - x_j(t)) \tag{16}$$

Define

$$\begin{aligned} \xi_i(t) &= x_i(t_c^i) - x_i(t) \\ \xi_{ij}(t) &= x_j(t_c^i) - x_j(t) \end{aligned} \tag{17}$$

and

$$\varepsilon_i(t) = \sum_{j=1}^{M} a_{ij}(x_i(t_c^i) - x_j(t_c^i)) \tag{18}$$

From (16) and (17), $\varepsilon_i(t)$ can be expressed as $\varepsilon_i(t) = \sum_{j=1}^{M} a_{ij}(x_i(t) + \xi_i(t) - x_j(t) - \xi_{ij}(t))$.

Then $u_i(t)$ can be expressed as

$$u_i(t) = -\left(\zeta_i(t) + c_4 y_i^{\alpha}(t) + c_5 y_i^{\beta}(t) + c_6 y_i(t)\right) \tag{19}$$

where $\zeta_i(t)$ is the measurement error which can be written as follows

$$\zeta_i(t) = c_4\varepsilon_i^{\alpha}(t) + c_5\varepsilon_i^{\beta}(t) + c_6\varepsilon_i(t) - c_4 y_i^{\alpha}(t) - c_5 y_i^{\beta}(t) - c_6 y_i(t) \tag{20}$$

Theorem 3. Consider the multi-agent systems, which composed of the controlled plant (1) and controller (15), the trigger function for all the agents has the following form

$$|\zeta_i(t)| \leq \frac{1}{2} c_6 |y_i(t)| \tag{21}$$

Thus for each i, an event is triggered when $|\zeta_i(t)| > \frac{1}{2} c_6 |y_i(t)|$. At any event instant, the agent i will update the control law and the measurement error will be set to zero. Furthermore, the multi-agent systems with nonlinear uncertainties can achieve fixed-time consensus if the following condition is satisfied

$$2\gamma \leq c_6\lambda_2 \tag{22}$$

where λ_2 is the second smallest eigenvalue of matrix L, and the fixed time T bounded as follows

$$T \leq T_{\max} := \frac{1}{c_4 2^{\frac{\alpha-1}{2}} \lambda_2^{\frac{\alpha+1}{2}} (1-\alpha)} + \frac{1}{c_5 2^{\frac{\beta-1}{2}} \lambda_2^{\frac{\beta+1}{2}} M^{\frac{1-\beta}{2}} (\beta-1)} \tag{23}$$

Remark 9. Similar as [29, 30], the relatively simple distributed event-triggered function $|\zeta_i(t)| \leq \frac{1}{2} c_6 |y_i(t)|$ is proposed. The distributed event-triggered algorithm is that the event-triggered functions estabilished in advance and then the suitable control inputs are designed to ensure the consensus property.

Proof.
Consider the Lyapunov candiate fuction as

$$\begin{aligned} V &= \frac{1}{2}x^T(t)Lx(t) \\ &= \frac{1}{4}\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))^2 \end{aligned} \tag{24}$$

Differentiating (24)

$$\begin{aligned} \dot{V} &= x^T(t)L\dot{x}(t) \\ &= -\sum_{i=1}^{M}y_i(t)\left(\zeta_i(t)+c_4y_i^{\alpha}(t)+c_5y_i^{\beta}(t)+c_6y_i(t)-f_i(t)\right) \\ &\le \sum_{i=1}^{M}|y_i(t)|\,|\zeta_i(t)|-c_6\sum_{i=1}^{M}y_i^2(t)-c_4\sum_{i=1}^{M}y_i^{\alpha+1}(t)-c_5\sum_{i=1}^{M}y_i^{\beta+1}(t)+\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))f_i(t) \\ &\le \sum_{i=1}^{M}|y_i(t)|\left(|\zeta_i(t)|-\frac{1}{2}c_6\,|y_i(t)|\right)-c_4\sum_{i=1}^{M}\left(y_i^2(t)\right)^{\frac{\alpha+1}{2}}-c_5\sum_{i=1}^{M}\left(y_i^2(t)\right)^{\frac{\beta+1}{2}} \\ &\quad +\frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))\left(f_i(t)-f_j(t)\right)-\frac{1}{2}c_6\sum_{i=1}^{M}y_i^2(t) \\ &\le -c_4\left(\sum_{i=1}^{M}y_i^2(t)\right)^{\frac{\alpha+1}{2}}-c_5M^{\frac{1-\beta}{2}}\left(\sum_{i=1}^{M}y_i^2(t)\right)^{\frac{\beta+1}{2}}+\frac{1}{2}\gamma\sum_{i=1}^{M}\sum_{j=1}^{M}a_{ij}(x_i(t)-x_j(t))^2-\frac{1}{2}c_6\sum_{i=1}^{M}y_i^2(t) \\ &\le -c_4(2\lambda_2V)^{\frac{\alpha+1}{2}}-c_5(2\lambda_2V)^{\frac{\beta+1}{2}}+2\gamma V-c_6\lambda_2V \\ &= (2\gamma-c_6\lambda_2)\,V-c_4(2\lambda_2)^{\frac{\alpha+1}{2}}V^{\frac{\alpha+1}{2}}-c_5M^{\frac{1-\beta}{2}}(2\lambda_2)^{\frac{\beta+1}{2}}V^{\frac{\beta+1}{2}} \end{aligned} \tag{25}$$

If inequation (22) is founded, one has $\dot{V}\le -c_4(2\lambda_2)^{\frac{\alpha+1}{2}}V^{\frac{\alpha+1}{2}}-c_5M^{\frac{1-\beta}{2}}(2\lambda_2)^{\frac{\beta+1}{2}}V^{\frac{\beta+1}{2}}$. According to lemma 2, one has $\tilde{\alpha}=c_4(2\lambda_2)^{\frac{\alpha+1}{2}}$, $\tilde{\beta}=c_5M^{\frac{1-\beta}{2}}(2\lambda_2)^{\frac{\beta+1}{2}}$, $\tilde{p}=\frac{\alpha+1}{2}$, $\tilde{q}=\frac{\beta+1}{2}$, and the fixed-time consensus can be achieved, the consensus time T satisfies

$$T\le T_{\max}:=\frac{1}{\tilde{\alpha}\,(1-\tilde{p})}+\frac{1}{\tilde{\beta}\,(\tilde{q}-1)}=\frac{1}{c_4 2^{\frac{\alpha-1}{2}}\lambda_2^{\frac{\alpha+1}{2}}(1-\alpha)}+\frac{1}{c_5 2^{\frac{\beta-1}{2}}\lambda_2^{\frac{\beta+1}{2}}M^{\frac{1-\beta}{2}}(\beta-1)} \tag{26}$$

Under the proposed strategy, the inter-event interval is lower bounded, so the fixed-time distributed event-triggered consensus protocol has no Zeno behaviour. This can be proven in the following theorem:

Theorem 4. Consider the multi-agent system (1) with nonlinear uncertainties, for any initial condition, the inter-event interval $\left\{t_{k+1}^i-t_k^i\right\}$ is lower bounded by the strictly positive τ_i as follows

$$\tau_i=\frac{c_6}{2(\eta_3+\eta_4)^2+c_6\,(\eta_3+\eta_4)} \tag{27}$$

where $\eta_3=c_4\alpha(\lambda_nV(0))^{(\alpha-1)/2}+c_5\beta(\lambda_nV(0))^{(\beta-1)/2}+c_6$, $\eta_4=\left|\sum_{j=1}^{M}l_{ij}\right|\left(c_4(\lambda_nV(0))^{(\alpha-1)/2}+c_5(\lambda_nV(0))^{(\beta-1)/2}+c_6\right)+\gamma$.

Proof.
Define

$$\psi_i(t)=|\zeta_i(t)|/|y_i(t)| \tag{28}$$

From (15), (16) and (20), one has

$$\dot{\psi}_i(t)=\frac{|\zeta_i(t)|'\,|y_i(t)|-|\zeta_i(t)|\,|y_i(t)|'}{|y_i(t)|^2}$$

$$
\begin{aligned}
&\leq \frac{\left|c_4\alpha y_i^{\alpha-1}(t)+c_5\beta y_i^{\beta-1}(t)+c_6\right||\dot{y}_i(t)|}{|y_i(t)|}-\frac{|\zeta_i(t)|\,|\dot{y}_i(t)|}{|y_i(t)|^2}\\
&\leq \left(c_4\alpha|y_i(t)|^{\alpha-1}+c_5\beta|y_i(t)|^{\beta-1}+c_6+\frac{|\zeta_i(t)|}{|y_i(t)|}\right)\frac{|\dot{y}_i(t)|}{|y_i(t)|}\\
&\leq (\eta_3+\psi_i(t))\frac{\left|\sum\limits_{j=1}^{M}a_{ij}(\dot{x}_i(t)-\dot{x}_j(t))\right|}{|y_i(t)|}\\
&\leq (\eta_3+\eta_4+\psi_i(t))^2
\end{aligned}
$$

The $\psi_i(t)$ satisfies the bound $\psi_i(t)\leq\phi_i\left(t,\phi_0^i\right)$, where $\phi_i\left(t,\phi_0^i\right)$ is the solution of $\dot{\phi}_i\leq(\eta_1+\eta_2+\psi_i(t))^2$, $\phi_i\left(0,\phi_0^i\right)=\phi_0$. The solution of the above differential equation is $\phi_i(\tau_i,0)=\frac{\tau_i(\eta_3+\eta_4)^2}{1-\tau_i(\eta_3+\eta_4)}$. Because the inter-event times are bounded from below by the time τ_i that satisfies $\phi_i(\tau_i,0)=\frac{1}{2}c_6$, one has $\tau_i=\frac{c_6}{2(\eta_3+\eta_4)^2+c_6(\eta_3+\eta_4)}$, and the proof is completed.

Remark 10. From the (21) and (25), it's easy to find that the controllers update the control inputs asynchronously and the consensus of the multi-agent system with nonlinear uncertainties can be reached in a fixed-time.

Remark 11. When $c_4\neq 0$, $c_5=0$, global finite-time event-triggered consensus will be achieved. If the $f(x_i(t),t)=0$, it will be the finite-time event-triggered consensus for the linear multi-agent systems studied in [29, 30]. The work of [29] is a special case of this paper.

Remark 12. When the number of agent nodes increases, as long as the communication topology meets Lemma 1, we can get the similar conclusion according to the fixed-time event-triggered consensus protocols. In addition, the change of agent nodes will lead to the transform of matrix L. With the raise of the second smallest eigenvalue of matrix L, the settling time will decrease.

5. Simulation results

In this section, we will list numerical simulations to demonstrate the effectiveness of the method proposed in this paper.

It is assumed that the multi-agent systems consist of five agents , and the relationship of all the agents is represented by the undirected graph which is shown in Fig.1.

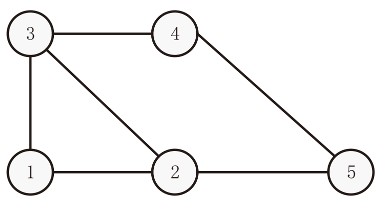

Figure 1: The undirected network topology

The dynamic models of the multi-agent systems with nonlinear uncertainties are described by

$$\dot{x}_i(t)=u_i(t)+f(x_i(t),t) \tag{29}$$

where $f(x_i(t),t)=0.2x_i(t)+0.6cos(t)$, it can be seen that the bounded disturbances f_i satisfies Assumption 1 with $\gamma=1$.

From the graph, we can get the Laplacian matrix:

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{bmatrix}$$

The second smallest eigenvalue λ_2 of the Laplacian matrix L under the undirected network topology is 1.38. The simulation is conducted by assuming that the initial states $x(0) = [-3 \ \ -2 \ \ \ 0 \ \ \ 2 \ \ \ 4]^T$.

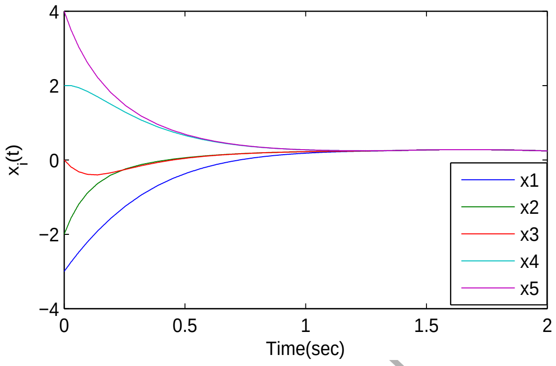

Figure 2: The states of the agents under the centralized event-triggered strategy.

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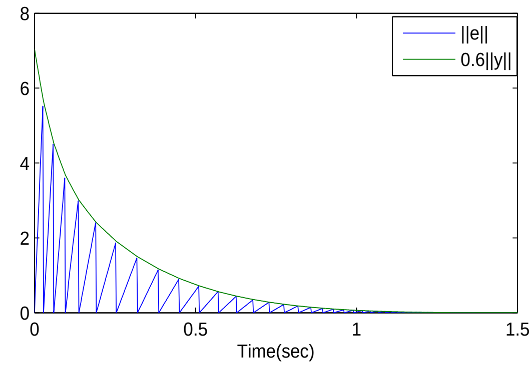

Figure 3: The fluttering of the measurement error $\|e(t)\|$ and the dotted line $\frac{1}{2}c_3 \|y(t)\|$.

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For the centralized event-triggered consensus algorithm, the control gains are chosen as $c_1 = 1$, $c_2 = 0.25$, $c_3 = 1.2$, $p = 3/5$, $q = 7/5$. The parameters satisfy the inequation (8). According to $\tilde{\alpha}=c_1(2\lambda_2)^{\frac{p+1}{2}}$, $\tilde{\beta}=c_2M^{\frac{1-q}{2}}(2\lambda_2)^{\frac{q+1}{2}}$, $\tilde{p} = \frac{p+1}{2}$ and $\tilde{q} = \frac{q+1}{2}$, we can find that these parameters satisfy the Lemma 2. Then, taking the parameters into the inequation (9), we can get the $T_{max} = 9.32$. According to the analysis of Theorem 1 and Fig.2, the settling time T satisfies that $T \le T_{max}$. Fig.3 illustrates the fluttering of the measurement error $\|e(t)\|$ and describes the dotted line $\frac{1}{2}c_3 \|y(t)\|$. The centralized control inputs are plotted in Fig.4. From the Fig.4, we can find that all the controllers update control inputs at the same time.

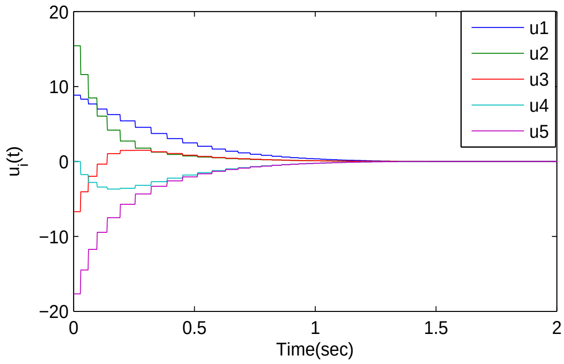

Figure 4: Control inputs of the agents under the centralized event-triggered strategy.

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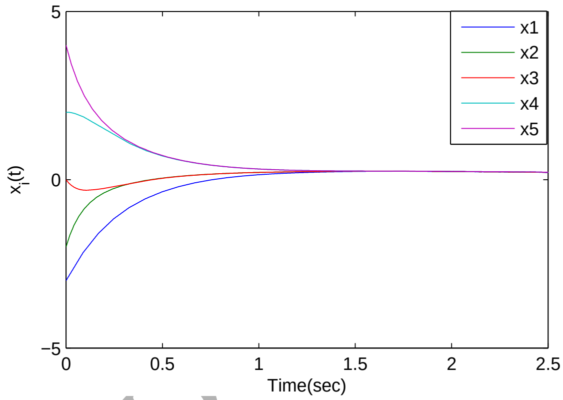

Figure 5: The states of the agents under the distributed event-triggered strategy.

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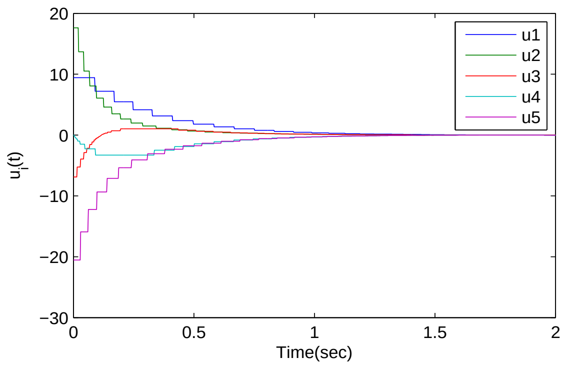

Figure 6: Control inputs of the agents under the distributed event-triggered strategy.

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For the distributed event-triggered consensus algorithm, the control gains are chosen as $c_4 = 0.5$, $c_5 = 0.5$, $c_6 = 1.2$, $\alpha = 3/5$, $\beta = 7/5$. According to $\tilde{\alpha}{=}c_4(2\lambda_2)^{\frac{\alpha+1}{2}}$, $\tilde{\beta}{=}c_5 M^{\frac{1-\beta}{2}}(2\lambda_2)^{\frac{\beta+1}{2}}$, $\tilde{p} = \frac{\alpha+1}{2}$ and $\tilde{q} = \frac{\beta+1}{2}$, we can find

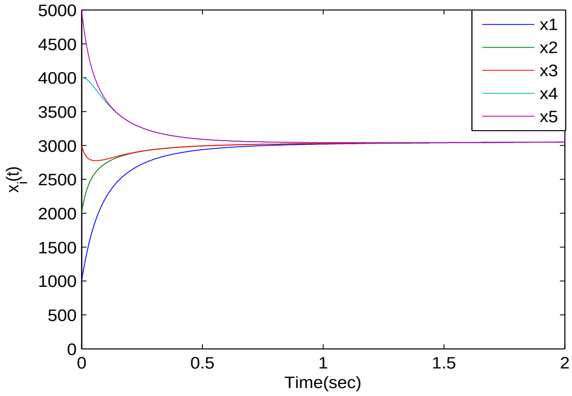

Figure 7: The states of the agents with very large initial states under the centralized event-triggered strategy.

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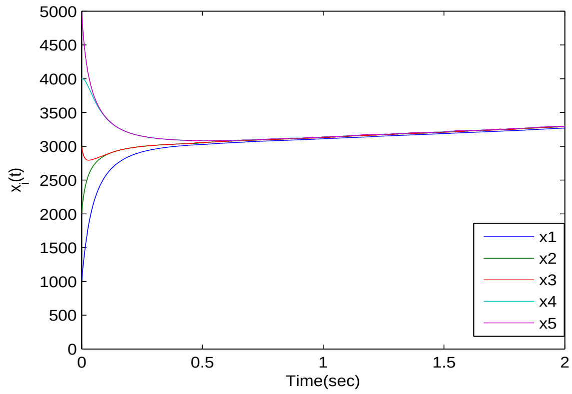

Figure 8: The states of the agents with very large initial states under the distributed event-triggered strategy.

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that these parameters satisfy the Lemma 2. Then, taking the parameters into the inequation (23), we can get the $T_{max} = 9.32$. According to the analysis of Theorem 2 and Fig.5, the settling time T satisfies that $T \leq T_{max}$. The distributed control inputs are plotted in Fig.6. From the Fig.6, we can find that all the controllers update the control inputs at its own event time.

Comparing Fig.4 and Fig.6, the common point is that their controllers are not updated continuously. However, the updates of control inputs are synchronous in centralized case and the updates are asynchronous in distributed case.

In order to demonstrate that the settling time is independent on the initial states of the agents, a simulation is also conducted with very large initial states $x(0) = [1000 \quad 2000 \quad 3000 \quad 4000 \quad 5000]^T$ for the both event-triggered consensus algorithms with the same dynamic models, network topology and control gains. The results can be proved according to the Fig.7 and Fig.8.

6. Conclusions

The fixed-time event-triggered consensus control algorithms for the multi-agent systems with nonlinear uncertainties are proposed in this paper. Both the centralized and the distributed consensus control strategies are considered. Compared with centralized event-triggered control, the distributed event-triggered strategy relieves the occupation of network abundantly and has better system property. With the event-triggered control approachs, the energy consumption and the frequency of the controller updates can be reduced significantly. The fixed-time consensus algorithms can ensure the settling time with arbitrary initial conditions of the agents. Finally, two examples for fixed-time event-triggered consensus control problem have been presented to show the effectiveness of the control algorithms. Fixed-time event-triggered consensus tracking for second-order multi-agent systems under the directed graph will be further researched in the future.

Acknowledgments

This work is supported by National Natural Science Foundation of China under Grants 61520106009, 61533008 and 61473324.

References

[1] W. Wang, J. Huang, C. Wen, H. Fan, Distributed adaptive control for consensus tracking with application to formation control of nonholonomic mobile robots, Automatica 50 (4) (2014) 1254–1263. doi:dx.doi.org/10.1016/j.automatica.2014.02.028.

[2] X. Dong, J. Xi, G. Lu, Y. Zhong, Formation control for high-order linear time-invariant multi-agent systems with time delays, IEEE Transactions on Control of Network Systems 1 (1) (2014) 232–240. doi:10.1109/TCNS.2014.2337972.

[3] J. Zhu, J. Lu, X. Yu, Flocking of multi-agent non-holonomic systems with proximity graphs, IEEE Transactions on Circuits Systems I Regular Papers 60 (1) (2013) 199–210. doi:10.1109/TCSI.2012.2215715.

[4] H. Pei, S. Chen, Q. Lai, A local flocking algorithm of multi-agent dynamic systems, International Journal of Control 88 (11) (2015) 1–23. doi:dx.doi.org/10.1080/00207179.2015.1039595.

[5] W. Yu, G. Chen, Z. Wang, W. Yang, Distributed consensus filtering in sensor networks., IEEE Transactions on Systems Man Cybernetics Part B 39 (6) (2009) 1568–77. doi:10.1109/TSMCB.2009.2021254.

[6] W. Ren, R. W. Beard, Information consensus in multivehicle cooperative control, IEEE Control Systems Magazine 27 (2) (2007) 71–82. doi:10.1109/MCS.2007.338264.

[7] S. Liu, L. Xie, H. Zhang, Distributed consensus for multi-agent systems with delays and noises in transmission channels, Automatica 47 (5) (2011) 920–934. doi:10.1109/CDC.2010.5717575.

[8] R. Olfati-Saber, R. M. Murray, Consensus problems in networks of agents with switching topology and time-delays, IEEE Transactions on Automatic Control 49 (9) (2015) 1520–1533. doi:10.1109/TAC.2004.834113.

[9] H. Li, X. Liao, T. Huang, W. Zhu, Y. Liu, Second-order global consensus in multi-agent networks with random directional link failure., IEEE Transactions on Neural Networks and Learning Systems 26 (3) (2015) 565–575. doi:10.1109/TNNLS.2014.2320274.

[10] C. Wang, Z. Ding, H_∞ consensus control of multi-agent systems with input delay and directed topology, IET Control Theory Applications 10 (6) (2016) 617–624. doi:10.1049/iet-cta.2015.0945.

[11] C. Sun, Q. Wang, Y. Yu, Robust output containment control of multi-agent systems with unknown heterogeneous nonlinear uncertainties in directed networks, International Journal of Systems Science (2016) 1–9.doi:dx.doi.org/10.1080/00207721.2016.1244306.

[12] S. P. Bhat, D. S. Bernstein, Finite-time stability of continuous autonomous systems, Siam Journal on Control Optimization 38 (3) (2006) 751–766. doi:10.1137/S0363012997321358.

[13] L. Wang, X. Feng, Finite-time consensus problems for networks of dynamic agents, IEEE Transactions on Automatic Control 55 (4) (2007) 950–955. doi:10.1109/TAC.2010.2041610.

[14] S. Li, H. Du, X. Lin, Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics, Automatica 47 (8) (2011) 1706–1712. doi:10.1016/j.automatica.2011.02.045.

[15] A. Polyakov, Nonlinear feedback design for fixed-time stabilization of linear control systems, IEEE Transactions on Automatic Control 57 (8) (2012) 2106–2110. doi:10.1109/TAC.2011.2179869.

[16] B. Zhang, Y. Jia, Fixed-time consensus protocols for multi-agent systems with linear and nonlinear state measurements, Nonlinear Dynamics 82 (4) (2015) 1683–1690. doi:10.1007/s11071-015-2269-9.

[17] Q. Wang, Y. Wang, C. Sun, Fixed-time consensus of multi-agent systems with directed and intermittent communications, Asian Journal of Control 19 (1) (2016) 1–11. doi:10.1002/asjc.1329.

[18] Y. Zhou, G. Jiang, C. Sun, Fixed time consensus of multi-agent systems, in: 34th Chinese Control Conference, 2015.

[19] M. Defoort, A. Polyakov, G. Demesure, M. Djemai, Leader-follower fixed-time consensus for multi-agent systems with unknown non-linear inherent dynamics, IET Control Theory Applications 9 (14) (2015) 2165–2170. doi:10.1049/iet-cta.2014.1301.

[20] D. V. Dimarogonas, K. H. Johansson, Event-triggered control for multi-agent systems, in: 28th Chinese Control Conference, 2009.

[21] D. V. Dimarogonas, E. Frazzoli, K. H. Johansson, Distributed event-triggered control for multi-agent systems, IEEE Transactions on Automatic Control 57 (5) (2012) 1291–1297. doi:10.1109/TAC.2011.2174666.

[22] H. Li, X. Liao, T. Huang, W. Zhu, Event-triggering sampling based leader-following consensus in second-order multi-agent systems, IEEE Transactions on Automatic Control 60 (7) (2015) 1998–2003. doi:10.1109/TAC.2014.2365073.

[23] D. Xie, S. Xu, Z. Li, Y. Zou, Event-triggered consensus control for second-order multi-agent systems, IET Control Theory Applications 9 (5) (2015) 667–680. doi:10.1049/iet-cta.2014.0219.

[24] W. Hu, L. Liu, G. Feng, Consensus of linear multi-agent systems by distributed event-triggered strategy, IEEE Transactions on Cybernetics 46 (1) (2016) 148–157. doi:10.1109/TCYB.2015.2398892.

[25] H. Li, G. Chen, T. Huang, W. Zhu, L. Xiao, Event-triggered consensus in nonlinear multi-agent systems with nonlinear dynamics and directed network topology, Neurocomputing 185 (C) (2015) 105–112. doi:dx.doi.org/10.1016/j.neucom.2015.12.047.

[26] D. Ding, Z. Wang, B. Shen, G. Wei, Event-triggered consensus control for discrete-time stochastic multi-agent systems, Automatica 62 (C) (2015) 284–291. doi:dx.doi.org/10.1016/j.automatica.2015.09.037.

[27] D. Ding, Z. Wang, B. Shen, H. Dong, Event-triggered distributedH_∞ state estimation with packet dropouts through sensor networks, IET Control Theory Applications 9 (13) (2015) 1948–1955. doi:10.1049/iet-cta.2014.1055.

[28] H. Li, X. Liao, G. Chen, D. J. Hill, Z. Dong, Event-triggered asynchronous intermittent communication strategy for synchronization in complex dynamical networks, Neural Networks 66 (C) (2015) 1–10. doi:dx.doi.org/10.1016/j.neunet.2015.01.006.

[29] Y. Zhu, X. Guan, X. Luo, S. Li, Finite-time consensus of multi-agent system via nonlinear event-triggered control strategy, IET Control Theory Applications 9 (17) (2015) 2548–2552. doi:10.1049/iet-cta.2014.0533.

[30] H. Zhang, D. Yue, X. Yin, S. Hu, C. X. Dou, Finite-time distributed event-triggered consensus control for multi-agent systems, Information Sciences 339 (2016) 132–142. doi:dx.doi.org/10.1016/j.ins.2015.12.031.

[31] R. Merris, A survey of graph laplacians, Linear Multilinear Algebra 39 (1) (2007) 19–31. doi:dx.doi.org/10.1080/03081089508818377.

[32] R. Olfati-Saber, R. M. Murray, Consensus problems in networks of agents with switching topology and time-delays, IEEE Transactions on Automatic Control 49 (9) (2004) 1520–1533.

[33] Z. Zuo, T. Lin, Distributed robust finite-time nonlinear consensus protocols for multi-agent systems, International Journal of Systems Science 47 (6) (2014) 1–10. doi:dx.doi.org/10.1080/00207721.2014.925608.

Jian Liu

Jian Liu Received his B.Eng. degree from the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, China, in 2015. He is a Ph.D. candidate in School of Automation and Electrical Engineering, University of Science and Technology Beijing. His current research interests include multi-agent systems, nonlinear control.

Yao Yu

Yao Yu Received her B.Sc. degree from Department of Control Science and Engineering, Huazhong University of Science and Technology in 2004, the M.S. and Ph.D. degrees from Department of Automation, Tsinghua University in 2010. She was a postdoctor with Tsinghua University. Currently, she is an Associate Professor with the School of Automation and Electrical Engineering, University of Science and Technology Beijing. Her current research interests include nonlinear control, robust control and time-delay systems.

Qing Wang

Qing Wang Received her B.Eng. degree from the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, China, in 2013. She is a Ph.D. candidate in School of Automation and Electrical Engineering, University of Science and Technology Beijing. Her current research interests include multi-agent systems, nonlinear systems, robust control and intelligent control.

Changyin Sun

Changyin Sun Received his bachelors degree with the College of Mathematics, Sichuan University, Chengdu, China, in 1996, and the M.S. and Ph.D. degrees in electrical engineering from the Southeast University, Nanjing, China, in 2001 and 2003, respectively. He is a distinguished Professor with the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, China and the School of Automation, Southeast University, Nanjing, China. His current research interests include intelligent control, flight control, pattern recognition, and optimal theory. Prof. Sun is an Associate Editor of the IEEE Transactions on Neural Networks and Learning Systems.