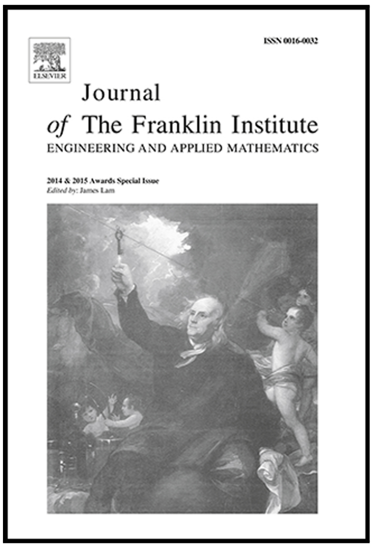
Accepted Manuscript



Observer-based Event-triggered Tracking Consensus of Non-ideal General Linear Multi-agent Systems

Shi-Ming Chen , Jun-Jie Guan , Yan-Li Gao , Huai-Cheng Yan

PII: S0016-0032(18)30331-4
DOI: 10.1016/j.jfranklin.2018.05.019
Reference: FI 3453

To appear in: *Journal of the Franklin Institute*

Received date: 31 October 2017
Revised date: 11 April 2018
Accepted date: 7 May 2018

Please cite this article as: Shi-Ming Chen , Jun-Jie Guan , Yan-Li Gao , Huai-Cheng Yan , Observer-based Event-triggered Tracking Consensus of Non-ideal General Linear Multi-agent Systems, *Journal of the Franklin Institute* (2018), doi: 10.1016/j.jfranklin.2018.05.019

Observer-based Event-triggered Tracking Consensus of Non-ideal General Linear Multi-agent Systems

Shi-Ming Chen[a,*], Jun-Jie Guan[a], Yan-Li Gao[a], Huai-Cheng Yan[b]
[a]School of Electrical and Automation Engineering, East China Jiao Tong University, Nanchang, 330013, China
[b]School of Information Science and Engineering, East China University of Science and Technology, Shanghai, 200237, China
E-mail: shmchen@ecjtu.jx.cn

Abstract

In this paper, the leader-following consensus problem of general linear multi-agent systems without direct access to real-time state is investigated. A novel observer-based event-triggered tracking consensus control scheme is proposed. In the control scheme, a distributed observer is designed to estimate the relative full states, which are used in tracking consensus protocol to achieve overall consensus. And an event-triggered mechanism with estimated state-dependent event condition is adopted to update the control signals so as to reduce unnecessary data communication. Based on the Lyapunov theorem and graph theory, the proposed event-triggered control scheme is proved to implement the tracking consensus when real-time state cannot direct obtain. Moreover, such scheme can exclude Zeno-behavior. Finally, numerical simulations illustrate the effectiveness of the theoretical results.

Index terms-tracking consensus, event-triggered, leader-following, multi-agent systems.

1、 Introduction

Recently, with the rapid development of computer technology, network technique and communication technology, the component unit of large scale group system gradually become agent with computing, execution and communication capabilities, which offers the potential for distributed cooperation control, such as formation control [1-2], flocking [3-4], distributed sensor network [5-6], smart grid system [7-8], et al. All these cooperate control can be unified into the information consensus framework of multi-agent systems (MASs). The basic idea of consensus is that each agent based on its own and its neighbor's state information updates information about themselves, so that all individual information eventually converges to a common value, namely to achieve consensus.

The study of consensus problems can be tracked back to the study of parallel computing and distributed decision-making in the 1980s by Tsitsiklis [9]. However, the study of consensus in system and control field has been greatly influenced by the research of Vicsek et al. in 1995 [10]. In [10], a simple discrete-time model of autonomous agents and simulated complex dynamics of the model was presented. 2003 Jadbabie et al. proposed an explicit discrete-time consensus protocol for the consensus behavior of the Vicsek model and the sufficient and necessary conditions are given for achieving leader-following consensus in [11]. Later Olfati-Saber and Murray proposed a theoretical framework for the consensus of the first-order integrator network, and the situation of constant, switching topology and communication delay are considered in [12]. Ren and Beard further promoted the result of literature [12] in [13], the sufficient and necessary conditions for reaching consensus of first-order MASs under directed graph were given, i.e. if and only if the communication topology has a directed spanning tree that the system can reach consensus. For second-order MASs, the consensus problem is also investigated extensively. It has been shown in [14] that, different from the first-order consensus problem, the second-order consensus might not be achieved even if the network topology has a directed spanning tree. For MASs with higher-order dynamics, Yu et al. derived some necessary and sufficient conditions for higher-order consensus in [15]. It has shown that the higher-order consensus can be reached if and only if all subsystems are asymptotically stable. The leader-following consensus is also knowing as tracking consensus which aims to design a consensus protocol make all followers can track the leader accurately through local interactions [16].

Previous studies above, to achieve the consensus, assumed that either agents get continuous communication with their neighbors or continuously update controllers via current states of agents. However, in practical application, each agent is equipped with a digital embedded micro-processor, thus the control protocols tend to be implemented on digital platforms. In this case, a common control strategy is time-scheduled scheme, that is, the control inputs are updated at periodically sampling times. A sample-date approach was proposed in [17]-[19]. Sampled-data control is essentially time-driven, in order to guarantee the performance of all operation point the constant sampling period usually taken a conservative value. Thus, the control execution is likely to be more frequent than necessary, and this could lead to unnecessary use of computational resources and communication resources. To overcome these drawbacks, the event-triggered consensus control has received wide attention in recent years. The event-triggered consensus control has been discussed in [20] for first-order dynamic system with state dependent trigger condition and prove that exist no Zeno behavior. For double-integrator leader-following MASs, event-based consensus with state-independent trigger functions was examined in [21]. Furthermore, the event-triggered consensus problem has been discussed in [22] for high-order dynamics system. the event-triggered consensus for general linear MASs under leader-following structure was considered in [23].

In many practical systems, due to the physical constrains or implementation costs, it is quite difficult to detect the real-time state. Design an estimator for each agent to estimate the agent's state is very necessary. Zhang et al. considered an observer-based control protocol which was related to the relative output information in [24]. Zhao et al. in [25] further studied the estimator-based scheme on the basis of the [24], designed a reduced-order observer-based scheme for continuous systems, and consensus with time delay also be considered. In [26], based on a distributed velocity estimation technique an observer-based consensus tracking control is designed for second-order leader-following MASs. An observer-based event-triggered control schemes with a static consensus protocol for general linear MAS was first investigated in [27].

In all cited studies above, the event-triggered consensus of general linear MASs mainly concentrated on leaderless consensus, event-triggered mechanism or estimator-based method. The event-triggered tracking consensus control under the assumption that the real-time states cannot be measured are rarely studied. In this paper, observer-based event-triggered tracking control of general linear MASs was considered. The main contribution of this paper can be summarized as follows. First, the Luenberger observers are designed for each agent to estimate the real-time state. Second, based on the estimated state, a novel distributed tracking consensus protocol for general linear MASs is given. Third, an estimated state dependent trigger condition is adopted to update the control input signals so as to reducing unnecessary data communications. Under the proposed control strategy all the following agents can ultimately track the leader accurately.

The rest of this paper is organized as follows: in section 2, a brief overview of algebraic graph theory is provided and the problem under study is formulated. In section 3, the main results of this paper are presented. Numerical simulation is carried out in section 4 to verify the effectiveness of the proposed theoretical result. Finally, some conclusions are drawn in section 5.

Notations: $\mathbb{R}^n$ is n-dimension real Euclidean space. A^T denotes the transpose of matrix A, I_N denotes the identity matrix of N order, let 1_N denotes the column vector with N dimensions, whose elements are ones. $\|x\|$ and $\|A\|$ are the Euclidean norm of vector x and matrix A, respectively. If a real matrix $Q>0$ ($Q<0$), that is to say, the matrix is positive (negative) define. The symbol $\otimes$ means the Kronecker product.

2、 Preliminary

2.1 Algebraic graph

Algebraic graph theory is a very important mathematical tool in the modeling and analysis of multi-agent systems. Consider a MASs composed of one leader agent and N following agent, the leader is labeled as node 0, and the followers are labeled node 1 to N. The communication among the followers can be described by a graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=\{\upsilon_1,\upsilon_2,\cdots,\upsilon_N\}$ is the node set and $\mathcal{E}=\left\{e_{ij}=(\upsilon_i,\upsilon_j):\ \upsilon_i,\upsilon_j\in\mathcal{V}\right\}$ is the edge set. A path on $\mathcal{G}$ from node υ_i to υ_j can be denoted by nonempty sequence $J=\upsilon_i\mathcal{E}_{ik}\upsilon_k\cdots\upsilon_j$. A graph is called connected if there exist a path between every pair of distinct nodes. The path (υ_j,υ_i) in edge set $\mathcal{E}$ denote agent i can receive information from j. Here, agent j is a neighbor of agent i. The neighbor set of agent i can be denoted by $N_i=(\upsilon_i\in\mathcal{V}:(\upsilon_i,\upsilon_j)\in\mathcal{E})$. The matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is said to be the weighed adjacency matrix of graph $\mathcal{G}$, where $a_{ij}>0$ if $(\upsilon_i,\upsilon_j)\in\mathcal{E}$, and $a_{ij}=0$ otherwise. For an undirected graph, we assume $a_{ij}=a_{ji}$. The in-degree matrix associated with $\mathcal{G}$ is defined as

$\mathcal{D} = diag\{d_i, d_2, \cdots, d_N\}$, where $d_i = \sum_{j\in N_i} a_{ij}$. The Laplacian matrix of graph $\mathcal{G}$ is expressed as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Suppose the graph is $\mathcal{G}$ connected, then we can find that the matrix $\mathcal{L}$ has a zero eigenvalue and the corresponding eigenvector is $\mathbf{1}_N$.

The communication graph among the leader-following system can described by a graph $\hat{\mathcal{G}} = (\hat{\mathcal{V}}, \hat{\mathcal{E}})$, where $\hat{\mathcal{V}} = \{\upsilon_0, \upsilon_1, \upsilon_2, \cdots, \upsilon_N\}$ is the node set, vertex 0 represents the leader and the rest are followers, $\hat{E}$ is the edge set. Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a subgraph of $\hat{\mathcal{G}}$. Suppose that the leader agent can send message to some of the followers, and the followers cannot send any message to the leader. Define a matrix $\mathcal{F} = diag(f_1, f_2, \cdots, f_N)$, if the leader is a neighbor of agent i then $f_i > 0$, and $f_i = 0$ otherwise. Let matrix $\mathcal{M} = \mathcal{L} + \mathcal{F}$.

2.2 Problem statement

Considering a MASs consist of one leader and N followers, which are equipped with general linear dynamics. Indexed the leader agent as node 0, and the rest following agent from node 1 to N. The dynamics of the i th following agent can be described as:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t) \\ y_i(t) &= Cx_i(t) \end{aligned} \qquad i \in \{1, 2, \cdots, N\}. \tag{1}$$

Where，$x_i(t) \in \mathbb{R}^n, u_i(t) \in \mathbb{R}^q, y_i(t) \in \mathbb{R}^r$ are the state，control input and measurement output of the i th agent, respectively; $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times q}, C \in \mathbb{R}^{r\times n}$ are constant matrices.

The dynamic of the leader vertex 0 is described as:

$$\begin{aligned} \dot{x}_0(t) &= Ax_0(t) \\ y_0(t) &= Cx_0(t) \end{aligned} \tag{2}$$

Where, $x_0(t) \in \mathbb{R}^n, y_0(t) \in \mathbb{R}^r$ are the state and output of leader agent, respectively.

The agent's full state may be unavailable in practical application due to physical constrains or implementation costs. To solve this problem, one possible way is to consider the following state observer:

$$\dot{\tilde{x}}_i(t) = A\tilde{x}_i(t) + Bu_i(t) + G\left(y_i(t) - \tilde{y}_i(t)\right) \tag{3}$$

where $\tilde{x}_i(t) \in \mathbb{R}^n$ is the observed sate of agent i, $G \in \mathbb{R}^{n\times r}$ is observation gain matrix to be determined.

The traditional distributed tracking consensus protocol was given as follows:

$$u_i(t) = K\left[\sum_{j\in N_i} a_{ij}\left(\tilde{x}_j(t) - \tilde{x}_i(t)\right) + f_i\left(x_0(t) - \tilde{x}_i(t)\right)\right], \quad t \ge 0, i \in V \tag{4}$$

Where $K \in \mathbb{R}^{m\times n}$ is feedback gain matrix to be determined.

Assumption 1: The followers' communication graph $\mathcal{G}$ is undirected, and there is a directed path from the leader to all followers.

Assumption 2: The matrix pair (A, B) of matrix in system (1) is controllable, (A, C) is observable。

Lemma 1: under assumption 1，matrix $\mathcal{M}$ is the positive define symmetric matrix, and the eigenvalues of $\mathcal{M}$ can be described as $0 < \lambda_1(\mathcal{M}) < \lambda_2(\mathcal{M}) < \cdots < \lambda_N(\mathcal{M})$.

Lemma 2: Given any $x, y \in R$, for any $\alpha > 0$,

$$x^T y \le \frac{1}{2\alpha} x^T x + \frac{\alpha}{2} y^T y$$

Definition 1: The tracking consensus of leader-following multi-agent system (1)-(2) is said to be achieved if, the control law for each followers can be designed to make sure that the closed-loop system satisfies

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, \quad \forall_i = 1, 2, \cdots, \mathrm{N}.$$

for any initial condition $x_i(0),\ i = 0, 1, \cdots, N$.

3、 Main results

In this section, the communication topology $\hat{\mathcal{G}}$ is assumed to contain a spanning tree with the leader as the root vertex. We will design a distributed event-triggered control strategy to guarantee all followers to track the leader finally and prove that no Zeno behavior will be exhibited. Note that in traditional control protocol the relative state will be updated continuously. However, it is unnecessary in practice and source wasting. We could update the relative state by some events. In order to develop the event-triggered strategy for agents. The measurement error for agent i can be defined as:

$$e_i(t) = e^{A(t-t_k^i)}\tilde{x}_i(t_k^i) - \tilde{x}_i(t), \quad t \in [t_k^i, t_{k+1}^i) \tag{5}$$

where t_k^i is the k th event-triggered instant for agent i. Besides, let the stack vector $e(t)$ defined by $e(t) = (e_1^T(t), e_2^T(t), \cdots, e_N^T(t))^T$.

For each agent i, we define:

$$\hat{x}_i(t) = e^{A(t-t_k^i)}\tilde{x}(t_k^i) \tag{6}$$

It should be emphasized that the leader agent is assumed to be moving independently and the trajectory of the leader cannot be affected by followers, thus it has no control input for the leader agent. Therefore $e_0(t) = 0$ for all the time, so as to $\hat{x}_0(t) = x_0(t)$.

We proposed an observer-based event-triggered tracking consensus control protocol for each follower as follows:

$$u_i(t) = K\left[\sum_{j\in N_i} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + f_i\left(x_0(t) - \hat{x}_i(t)\right)\right] \tag{7}$$

The closed-loop systems of the followers can be described as:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + BK\left[\sum_{j\in N_i} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + f_i\left(x_0(t) - \hat{x}_i(t)\right)\right] \\ \dot{\tilde{x}}_i(t) &= A\tilde{x}_i(t) + BK\left[\sum_{j\in N_i} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + f_i\left(x_0(t) - \hat{x}_i(t)\right)\right] + GC\left(x_i(t) - \tilde{x}_i(t)\right) \end{aligned} \tag{8}$$

Let $h_i(t) = x_i(t) - \tilde{x}_i(t)$ be the error between observed state and real-time state of i th follower agent. Then

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= A\tilde{x}_i(t) + BK\left[\sum_{j\in N_i} a_{ij}\left(\tilde{x}_j(t) + e_j(t) - \tilde{x}_i(t) - e_i(t)\right) + f_i\left(x_0(t) - \tilde{x}_i(t) - e_i(t)\right)\right] + GCh_i(t) \\ \dot{h}_i(t) &= (A - GC)h_i(t) \end{aligned} \tag{9}$$

Define $\delta_i(t) = \tilde{x}_i(t) - x_0(t)$ be the i th agent's tracking error. Then

$$\dot{\delta}_i(t) = A\delta_i(t) + BK\left[\sum_{j\in N_i} a_{ij}\left(\delta_j(t) - \delta_i(t) + e_j(t) - e_i(t)\right) - f_i\left(\delta_i(t) + e_i(t)\right)\right] + GCh_i(t) \tag{10}$$

Theorem 1: consider system (1) and (2) under assumption 1-2. Design observation gain matrix G in observer (3) make $A - GC$ is Hurwitz, If there exist matrix $P > 0$ and scalar $\beta > 0$ such that the following Riccati matrix inequality

$$A^T P + PA - 2\beta PBB^T P + \beta I < 0 \tag{11}$$

holds, and the triggering condition is defined as:

$$\|e_i(t)\| \le \sqrt{\frac{\kappa}{(\theta - \lambda_1(1-\alpha)/2)}}\,\|\hat{\delta}_i(t)\| \tag{12}$$

where $\theta \ge \|\mathcal{M} \otimes PBB^T P\|$, $0 < \kappa < \frac{\lambda_1}{2}(1 - \frac{1}{\alpha})$. Then, N followers in (1) will track the leader (2) under the control law (7) with $K = B^T P$. Furthermore, for any initial conditions in $\mathbb{R}^n$ and any time $t>0$, no agent will exhibit the Zeno behavior.

In (12) $e_i(t)=e^{A(t-t_k^i)}\tilde{x}_i(t_k^i)-\tilde{x}_i(t)$ present the deference between k th event-triggered instant state and current estimated state of agent i. $\hat{\delta}_i(t)=\delta_i(t_k)$ be the k th triggered instant tracking error, λ_1 is the minimum eigenvalue of matrix $\mathcal{M}$, $\|\cdot\|$ represents the Euclidean norm.

Proof: According to (9) and (10), the closed-loop system can be written in a compact form：

$$\dot{\delta}(t)=(I_N\otimes A-\mathcal{M}\otimes BK)\delta(t)-(\mathcal{M}\otimes BK)e(t)+(I_N\otimes GC)h(t) \tag{13}$$

$$\dot{h}(t)=(I_N\otimes(A-GC))h(t) \tag{14}$$

If G is designed to make matrix $A-GC$ be Hurwitz, then $h_i(t)$ will approach zero asymptotically. From (13) and (14), one can find that the estimation error $h(t)$ is decoupled from the tracking error dynamics $\delta(t)$; thus the stability of (13) is equal to the stability of the following system:

$$\dot{\delta}(t)=(I_N\otimes A-\mathcal{M}\otimes BK)\delta(t)-(\mathcal{M}\otimes BK)e(t) \tag{15}$$

Choosing Lyapunov function $V(t)=\delta(t)^T(I_N\otimes P)\delta(t)$

Differentiating $V(t)$ along the solution of (15), we have

$$\dot{V}(t)=\delta^T(t)(I_N\otimes(PA+A^TP)-\mathcal{M}\otimes 2PBB^TP)\delta(t)-\delta^T(t)(\mathcal{M}\otimes 2PBB^TP)e(t) \tag{16}$$

Under Lemma 1, $\mathcal{M}$ can be written as $\mathcal{M}=\mathcal{H}^T\mathcal{H}$. Substituting in the second part of (16)

$$\begin{aligned}-\delta^T(t)(\mathcal{M}\otimes 2PBB^TP)e(t)&=-\delta^T(t)(\mathcal{H}^T\mathcal{H}\otimes 2PBB^TP)e(t)\\&=-2((\mathcal{H}\otimes B^TP)\delta(t))^T((\mathcal{H}\otimes B^TP)e(t))\end{aligned} \tag{17}$$

According to Lemma 2, let $\alpha=1$, we have

$$\delta^T(t)(\mathcal{M}\otimes 2PBB^TP)e(t)\le\delta^T(t)(\mathcal{M}\otimes PBB^TP)\delta(t)+e^T(t)(\mathcal{M}\otimes PBB^TP)e(t) \tag{18}$$

then we can get:

$$\dot{V}(t)\le\delta^T(t)(I_N\otimes(PA+A^TP)-\mathcal{M}\otimes PBB^TP)\delta(t)+e^T(t)(\mathcal{M}\otimes PBB^TP)e(t) \tag{19}$$

From Lemma 1 once can get:

$$\dot{V}(t)\le\delta^T(t)(I_N\otimes(PA+A^TP)-I_N\otimes\lambda_1 PBB^TP)\delta(t)+e^T(t)(\mathcal{M}\otimes 2PBB^TP)e(t) \tag{20}$$

Substitute $\delta(t)=\hat{\delta}(t)-e(t)$ in previous inequality, we have

$$\dot{V}(t)\le(\hat{\delta}^T(t)-e^T(t))(I_N\otimes(PA+A^TP-\lambda_1 PBB^TP))(\hat{\delta}(t)-e(t))+e^T(t)(\mathcal{M}\otimes PBB^TP)e(t) \tag{21}$$

Noting that $A^TP+PA-2\beta PBB^TP+\beta I<0$, such that $\beta=\lambda_1/2$, the inequality (21) can be expressed as

$$\dot{V}(t)\le-\frac{\lambda_1}{2}(\hat{\delta}^T(t)-e^T(t))(\hat{\delta}(t)-e(t))+e^T(t)(\mathcal{M}\otimes P_1BB^TP_1)e(t) \tag{22}$$

By developing the product in the previous inequality, we obtain:

$$\dot{V}(t)\le-\frac{\lambda_1}{2}(\hat{\delta}^T(t)\hat{\delta}(t)-\hat{\delta}^T(t)e(t)-\hat{\delta}(t)e^T(t)+e^T(t)e(t))+e^T(t)(\mathcal{M}\otimes P_1BB^TP_1)e(t) \tag{23}$$

By using Lemma 2 such that $\alpha>0$, one has

$$\hat{\delta}^T(t)e(t)+e^T(t)\hat{\delta}(t)=2\hat{\delta}^T(t)e(t)\le\frac{1}{\alpha}\hat{\delta}^T(t)\hat{\delta}(t)+\alpha e^T(t)e(t) \tag{24}$$

Therefore

$$(\hat{\delta}^T(t)\hat{\delta}(t)-\hat{\delta}^T(t)e(t)-\hat{\delta}(t)e^T(t)+e^T(t)e(t))\ge(1-\frac{1}{\alpha})\hat{\delta}^T(t)\hat{\delta}(t)+(1-\alpha)e^T(t)e(t) \tag{25}$$

So, the first term of inequality (22) called $R(t)$ can be rewritten like

$$R(t)\le-\frac{\lambda_1}{2}((1-\frac{1}{\alpha})\left\|\hat{\delta}(t)\right\|^2+(1-\alpha)\left\|e(t)\right\|^2) \tag{26}$$

It is obviously that

$$e^T(t)(\mathcal{M}\otimes P_1BB^TP_1)e(t)\le\left\|\mathcal{M}\otimes P_1BB^TP_1\right\|\left\|e(t)\right\|^2 \tag{27}$$

Substituting these inequalities in (22), we obtain

$$\dot{V}(t)\le-\frac{\lambda_1}{2}((1-\frac{1}{\alpha})\left\|\hat{\delta}(t)\right\|^2+(\left\|\mathcal{M}\otimes P_1BB^TP_1\right\|-\frac{\lambda_1}{2}(1-\alpha))\left\|e(t)\right\|^2 \tag{28}$$

Then, by choosing $\alpha>1$, $\kappa-\frac{\lambda_1}{2}((1-\frac{1}{\alpha})<0$ and recall the trigger condition

$$\left\|e_i\right\|\le\sqrt{\frac{\kappa}{(\theta-\lambda_1(1-\alpha)/2)}}\left\|\hat{\delta}_i(t)\right\| \tag{29}$$

Such that $\theta\ge\left\|\mathcal{M}\otimes PBB^TP\right\|$. which leads to

$$\left\|e_i\right\|\le\sqrt{\frac{\kappa}{(\left\|\mathcal{M}\otimes PBB^TP\right\|-\lambda_1(1-\alpha)/2)}}\left\|\hat{\delta}_i(t)\right\| \tag{30}$$

Then

$$\dot{V}(t)\le\kappa-\frac{\lambda_1}{2}((1-\frac{1}{\alpha})\left\|\hat{\delta}_i(t)\right\|^2 \tag{31}$$

Which yields, $\dot{V}(t)\le 0$.

From Lyapunov theorem $V(t)>0$ and $\dot{V}(t)\le 0$ then, $\lim_{t\to\infty}\delta(t)=0$, i.e. $\lim_{t\to\infty}\left\|x_i(t)-x_0(t)\right\|=0$ for all $i\in V$, which implies that all the followers can track the leader for any initial state and without any continuous communication.

Next, we will show that the proposed control strategy can eliminate Zeno behavior. Denote $z(t)=\left\|e(t)\right\|/\left\|\hat{\delta}(t)\right\|$. In each $t\in[t^i_{k_i},t^i_{k_i+1})$, one has

$$\left\|\dot{z}(t)\right\|\le\frac{\left\|\dot{e}(t)\right\|}{\left\|\hat{\delta}(t)\right\|}+z(t)\frac{\left\|\dot{\hat{\delta}}(t)\right\|}{\left\|\hat{\delta}(t)\right\|} \tag{32}$$

Since $\dot{\hat{\delta}}(t)=\dot{\hat{x}}(t)-\dot{\hat{x}}_0(t)$, then $\dot{\hat{\delta}}(t)=(I_N\otimes A)\hat{x}(t)-(I_N\otimes A)\hat{x}_0(t)=(I_N\otimes A)\hat{\delta}(t)$
Therefore

$$\left\|\dot{\hat{\delta}}(t)\right\|/\left\|\hat{\delta}(t)\right\|\le\mu \tag{33}$$

such that $\mu=\left\|I_N\otimes A\right\|$.

Rewrite $e(t)$ in the compact form

$$e(t)=e^{(I_N\otimes A)(t-t^i_k)}\tilde{x}(t^i_k)-\tilde{x}(t) \tag{34}$$

Then $\dot{e}(t)=(I_N\otimes A)(e(t)+\tilde{x}(t))-\dot{\tilde{x}}(t)$

It follows from (13) that

$$\dot{e}(t)\text{=}(I_N\otimes A+\mathcal{M}\otimes BK)e(t)+(\mathcal{M}\otimes BK)\hat{\delta}(t)+(I_N\otimes GC)h(t) \tag{35}$$

It can be observed from (35) that $\left\|\hat{\delta}(t)\right\|$ will not approach zero unless $\left\|h(t)\right\|$ approaches zero. thus, there exists a finite positive scalar γ such that $\left\|h(t)\right\|/\left\|\hat{\delta}(t)\right\|<\gamma$.

Consequently

$$\left\|\dot{e}(t)\right\|\le p\left\|e(t)\right\|+q\left\|\hat{\delta}(t)\right\| \tag{36}$$

with $p=\left\|I_N\otimes A+\mathcal{M}\otimes BK\right\|$ and $q=\left\|\mathcal{M}\otimes BK\right\|+\gamma\left\|I_N\otimes GC\right\|$.

Let $r=p+\mu$. Combining (33) and (36), we can obtain

$$\dot{z}(t)\le q+rz(t) \tag{37}$$

Thus, the evolution of $z(t)$ for $t\in[t^i_{k_i},t^i_{k_i+1})$ satisfies the bound $z(t)\le\phi(t,\phi_0)$, where

$\phi(t,\phi_0)$ is the solution of follow equation:

$$\dot{\phi}(t,\phi_0) = q + r\phi(\tag{38}$$

The corresponding solution of (38) during $t \in [t_{k_i}^i, t_{k_i+1}^i)$ is given by:

$$\phi(t,\phi_0) = ce^{rt} - \frac{q}{r} \tag{39}$$

Where c is a constant. Since $e(t^*) = 0$, therefore $\phi(t^*) = 0$ then $f = \frac{q}{r}$. The inter-event time intervals are bounded from below by τ ,which satisfies $\phi(t,\phi_0) = fe^{rt} - q/r$. On the other hand, the event will not be triggered before (10) is crossing zero, i.e., before $\phi(t,0) = d$, where $d = \sqrt{\kappa/(\theta - \lambda_1(1-\alpha)/2)}$. By solving this equation, one has $\tau = \frac{1}{r}\ln(1+\frac{rd}{q})$. Because $p, r, d > 0$, then $\tau > 0$. Therefore, the inter-event time interval is a lower bounded by a strictly positive value. This excludes the existence of Zeno behavior under the proposed control strategy. This completes the proof.

Remark 1 The matrix pair (A,C) is completely observable so there exist a matrix $Q > 0$ such that $QA + A^TQ - 2C^TC < 0$. At the same time G is designed to make matrix $A - GC$ be Hurwitz, so there also exist a matrix $Q > 0$ such that $Q(A-GC) + (A-GC)^TQ < 0$. Let $G = Q^{-1}C^T$.

4、 Simulation example

In this section, to illustrate the validity of the proposed theoretical result, we Consider multi-agent systems consist of one leader agent and four following agents whose communication graph is given as in figure 1.

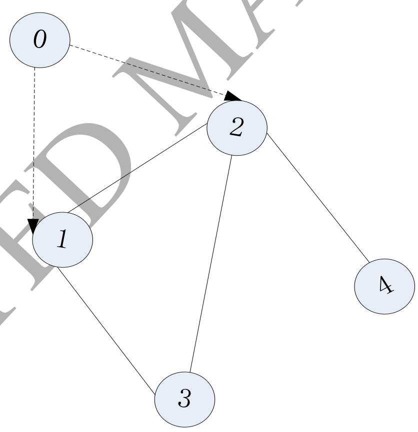


Fig. 1. Communication topology graph

The dynamics of the agents are given by (1) and (2), with $x_i = [x_{i1}, x_{i2}]^T$ and

$$A = \begin{bmatrix} 0 & 1 \\ -9.8 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

One can verify the matrix pairs (A,B) is controllable, and (A,C) is observable. Solve the inequality $QA + A^TQ - 2C^TC < 0$ and equality $G = Q^{-1}C^T$ we can get the observation gain matrix $G = \begin{bmatrix} 0.8960 & 0.3945 \end{bmatrix}^T$, it is obviously that $A - GC$ is Hurwitz. According to theorem 1, solve the Riccati matrix inequality (9), we have:

$$p = \begin{bmatrix} 8.4320 & 0.3945 \\ 0.3945 & 0.8960 \end{bmatrix},\ K = [-0.9989 \quad -2.2685]$$

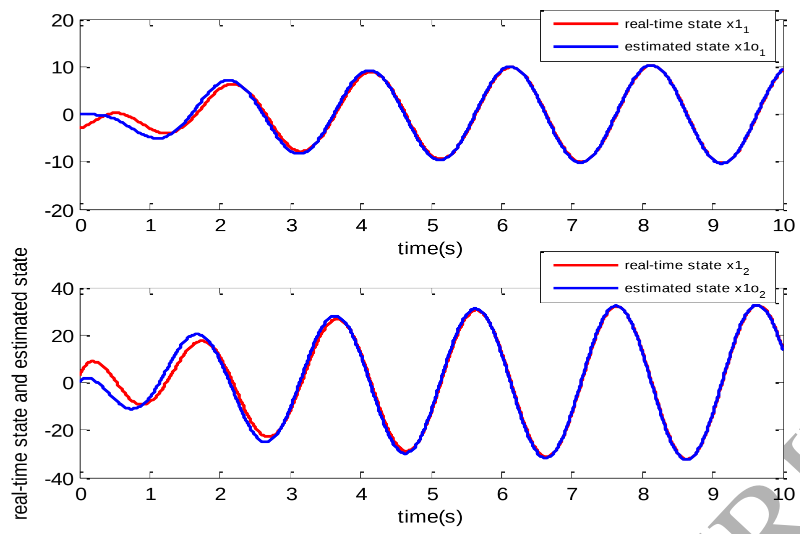


Fig. 2. The estimated state and real-time state

The estimated state and real-time state of agent i as showing in figure 2. It can be proved that the designed observer can reconstruct the agent's state effectively. The state trajectories are presented in figure 3, and the tracking error of four followers are asymptotically converge to zero as showing in figure 4. It is shown that under the proposed observer-based event-triggered distributed control strategy, all the followers can ultimately track the leader accurately. In general, continuous communication will happens in conventional leader-following consensus strategies. Moreover, figure 5 shows event-triggered time interval for each agent, which indicate that the proposed control strategy can lead to a significant reduction of communication burden and save communication resource greatly.

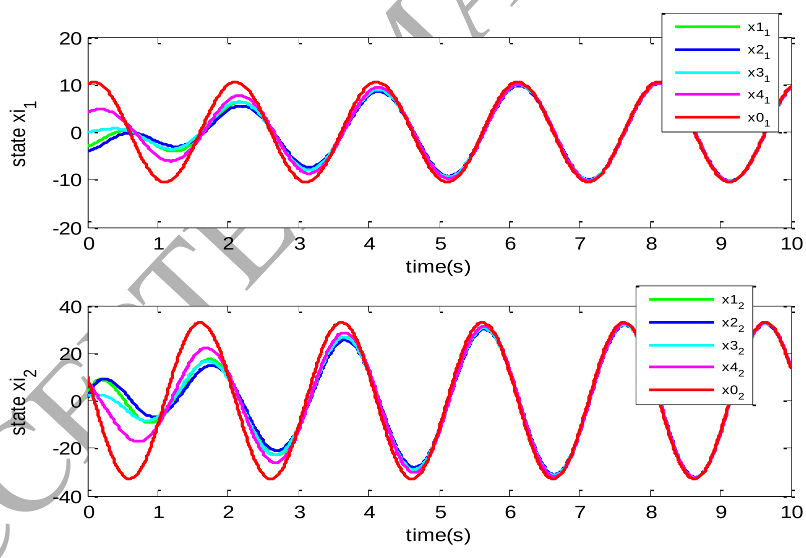


Fig. 3. The state trajectories of agent under the proposed control strategy

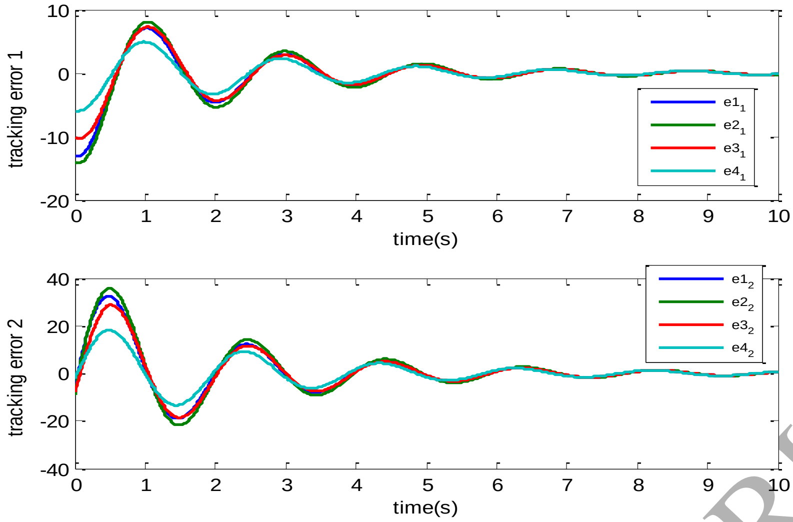


Fig. 4. The tracking error

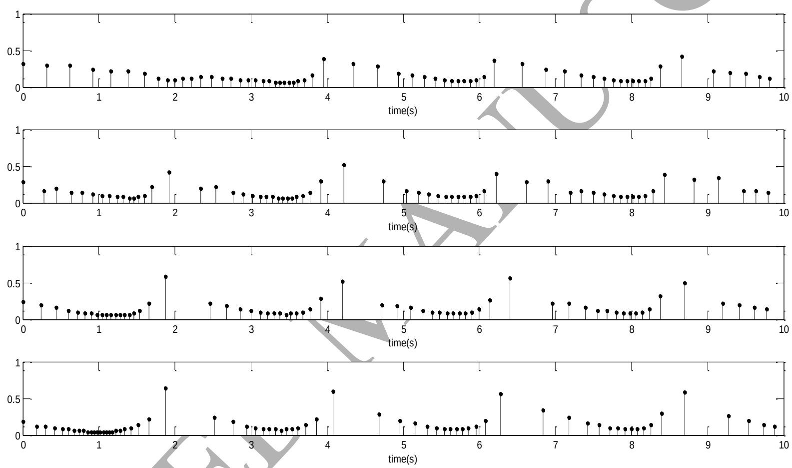


Fig. 5. The event-triggered time interval

The numbers of event trigger are totally 84, 92, 78, and 96 times, respectively, during [0s 10s] and the mean time interval (s) is 0.119, 0,108, 0.128, 0.104, which are presented in table 1. The distributed-ETS reduces substantially the frequency of information transmission and control updated compared with that without event-triggered case (see Table 1).

Table 1 Event Time Intervals for Each Followers

case	followers	Nos of event trigger	Mean time interval
Event-triggered	1	84	0.119
	2	92	0.108
	3	78	0.128
	4	96	0.104
Without event-triggered	1&2&3&4	501	0.020

5、 Conclusion

In this paper, we have investigated the event-triggered tracking consensus control issue for a class of general linear MASs without direct access to real-time state. A Luenberger observer is designed to estimate the relevant full state, based on which an event-triggered tracking control protocol has been proposed to guarantee the leader-following consensus, where the state-dependent trigger condition has been adopted to significantly reduce the communication burden. With the aid of eigenvalues and eigenvectors of Laplacian matrix, some sufficient conditions have been derived to guarantee the expected tracking consensus. In terms of solve the algebraic Riccati inequality, the controller gains have been designed which is independent of the number of agents. Furthermore, it can be proved that Zeno-behavior does not exist. The simulation verifies the superiority of the proposed control strategy in reducing the communication burden.

Acknowledgement:

This work was supported by the Natural Science Foundation of P. R. China under Grant 61364017, 11662002, and Science Foundation of Jiangxi Province under Grant 20165BCB19011.

6、 References

[1] X. W. Dong, Q. D. Li, Z. Ren, et al, Formation-containment control for high-order linear time-invariant multi-agent systems with time delays, Journal of the Franklin Institute 352 (9) (2015) 3564-3584.

[2] J. Lü, F. Chen, G. Chen, Nonsmooth leader-following formation control of nonidentical multi-agent systems with directed communication topologies, Automatica 64 (C) (2016) 112-120.

[3] H. Pei, S. Chen, Q. Lai, A local flocking algorithm of multi-agent dynamic systems, International Journal of Control 88(11) (2015) 2242-2249.

[4] H. Pei, S. Chen, Q. Lai, Multi-target consensus circle pursuit for multi-agent systems via a distributed multi-flocking method, International Journal of Systems Science 47 (16) (2016) 3741-3748.

[5] C. Shen, Y. Chen, X. Guan, Performance evaluation of implicit smartphones authentication via sensor-behavior analysis, Information Sciences 430 (2018) 538-553.

[6] C. Shen, Y. Li, Y. Chen, X. Guan, R. Maxion, Performance Analysis of Multi-Motion Sensor Behavior for Active Smartphone Authentication, IEEE Transactions on Information Forensics and Security 13 (1) (2018) 48-62.

[7] V. Loia, V. Terzija, A. Vaccaro, An affine-arithmetic-based consensus protocol for smart-grid computing in the presence of data uncertainties, IEEE Transactions on Industrial Electronics 62 (5) (2015) 2973-2982.

[8] C. Zhao, J. V, P. Cheng, Consensus-based energy management in smart grid with transmission losses and directed communication, IEEE Transactions on Smart Grid 8 (5) (2017) 2049-2061.

[9] J. Tsitsiklis, M. Athans, On the complexity of decentralized decision making and detection problems, IEEE Transactions on Automatic Control 30 (5) (1985) 440-446.

[10] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, O. Shochet, Novel type of phase transition in a system of self-driven particles, Physical Review Letters 75 (6) (1995) 1226-1229.

[11] A. Jadbabaie, J. Lin, A. S. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules, IEEE Transactions on Automatic Control 48 (6) (2003) 988-1001.

[12] R. Olfati-Saber, R. M. Murray, Consensus problems in networks of agents with switching topology and time-delays, IEEE Transactions on Automatic Control 49 (9) (2004) 1520-1533.

[13] W. Ren, R. Beard. Consensus seeking in multi-agent systems under dynamically changing interaction topologies, IEEE Transactions on Automatic Control 50 (5) (2005) 655-661.

[14] W. Ren, E. Atkins, Distributed multi- vehicle coordinated control via local information exchange, International Journal of Robust & Nonlinear Control 17 (10) (2010) 1002-1033.

[15] W. Yu, G. Chen, W. Ren, Distributed higher order consensus protocols in multiagent dynamical systems, IEEE Transactions on Circuits & Systems 58 (8) (2011)1924-1932.

[16] S. Chen, H. Pei, Q. Lai, H. Yan, Multitarget tracking control for coupled heterogeneous inertial agents systems based on flocking behavior, IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2018, DOI: 10.1109/TSMC.2017.2789335.

[17] G. Wen, Z. Duan, W. Yu, G. Chen, Consensus of multi-agent systems with nonlinear dynamics and sampled data information: A delayed-input approach, International Journal of Robust & Nonlinear Control 23 (6) (2013) 602–619.

[18] L. Ding, G. Guo, Sampled-data leader-following consensus for nonlinear multi-agent systems with Markovian switching topologies and communication delay, Journal of the Franklin Institute 352(1) (2015) 369-383.

[19] W. Zhang, T. Yang, T. Huang, Sampled-Data consensus of linear multi-agent systems with packet losses, IEEE Transactions on Neural Networks & Learning Systems 28 (11) (2017)

2516-2527.
[20] D. V. Dimarogonas, E. Frazzoli, Johansson K H. Distributed event-triggered control for multi-agent systems, IEEE Transactions on Automatic Control 57 (5) (2012) 1291-1297.
[21] H. Li, X. Liao, T. Huang, Event-triggering sampling based leader-following consensus in second-order multi-agent systems, IEEE Transactions on Automatic Control 60 (7) (2015) 1998-2003.
[22] Z. Wu, Y. Wu, Z. Wu, J. Lu, Event-based synchronization of heterogeneous complex networks subject to transmission delays, IEEE Transactions on Systems, Man and Cybernetics: Systems, 2018, DOI: 10.1109/TSMC.2017.2723760.
[23] Z. Wu, Y. Xu, R. Wu, T. Huang, Event-triggered control for consensus of multi-agent systems with fixed/switching topologies, IEEE Transactions on Systems, Man and Cybernetics: Systems, 2017, DOI: 10.1109/TSMC.2017.2744671.
[24] H. Zhang, G. Feng, H. Yan, Q. Chen, Observer-based output feedback event-triggered control for consensus of multi-agent systems, IEEE Transactions Industrial Electronics 61 (9) (2014) 4885–4894.
[25] L. Gao, B. Xu, J. Li, Distributed reduced-order observer-based approach to consensus problems for linear multi-agent systems, Control Theory & Applications 9 (5) (2015) 784-792.
[26] J. Hu, J. Geng, H. Zhu, An observer-based consensus tracking control and application to event-triggered tracking, Communications in Nonlinear Science & Numerical Simulation 20 (2) (2015) 559-570.
[27] W. Liu, C. Yang, Y. Sun, Observer-based event-triggered tracking control of leader-follower systems with time delay, Journal of Systems Science & Complexity 29 (4) (2016) 865-880.