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Designing fixed-time tracking consensus protocols for networked Euler-Lagrangian systems with directed graphs

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In this paper, a class of disturbed networked Euler-Lagrangian systems is investigated to track a general virtual signal under a general directed communication network. Firstly, a class of fixed-time distributed observer is constructed to estimate the leader's state. Secondly, a local anti-disturbance tracking control based on the previous distributed observer is proposed for each follower to achieve the tracking consensus in a fixed time. A simulation example is finally conducted to verify the proposed algorithm.

fixed-time tracking, networked Euler-Lagrangian system, anti-disturbance control, directed network

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1 Introduction

During the past two decades, cooperative control [1] of multi-agent system has attracted much attention from not only academic community but also industrial community, owing to its widespread application in various fields such as mart grid [2, 3], unmanned air vehicle [4], and multi-vessels [5], and theoretical challenges aroused from these applications. Cooperative control problem generally refers to controlling a group of multi-agent system to achieve a common goal. As a fundamental research topic of cooperative control, consensus problem has been widely investigated recently. Consensus means that a collection of agents achieve an agreement on some certain variables through local interaction [6–9].

Generally speaking, consensus problems can be categorized into three classes: leaderless consensus without leader, tracking consensus with one leader, and containment control with two or more leaders [10–13]. Usually, a group of agents should be driven to follow an appointed agent or a predefined virtual signal so as to fulfill a specific task. Hence, in this paper, the authors mainly focus on the tracking consensus problem. A critical indicator to assess the performance of a consensus algorithm is the convergence rate. It has been revealed that the convergence rate of a consensus algorithm depends not merely on control gains and dynamics, but also on the communication network among all the agents and the number of the agents in the network [6, 14]. Massive effort has been made to improve convergence rate, among which finite-time consensus algorithm [15–19] is a significant class since in most real applications consensus should be achieved in finite time. Nevertheless, the convergence time (called settling time) of a finite-time consensus algorithm generally depends explicitly on initial state information, which hinders the application of such algorithm especially when initial state information is unavailable. To overcome such kind of weakness, the concept of fixed-time stability [20] was proposed. Since then, numerous research works concerning fixed-time consensus have emerged [21–24]. The dynamics considered in refs. [21, 22, 24] are integral-type models, while general linear model in ref. [23]. Meanwhile, fixed-time consensus

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problem with general nonlinear dynamics brings more challenges in comparison to the problem with integral-type and general linear dynamics.

Euler-Lagrangian system is a representative nonlinear system that can describe lots of real mechanical systems like robotic manipulators and rigid bodies [25–27]. Recently, finite-time [28, 29] and fixed-time [30, 31] consensus problem of networked Euler-Lagrangian systems have been investigated. In ref. [31], a master-slave bilateral telerobotics system was considered with asymmetric time-varying delays. In ref. [30], a second-order fixed-time lag tracking consensus algorithm was investigated and the algorithm was applied to solve the lag tracking problem for multiple robotic manipulators, where the leader has bounded input and the communication network among all the followers was undirected.

Based on the points discussed above, we mainly focus on the fixed-time tracking consensus problem for a class of disturbed networked Euler-Lagrangian system, where the leader is subject to general nonlinearity and the communication network is generally directed. The difficulties mainly arise from the nonlinearity of the leader's dynamics and the asymmetric communication structure. The main contributions of our paper are twofold. First of all, a novel class of distributed observer is investigated for each follower to cooperatively observe the leader's state subject to Lipschitz-type dynamics in fixed time while the communication network is generally directed. Secondly, a local tracking control based on integral sliding mode control is proposed to steer each follower to track the state of the local observer in fixed time. The total convergence time can be adjusted to arbitrary positive values by appropriately selecting control gains.

Following is the outline of this paper. Preliminaries about graph theory and some important lemmas are listed in Sect. 2. The distributed observer and the tracking control protocol are presented in Sect. 3. A numerical example is then conducted in Sect. 4 to verify the main results. Sect. 5 finally concludes the paper.

Notations: given a vector $\xi = [\xi_1, \xi_2, \cdots, \xi_n]^{\mathrm{T}}$, denote $\|\xi\|$ the Euclidean norm of ξ. Besides, define an exponential-like function as $\xi^{[k]} = \left[\mathrm{sign}(\xi_1)|\xi_1|^{\mathrm{k}}, \mathrm{sign}(\xi_2)|\xi_2|^{k}, \cdots, \mathrm{sign}(\xi_n)|\xi_n|^{k}\right]^{\mathrm{T}}$, where k is a positive real number.

2 Preliminaries

First of all, basic concepts and results with respect to algebraic graph theory [32] are listed here.

A communication network with N nodes can be represented by a directed graph $\mathcal{G} = (\nu, \varepsilon)$, where $V = \{\nu_1, \nu_2, \nu_3, \cdots, \nu_N\}$ represent the node set and $\varepsilon \subseteq V\times V$ stands for the edge set. A directed edge $(i, j) \in \varepsilon$ of the directed graph $\mathcal{G}$ indicates the directed information flow from agent j to agent i. The inner neighbor set of agent i can be expressed as $N_i = \{j \in V\,|(j,i) \in \varepsilon\}$. For any two nodes i_l and i_s in a directed graph $\mathcal{G}$, if there is a sequence of ordered edges $(i_k, i_{k+1}) \in \varepsilon, k = 1, 2, 3, \cdots, s-1$, then it is said that there exists a directed path from node i_l to node i_s.

Usually, a directed graph $\mathcal{G}$ with N nodes is assigned with an adjacency matrix defined as $A = \left(a_{ij}\right) \in R^N \times R^N$, of which $a_{ij} > 0$ if $(j, i) \in \varepsilon$ and $a_{ij} = 0$ otherwise. And the Laplacian matrix of graph $\mathcal{G}$ is defined as $L = (l_{ij})$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$; $l_{ij} = -a_{ij}, i \neq j, i, j = 1, 2, \cdots, N$.

Lemma 1 [14] Denote L the Laplacian matrix of a directed graph $\mathcal{G}$ with N nodes. Then the eigenvalues of L consist of a simple eigenvalue zero and $N-1$ eigenvalues with positive real parts when and only when $\mathcal{G}$ contains a directed spanning tree.

For a directed leader-follower network with $N + 1$ nodes where the leader has no inner-neighbors, the Laplacian matrix can be rearranged as

$$L = \begin{bmatrix} 0 & 0_{1\times N} \\ b & L_B \end{bmatrix}, \quad L_B \in R^{N\times N},\ b \in R^{\mathrm{N}}, \tag{1}$$

where $b = (b_1, b_2, \cdots, b_N)^{\mathrm{T}}$ and $b_i > 0$ means that the i-th follower can obtain information from leader.

Throughout this paper, the following assumption on the communication network is required.

Assumption 1 The directed leader-follower network contains a directed spanning tree with the leader being the root.

Lemma 2 [34] With Assumption 1, there exists a positive diagonal matrix $\Omega = \mathrm{diag}(\omega_1, \cdots, \omega_N)$ such that $\Omega L_B + L_B^{\mathrm{T}}\Omega > 0$.

Lemma 3 Consider the following differential dynamics:

$$\dot{z}(t) = g(z(t)), \quad z(0) = z_0, \tag{2}$$

where $z = [z_1, z_2, ..., z_N]^{\mathrm{T}} \in R^N$, $g(z) : R^N \to R^N$ is continuous on R^N, and $f(0) = 0$. The origin is a globally fixed-time stable equilibrium of eq. (2) if there exits a function $V(z) : R^N \to R$ which is continuous positive definite and $k > 0$, $l > 0$, $\alpha \in (0, 1)$ and $\beta > 1$, such that

$$\dot{V}(z) + k(V(z))^{\alpha} + l(V(z))^{\beta} \leqslant 0, \quad z \in R^N \backslash \{0\}.$$

Furthermore, it holds that any solution of eq. (2) satisfies $z(t, z_0) = 0$, $\forall t \geqslant T = \frac{1}{k(1-\alpha)} + \frac{1}{l(\beta-1)}$.

Lemma 4 [35] Consider the following second-order dynamics:

$$\begin{aligned} \dot{x} &= \nu, \\ \dot{\nu} &= u. \end{aligned} \tag{3}$$

Choose constants k_i and $\bar{k}_i$ (i=1,2) such that $z^2+k_2z+k_1$ and $z^2+\bar{k}_2z+\bar{k}_1$ are Hurwitz. Then, the origin of eq. (3) is fixed-time stable under the following control input:

$$u=-k_1x^{[\rho_1]}-k_2v^{[\rho_2]}-\bar{k}_1x^{[\bar{\rho}_1]}-\bar{k}_2v^{[\bar{\rho}_2]}, \tag{4}$$

of which

$$\rho_1=\frac{\rho_2}{2-\rho_2},\ \rho_2\in(1-\epsilon,1),$$
$$\bar{\rho}_1=\frac{\bar{\rho}_2}{2-\bar{\rho}_2},\ \bar{\rho}_2\in(1,1+\epsilon),$$

with $\epsilon>0$ being sufficiently small. Furthermore, the convergence time is bounded as

$$T\leqslant\frac{\rho_2\lambda_{\max}^{\frac{1}{\rho_2}}(P)}{(1-\rho_2)\lambda_{\min}(Q)}+\frac{\bar{\rho}_2\lambda_{\max}^{\frac{1}{\bar{\rho}_2}}(\bar{P})}{(\bar{\rho}_2-1)\lambda_{\min}(\bar{Q})}, \tag{5}$$

where $P=P^{\mathrm{T}}>0$, $Q=Q^{\mathrm{T}}>0$, $\bar{P}=\bar{P}^{\mathrm{T}}>0$, and $\bar{Q}=\bar{Q}^{\mathrm{T}}>0$ satisfy $PA+A^{\mathrm{T}}P=-Q$ and $\bar{P}\bar{A}+\bar{A}^{\mathrm{T}}\bar{P}=-\bar{Q}$ with $A=\begin{bmatrix}0 & 1\\ -k_1 & -k_2\end{bmatrix}$ and $\bar{A}=\begin{bmatrix}0 & 1\\ -\bar{k}_1 & -\bar{k}_2\end{bmatrix}$.

Lemma 5 [36] Let $\xi_1,\xi_2,\ldots,\xi_n\geqslant0$. If $0<p\leqslant1$, then $\sum_{i=1}^n\xi_i^p\geqslant\left(\sum_{i=1}^n\xi_i\right)^p$; If $p>1$, it holds $\sum_{i=1}^n\xi_i^p\geqslant n^{1-p}\left(\sum_{i=1}^n\xi_i\right)^p$.

3 Main results

Consider a networked Euler-Lagrangian dynamics described by

$$M_i(q_i)\ddot{q}_i+C_i(q_i,\dot{q}_i)\dot{q}_i+G_i(q_i)=\tau_i+d_i, i=1,2\ldots,N, \tag{6}$$

of which $q_i\in R^n$ is a vector of generalized coordinates and the measuring output, $d_i\in R^n$ is a disturbance force, $M_i(q_i)$ represents the inertia matrix agent i, $C_i(q_i,\dot{q}_i)\dot{q}_i$ includes the Coriolis and centrifugal forces, and $G_i(q_i)$ denotes the gravitational force. Furthermore, the virtual signal to be tracked is denoted as $q_0\in R^n$ subject to

$$\ddot{q}_0=f(q_0,\dot{q}_0). \tag{7}$$

To obtain the main results, the following standard assumption for each Euler-Lagrangian system is required.

Assumption 2 The inertia matrices $M_i(q_i)$ are symmetric positive-definite with $0<k_m\leqslant\|M_i(q_i)\|\leqslant\bar{k}_m$, the Coriolis and centrifugal forces satisfy $\|C_i(q_i,\dot{q}_i)\|\leqslant k_c\|\dot{q}_i\|, k_c>0$. Furthermore, $\dot{M}_i(q_i)-2C_i(q_i,\dot{q}_i)$ are skew-symmetric matrices.

Besides, the virtual signal and disturbance force acted on each follower satisfy the following constraint conditions.

Assumption 3 There exists a Lipshitz constant $l>0$ such that for any vectors $y,y',z,z'\in R^n$,

$$\left\|f(y,z)-f(y',z')\right\|\leqslant l\left(\left\|y-y'\right\|+\left\|z-z'\right\|\right). \tag{8}$$

Assumption 4 The disturbance force d_i is uniformly bounded, i.e.,

$$\|d_i(t)\|\leqslant D_i, \tag{9}$$

of which D_i is a positive constant, $i=1,2,\ldots,N$.

Definition 1 The fixed-time tracking consensus is said to be solved for the networked Euler-Lagrangian system (6) with the leader (7) for any initial value $q_i(0)$ and $q_0(0)$, if there exist control protocols τ_i, $i=1,\ldots,N$, and a fixed constant $T_{\max}>0$ irrelevant to initial values such that

$$\begin{cases}\lim_{t\to T_{\max}}|q_i(t)-q_0(t)|=0,\\ q_i(t)=q_0(t),\ \forall t\geqslant T_{\max},\ i=1,2,\ldots,N.\end{cases}$$

To solve the fixed-time tracking consensus problem, a two-step control design will be developed. In the first step, since not all the followers are available to the leader's state information, a class of distributed observer will be designed for each follower so as to cooperatively estimate q_0 and $\dot{q}_0$ within a fixed convergence time. In the second step, a local control protocol will be designed for each follower to track its own estimated state of the leader in fixed time.

Step 1 Distributed fixed-time observer design.

Construct the following observer for each follower:

$$\begin{aligned}\dot{\xi}_i=&\eta_i-\alpha\left[\sum_{j=1}^n a_{ij}(\xi_i-\xi_j)+b_i(\xi_i-q_0)\right]^{[\mu]}\\&-\alpha\left[\sum_{j=1}^n a_{ij}(\xi_i-\xi_j)+b_i(\xi_i-q_0)\right]^{[\nu]},\\ \dot{\eta}_i=&-\beta\left[\sum_{j=1}^n a_{ij}(\eta_i-\eta_j)+b_i(\eta_i-\dot{q}_0)\right]^{[\mu]}\\&-\beta\left[\sum_{j=1}^n a_{ij}(\eta_i-\eta_j)+b_i(\eta_i-\dot{q}_0)\right]^{[\nu]}+f(\xi_i,\eta_i),\end{aligned} \tag{10}$$

of which $\alpha,\beta>0$, and $0<\mu<1<\nu=2-\mu$.

Theorem 1 Consider the distributed observer (10). Suppose that Assumption 1 is satisfied, then ξ_i and η_i provide precise estimation for q_0 and $\dot{q}_0$ in fixed time with some properly chosen α and β.

Proof. Firstly, define the relative estimation errors as $\tilde{\xi}=[\tilde{\xi}_1^{\mathrm{T}},\ldots,\tilde{\xi}_N^{\mathrm{T}}]^{\mathrm{T}}=(L_B\otimes I_n)\cdot[\xi_1^{\mathrm{T}}-q_0^{\mathrm{T}},\ldots,\xi_N^{\mathrm{T}}-q_0^{\mathrm{T}}]^{\mathrm{T}}$ and $\tilde{\eta}=[\tilde{\eta}_1^{\mathrm{T}},\ldots,\tilde{\eta}_N^{\mathrm{T}}]^{\mathrm{T}}=(L_B\otimes I_n)\cdot[\eta_1^{\mathrm{T}}-\dot{q}_0^{\mathrm{T}},\ldots,\eta_N^{\mathrm{T}}-\dot{q}_0^{\mathrm{T}}]^{\mathrm{T}}$. Then, the dynamics for the relative estimation errors can be written as

$$\begin{aligned}\dot{\tilde{\xi}} &= \tilde{\eta} - \alpha (L_B \otimes I_n)\left(\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right),\\ \dot{\tilde{\eta}} &= -\beta (L_B \otimes I_n)\left(\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]} + \tilde{F}(\tilde{\xi},\tilde{\eta})\right),\end{aligned} \tag{11}$$

of which $\tilde{F}(\tilde{\xi},\tilde{\eta}) = [f(\xi_1,\eta_1)^{\mathrm{T}} - f(q_0,\dot{q}_0)^{\mathrm{T}},\cdots,f(\xi_N,\eta_N)^{\mathrm{T}} - f(q_0,\dot{q}_0)^{\mathrm{T}}]^{\mathrm{T}}$.

Select the Lyapunov function candidate $V = V_1 + V_2$ for the error system (11):

$$\begin{aligned}V &= V_1 + V_2,\\ V_1 &= \sum_{i=1}^{N}\left(\frac{\omega_i}{1+\mu}\left\|\tilde{\xi}_i\right\|_{1+\mu}^{1+\mu} + \frac{\omega_i}{1+\nu}\left\|\tilde{\xi}_i\right\|_{1+\nu}^{1+\nu}\right),\\ V_2 &= \sum_{i=1}^{N}\left(\frac{\omega_i}{1+\mu}\|\tilde{\eta}_i\|_{1+\mu}^{1+\mu} + \frac{\omega_i}{1+\nu}\|\tilde{\eta}_i\|_{1+\nu}^{1+\nu}\right),\end{aligned} \tag{12}$$

where ω_i is defined in Lemma 2. It follows that

$$\begin{aligned}V \leqslant& \frac{\omega_{\max}}{1+\mu}\sum_{i=1}^{N}\sum_{j=1}^{n}\left(\left|\tilde{\xi}_{ij}\right|^{1+\mu} + \left|\tilde{\xi}_{ij}\right|^{1+\nu}\right.\\ &\left.+ \left|\tilde{\eta}_{ij}\right|^{1+\mu} + \left|\tilde{\eta}_{ij}\right|^{1+\nu}\right) \triangleq \frac{\omega_{\max}}{1+\mu} I_0,\end{aligned} \tag{13}$$

where $\omega_{\max} = \max\{\omega_i\}$, $\tilde{\xi}_i = [\tilde{\xi}_{i1},\ldots,\tilde{\xi}_{in}]^{\mathrm{T}}$, and $\tilde{\eta}_i = [\tilde{\eta}_{i1},\ldots,\tilde{\eta}_{in}]^{\mathrm{T}}$.

The derivative of V_1 can be computed as

$$\begin{aligned}\dot{V}_1 =& \left(\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right)^{\mathrm{T}}(\Omega\otimes I_n)\dot{\tilde{\xi}}\\ =& \left(\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right)^{\mathrm{T}}\tilde{\eta} - \alpha\left(\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right)^{\mathrm{T}}\hat{L}\left(\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right)\\ \leqslant& \frac{\alpha\hat{\lambda}_{\min}}{2}\left\|\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right\|^2 + \frac{1}{2\alpha\hat{\lambda}_{\min}}|\tilde{\eta}|^2\\ &- \alpha\hat{\lambda}_{\min}\left\|\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right\|^2\\ \leqslant& -\frac{\alpha\hat{\lambda}_{\min}}{2}\left\|\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right\|^2 + \frac{1}{2\alpha\hat{\lambda}_{\min}}\|\tilde{\eta}\|^2.\end{aligned} \tag{14}$$

of which $\Omega = \mathrm{diag}(\omega_1,\cdots,\omega_N)$, $\hat{\lambda}_{\min} > 0$ denotes the minimum eigenvalue of $\hat{L} = \frac{1}{2}(\Omega L_B + L_B^{\mathrm{T}}\Omega)$. Similarly, one has

$$\begin{aligned}\dot{V}_2 =& \left(\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right)^{\mathrm{T}}(\Omega\otimes I_n)\dot{\tilde{\eta}}\\ =& -\beta\left(\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right)^{\mathrm{T}}\hat{L}\left(\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right)\\ &+ \left(\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right)^{\mathrm{T}}(\Omega L_B\otimes I_n)\tilde{F}(\tilde{\xi},\tilde{\eta})\\ \leqslant& -\beta\hat{\lambda}_{\min}\left\|\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right\|^2\\ &+ \frac{\beta\hat{\lambda}_{\min}}{2}\left\|\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right\|^2 + \frac{\bar{\lambda}_{\max}}{2\beta\hat{\lambda}_{\min}}\left\|\tilde{F}(\tilde{\xi},\tilde{\eta})\right\|^2\\ =& -\frac{\beta\hat{\lambda}_{\min}}{2}\left\|\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right\|^2 + \frac{\bar{\lambda}_{\max}}{2\beta\hat{\lambda}_{\min}}\left\|\tilde{F}(\tilde{\xi},\tilde{\eta})\right\|^2,\end{aligned} \tag{15}$$

where $\bar{\lambda}_{\max} > 0$ is the maximum eigenvalue of $\Omega L_B L_B^{\mathrm{T}}\Omega$. Noticing that

$$\begin{aligned}\left\|\tilde{F}(\tilde{\xi},\tilde{\eta})\right\|^2 =& \sum_{i=1}^{N}\|f(\xi_i,\eta_i) - f(q_0,\dot{q}_0)\|^2\\ \leqslant& 2l^2\sum_{i=1}^{N}\left(\|\xi_i - q_0\|^2 + \|\eta_i - \dot{q}_0\|^2\right)\\ \leqslant& \frac{2l^2}{\check{\lambda}_{\min}}\left(\left\|\tilde{\xi}\right\|^2 + \|\tilde{\eta}\|^2\right),\end{aligned} \tag{16}$$

where $\check{\lambda}_{\min}$ denotes the minimum eigenvalue of $L_B^{\mathrm{T}}L_B$.

Since $\mu+\nu=2$, it is not difficult to see that $\|x^{[\mu]} + x^{[\nu]}\|^2 \geqslant 4\|x\|^2$, $\forall x\in R^n$. Then, combining eqs. (14)–(16), one has

$$\begin{aligned}\dot{V} \leqslant& -\left(\frac{\alpha\hat{\lambda}_{\min}}{2} - \frac{\bar{\lambda}_{\max}l^2}{\beta\hat{\lambda}_{\min}\check{\lambda}_{\min}}\right)\left\|\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right\|^2\\ &- \left(\frac{\beta\hat{\lambda}_{\min}}{2} - \frac{1}{2\alpha\hat{\lambda}_{\min}} - \frac{\bar{\lambda}_{\max}l^2}{\beta\hat{\lambda}_{\min}\check{\lambda}_{\min}}\right)\left\|\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right\|^2.\end{aligned}$$

Choose $\beta \geqslant \alpha > \sqrt{\frac{1}{\hat{\lambda}_{\min}^2} + \frac{2\bar{\lambda}_{\max}l^2}{\hat{\lambda}_{\min}^2\check{\lambda}_{\min}}}$, then $\gamma = \min\left\{\frac{\alpha\hat{\lambda}_{\min}}{2} - \frac{\bar{\lambda}_{\max}l^2}{\beta\hat{\lambda}_{\min}\check{\lambda}_{\min}}, \frac{\beta\hat{\lambda}_{\min}}{2} - \frac{1}{2\alpha\hat{\lambda}_{\min}} - \frac{\bar{\lambda}_{\max}l^2}{\beta\hat{\lambda}_{\min}\check{\lambda}_{\min}}\right\} > 0$. It follows that

$$\dot{V} \leqslant -\gamma\left(\left\|\tilde{\xi}^{[\mu]} + \tilde{\xi}^{[\nu]}\right\|^2 + \left\|\tilde{\eta}^{[\mu]} + \tilde{\eta}^{[\nu]}\right\|^2\right) \triangleq -\gamma I_1. \tag{17}$$

Here, I_1 can be further expressed as

$$\begin{aligned}I_1 =& \sum_{i=1}^{N}\sum_{j=1}^{n}\left(\left|\tilde{\xi}_{ij}\right|^{2\mu} + \left|\tilde{\xi}_{ij}\right|^{2\nu} + 2\left|\tilde{\xi}_{ij}\right|^2\right.\\ &\left.+ \left|\tilde{\eta}_{ij}\right|^{2\mu} + \left|\tilde{\eta}_{ij}\right|^{2\nu} + 2\left|\tilde{\eta}_{ij}\right|^2\right).\end{aligned}$$

One the one hand, according to Lemma 5 and $\frac{2}{\nu+1} < 1$, one can estimate I_0 as

$$I_0^{\frac{2}{\nu+1}} \leqslant \sum_{i=1}^{N}\sum_{j=1}^{n}\left(\left|\tilde{\xi}_{ij}\right|^{\frac{2(1+\mu)}{\nu+1}} + \left|\tilde{\xi}_{ij}\right|^2 + \left|\tilde{\eta}_{ij}\right|^{\frac{2(1+\mu)}{\nu+1}} + \left|\tilde{\eta}_{ij}\right|^2\right).$$

Noticing $2\mu < \frac{2(1+\mu)}{\nu+1} < 2\nu$, one can obtain that for any scale $a \geqslant 0$, it holds $a^{\frac{2(1+\mu)}{\nu+1}} \leqslant a^{2\nu} + a^{2\mu}$. It follows that

$$I_0^{\frac{2}{\nu+1}} \leqslant I_1. \tag{18}$$

On the other hand, by invoking Lemma 5 with $\frac{2\nu}{\nu+1} > 1$, one can estimate I_0 as

$$\begin{aligned}&(4nN)^{1-\frac{2\nu}{\nu+1}} I_0^{\frac{2\nu}{\nu+1}}\\ \leqslant& \sum_{i=1}^{N}\sum_{j=1}^{n}\left(\left|\tilde{\xi}_{ij}\right|^{2\nu\frac{\mu+1}{\nu+1}} + \left|\tilde{\xi}_{ij}\right|^{2\nu} + \left|\tilde{\eta}_{ij}\right|^{2\nu\frac{\mu+1}{\nu+1}} + \left|\tilde{\eta}_{ij}\right|^{2\nu}\right).\end{aligned}$$

With simple calculation, one has $2\mu < 2\nu\frac{\mu+1}{\nu+1} < 2$. Then, for any scale $a \geqslant 0$, it holds $a^{2\nu\frac{\mu+1}{\nu+1}} \leqslant a^{2\mu} + a^2$. Hence,

$$(4nN)^{1-\frac{2\nu}{\nu+1}} I_0^{\frac{2\nu}{\nu+1}} \leqslant I_1. \tag{19}$$

Combining eqs. (13), (17), (18) and (19), one can obtain that

$$\dot{V} \leqslant -\frac{\gamma}{2}\left(\frac{1+\mu}{\omega_{\max}}V\right)^{\frac{2}{1+\nu}} - \frac{\gamma}{2}(4nN)^{1-\frac{2\nu}{\nu+1}}\left(\frac{1+\mu}{\omega_{\max}}V\right)^{\frac{2\nu}{1+\nu}}.$$

Since $0 < \frac{2}{1+\nu} < 1 < \frac{2\nu}{1+\nu}$, according to Lemma 3, V reaches zero in fixed time, i.e., ξ_i and η_i can respectively estimate the exact values of q_0 and $\dot{q}_0$ in fixed time, and the fixed time can be bounded as $T_1 \leqslant \frac{2(\nu+1)}{\gamma(\nu-1)}\left[(4nN)^{\frac{\nu-1}{\nu+1}}\left(\frac{\omega_{\max}}{1+\mu}\right)^{\frac{2\nu}{\nu+1}} + \left(\frac{\omega_{\max}}{1+\mu}\right)^{\frac{2}{\nu+1}}\right]$. It is not difficult to see that the upper bound of T_1 can be reduced by increasing γ, which can be achieved by enlarging the values of α and β according to the definition of γ above eq. (17).

Remark 1 In the observer design, ν is chosen as $2-\mu$ to simplify the proof. However, ν can be any positive number larger than 1.

After the design of distributed fixed-time observer for the leader, it is time to present the local fixed-time tracking control design.

Step 2 Local fixed-time tracking control design.

After time T_1, one has $\xi_i = q_0$ and $\eta_i = \dot{q}_0$. Hence, after T_1 the relative state information $e_i = q_i - q_0$ and $\bar{e}_i = \dot{e}_i = \dot{q}_i - \dot{q}_0$ can be utilized in the local tracking control design.

Consider the following integral sliding mode variable:

$$\begin{aligned}\dot{r}_i &= k_1 e_i^{[\rho_1]} + k_2 \bar{e}_i^{[\rho_2]} + \bar{k}_1 e_i^{[\bar{\rho}_1]} + \bar{k}_2 \bar{e}_i^{[\bar{\rho}_2]},\\ s_i &= \bar{e}_i + r_i,\end{aligned}$$

where k_i, $\bar{k}_i$, ρ_i, and $\bar{\rho}_i$ (i=1,2) are chosen as in Lemma 4. It is easy to see that $\dot{s}_i = 0$ is equivalent to $\ddot{e}_i = -\left(k_1 e_i^{[\rho_1]} + k_2 \bar{e}_i^{[\rho_2]} + \bar{k}_1 e_i^{[\bar{\rho}_1]} + \bar{k}_2 \bar{e}_i^{[\bar{\rho}_2]}\right)$. According to Lemma 4, if $\dot{s}_i = 0$, then e_i and $\bar{e}_i$ will reach zero in fixed time. With this observation, one has the following result.

Theorem 2 Suppose that Assumptions 1 and 4 hold. Then, multiple Euler-Lagrangian system (6) can track the virtual signal (7) in fixed time under the following control input:

$$\begin{aligned}\tau_i =& G_i - C_i(r_i - \eta_i) - M_i(\dot{r}_i - f(\xi_i, \eta_i))\\ &- D_i \mathrm{sign}(s_i) - \alpha_i s_i^{[\mu_i]} - \beta_i s_i^{[\nu_i]},\end{aligned} \tag{20}$$

with α_i, $\beta_i > 0$ and $0 < \mu_i < 1 < \nu_i$, $i = 1, \cdots, N$.

Proof. For each follower, consider the Lyapunov function candidate $\bar{V}_i = s_i^{\mathrm{T}} M_i s_i$. Then $\bar{V}_i \leqslant \bar{k}_m \|s_i\|^2$. Taking the derivative of $\bar{V}_i$:

$$\begin{aligned}\dot{\bar{V}}_i =& 2 s_i^{\mathrm{T}} M_i \dot{s}_i + s_i^{\mathrm{T}} \dot{M}_i s_i\\ =& 2 s_i^{\mathrm{T}} M_i(\dot{r}_i + \ddot{q}_i - \ddot{q}_0) + s_i^{\mathrm{T}} \dot{M}_i s_i\\ =& 2 s_i^{\mathrm{T}} M_i(\dot{r}_i - \ddot{q}_0) + 2 s_i^{\mathrm{T}}(\tau_i + d_i - G_i - C_i\dot{q}_i) + s_i^{\mathrm{T}} \dot{M}_i s_i.\end{aligned}$$

Noticing $\dot{M}_i - 2C_i$ is skew-symmetric, one has

$$\dot{\bar{V}}_i \leqslant 2 s_i^{\mathrm{T}} M_i(\dot{r}_i - \ddot{q}_0) + 2 s_i^{\mathrm{T}}\left[\tau_i + d_i + C_i(s_i - \dot{q}_i)\right].$$

Taking eq. (20) into consideration and noticing the fact that after time T_1 it holds $\xi_i = q_0$ and $\eta_i = \dot{q}_0$, one further has

$$\begin{aligned}\dot{\bar{V}}_i \leqslant& -2\alpha_i \|s_i\|_{\mu_i+1}^{\mu_i+1} - 2\beta_i \|s_i\|_{\nu_i+1}^{\nu_i+1}\\ \leqslant& -2\alpha_i\left(\frac{\bar{V}_i}{\bar{k}_m}\right)^{\frac{\mu_i+1}{2}} - 2\beta_i n^{1-\frac{\nu_i+1}{2}}\left(\frac{\bar{V}_i}{\bar{k}_m}\right)^{\frac{\nu_i+1}{2}}.\end{aligned} \tag{21}$$

According to Lemma 3, after T_1, $\bar{V}_i$ reaches zero in fixed time, i.e., s_i reaches zero in fixed time, and the convergence time is bounded as

$$T_{2i} \leqslant \frac{\bar{k}_m^{\frac{\mu_i+1}{2}}}{\alpha_i(1-\mu_i)} + \frac{\bar{k}_m^{\frac{\nu_i+1}{2}}}{\beta_i(\nu_i-1)}. \tag{22}$$

Similar to T_1, one can also reduce T_{2i} by increasing the values of α_i and β_i.

To this end, one can conclude that after T_1, each follower can estimate the accurate state information q_0 and $\dot{q}_0$ of the virtual signal; after $T_1 + T_{2i}$, the sliding surface $s_i = 0$ is reached which means that $\dot{s}_i = 0$ is also reached after $T_1 + T_{2i}$; by invoking Lemma 3, after $T_1 + \max\{T_{2i} : i\} + T_3$ with $T_3 = \frac{\rho_2 \lambda_{\max}^{\frac{1}{\rho_2}}(P)}{(1-\rho_2)\lambda_{\min}(Q)} + \frac{\bar{\rho}_2 \lambda_{\max}^{\frac{1}{\bar{\rho}_2}}(\bar{P})}{(\bar{\rho}_2-1)\lambda_{\min}(\bar{Q})}$, each follower can finally track the virtual signal. As discussed above, T_1 and T_{2i} can be decreased by adjusting some control parameters. With regards to T_3, notice that $PA + A^{\mathrm{T}}P = -Q$ and $\bar{P}\bar{A} + \bar{A}^{\mathrm{T}}\bar{P} = -\bar{Q}$. If one increases the real part of the eigenvalues of $-A$ and $-\bar{A}$ by selecting k_i and $\bar{k}_i$, then both $\frac{\lambda_{\max}^{\frac{1}{\rho_2}}(P)}{\lambda_{\min}(Q)}$ and $\frac{\lambda_{\max}^{\frac{1}{\bar{\rho}_2}}(\bar{P})}{\lambda_{\min}(\bar{Q})}$ can be decreased, which leads to the decrease of T_3.

Remark 2 During the time interval $[0, T_1]$, the boundedness of q_i and $\dot{q}_i$ are not discussed. In fact, we can apply the following local control law for each follower during $[0, T_1]$: $\tau_i = G_i - q_i - k\dot{q}_i$ with k being a positive number. For this control law, choose the Lyapunov function candidate $\tilde{V}_i = \frac{1}{2}q_i^{\mathrm{T}} q_i + \frac{1}{2}\dot{q}_i^{\mathrm{T}} M_i \dot{q}_i$. Taking the derivative of $\tilde{V}_i$ shows $\dot{\tilde{V}}_i = q_i^{\mathrm{T}}\dot{q}_i + \frac{1}{2}\dot{q}_i^{\mathrm{T}}\dot{M}_i\dot{q}_i + \dot{q}_i^{\mathrm{T}} M \ddot{q}_i = q_i^{\mathrm{T}}\dot{q}_i + \frac{1}{2}\dot{q}_i^{\mathrm{T}}\dot{M}_i\dot{q}_i + \dot{q}_i^{\mathrm{T}}(d_i + G_i - q_i - k\dot{q}_i - G_i - C_i\dot{q}_i) \leqslant -\|\dot{q}_i\|(k\|\dot{q}_i\| - D_i) \leqslant \frac{D_i^2}{2} + \frac{\|\dot{q}_i\|^2}{2} \leqslant \frac{D_i^2}{2} + \frac{\tilde{V}_i}{k_m}$. With simple calculation, one can show that $\tilde{V}_i(t) \leqslant \left(\frac{D_i^2}{2} + \frac{\tilde{V}_i(0)}{k_m}\right)e^{\frac{t}{k_m}} - \frac{D_i^2}{2}$, which indicates that $\tilde{V}_i$ is bounded during $[0, T_1]$. After the fixed time T_1, the local control switches to the distributed control (20).

4 Simulation example

In this section, one example is conducted to verify the theoretical results. Consider one virtual signal labeled 0, and 4 followers labeled from 1 to 4. The communication network is illustrated in Figure 1 with all the weights indicated on the edges.

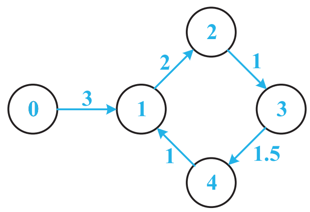

Figure 1 Communication network with 1 leader and 4 followers.

Suppose the leader has the following dynamics:

$$\ddot{q}_0 = f(q_0, \dot{q}_0) = A(q_0 + \dot{q}_0), \tag{23}$$

where $q_0 = [q_{01}, q_{02}]^{\mathrm{T}}$ and $A = -\begin{bmatrix} \frac{3}{7} & \frac{5}{14} \\ -\frac{5}{7} & \frac{4}{7} \end{bmatrix}$. Under simple calculation, one can obtain that

$$\left\|f(y,z) - f(y',z')\right\|^2 = \left\|A(y-y') + A(z-z')\right\|^2$$
$$\leqslant 2\left\|A\right\|^2 \left(\left\|y-y'\right\|^2 + \left\|z-z'\right\|^2\right).$$

Hence the Lipschitz constant defined in eq. (8) can be taken as $l = \sqrt{2}\left\|A\right\| = 1.8502$. The followers are two-link robot arms of dynamics (6) with:

$$M_i(q_i) = \begin{pmatrix} m_{i1} + m_{i2} + 2m_{i3}\cos q_{i2} & m_{i2} + m_{i3}\cos q_{i2} \\ m_{i2} + m_{i3}\cos q_{i2} & m_{i2} \end{pmatrix},$$

$$C_i(q_i, \dot{q}_i) = -\begin{pmatrix} m_{i3}(\sin q_{i2})\dot{q}_{i2} & m_{i3}(\sin q_{i2})(\dot{q}_{i1} + \dot{q}_{i2}) \\ -m_{i3}(\sin q_{i2})\dot{q}_{i1} & 0, \end{pmatrix},$$

$$G_i(q_i) = \begin{pmatrix} m_{i4}\cos q_{i1} + m_{i5}g\cos(q_{i1} + q_{i2}) \\ m_{i5}\cos(q_{i1} + q_{i2}) \end{pmatrix},$$

$$d_i = \begin{pmatrix} \sin(it + \pi/4) \\ 1 - \cos((3-i)t + \pi/6) \end{pmatrix},$$

where $q_i = [q_{i1}, q_{i2}]^{\mathrm{T}}$, and $\Sigma_i = [m_{i1}, ..., m_{i5}]^{\mathrm{T}}$ are system parameters, $i = 1, ..., 4$. Choose the system parameters for the followers as $\Sigma_1 = [0.7, 1.5, 0.3, 0.5, 0.2]^{\mathrm{T}}$, $\Sigma_2 = [0.8, 1.3, 0.2, 0.3, 0.4]^{\mathrm{T}}$, $\Sigma_3 = [0.5, 1.6, 0.4, 0.4, 0.3]^{\mathrm{T}}$, $\Sigma_4 = [0.6, 1.8, 0.1, 0.6, 0.1]^{\mathrm{T}}$. Set the initial values as $q_0(0) = [2, -1]^{\mathrm{T}}$, $q_1(0) = [4, 6]^{\mathrm{T}}$, $q_2(0) = [2, 1]^{\mathrm{T}}$, $q_3(0) = [8, 9]^{\mathrm{T}}$, $q_4(0) = [9, 5]^{\mathrm{T}}$, $\dot{q}_i(0) = [1, 3]^{\mathrm{T}}$, $i = 1, ..., 4$. In the observer design, choose the parameters as $\alpha = \beta = 10$, $\mu = 0.7$, and $\nu = 1.3$. Furthermore, the parameters in the controller design are chosen as $\alpha_i = \beta_i = 1$, $\mu_i = 0.3$, $\nu_i = 3.1$, $k_1 = \bar{k}_1 = 1$, $k_2 = \bar{k}_2 = 2$, $\rho_2 = 0.97$, $\rho_1 = \frac{\rho_2}{2-\rho_2}$, $\bar{\rho}_2 = 1.03$, and $\bar{\rho}_1 = \frac{\bar{\rho}_2}{2-\bar{\rho}_2}$, $i = 1, ..., 4$. Figures 2 and 3 are the estimation errors of $\xi_i - q_0$ and $\eta_i - \dot{q}_0$, respectively. One can see that the observer eq. (10) can estimate the accurate state information q_0 and $\dot{q}_0$ in fixed time. Figures 4 and 5 depict the state and velocity trajectories of the virtual signal and 4 followers, which show that all the followers finally can track the virtual signal in fixed time.

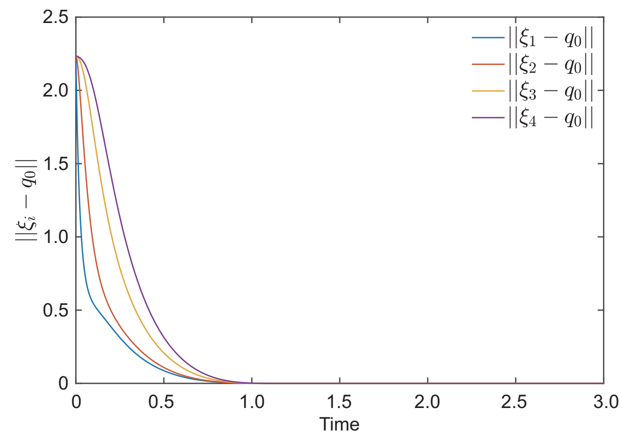

Figure 2 The norm of the estimation error $\xi_i - q_0$, $i = 1, ..., 4$.

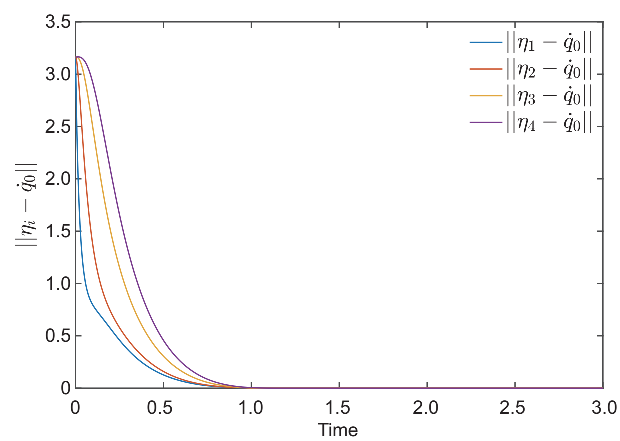

Figure 3 The norm of the estimation error $\eta_i - \dot{q}_0$, $i = 1, ..., 4$.

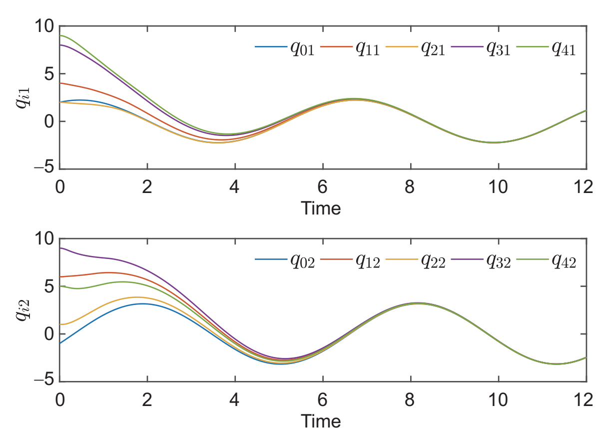

Figure 4 The state trajectories q_i of the leader and 4 followers.

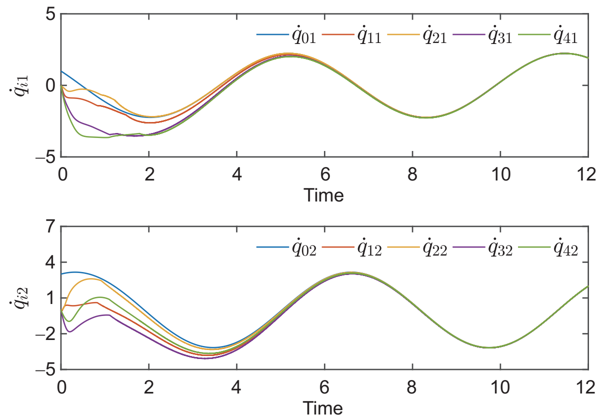

Figure 5 The velocity trajectories $\dot{q}_i$ of the leader and 4 followers.

5 Conclusions

This paper mainly investigated the distributed tracking consensus problem for networked Euler-Lagrangian systems, where the leader is subject to nonlinear dynamics, and the followers are influenced by bounded disturbances. First of all, a class of distributed observer was designed to cooperatively estimate the state of the nonlinear leader in fixed time. Then, a local control protocol based on integral sliding mode was constructed for each follower to track the observer in fixed time. Further research directions include fixed-time leaderless consensus problem for time-delayed networked Euler-Lagrangian systems and fully distributed fixed-time consensus problem for networked Euler-Lagrangian systems with general directed communication networks.

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