Telecommunications and RF

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1 Telecommunications Systems

Telecommunication is the transmission of information over a distance through some technology.
Telecommunications systems are designed to transmit information with as little **deterioration** as possible while satisfying design constraints, such as allowable transmittable energy and signal bandwidth.
Signal deterioration commonly measures:

- For Analog Systems: **Signal-to-Noise Ratio** (SNR) at the receiver output — the ratio of the signal power to the noise power
- For Digital Systems: **Bit Error Rate** (BER) at the receiver output — the ratio of the number of bits received in error to the total number of bits transmitted

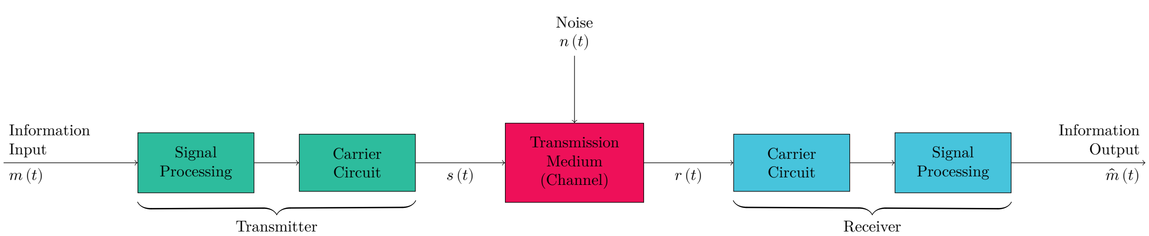
1.1 Analog and Digital Signals

A system is either **analog** or **digital** based on the possible amplitudes of waveforms it can handle.

- **Analog Information Sources** produce values defined on a continuum:
  - Human voice and other sounds
- **Digital Information Sources** produce a finite set of possible symbols:
  - Computer data
  - MP3 encoder output

1.2 Communications System

A communications system can be summarised by the following block diagram:



where

- $m(t)$ is the information message signal (prior to conditioning for transmission)
- $s(t)$ is the conditioned signal for transmission
- $n(t)$ contains channel noise and interference
- $r(t)$ is the received signal
- $\hat{m}(t)$ is the reconstructed received message signal (where $\hat{m}(t)$ usually approximates $m(t)$)

1.2.1 Transmitter

The transmitter carries out **signal conditioning** by transforming the signal to a more appropriate form before transmission through the channel. Some examples of signal conditioning techniques are prodived below.

- **Low Pass Filtering** (LPF) restricts signal bandwidth to avoid wasting signal power on frequencies that are not transmitted by the channel and to avoid interference with other signals
- **Analog to Digital Conversion** (ADC) produces a digital word which represents a sample of the analog message waveform
- **Carrier Modulation** transfers the signal to a frequency band that is suitable for transmission through the channel

1.2.2 Channel

A communication channel refers to the physical medium that carries the signal from the transmitter to the receiver. There are two types of channels:

- **Wired**: twisted pair copper telephone lines, waveguides, coaxial cables, fibre-optic cables
- **Wireless**: air, vacuums, sea water, optical fibres

General principles of communications always apply regardless of the type of channel. However, certain conditioning methods are better suited to certain channels.
Channels often **attenuate** signals (reduce their amplitude or strength) through

- random noise
- interference from other sources

and therefore it is a key consideration in the design of a communications system.

1.2.3 Receiver

The receiver acts as the inverse of a transmitter. The receiver:

1. **Demodulates** the received signal by stripping the carrier from the received signal $r\left(t\right)$.
2. **Filters** out noise and interference from the demodulated signal.
3. **Reconstructs** an estimate of the original message signal $\hat{m}\left(t\right)$.

Due to the finite nature of the SNR, the estimated output of an analog signal can never be exactly equal to the original signal[1]. However, it is often possible to reconstruct a digital signal exactly using error detection and correction techniques at the receiver.

[1] A perfect reconstruction requires an infinite SNR which is impractical.

1.2.4 Information Sources

As discussed before, an information source can be classified as either **analog** or **digital**. Analog signals can be modulated or transmitted directly, or converted to digital data and transmitted using digital modulation techniques.
An analogue signal to be transmitted is called the **message signal** and is denoted $m(t)$. The spectral components of this signal lie within a finite bandwidth W, such that $M(f) = 0$ for $|f| > W$, where $M(f)$ is the Fourier Transform of $m(t)$. This signal's bandwidth is limited to prevent interference with other signals.
Many kinds of message signals can be considered:

- Audio
- Video
- Computer data
- Telemetry (measurements)
- Soundings (RADAR, SONAR)
- A mixure of the above (i.e., data over voice)

1.3 Modulation

The process of modulation produces a signal that is suitable for transmission through the channel by transforming the message signal $m(t)$ to a new signal $s(t)$.
Modulation is often performed with respect to another signal, called the **carrier** signal $c(t)$. Here the message *modulates* the carrier to produce the transmitted signal $s(t)$.

1.3.1 Benefits of Modulation

- Modulation shifts the spectral content of a message onto a suitable band. As the size of an antenna is related to the wavelength of a signal, higher carrier frequencies require smaller antennas.
- Modulation facilitates multiplexing, where multiple signals are transmitted over the same spectrum. This is because the frequency spectrum can be divided into non-overlapping frequency bands, each of which can carry a separate signal.
- Modulation provides some control over noise and interference by choosing a bandwidth that is smaller than the allocated channel bandwidth.

1.3.2 Convolution and Modulation

Consider two time domain signals $m(t)$ and $c(t)$ and their Fourier Transforms $M(f)$ and $C(f)$ respectively.
The **Convolution Property** demonstrates:

$$
\begin{aligned}
m(t) * c(t) &= y(t) && \text{Convolution in Time Domain} \\
M(f)\,C(f) &= Y(f) && \text{Multiplication in Frequency Domain}
\end{aligned}
$$

The **Modulation property** demonstrates:

$$m(t)\,c(t) = y(t) \qquad \text{Multiplication in Time Domain}$$
$$M(f) * C(f) = Y(f) \qquad \text{Convolution in Frequency Domain}$$

1.3.3 Carrier Signal

The carrier signal $c(t)$ is a sinusoidal signal of the form:

$$c(t) = A_c \cos(2\pi f_c t + \phi)$$

where A_c is the amplitude, f_c is the frequency and ϕ is the phase of the carrier signal.

1.3.4 Signal Properties

The **energy** of a signal $x(t)$ is defined as:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \,\mathrm{d}t$$

The rate at which energy is transmitted is called the **power** of the signal, and is defined as:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \,\mathrm{d}t$$

where T is the time period over which the power is measured.

1.4 Amplitude Modulation Schemes

A message signal $m(t)$ with bandwidth W can be amplitude modulated by mixing (multiplying) the signal with a carrier signal $c(t)$:

$$c(t) = A_c \cos(2\pi f_c t)$$

with amplitude A_c and carrier frequency $f_c \gg W$.
This section will consider:

- Double Side Band — Suppressed Carrier (DSB-SC) modulation
- Double Side Band — Full Carrier (DSB-FC) modulation, or simply conventional Amplitude Modulation (AM)
- Single Side Band (SSB) modulation

1.4.1 Double Side Band — Suppressed Carrier (DSB-SC)

The transmitted signal is defined as:

$$s(t) = m(t)\,c(t) = A_c m(t) \cos(2\pi f_c t).$$

In the frequency domain, the message signal is shifted to the carrier frequency at $\pm f_c$ and the magnitude is halved.

$$S(f) = \frac{1}{2} A_c \left[M(f - f_c) + M(f + f_c) \right]$$

Assuming no channel noise, the received signal can be represented as:

$$r(t) = s(t) = A_c m(t) \cos(2\pi f_c t)$$

This signal can be demodulated coherently by multiplying a sinusoidal signal of the same frequency as the carrier. Suppose we generate the sinusoidal signal $\cos(2\pi f_c t + \phi)$ where ϕ is the phase of the sinusoid.

$$\begin{aligned} y(t) &= r(t) \cos(2\pi f_c t + \phi) \\ &= A_c m(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) \\ &= \frac{1}{2} A_c m(t) \left[\cos(4\pi f_c t + \phi) + \cos(-\phi) \right] \\ &= \underbrace{\frac{1}{2} A_c m(t) \cos(\phi)}_{\text{filter out}} + \underbrace{\frac{1}{2} A_c m(t) \cos(4\pi f_c t + \phi)}_{\text{reject}} \end{aligned}$$

As the frequency content of the message signal $m(t)$ is limited to W, a lowpass filter can be used to remove the high frequency component centred at $2f_c$. The output of this filter is then

$$\hat{m}(t) = \frac{1}{2} A_c m(t) \cos(\phi).$$

Note that $m(t)$ is multiplied by $\cos(\phi)$, meaning the power of the message signal is reduced by a factor of $\cos^2(\phi)$. It is therefore important to choose ϕ such that $\cos(\phi)$ is as close to 1 as possible. This demonstrates a need for a phase-coherent or synchronous demodulator, i.e., the phase of the locally generated sinusoid should be identical to the received carrier signal.

1.4.2 Double Side Band — Full Carrier (DSB-FC)

This scheme is similar to DSB-SC, however the carrier is sent along with the modulated signal. We begin by defining an envelope signal $g(t)$ by amplifying and biasing the message signal, so that:

$$g(t) = 1 + \mu m_n(t)$$

where the **modulation index** $0 < \mu \leqslant 1$ is chosen to ensure $g(t) > 0$. $m_n(t)$ is the normalised message signal defined as

$$m_n(t) = \frac{m(t)}{\max|m(t)|}$$

such that $|m_n(t)| \leqslant 1$. The transmitted signal is then defined as:

$$s(t) = g(t)\,c(t) = A_c\left[1 + \mu m_n(t)\right]\cos\left(2\pi f_c t\right).$$

In the frequency domain, we observe similar behaviour, with impulses at $\pm f_c$.

$$S(f) = \frac{1}{2}A_c\mu\left[M(f - f_c) + M(f + f_c)\right] + \frac{1}{2}A_c\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

The modulation index can be determined from the AM signal using the following equation:

$$\mu = \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}} = \frac{\max|m(t)|}{\max|c(t)|}$$

where A_{max} and A_{min} are the maximum and minimum amplitudes of the envelope signal $g(t)$. This value is chosen to be as close to 1 as possible to maximise the power of the transmitted signal. Increasing μ beyond 1 overmodulates the signal, resulting in a full-wave rectified version of the signal.

Modulation efficiency is the percentage of the total power of the modulated signal that conveys information

$$\eta = \frac{\text{sideband power}}{\text{total power}} \times 100\%$$

As the total power comprises of both the sideband power and carrier power, the DSB-FC scheme is very inefficient. This is because the carrier signal does not contain any useful information, and therefore the power of the carrier is wasted. This can be seen when transmitting a sinusoidal message signal,

- Carrier power: $\frac{1}{2}A_c^2$
- Sideband power: $\frac{1}{4}A_c^2\mu^2$
- Total power: $\frac{1}{2}A_c^2\left(1 + \frac{1}{2}\mu^2\right)$
- Modulation efficiency: $\frac{\frac{1}{4}A_c^2\mu^2}{\frac{1}{2}A_c^2\left(1+\frac{1}{2}\mu^2\right)} = \frac{\mu^2}{2+\mu^2}$

The maximum efficiency is achieved when $\mu = 1$, i.e., which is 33%. This scheme is often used in AM medium-wave (MW) systems.

To demodulate a DSB-FC signal, we can use an envelope detector. In this circuit, we need to calculate the minimum and maximum frequency that the tuned circuit can respond to:

$$f_0 = \frac{1}{2\pi\sqrt{LC_1}}$$

and calculate the break frequency of the lowpass filter:

$$f_b = \frac{1}{2\pi RC_2}$$

1.4.3 Single Side Band (SSB)

As the double side band schemes produce a signal with twice the bandwidth of the message signal, consider either the lower or upper sidebands of the modulated signal. We can attempt to use a filter to remove the unwanted sideband, however there are two issues with this approach:

1. The filter is particularly difficult to implement when $m(t)$ has a large concentration of power close to $f = 0$
2. The filter must have a very sharp cutoff in the vicinity of the carrier frequency

Instead, we can use a phase shift method through the use of a Hilbert transform. The Hilbert transform of a signal $m(t)$ is defined as

$$\hat{m}(t) = m(t) * \frac{1}{\pi t}$$

The result of this operation is more easily understood in the frequency domain.

$$\hat{M}(f) = -j\,\mathrm{sgn}(f) M(f) = \begin{cases} -jM(f) & f > 0 \\ jM(f) & f < 0 \end{cases}$$

so that negative frequency components of m are phase shifted by $+90^\circ$ $(+\pi/2)$ and positive frequency components are phase shifted by -90° $(-\pi/2)$. The magnitude remains unchanged.
Using this transform, we can define SSB signals as:

$$\begin{aligned} s_{\text{USB}}(t) &= A_c \left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right] \\ s_{\text{LSB}}(t) &= A_c \left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right] \end{aligned}$$

or compactly,

$$s(t) = m(t)\,c(t) \pm \hat{m}(t)\,\hat{c}(t)$$

These signals can be demodulated coherently using the same process as the DSB-SC scheme. Again assuming that there is no channel noise,

$$r(t) = s(t) = A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t)$$

$$\begin{aligned} y(t) &= r(t)\cos(2\pi f_c t + \phi) \\ &= \left[A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t + \phi) \\ &= A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) \pm A_c \hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \phi) \\ &= \frac{1}{2}A_c m(t)\left[\cos(\phi) + \cos(4\pi f_c t + \phi)\right] \pm \frac{1}{2}A_c \hat{m}(t)\left[-\sin(\phi) + \sin(4\pi f_c t + \phi)\right] \\ &= \underbrace{\frac{1}{2}A_c\left[m(t)\cos(\phi) \mp \hat{m}(t)\sin(\phi)\right]}_{\text{filter out}} + \underbrace{\frac{1}{2}A_c\left[m(t)\cos(4\pi f_c t + \phi) \pm \hat{m}(t)\sin(4\pi f_c t + \phi)\right]}_{\text{reject}} \end{aligned}$$

This gives the output

$$\hat{m}(t) = \frac{1}{2}A_c\left[m(t)\cos(\phi) \mp \hat{m}(t)\sin(\phi)\right]$$

Note the $\hat{m}(t)$ term on the right hand side of the equation is referring to the Hilbert transform of $m(t)$, not the output of the demodulator.

2 Angle Modulation

AM suffers from poor noise performance, as amplitude variations in the received signal cannot be removed from the demodulated signal. Angle modulation schemes overcome this issue by modulating the phase or frequency of the carrier signal.

2.1 Carrier Signal

In these schemes, the message signal is modulated onto the angle $\theta\left(t\right)$ of the carrier signal:

$$\begin{aligned} s\left(t\right) &= A_c \cos\left(\theta\left(t\right)\right) \\ \theta\left(t\right) &= 2\pi f_c t + \phi\left(t\right) \end{aligned}$$

In **phase modulation** (PM), variations in the message signal are encoded into the phase:

$$\phi\left(t\right) = k_p m\left(t\right)$$

where k_p is the **phase deviation** constant.
Frequency modulation (FM) considers the frequency deviation from the modulation frequency f_c:

$$f_i - f_c = k_f m\left(t\right)$$

where f_i is the instantaneous frequency of the carrier signal and k_f is the **frequency deviation** constant. To determine the phase ϕ, consider the time derivative of the angle $\theta\left(t\right)$ with arbitrary frequency f

$$\begin{aligned} \frac{\mathrm{d}\theta\left(t\right)}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}\left(2\pi f t + \phi\right) \\ \frac{\mathrm{d}\theta\left(t\right)}{\mathrm{d}t} &= 2\pi f_i \\ \theta\left(t\right) &= 2\pi \int_0^t f_i \,\mathrm{d}\tau \\ \theta\left(t\right) &= 2\pi \int_0^t f_c + k_f m\left(\tau\right) \mathrm{d}\tau \\ \theta\left(t\right) &= 2\pi f_c t + 2\pi k_f \int_0^t m\left(\tau\right) \mathrm{d}\tau \end{aligned}$$

2.1.1 Phase Modulation

A phase modulated signal is defined as:

$$s\left(t\right) = A_c \cos\left(2\pi f_c t + k_p m\left(t\right)\right)$$

with

$$\theta\left(t\right) = 2\pi f_c t + k_p m\left(t\right)$$

where k_p is the **phase deviation** constant.

2.1.2 Frequency Modulation

A frequency modulated signal is defined as:

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,\mathrm{d}\tau\right)$$

with

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,\mathrm{d}\tau$$

where k_f is the **frequency deviation** constant.

2.1.3 Instantaneous Frequency

The instantaneous frequency f_i may be determined from these signals by considering the time derivative of θ.
In PM,

$$\begin{aligned}\frac{\mathrm{d}\theta}{\mathrm{d}t} &= 2\pi f_c + k_p \frac{\mathrm{d}}{\mathrm{d}t} m(t) \\ &= 2\pi\left(f_c + \frac{k_p}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t} m(t)\right) \implies f_i = f_c + \frac{k_p}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t} m(t)\end{aligned}$$

In FM,

$$\begin{aligned}\frac{\mathrm{d}\theta}{\mathrm{d}t} &= 2\pi f_c + 2\pi k_f m(t) \\ &= 2\pi\left(f_c + k_f m(t)\right) \implies f_i = f_c + k_f m(t)\end{aligned}$$

2.1.4 Modulation Index

The modulation index β is defined as the ratio of the maximum frequency deviation to the modulating frequency of the message signal. In PM, if we rewrite the phase $\phi(t)$ as,

$$\begin{aligned}\phi(t) &= k_p \max|m(t)| \frac{m(t)}{\max|m(t)|} \\ &= \beta_p m_n(t)\end{aligned}$$

then β_p is the modulation index for PM. By definition, FM has a modulation index of $\beta_f = \frac{k_f \max|m(t)|}{B}$ where B is the bandwidth of the message signal.

2.1.5 Maximum Phase and Frequency Deviation

The maximum phase deviation Δp and maximum frequency deviation Δf are defined as:

$$\Delta p = k_p \max|m(t)| \qquad \Delta f = k_f \max|m(t)|$$

2.1.6 Carson's Rule

Phase and frequency modulation generally expand the bandwidth of a signal. The modulated signals bandwidth W may be approximated by Carson's rule:

$$W = 2B(\beta + 1)$$

2.1.7 Summary of Angle Modulation Definitions

	Phase Modulation	**Frequency Modulation**
Modulated Signal $s(t)$	$A_c \cos\left(2\pi f_c t + k_p m(t)\right)$	$A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,\mathrm{d}\tau\right)$
Phase $\phi(t)$	$k_p m(t)$	$2\pi k_f \int_0^t m(\tau)\,\mathrm{d}\tau$
Instantaneous Frequency f_i	$f_c + \frac{k_p}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t} m(t)$	$f_c + k_f m(t)$
Modulation Index	$\beta_p = k_p \max\lvert m(t)\rvert$	$\beta_f = \frac{k_f \max\lvert m(t)\rvert}{B}$
Maximum Deviation	$\Delta p = k_p \max\lvert m(t)\rvert$	$\Delta f = k_f \max\lvert m(t)\rvert$

2.2 Narrowband Angle Modulation

When $\beta \ll 1$, angle modulation is referred to as narrowband angle modulation. The modulated signal can be approximated as:

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t + \phi(t)) \\ &= A_c \cos(2\pi f_c t)\cos(\phi(t)) - A_c \sin(2\pi f_c t)\sin(\phi(t)) \\ &= A_c \cos(2\pi f_c t) - A_c \phi(t)\sin(2\pi f_c t) \end{aligned}$$

This is equivalent to a DSB-FC signal with a phase modulated carrier, as the message signal is now modulated onto a sine carrier instead of cosine. The modulated signal bandwidth W is approximately $2B$.
Narrowband angle modulation (low-index angle modulation) does not provide better noise immunity than AM, and is seldom used on its own in practical communication systems.

2.3 Wideband Angle Modulation

Assuming the message signal is a sinusoidal signal, $\phi(t) = \beta \sin(2\pi f_m t)$,

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \\ &= A_c \cos(2\pi f_c t)\cos(\beta \sin(2\pi f_m t)) - A_c \sin(2\pi f_c t)\sin(\beta \sin(2\pi f_m t)) \end{aligned}$$

To simplify this expression, we can use Bessel functions. Notably

$$\cos\left(\beta\sin\left(2\pi f_m t\right)\right) = J_0\left(\beta\right) + 2\sum_{n=1}^{\infty} J_{2n}\left(\beta\right)\cos\left(2\pi\left(2n\right)f_m t\right)$$
$$\sin\left(\beta\sin\left(2\pi f_m t\right)\right) = 2\sum_{n=1}^{\infty} J_{2n-1}\left(\beta\right)\sin\left(2\pi\left(2n-1\right)f_m t\right)$$

where $J_n\left(\beta\right)$ are Bessel functions of the first kind and order n, evaluated at β.
These results can be used to show that a wideband FM modulated signal can be expressed as:

$$s\left(t\right) = \sum_{n=-\infty}^{\infty} A_c J_n\left(\beta\right)\cos\left(2\pi\left(f_c + n f_m\right)t\right)$$

so that the carrier frequency is located at $2\pi f_c$ with magnitude $A_c J_0\left(\beta\right)$, with an infinite number of sidebands at $2\pi\left(f_c \pm nfm\right)$ with magnitudes $A_c J_{\pm n}\left(\beta\right)$.
When deciding the number of sidebands n to include, consider

$$\left|J_{\pm n}\left(\beta\right)\right| \geqslant 0.1$$

For large values of β, $n \approx \beta$ is sufficient.
According to Carson's rule, $W = 2f_m\left(\beta + 1\right)$ contains at least 98% of the signal power.

2.3.1 Bessel Function Properties

The Bessel functions of the first kind $J_n\left(x\right)$ are defined as the solutions to the Bessel differential equation

$$x^2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} + \left(x^2 - n^2\right)y = 0$$

- $J_n\left(\beta\right)$ is a real function
- $J_n\left(\beta\right) = J_{-n}\left(\beta\right)$
- $\sum_{n=-\infty}^{\infty} J_n^2\left(\beta\right) = 1$

For small values of β,

- $J_0\left(\beta\right) = 1$
- $J_1\left(\beta\right) = \beta/2$
- $J_n\left(\beta\right) = 0$ for $n > 2$

2.4 Effect of Bandwidth

Rewriting Carson's rule using β,

$$W = 2B\left(\beta + 1\right) = \begin{cases} 2B\left(k_p \max\left|m\left(t\right)\right| + 1\right) & \text{PM} \\ 2\left(k_f \max\left|m\left(t\right)\right| + B\right) & \text{FM} \end{cases}$$

From these equations

- Increasing the amplitude of the modulating signal has the same effect in both PM and FM.
- Increase the message signal bandwidth B has a greater effect on the bandwidth of a PM signal than for FM.

2.5 Angle Modulator Implementation

FM can be generated using a Voltage Controlled Oscillator (VCO). A varactor diode is a capacitor whose capacitance changes with applied voltage. This capacitor can be used in the tuned circuit of an oscillator. If the message signal is applied to the varactor, the output frequency of the oscillator will change in accordance with the message signal.
The time varying capacitance of the varactor diode is given by

$$C_v(t) = C_a + k_0 m(t)$$

- When $m(t) = 0$, the frequency of the tuned circuit is given by

$$f_i = f_c = \frac{1}{2\pi\sqrt{LC_a}}$$

- When $m(t) \neq 0$, the frequency of the tuned circuit is given by

$$f_i = \frac{1}{2\pi\sqrt{L(C_a + k_0 m(t))}} = \frac{1}{2\pi\sqrt{LC_a(1 + k_0 m(t)/C_a)}} = f_c\frac{1}{\sqrt{1 + k_0 m(t)/C_a}}$$

Narrowband FM can be converted to wideband using a narrowband-to-wideband convertor by multiplying the frequencies of the narrowhand signal.

2.6 Demodulating Angle Modulated Signals

FM demodulators are implemented by generating an AM signal whose amplitude is proportional to the instantaneous frequency of the FM signal. We can then use an AM demodulator to recover the message signal.
In such a circuit, the LTI system is a differentiator whose frequency response is approximately a straight line in the frequency band of the FM signal. $|H| = 2\pi f$.

2.7 AM vs. FM

- FM capture effect: When two FM signals are received, the stronger signal is demodulated and the weaker signal is ignored.
  - The complete suppression of the weaker signal occurs at the receiver limited, where it is treated as noise and rejected.
  - When both signals are of equal strength, the receiver may switch between the two signals.
- FM requires a higher bandwidth $W_{\text{AM}} < W_{\text{FM}}$.
- FM rejects amplitude noise cause by lightning and other man-made noise.
- AM demodulators: envelope detector, product detector.
- FM demodulators: PLL, ratio detector, frequency discriminator, slope detector.