

Electrical Engineering Mathematics

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Prof Scott McCue

Tarang Janawalkar

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1 Infinite Series

1.1 Sequences

A sequence is an **ordered list** of numbers

$$a_1, a_2, a_3, \dots, a_n, \dots$$

denoted $\{a_n\}_{n=1}^{\infty}$, where n is the index of the sequence. A sequence can be **finite** or **infinite**.

1.2 Limits of Sequences

An infinite sequence $\{a_n\}$ has a limit L if a_n approaches L as n approaches infinity:

$$\lim_{n \rightarrow \infty} a_n = L$$

If such a limit exists, the sequence **converges** to L . Otherwise, the sequence **diverges**. Sequences that oscillate between two or more values do not have a limit.

1.3 Series

Given a sequence $\{a_n\}$, we can construct a sequence of **partial sums**,

$$s_n = a_1 + a_2 + \dots + a_n$$

denoted $\{s_n\}$, such that when $\{s_n\}$ converges to a finite limit L , that is,

$$\lim_{n \rightarrow \infty} s_n = L$$

the **infinite series** $\sum_{n=1}^{\infty} a_n$ converges to L . Otherwise, the series $\sum_{n=1}^{\infty} a_n$ diverges.

1.3.1 Common Series

Below are a list of common series that converge to a finite limit:

- **Geometric Series:** A sum of the geometric progression

$$\sum_{n=0}^{\infty} ar^n$$

converges when $|r| < 1$, and diverges otherwise. When $|r| < 1$,

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

- **Harmonic Series:** A sum of the reciprocals of natural numbers

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

always diverges.

- **p -Series:** A sum of the reciprocals of p -powers of natural numbers

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges when $p > 1$, and diverges otherwise. This series is closely related to the **Riemann Zeta Function**, and has exact values for even integers p .

1.4 Convergence Tests

There are several tests to determine the convergence of an infinite series. Note that these tests do not determine the value of the limit.

1.4.1 Ratio Test

Given the infinite series $\sum_{n=1}^{\infty} a_n$, with

$$\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

- (1) If $\rho < 1$, the series converges.
- (2) If $\rho > 1$, the series diverges.
- (3) If $\rho = 1$, the test is inconclusive.

1.4.2 Alternating Series Test

Given the infinite series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$, the alternating series converges if the following conditions are met:

- (1) $b_n > 0$ for all n .
- (2) $b_{n+1} \leq b_n$ for all n .
- (3) $\lim_{n \rightarrow \infty} b_n = 0$.