FLP Lab 6 (Seminar) 25.05.2022

$\mathcal{F}$ mulțime simboluri

x, y, z. variabile, a b c constante, f g h funcții

~~$p(a, x, h(g(y))) = p\, z,$~~

S	R	
$\emptyset$	$p(a, x, h(g(y))) \doteq p(z, h(z), h(u))$	Descompunere
$\emptyset$	$a \doteq z,\ x \doteq h(z),\ h(g(y)) \doteq h(u)$	Rezolvă
$z \doteq a$	$x \doteq h(a),\ h(g(y)) \doteq h(u)$	Descompune
$z \doteq a$	$x \doteq h(a),\ g(y) \doteq u$	Rezolvă
~~$z \doteq a$~~ ~~$x \doteq$~~ $x \doteq h(a)$, $z \doteq a$	$g(y) \doteq u$	Rezolvă
$g(y) \doteq u$, $x \doteq h(a)$, $z \doteq a$	$\emptyset$	

$\sigma = \{ z \mapsto a,\ u \mapsto g(y),\ x \mapsto h(a) \} \Rightarrow$ cgu

(*) Obs variabilă = ...

5. ~~$f\ x,\ f(x,x)$~~

S	R	
$\emptyset$	$f(x, f(x,x)) \doteq f(g(y), f(z, g(z)))$	Desc
$\emptyset$	$x \doteq g(y),\ f(x,x) \doteq f(z, g(z))$	Desc
$\emptyset$	$x \doteq g(y),\ x \doteq z,\ x \doteq g(z)$	Reel.
$x \doteq z$	$z = g(y),\ z = g(z)$	Ree
$z = g(y)$ $x = g(y)$	$g(y) = g(z)$	Desc.
$z = g(y)$ $x = g(y)$	$y = z.$	Re.
~~$z = g(y)$~~ $y = z.$ $z = g(z)$ $x = g(z)$	$\emptyset$	

$\sigma = \{ y \mapsto z,\ z \mapsto g(z),\ x \mapsto g(z) \} \Rightarrow$ Cgc

2.

S	R	
$\emptyset$	$f(h(z), g(x)) = f(y, y)$	Desc.
$\emptyset$	$h(z) = y,\ g(x) = y$	Rez.
$y \doteq h(z)$	$g(x) = h(z)$	ESEC ~~Act~~
$\emptyset$		

nu există cgu

3.

S	R	
$\emptyset$	$p(a, x, g(x)) = p(a, y, y)$	Dec
$\emptyset$	$a = a,\ x = y,\ g(x) = y$	Scoate
$\emptyset$	$x = y,\ g(x) = y$	Rezolv.
$x \doteq y$	$g(y) = y$	Esec.

nu $\exists$ cgu.

4.

S	R	
$\emptyset$	$p(x,y,z) \doteq p(u, f(v,v), u)$	Desc.
$\emptyset$	$x \doteq u,\ y \doteq f(v,v),\ z \doteq u$	Rezolv
$x \doteq u$	$y \doteq f(v,v),\ z \doteq u$	Rezolv
$z \doteq u$, $x \doteq u$	$y \doteq f(v,v)$	Rezolv
$z \doteq u$, $x \doteq u$, $y \doteq f(v,v)$	$\emptyset$	

$$\nabla = \{ z \mapsto u,\ x \mapsto u,\ y \mapsto f(v,v) \}$$

$$p(u, f(v,v), u) = p(u, f(v,v), u) \checkmark$$

$$\nabla = \{ y \mapsto f(v,v),\ z \mapsto u,\ x \mapsto u \}$$

6.

S	R	
$\emptyset$	$x+(y*y) = (y*y)+z$	Desc
$\emptyset$	$x=(y*y),\ (y*y)=z$	Rel
$x=(y*y)$	$(y*y)=z$	Rel
$(y*y)=z$, $x=(y*y)$	$\emptyset$	

$\sigma = \{ z \mapsto (y*y) ;\ x \mapsto (y*y) \}$

7

S	R	
$\emptyset$	$(x*y)*z = u*u^{-1}$	Desc.
$\emptyset$	$(x*y)=u,\ z=u^{-1}$	Re2
$(x*y)=u$	$z=(x*y)^{-1}$	Re2
$(x*y)=u$, $z=(x*y)^{-1}$	$\emptyset$	

$\sigma = \{ u \mapsto (x*y),\ z \mapsto (x*y)^{-1} \}$

21.

S	R	
$\emptyset$	$f(f(g(x), h(y)), h(z)) = f(v, w)$ $f(f(u, h(h(x))), h(y)) = f(v, w)$	Act.
$\emptyset$	$\cancel{f(g(x), h(y)) = v,\ h(z) = w}$ $\cancel{f(u, h(h(x))) = v}$ $f(g(x), h(y)) = v,\ h(z) = w$ $f(u, h(h(x))) = v$ $h(y) = w$	Res.
$w = h(z)$	$f(g(x), h(y)) = v$ $f(u, h(h(x))) = v$ $h(y) = h(z)$	~~Act.~~ Res.
$v = f(g(x), h(y))$	$f(u, h(h(x))) = f(g(x), h(y))$ $h(y) = h(z)$	Act.
$v = f(g(x), h(y))$	$u = g(x),\ h(h(x)) = h(y)$ $y = z$	Res.
$\cancel{v = h}\ y \doteq z$ $v = f(g(x), h(z))$	$u = g(x),\ h(h(x)) = h(z)$	Res.
$u = g(x)$ $y = z$ $v = f(g(x), h(z))$	$h(h(x)) = h(z)$	Act.

$u = g(x)$

$y = z$

$v = f(g(x), h(z))$

$h(x) = z$

Res.

$\cancel{z = h(x)}$

$u = g(x)$

$y = z$

$v = f(g(x), h(z))$

$\cancel{z = h(x)}$

$\cancel{\emptyset}$

$\cancel{\sigma = \{ u \mapsto g(x), y \mapsto z, v \mapsto f(g(x), h(z)), z \mapsto h(x) \}}$

$z = h(x)$

$u = g(x)$

$y = h(x)$

$v = f(g(x), h(h(x)))$

$w = h(h(x))$

01.05.2022

FLP seminar 7.

formulă = atunci când e completă
expresii = „nu e completă"

Teoremă $\vdash \varphi$ sintactic
$\Gamma \vdash \varphi$
mulțime premise

Tautologie $\vDash \varphi$ adevărat indiferent de valoare

$\hookrightarrow e : Var \to \{0,1\}$

Fie: $e(p) = 1$
$e(q) = 0$
$e(v) = 1 \ (\forall)\ v \in Var \setminus \{p, q\}$

$e^+ : Formule \to \{0,1\}$

$$e^+(\neg v \to p) = e^+(\neg v) \to e^+(p) = \neg e^+(v) \to e(p)$$
$$= \neg e(v) \to e(p) = \neg 1 \to 1 = 0 \to 1 = 1$$

$\vDash \neg v \to p$? (verificăm că formula e tautologie)

v	p	$\neg v$	$\neg v \to p$
1	1	0	1
1	0	0	1
0	1	1	1
0	0	1	0

$\Rightarrow$ nu e tautologie
(trebuie să dea toată coloana 1)

S 2.1 $(v_1 \vee v_2 \to v_3) \leftrightarrow ((v_1 \to v_3) \wedge (v_2 \to v_3))$

este tautologie (tabel)

v_1	v_2	v_3
0	0	1
0	0	0
0	1	1
0	1	0
1	0	1
1	0	0
1	1	1
1	1	0

Th. 1 (Th deducție) $\Gamma \vdash \varphi \to \psi \Leftrightarrow \Gamma \cup \{\varphi\} \vdash \psi$

(orice se poate demonstra sintactic se poate demonstra și semantic)

$\vdash \varphi \Leftrightarrow \vDash \varphi$

$\Gamma \vDash \varphi \to \psi \Leftrightarrow \Gamma \cup \{\varphi\} \vDash \psi$

T dacă și φ e 1 și $\varphi \to \psi$ este T

Deducție naturală

$(p \Rightarrow q) \Leftrightarrow (q \Leftrightarrow p)$

$\boxed{\begin{matrix} p \\ \vdots \\ x \end{matrix}}$ ipoteze

$\underbrace{\varphi_1, \dots, \varphi_n}_{\text{premise}} \vdash \underbrace{\psi}_{\text{concluzie}}$ – secvent

$\underbrace{p \wedge q,\ r}_{\text{premise}} \vdash \underbrace{q \wedge r}_{\text{concluzie}}$

1	$p \wedge q$	premise
2.	r	
3.	q	$(\wedge e_2)$, 1.
4.	$q \wedge r$	$(\wedge i)$, 2, 3.

$p \wedge q,\ r \vDash q \wedge r.$

Fie e – model pt. $\{p \wedge q, r\}$

Vrem să demonstrăm că e este model pt $q \wedge r$

$e \vDash \{p \wedge q, r\} \Leftrightarrow e^+(p \wedge q) = 1,\ e^+(r) = 1$

$e(p) = e(q) = 1 \quad , \quad e(r) = 1$

$e^+(q \wedge r) = e^+(q) \wedge e^+(r) = e(q) \wedge e(r) = 1 \wedge 1 = 1$

$\Rightarrow e$ este model pt $q \wedge r$

$\underline{S2.2}$ $(p \wedge q) \wedge r,\ s \wedge t \vdash q \wedge s.$

1. $(p \wedge q) \wedge r$ premisa
2. $s \wedge t$ premisa.
3. $p \wedge q$ $(\wedge e_1)$, 1.
4. q $(\wedge e_2)$, 3.
5. s $(\wedge e_1)$, 2.
6. $q \wedge s$ $(\wedge i)$, 4,5.

S2.3. $p,\ \neg\neg(q \wedge r) \vdash \neg\neg p \wedge r$

1. p premisa
2. $\neg\neg(q \wedge r)$ premisa.
3. $q \wedge r$ $(\neg\neg e)$, 2.
4. r $(\wedge e_2)$, 3.
5. ~~$p \wedge r$ $(\wedge i)$, 1,4~~
6. ~~$\neg\neg(p \wedge r)$~~

5. $\neg\neg p$ $(\neg\neg i)$ 1.
6. $\neg\neg p \wedge r$ $(\wedge i)$, 4,5

$\underbrace{p \wedge q \to r}$ $\vdash p \to (q \to r)$

$p \wedge q \to r \vdash p \to (q \to r)$

1.	$p \wedge q \to r$	premise
2.	p	ipoteza
3	q	ipoteza
4	p	copiere
5	$p \wedge q$	$(\wedge i)$, 3, 3
6	r	$(\to i)$, 5, 1
7	$q \to r$	$(\to i)$, 3-6
8	$p \to (q \to r)$	$(\to i)$, 2-7

$q \to r \vdash (p \lor q) \to (p \lor r)$

1	$q \to r$	premisă
2.	$p \lor q$	ipoteză
3	p	ipoteză
4	$p \lor r$	$(\lor i_1)$, 3
5	q	ipoteză
6	r	$(\to e)$, 5,
7	$p \lor r$	$(\lor i_2)$, 6
8	$p \lor r$	$(\lor e)$, 2, 3–4, 5–7
9	$(p \lor q) \to (p \lor r)$	$(\to i)$, 2–8

(4) $p \wedge (q \vee r) \rightarrow (p \wedge q) \vee (p \wedge r)$

1. $p \wedge (q \vee r)$ premissa
2. p $(\wedge e_1)$, 1
3. $q \vee r$ $(\wedge e_2)$, 1
4. q ipotesi
5. $p \wedge q$ $(\wedge i)$, 2, 4
6. $(p \wedge q) \vee (p \wedge r)$ $(\vee i_1)$, 5
7. r ipotesi
8. $p \wedge r$ $(\wedge i)$, 2, 7
9. $(p \wedge q) \vee (p \wedge r)$ $(\vee i_2)$, 8
10. $(p \wedge q) \vee (p \wedge r)$ $(\vee e)$, 3, 4-6, 7-9

materiale MOODLE

FLP seminar 2 (cont) 08.04.2022

$\vdash p \to p \iff \models p \to p$

p	$p \to p$
0	1
1	1

$\Rightarrow$ tautologie

$(p \wedge q) \wedge r,\ s \wedge t \models q \wedge s \iff$

$\iff$ fie e un model pt $\{(p \wedge q) \wedge r,\ s \wedge t\}$

$\iff e^+((p \wedge q) \wedge r) = 1$

$e^+(s \wedge t) = 1 \iff e^+(s) \wedge e^+(t) = 1$

$\iff e(s) \wedge e(t) = 1$

$\iff e(s) = e(t) = 1$

$e : Var \to \{0,1\}$

$e^+ : Form \to \{0,1\}$

$e^+((p \wedge q) \wedge r) = 1$

$e^+(p \wedge q) \wedge e^+(r) = 1$

$e^+(p) \wedge e^+(q) \wedge e^+(r) = 1$

$e(p) \wedge e(q) \wedge e(r) = 1$

$e(p) = e(q) = e(r) = 1$

Dem. că e este model pt formula

$q \wedge s \iff e^{+}(q \wedge s) = 1$.

Folosim ce am aflat mai devreme

$e^{+}(q \wedge s) = e^{+}(q) \wedge e^{+}(s)$

$e^{+}(q \wedge s) = e(q) \wedge e(s)$ ~~și~~ $e(q) = 1$

$e(s) = 1$

$e^{+}(q \wedge s) = 1 \wedge 1$

$e^{+}(q \wedge s) = 1$

Deci $\{(p \wedge q) \wedge r,\ s \wedge t\} \models q \wedge s$.

S 2.3

$$\frac{\varphi \to \psi, \quad \neg \psi}{\neg \varphi} \quad \text{Modus Tolens}$$

1	$\varphi \to \psi$	premisă
2	$\neg \psi$	premisă
3	φ	ipoteză
4	ψ	$(\to e)$, 1, 3
5	$\neg \psi$	copiere
6	$\perp$	$(\neg e)$, 4, 5
7	$\neg \varphi$	$(\neg i)$, 3, 6

$$\frac{\boxed{\begin{array}{c}\neg \varphi \\ \vdots \\ \bot\end{array}}}{\varphi}\ \text{RAA}$$

1	$\neg\varphi \to \bot$	premisă
2.	$\neg\varphi$	ipoteză
3.	$\bot$	$(\to e)$, 1, 2.
4	$\neg\neg\varphi$	$(\neg i)$, 2–3
5	φ	$(\neg\neg e)$, 4.

S2.4. Fie $n \geq 1$ și $\varphi_1, \dots \varphi_n, \varphi$ formule.

Știm că $\vdash \varphi_1 \to (\varphi_2 \to (\dots(\varphi_n \to \varphi)\dots))$

Vrem să dem că $\varphi_1 \dots \varphi_n \to \varphi$

Considerăm $\varphi_1 \dots \varphi_n$ premise.

1	φ_1	premisă
2	$\varphi_1 \to (\varphi_2 \to (\dots(\dots(\varphi_n \to \varphi)\dots))$	teoremă
3.	$\varphi_2 \to (\dots(\varphi_n \to \varphi))\dots)$	$(\to e)$, 1, 2.
4.	φ_2	
.....	Aplicăm de $(n-1)$ ori $(\to e)$	
x	φ	$(\to e)$

S25. $\varphi \leftrightarrow \psi = (\varphi \to \psi) \wedge (\psi \to \varphi)$

$$\dfrac{\boxed{\begin{array}{c}\varphi\\ \vdots\\ \psi\end{array}} \quad \boxed{\begin{array}{c}\psi\\ \vdots\\ \varphi\end{array}}}{\begin{array}{c}\varphi\to\psi \quad \psi\to\varphi\end{array}}$$

$$\dfrac{\varphi\to\psi \qquad \psi\to\varphi}{\cancel{(\varphi\to\psi)\wedge(\psi\to\varphi)}} \ (\leftrightarrow i)$$

$\varphi \leftrightarrow \psi$

$$\dfrac{\dfrac{\varphi\leftrightarrow\psi \qquad \varphi}{\varphi\to\psi\ (\wedge e) \qquad \varphi}}{\psi}\ (\to e) \qquad\qquad \dfrac{\varphi\leftrightarrow\psi}{\psi\to\varphi}$$

1	$\varphi \leftrightarrow \psi$	premisă
2	φ	premisă
3	$\varphi \to \psi$	$(\wedge e_1)$, 1
4	ψ	$(\to e)$, 2, 3

$$\dfrac{\varphi\leftrightarrow\psi \qquad \varphi}{\psi}$$

$\Rightarrow$ mai e și varianta cu φ

S 2.6 scriitori
writer

i_1 Toți scriitorii care înțeleg natura umană sunt înțelepți

i_2 Un scriitor (Writer $\wedge$ HumanNat) $\rightarrow$ Clever.

i_2. Un scriitor care este poet adevărat poate trezi sentimente puternice
writer poet Heart

$$Writer \rightarrow (Poet \rightarrow Heart) = (Writer \wedge Poet) \rightarrow Heart$$

$$W \rightarrow (P \rightarrow H) = (W \wedge P) \rightarrow H$$

i_3 Shakespeare este scriitorul care a scris "Hamlet"

Shakespeare $\rightarrow$ (Writer $\wedge$ Hamlet)

i_4. (Writer $\wedge$ Heart) $\rightarrow$ HumanNat.

i_5. Hamlet $\rightarrow$ Poet.

1

2

3

4

1

2

3 Shakespeare → Clever.

4

5

6. Shakespeare ipoteză

Writer ∧ Hamlet (→e), 3, 6

8 Writer. (∧ e_1), 7

9 Hamlet (∧ e_2), 7

10 Poet (→e), 5

11 Writer ∧ Poet (∧ i_1) 8, 10

12. Heart (→e), 2, 11

13. Writer∧Heart (∧i) 8, 12.

14. HumanHat (→e) 4, 13.

15. Writer ∧ HumanHat (∧i), 8, 14

16 Clever. (→e), 1, 15.

17. Shakespeare → Clever (→i), 6-16.

08.05.2022

FLP seminar 3

Puncte fixe.

(1) $f: \mathcal{P}(\{1,2,3\}) \to \mathcal{P}(\{1,2,3\})$

$f_1(Y) = Y \cup \{1\}$

A pct fix $\Leftrightarrow f_1(A) = A$

$Y \in \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Puncte fixe : $\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}$

Cel mai mic pct fix. B a.î. $B = f(B)$, pt. orice alt pct fix A, $B \subseteq A$

$\Rightarrow \{1\}$ cel mai mic punct fix pt f_1.

(2) $f_2(Y) = \begin{cases} \{1\}, & \text{dacă } 1 \in Y \\ \emptyset, & \text{altfel.} \end{cases}$

$f_2(\emptyset) = \emptyset$ – pct fix

$f_2(\{1\}) = \{1\}$ – pct fix

$\emptyset \subseteq \{1\} \Rightarrow \emptyset$ cel mai mic punct fix pt f_2.

$f_2(\{2\}) = \emptyset$

$f_2(\{3\}) = \emptyset$

$f_2(\{1,2\}) = \{1\}$

$f_2(\{1,3\}) = \{1\}$

$f_2(\{2,3\}) = \emptyset$

$f_2(\{1,2,3\}) = \{1\}$

(3) $f_3(Y) = \begin{cases} \emptyset, & \text{dacă } 1 \in Y \\ \{1\}, & \text{altfel} \end{cases}$

$f_3(\emptyset) = \{1\}$

$f_3(\{1\}) = \emptyset$

$f_3(\{2\}) = \{1\}$

$f_3(\{3\}) = \{1\}$

$f_3(\{1, 2\}) = \emptyset$

$\Rightarrow$ f_3 nu are puncte fixe

15.05.2022

FLP ~~lab~~ Seminar 5?

MONOTONIC

$f(P\{1,2,3\}) \to P(\{1,2,3\})$

$f(Y) = Y \cup \{1\}$

Def: $f : A \to B$

$a_1 \leq_A a_2 \Rightarrow f(a_1) \leq_B f(a_2)$

$\{1\} \subseteq_A \{1,2\}$ | $\{2\} \subseteq \{2,3\}$

$f(\{1\}) \subseteq f(\{1,2\})$ | $\{1,2\} \subseteq \{1,2,3\}$

$\{1\} \subseteq \{1,2\}$

$M \subseteq N \iff \forall m \in M \Rightarrow m \in \cancel{N}$

CONTINUĂ

$f_i : P(\{1,2,3\}) \to P(\{1,2,3\}) \quad i \in \{1,2,3\}$

$$f(Y) = \begin{cases} \emptyset, & 1 \in Y \\ \{1\}, & \text{altfel} \end{cases}$$

$\emptyset \subset \{1\}$

$\emptyset \cup \{1\} = \{1\}$

Pp. că f este cont $\iff$

$\iff f(\bigcup_i M_i) = \bigcup_i f(M_i)$

$f(\emptyset \cup \{1\}) = f(\emptyset) \cup f(\{1\})$

$f(\{1\}) = \{1\} \cup \emptyset$

$\emptyset = \{1\}$ Fals $\Rightarrow$ f nu este continuă

Teoremă pct. fix. pct fix $f(A) = A$

$f_S : \mathcal{P}(At) \to \mathcal{P}(At)$

f_S monotonă $\Leftrightarrow$ $(\forall)\ Y_1, Y_2 \in \mathcal{P}(At)$ a.î. $Y_1 \subseteq Y_2$

at. $f_S(Y_1) \subseteq f_S(Y_2)$

Fie $Y_1, Y_2 \in \mathcal{P}(At)$ a.î. $Y_1 \subseteq Y_2$.

Vrem să dem că $f_S(Y_1) \subseteq f_S(Y_2)$

$\Leftrightarrow \cancel{f(Y_1)} = \underline{Y_1} \cup B_{azz} \cup \underbrace{\{a \in At \mid (s_1 \wedge \dots \wedge s_n \to a) \text{ este în } S,\ s_1 \in Y_1, \dots, s_n \in Y_1\}}_{=M_1}$

$\subseteq \underline{Y_2} \cup B_{azz} \cup \underbrace{\{a \in At \mid (s_1 \wedge \dots \wedge s_n \to a) \text{ este în } S,\ s_1 \in Y_2, \dots, s_n \in Y_2\}}_{=M_2}$

Vrem să dem că $M_1 \subseteq M_2$.

Fie $a \in M_1 \Rightarrow \exists\ s_1 \in Y_1 \dots s_n \in Y_1$ a.î.

$s_1 \wedge \dots \wedge s_n \to a$ se găsește în S

Știm că $Y_1 \subseteq Y_2 \Rightarrow \exists\ s_1 \in Y_2, \dots, s_n \in Y_2$ a.î. $s_1 \wedge \dots \wedge s_n \to a$ se găsește în S $\Bigg\} \Rightarrow$

$\Rightarrow a \in M_2 \Rightarrow M_1 \subseteq M_2$. Prin urmare $f_S(Y_1) \subseteq f_S(Y_2)$

⑦ Cel mai mic punct fix.

$At = \{cold, wet, windy, dry, scotland\}$

$Baza = \{cold\}$

$f_s(\emptyset) = \{\emptyset\} \cup \{cold\} = \{cold\}$

nimic nu poate implica $\emptyset$

$f_s(f_s(\emptyset)) = f_s(\{cold\}) = \{cold\} \cup \{cold\} \cup \{wet\}$
$= \{cold, wet\}$

$f_s(f_s(f_s(\emptyset))) = f_s(\{cold, wet\}) = \{cold, wet\} \cup \{cold\} \cup \{scotland\}$
$= \{cold, wet, scotland\}$

$f_s^4(\emptyset) = f_s(\{cold, wet, scotland\}) = \{cold, wet, scotland\} \cup \{cold\}$
$\cup \emptyset$

$\Rightarrow \{cold, wet, scotland\}$ cel mai mic punct fix.

$S_{3.3}$ f_S, $i \in \{1,2,3\}$

$S_1 = \{ x_1 \wedge x_2 \to x_3,$
$x_4 \wedge x_2 \to x_5,$
$x_6 \to x_1,$
$x_2.,$
$x_6. \}$

$At = \{x_1, x_2, \dots, x_6\}$

$B_{S_1} = \{x_2, x_6\}$

$f_S(\emptyset) = \emptyset \cup \{x_2, x_6\} \cup \emptyset = \{x_2, x_6\}$

$f_S(f_S(\emptyset)) = f_S(\{x_2, x_6\}) = \{x_2, x_6\} \cup \{x_2, x_6\} \cup \{x_1\}$
$= \{x_1, x_2, x_6\}$

$f_S^3(\emptyset) = f_S(\{x_1, x_2, x_6\}) = \{x_1, x_2, x_6\} \cup \{x_2, x_6\} \cup \{x_3\}$
$= \{x_1, x_2, x_3, x_6\}$

$f_S^4(\emptyset) = f_S(\{x_1, x_2, x_3, x_6\}) = \{x_1, x_2, x_3, x_6\} \cup B_{S_1} \cup \emptyset$
$= \{x_1, x_2, x_3, x_6\}$

$\Rightarrow \{x_1, x_2, x_3, x_6\}$ cel mai mic pct. fix

$S_2 = \{ x_1 \wedge x_2 \to x_3$ $\qquad At = \{x_1 \dots x_5\}$

$x_4 \to x_1$ $\qquad Baza = \{x_4\}$

$x_5 \to x_2$

$x_2 \to x_5$

$x_4 \}$

$f_S(\emptyset) = \emptyset \cup \{x_4\} \cup \emptyset = \{x_4\}$

$f_S^2(\emptyset) = \{x_4\} \cup \{x_4\} \cup \{x_1\} = \{x_1, x_4\}$

$f_S^3(\emptyset) = \{x_1, x_4\} \cup \{x_4\} \cup \emptyset = \{x_1, x_4\} \to$ cel mai mic punct fix

$S_3 = \{ x_1 \to x_2$ $\qquad At = \{x_1, x_2, x_3\}$

$x_1 \wedge x_3 \to x_1$ $\qquad Baza = \{x_3\}$

$x_3 \}$

$f_S(\emptyset) = \emptyset \cup \{x_3\} \cup \emptyset = \{x_3\}$

$f_S^2(\emptyset) = \{x_3\} \cup \{x_3\} \cup \emptyset = \{x_3\} \longrightarrow$ cel mai mic pct. fix

Seminar 4

Rezoluții

$a \to b \quad = \quad \neg a \lor b$

$\cancel{a \land c \to b \quad = \quad \neg(a \land c) \lor b}$

$\cancel{= \neg a \lor}$

$a \land c \to b \quad = \quad \neg(a \land c) \lor b$

$\neg a \lor \neg c \lor b$

$$Q :- P_1, P_2, \dots, P_n \iff P_1 \land P_2 \land \dots \land P_n \to Q$$

$$\iff$$

$$\neg(P_1 \land P_2 \land \dots \land P_n) \lor Q$$

$$\iff$$

$$\neg P_1 \lor \neg P_2 \lor \neg P_3 \lor \dots \lor \neg P_n \lor Q$$

$S_{4.1.}$

1. $r :- p, q \quad = \quad \neg p \lor \neg q \lor r$
2. $s :- p, q \quad = \quad \neg p \lor \neg q \lor s$
3. $\neg t \lor \neg u \lor v$
4. $\neg v \lor \neg s \lor w$
5. t
6. q
7. u
8. p
9. $?- w$

$\delta \; \delta \delta \delta \delta$

$G_0 = \cancel{t} \; \neg w$

$G_1 = \underline{\neg v} \vee \neg s$ (4)

$G_2 = \neg t \vee \neg U \vee \neg s$ (3)

$G_3 = \neg U \vee \neg s$ (5)

$G_4 = \neg s$ (7)

$G_5 = \neg p \vee \neg q$ (2)

$G_6 = \neg q$ (8)

$G_7 = \square$ (6)

de la găsit pe t sigur și îl înlocuim cu "birit"

5) 1. $\neg q(y, x) \vee \neg q(y, f(f(y))) \vee q(x, y)$

2. $q(a, f(f(x)))$

3. ? $- q(f(z), a)$

$\theta(x) = f(z)$

$\theta(y) = a.$

$G_0 = \neg q(f(z), a)$

$G_1 = \neg q(\cancel{f}a, f(z)) \vee \neg q(a, ff(a)))$ (1)

$G_2 = \neg q(a, f(z))$ (2)

$\theta(x) = a$

$\theta(z) = f(x)$

$G_3 = \neg q(a, f(f(x)))$ (2)

$G_4 = \square$

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c) 1. $\neg q(x, f(y)) \vee \neg r(z) \vee p(x)$

2. $\neg r(x) \vee p(x)$ $\qquad p(x) \wedge q(y,z)$

3. $\neg p(y) \vee q(x,y)$

4. $\neg q(x,y) \vee r(x)$

5. $r(f(b))$

$G_0 = \neg p(x) \vee \neg q(y,z)$

$G_1 = \neg r(x_1) \vee \neg q(y,z)$ $\quad$ (2 cu $\theta(x) = x_1$)

~~$G_2 = \neg r(x_1) \vee \neg q(y_1, z)$~~ $\quad$ ~~(4 cu $\theta(x) = x_1$, $\theta(y) = y_1$)~~

~~$G_2 = \neg q(x_1, y_1) \vee \neg q(y_1, z)$~~ $\quad$ ~~(3 cu $\theta(x) = x_1$, $\theta(y) = y_1$)~~

~~$G_3 = \neg p(y_1) \vee \neg q(y_1, z)$~~

~~$G_4 = \neg r(x_1) \vee \neg q(y_1, z)$~~ $\quad$ ~~2 cu $\theta(x) = y$~~

~~$G_5 =$~~ $\quad$ ~~5 cu $\theta(x) = f(b)$~~

$G_2 = \neg q(y,z)$ $\quad$ (5 cu $\theta(x_1) = f(b)$)

$G_3 = \neg p(z_1)$ $\quad$ (3 cu $\theta(x) = y_1$, $\theta(y) = z_1$)

$G_4 = \neg r(z_1)$ $\quad$ 2 cu $\theta(x) = z_1$

$G_5 = r(f(b))$ $\quad$ (5 cu $\theta(z_1) = f(b)$)

$G_6 = \square$

§4.2

1. $\neg q(X,Z) \vee \neg r(Z,Y) \vee p(X,Y)$

2. $\neg s(X) \vee p(X,X)$

3. $q(X,b)$

4. $q(b,a)$

5. $\neg r(a,X) \vee q(X,a)$

6. $r(b,a)$

7. $\neg t(X,a) \vee s(X)$

8. $\neg t(X,b) \vee s(X)$

9. $\neg t(X,X) \vee s(X)$

10. $t(a,b)$

11. $t(b,a)$