Recap examen FLP

C3. Unificare

Notații:

- x, y, z.... - variabile
- a, b, c.... - constante
- f, g, h... - simboluri de fct. arbitrare
- s, t, u... - termeni
- var(t) - mult. variabilelor care apar în t
- ecuații $s = t$ pt o pereche de termeni

ex: $f(x, g(x,a), y)$ - termen (f aritate 3; g aritate 2; a const; x, y var)

$var(f(x, g(x,a), y)) = \{x, y\}$

substituție: funcție de la variabile la term.

ex: $\sigma = \{x/a, y/g(w), z/b\}$

$\sigma = \{x \mapsto a, y \mapsto g(w), z \mapsto b\}$

! De văzut ex cu cgu.

ex: Ecuațiile $\{g(y) = x, f(x, h(x), y) = f(g(z), w, z)\}$ au cgu?

S	R	
$\emptyset$	$g(y)=x, f(x, h(x), y) = f(g(z), w, z)$	R
$x \doteq g(y)$	$f(g(y), h(g(y)), y) = f(g(z), w, z)$	D
$x \doteq g(y)$	$g(y) \doteq g(z), h(g(y)) \doteq w, y \doteq z$	R
$w \doteq h(g(y)), x \doteq g(y)$	$g(y) \doteq g(z), y \doteq z$	R
$y \doteq z, w \doteq h(g(z)), x \doteq g(z)$	$g(z) \doteq g(z)$	S

$\sigma = \{y \mapsto z, w \mapsto h(g(z)), x \mapsto g(z)\}$

Ecuațiile $\{g(y) \doteq x, f(x, h(y), y) \doteq f(g(z), b, z)\}$ au cgu?

S	R	
$\emptyset$	$g(y) \doteq x, f(x, h(y), y) \doteq f(g(z), b, z)$	R
$x \doteq g(y)$	$f(g(y), h(y), y) \doteq f(g(z), b, z)$	D
$x \doteq g(y)$	$g(y) \doteq g(z), \underline{h(y) \doteq b}, y \doteq z$	EȘEC

$g(y) \doteq x; f(x, h(x), y) \doteq f(y, w, z)$ au cgu?

S	R	
$\emptyset$	$g(y) \doteq x, f(x, h(x), y) \doteq f(y, w, z)$	R
$x \doteq g(y)$	$f(g(y), h(g(y)), y) \doteq f(y, w, z)$	D
$x \doteq g(y)$	$\cancel{g(y) \doteq y}, h(g(y)) = w, y \doteq z$	EȘEC

$p(a, x, h(g(y))) = p(z, h(z), h(u))$

S	R	
$\emptyset$	$p(a, x, h(g(y))) = p(z, h(z), h(u))$	D
$\emptyset$	$a \doteq z, x \doteq h(z), h(g(y)) \doteq h(u)$	R
$z \doteq a$	$x \doteq h(a), h(g(y)) \doteq h(u)$	D
$z \doteq a$	$x \doteq h(a), g(y) \doteq u$	R
$x \doteq h(a), z \doteq a$	$g(y) \doteq u$	R
$u \doteq g(y), x \doteq h(a), z \doteq a$	$\emptyset$	

$f(x, f(x,x)) = f(g(y), f(z, g(a)))$

S	R	
$\emptyset$	$f(x, f(x,x)) = f(g(y), f(z, g(a)))$	D
$\emptyset$	$x \doteq g(y),\ f(x,x) \doteq f(z, g(a))$	D
$\emptyset$	$x \doteq g(y),\ x \doteq z,\ x \doteq g(a)$	R
$x \doteq z$	$z \doteq g(y),\ z \doteq g(a)$	R
$z \doteq g(y),\ x \doteq g(y)$	$g(y) \doteq g(a)$	D
—||—	$y \doteq a$	R
$y \doteq a,\ z \doteq g(a),\ x \doteq g(a)$	$\emptyset$	

$\sigma = \{y \mapsto a,\ z \doteq g(a),\ x \doteq g(a)\}$ este cgu.

$f(h(a), g(x)) = f(y, y)$

S	R	
$\emptyset$	$f(h(a), g(x)) = f(y,y)$	D
$\emptyset$	$h(a) \doteq y,\ g(x) \doteq y$	R
$y \doteq h(a)$	$g(x) \doteq h(a)$	EȘEC

Nu $\exists$ cgu.

$p(a, x, g(x)) = p(a, y, y)$

S	R	
$\emptyset$	$p(a, x, g(x)) = p(a, y, y)$	D
$\emptyset$	$a \doteq a,\ x \doteq y,\ g(x) \doteq y$	S
$\emptyset$	$x \doteq y,\ g(x) \doteq y$	R
$x \doteq y$	$g(y) \doteq y$	EȘEC

Nu $\exists$ cgu

$$p(x, y, z) = p(u, f(\ , v), u)$$

S	R	
$\emptyset$	$p(x,y,z) = p(u, f(v,v), u)$	D
$\emptyset$	$x \doteq u,\ y \doteq f(v,v),\ z \doteq u$	R
$x \doteq u$	$y \doteq f(v,v),\ z \doteq u$	R
$z \doteq u$	$y \doteq f(v,v)$	R
$x \doteq u$, $y \doteq f(v,v)$, $z \doteq u$, $x \doteq u$	$\emptyset$	

$$x + (y * y) = (y * y) + z$$

S	R	
$\emptyset$	$x + (y*y) = (y*y) + z$	D
$\emptyset$	$x \doteq y*y,\ y*y \doteq z$	R
$x \doteq y*y$	$y*y \doteq z$	R
$x \doteq y*y$, $z \doteq y*y$	$\emptyset$	

$\sigma = \{x \mapsto y*y,\ z \mapsto y*y\}$ este cgu.

$$(x*y)*z = u * u^{-1}$$

S	R	
$\emptyset$	$(x*y)*z = u*u^{-1}$	D
$\emptyset$	$x*y \doteq u,\ z \doteq u^{-1}$	R
$u \doteq x*y$	$z \doteq (x*y)^{-1}$	R
$u \doteq x*y$, $z \doteq (x*y)^{-1}$	$\emptyset$	

$f(f(g(x), h(y)), h(z)) = f(v, w)$
$f(f(u, h(h(x))), h(y)) = f(v, w)$

S	R	
$\emptyset$	$f(f(g(x), h(y)), h(z)) = f(v, w)$ $f(f(u, h(h(x))), h(y)) = f(v, w)$	D
$\emptyset$	$f(g(x), h(y)) \doteq v,\ h(z) \doteq w$ $f(u, h(h(x))) \doteq v,\ h(y) \doteq w$	R
$w \doteq h(z)$	$f(g(x), h(y)) \doteq v$ $f(u, h(h(x))) \doteq v,\ h(y) \doteq h(z)$	D
$w \doteq h(z)$	$-\|\|-\quad y \doteq z$	R
$y \doteq z,\ w \doteq h(z)$	$f(g(x), w) \doteq v$ $f(u, h(h(x))) \doteq v$	R
$-\|\|-$ $v \doteq f(g(x), w)$	$f(u, h(h(x))) \doteq f(g(x), w)$	D
$-\|\|-$	$u \doteq g(x),\ h(h(x)) \doteq h(z)$	R
$-\|\|-\ u \doteq g(x)$	$h(h(x)) \doteq h(z)$	D
$-\|\|-$	$h(x) \doteq z$	R
$-\|\|-\ z \doteq h(x)$	$\emptyset$	

$\sigma = \{w \mapsto h(h(x)),\ y \mapsto h(x),\ v \mapsto f(g(x), h(h(x))),$
$u \mapsto g(x),\ z \mapsto h(x)\}$ este cgu

Deducție naturală

$\models ((v_1 \vee v_2) \rightarrow v_3) \Longleftrightarrow ((v_1 \rightarrow v_3) \wedge (v_2 \rightarrow v_3))$?

v_1	v_2	v_3	$v_1 \vee v_2$	$(v_1 \vee v_2 \rightarrow v_3)$	$(v_1 \rightarrow v_3) \wedge (v_2 \rightarrow v_3)$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	0	0
1	1	1	1	1	1

$\Rightarrow$ tautologie ($\models$)

Demonstrați că urm. secvenți sunt valizi:

$(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$

1. $(p \wedge q) \wedge r$ premisă
2. $s \wedge t$ premisă
3. $p \wedge q$ $(\wedge e_1)$, 1
4. q $(\wedge e_2)$, 3
5. s $(\wedge e_1)$, 2
6. $q \wedge s$ $(\wedge i)$, 4, 5

$p, \neg\neg(q \wedge r) \vdash \neg\neg p \wedge r$

1. p premisă
2. $\neg\neg(q \wedge r)$ premisă
3. $q \wedge r$ $(\neg\neg e)$, 2
4. r $(\wedge e_2)$, 3
5. $\neg\neg p$ $(\neg\neg i)$, 1
6. $\neg\neg p \wedge r$ $(\wedge i)$, 5, 4

$p \land q \to r \vdash p \to (q \to r)$

1. $p \land q \to r$ premisă
2. p ipoteză
3. q ipoteză
4. p copiere
5. $p \land q$ $(\land_i)$, 4, 3
6. r $(\to_e)$, 5, 1
7. $q \to r$ $(\to_i)$, 3-6
8. $p \to (q \to r)$ $(\to_i)$, 2-7

$p \land (q \lor r) \vdash (p \land q) \lor (p \land r)$

1. $p \land (q \lor r)$ premisă
2. p $(\land_{e_1})$, 1
3. $q \lor r$ $(\land_{e_2})$, 1
4. q ipoteză
5. $p \land q$ $(\land_i)$, 2, 4
6. $(p \land q) \lor (q \lor r)$ $(\lor_i)$, 5, 3
7. r ipoteză
8. $p \land r$ $(\land_i)$, 2, 7
9. $(q \lor r) \lor (p \land r)$ $(\lor_i)$, 3, 5
10. $(p \land q) \lor (p \land r)$ $(\lor_e)$, 3, 4-6, 7-9

$p \to q, p \to \neg q \vdash \neg p$

1. $p \to q$ premisă
2. $p \to \neg q$ premisă
3. p ipoteză
4. q $(\to_e)$, 1, 3

5. $\neg q$ $(\to e), 2, 3$
6. $\perp$ $(\neg e), 4, 5$

7. $\neg p$ $(\neg i), 3-6$

$$(p \wedge q) \wedge r,\ s \wedge t \overset{\text{model}}{\vDash} q \wedge s \ ?$$

Fie e model pt fiecare $\{(p\wedge q)\wedge r,\ s \wedge t\} \Leftrightarrow$

$$\begin{cases} e^{+}((p\wedge q)\wedge r) = 1 \\ e^{+}(s \wedge t) = 1 \end{cases}$$

$$\begin{aligned} e^{+}((p\wedge q)\wedge r) = 1 &\Leftrightarrow e^{+}(p \wedge q) \wedge e^{+}(r) = 1 \\ &\Leftrightarrow e^{+}(p\wedge q) \wedge e(r) = 1 \\ &\Leftrightarrow \begin{cases} e^{+}(p\wedge q) = 1 \Leftrightarrow e^{+}(p) \wedge e^{+}(q) = 1 \\ \qquad \Leftrightarrow e(p) \wedge e(q) = 1 \\ \qquad \Leftrightarrow e(p) = e(q) = 1 \\ e(r) = 1 \end{cases} \end{aligned}$$

$$\begin{aligned} e^{+}(s\wedge t) = 1 &\Leftrightarrow e^{+}(s) \wedge e^{+}(t) = 1 \\ &\Leftrightarrow e(s) \wedge e(t) = 1 \\ &\Leftrightarrow e(s) = e(t) = 1 \end{aligned}$$

Dem că e este model pt $q \wedge s \Leftrightarrow e^{+}(q \wedge s) = 1.$

$$e^{+}(q\wedge s) = e^{+}(q) \wedge e^{+}(s) = e(q) \wedge e(s) = 1 \wedge 1 = 1.$$

Deci, $(p\wedge q)\wedge r,\ s\wedge t \vDash q \wedge s.$

Demonstrați:

$$\frac{\varphi \to \psi,\ \neg \psi}{\neg \varphi} \Leftrightarrow \varphi \to \psi,\ \neg\psi \vdash \neg \varphi$$

1. $\varphi \to \psi$ premisă
2. $\neg \psi$ premisă
3. φ ipoteză
4. ψ $(\to e), 1, 3$

5. $\neg\psi$ copiere
6. $\perp$ $(\neg e)$,4,5

7. $\neg\varphi$ $(\neg i)$,3-6

$$\dfrac{\begin{array}{c}\neg\varphi\\ \vdots\\ \perp\end{array}}{\varphi} \iff \neg\varphi \to \perp \vdash \varphi$$

1. $\neg\varphi \to \perp$ premisă
2. $\neg\varphi$ ipoteză
3. $\perp$ $(\to e)$, 1,2

4. $\neg\neg\varphi$ $(\neg i)$, 2-3
5. φ $(\neg\neg e)$, 4

Prob. Shakespeare:

$i_1 : (w \land hn) \to c$
$i_2 : (w \land p) \to he$
$i_3 : s \to (w \land h)$
$i_4 : (w \land he) \to hn$
$i_5 : h \to p$
$c : s \to c$

1. $(w \land hn) \to c$ premisă
2. $(w \land p) \to he$ premisă
3. $s \to (w \land h)$ premisă
4. $(w \land he) \to hn$ premisă
5. $h \to p$ premisă

6. s ipoteză
7. $w \land h$ $(\to e)$,3,6
8. w $(\land e_1)$,7

9.	h	$(\wedge e_2, 7)$
10.	p	$(\to e), 5, 9$
11.	$v \wedge p$	$(\wedge i), 8, 10$
12.	he	$(\to e), 2, 11$
13.	$v \wedge he$	$(\wedge i), 8, 12$
14.	hn	$(\to e), 4, 13$
15.	$v \wedge hn$	$(\wedge i), 8, 14$
16.	c	$(\to e), 1, 15$
17.	$s \to c$	$(\to i), 6-16$

Ex. echivalență logică

$\varphi \leftrightarrow \psi = (\varphi \to \psi) \wedge (\psi \to \varphi)$

Găsiți regulile de introducere/eliminare pt $\leftrightarrow$

$$\varphi \leftrightarrow \psi = (\varphi \to \psi) \wedge (\psi \to \varphi) \Leftrightarrow \frac{\boxed{\begin{array}{c}\varphi \\ \vdots \\ \psi\end{array}} \quad \boxed{\begin{array}{c}\psi \\ \vdots \\ \varphi\end{array}}}{\varphi \leftrightarrow \psi}$$

1. $\varphi \leftrightarrow \psi$ premisă
2. φ premisă
3. $\varphi \to \psi$ $(\wedge e_1), 1$
4. ψ $(\to e), 2, 3$

$$\frac{\dfrac{\varphi \leftrightarrow \psi}{\varphi \to \psi}(\wedge e) \quad \varphi}{\psi}(\to e) \qquad \frac{\dfrac{\varphi \leftrightarrow \psi}{\psi \to \varphi}(\wedge e) \quad \psi}{\varphi}(\to e)$$

$$\Rightarrow \frac{\varphi \leftrightarrow \psi \quad \varphi}{\psi}(\leftrightarrow e) \qquad \frac{\varphi \leftrightarrow \psi \quad \psi}{\varphi}(\leftrightarrow e)$$

Ex 2.4 Fie $n \geq 1$ și $\varphi_1, \dots, \varphi_n, \varphi$ formule. Dem. că dacă $\vdash \varphi_1 \to (\varphi_2 \to (\dots \to (\varphi_n \to \varphi) \dots))$ este valid, at. $\varphi_1, \dots, \varphi_n \vdash \varphi$ este valid

- Considerăm $\varphi_1, \dots, \varphi_n$ premise.

1. φ_1 premisă
2. $\varphi_1 \to (\varphi_2 \to (\dots \to (\varphi_n \to \varphi) \dots))$ teoremă
3. $\varphi_2 \to (\dots (\varphi_n \to \varphi) \dots)$ $(\to e), 1, 2$
4. φ_2 premisă

⋮ Aplicăm de $(n-1)$ ori $(\to e)$

x_i. φ

Puncte fixe

! Un elem. $a \in C$ este pct. fixe al unei funcții $f: C \to C$, dc $f(a) = a$

! Un elem. $lfp \in C$ este cel mai mic pct. fixe al unei funcții $f: C \to C$ dc este pct fixe și pt $\forall a \in C$ al lui f avem $lfp \leq_C a$.

Care sunt pct fixe ale urm. funcții?

$f_1: \mathcal{P}(\{1,2,3\}) \to \mathcal{P}(\{1,2,3\})$, $f_1(Y) = Y \cup \{1\}$

A pct fixe $\Leftrightarrow f_1(A) = A$

$Y \in \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Pct fixe: $\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}$

Cel mai mic pct fixe: $\{1\}$

$f_2(Y) = \begin{cases} \{1\}, & \text{dc } 1 \in Y \\ \emptyset, & \text{altfel} \end{cases}$

$f_2(\emptyset) = \emptyset \Rightarrow \emptyset$ pct. fixe

$f_2(\{1\}) = \{1\} \Rightarrow \{1\}$ pct fixe

$f_2(\{2\}) = \emptyset$

$f_2(\{3\}) = \emptyset$

$f_2(\{1,2\}) = \{1\}$

$\vdots$

$\emptyset \subseteq \{1\}$; $\emptyset \leq_C \{1\} \Rightarrow \emptyset$ este cel mai mic pct. fixe

$f_3(Y) = \begin{cases} \emptyset, & \text{dc } 1 \in Y \\ \{1\}, & \text{altfel} \end{cases}$

$f_3(\emptyset) = \{1\}$

$f_3(\{1\}) = \emptyset$ $\dots \Rightarrow f_3$ nu are pct. fixe

$f_S : \mathcal{P}(At) \to \mathcal{P}(At)$

f_S este monotonă $\Leftrightarrow (\forall)\ y_1, y_2 \in \mathcal{P}(At)$ a.î. $y_1 \subseteq y_2$ at. $f_S(y_1) \subseteq f_S(y_2)$.

Fie $y_1, y_2 \in \mathcal{P}(At)$ a.î. $y_1 \subseteq y_2$. Vrem să dem. că $f_S(y_1) \subseteq f_S(y_2) \Leftrightarrow y_1 \cup Baza \cup \{a \in At | \ldots$

Ex curs 6 slide 28: $At = \{cold, wet, windy, scotland, dry\}$

$Baza = \{cold\}$

$f_S(\emptyset) = \emptyset \cup \{cold\} = \{cold\}$

$f_S^2(\emptyset) = f_S(\{cold\}) = \{cold\} \cup \{cold\} \cup \{wet\}$
$= \{cold, wet\}$

$f_S^3(\emptyset) = f_S(\{cold, wet\}) = \{cold, wet, scotland\}$

$f_S^4(\emptyset) = f_S(\{\underline{cold, wet, scotland}\}) = \{cold, wet, scotland\} = f_S^3(\emptyset)$

cel mai mic pct. fix pt. f_S

Calc. cel mai mic pct. fix pt funcțiile:

$S_1 = \{x_1 \wedge x_2 \to x_3, x_4 \wedge x_2 \to x_5, x_2, x_6, x_6 \to x_1\}$

$At = \{x_1, x_2, x_3, x_4, x_5, x_6\}$

$Baza = \{x_2, x_6\}$

$f_{S_1}(\emptyset) = \emptyset \cup \{x_2, x_6\} = \{x_2, x_6\}$

$f_{S_1}^2(\emptyset) = f_{S_1}(\{x_2, x_6\}) = \{x_2, x_6\} \cup \{x_2, x_6\} \cup \{x_1\}$
$= \{x_1, x_2, x_6\}$

$f_{S_1}^3(\emptyset) = f_{S_1}(\{x_1, x_2, x_6\}) = \{x_1, x_2, x_3, x_6\}$

$f_{S_1}^4(\emptyset) = f_{S_1}^3(\emptyset) \Rightarrow \{x_1, x_2, x_3, x_6\}$ cel mai mic pct fix pt f_{S_1}.

$S_2 = \{x_1 \wedge x_2 \to x_3, x_4 \to x_1, x_5 \to x_2, x_2 \to x_5, x_4\}$

$At = \{x_1, x_2, x_3, x_4, x_5\}$

$Baza = \{x_4\}$

$f_{S_2}(\emptyset) = \emptyset \cup \{x_4\} = \{x_4\}$

$f_{S_2}(f_{S_2}(\emptyset)) = f_{S_2}(\{x_4\}) = \{x_4\} \cup \{x_4\} \cup \{x_1\}$
$= \{x_1, x_4\}$

$f_{S_2}(\{x_1, x_4\}) = \underline{\{x_1, x_4\}} = f_{S_2}(\{x_4\})$

↳ cel mai mic pct fix pt f_{S_2}

$S_3 = \{x_1 \to x_2, x_1 \wedge x_3 \to x_1, x_3\}$

$At = \{x_1, x_2, x_3\}$

$Baza = \{x_3\}$

$f_{S_3}(\emptyset) = \emptyset \cup \{x_3\} = \{x_3\}$

$f_{S_3}(\{x_3\}) = \{x_3\} \cup \{x_3\} \cup \emptyset = \underline{\{x_3\}} = f_{S_3}(\emptyset)$

↳ cel mai mic pct. fix

Ex 3.2: Este $f_S(Y) = Y \cup Baza \cup \{a \in A \mid (s_1 \wedge \ldots \wedge s_n \to a)$ este în S, $s_1 \in Y, \ldots, s_n \in Y\}$ monotonă?

f_S monotonă $\Leftrightarrow \exists\, y_1, y_2 \in \mathcal{P}(A)$ a.î. $y_1 \subseteq y_2$ atunci $f_S(y_1) \subseteq f_S(y_2)$

$\Leftrightarrow y_1 \cup Baza \cup \{a \in A \mid (s_1 \wedge \ldots \wedge s_n) \to a$ este în S, $s_1 \in y_1, \ldots, s_n \in y_1\} \subseteq$
$y_2 \cup Baza \cup \{a \in A \mid (s_1 \wedge \ldots \wedge s_n) \to a$ este în S, $s_1 \in y_2, \ldots, s_n \in y_2\}$

Vrem să dem. $M_1 \subseteq M_2$.

Fie $a \in M_1 \Rightarrow \exists\, s_1 \wedge \ldots \wedge s_n \to a \in S$ și $s_1 \wedge \ldots \wedge s_n \in y_1$

Cum $y_1 \subseteq y_2 \Rightarrow s_1 \ldots s_n \in y_2 \Rightarrow a \in M_2$

Prin urmare, $f_S(y_1) \subseteq f_S(y_2) \Rightarrow f_S$ este monotonă.

Rezoluție SLD

Găsiți o SLD - respingere pt. urm. prog. Prolog

a)
1. r :- p, q.
2. s :- p, q.
3. v :- t, u.
4. w :- v, s.
5. t.
6. q.
7. u.
8. p.

?- w.

$r :- p, q \Leftrightarrow p \wedge q \rightarrow r \Leftrightarrow \neg(p \wedge q) \vee r \Leftrightarrow \neg p \vee \neg q \vee r$

1. $\neg p \vee \neg q \vee r$
2. $\neg p \vee \neg q \vee s$
3. $\neg t \vee \neg u \vee v$
4. $\neg v \vee \neg s \vee w$
5. $t.$
6. q
7. u
8. p

?- w

$G_0 = \neg w$

$G_1 = \neg v \vee \neg s$ (4)

$G_2 = \neg t \vee \neg u \vee \neg s$ (3)

$G_3 = \neg u \vee \neg s$ (5)

$G_4 = \neg s$ (7)

$G_5 = \neg p \vee \neg q$ (2)

$G_6 = \neg q$ (8)

$G_7 = \square$ (6)

b)
1. $q(X,Y) :- q(Y,X), q(Y, f(f(Y)))$.
2. $q(a, f(f(X)))$.

?- $q(f(Z), a)$.

$q(X,Y) :- q(Y,X), q(Y, f(f(Y))) \Leftrightarrow$

$q(Y,X) \wedge q(Y, f(f(Y))) \rightarrow q(X,Y). \Leftrightarrow$

$\neg(q(Y,X) \wedge q(Y, f(f(Y)))) \vee q(X,Y) \Leftrightarrow$

1. $\neg q(Y,X) \vee \neg q(Y, f(f(Y))) \vee q(X,Y)$.
2. $q(a, f(f(X)))$.

?- $q(f(Z), a)$.

$G_0 = \neg q(f(Z), a)$. $\theta(X) = f(Z)$, $\theta(Y) = a$.

$G_1 = \neg q(a, f(z)) \vee \neg q(a, f(f(a)))$ (1) $\theta(x)=a$

$G_2 = \neg q(a, f(z))$ (2) $\theta(z) = f(x)$

$G_3 = \square$ (2)

c) 1. p(X) :- q(X, f(Y)), r(a).
2. p(X) :- r(X).
3. q(X, Y) :- p(Y).
4. r(X) :- q(X, Y).
5. r(f(b)).
?- p(X), q(Y, Z).

1. $\neg q(X, f(Y)) \vee \neg r(a) \vee p(X)$.
2. $\neg r(X) \vee p(X)$.
3. $\neg p(Y) \vee q(X, Y)$.
4. $\neg q(X, Y) \vee r(X)$.
5. $r(f(b))$.

?- $p(X) \wedge q(Y, Z)$

$G_0 = \neg p(X) \vee \neg q(Y, Z)$.

$G_1 = \neg r(X) \vee \neg q(Y, Z)$. (2) $\theta(X) = f(b)$.

$G_2 = \neg q(Y, Z)$. (5) $\theta(X) = Y$, $\theta(Y) = Z$.

$G_3 = \neg p(Z)$. (3) $\theta(X) = Z$

$G_4 = \neg r(Z)$ (2) $\theta(Z) = f(b)$

$G_5 = \square$ (5)

Desenați arborele SLD pt. prog Prolog de mai jos și ținta ?- p(X, X).

1. p(X, Y) :- q(X, Z), r(Z, Y).
2. p(X, X) :- s(X).
3. q(X, b).
4. q(b, a).
5. q(X, a) :- r(a, X).
6. r(b, a).
7. s(X) :- t(X, a).
8. s(X) :- t(X, b).

9. $s(X) :- t(X,X).$

10. $t(a,b).$ 11. $t(b,a).$

1. $\neg q(X,Z) \vee \neg r(Z,Y) \vee p(X,Y)$
2. $\neg s(X) \vee p(X,X).$
3. $q(X,b).$
4. $q(b,a).$
5. $\neg r(a,X) \vee q(X,a).$
6. $r(b,a).$
7. $\neg t(X,a) \vee s(X).$
8. $\neg t(X,b) \vee s(X).$
9. $\neg t(X,X) \vee s(X).$
10. $t(a,b).$
11. $t(b,a).$