Recurrence Quantification Analysis

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Research question

Is there an association between the performance of a group and the patterns and recurrence of behavior and deeper level measurements of the interactions in a CPS environment?

Using categorical recurrence quantification analysis to capture the patterns of behavior codes in a CPS conversation, and predicting performance using measures related to the recurrence of the codes.

Data/Codes

|	task	team_id	mellon_id	event_start_time	event_end_time	event_result	Primary code	Primary Sub-code	Secondary Code
0	apartment	179706_162850_162936_182908	162850	2019-09-28 17:14:21+00:00	2019-09-28 17:14:21+00:00	Hi	SMC	SMC-R	NaN
1	apartment	179706_162850_162936_182908	182908	2019-09-28 17:14:39+00:00	2019-09-28 17:14:39+00:00	hey	SMC	SMC-R	NaN
2	apartment	179706_162850_162936_182908	162936	2019-09-28 17:14:47+00:00	2019-09-28 17:14:47+00:00	Hey	SMC	SMC-R	NaN
3	apartment	179706_162850_162936_182908	179706	2019-09-28 17:14:48+00:00	2019-09-28 17:14:48+00:00	hola	SMC	SMC-R	NaN
4	apartment	179706_162850_162936_182908	162850	2019-09-28 17:14:56+00:00	2019-09-28 17:14:56+00:00	Is everyone here?	CM	CM-G	NaN
...	...	...	...	...	...	...	...	...	...

Codes

- Social
  - 'SMC', 'SSI', 'SESU', 'SN'
- Cognitive
  - 'CM', 'CE', 'CRF', 'CP'

Recurrence rate

The rate of recurrence of interaction which can be interpreted by dividing the number of ones by total number of cells in the matrix, which is the number of total messages squared.

In our example, it's the rate of the reoccurrence of the same code in a CPS conversation. Meaning that the higher the RR, the more same actions recur throughout the conversation.

$$\mathbf{RR} = \frac{1}{N^2} \sum_{i,j=1}^{N} \mathbf{R}(i,j).$$

Recurrence Plot of 18-15
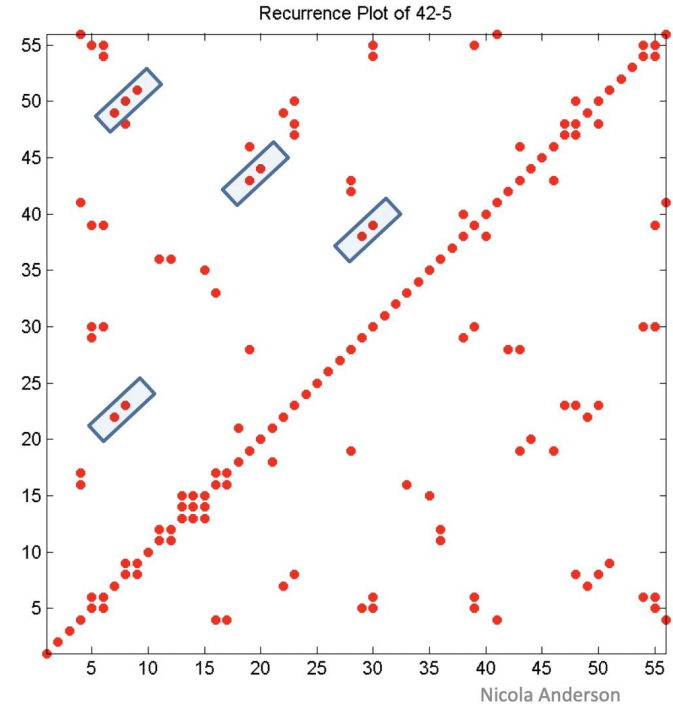
Σ

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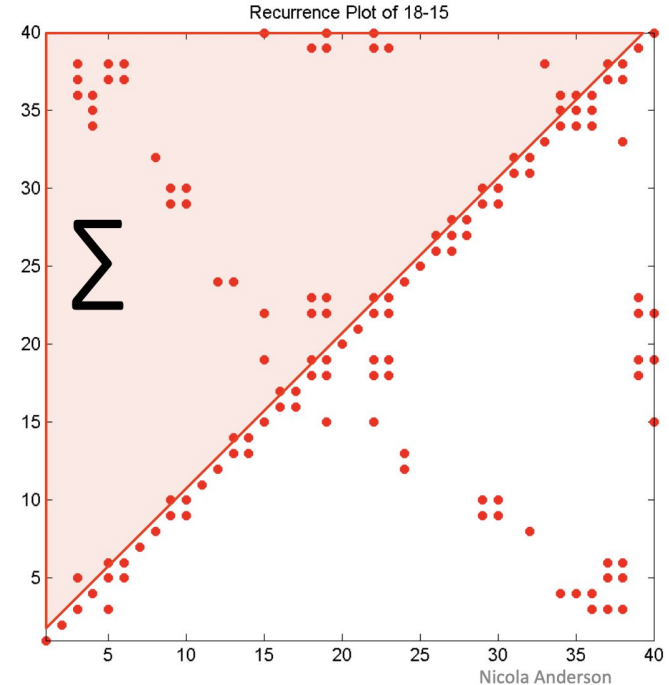
Determinism

Determinism is a measure of distribution of recurrence points across diagonal lines. In simpler words, it measures the percentage of recurrence points which form diagonal lines.

Forming a diagonal line means the same code of interaction has been used in back to back messages, and the more often the recurrence points align in a diagonal line the higher determinism that conversation has.

$$\text{DET} = \frac{\sum_{\ell=\ell_{\min}}^{N} \ell\, P(\ell)}{\sum_{\ell=1}^{N} \ell P(\ell)}$$


Recurrence Plot of 42-5

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Laminarity

Laminarity is a measure of distribution of recurrence points across vertical lines. In simpler words, it measures the percentage of recurrence points which form vertical lines.

Laminarity captures the return from time stamps into the same previously seen location. Higher laminarity in our instance shows the same code is repeated from a previously seen code within the conversation.

$$\text{LAM} = \frac{\sum_{v=v_{\min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)}$$
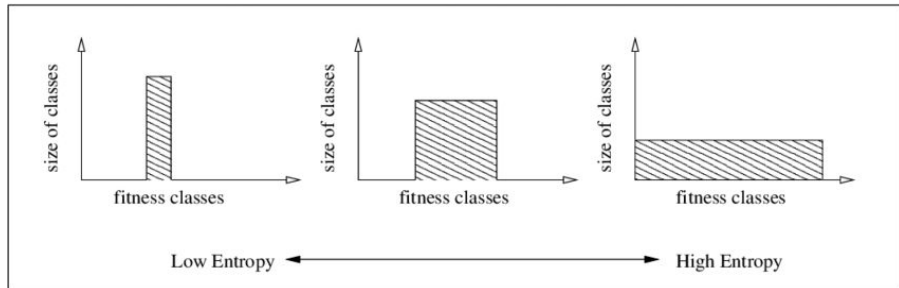

Recurrence Plot of 18-15
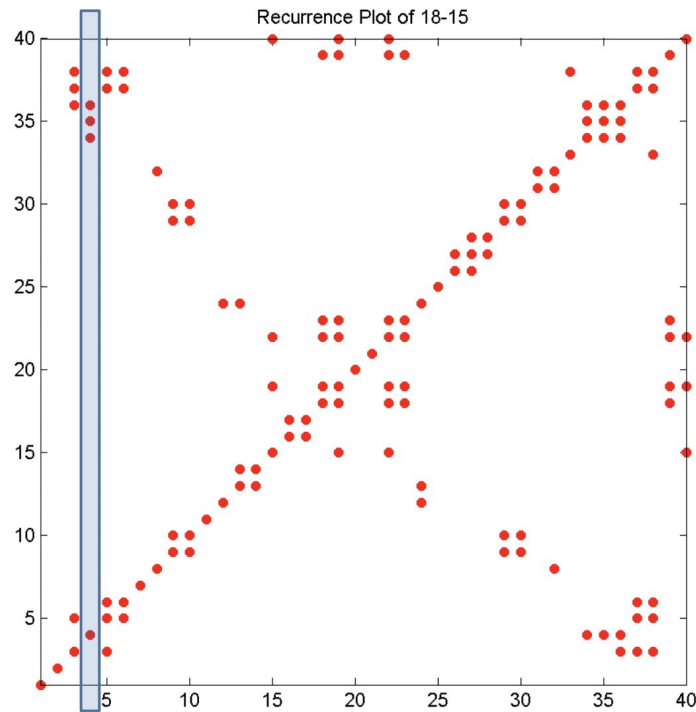
Divergence

The inverse of longest diagonal line.

Related with the KS entropy of the system, i.e. with the sum of the positive **Lyapunov exponents**.

$$DIV = \frac{1}{L_{max}}$$

Entropy

The Shannon entropy of the diagonal lines contained in the recurrence matrix, ruling out the lines shorter than l_{min} (2 by default)

$$\text{ENTR} = -\sum_{\ell=\ell_{min}}^{N} p(\ell) \ln p(\ell),$$

While *p(l)* is the probability of a diagonal line having a length *l* .

Entropy is a measure of uncertainty, complexity and randomness.

Results

- Experiment
  - Predicting performance based on the RQA measurements
  - Binary outcome of high performance is used as an outcome. The variable indicates whether a group has above average performance or not. And the input are the discussed measurements along with lower level measures.

Logit Regression Results

Dep. Variable:	y	**No. Observations:**	821
Model:	Logit	**Df Residuals:**	814
Method:	MLE	**Df Model:**	6
Date:	Thu, 13 May 2021	**Pseudo R-squ.:**	0.1252
Time:	14:56:17	**Log-Likelihood:**	-244.62
converged:	False	**LL-Null:**	-279.63
Covariance Type:	nonrobust	**LLR p-value:**	4.063e-13

	coef	std err	z	P>|z|	[0.025	0.975]
determinism	-17.3767	4.509	-3.854	0.000	-26.213	-8.540
recurrence_rate	26.5851	7.582	3.506	0.000	11.724	41.446
entropy	0.7676	1.442	0.532	0.595	-2.059	3.594
longest_diagonal_line	-0.0013	7.82e+04	-1.69e-08	1.000	-1.53e+05	1.53e+05
laminarity	2.2270	2.374	0.938	0.348	-2.426	6.880
divergence	-29.1490	7.583	-3.844	0.000	-44.011	-14.287
num_messages	-0.0015	7.82e+04	-1.96e-08	1.000	-1.53e+05	1.53e+05