8.6 *Lab*: Perfect Numbers

A whole number is called *perfect* if it is equal to the sum of all of its divisors, including 1 (but excluding the number itself). For example, 28 = 1 + 2 + 4 + 7 + 14. Perfect numbers were known in ancient Greece. In Book VII of *Elements*, Euclid (300 BC) defined a perfect number as one "which is equal to its own parts."

Nicomachus, a Greek mathematician of the first century, wrote in his *Introduction to Arithmetic* (around A.D. 100):

> *In the case of the too much, is produced excess, superfluity, exaggerations and abuse; in the case of too little, is produced wanting, defaults, privations and insufficiencies. And in the case of those that are found between the too much and the too little, that is, in equality, is produced virtue, just measure, propriety, beauty and things of that sort - of which the most exemplary form is that type of number which is called perfect.*

Unfortunately, Nicomachus had many mistakes in his book. For example, he stated erroneously that the *n*-th perfect number has *n* digits and that perfect numbers end alternately in 6 and 8. He knew of only four perfect numbers and jumped to conclusions.

Write a program to find the first four perfect numbers.

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You might be tempted to use your program to find the fifth perfect number. Then you'd better be patient: on a relatively fast computer, it could take almost an hour. There is a better strategy. Euclid proved that if you find a number of the form $2^n - 1$ that is a prime, then $2^{n-1}(2^n - 1)$ is a perfect number! For example, $(2^3 - 1) = 7$ is a prime, so $28 = 2^2(2^3 - 1)$ is a perfect number. Many centuries later, Euler proved that any <u>even</u> perfect number must have this form. Therefore the search for even perfect numbers can be reduced to the search for primes that have the form $2^n - 1$. Such primes are called *Mersenne primes*, after the French math enthusiast Marin Mersenne (1588-1648) who made them popular.

In 1996, George Woltman, a software engineer, started The Great Internet Mersenne Prime Search project (GIMPS). In this project, volunteers contribute idle CPU time on their personal computers for the search. The 42nd Mersenne prime, $2^{25,964,951} - 1$, was found on February 18, 2005. (It has 7,816,230 digits.)

Write a program to find the first six Mersenne primes, and use them to calculate the first six perfect numbers. Note that while the sixth Mersenne is still well within the Java `int` range, the sixth perfect number, 8,589,869,056, is not. Use a `long` variable to hold it.

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It is unknown to this day whether any odd perfect numbers exist. It has been shown that such a number must have at least 300 digits!