Maxwell equations

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Definition of average energy -

$$\overline{E} = \frac{\int_0^\infty n(E)EdE}{\int_0^\infty n(E)dE} \tag{1}$$

$$n(E) = n_0 \frac{2\pi}{(\pi kT)^{\frac{3}{2}}}\ \sqrt{E}\ e^{-\frac{E}{kT}} \tag{2}$$

Denominator first -

$$\int_0^\infty n(E)dE \equiv F(E) = a\ n_0 \int_0^\infty \sqrt{E}\ e^{-bE}dE \tag{3}$$

$$a \equiv \frac{2\pi}{(\pi kT)^{\frac{3}{2}}}$$

$$b \equiv \frac{1}{kT}$$

$$x = \sqrt{E}$$
$$2x\ dx = dE$$

$$F(E) \to F(x) = a\ n_0 \int_0^\infty x\ e^{-bx^2}\ 2xdx \tag{4}$$

$$\int u\ dv = uv - \int v\ du$$

$$u = x$$
$$du = dx$$

$$dv = 2x\ e^{-bx^2}\ dx$$
$$v = -\frac{1}{b}\ e^{-bx^2}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x\ e^{-bx^2} + \frac{1}{b}\ \int_0^\infty e^{-bx^2}\ dx] \tag{5}$$

$$w = x\sqrt{b}$$
$$\frac{1}{\sqrt{b}}\ dw = dx$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x\ e^{-bx^2} + \frac{1}{b}\ \int_0^\infty e^{-w^2}\ \frac{1}{b}\ dw] \tag{6}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x\ e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}}\ \int_0^\infty e^{-w^2}\ dw] \tag{7}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x\ e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}}\ \frac{\sqrt{\pi}}{2}\ \int_0^\infty \frac{2}{\sqrt{\pi}}\ e^{-w^2}\ dw] \tag{8}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x\ e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}}\ \frac{\sqrt{\pi}}{2}\ \mathrm{erf}(w)] \tag{9}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x\ e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}}\ \frac{\sqrt{\pi}}{2}\ \mathrm{erf}(x\sqrt{b})] \tag{10}$$

$$F(x) = a\ n_0\ [-\frac{1}{b}\ x\ e^{-bx^2} + \frac{1}{b^{\frac{3}{2}}}\ \frac{\sqrt{\pi}}{2}\ \mathrm{erf}(x\sqrt{b})]\Bigg|_{0}^{\infty} \tag{11}$$

$$F(E) = a\ n_0\ [-\frac{1}{b}\ \sqrt{E}\ e^{-bE} + \frac{1}{b^{\frac{3}{2}}}\ \frac{\sqrt{\pi}}{2}\ \operatorname{erf}(\sqrt{bE})]\Big|_0^{\infty} \tag{12}$$

$$F(E) = a\ n_0[(0 + \frac{1}{b^{\frac{3}{2}}}\ \frac{\sqrt{\pi}}{2}\ 1) - (0 + 0)] \tag{13}$$

$$F(E) = \frac{2\pi}{\sqrt{\pi^3 k^3 T^3}}\ n_0[\frac{k^3 T^3}{}\ \frac{\sqrt{\pi}}{2}] \tag{14}$$

$$\int_0^{\infty} n(E) dE = n_0 \tag{15}$$

Now numerator -

$$\int_0^\infty n(E)EdE \equiv F(E) = a\ n_0 \int_0^\infty \sqrt{E}E\ e^{-bE}dE \tag{16}$$

$$a \equiv \frac{2\pi}{(\pi kT)^{\frac{3}{2}}}$$

$$b \equiv \frac{1}{kT}$$

$$x = \sqrt{E}$$
$$2x\ dx = dE$$

$$F(E) \to F(x) = a\ n_0 \int_0^\infty x\ x^2\ e^{-bx^2}\ 2xdx \tag{17}$$

$$F(E) \to F(x) = a\ n_0 \int_0^\infty x^3\ e^{-bx^2}\ 2xdx \tag{18}$$

$$\int u\ dv = uv - \int v\ du$$

$$u = x^3$$
$$du = 3x^2\ dx$$

$$dv = 2x\ e^{-bx^2}dx$$
$$v = -\frac{1}{b}\ e^{-bx^2}$$

$$F(x) = a\ n_0[-\frac{1}{b}x^3\ e^{-bx^2} + \frac{3}{b}\int_0^\infty x^2\ e^{-bx^2}\ dx] \tag{19}$$

$$F(x) = a\ n_0[-\frac{1}{b}x^3\ e^{-bx^2} + \frac{3}{b}(\int_0^\infty x\ x\ e^{-bx^2}\ dx)] \tag{20}$$

$$u = x$$
$$du = dx$$

$$\begin{aligned} dv &= x\ e^{-bx^2}\ dx \\ v &= -\frac{1}{2b}\ e^{-bx^2} \end{aligned} \tag{21}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x^3\ e^{-bx^2} + \frac{3}{b}(-\frac{1}{2b}\ x\ e^{-bx^2} + \frac{1}{2b}\int_0^\infty e^{-bx^2}\ dx)] \tag{22}$$

$$w = x\sqrt{b}$$
$$\frac{1}{\sqrt{b}}\ dw = dx$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x^3\ e^{-bx^2} + \frac{3}{b}(-\frac{1}{2b}\ x\ e^{-bx^2} + \frac{1}{2b}\int_0^\infty e^{-w^2}\ \frac{1}{\sqrt{b}}\ dw)] \tag{23}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x^3\ e^{-bx^2} + \frac{3}{b}(-\frac{1}{2b}\ x\ e^{-bx^2} + \frac{1}{2b^{\frac{3}{2}}}\int_0^\infty e^{-w^2}\ dw)] \tag{24}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x^3\ e^{-bx^2} - \frac{3}{2b^2}\ x\ e^{-bx^2} + \frac{3}{2b^{\frac{5}{2}}}\int_0^\infty e^{-w^2}\ dw] \tag{25}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x^3\ e^{-bx^2} - \frac{3}{2b^2}\ x\ e^{-bx^2} + \frac{3}{2b^{\frac{5}{2}}}\frac{\sqrt{\pi}}{2}\int_0^\infty \frac{2}{\sqrt{\pi}}\ e^{-w^2}\ dw] \tag{26}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x^3\ e^{-bx^2} - \frac{3}{2b^2}\ x\ e^{-bx^2} + \frac{3}{2b^{\frac{5}{2}}}\frac{\sqrt{\pi}}{2}\,\mathrm{erf}(w)] \tag{27}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ x^3\ e^{-bx^2} - \frac{3}{2b^2}\ x\ e^{-bx^2} + \frac{3}{2b^{\frac{5}{2}}}\frac{\sqrt{\pi}}{2}\,\mathrm{erf}(x\sqrt{b})] \tag{28}$$

$$F(x) = a\ n_0[-\frac{1}{b}\ \sqrt{E^3}\ e^{-bE} - \frac{3}{2b^2}\ \sqrt{E}\ e^{-bE} + \frac{3}{2b^{\frac{5}{2}}}\frac{\sqrt{\pi}}{2}\,\mathrm{erf}(\sqrt{bE})] \tag{29}$$

$$F(E) = a\ n_0\ [-\frac{1}{b}\ \sqrt{E^3}\ e^{-bE} - \frac{3}{2b^2}\ \sqrt{E}\ e^{-bE} + \frac{3}{2b^{\frac{5}{2}}}\frac{\sqrt{\pi}}{2}\,\mathrm{erf}(\sqrt{bE})]\Bigg|\begin{matrix}\infty\\0\end{matrix} \tag{30}$$

$$F(E) = a\ n_0[(0 - 0 + \frac{3}{2b^{\frac{5}{2}}}\frac{\sqrt{\pi}}{2}) - (0 - 0 + 0)] \tag{31}$$

$$F(E) = a\ n_0\ \frac{3}{2b^{\frac{5}{2}}}\frac{\sqrt{\pi}}{2} \tag{32}$$

$$F(E) = \frac{2\pi}{\sqrt{\pi^3 k^3 T^3}}\ n_0\ \frac{3\sqrt{k^5T^5}}{2}\frac{\sqrt{\pi}}{2} \tag{33}$$

$$F(E) = \frac{}{\sqrt{k^3T^3}}\ n_0\ \frac{3\sqrt{k^5T^5}}{2} \tag{34}$$

$$F(E) = \frac{3}{2}\ n_0\ kT \tag{35}$$

$$\overline{E} = \frac{\int_0^\infty n(E) E dE}{\int_0^\infty n(E) dE} \tag{36}$$

$$\overline{E} = \frac{\frac{3}{2}\; n_0\; kT}{n_0} \tag{37}$$

$$\overline{E} = \frac{3}{2} kT \tag{38}$$