# Principles of nuclear engineering

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R. A. Borrelli

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# About the author

**Prof. R. A. Borrelli** (*Bob*) is currently and Assistant Professor in the Department of Nuclear Engineering and Industrial Management at the University of Idaho – Idaho Falls Center for Higher Education since 2015.

**Educational background**
Ph.D – University of California-Berkeley (2006)
*Coupled modeling of radionuclide transport with bentonite extrusion of a nuclear waste repository*

MS – Worcester Polytechnic Institute, Civil/Environmental Engineering (1999)
*Characterization of background radiation in the environment*

BS – Worcester Polytechnic Institute, Mechanical Engineering (1996)
*Real time PLC-based reactivity modeling by inverse point kinetics*

**Sketch**
Bob received a BS in Mechanical and Nuclear Engineering and then an MS in Civil and Environmental Engineering at Worcester Polytechnic Institute, where he was also an NRC licensed Senior Reactor Operator at the Leslie C. Wilbur Nuclear Reacotr Facility. He received his Ph.D. in Nuclear Engineering at UC-Berkeley, working with Prof. Joonhong Ahn in high level nuclear waste management. He continued study in this field at The University of Tokyo, and then returned to UC-Berkeley for a second postdoctorate position in the field of safeguards, safety, and security with Profs. Joonhong Ahn and Per Peterson. When this position ended, Bob taught engineering at Diablo Valley Community College in the SF Bay area. He then joined the faculty at the University of Idaho in his current position.

His broad research interests include advanced fuel cycle analysis and risk assessment. This includes developing methodologies to optimize proliferation resistance and physical protection with the safety and security for the advanced nuclear fuel cycle, nuclear hybrid energy systems modeling and transportation of advanced nuclear reactor components, nuclear cybersecurity and plant modernization, RTG satellite design for deep space missions, and risk assessment of disruptions to the grid.

He restarted the Student Section of the American Nuclear Society at the University of Idaho in 2015. The Section won second place for the Samuel Glasstone Award for service in 2019. He is an active member of the Idaho Section of the American Nuclear Society, donating time for various community service projects, as well as serving in various Professional Divisions of the American Nuclear Society on the national level, including the Fuel Cycle and Waste Management and Nuclear Nonproliferation Divisions and the Student Sections Committee.

**CV**

# Preface

**Nuclear engineering**
Nuclear engineering encompasses many different fields. Therefore, there is not really any singular textbooks encompassing the discipline. Currently, there are three main textbooks used in the United States –

> Introduction to nuclear engineering – John R. Lamarsh
> Nuclear reactor analysis – James Duderstadt
> Fundamentals of nuclear science and engineering – J. Kenneth Shultis, Richard E. Faw
> Nuclear chemical engineering – M. Benedict, T. H. Pigford, H. W. Levi

Typically, a beginning nuclear engineering course will use either of the first three textbooks. I use Lamarsh. The fourth book is the top reference text for chemical engineering in the discipline.

I do recommend at least owning one of them for reference. However, these books can be very costly, so owning all is difficult to ask of a student.

**Motivation**
In addition to the prohibitive cost, over the last decade, the discipline has been quite dynamic. Advanced reactors are being developed with over forty startup companies in the United States, particularly microreactors. The Department of Energy is funding some of them to demonstrate these advanced concepts. NuScale Power, LLC, has developed a small modular reactor and will deploy them on the Idaho National Laboratory site by the end of the 2020s. That is just reactors. There have been major developments in regards to international safeguards. Digitization of instrumentation for the large scale power plants will introduce new vulnerabilities that will challenge cybersecurity. Similarly, these new advanced concepts will feature total digital controls. Some will even be fully automated. This is only a cursory overview of the discipline. There includes other interesting advancements, including new fuel designs, storage, transportation, hybrid energy systems, human factors, and economics.

While neutrons actually have behaved the same since the beginning of time, and there will still be a need for understanding fundamental concepts contained in the above cited list of textbooks, the dynamic landscape of nuclear engineering would require a new text every year. This is completely unreasonable for student and faculty.

Therefore, I have compiled this OER of online, publicly available resources to enhance and supplement fundamental knowledge in nuclear engineering, in order to provide a more comprehensive learning experience for the student, in an effort to prepare them for professional careers in the discipline.

**Target**
Students using this OER should have at least completed or be at the junior undergraduate level, in terms of background knowledge needed. The OER should be used as a supplementary resource for an introductory nuclear engineering course in companion to one of the textbooks listed above. It is intended that the OER will be a 'living document', evolving with the discipline in order to provide use beyond the classroom.

**Structure and content**
The OER is broadly structured as follows –

> Lectures and slides
> Reports and papers
> Videos and other media

Sections will invariably have some overlap – resources in one section contain information that is applicable to another section. The intent at this version of organization is to just make it easier for the student to find the information they need.

I

# 1.0. Nuclear engineering basics

Chapter 1 covers the essential foundation for higher level nuclear engineering material Some of this content could be review for junior undergraduates and up.

## Learning Objectives

- Demonstrating fundamental concepts in nuclear physics
- Analyzing decay data to identify radionuclides
- Summarizing different decay mechanisms

Prof. Borrelli slides

# 1

# Atomic structure

In nuclear engineering, the shell model of the atom is used. This posits the nucleus is fixed in the center of the atom, surrounded by shells corresponding to electron energy levels. The shell model was developed by Niels Bohr.

**Additional notes**

- MIT Open Courseware – Introduction to Solid State Chemistry
- Brock University – Electromagnetism, Optics, and Modern Physics
- Portland State University – PHY381

# 2

# Energy and matter

A more famous equation there may never be –

[latex] e = mc^2 [/latex].

Its simple elegance gives the equivalence between matter and energy. The Einstein equation is step one in learning nuclear engineering because uranium will be fissioned to release huge amounts of energy for many different purposes.

**Additional notes**

- MIT Open Courseware – Introduction to Applied Nuclear Physics
- MIT Open Courseware – Relativity
- Stanford Encyclopedia of Philosophy – The equivalence of mass and energy
- Einstein archives online
- Albert Einstein Center for Fundamental Physics – Lecture notes on general relativity
- Feynman Hughes Lectures
  1966 – 1971

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# Nuclear physics

Nuclear physics is a very wide field. It is also essential to the entire fuel cycle, from reactor design, to final disposal. Nuclear physics is a important to detector design as well. We don’t need to be nuclear physicists, however, to be nuclear engineers. We need to know how to apply these concepts.

**Additional notes**

- MIT Open Courseware – Introduction to Applied Nuclear Physics
- Michigan State University – Nuclear Shell Model Applications
- The University of Manchester – Introduction to Nuclear and Particle Physics
- Online books by Enrico Fermi
- University of Oxford – Nuclear physics lecture notes
- SMU – Introduction to modern physics
- North Dakota State University – PHY120: Fundamentals of Physics
- San Francisco State University – PHYSICS 430 Lecture Notes on Quantum Mechanics

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# Radioactive decay

Many atoms are unstable even if they occur in nature. This can mean the nucleus has either too many protons or too many neutrons. Therefore, a particle transformation takes place; e.g., a neutron is converted to a proton, or particles are emitted, and then energy is released as gamma rays or x rays. This is called radioactive decay.

## common forms of decay

- Alpha particle decay is the ejection of a helium atom from a heavy nucleus.

[latex] ^{238}_{92}U \rightarrow ^{234}_{90}Th + ^4_2\alpha [/latex]

- Beta(-) particle decay occurs for unstable nuclei with excessive neutrons.

[latex] ^{19}_{8}O \rightarrow ^{19}_{9}F + \beta^- + \bar{\nu} [/latex]

- Beta(+) particle decay occurs for unstable nuclei with deficient neutrons.

[latex] ^{11}_{6}C \rightarrow ^{11}_{5}B + \beta^+ + \nu [/latex]

- Electron capture also occurs for unstable nuclei with deficient neutrons.

[latex] ^{22}_{11}Na + \epsilon \rightarrow ^{22}_{10}Ne + \nu [/latex]

Alpha decay emits particles in a discrete energy spectrum, and both forms of beta decay emits particles in a continuous energy spectrum.

## Electron capture and beta(+) are competing processes

- [latex] Q > 1.022 \; MeV [/latex] positron favored
- [latex] Q < 1.022 \; MeV [/latex] electron capture favored

Why?

A radioactive isotope decays with a unique characteristic time. Decay is stochastic, characterized by a Poisson probability density function.

[latex] P(n) = e^{-\mu} \cdot \frac{\mu^n}{n!} [/latex]

where

[latex] \mu \equiv \lambda \Delta t [/latex]

and

[latex] \lambda [/latex] is the characteristic time.

Students should be able to compute the expected value and variance of statistical distributions.

Derivation of the change of atoms in time due to decay can then be derived.

[latex] -dn(t) = \lambda n(t) dt [/latex]

[latex] n(t) = n_0 e^{-\lambda t} [/latex]

Students are also expected to be able to derive this solution.

**Additional notes**

- University of Michigan – NERS 311: Elements of Nuclear Engineering and Radiological Sciences I
  *See course library*
- University of Michigan – NERS 312: Elements of Nuclear Engineering and Radiological Sciences II
  *See course library*
- Eastern Mediterranean University, Famagusta, North Cyprus – PHY111
- MIT Open Courseware – Nuclear Systematics
- Colorado School of Mines – PHGN 422: Nuclear Physics
- Arizona State University – EEE460: Nuclear Power Engineering
- Harry Bateman Papers

# II

## 2.0. Interaction of radiation with matter

Chapter 2 covers the basic nuclear engineering concepts. Most of this will be new material for students just entering the field.

### Learning Objectives

- Explain the different ways radiation interacts with matter
- Apply the concept of cross sections to neutron interactions with matter
- Derive neutron scattering relationships

Prof. Borrelli slides

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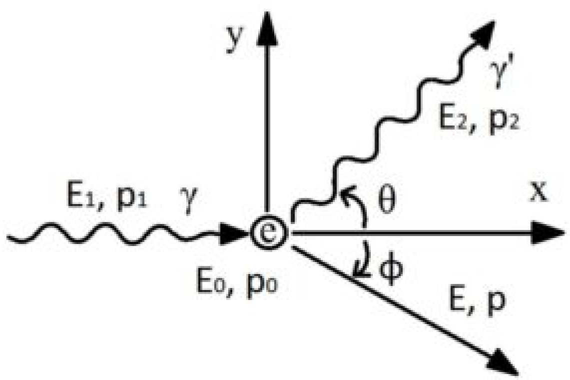
# Gamma rays

Gamma rays interact with matter in very complicated ways. The three main modes of interaction are –

- *Photoelectric effect*
- *Pair production*
- *Compton scattering*

## Compton scattering

Compton scattering is elastic scattering of a photon by an electron.
Momentum and energy are conserved.
All angles of scattering are possible –
[latex] E_2 = \frac{E_1 E_{\epsilon}}{E_1(1-cos \theta) + E_{\epsilon}} [/latex]




Based on the equations for energy conservation and momentum conservation, students should be able to derive the result for [latex] E_2 [/latex], as well as other important quantities, such as maximum scattered energy.

**Additional notes**

- Trinity College Dublin – PY2P20 Quantum Physics
- University of Florida – PHY 4803L Advanced Physics Laboratory
- Rochester Institute of Technology – PHYS314 Introduction to modern physics
- MIT Open Courseware – Physical chemistry
- MIT Open Courseware – Principles of chemical science
- Open Yale Courses – PHYS201 Fundamentals of physics II
- UIUC – NPRE402 Nuclear power engineering
- MIT Open Courseware – Principles of radiation interactions
- Michigan State University – CEM 485 Modern nuclear chemistry
- University of Washington – Physics 575 Nuclear physics: Sources, detectors, and safety
- University of Wisconsin – Physics 407 advanced laboratory

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# Cross sections

A cross section tells us basically the probability of an interaction (with something). This could be fission, absorption, scattering, etc. It is not a probability in the formal sense in that the range of cross section values does not rang from 0 to 1, but the higher the number, the more probable the reaction. For example, [latex] ^{10}B [/latex] has a neutron absorption cross section of 2 million barns (more on the units later). That means it is really likely the neutron is going to be absorbed by the boron.

The unit of the cross section is [latex] barns [/latex], where [latex] 1 \; b = 10^{-24} \; cm^2 [/latex]. So, the cross section is effectively an area.

Since there is a lot of space in between nuclei in a particular medium, some neutrons just fly by or through the material.

The radius of a nucleus [latex] \sim 10^{-12} \; cm^2 [/latex], so the cross sectional area is [latex] \sim 10^{-24} \; cm^2 [/latex]. The probability of interaction is the ratio of the total surface area of the atoms to the total area of the medium, which is then defined as the cross section ([latex] \sigma [/latex]).

**Additional notes**

- MIT Open Courseware – Neutron interactions and applications
- MIT Open Courseware – Neutron science and reactor physics
- NIST – Summer school on methods and applications of neutron spectroscopy
- University of Illinois – Dr. Magdi Ragheb
- Oregon State University – Nuclear chemistry 418/518
- University of Rochester – CHM465/PHY465 – Nuclear structure and reactions
- MIT Open Courseware – Applied nuclear physics

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# Fission

Quite simply, fission is the splitting of the atom by a neutron to release energy. Some atoms split more readily than others.

## Splitting uranium

[latex] ^{1}_{0}n + ^{235}_{92}U \rightarrow ^{92}_{36}Kr + ^{141}_{56}Ba + ^{1}_{0}n + ^{1}_{0}n + ^{1}_{0}n [/latex]

Uranium is widely used as nuclear fuel for fission, but other TRUs can be split as well. Actually, any atom can be split, but the selection of the atom should be one such that the most energy could be released.

A straightforward calculation of the *Q value* shows that [latex] Q = 173.3 \; MeV [/latex]. In contrast, burning a carbon atom yields [latex] 4 \; eV [/latex].

In the fission reaction for [latex] ^{235}_{92}U [/latex] above, two fission products and three neutrons are released. This is not the same exact reaction every time, but *fission product yields* for a given atom are readily obtained.

Similarly, on average 2 to 3 neutrons are released. Again, the *neutrons emitted per fission* ([latex] \nu [/latex]) is also readily obtained.

This is the whole point of fission. With additional neutrons produced, a chain reaction of fission can be controlled, releasing large amounts of energy in order to generate electricity.

**Additional notes**

- University of Southampton – PHY3002 Nuclei and particles
- MIT Open Courseware – Introduction to sustainable energy
- OpenStax – Physics

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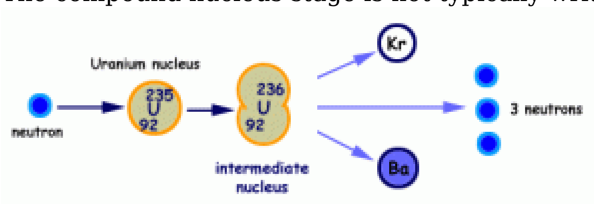
# Neutron interactions

Neutrons will pass through electron cloud and react directly with the nucleus.

Sometimes the compound nucleus is left in an excited state, before the reaction proceeds, but this can be a very short time.

[latex] ^1_0n + ^{238}_{92}U \rightarrow ^{239}_{92}U^* \rightarrow \ldots [/latex]

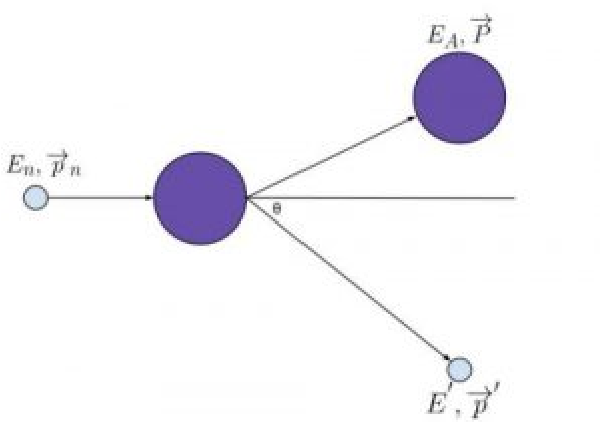
The compound nucleus stage is not typically written as part of the reaction equation.




## Neutron interactions

- Elastic scattering
- Inelastic scattering
- Radiative capture/neutron absorption
- Charged-particle reactions
- Neutron producing reactions

## Elastic scattering




With elastic scattering, energy and momentum are conserved.
[latex] E^{‘}=\frac{E_n}{(A+1)^2} \cdot [cos \theta \sqrt{A^2-sin^2 \theta}]^2 [/latex]
Students again should be able to derive this equation.

**Additional notes**

- An Introduction to Neutron Scattering – A special topics course for UCSB graduate students
- Institute of Nuclear Physics Universität zu Köln – Nuclear reactions – An introduction
- Paul Scherrer Institut – Lecture Notes of the First Summer School on Neutron Scattering

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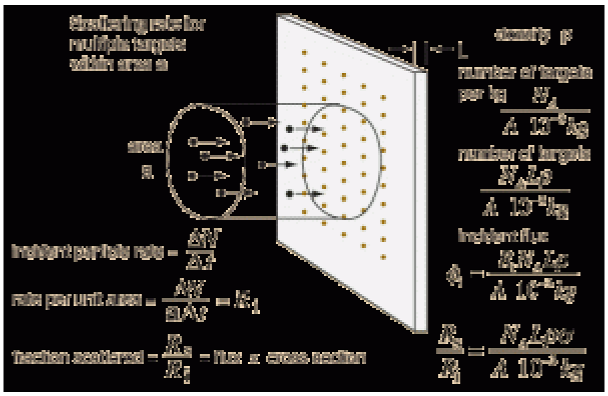
# Neutron attenuation

If a flux of neutrons ([latex] \phi_0 [/latex]) is impinged on a target, some neutrons will interact with the medium, but many will pass through with a resultant flux ([latex] \phi [/latex]) on the other side.

Click on the picture for a clearer image.




The probability that the neutrons interact with the medium is the cross section ([latex] \sigma [/latex]).

If the macroscopic cross section is defined as –
[latex] \Sigma = \frac{\rho N_A}{A} [/latex], the units of the quantity are in [latex] \frac{1}{cm} [/latex].

Then, the mean free path can be defined as –
[latex] \lambda \equiv \frac{1}{\Sigma} [/latex], which are in units of [latex] cm [/latex].

The mean free path then can be considered as the expected value of the distance a neutron travels between interactions.

To solve for flux, it should be intuitive that the resultant flux will be less than the initial flux –
[latex] -d\phi = \Sigma_T \phi dx [/latex],

with the solution –
[latex] \phi(x) = \phi_0 e^{-\Sigma_T x} [/latex] if [latex] \phi(0) = \phi_0 [/latex].

**Additional notes**

- Oak Ridge National Laboratory – 15th National School on Neutron & X-ray Scattering
- Applications of X-Ray and Neutron Scattering in Biology, Chemistry and Physics
- NIST Center for Neutron Research – Summer School on Methods and Applications of Neutron Spectroscopy

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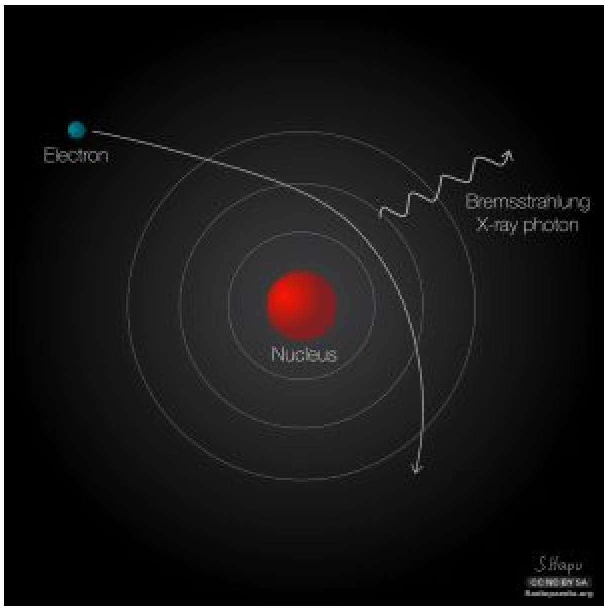
# Bremsstrahlung radiation

A consequence of charged particle reactions is the Bremsstrahlung radiation. Literally translated, it is 'braking radiation'. As a charged particle interacts with a medium, this radiation is emitted as the particle slows down.

A photon is emitted from interaction with the electron cloud or nucleus with the charged particle.




The moving particle loses energy and this is in the form of a photon.

Bremsstrahlung radiation is used to identify galaxy clusters.

- University of Maryland – ASTR 601 Radiative Processes
- Alessandro Patruno – Radiative Processes 2016-2017

III

# 3.0. Nuclear fuel cycle

Chapter 3 covers the nuclear fuel cycle from a macroscopic perspective. This material *should* be general knowledge to a point, and it was put together to help students communicate nuclear energy information with friends and family as well as a stepping stone into more specialized knowledge conveyed in a graduate class.

## Learning Objectives

- Demonstrating how the nuclear fuel cycle is an holistic system
- Explaining why the nuclear fuel cycle is what it is
- Model a nominal solvent extraction system

Prof. Borrelli slides

# Nuclear fuel cycle overview

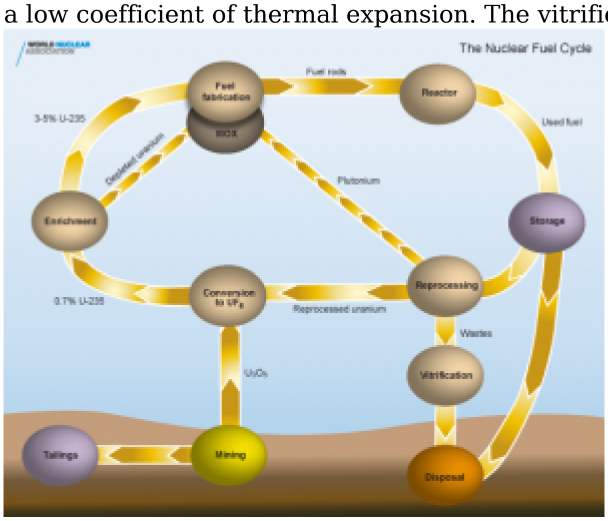
We normally think about the nuclear fuel cycle as putting fuel in the reactor and then taking it out. Sometimes enrichment appears in the news, but not usually in a positive context.

The current nuclear fuel cycle consists of –

- Mining ore from the ground
- Conversion to [latex] UF_6 [/latex]
- Enrichment from natural [latex] ^{235}U [/latex] abundance to 3% – 5%
- Fuel fabrication [latex] UF_6 \rightarrow UO_2 [/latex]
- Burning in reactor – fission
- Storage – wet and dry
- Reprocessing
- Fuel fabrication – [latex] MOX [/latex]
- Burning [latex] MOX [/latex]
- Back to storage

The United States does not currently reprocess used fuel. Once burned, fuel is stored in the used fuel pool for a time, then into dry storage. This is the same for any nation. Then, in the United States, it is expected that the stored fuel will be directly disposed in a repository. In a nation like France, the used fuel is chemically treated by the PUREX process to separate fission products from fissionable material; namely [latex] ^{235}U [/latex] and [latex] ^{239}Pu [/latex]. Then, these are fabricated to a new, mixed oxide or [latex] MOX [/latex] fuel to be fissioned in the reactor and repeat.

There are waste streams produced in the nuclear fuel cycle. Mill tailings result from the uranium ore extraction process. These are stored in ponds. For reprocessing, PUREX results in several liquid waste streams. These are vitrified into borosilicate glass. This is a material with over a half century of characterization. It is very stable with a low coefficient of thermal expansion. The vitrified waste is then disposed in the repository.




Advanced nuclear reactor concepts may have a different fuel cycle, but overall, is largely the same. The main difference would be the fuel fabrication for a molten salt or sodium fast reactor would be different than making uranium oxide fuel. Disposal will also always be needed, but the matrix; e.g., the borosilicate glass, may be a different material. Recycling processes could also be different, as production of metallic fuel or molten salt reactor fission product removal may use pyroprocessing, which uses electrochemistry to take advantage of the differences in the Gibbs free energy to extract fission products and TRUs from the used fuel.

**Additional notes**

- MIT Open Courseware – Systems analysis of the nuclear fuel cycle
- UC-Berkeley – Energy and society
- World Nuclear Association – Nuclear fuel cycle overview
- MIT Open Courseware – Nuclear reactor safety
- MIT Open Courseware – Integration of reactor design, operations, and safety</li
- MIT Open Courseware – Nuclear science and engineering

**Fuel cycle processes directed self-study course**

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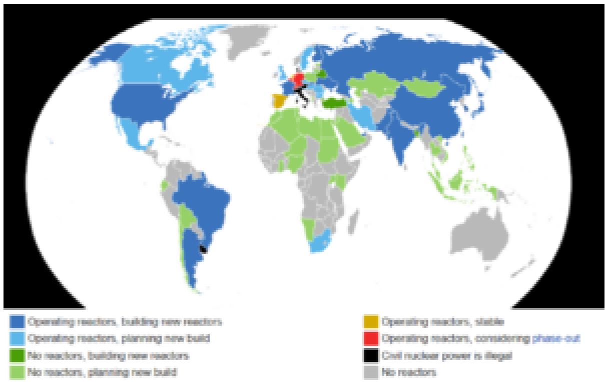
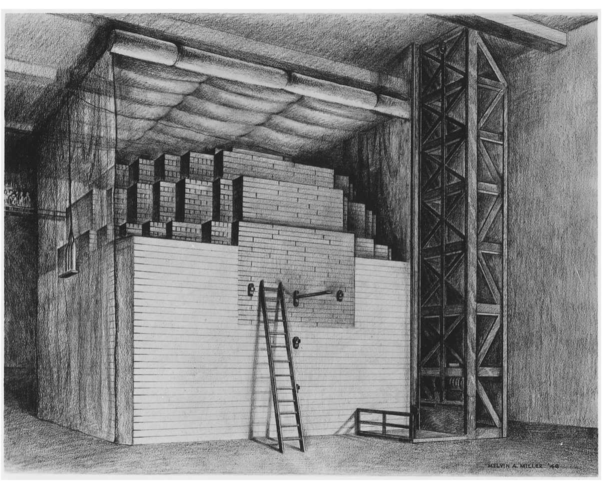
# Nuclear reactors

There are just under 100 operating commercial reactors in the United States.

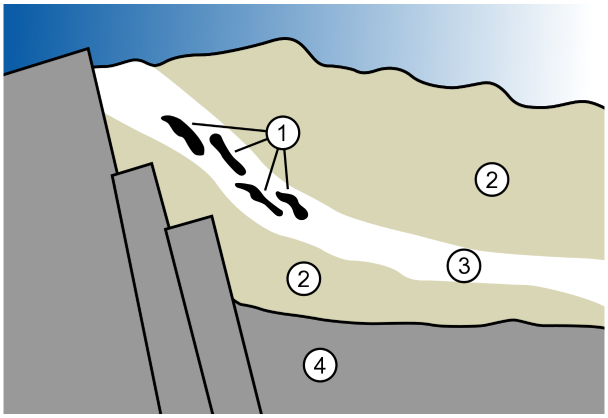



**The first reactors**
The first man made reactor was constructed at University of Chicago in 1942. It was made of uranium and graphite blocks with cadmium coated control rods.

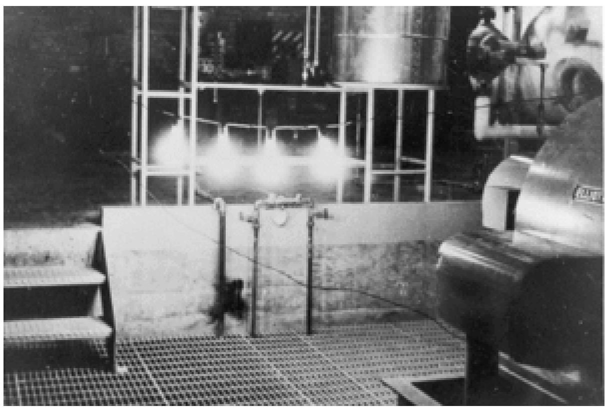


The Hanford B Reactor is the same ‘pile’ design. It was used to produce plutonium during the Cold War.

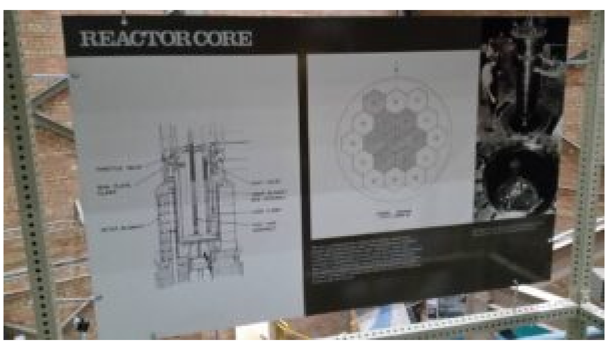
**Natural reactors**
A natural reactor actually happened about 2 billion years ago. Sixteen sites operated for about [latex] 10^5 [/latex] years at 100 kW thermal energy. Uranium rich mineral deposits were infiltrated by groundwater which served as moderator.

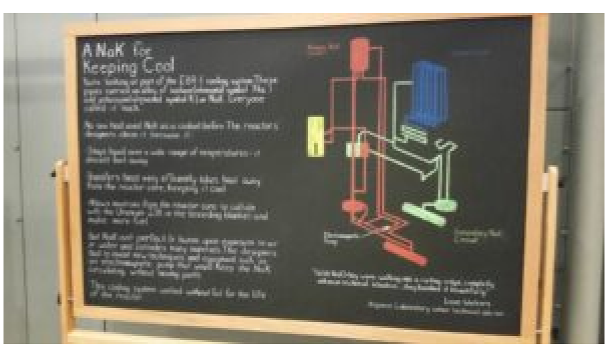



**EBR-I**



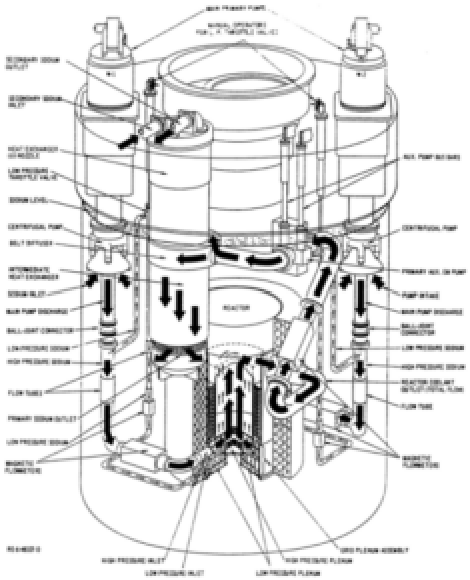
EBR-I was the first reactor to generate electricity in 1951. It is now a museum, open to the public for free from Memorial Day to Labor Day. EBR-I was a breeder reactor and the first of its kind. This proved Fermi's theory that a reactor can be build to generate fissile material.




**EBR-II**

EBR-II is an sodium fast reactor design for 62.5 MW that operated from 1964 to 1994. It produced 19 MW electricity operated for 30 years with no accidents. EBR-II was also a breeder reactor that reprocessed fuel onsite by pyroprocessing. The design featured the most advanced passive safety systems.



**TREAT**
TREAT is an air cooled, graphite moderated thermal reactor that operated from 1959 to 1994. It was used for transient reactor tests to simulate all sorts of reactor accidents. The reactor can generate neutron pulses up to 19 GW. It was restarted in 2018 to support the Accident Tolerant Fuel program.

**Military**
Nuclear reactors are used on submarines and aircraft carriers. The lifecycle of the reactors is about 30 years. The first nuclear powered submarine was the Nautilus, launched in 1954, with a 25 year life span. The reactor designs are classified, but they are known to be light water reactors.

**Additional notes**

- The Nuclear Fuel Cycle: Waste, Risk, and Economics
- Interactive map
- Plant closures
- Nuclear power around the world
- Planned construction

# 13

# Back end management

Back end management is a blanket term for after the fuel is taken out of the reactor; i.e., the fuel is burned and has to be removed. Typically, the fuel is stored in a large cooling pool while the short lived fission products decay. The pools serve dual purpose – heat dissipation due to decay of the fission products and shielding. After a varying period of time, anywhere from 3 to 20 years, fuel is moved to dry casks. Several assemblies are placed in each cask depending on the design.

The back end management 'policy' in the United States is to store the used fuel on the dry cask pads onsite until a repository is constructed for disposal.

The United States does have an actively operating repository – the Waste Isolation Pilot Plant operated by the Department of Energy in Carslbad, New Mexico, for TRU waste stemming from nuclear weapons production. The WIPP facility is a salt formation.

Yucca Mountain, about 100 miles north of Las Vegas was designated the repository for commercial high-level waste; i.e., used fuel, in 1987, but legal issues and local opposition has held it up since. Unfortunately, many administrations since have used it as a political wedge issue. Ironically, based on the used fuel inventory in the United State, Yucca Mountain would fill up if it opened tomorrow. The capacity is only set by statute, but collectively, it is agreed that a second repository is needed.

**How the repository is supposed to work**
Whether high-level waste or used fuel, the repository is a geologic formation that is first meticulously characterized in terms of hydrology, geology, chemistry, meteorology, etc. If there is an -ology, it will be studied. Characterization will take at least ten years to even thirty years.

The design of the repository is an example of defense-in-depth. Waste is contained in a matrix – borosilicate glass, or the uranium oxide itself. This, in turn, is then placed in a canister. For an unsaturated repository; i.e., above the water table, like Yucca Mountain, the cans are placed underneath drip shields. For the rest of the world, the repository concepts are for saturated conditions; i.e., under the water table. Typically, the canisters would then be surrounded by bentonite. This is a clay material that absorbs water and expands. Ever see kitty litter clump? That is basically the physical process.

Eventually, the canisters will degrade. Modelers assume canisters last about 1000 years. With the borosilicate glass matrix, dissolution of that takes even longer. Radionuclides will leak out, but some, due to geochemistry will precipitate out and remain there. Others will dissolve in water or seep through the vadose zone to the water table. The approach is called 'dilute and disperse' in that the water volume is large enough to render the concentration of any particular radionuclide to be low.

**Operate**
Most repositories are expected to 'operate'; i.e., people actually working there, for up to 300 years. After that, the repository enters the 'post closure period' where everyone leaves. There is considerable debate as to whether some sort of signage should be installed.

**Additional notes**

- Managing Nuclear Technology

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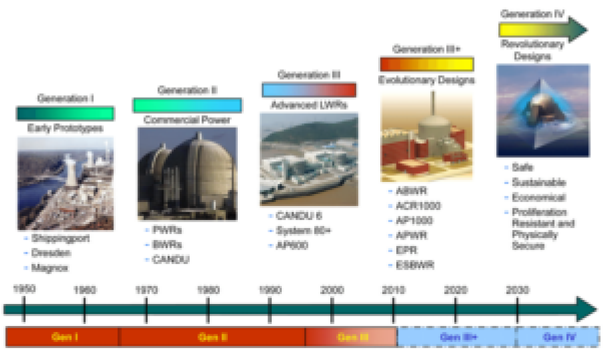
# Commercial nuclear power

**Generations of nuclear energy**

Commercial reactors, designed to generate electricity, were first built in the 1950s. These are the so-called ‘Generation I’ concepts. Generations II, III, and III+ are the reactors operating around the world today. These are considered evolutionary designs; they are all water-cooled reactor designs. Each generation featured advanced safety systems. The most advanced reactors are the Generation III+ AP1000 (USA) and APR1400 (ROK). These have more passive safety systems and emergency cooling.

Generation IV concepts are considered ‘revolutionary’ designs. Many are just Generation I designs on a larger scale. Most are non-water cooled designs. A top billing for the Generation IV systems is ‘enhanced proliferation resistance’.



# IV

## 4.0. Nuclear reactor theory

Chapter 4 covers the technical knowledge governing critical nuclear core design, operations, and neutron transport theory. This is more specialized knowledge that would be in an upper division undergraduate course or introductory graduate level course.

### Learning Objectives

- Design a critical nuclear reactor configuration
- Derive steady state neutron transport equation
- Demonstrate transient reactor behavior

Prof. Borrelli slides

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# Neutron multiplication factor

Designing a nuclear reactor is about controlling the neutron chain reaction. The underlying neutron physics allows this control.

The neutron multiplication factor ([latex] k [/latex]) describes the chain reaction. It is defined as the ratio of neutrons in generation [latex] (n + 1) [/latex] to the number of neutrons in generation [latex] n [/latex].

Then, if [latex] k < 1 [/latex], the chain reaction decreases in time. If [latex] k > 1 [/latex], the chain reaction is increasing. These are defined as subcritical and supercritical, respectively. However, if [latex] k = 1 [/latex], then the chain reaction is self-sustaining. This state is called critical. In terms of reactor operation, [latex] k [/latex] is manipulated to obtain a state of criticality at a designated power level.

The four factor formula can be calculated to obtain the neutron multiplication factor for an ‘infinite reactor’. This means that the reactor is theoretically large such that none of the neutrons ‘leak’. [latex] k [/latex] then can be applied to determine the size of the critical reactor, based on fuel type, coolant, geometry, etc.

## Four factor formula

[latex] k \equiv \eta f \epsilon p [/latex]

- [latex] \eta \equiv \; [/latex] neutron reproduction factor
- [latex] f \equiv \; [/latex] fuel utilization factor
- [latex] \epsilon \equiv \; [/latex] fast fission factor
- [latex] p \equiv \; [/latex] resonance escape probability

Remember, if the goal is for [latex] k = 1 [/latex], then think of the range of values for each parameter when studying the factors below.

## Neutron reproduction factor

[latex] \eta \equiv \frac{\nu\Sigma_F}{\Sigma_A} [/latex]

- [latex] \nu \equiv \; [/latex] average number of neutrons released per fission (dependent on fissonable material)
- [latex] \Sigma_F \equiv \; [/latex] macroscopic fission cross section
- [latex] \Sigma_A \equiv \; [/latex] macroscopic absorption cross section

The neutron reproduction factor gives a ratio of the ‘usable’ neutrons to neutrons absorbed per birth of neutrons.

## Fuel utilization factor

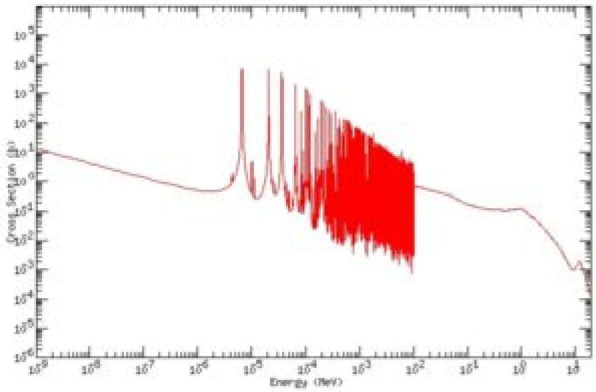
[latex] f \equiv \frac{\Sigma^{fuel}_{A}}{\Sigma^{fuel}_{A} + \Sigma^{mod}_{A}} [/latex]

- [latex] \Sigma^{fuel}_{A} \equiv \; [/latex] macroscopic absorption cross section in the fuel
- [latex] \Sigma^{mod}_{A} \equiv \; [/latex] macroscopic absorption cross section in the moderator

The fuel utilization factor basically shows the 'miles per gallon' for the core. Neutrons absorbed in the moderator will not produce any more fissions.

## Resonance escape probability

Resonance escape ([latex] p [/latex]) is the probability neutron is not absorbed in the resonance region. Most neutrons are absorbed by [latex] ^{238}U [/latex] when slowing down in commercial reactors. Empirical results are typically used because it is extremely difficult to compute.




## Fast fission factor

The fast fission factor is ratio of the total number of fast and thermal neutrons produced to the number produced by just thermal fission. Empirical relationships are again used because it is a really difficult parameter to calculate.

Lastly, neutron leakage has to be considered. These are the neutrons that completely leave the system (reactor vessel). There is leakage of fast neutrons and thermal neutrons. The parameter of interest then is the 'non leakage' probability; that is, the probability of the neutrons that do not leak. This value is needed to compute [latex] k [/latex] for the critical reactor. Leakage is largely a function of geometry, but the choice of reflector is also important.

The following image illustrates how [latex] k [/latex] can be calculated.

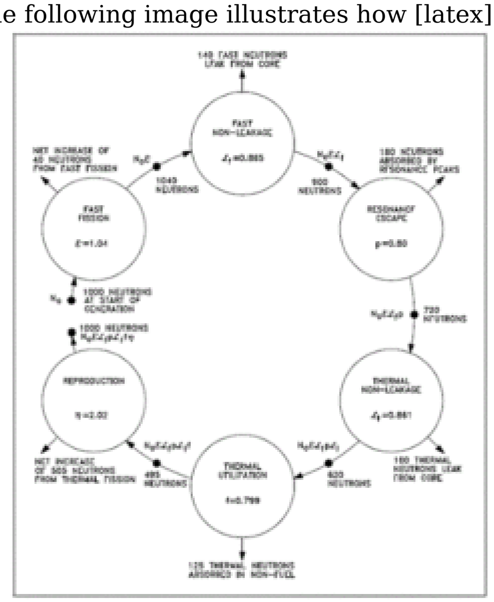



Figure 1 Neutron Life Cycle with $k_{eff} = 1$

**Additional notes**

- Nuclear Reactor Physics

- Nuclear Chain Reactions
- Reactor Physics Review
- Fundamentals of Physics
- Elementary physics of reactor control
- Nuclear Criticality
- Nuclear reactor physics
- Reactor physics: Diffusion of neutrons
- Neutron science and reactor physics lecture notes

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# Burnup

Burnup is a measure of the total energy released in fission by the fuel. Try to think of it as ‘fissionability’. If fresh fuel only has a certain amount of fissions, the more means more energy. This is only a very loose analogy.

The units of burnup are [latex] GWD/MTU [/latex].

Early designs of Generation II and III reactors were in the 30 GWD/MTU range, but Generation III+ and IV can reach 50+ GWD/MTU.

**Additional notes**

- Aalto University – PHYS-E0562 – Nuclear engineering
- Dr. John R. White, Emeritus Professor – UMass – Lowell

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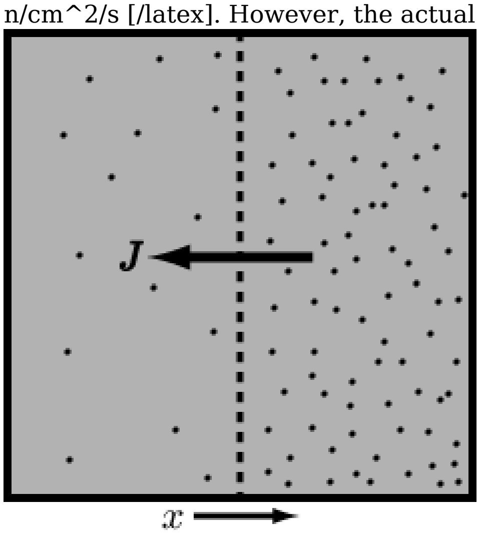
# Neutron diffusion

Understanding the physical process of and deriving neutron diffusion is crucial to understanding higher level concepts that will be used to determine critical core size, volume, and mass.

Neutron diffusion is assumed to follow *Fick's law* because just about every physical diffusion process does – [latex] J_i = -D\frac{d\phi}{di} [/latex]

The diffusion coefficient [latex] D [/latex] in nuclear engineering is called the *diffusion length* because the units of [latex] D [/latex] are in length here, where as in something like environmental fate and transport, [latex] D [/latex] is typically in units of [latex] L^2/T [/latex]. This is because the flux [latex] \phi [/latex] here is in units of [latex] n/cm^2/s [/latex]. However, the actual process of neutron diffusion is the same as any other diffusion.




In nuclear engineering [latex] J [/latex] is called *current*.

Fick's law is not valid for neutron diffusion if –

- The medium is strongly sorbing.
- There are more than three mean free paths from the source to the medium surface.
- Scattering is anisotropic.

**Additional notes**

- Walter Ambrosini – Università di Pisa – Dipartimento di Ingegneria Meccanica, Nucleare e della Produzione
- BYU – ChE412 – Introductory Nuclear Engineering

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# Equation of continuity

An equation of continuity generally describes the rate of change of a selected parameter in a defined environment. It is a mass balance of the parameter in time through a control volume. This could be temperature in a medium, fluid flow in a channel, chemical compounds through soil, etc.

Here, the parameter of interest is of course neutrons.

In words this is –

`[rate of change of neutrons] = [production rate] - [absorption rate] - [leakage rate]`

Typically, a fixed volume is assumed. Depending on the phenomenon to be modeled, there could be other terms. For example, a chemical reaction rate could be added for chemical compounds or a decay rate for radionuclides.

## Equation of continuity

[latex] \frac{d}{dt}\int_V n dV \; = \; \int_V s dV \; - \; \int_V \Sigma_A \phi dV \; - \; \int_V \underline{J} \cdot \underline{n} dA [/latex]

- [latex] \int_V n dV \equiv \; [/latex] total number of neutrons
- [latex] \frac{d}{dt}\int_V n dV \equiv \; [/latex] rate of change of the total number of neutrons
- [latex] \int_V s dV \equiv \; [/latex] production rate
- [latex] \int_V \Sigma_A \phi dV \equiv \; [/latex] absorption rate
- [latex] \int_A \underline{J} \cdot \underline{n} dA \equiv \; [/latex] leakage rate

[latex] \phi [/latex] is the neutron flux in the reactor. Note that current ([latex] \underline{J} [/latex]) appears in the leakage term. The ([latex] \underline{n} [/latex]) is just the unit normal vector. Essentially, this term describes the flux of neutrons leaking out of a 'face' of the control volume.

With applying the following known, mathematical relationships –
[latex] \frac{d}{dt}\int_V n dV = \int_V \frac{\partial{n}}{\partial{t}} dV [/latex]

and

[latex] \int_A \underline{J} \cdot \underline{n} dA = \int_V \nabla \underline{J} dV [/latex]

where the first is a form of the Leibniz integral rule, and the second is a form of Gauss' divergence theorem.

Then –
[latex] \int_V \frac{\partial{n}}{\partial{t}} dV \; = \; \int_V s dV \; - \; \int_V \Sigma_A \phi dV \; - \; \int_V \nabla \underline{J} dV [/latex]

The integrals essentially cancel because integration is performed over the same control volume.

Therefore –
[latex] \frac{\partial{n}}{\partial{t}} \; = \; s \; - \; \Sigma_A \phi \; - \; \nabla \underline{J} [/latex]

This is the equation used to describe reactor operations and other neutron transport phenomenon.

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# Nuclear reactor kinetics

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# Heat transfer

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# Criticality safety

Sensitivity-Uncertainty Based Nuclear Criticality Safety Validation

**Nuclear criticality safety directed self-study course**

- Module 1 – NRC’s nuclear criticality safety mission
- Module 2 – Nuclear criticality safety standards
- Module 3 – Nuclear theory
- Module 4 – Nuclear criticality safety standards
- Module 5 – Historical accidents
- References

**Nuclear criticality safety engineer training**

- Module 1 – Introductory Nuclear Criticality Physics
- Module 2 – Neutron Interactions
- Module 3 – Fission Chain Reactions
- Module 4 – Neutron Scattering
- Module 5 – Criticality Limits
- Module 6 – Diffusion and Transport Theory – Part I
- Module 8 – Hand Calculation Methods – Part I
- Module 10 – Criticality Safety in Material Processing Operations – Part 1
- Module 11 – Criticality Safety in Material Processing Operations – Part 2
- Module 13 – Measurement and Development of Cross Section Sets

V

# 5.0. Monte Carlo methods and MCNP

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# Monte Carlo methods

VI

# 6.0. Back end management

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# Analytical solution to the general radionuclide transport equation

## General form of the transport equation

The general form of the radionuclide transport equation is:
[latex] \frac{\partial}{\partial t}(\epsilon K N)+ \nabla (\epsilon v N) - \nabla^2 (\epsilon D N)+\epsilon K \lambda N =0. [/latex]

This governing equation is used not only for radionuclides, but for any dissolved contaminant in a saturated, porous geological media. This is the general version of the advection-diffusion-reaction transport equation. Depending on the discipline, different symbols may be used in the equations. For example, the radionuclide concentration in the groundwater (N) may be shown as (c). For nuclear engineering problems, the reaction constant (λ) is simply the decay constant of the radionuclide of interest.

## Simplifying assumptions

In reality, parameters such as fluid porosity or advection may be both spatially and temporally dependent. The transport equation would then be solved numerically. It is preferred, however, that an analytical solution be obtained if possible, if reasonable, simplified assumptions can be made. We can assume a one dimensional spatial geometry, for example, to represent a parallel, planar fracture. If we further assume that the length of the fracture is sufficiently short such that there is not much variation in the parameters, then they can also assumed to be constant.

The transport equation reduces to:
[latex] \epsilon K \frac{\partial N}{\partial t}+\epsilon v \frac{\partial N}{\partial x}-\epsilon D \frac{\partial^2 N}{\partial x^2}+\epsilon K \lambda N=0. [/latex]

For a semi-infinite medium, the following side conditions are applied:
[latex] N(x,0)=0,~~ 0<x<\infty, \newline N(0,t)=N^*,~~ t>\infty, \newline N(\infty,t)=0,~~ t>\infty. [/latex]

## Integral transformation

For this governing equation, with these conditions, an analytical solution can be obtained. It is admittedly somewhat of an abstraction of reality, but results do give an indication of the physical behavior of radionuclide migration through a porous medium. This is very important to repository performance assessment, as we are concerned with release rates into the far field.

Apply the following integral transformation:
[latex] N(x,t)\equiv\lambda\int_0^te^{-\lambda\tau}u(x,\tau)d\tau+e^{-\lambda t}u(x,t). [/latex]

Then, compute the derivatives of the above expression:
[latex] \frac{\partial N}{\partial t}=e^{-\lambda t}\frac{\partial u}{\partial t} \newline \frac{\partial N}{\partial x}=\lambda\int_0^te^{-\lambda\tau}\frac{\partial u}{\partial x}d\tau+e^{-\lambda t}\frac{\partial u}{\partial x} \newline \frac{\partial^2 N}{\partial x^2}=\lambda\int_0^te^{-\lambda\tau}\frac{\partial^2 u}{\partial x^2}d\tau+e^{-\lambda t}\frac{\partial^2 u}{\partial x^2} [/latex]

The subsequent procedure is to substitute the derivatives into the governing equation and then apply some basic algebra and rearrange the terms.
[latex] e^{-\lambda t}K\frac{\partial u}{\partial t}+ \lambda\int_0^te^{-\lambda\tau}v\frac{\partial u}{\partial x}d\tau+ e^{-\lambda t}v\frac{\partial u}{\partial x}- \lambda\int_0^te^{-\lambda\tau}D\frac{\partial^2 u}{\partial x^2}d\tau- e^{-\lambda t}D\frac{\partial^2 u}{\partial x^2} +K\lambda^2\int_0^te^{-

\lambda\tau}u d\tau+ e^{-\lambda t} K\lambda u=0. [/latex]

Then:
[latex] e^{-\lambda t}(K\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial x}-D\frac{\partial^2 u}{\partial x^2}+K\lambda u) +\lambda\int_0^t[e^{-\lambda\tau}(v\frac{\partial u}{\partial x}-D\frac{\partial^2 u}{\partial x^2})]d\tau \newline +K\lambda^2\int_0^te^{-\lambda\tau}ud\tau=0. [/latex]

Evaluate the integral for the last term:
[latex] K\lambda^2\int_0^te^{-\lambda\tau}ud\tau=- e^{-\lambda t}K\lambda u+\lambda\int_0^te^{-\lambda\tau} K\frac{\partial u}{\partial t} d\tau. [/latex]

Substitute back into the governing equation and apply some more algebra:
[latex] e^{-\lambda t}(K\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial x}-D\frac{\partial^2 u}{\partial x^2}+K\lambda u-K\lambda u) \newline +\lambda\int_0^t[e^{-\lambda\tau}(K\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial x}-D\frac{\partial^2 u}{\partial x^2})]d\tau=0. [/latex]
[latex] e^{-\lambda t}(K\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial x}-D\frac{\partial^2 u}{\partial x^2}) +\lambda\int_0^t[e^{-\lambda\tau}(K\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial x}-D\frac{\partial^2 u}{\partial x^2})]d\tau=0. [/latex]

The solution to:
[latex] K\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial x}-D\frac{\partial^2 u}{\partial x^2}=0, [/latex]
will satisfy the above mathematical expression with transformed side conditions:
[latex] u(x,0)=0,~~ 0<x<\infty, \newline u(0,t)=u^*,~~ t>\infty, \newline u(\infty,t)=0,~~ t>\infty. [/latex]

Integral transformations are common in higher level applied math. This procedure for the radionuclide transport equation was applied because the transformed expression for the transformed variable (u) is known to have an analytical solution that can be readily obtained with the Laplace transform. It is a common mathematical model that is valid for a wide range of physical problems.

## Using the Laplace transform

By definition, the Laplace transform is:

[latex] \tilde{f}(x,s)\equiv\int_0^\infty f(x,t)e^{-st}dt. [/latex]

The Laplace transform is applied to the time derivative in the transformed radionuclide transport equation to obtain:
[latex] sK\tilde u+v\frac{\partial \tilde u}{\partial x}-D\frac{\partial^2 \tilde u}{\partial x^2}=0. [/latex]

It is typical to rearrange the differential equation as:
[latex] D\frac{\partial^2 \tilde u}{\partial x^2}-v\frac{\partial \tilde u}{\partial x}-sK\tilde u=0. [/latex]

It is typical to rearrange the differential equation as:
[latex] \frac{\partial^2 \tilde u}{\partial x^2}-\frac{v}{D}\frac{\partial \tilde u}{\partial x}-\frac{sK}{D}\tilde u=0. [/latex]

The transformed side conditions are:
[latex] \tilde{u}(0,s)=\frac{u^*}{s}, \newline \tilde{u}(\infty,s)=0. [/latex]

What are the limits on s?

For this mathematical model, the solution is known to be:
[latex] \tilde u=Ae^{x(\frac{v}{2D}+\sqrt{\frac{v^2}{4D^2}+\frac{sK}{D}})}+Be^{x(\frac{v}{2D}-\sqrt{\frac{v^2}{4D^2}+\frac{sK}{D}})}, [/latex]
where A and B are the constants of integration.

Because the solution must be finite it is intuitive that A = 0. Students should be able to prove this mathematically. Subsequently, with A = 0, at x = 0:
[latex] B=\frac{u^*}{s}, [/latex]
the solution to the Laplace transformed model is then:
[latex] \tilde u=\frac{u^*}{s}e^{x(\frac{v}{2D}-\sqrt{\frac{v^2}{4D^2}+\frac{sK}{D}})}. [/latex]

# Inverting the Laplace solution into the real time domain

Obtaining the solution to the Laplace transformed differential equation is fairly straightforward because these forms of differential equations have well known solutions. Inversion to the real time domain is not exactly as straightforward. This is why math is considered an art. Obtaining the solution to the transport equation or any similarly structured mathematical problem requires some knowledge of the actual solution in order to arrange the Laplace solution into a form that is readily invertible.

The inverse of the Laplace transform is defined as:
[latex] f(t)\equiv\frac{1}{2\pi i}\lim_{\tau\rightarrow\infty}\int_{\gamma -i\tau}^{\gamma +i\tau}e^{st}\tilde{f}(t)ds. [/latex]
This is a line integral in the complex plane (sort of), where the line integral must be computed to avoid singularities. Fortunately, for engineers, there are tables of inverse Laplace transforms, so the integral does not have to be computed all the time. However, some skill is needed in order to manipulate any given Laplace solution into a recognizable form such that the tables can be used. Which, again, requires some knowledge of the real solution.

# Manipulating the Laplace solution into an invertible form

Rearrange the Laplace solution as:
[latex] \tilde u=u^*e^{(\frac{xv}{2D})} \cdot \frac{1}{s}e^{(-x\sqrt{\frac{v^2}{4D^2}+\frac{sK}{D}})}. [/latex]
Note that the first factor is only x dependent. The second factor contains the Laplace variable (s). This is a common first step in setting up the solution into an invertible form.

The next step is to eliminate the square root that contains the Laplace variable (s). Yet another integral transformation is needed.
[latex] \frac{\sqrt\pi}{2}e^{-2\sigma}=\int_0^\infty e^{(-\xi^2-\frac{\sigma^2}{\xi^2})}d\xi. [/latex]
Note that the squared terms will help to this end.

The argument in the exponential function that contains the Laplace variable (s) then can be conveniently defined as:
[latex] \sigma\equiv \frac{x}{2} \sqrt{\frac{v^2}{4D^2}+\frac{sK}{D}}. [/latex]
Substitute this all into the Laplace solution.
[latex] \tilde u=u^*e^{(\frac{xv}{2D})} \cdot \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{1}{s}e^{(-\xi^2-\frac{x^2}{4\xi^2}(\frac{v^2}{4D^2}+\frac{sK}{D}))}d\xi. [/latex]
Similarly, group the terms containing the Laplace variable (s) together.
[latex] \tilde u=u^*e^{(\frac{xv}{2D})} \cdot \frac{2}{\sqrt{\pi}} \int_0^\infty e^{(-\xi^2-\frac{x^2}{4\xi^2}\frac{v^2}{4D^2})}\cdot \frac{1}{s}e^{(-\frac{Kx^2}{4D\xi^2}s)}d\xi. [/latex]
Now, the model is in an invertible form.

# Inverting the Laplace solution

The last factor in the Laplace solution is a known form of the Heaviside step function.
[latex] \tilde{f}(s)=\frac{1}{s}e^{-\mu s} \newline f(t)=h(t-\mu) [/latex]
Then, the second factor in the integral can be inverted.
[latex] u=u^*e^{(\frac{xv}{2D})} \cdot \frac{2}{\sqrt{\pi}} \int_0^\infty e^{(-\xi^2-\frac{x^2}{4\xi^2}\frac{v^2}{4D^2})}h(t-\frac{Kx^2}{4D\xi^2})d\xi. [/latex]
[latex] u=u^*e^{(\frac{xv}{2D})} \cdot \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{Kx^2}{4Dt}}}^\infty e^{(-\xi^2-\frac{x^2}{4\xi^2}\frac{v^2}{4D^2})}d\xi. [/latex]
[latex] u=u^*e^{(\frac{xv}{2D})} \cdot \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{Kx^2}{4Dt}}}^\infty e^{(-\xi^2-\frac{x^2v^2}{16D^2\xi^2})}d\xi. [/latex]
The solution has been transformed back into the real space (x,t), but the integral needs to be evaluated now in order to obtain the formal solution for the transformed radionuclide transport solution (u).

# Obtaining the transformed radionuclide transport solution

The integral in the transformed solution is also a known form:
[latex] \int e^{(-a^2\xi^2-

\frac{b^2}{\xi^2})}d\xi=\frac{\sqrt{\pi}}{4a}(e^{2ab}~erf(a\xi+\frac{b}{\xi})+e^{-2ab}~erf(a\xi-\frac{b}{\xi})). [/latex]
For this problem:
[latex] a=1,~b=\frac{xv}{4D}. [/latex]
The general form of the transformed solution is then:
[latex]
u=[u^*e^{(\frac{xv}{2D})}~\frac{2}{\sqrt{\pi}}][\frac{\sqrt{\pi}}{4}(e^{\frac{xv}{2D}}~erf(\xi+\frac{vx}{4D\xi})+e^{-\frac{xv}{2D}}~erf(\xi-\frac{xv}{4D\xi}))]. [/latex]
Evaluate the expression by applying the integration limits. This only involves straightforward algebra, and left to the student. The transformed solution is then:
[latex] u=\frac{u^*}{2}e^{(\frac{xv}{2D})}~[e^{\frac{xv}{2D}}~erfc(\frac{Kx+vt}{\sqrt{4KDt}})+e^{-\frac{xv}{2D}}~erfc(\frac{Kx-vt}{\sqrt{4KDt}})], [/latex]
Making use of:
[latex] erfc(\mu)\equiv 1-erf(\mu). [/latex]

## Solution to the radionuclide transport equation

The first step in obtaining the radionuclide transport solution makes use of the boundary condition at x = 0.
[latex] u=\frac{N^*}{2}e^{(\frac{xv}{2D})}~[e^{\frac{xv}{2D}}~erfc(\frac{Kx+vt}{\sqrt{4KDt}})+e^{-\frac{xv}{2D}}~erfc(\frac{Kx-vt}{\sqrt{4KDt}})] [/latex]
Recall the initial integral transformation:
[latex] N(x,t)\equiv\lambda\int_0^te^{-\lambda\tau}u(x,\tau)d\tau+e^{-\lambda t}u(x,t). [/latex]
Therefore, the solution for (N) can be obtained.
[latex] e^{-\lambda t}u(x,t)=\frac{N^*}{2}e^{(\frac{xv}{2D}-\lambda t)}~[e^{\frac{xv}{2D}}~erfc(\frac{Kx+vt}{\sqrt{4KDt}})+e^{-\frac{xv}{2D}}~erfc(\frac{Kx-vt}{\sqrt{4KDt}})], [/latex]
[latex] e^{-\lambda t}u(x,t)=\frac{N^*}{2}[e^{(\frac{xv}{D}-\lambda t)}~erfc(\frac{Kx+vt}{\sqrt{4KDt}})+e^{-\lambda t}~erfc(\frac{Kx-vt}{\sqrt{4KDt}})]. [/latex]
The solution is therefore:
[latex] N(x,t)=\frac{\lambda N^*}{2}\int_0^t [e^{(\frac{xv}{D}-\lambda\tau)}~erfc(\frac{Kx+v\tau}{\sqrt{4KD\tau}})+e^{-\lambda\tau}~erfc(\frac{Kx-v\tau}{\sqrt{4KD\tau}})]d\tau+\frac{N^*}{2}[e^{(\frac{xv}{D}-\lambda t)}~erfc(\frac{Kx+vt}{\sqrt{4KDt}})+e^{-\lambda t}~erfc(\frac{Kx-vt}{\sqrt{4KDt}})]. [/latex]

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# Analytical solution to the diffusion reaction equation

## General form of the transport equation

The general form of the radionuclide transport equation is:
[latex] \frac{\partial}{\partial t}(\epsilon K N)+ \nabla (\epsilon v N) - \nabla^2 (\epsilon D N)+\epsilon K \lambda N =0. [/latex]

## Simplifying assumptions

Assuming v = 0, we have the time-dependent diffusion reaction equation.

The transport equation reduces to:
[latex] \epsilon K \frac{\partial N}{\partial t}-\epsilon D \frac{\partial^2 N}{\partial x^2}+\epsilon K \lambda N=0. [/latex]

For a semi-infinite medium, the following side conditions are applied:
[latex] N(x,0)=0,~~ 0<x<\infty, \newline N(0,t)=N^*,~~ t>\infty, \newline N(\infty,t)=0,~~ t>\infty. [/latex]

## General procedure

Apply the Laplace transformation as before.
Obtain the general solution in the Laplace space.
Apply the side conditions to the general solution in the Laplace space.
Use the 'sigma transformation':
[latex] \frac{\sqrt{\pi}}{2}e^{-2\sigma}=\int_{0}^{\infty}e^{-\xi^2-\frac{\sigma^2}{\xi^2}}d\xi [/latex]
Rearrange the integral similar to the general transport equation to obtain the solution in the real space, N(x,t).
Apply the exponential/error function transformation:
[latex] \int e^{-a^2\xi^2-\frac{b^2}{\xi^2}}d\xi=\frac{\sqrt{\pi}}{4a}[e^{2ab}erf(a\xi+\frac{b}{\xi})+e^{-2ab}erf(a\xi-\frac{b}{\xi})] [/latex]
Evaluate the expression using the limits of integration, rearrange, and apply the erfc.

The final solution is:
[latex]N(x,t)=\frac{N^*}{2}[e^{x\sqrt{\frac{K\lambda}{D}}}erfc(x\sqrt{\frac{K}{4Dt}}+2\sqrt{\lambda t}+e^{-x\sqrt{\frac{K\lambda}{D}}}erfc(x\sqrt{\frac{K}{4Dt}}-2\sqrt{\lambda t})] [/latex]

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# Waste classifications

- NRC 10 CFR 61.55 Waste classification
- Classifications of Nuclear Waste
- Radioactive Waste Management
- Disposal of High-Level Nuclear Waste

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- Storage and Disposal of Radioactive Wastes
- The disposal of high-level radioactive waste

# VII

## 7.0. Literature

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## General

- Cornell University – arXiv.org
- National Programme on Technology Enhanced Learning
- U. S. government information
- CANTEACH – Publication library

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# Nuclear physics

- DOE nuclear physics handbook V1
- DOE nuclear physics handbook V2
- Brookhaven National Laboratory – Introduction to Nuclear Engineering 101
- AK Lectures – Nuclear physics
- Washington University – CHEM436: Introduction to the atomic nucleus (lecture notes)

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# Neutronics

- Minimum mass of moderator required for criticality of homogeneous low-enriched uranium systems
- Reactor physics constants
- ORAU Museum of Radiation and Radioactivity
- Lecture notes for criticality safety
- Some topics in neutron diffusion theory
- Oklo reactors
- Enrico Fermi and the physics and engineering of a nuclear pile: The retrieval of novel documents
- Multi-species neutron transport equation
- Physics with neutrons
- Three methods for solving the low energy neutron Boltzmann equation
- The application of the finite element method to the neutron transport equation

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# Nuclear fuels

- High burnup fuels
- HTGR Technology Course
- International Project on Innovative Nuclear Reactors and Fuel Cycles (INPRO)

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# Advanced nuclear reactor companies

- NuScale
- Kairos Power
- Terrestrial Energy
- ThornCon Power
- TerraPower
- Flibe Energy
- Transatomic Power
- Fluor Corporation

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# Nuclear fuel resources and performance

- National Uranium Resource Evaluation (NURE) Hydrogeochemical and Stream Sediment Reconnaissance data
- National Geochemical Database reformatted data from the National Uranium Resource Evaluation (NURE) Hydrogeochemical and Stream Sediment Reconnaissance (HSSR) program
- Online spatial data interactive map
- Summary history of domestic uranium procurement under U.S. Atomic Energy Commission contracts: Final report
- USGS Landsat missions
- EarthExplorer
- Uranium, potassium, and thorium contour maps derived from a helicopter gamma-ray spectrometer survey of the Getchell Trend, Humboldt County, Nevada
- Helicopter-borne magnetic and radiometric geophysical survey at Gratangen and Sørreisa, Troms county
- Borehole logging for uranium exploration
- Manual of acid in situ leach uranium mining technology
- Exxon study on uranium extraction from seawater
- Introduction of Thorium in the Nuclear Fuel Cycle
- Classification of uranium reserves/resources
- Accident tolerant fuels
- Light Water Reactor Accident Tolerant Fuel Performance Metrics

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# Nuclear fuel fabrication

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# Front-end of the fuel cycle

- Unraveling the A. Q. Khan and Future Proliferation Networks
- Experimental Breeder Reactor-II
- Transient Reactor Test Facility
- Restart of the Transient Reactor Test Facility (TREAT ) and Resumption of Transient Testing
- Introduction to CANDU
- Liquid-Liquid Extraction (LLE)
- Principles of Industrial Solvent Extraction
- Separation techniques
- Gas Centrifuge Theory and Development: A Review of U.S. Programs
- Uranium chemistry
- Reactor theory and power reactors

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# Countercurrent solvent extraction

- Simulation of Multistage Countercurrent Liquid-Liquid Extraction
- Countercurrent equilibrium extraction – Excerpt from Pigford et al.
- Nuclear fuel reprocessing
- The PUREX process
- Spent Nuclear Fuel Reprocessing

# Generation IV systems

Generation IV international forum

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- Introdution to Generation IV Nuclear Plants
- Very-High-Temperature Reactor
- Supercritical-Water-Cooled Reactor
- Gas-Cooled Fast Reactor
- Lead-Cooled Fast Reactor
- Sodium-Cooled Fast Reactor
- Molten Salt Reactor

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# SWU

- SWU derivation
- Separation of isotopes and thermal diffusion
- Extensive notes on thermodynamics
- The theory of uranium enrichment by the gas centrifuge

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# Back-end of the fuel cycle

- Six Overarching Recommendations for How to Move the Nation's Nuclear Waste Management Program Forward
- Sorption behavior of Np(IV), Np(V), and Am(III) in the disturbed zone between engineered and natural barriers
- Integrated radionuclide transport model for a high-level waste repository in water-saturated geologic formations
- Influence of exchangeable cations on hydraulic conductivity of compacted bentonite
- Transport model derivation
- Migration of radionuclides through sorbing media I
- Migration of radionuclides through sorbing media II
- Overview of the BRC draft report
- The environmental and ethical basis of geological disposal of long-lived radioactive wastes
- The disposal of radioactive waste on land
- Yucca mountain license
- Geological criteria for repositories for high-level radioactive wastes
- Yucca mountain performance assessment summary by Peter Swift
- Blue ribbon commission on America's nuclear future
- The Back-End of the Nuclear Fuel Cycle: An Innovative Storage Concept
- Radioactive Waste: Production, Storage, Disposal
- Yucca Mountain
- Consent based siting
- An atomic garbage dump for Kansas
- Yucca mountain: A case study

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# MCNP

There is a wealth of information on MCNP due to such a long history of use. This is just a small sample of information.

- MCNP neutron benchmark problems
- Handbook of Monte Carlo Methods
- MCNP criticality calculations
- MCNP tutorial
- Monte Carlo methods
- Fundamentals of Monte Carlo particle transport
- Compendium of Material Composition Data for Radiation Transport Modeling
- Compendium of Material Composition Data for Radiation Transport Modeling Revision 1
- LANL Monte Carlo code group
- Use of Monte Carlo codes
- MCNP primer
- MCNP criticality safety benchmark problems
- MCNP Reference Collection
- Vised help
- Introduction to MCNP
- MCNP v6.2 user's manual
- Radiation Detection Computational Benchmark Scenarios

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# Fuel cycle analysis

- An Environmental Impact Measure for Nuclear Fuel Cycle Evaluation
- USA-ROK 123 agreement
- Can nuclear power and renewables be friends?
- Reflections on the Fukushima Daiichi nuclear accident
- BORAX nuclear reactor and the EBR-I Meltdown

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# Fuel mangement

- Generic environmental impact statement for mixed oxide fuel
- In-core fuel management: reloading techniques
- Good practices
  for outage management in nuclear power plants
- Management strategies for nuclear power plant outages
- Solving the Nuclear Waste Storage Dilemma

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# Burnup

# 43

# Criticality safety

- The chilling story of the demon core and the scientists who became its victims
- The demon core and the strange death of Louis Slotin
- The S-1 Committee
- Nuclear criticality experiments from 1943 to 1978
- NRC review of the Tokaimura criticality accident
- Lessons learned from criticality accident in Tokaimura
- Lessons learned from the JCO nuclear criticality accident in Japan in 1999
- Report on the preliminary fact finding mission flowwing the accident at the nuclear fuel processing facility in Tokaimura Japan
- An analysis of Tokaimura nuclear criticality accident: A systems approach
- Japan Arrests Six in Nuclear Accident that Killed Two
- DOE-STD-3007-2017, Preparing Criticality Safety Evaluations at Department of Energy Nonreactor Nuclear Facilities
- Criticality Safety Basics for INL FMHs and CSOs
- Defense Nuclear Facilities Safety Board 27th Annual Report to Congress April 2017
- A near-disaster at a federal nuclear weapons laboratory takes a hidden toll on America's arsenal
- 2016 Annual metrics report to Defense Nuclear Facilities Safety Board
- A review of criticality accidents
- The criticality accident in Sarov
- Compilation of criticality data involving thorium or U233 and light water moderation
- Nuclear criticality safety guide
- Handbook of critical experiments benchmarks
- Hand calculation methods for criticality safety – A primer
- Critical dimensions of systems containing U235, Pu239, and U233
- A huge and gigantic compilation of criticality safety slides
- Y-12 handbook
- Minimum critical masses at the Portsmouth gaseous diffusion plant
- DOE O 420.1C, Facility Safety
- Nuclear Criticality Safety Evaluation for Contact-Handled Transuranic Waste at the Waste Isolation Pilot Plant
- Preparation of nuclear criticality safety evaluations
- WIPP Shielded Containers – October 29, 2008 DOE Submission
- 10CFR830 Subpart B – Safety basis requirements
- DOE-STD-3009-2014, Preparation of nonreactor nuclear facility documented safety analysis
- DOE-STD-1135-99, Guide for nuclear criticality safety engineer training and qualification
- 10 CFR 830 Subpart A, Quality assurance requirements
- 10 CFR Part 50 Appendix B, Quality Assurance Criteria for Nuclear Power Plants and Fuel Reprocessing Plants
- DOE Order 227.lA, Independent Oversight Program
- DOE Office of Nuclear Safety and Environmental Assessments
- DOE – EA – CRAD 31-30, Criticality Safety Program and Criticality Safety Controls Implementation Criteria and Review Approach Document
- Seventy-Five Years of Nuclear Criticality Safety Documents – A Bibliography
- Anomalies of nuclear criticality
- Enterprise Assessments Review of the Los Alamos National Laboratory Plutonium Facility Restart of Fissile Material Operations – January 2016
- ARH criticality handbook
- Regulatory Guide 3.71, Nuclear Criticality Safety Standards for Fuels and Material Facilities
- Nuclear safety guide TID-7016
- NUREG/CR-6698, Guide for Validation of Nuclear Criticality Safety Calculational Methodology

- Large nuclear criticality safety evaluation of large cylinder cleaning operations in X-705, Portsmouth gaseous diffusion plant
- MCNP6 for Criticality Accident Alarm Systems — A Primer
- Brookhaven National Laboratory – Criticality safety

# 44

# Pyroprocessing

- EXPERIMENTAL BREEDER REACTOR II (EBR-II) Presentation to National Research Council Committee on DOE Category A Reactors

# 45

# Safeguards

- Material accountability – Theory, verification, applications
- Treaty on the Non-Proliferation of Nuclear Weapons
- INFCIRC/153/Corr
- INFCIRC/66
- INFCIRC/540
- Introduction to international safeguards
- Course on safeguards and nonproliferation
- IAEA Facility-Level Safeguards and Implementation
- Nuclear Safeguards, Monte Carlo Method, and MCNP – A Brief Review of Our 70 Year History
- Lessons Learned in International Safeguards—Implementation of Safeguards at the Rokkasho Reprocessing Plant
- Designing and Operating for Safeguards: Lessons Learned From the Rokkasho Reprocessing Plant (RRP)
- Implementing safeguards-by-design
- Fundamentals of materials accounting for nuclear safeguards
- Advanced Safeguards Approaches for New Reprocessing Facilities
- DOE standard – Nuclear materials control and accountability
- The Attractiveness of Materials in Advanced Nuclear Fuel Cycles for Various Proliferation and Theft Scenarios
- DOE graded safeguards
- Improving the Assessment of the Proliferation Risk of Nuclear Fuel Cycles
- An assessment of the proliferation resistance of materials in advanced nuclear fuel cycles
- Statistical Methods in Nuclear Material Control
- Statistical Methods for Nuclear Material Management
- Training Manual on Statistical Methods for Nuclear Material Management

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# Neutron transport

- Case studies in neutron transport theory
- Transport theory

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# LLW management

- LLW disposal technologies
- 10CFR61
- History and Framework of Commercial Low-Level Radioactive Waste Management in the United States
- Low level radioactive waste management and disposition: Background information
- Radioactive waste: Production, storage, disposal
- Civilian Nuclear Waste Disposal
- Overview of WCS' Low-Level Radioactive Waste Disposal Site
- Appendix G to Part 20—Requirements for Transfers of Low-Level Radioactive Waste Intended for Disposal at Licensed Land Disposal Facilities and Manifests
- Commercial Low-Level Radioactive Waste Disposal In South Carolina
- NUREG-1573
- Practical Considerations for Feature, Event, and Process (FEP) Analysis
- Features, Events and Processes (FEPs) for Geologic Disposal of Radioactive Waste
- Guidance for Conducting Technical Analyses for 10 CFR Part 61
- Probabilistic safety assessment model for near surface radioactive waste disposal facilities

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# DOE waste disposal facilities

- Cleaning up America’s nuclear weapons complex
- Assessment of Disposal Options for DOE-Managed High-Level Radioactive Waste and Spent Nuclear Fuel
- WIPP accident report
- WIPP accident report Phase 2
- WIPP accident description
- Management and Disposal of U.S. Department of Energy Spent Nuclear Fuel
- Disposal of surplus plutonium in the waste isolation pilot plant
- Reassess New Mexico’s nuclear-waste repository
- WIPP Performance Assessment: Surplus Pu Disposition
- WIPP regulatory compliance
- Brief history of WIPP
- WIPP site incident independent review
- Feasibility and Risks of Human Intrusion in WIPP

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# Waste classification

- High-level radioactive waste interpretation
- Waste Disposition: A New Approach to DOE's Waste Management Must Be Pursued

# 50

# Math skills

- University of Houston – MATH3321 Engineering Mathematics
- Handbook of mathematical functions

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# Nuclear reactor kinetics

- Nuclear reactor kinetics lecture series
- Point kinetics equations
- More point kinetics equations
- Doppler effect
- Physics of the Doppler effect
- Evaluation and Application of Delayed Neutron Precursor Data
- Nuclear reactor physics

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# Radioactive decay

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# Interactions of radiation with matter

- Diagnostic radiology physics
- Los Alamos National Laboratory – Basics of gamma ray detection
- UC-Berkeley – Electron trajectory reconstruction for advanced compton imaging of gamma rays

54

# Cross sections

- Nuclear cross sections and technology – Volume I
- Nuclear cross sections and technology – Volume II
- Nuclear reactions – An introduction
- Handbook on nuclear activation cross sections

# 55

# Nuclear fuel cycle overview

**Reactors in nature**

- The Workings of an Ancient Nuclear Reactor
- Oklo's Natural Fission Reactors

**Reactor concepts**

- Hanford B reactor history
- Vision and reality: The EBR-II story
- NRC – Issued Design Certification – Advanced Boiling-Water Reactor (ABWR)
- ABWR

**Mining and milling**

- Environmental Damage and Public Health Risks From Uranium Mining in the American West
- Uranium Mining Overview
- Uranium extraction technology

# 8.0. Media

## 56

# General

- Khan Academy
- Yale University – Open Yale courses
- AK lectures

# 57

**Video lectures**

- Motion Mountain – The free physics textbook
- MIT Open Courseware – Quantum physics I
- Series of lectures from UC-Berkeley on nuclear engineering basics
- Series of lectures from MIT on nuclear physics
- Mass-energy conversion, mass defect, and nuclear binding energy
- Antiparticles and mass-energy equivalence
- Nuclear physics decay rates and half lives
- Nuclear decay
- Mass defect and binding energy
- Radioactive decay
- Radiation and Radioactive Decay
- Chart of nuclides introduction
- Introduction to Radioactivity, Fission, and Fusion

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# Interaction of radiation with matter

- AK Lectures – Photoelectric effect
- BYJU's Classes – The photoelectric effect
- Photoelectric effect
- Photoelectric Effect: Tennis Ball
- Photon Pair Production
- Physics – Modern Physics – Particle Pair Production
- Pair Production
- Pair production and annihilation
- Compton Scattering
- Compton Scattering
- The Interaction of Radiation and Matter
- MIT Open Courseware – Introduction to Nuclear Engineering and Ionizing Radiation

# 59

- Neutron scattering parameters
- Neutron Interactions With Matter
- Interaction of Neutrons with Matter
- Neutron capture
- Neutron dance
- Comparing attenuation for gamma and neutrons

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# Neutron attentuation

- Neutron mean free path in tissue

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# Nuclear reactor kinetics

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# Bremsstrahlung radiation

- Bremsstrahlung radiation
- Thermal Bremsstrahlung / Free-Free Emission

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# Cross sections

- AK Lectures – Nuclear cross sections
- Nuclear Cross Section
- Cross sections & Maxwell Boltzman distribution
- Nuclear Reactions, Compound Nuclear Reactions, Direct Reactions
- Macroscopic cross sections and lethargy
- Klein-Nishina Formula

## 64

# Fission

- Nuclear Fission reaction explained
- Fission And Fusion
- Nuclear fission, fusion, and radiation
- Nuclear Fission

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# Nuclear fuel cycle overview

Some of this is a little dated, and perspectives might have changed.

- Atoms for peace
- Pandora's promise
- Nuclear Fuel Cycle
- MIT Study: The Future of the Nuclear Fuel Cycle (This was a big deal when it came out.)
- The Back End of the Nuclear Fuel Cycle-Reprocessing
- Nuclear Security & Safeguards Education Portal
- The Back End of the Nuclear Fuel Cycle:Disposal:DOE Yucca Mountain Video
- Advanced Nuclear Power Systems for Long-term Energy and Climate Security – ANU Lecture
- The Nuclear Fuel Cycle and North Korea: How did North Korea Develop its Nuclear Capacity?(Highly recommended. I know the speaker.)
- Exelon shutting down Clinton and Cordova nuclear plants
- Closure threat looms over Oswego County's Nine Mile 1 nuclear reactor
- Shutdown battle: A tale of 2 nuclear power plants Fitzpatrick has since been saved.
- Diablo Canyon Nuclear Plant Closure Approved
- Diablo Canyon Closure
- Communities react to proposed Fort Calhoun Power Plant closure
- Chernobyl and Hanford – "A Tale of Two Cities" Lecture
- UC Berkeley Nuclear Engineering weekly Colloquium
- Pandora's Promise Tour – Nuclear Communication -by Robert Stone
- Basic principles of operation of nuclear power plants
- The Molten-Salt Reactor Experiment

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# Neutronics

- Neutron Transport Equation
- Four and Six Factor Formulas
- Neutron Life Cycle

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# Used fuel storage

- Spent fuel storage at Diablo Canyon

Public domain content

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# Front-end of the fuel cycle

- Chicago pile 1
- SL-1 accident
- SL-1 documentary
- Plutonium production at Hanford
- Lasting legacy: Hanford's B-Reactor
- Overview of the Nuclear Fuel Cycle and Its Chemistry
- Safeguards for Uranium Enrichment
- LFTRs in 5 minutes – Thorium Reactors
- Fukushima: Timeline, Facts, & Implications for Nuclear Power
- EBR1 Haroldsen Tour Pt 1
- Making a Contribution: The Story of EBR-II
- How Nuclear Power Plants Work
- Liquid liquid extraction – worked solution
- Liquid Liquid Extraction Problem 1
- Liquid Liquid Extraction Problem 5
- The EBR-II Fuel Facility

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# Back-end of the fuel cycle

- Solving the Nuclear Waste Storage Dilemma
- Into enternity

# 70

# Criticality safety

The troubling story of the nuclear demon core

Tokaimura nuclear accident

Tokaimura: The forgotten criticality – WARNING GRAPHIC TOWARDS THE END

71

# MCNP

Karim Hossny's MCNP videos

MCNP6 tutorial

# 72

# Safeguards

- Nuclear Non-Proliferation Technical Primer
- Nuclear Security & Safeguards Education Portal

# IX

# Appendices

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# Python

- Anaconda
- Learn python the hard way
- Command line crash course
- Numerical methods lecture notes (with code)
- PyNE – The Nuclear Engineering Toolkit

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# Related material

- vim Editor
- Jupyter notebook
- Wolfram alpha
- Chart of Nuclides
- TheDoctorRAB GitHub

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# Apps and utilities

- LibreOffice
- Equation editor
- Function grapher
- piazza iOS
- piazza android
- prezi iOS
- prezi android

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# Other Lumen OERs

- Introductory Chemistry – Lecture & Lab
- Boundless chemistry
- OER Overview & Userguide