NE585
NUCLEAR FUEL CYCLES
Nuclear reactor theory
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Learning objectives

Design a critical nuclear reactor configuration

Derive steady state neutron transport equation

Demonstrate transient reactor behavior

Watch videos on different reactors and historical events

Learning nodes

Review

Neutron chain reaction

Neutron multiplication factor

Four factor formula

Neutron reproduction factor

Fuel utilization factor

Resonance escape probability

Fast fission factor

Critical and subcritical configurations

Burnup

Neutron interaction rate

Neutron diffusion

Equation of continuity

Diffusion equation

Diffusion length

Group diffusion theory

More learning nodes

Nuclear reactor design
One group reactor equation
Buckling
Leakage
Criticality for thermal reactors
Two group theory
Six factor formula
Reflected reactor
Multigroup theory
Heterogeneity

Reactor kinetics
Prompt neutrons
Delayed neutrons
Reactor period
Point kinetics equations
Prompt critical

Temperature coefficient

Doppler broadening

Void coefficient

Fission product poisons

Even more learning nodes

Heat removal

Heat removal rate

Heat production rate

Conduction

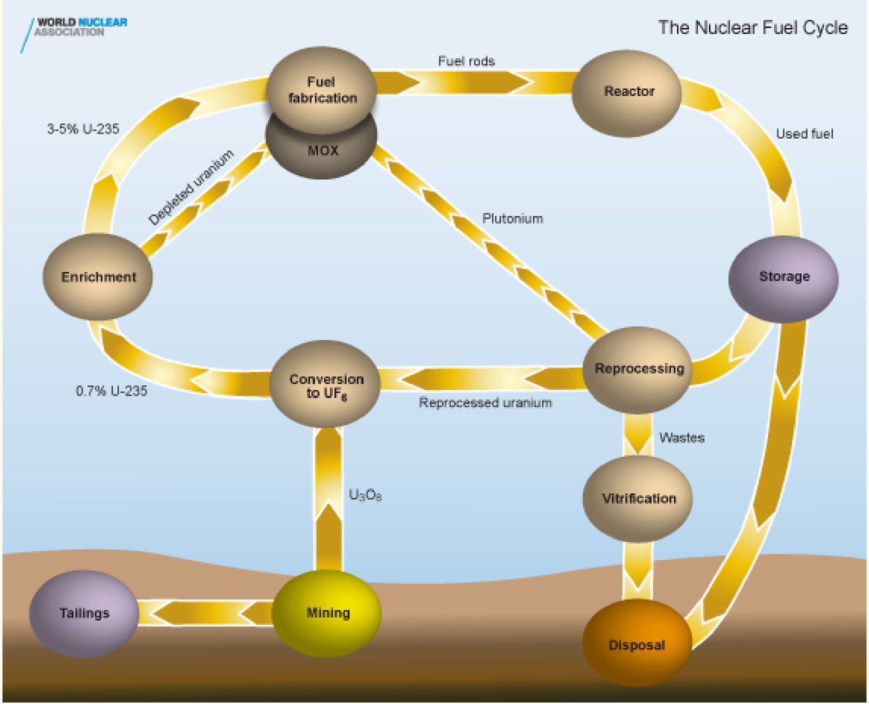
Convection

Dimensionless heat transfer numbers

Boiling

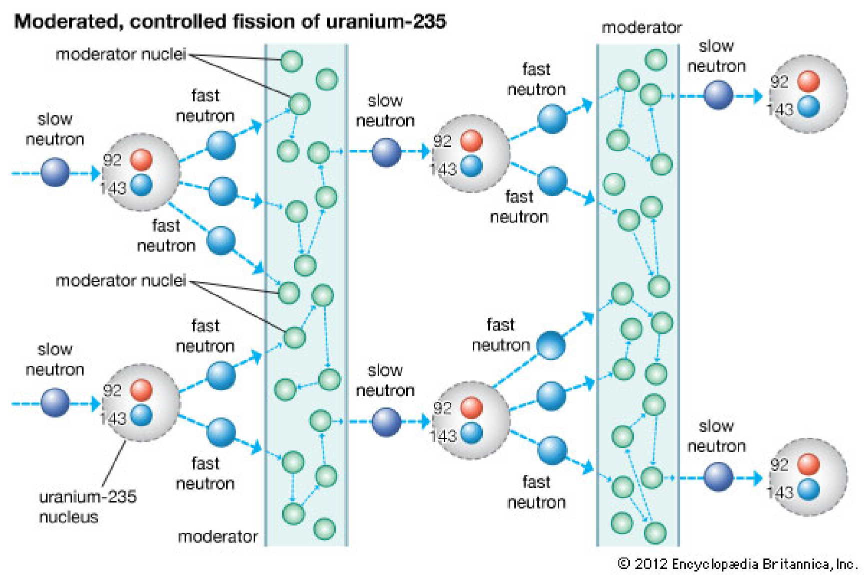
Meltdown

Nuclear fuel cycle review


WORLD NUCLEAR ASSOCIATION
The Nuclear Fuel Cycle
Fuel fabrication
MOX
Fuel rods
Reactor
Used fuel
3-5% U-235
Depleted uranium
Plutonium
Enrichment
Storage
Reprocessing
0.7% U-235
Conversion to UF_6
Reprocessed uranium
Wastes
U_3O_8
Vitrification
Tailings
Mining
Disposal

Neutron chain reaction

Designing a nuclear reactor is about controlling the neutron chain reaction



Neutron multiplication factor

The neutron multiplication factor describes a chain reaction

Ratio of fissions in generation $(n + 1)$ to fissions in generation (n)

$$k < 1 \rightarrow ? \tag{1}$$

$$k = 1 \rightarrow ? \tag{2}$$

$$k > 1 \rightarrow ? \tag{3}$$

Four factor formula

$$k_\infty \equiv \eta f \epsilon p$$

Neutron reproduction factor

The neutron multiplication factor describes a chain reaction

$$\eta \equiv \frac{\nu \Sigma_F}{\Sigma_A} \tag{4}$$

Average number of neutrons released per fission is dependent on material

Prompt and delayed neutrons (important to control)

Average number of neutrons per thermal fission times the probability a fission occurs when a thermal neutron is absorbed by the fuel

Is η less than or greater than 1? – Why?

Fuel utilization factor

Fuel utilization factor is ratio of neutrons absorbed in fuel to fuel + moderator

$$f \equiv \frac{\Sigma_A^{FUEL}}{\Sigma_A^{FUEL} + \Sigma_A^{MOD}} \tag{5}$$

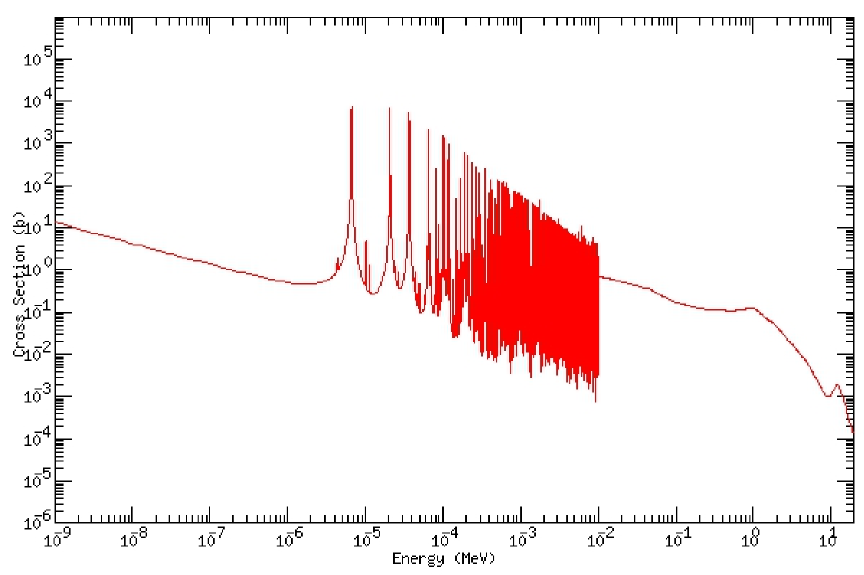
$$0 \leq f \leq 1 \tag{6}$$

Resonance escape probability

Resonance escape is probability neutron is not absorbed in the resonance region

Most neutrons are absorbed by ^{238}U when slowing down in commercial reactors

Empirical results are typically used because it is extremely difficult to compute



Cross Section (b)
Energy (MeV)
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}

Fast fission factor

Fast fission factor is ratio of the total number of fast and thermal neutrons produced to number produced by just thermal fission

Again, very hard to calculate

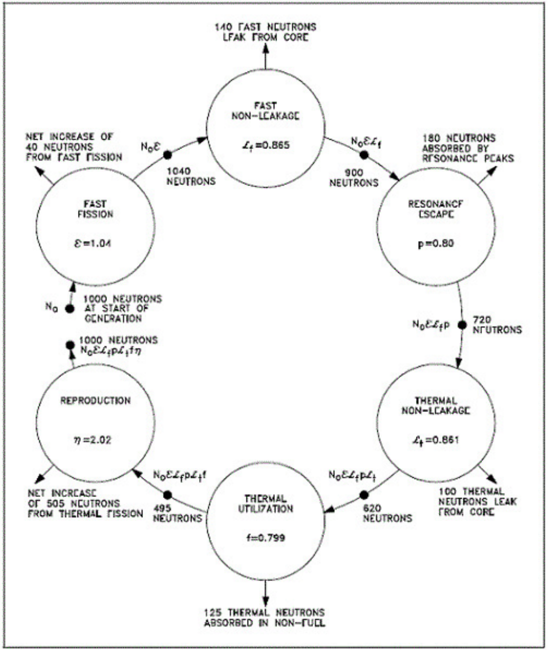


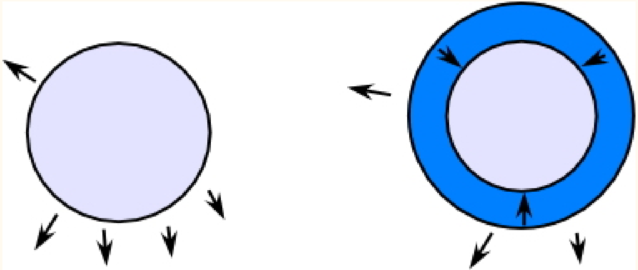
Figure 1 Neutron Life Cycle with k_{eff} = 1

Critical and subcritical configurations

The critical mass is the minimum amount of material required to maintain $k = 1$

Critical size is determined based on material and geometry

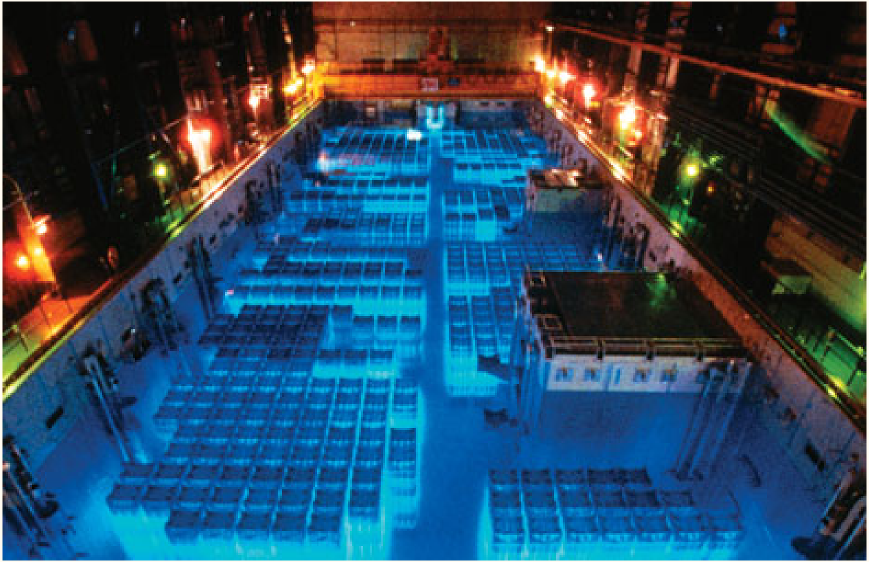
If the critical size of a Pu bare sphere is 10 cm, how do you make this smaller?



k is used to determine subcritical assemblies as well

Like used fuel pool storage

Or any kind of storage



Burnup

Burnup is a measure of the total energy released in fission by the fuel

Typically given as GWD/MTU

1.05 g ^{235}U = 1 MWD

Also called depletion analysis

Generation III+ designs – 55 GWD/MTU

Higher burnup means more fissions, more fuel consumed

But the build up of fission product poisons means that refueling is needed

^{239}Pu is actually produced in the reactor due to ^{238}U neutron absorption

Quantity depends on burnup

At the end of a fuel campaign, some Pu is fissioning as well

This is extracted by reprocessing to make recycled Mixed Oxide (MOX) fuel

Taking advantage of this, reactors can be designed to make plutonium

Fast reactors are used

Neutron interaction rate

Define energy dependent neutron interaction rate

$$F \equiv \int_0^{\infty} \Sigma_T(E)\phi(E)dE \tag{7}$$

Total interaction rate over all neutron energies

Typically assume monoenergetic neutrons

Derived in 5.1 – whatever chapter is called ‘neutron diffusion’

Neutron diffusion

We assume neutron diffusion follows Fick's law

Which is a good assumption because nearly everything does

The book calls J 'current'

$$J_i \equiv -D\frac{d\phi}{di} \tag{8}$$

$$D \equiv [L] \tag{9}$$

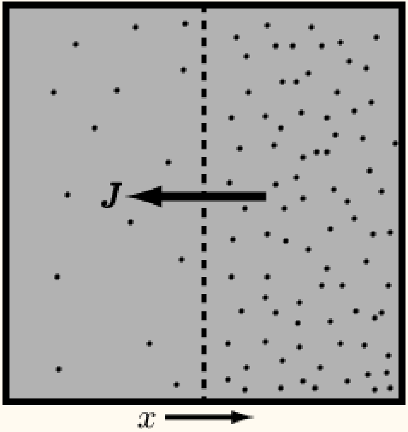
$$\underline{J} \equiv \underline{\underline{D}}\nabla\phi \tag{10}$$

There are conditions when Fick's law is not valid

Strongly absorbing medium

Three mean free paths of source or medium surface

Anisotropic scattering


J
x

Equation of continuity

Equation of continuity

Concept applied to describe many physical phenomena

Material passing through a control volume must be accounted for

How the control volume is defined is important

Typically just a fixed volume

[rate of change of neutrons] = [production rate] - [absorption rate] - [leakage rate]

$$\frac{d}{dt}\int_V n dV = \int_V s dV - \int_V \Sigma_A \phi dV - \int_A \underline{J} \cdot \underline{n} dA$$

[rate of change of neutrons] = [production rate] - [absorption rate] - [leakage rate]

$\int_V n dV$ – Total number of neutrons

$\frac{d}{dt}\int_V n dV = \int_V \frac{\partial n}{\partial t} dV$ – Rate of change

$\int_V s dV$ – Production rate

$\int_V \Sigma_A \phi dV$ – Absorption rate

$\int_A \underline{J} \cdot \underline{n} dA = \int_V \nabla \underline{J} dV$ – Leakage rate

$$\int_V \frac{\partial n}{\partial t} dV = \int_V s dV - \int_V \Sigma_A \phi dV - \int_V \nabla \underline{J} dV$$

$$\frac{\partial n}{\partial t} = s - \Sigma_A \phi - \nabla \underline{J} \tag{11}$$

Diffusion equation

Use the equation of continuity to obtain the diffusion equation

$$\frac{\partial n}{\partial t} = s - \Sigma_A \phi - \nabla \underline{J} \tag{12}$$

Substitute in Fick's law

$$D\nabla^2 \phi - \Sigma_A \phi + s = \frac{\partial n}{\partial t} \tag{13}$$

$$\phi = nv \tag{14}$$

$$D\nabla^2 \phi - \Sigma_A \phi + s = \frac{1}{v}\frac{\partial \phi}{\partial t} \tag{15}$$

$$\nabla^2 \phi - \frac{\Sigma_A}{D}\phi + \frac{s}{D} = \frac{1}{D}\frac{1}{v}\frac{\partial \phi}{\partial t} \tag{16}$$

$$\nabla^2 \phi - \frac{1}{L^2}\phi + \frac{s}{D} = \frac{1}{D}\frac{1}{v}\frac{\partial \phi}{\partial t} \tag{17}$$

$$\nabla^2 \phi - \frac{1}{L^2}\phi + \frac{s}{D} = 0 \tag{18}$$

Diffusion length

L is defined as the 'diffusion length' (5.7)

Average(ish) distance traveled by neutron before absorption

Not quite the same as mean free path

There are several typical solutions in 5.6 based on geometry

$s = 0$ since the medium itself does not produce neutrons

We are basically talking about moderator behavior

With more math, these are valid for thermal neutrons

Group diffusion theory

Neutrons have an energy distribution

Emitted in fission with continuous energy spectrum

To get around this ranges of neutrons grouped into 'bins'

Group diffusion theory

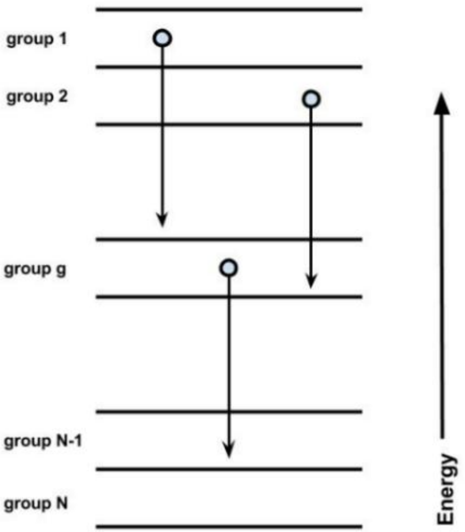
Each group has averaged parameters

Continuity equation then needs more terms

Scatter out of the group

Scatter into the group

Three group diffusion, four group, five, etc.


group 1
group 2
group g
group N-1
group N
Energy

Group theory useful for fast neutrons and thermal neutrons

$$D_F \nabla^2 \phi_F - \Sigma^S_{F \to T} \phi_F = 0 \tag{19}$$

$$\nabla^2 \phi_T - \frac{1}{L^2} \phi_T + \frac{\Sigma^S_{F \to T} \phi_F}{D_T} = 0 \tag{20}$$

Describe in words

Nuclear reactor design

One group reactor equation

Consider a bare fast reactor, homogeneous mix of fuel and coolant

Bare = no reflector

$$D\nabla^2\phi - \Sigma_A\phi + s = 0 \tag{21}$$

This time $s \neq 0$

Cross section is for the mixture

Source neutrons emitted due to fission not absorbed

$$\therefore s = \eta f \Sigma_A \phi \tag{22}$$

Assume an infinite reactor

$$\therefore k_\infty = \eta f \tag{23}$$

$$D\nabla^2\phi - \Sigma_A\phi + k_\infty\Sigma_A\phi = 0 \tag{24}$$

One group reactor equation

$$\nabla^2\phi + B^2\phi = 0 \tag{25}$$

$$B^2 \equiv \frac{k_\infty - 1}{L^2} \tag{26}$$

Someone solve for B^2 on the board

Buckling

Solve buckling for different geometries

B^2 is the eigenvalue

What does this mean?

For a sphere – $B^2 = (\frac{\pi}{R})^2$

For a slab – $B^2 = (\frac{\pi}{a})^2$

Find the table in Lamarsh

Buckling determines critical geometry

Solve for whatever geometry

Integrate over the volume for power

Use power to find the constant of integration

$$P = E_R\Sigma_F \int \phi dV \tag{27}$$

$$E_R = 3.2 \times 10^{-11} J \; per \; fission \sim 200 \; MeV \tag{28}$$

Leakage

‘Real’ reactors have leakage

Neutrons either leak out or absorbed

Even neutrons absorbed can birth the next generation

Leaked neutrons are just gone

$B^2 = \frac{k_\infty - 1}{L^2}$ is a necessary condition for critical reactor

$\frac{k_\infty}{1+B^2L^2} = 1$ just rearrange the equation to see the leakage term

$k_{EFF} = k_\infty \cdot P_L$ is one group critical equation for a bare reactor

With P_L being a general nonleakage probability term

But we want to account for all the neutrons that will not leak

Why

Go back to the continuity equation

$$\frac{d}{dt}\int_V ndV = \int_V sdV - \int_V \Sigma_A \phi dV - \int_A \underline{J} \cdot \underline{n}dA \tag{29}$$

Identify losses – $\int_V \Sigma_A \phi dV$

Identify leakage – $-\int_V D\nabla^2 \phi dV$

So the probability of absorption; i.e., non leakage is –

$$P_L \equiv \frac{\int_V \Sigma_A \phi dV}{\int_V \Sigma_A \phi dV - \int_V D\nabla^2 \phi dV} \tag{30}$$

$$\therefore P_L = \frac{\Sigma_A}{\Sigma_A + DB^2} = \frac{1}{1 + B^2 L^2} \tag{31}$$

Because – $-\int_V D\nabla^2 \phi dV = \int_V DB^2 \phi dV$

Criticality for thermal reactors

Criticality is technically different for thermal reactors

$$\eta_T \equiv \frac{\int \eta(E)\sigma_F \phi(E) dE}{\int \eta(E)\sigma_A \phi(E) dE} \tag{32}$$

Slowly varying with temperature

Slowly varying with T

But just look it up

It's the same procedure = FUEL + MODERATOR

Four factor formula is the same

Resonance escape is an important parameter

Why?

Two group theory

Need two group theory for thermal criticality

This is like two group diffusion except the source is now nonzero

$$D_1 \nabla^2 \phi_1 - \Sigma_1 \phi_1 + s_1 = 0 \tag{33}$$

$$D_T \nabla^2 \phi_T - \Sigma_A \phi_T + s_T = 0 \tag{34}$$

Assume most fissions are induced by thermal neutrons

Derive the source terms

Thermal fission neutron birth rate

$$\eta_T f \epsilon \Sigma_A \phi_T = \frac{k_\infty}{p} \Sigma_A \phi_T = s_1 \tag{35}$$

Scattered neutrons from fast group are source in thermal group

But only those that escape resonances

$$s_T = p \Sigma_1 \phi_1 \tag{36}$$

For bare thermal reactor

$$D_1 \nabla^2 \phi_1 - \Sigma_1 \phi_1 + \frac{k_\infty}{p} \Sigma_A \phi_T = 0 \tag{37}$$

$$D_T \nabla^2 \phi_T - \Sigma_A \phi_T + p \Sigma_1 \phi_1 = 0 \tag{38}$$

Solving it is not as hard as it might seem

Assume all group fluxes have same spatial dependence in bare reactor

$$\phi_1 = A_1\phi \tag{39}$$

$$\phi_T = A_2\phi \tag{40}$$

Necessary condition for criticality

$$\nabla^2\phi + B^2 = 0 \tag{41}$$

Substitute these into the two group equations

$$-(D_1 B^2 + \Sigma_1)A_1 + \frac{k_\infty}{p}\Sigma_A A_2 = 0 \tag{42}$$

$$p\Sigma_1 A_1 - (D_T B^2 + \Sigma_A)A_2 = 0 \tag{43}$$

Apply Cramer's rule

$$\therefore \frac{k_\infty}{(1 + B^2 L^2)(1 + B^2\tau)} = 1 \tag{44}$$

Where – $\tau \equiv \frac{D_1}{\Sigma_1}$ is the 'neutron age'

Six factor formula

The result is the six factor formula

$$P_F \equiv \frac{1}{1 + B^2\tau} \tag{45}$$

Probability that the fission neutron does *not* escape while slowing down

$$\therefore k = k_\infty P_L P_F \tag{46}$$

Or multiply out the denominator from before

$$\frac{k_\infty}{1 + B^2 M^2} = 1 \tag{47}$$

Thermal migration area

See cases in section 6.5

Reflected reactor

A reflector is added to make the core smaller

$$\nabla^2 \phi_C + B^2 \phi_C = 0 \ core \tag{48}$$

$$\nabla^2 \phi_R - \frac{1}{L_R^2} \phi_C = 0 \ reflector \tag{49}$$

With boundary conditions –

$$\phi_C(R) = \phi_R(R) \tag{50}$$

$$D_C \phi'_C(R) = D_R \phi'_R(R) \tag{51}$$

$$\therefore BR \cdot cot(BR) - 1 = -\frac{D_R}{D_C}(\frac{R}{L_R} + 1) \ sphere \tag{52}$$

Reflected reactor

Multigroup theory is applied in the same way (as before)

More groups gives a more accurate flux

Obviously gets way more complicated to solve

Procedure is the same

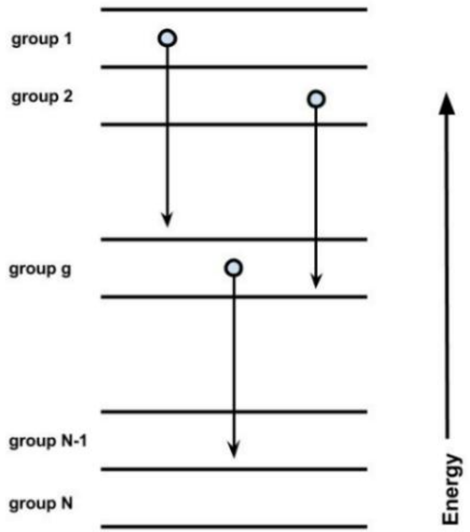
For the two group equation –

$$D_1 \nabla^2 \phi_1 - \Sigma_1 \phi_1 + s_1 = 0 \tag{53}$$

$$D_T \nabla^2 \phi_T - \Sigma_A \phi_T + s_T = 0 \tag{54}$$

Now we want N groups

So we derive for an arbitrary group g which is next to a group h


group 1
group 2
group g
group N-1
group N
Energy

Multigroup theory is applied in the same way (as before)

Σ_F^g – group averaged fission cross section

ν_g – fission neutrons released per induced in the group

χ_g – fraction of fission neutrons emitted in the group

$\Sigma_F^h \phi_h$ – fission density in h group (adjacent to g)

$\nu_h \Sigma_F^h \phi_h$ – neutrons released from h group fissions

$\sum \nu_h \Sigma_F^h \phi_h$ – total neutrons emitted due to fission all groups

$s_g = \sum_{h=1}^{N} \nu_h \Sigma_F^h \phi_h$ – source

Then put that all together

$$D_g\nabla^2\phi_g - \Sigma_A^g\phi_g - \sum_{h=g+1}^{N}\Sigma_{g\rightarrow h}\phi_g + \sum_{h=1}^{g-1}\Sigma_{h\rightarrow g}\phi_h + \chi_g\sum_{h=1}^{N}\nu_h\Sigma_F^h\phi_h = 0 \quad (55)$$

Can anyone do it out for 3 groups?

UGM

Multi-group Reactor Equation

The "transfer" of neutrons between groups is accounted for by:

- Scattering Cross-sections (Transfer X_sections)
- Fission Spectrum

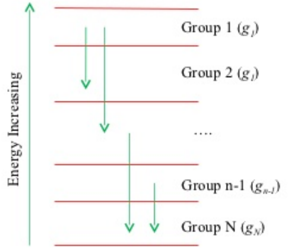
The Multi-group diffusion equations are:

$$D_g \nabla^2 \phi_g - \Sigma_{ag}\phi_g - \underbrace{\boxed{\sum_{h=g+1}^{N} \Sigma_{g\rightarrow h}\phi_g}}_{\textit{Transfer out of } g} + \underbrace{\boxed{\sum_{h=1}^{g-1} \Sigma_{h\rightarrow g}\phi_h}}_{\textit{Transfer into } g} + s_g = 0$$

$$g = 1, \dots, N$$

For Fluxes and X-sections defined as:

$$\phi_g = \int_g \phi(E)\, dE \qquad \Sigma_g = \frac{1}{\phi_g}\int_g \Sigma(E)\,\phi(E)\, dE$$



Energy Groups for a N-Group Diffusion Calculation

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Heterogeneity

Unfortunately, real reactors aren't homogeneous and it makes calculating k hard

Fortunately, the theory is the same

Current LWR fuel is enriched uranium dioxide

And they're all thermal

$$\eta_T = \frac{\nu^{25}\Sigma_F^{25}}{\Sigma_A^{25} + \Sigma_A^{28}} \tag{56}$$

And for fuel utilization

Neutron absorption rate in fuel

$$\int_{V_F} \Sigma_A^F \phi_T dV \tag{57}$$

Neutron absorption rate in moderator

$$\int_{V_M} \Sigma_A^M \phi_T dV \tag{58}$$

And for fuel utilization

By definition –

$$f = \frac{\Sigma_A^F \overline{\phi}_T^F V_F}{\Sigma_A^F \overline{\phi}_T^F V_F + \Sigma_A^M \overline{\phi}_T^M V_M} \tag{59}$$

Hard to compute for real because of flux, so they developed approximations

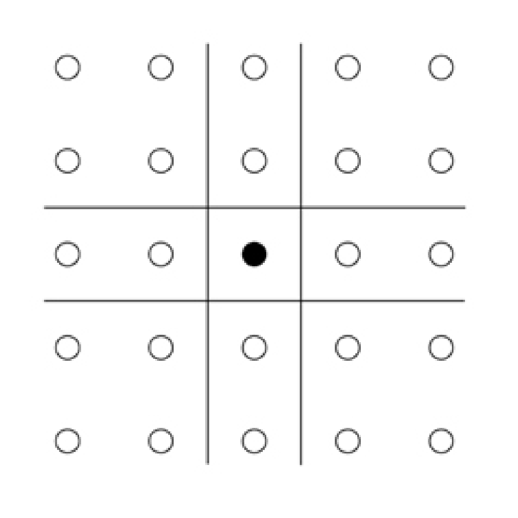
$$\frac{1}{f} = \frac{\Sigma_A^M V_M}{\Sigma_A^F V_F} \cdot F + E \tag{60}$$

Of course they had to use F to confuse everyone

Lattice constants, Bessel functions p264 for F,E

p265 for lattice and unit cell

Wigner-Seitz equivalent cylindrical cell



Then for resonance escape

$$p = e^{-\frac{N_F V_F I}{\xi_M \Sigma_S^M V_M}} \tag{61}$$

N_F is the fuel atom density

ξ_M is the average increase in lethargy in the moderator

$I = A + \frac{C}{\sqrt{a\rho}}$ is the resonance integral

A, C are constants, a is fuel rod radius, ρ is fuel density – p266

So basically there are a bunch of semi empirical expressions needed

Very typical in engineering

Reactor kinetics

Reactor kinetics is about what happens when the reactor shuts down or starts up

$$\therefore \frac{\partial n}{\partial t} \neq 0 \tag{62}$$

Changes in temperature affect changes in neutron multiplication

Reactor is initially loaded with more than the minimum critical mass due to burnup

Criticality affected by fission products

Many are gaseous and have to be trapped

Prompt neutrons

There are different kinds of neutrons to account for

Prompt neutrons are emitted at the instant of fission

Delayed neutrons emitted after the fission event

These control reactor kinetics

Prompt neutron lifetime is the average time between emission and absorption

Time for neutron to slow to thermal is small compared to time as thermal

Prompt neutron lifetime l_P = mean diffusion time t_D in the infinite thermal reactor

Reason out the reactor physics to derive t_D

Neutron travels an absorption mean free path before actually being absorbed

Energy dependent to start

$$t(E) = \frac{\lambda_A(E)}{v(E)} \tag{63}$$

$$t(E) = \frac{1}{\Sigma_A(E)v(E)} \tag{64}$$

$$t_D \equiv \overline{t(E)} \tag{65}$$

Assuming Maxwell in the thermal region – $\frac{1}{v}$

$t(E)$ isn't energy dependent anymore because $E_0 = 0.0253eV$ and $v_0 = 2200m/s$

Reason out the reactor physics to derive t_D

$$\therefore t_D = \frac{\sqrt{\pi}}{2} \cdot \frac{1}{\Sigma_A v_T} \tag{66}$$

For fuel and moderator –

$$t_D = \frac{\sqrt{\pi}}{2 v_T} \cdot \frac{1}{\Sigma_A^F + \Sigma_A^M} \tag{67}$$

Or –

$$t_D = \frac{\sqrt{\pi}}{2 v_T} \cdot \frac{\Sigma_A^M}{\Sigma_A^M} \cdot \frac{1}{\Sigma_A^F + \Sigma_A^M} \tag{68}$$

Reason out the reactor physics to derive t_D

$$t_D = \frac{\sqrt{\pi}}{2v_T\Sigma_A^M} \cdot \frac{\Sigma_A^M}{\Sigma_A^F + \Sigma_A^M} \tag{69}$$

$$\therefore t_D = \frac{\sqrt{\pi}}{2v_T\Sigma_A^M}(1 - f) \tag{70}$$

$\sim 10^4$ s for water

Prompt neutron lifetime is much shorter in fast reactors than thermal $\sim 10^{-7}$ s

What role do neutrons play in reactor kinetics?

Consider an infinite thermal reactor with only prompt neutrons

Prompt neutron lifetime l_P then is time between successive neutron generations

Absorption of a neutron at $t = t_0$ means absorption of k_∞ neutrons at $t = t_0 + l_P$

How to we derive a measure of this time?

$$N_F(t + l_P) = k_\infty N_F(t) \tag{71}$$

Take a Taylor expansion of the left side

$$N_F(t + l_P) \approx N_F(t) + l_P \frac{dN_F(t)}{dt} \tag{72}$$

Substitute back in

$$\frac{dN_F(t)}{dt} = \frac{k_\infty - 1}{l_P} N_F(t) \tag{73}$$

Reactor period – Prompt

How to we derive a measure of this time?

$$N_F(t) = N_F(0)e^{\frac{t}{T}} \tag{74}$$

$$T = \frac{l_P}{k_\infty - 1} \tag{75}$$

T is called the reactor period (in the absence of delayed neutrons)

What is this telling us?

What is the period physically describing?

Picture break


CAUTION

Delayed neutrons

Delayed neutrons control reactor operation

Six groups of delayed neutron precursors with characteristic half life

Fission products that produce neutrons as part of decay process

For infinite homogeneous thermal reactor (not necessarily critical) one group of delayed neutrons

The diffusion equation for thermal neutrons is (5.21)

$$\frac{dn}{dt} = s_T - \Sigma_A \phi_T \tag{76}$$

Assume thermal flux is independent of position

Point kinetics equations

Deriving rate of change for delayed neutrons

From 5.9 based on Maxwellian distribution

$$\phi_T = \frac{2}{\sqrt{\pi}} n v_T \tag{77}$$

$$l_P \approx t_D = \frac{\sqrt{\pi}}{2} \cdot \frac{1}{\Sigma_A v_T} \tag{78}$$

$$\frac{dn}{dt} = s_T - \Sigma_A \phi_T \tag{79}$$

Substitute back in

$$l_P \frac{d\phi_T}{dt} = \frac{s_T}{\Sigma_A} - \phi_T \tag{80}$$

Now derive s_T

The source has two contributions – Prompt and delayed

If $\beta \equiv$ fraction of fission neutrons that are delayed, then the prompt contribution –

$$s_T^P = (1 - \beta) k_\infty \Sigma_A \phi_T \tag{81}$$

Delayed neutrons slow down quick after emission

$$s_T^D = p\lambda C \tag{82}$$

$p \equiv$ resonance escape and λC is decay of precursor (like Bateman)

This means the source is based on the production from the precursor and the probability it escaped through the resonance region

Combining everything

$$l_P \frac{d\phi_T}{dt} = (1-\beta)k_\infty \Sigma_A \phi_T + \frac{p}{\Sigma_A}\lambda C - \phi_T \qquad (83)$$

Rate of change of thermal flux based on one group of delayed neutrons

C is the precursor concentration

We need another equation

Derive the precursor equation

Fission neutron production rate –

$$\eta_T \epsilon f \Sigma_A \phi_T = \frac{1}{p} k_\infty \Sigma_A \phi_T \tag{84}$$

Delayed neutron production rate is then –

$$\beta \cdot \frac{1}{p} k_\infty \Sigma_A \phi_T \tag{85}$$

$$\therefore \frac{dC}{dt} = \beta \cdot \frac{1}{p} k_\infty \Sigma_A \phi_T - \lambda C \tag{86}$$

Point kinetics describes reactor transient behavior

$$l_P \frac{d\phi_T}{dt} = (1-\beta)k_\infty \Sigma_A \phi_T + \frac{p}{\Sigma_A}\lambda - \phi_T \tag{87}$$

$$\frac{dC}{dt} = \beta \cdot \frac{1}{p} k_\infty \Sigma_A \phi_T - \lambda C \tag{88}$$

Leo M. Bobek, R. A. Borrelli, PLC-based reactivity measurements using inverse point kinetics, Transactions of the American Nuclear Society, 74, Annual meeting of the American Nuclear Society, 16-20 June, 1996, Reno, Nevada.

How is a solution obtained?

Solve the system simultaneously

Assume $k_\infty = 1$ at $t = 0$

Step change then made to change k_∞

Assume –

$$\phi = Ae^{\omega t} \rightarrow \frac{d\phi}{dt} = \omega Ae^{\omega t} \tag{89}$$

$$C = C_0 e^{\omega t} \rightarrow \frac{dC}{dt} = \omega C_0 e^{\omega t} \tag{90}$$

Substitute –

$$\omega C_0 e^{\omega t} = (\beta \frac{1}{p} k_\infty \Sigma_A)(Ae^{\omega t}) - \lambda(C_0 e^{\omega t}) \tag{91}$$

Solve the system simultaneously

Continuing gives us reactivity equation for one group of delayed neutrons

$$\rho = \frac{\omega l_P}{1 + \omega l_P} + \frac{\omega}{1 + \omega l_p} \frac{\beta}{\omega + \lambda} \tag{92}$$

$$\rho \equiv \frac{k - 1}{k} \tag{93}$$

What is the range of ρ?

$$\rho = \frac{\omega l_P}{1 + \omega l_P} + \frac{\omega}{1 + \omega l_p} \sum_{i=1}^{6} \frac{\beta_i}{\omega + \lambda_i} \tag{94}$$

Reactor period – Delayed

We still need to figure out ω

Figure 7.1

$$\phi_T = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} \tag{95}$$

For either $\rho < 1$ or $\rho > 1$ the second term dies out

$$\phi_T \rightarrow e^{\omega_1 t} \tag{96}$$

This gives reactor period as –

$$T = \frac{1}{\omega_1} \tag{97}$$

Prompt critical

Let's look at the prompt critical reactor state

If the reactor would be critical only on prompt neutrons –

$$(1-\beta)k = 1 \tag{98}$$

The period is very short and you can't control the reactor

The reactivity corresponding to prompt critical is just –

$$\rho = \beta \tag{99}$$

What is k for this condition for ^{235}U?

See table 7.2

Although, can you really go prompt critical?

We need the reactivity insertion to actually control the reactor

If there isn't much to 'give', there isn't proper reactor kinetic control

Units of dollars are used (because of course they are) to normalize reactivity per prompt critical

South Korean research reactor experiment at ATR

Insertion of reactivity gives a sudden rise in the flux and vice versa

$$\phi_T = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} \tag{100}$$

With 7 exponentials if all groups are concerned

Those terms die away liked we talked about before

Then stable period is achieved

We want to know what that rise/drop is (prompt jump approximation)

The rapid die-away gives sudden rise/drop to the flux before stability

Assume precursor concentrations do not change over the rise/drop

$$\therefore \frac{dC}{dt} = 0 \tag{101}$$

$$C = \beta \frac{1}{p} \frac{1}{\lambda} \Sigma_A \phi_T^0 \tag{102}$$

Substitute –

$$l_P \frac{d\phi_T}{dt} = (1 - \beta) k_\infty \Sigma_A \phi_T + \frac{p}{\Sigma_A} \lambda C - \phi_T \tag{103}$$

Solve for flux

$$l_P\frac{d\phi_T}{dt} = (1-\beta)k_\infty\Sigma_A\phi_T + \frac{p}{\Sigma_A}\lambda C - \phi_T \tag{104}$$

$$l_P\frac{d\phi_T}{dt} = [(1-\beta)k_\infty - 1]\phi_T + \beta\phi_T^0 \tag{105}$$

$$\phi_T = \phi_T^0 e^{\frac{(1-\beta)k_\infty - 1}{l_P}t} + \frac{\beta\phi_T^0}{1-(1-\beta)k_\infty}[1 - e^{\frac{(1-\beta)k_\infty - 1}{l_P}t}] \tag{106}$$

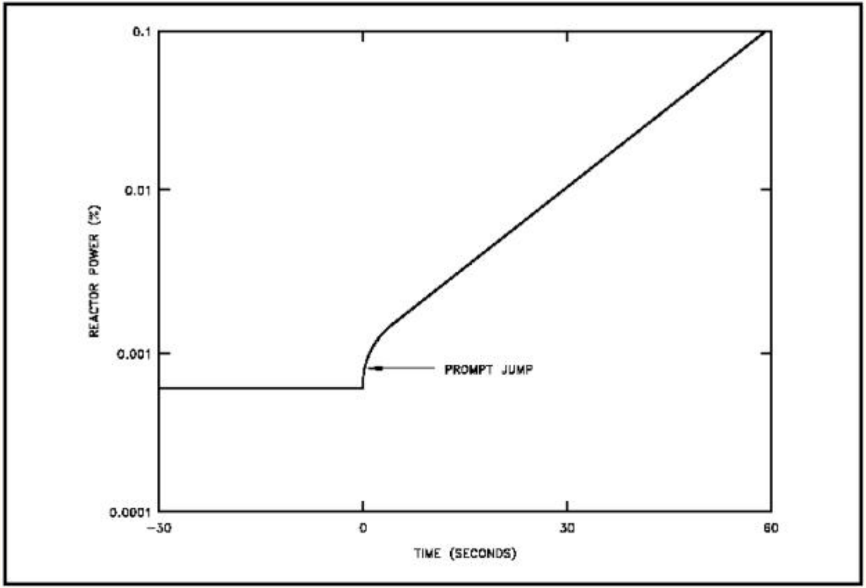
For reactivity less than prompt critical the exponentials die out

$$\phi_T = \frac{\beta}{1-(1-\beta)k_\infty}\phi_T^0 \tag{107}$$

Then –

$$\phi_T = \frac{\beta(1-\rho)}{\beta-\rho}\cdot\phi_T^0 \tag{108}$$

Figure 7.3 p287



What does this mean?

What does this mean?

What happens when there is positive reactivity insertion?

What happens with a negative reactivity insertion?

Control rod worth

What does this mean?

Addition of a control rod to a finite geometry reactor requires solving the reactor equation twice (coupled) because buckling is different given the insertion of a rod(s)

Usual diffusion theory doesn't work

Solving this analytically can be overly much

From a design standpoint, the arrangement of rods needs to let the neutron flux be as uniform as possible over the core

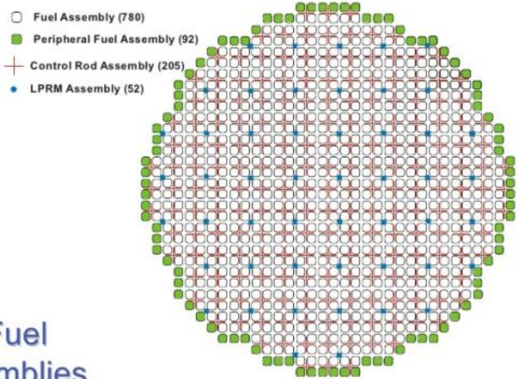
Boric acid is often introduced into the coolant to affect criticality

Changes thermal utilization (f)

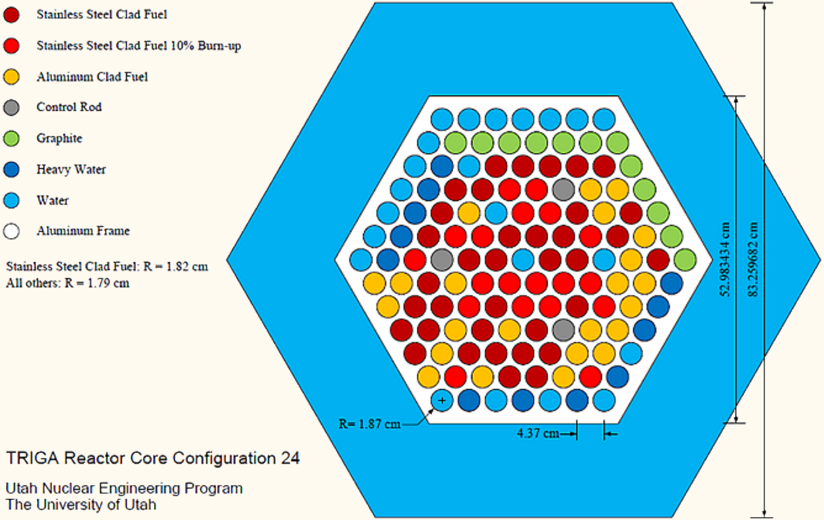
Figure 7.10 experiment – Rod worth curve

Picture break

ABWR Core Configuration



872 Fuel Assemblies



TRIGA core configuration

Temperature coefficient

Many parameters that contribute to neutron multiplication are temperature dependent which changes reactivity in the system

$$\alpha \equiv \frac{d\rho}{dT} \approx \frac{1}{k}\frac{dk}{dT} \tag{109}$$

With a positive coefficient, and increase in temperature leads to meltdown, because of uncontrollable feedback loop

Increase in T = increase in k and vice versa for positive coefficient

With negative coefficient the feedback returns the reactor to its original state

Increase in T = decrease in k

Cannot obtain a license otherwise

Doppler broadening

Doppler broadening is the change in resonance with temperature

Basically about trying to describe changes due to thermal motion of atoms with temperature

Changes the resonance region and affects cross sections (absorption)

Resonance peaks broaden due to vibration of nuclei

^{238}U absorbs more neutrons without causing fission as reactor temperature increases

Increase in reactor temperature leads to a fall in reactivity

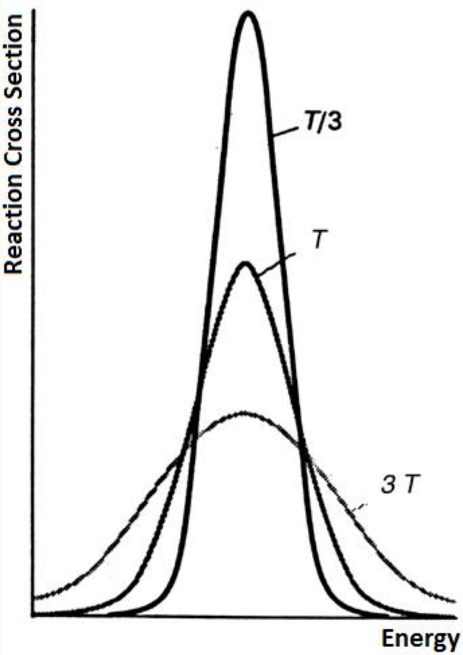
Increase in reactor temperature increases resonance absorption, decreases k

So, the point is to apply the Doppler effect to assure a negative coefficient

Affects real time control of the reactor

Figure 7.12 on p309

Inverse relationship with flux


Reaction Cross Section
T/3
T
3 T
Energy

Void coefficient

The void coefficient describes change in reactivity to void fraction

$$\alpha \equiv \frac{d\rho}{dx} \tag{110}$$

Void basically means the volume occupied by vapor in the coolant upon boiling

Void fraction increases reactivity, power, boiling, uncontrolled feedback loop

So, void coefficient needs to be negative (why?)

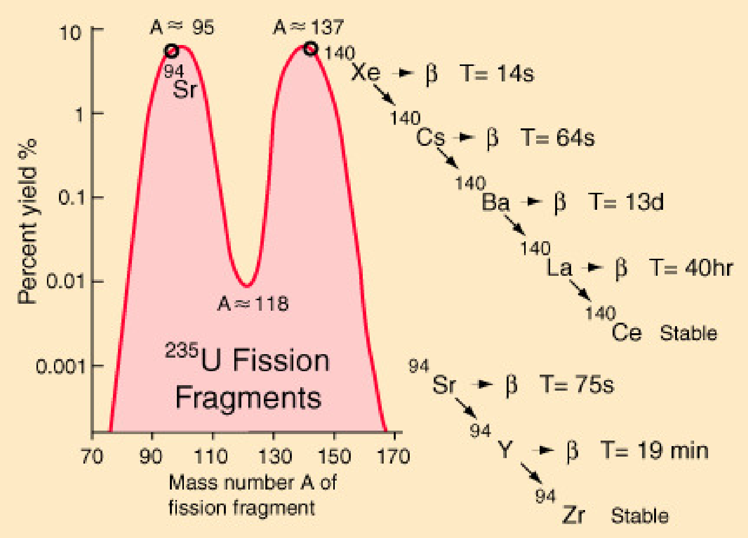
Voids affect moderator/coolant density

More voids decrease density

Important for BWR control

Fission product poisons

Fission product poisons accumulate with burnup



Fission products absorb neutrons

What does this do to the multiplication factor?

$$f = \frac{\Sigma_A^F}{\Sigma_A^F + \Sigma_A^M + \Sigma_A^P} \tag{111}$$

Using the definition of neutron multiplication

$$\rho = -\frac{\Sigma_A^P}{\Sigma_F} \cdot \frac{1}{\nu p \epsilon} \tag{112}$$

What does this mean for reactor operation?

^{135}Xe is a huge poison because $\Sigma_A \approx 2 \times 10^6\ b$

That's a lot of cows

Xenon is produced by ^{135}I decay but also a fission product

$$\frac{dI}{dt} = \gamma_I \Sigma_F \phi_T - \lambda_I I \tag{113}$$

$$\frac{dX}{dt} = \lambda_I I + \lambda_X \Sigma_F \phi_T - \sigma_A \phi_T X - \lambda_X X \tag{114}$$

On shutdown, flux is zero and production of Xe is only due to I decay

Reactor cannot be restarted (unless you fool it with a cold start) until all the *Xe* decays

Due to the high negative reactivity


Poison

This is basically what goes into making a power reactor

Fortunately, we have codes to do this for us

But understanding what is going on and being able to explain it is critical (*rimshot*) to being a nuclear engineer even if it's not your primary field