NE585
NUCLEAR FUEL CYCLES
Back end of the nuclear fuel cycle
6

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Learning objectives

Interpreting back end of the fuel cycle as an equal component with the front end

Explaining how physical processes in the repository work

Criticizing current back end ‘policy’

Generating an approach to consent-based siting

Material from my Berkeley courses & Todai summer schools

Learning nodes

Back-end management overview
Geologic disposal
Classes of waste
Nuclear Waste Policy Act (NWPA)
Waste Isolation Pilot Plant (WIPP)
Blue Ribbon Commission on America's Nuclear Future (BRC)
Energy policy act

Disposal options
History
Yucca mountain
Total System Performance Assessment (TSPA)

Technical basis
Design
Multibarriers & Defense-in-depth
Linear programming

Siting
Technical
Consent
Public engagement

Safety

Technical & political

Back-end management is the most socially science based in the fuel cycle

Some engineers tend to think the public is ignorant

‘Convincing them’ is not a solution

Approaches, criteria, for engineering design are affected by social factors

But, to what extent should nuclear engineers understand social factors?

What is the role of engineers in social decision making process?

Understand the social system correctly

Listen to the public

Develop engineering options to explore optimized solution for the people

What do you do with the fuel when you take it out of the reactor?

Either the used fuel is reprocessed or disposed

But there is always going to be some high-level waste

High-level waste = highly radioactive, some long-lived, toxic

So far in USA, used fuel is stored in the pool and in dry casks

What's the problem with keeping the spent fuel just lying around?

Who has 'active' repositories? How is safety determined?

Watch 'Into Eternity' if you can

Why not just hang onto waste until we figure out something better?

Ethics, which is not nearly taken into consideration enough

> *. . . from an ethical standpoint, including long-term safety considerations, our responsibilities to future generations are better discharged by a strategy of final disposal than by reliance on stores which require surveillance, bequeath long-term responsibilities of care, and may in due course be neglected by future societies whose structural stability should not be presumed . . .*

The Environmental and Ethical Basis of Geological Disposal of Long-Lived Radioactive Wastes, OECD Nuclear Energy Agency (1995)

Waste

Building the reactors is easy

What's the difference between Oklo or Ultra Safe?

The point is to standardize reactor designs for efficient deployment

What's the difference between Yucca Mountain and Onkalo?

Repositories are variable across nations and the world

Acceptance for siting is complex

There are different kinds of nuclear waste

High-Level Radioactive Waste (HLW)

Unreprocessed spent fuel assemblies

Highly radioactive primary waste stream from reprocessing

Containing virtually all fission products and most transuranics except plutonium

Transuranic Waste (TRU)

Non HLW contaminated with long-lived transuranics above $100nCi/g$

Uranium mill tailings

Residues from uranium mining and milling operations containing low concentrations of naturally occurring radioactive materials

Low Level Waste (LLW)

All non HLW, non TRU; wide variation in physical and chemical forms, activity levels, gloves, etc.

Decontamination & Decommissioning (D&D)

Waste contaminated with small amounts of radioactivity from D&D (mostly LLW)

Mixed waste

Contains both radioactive materials and hazardous chemicals

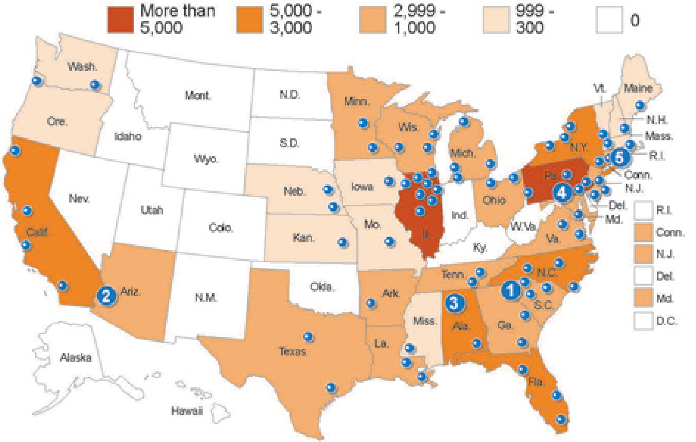
Effluents

Contaminated materials below ‘de minimus’ levels permitting direct discharge to environment

Spent Nuclear Fuel Awaits Permanent Home

The U.S. has about 70,000 metric tons of spent nuclear fuel stored at 75 active and decommissioned reactor sites in 33 states.

Spent nuclear fuel accumulated by state, in metric tons



Top sites for spent fuel *(metric tons)*

1. Oconee, S.C. *(1,986)*
2. Palo Verde, Ariz. *(1,955)*
3. Browns Ferry, Ala. *(1,939)*
4. Peach Bottom, Pa. *(1,694)*
5. Millstone, Conn. *(1,630)*

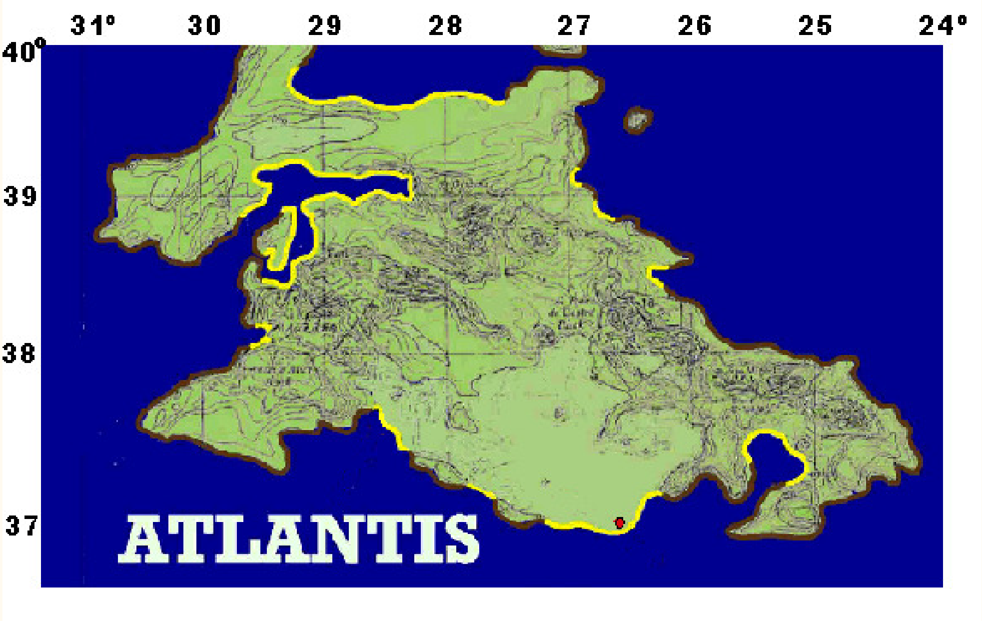
Note: Does not include about 13,000 metric tons of nuclear waste, primarily from U.S. weapons programs, that the Department of Energy manages in several states.

Sources: U.S. Government Accountability Office, Nuclear Energy Institute, Union of Concerned Scientists

Graphic: Dave Merrill
BGOVgraphics@bloomberg.com

Bloomberg GOVERNMENT

Where else can we put the waste?


31°
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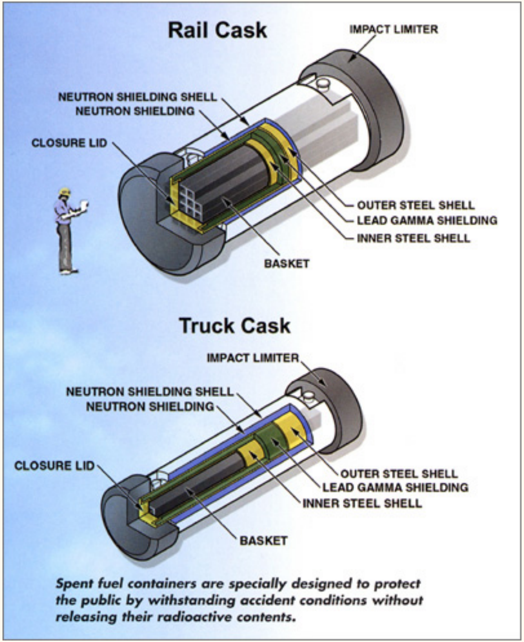


Rail Cask
IMPACT LIMITER
NEUTRON SHIELDING SHELL
NEUTRON SHIELDING
CLOSURE LID
OUTER STEEL SHELL
LEAD GAMMA SHIELDING
INNER STEEL SHELL
BASKET
Truck Cask
IMPACT LIMITER
NEUTRON SHIELDING SHELL
NEUTRON SHIELDING
CLOSURE LID
OUTER STEEL SHELL
LEAD GAMMA SHIELDING
INNER STEEL SHELL
BASKET
Spent fuel containers are specially designed to protect the public by withstanding accident conditions without releasing their radioactive contents.


OVERSIZE LOAD
MACK
www.dexigner.com

What are the options to dispose of nuclear waste?

What are the technical limitations of these options?

Near surface engineered storage (LLW)

Geologic repository

Deep borehole

Ice sheet disposal

Sub seabed disposal

Spaaaaaacceeeee!

Interim aboveground

The National Academy of Science judged geologic repositories to be the way waste should be disposed

What do you need to know to dispose the waste in the repository?

History

Waste disposal was thought about from the start

Commercial nuclear development followed Eisenhower's Atoms for Peace speech in 1953

Mid-50s – US made a decision to take naval nuclear reactor technology and apply it to commercial generation of electrical power using civilian owned reactors

First commercial nuclear power plant was at Shippingport PA

United States was reprocessing nuclear fuel materials until Carter

1955 – National Academy of Sciences (NAS) was asked what to do about waste

1957 – Disposal in cavities mined in salt is suggested as the possibility promising the most practical immediate solution of the problem

Waste management is a failure because it was not treated as a social problem

NAS in 1957 recommended deep disposal in salt

Lyons Kansas was investigated 1957–72 but scrapped due to public opposition

WIPP authorized in 1980 and received waste first in 1999

WIPP – TRU waste from weapons program (1992)

1972 – US abandons repository project at a salt mine in Lyons, KA. Promotes Retrievable Surface Storage Facility as 100-year interim solution

Waste management lacked a policy direction

1975 – Geologic disposal adopted as preferred alternative

1977 – Spent fuel reprocessing indefinitely deferred

1982 – NWPA lays out comprehensive screening process leading to 2 sites in West and East

Establishes Nuclear Waste Fund, financed by 0.1 cent/kWh nuclear electricity levy

Directs DOE to begin accepting spent fuel from utilities in 1998

That Nuclear Waste Fund has a lot of money now

1985 – President Reagan abandons site search in east

1987 – NWPA Amendments direct DOE to focus all site investigation at Yucca Mountain, NV

Ended 2nd repository screening activity

The saga of disposal is far from over

1987 – Nevada opposed DOE site characterization and lose in court

1998 – DOE ‘Viability Assessment’ finds no technical ‘showstoppers’ to proceeding with Yucca Mountain site

1999 – DOE issues Draft Environmental Impact Statement concluding that disposal at Yucca Mountain would be safer than leaving the waste where it is

2002 – President approves proceeding with Yucca mountain as nation’s first repository

Complex national geologic repository site selection process initiated, then abandoned

Yucca Mountain picked instead

DOE didn't do so well either

DOE contracts with utilities to take possession of utility spent fuel beginning in 1998, but fails to do so

Leaks of high-level radioactive waste from tanks at DOE sites in Washington and South Carolina

Disclosures of contamination and excessive radiation doses to workers throughout DOE nuclear complex over a period of decades

Continuing conflict between federal, state, and local jurisdictions over siting, regulatory issues

There is a lot of work to do to restore public trust

Yucca mountain is not going to be a waste repository

Disposing used fuel is a wasted resource

Some will have to be disposed even if we recycle

Reprocessing is a sound technology that has half a century of operational history

And reduces waste volume

Current onsite storage in dry casks is relatively secure

Consolidated interim storage facilities can offer benefits

Pros

Jobs

Growth

Cons

Will become a de facto alternative to disposal

Could write into law that it cannot; give authority to State to make decsion

Will be no easier to site than a repository

That's why they gave us $2M

Will reduce momentum to develop a repository

Everyone agrees we need one

Back end management needs to be treated as a social problem or there will be no success

WIPP is the first mined repository to receive waste

WIPP authorized in 1980

Received waste first in 1999. From Jeff Terry!

Zero accidents recorded, not counting the moron who lost his badge when we were there

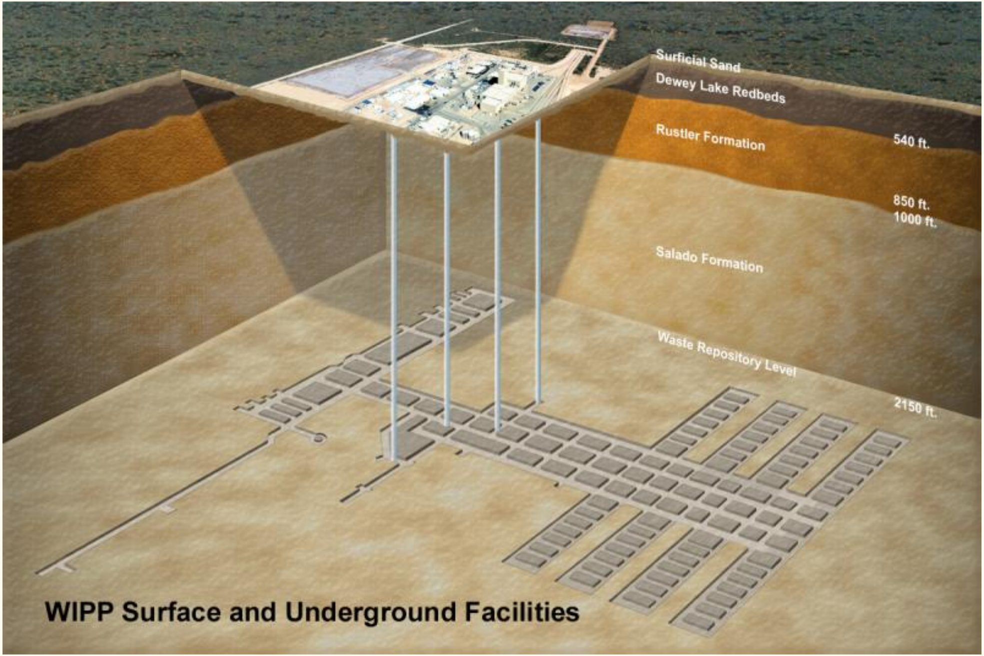
WIPP = TRU waste

WIPP accident


United States
Department of Energy
Waste Isolation Pilot Plant


BH 032


158
188
167
341


Surficial Sand
Dewey Lake Redbeds
Rustler Formation
540 ft.
850 ft.
1000 ft.
Salado Formation
Waste Repository Level
2150 ft.
WIPP Surface and Underground Facilities

Nuclear Waste Policy Act (NWPA)

Nuclear Waste Policy Act is the basis for HLW management

Established disposal in geologic formation

DOE sites, constructs, operates the repository

Sites studied: Hanford, Yucca Mountain, Texas

1987 – NWPA amended to focus only on Yucca at 70000 MTHM

Supposed to start in 1998

A license application was submitted to NRC in 2008

Obama Administration withdrew any further funding

2013 – Circuit Court ruled for NRC to continue safety evaluation

2015 – NRC Safety Evaluation Report

2016 – Working Environmental Impact Statement

But what happens if the license is granted?

How should nuclear facilities be sited?

Blue Ribbon Commission on America’s Nuclear Future on America’s nuclear future

Blue Ribbon Commission on America's Nuclear Future was formed to review back-end policy

Really, because there wasn't any policy

(1) A new, consent-based approach to siting future nuclear waste management facilities.

(2) A new organization dedicated solely to implementing the waste management program and empowered with the authority and resources to succeed.

(3) Access to the funds nuclear utility ratepayers are providing for the purpose of nuclear waste management.

(4) Prompt efforts to develop one or more geologic disposal facilities.

(5) Prompt efforts to develop one or more consolidated storage facilities.

Blue Ribbon Commission on America’s Nuclear Future was formed to review back-end policy

(6) Prompt efforts to prepare for the eventual large-scale transport of spent nuclear fuel and high-level waste to consolidated storage and disposal facilities when such facilities become available.

(7) Support for continued U.S. innovation in nuclear energy technology and for workforce development.

(8) Active U.S. leadership in international efforts to address safety, waste management, non-proliferation, and security concerns.

Not particularly revolutionary (Peterson), but what other nations are doing around the world

The point of the BRC was to establish consensus

Energy Policy Act

The Energy Policy Act (1992) set regulatory framework

40CFR197 – EPA radioactivity releases

Individual protection standard

Groundwater protection standard

Human intrusion standard

No greater than 15 mrem per year for the maximally-exposed individual during first 10000 years

(National average background is 300 mrem)

Parameters describing human activity and behavior ‘will remain as there are today’

NRC verifies compliance with performance assessments, risk-informed approach

10CFR63 – Construction, operation, closure, 15 mrem limit

1995 – NAS review

Extended time frame of peak risk to (of adverse health effects) *1 million years*

The Energy Policy Act (1992) had **NAS** to review the whole issue of disposal

Can scientifically justifiable analyses of repository behavior over many thousands of years in the future be made?

Recommended the use of a standard that sets a limit on the risk to individuals of adverse health effects from releases from the repository is recommended

Extension of time frame from 10000 to a million years

Based on long term stability of the geologic environment

Functional components and technical basis

What are the functional components for repository performance?

Waste cannot be released to biosphere at unacceptable concentrations

Waste must be removed and isolated from catastrophic natural events or human activity

Must be able to use current technology and engineering at reasonable cost

Widely acceptable performance assessment modeling

Into Eternity

Waste is inevitably going to get out

Contain (isolate) waste as long as possible

Maintain engineered barrier system for 10^5 y

Physically isolated in a matrix/waste form (vitrified glass)

Then put that in a canister

During containment, we rely on decay to wipe out most of the radionuclides (at least 300 years)

Which are the two most important in the near term?

Engineers generally agree that maintaining integrity to about 1000 years is possible

When waste does get out, we want controlled release and dispersal based on natural processes

Controlled release = multi barrier concept

Multi barrier concept aka defense in depth

Very common approach in all fields of risk assessment

Redundancy and diversity

Common examples of this?

For the repository, a series of engineered and natural barriers

Multi barrier is an holistic concept

Act in concert

Delay and decay

Host rock

Groundwater

Waste form

Canister

Backfill

Should not have common failure modes

Delay and decay means contain and extend transport

If containment time is $10\times$ greater than half life, then all set

Containment for $300 - 1000\ y$ is effective in eliminating Co-60, Sr-90, Cs-137

Repository will also approach ambient temperature by this time

Non sorbing radionuclides are problematic though

Similarly if transport time is greater than $10\times$, also set

Want diffusion to dominate over advection

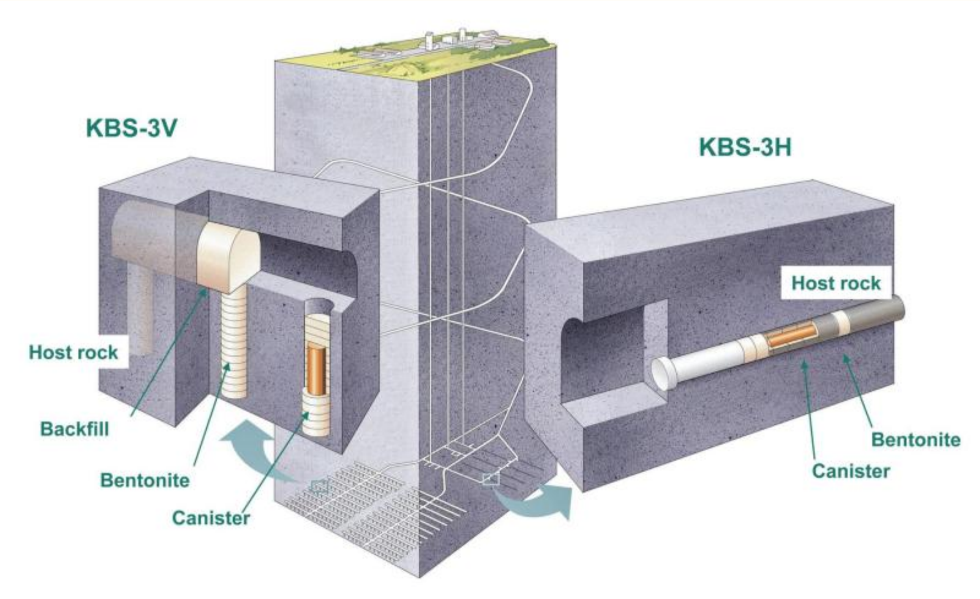
How can this be achieved?

How are multi barriers applied to repository design?

Waste package is the waste form and canister
Resistant to leaching by groundwater, chemically and physically

Backfill
Placed around everything once its put in there
Inhibits groundwater movement
Chemically inhibitive; clay is negatively charged, radionuclides are postive

Geologic structure
Isolates waste from humans
Chemically, mechanically stable
Prevents/restricts circulating groundwater
Slows releases to biosphere


KBS-3V
Host rock
Backfill
Bentonite
Canister
KBS-3H
Host rock
Bentonite
Canister

Across the world, there are different repository concepts

Switzerland – granite

France – clay

Sweden – granite

Belguim – clay

Finland – granite

United Kingdom – granite

Japan – granite

Korea – granite

Each country legally responsible for waste management and disposal

All saturated

Most all have the quasi-federal companies

Do we have similar formations in the USA?

What is the best formation?

Siting is a decades long process

Before you even start

Assuming a locality volunteers

Public acceptance, both local and national

In USA, we have extra layer of state government

Geology

Structurally stable geological block

Not near a tectonic boundary

Avoid faults along which rupture could occur

Avoid areas with abnormally high geothermal

Gradients or with evidence of relatively recent volcanic activity

Mechanical properties should assure stability during operation

Competing land use like parks, oil, gas

Transportation requirements

Hydrology

Fluid transport should not move hazardous material to the biosphere in amounts and rates above prescribed limits

Low permeability

Low flow

System should be capable of being sealed when the repository is closed

Geological record should support predictions favorable for long-term hydrological isolation of the repository site in a perturbed geological environment

Most places in Europe are not like that

Geochemistry

Heat, radiation should not produce physical, chemical reactions in the rock that would compromise containment

Low oxygen

Low redox potential (*Eh*)

Near neutral *pH*

Low concentrations of complexing anions and organics

Conditions should minimize the rate of dissolution of the waste form (typically reducing)

Water in the repository should not react to increase permeability

Limit mobility of radionuclides and delay or prevent their migration to the biosphere

No area with record of resource extraction

What type of places can you actually site?

How long is long term?

What are the oldest human constructed formations?

Multiple barriers are engineered to provide defense in depth

This is the basis for any risk management approach

What actually happens in the repository?

The whole point is that after everything gets put in you close the repository and walk away

It is supposed to function on its own

Why?

Limit exposures in the long term

Compliance demonstrated by the performance assessment

Know your materials science

Corrosion resistant metals with inert oxide surface layer

Low rate of corrosion in water

Advanced materials

New alloy made for Yucca Mountain casks

Compatible with repository environment

Buffer assures slow diffusion and high sorption

Diffusion dominated transport

Long transit time, delay and decay

Constrain corrosion rate

Seal fractures due to construction

Saturated repositories use bentonite

Each has different chemical make up Na^{+}, Ca^{++} interlayer ions

For unsaturated conditions, use sand over gravel

Other processes to consider

Thermal management

Subcritical configuration

Waste form dissolution

Diffusion

Solubility

Corrosion

What is each barrier expected to do?

Geologic formation
isolate the EBS from biosphere
geomechanical stability
favorable geochemistry
limit groundwater flow

Backfill
contain EBS
limit fast release pathways
mechanical stability of tunnels

Buffer

diffusive dominated transport

isolate canister from formation

conduct heat

Canister

isolate waste for long time period

mechanical stability

conduct heat

We want to draw on accepted methods and design bottom-up

Readily available technologies

Retrievability

Based on well accepted models

Risk informed approach

Environmental modeling

We want to use what we know well so we are confident in results

Waste must be contained ‘as long as necessary’

This depends on making a robust waste form

Borosilicate glass or ceramic matrix

‘Long enough’ so most radionuclides decay away $\approx 1000\ y$

Significant heat comes from ^{137}Cs and ^{90}Sr

Actinides like Pu, Cm, etc., are long lived, but low radioactivity

They are really toxic though because they are heavy metals

Oxidation state of Np makes it very mobile

Iodine is a negative ion so also very mobile

When there is release, it must be diluted and dispersed

Clay can trap the radionuclides ($\pm$ charges)

A lot of water can dilute the contaminant plume

The multi-barrier concept is a series of engineered and natural barriers to delay release

No common failure modes

Waste form – dissolution and solubility modeling

Waste container

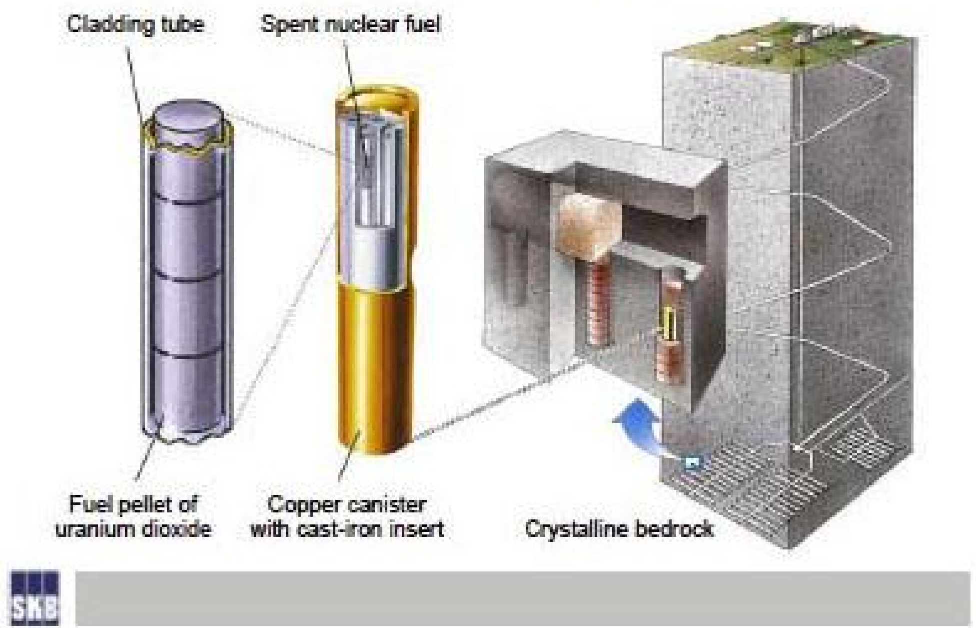
Buffer – Diffusion of radionuclides, sorption

Backfill – Diffusion of radionuclides, sorption

Geological structure – requires decades long studies

Diffusion, dispersion, advection, sorption modeling, ion exchange

The KBS-concept

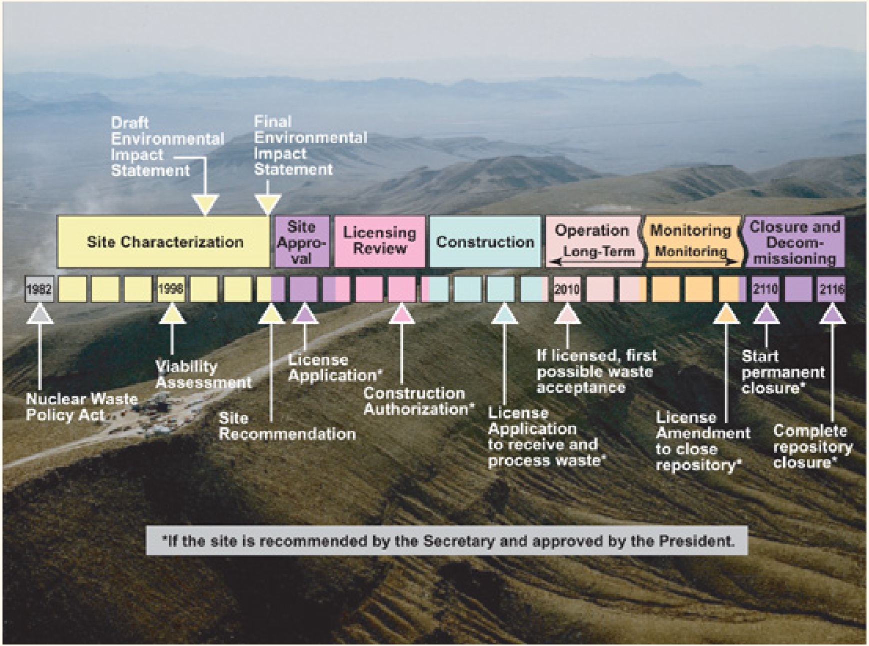


Yucca Mountain
Total System Performance Assessment (TSPA)

The TSPA models all possible scenarios for release

Summary by Peter Swift


Proposed route to Yucca Mountain
Exisiting rail line that could be used
Exisiting rail line
Caliente Corridor
Yucca Mountain
Nevada Test & Training Range
Winnemucca
Elko
Wells
Carlin
Battle Mountain
Reno
Fallon
Austin
Eureka
Ely
Hawthorne
Tonopah
Warm Springs
Goldfield
Scottys Junction
Beatty
Rachel
Hiko
Caliente
Pioche
Panaca
Alamo
Ash Springs
Amargosa Valley
Indian Springs
Pahrump
Las Vegas
Goodsprings
Jean
Primm
Boulder City
Searchlight
Glendale
Moapa
Mesquite
Oregon
Idaho
California
Arizona
Utah


Draft Environmental Impact Statement
Final Environmental Impact Statement
Site Characterization
Site Appro-val
Licensing Review
Construction
Operation
Long-Term
Monitoring
Monitoring
Closure and Decom-missioning
1982
1998
2010
2110
2116
Nuclear Waste Policy Act
Viability Assessment
Site Recommendation
License Application*
Construction Authorization*
License Application to receive and process waste*
If licensed, first possible waste acceptance
License Amendment to close repository*
Start permanent closure*
Complete repository closure*
*If the site is recommended by the Secretary and approved by the President.

Aside – DOE & HLW is stored at Hanford and Savannah river

Really made a mess of it

Tanks have leaked into soil

People are worried about toxicity in Columbia River

First large scale plutonium production reactor

Legacy of the cold war

Yucca mountain has a unique geology compared other repository concepts

100 miles northwest of Las Vegas (the wiseguys not happy)

Volcanic tuff

Layers of consolidated, compacted ashfalls from volcanic eruptions occurring more than 10 million years ago

Underlying the tuff is sedimentary carbonate rock

Repository horizon in unsaturated zone, about 300 meters below the surface, and 300-500 meters above the water table

Two major aquifers in the saturated zone below Yucca Mountain, one in tuff, one in carbonate rock

Which leads to different operating concepts

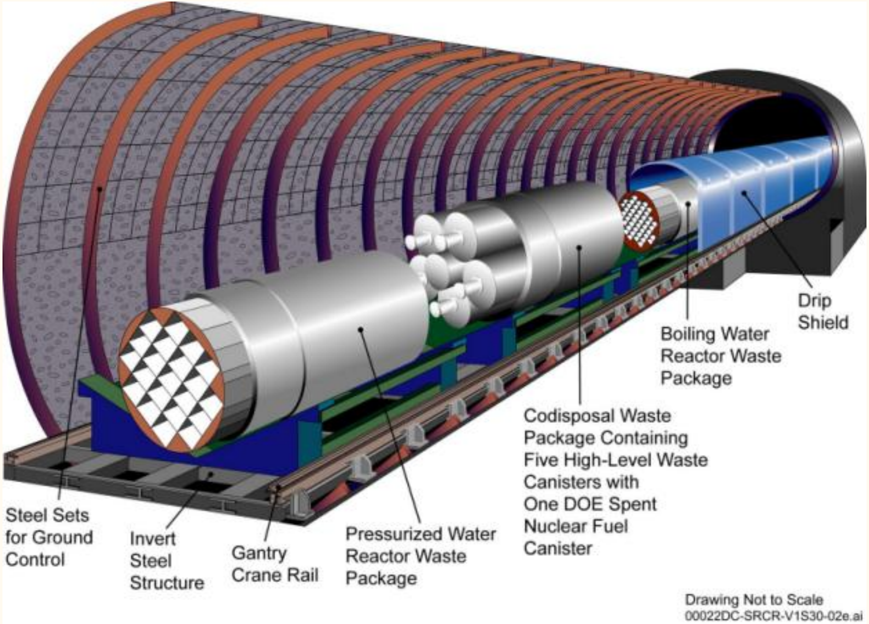
Yucca Mountain would remain open with active ventilation for at least a hundred years

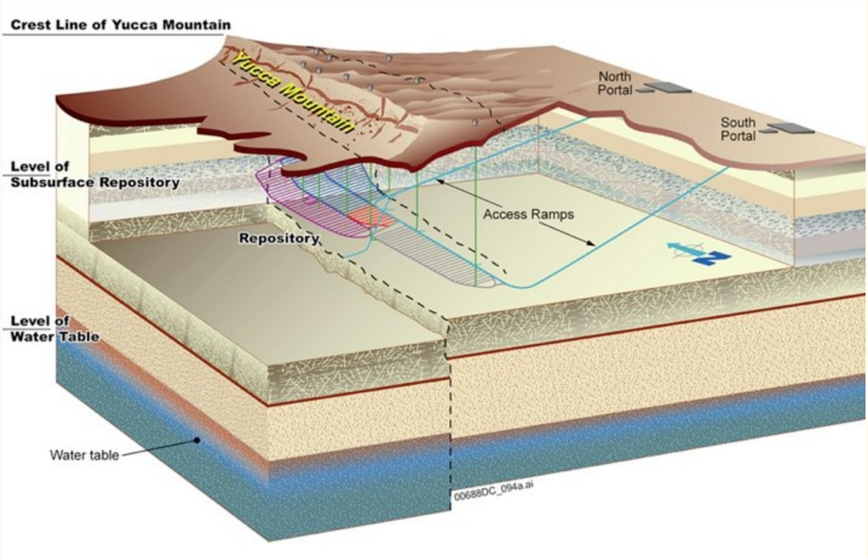
This allows a significant fraction of decay heat to go to the atmosphere and reduces peak post closure temperatures

That's easier to do because the repository is not saturated

Unsaturated zone design allows long term monitoring and, if necessary, retrieval, modification or repair

The design is also different than the other repositories




Crest Line of Yucca Mountain
Yucca Mountain
North Portal
South Portal
Level of Subsurface Repository
Access Ramps
Repository
Level of Water Table
Water table
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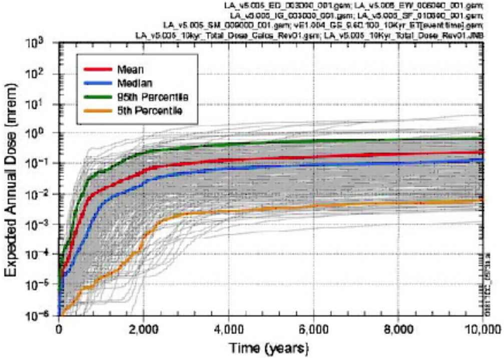
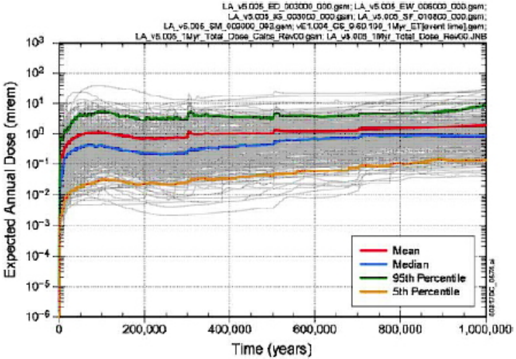
What are the failure modes for the repository?

What is the world going to look like in $10^3,\ 10^4,\ 10^5$ years?

Total Mean Annual Dose



10,000 years



1,000,000 years

What are the failure modes for the repository?

Degradation over time

Natural and engineered barriers

Dissolution and transport of radionuclides in groundwater

Tectonic processes – folding, faulting, magmatic intrusions, volcanism

Erosion – wind, water, glaciation

Breaching of barriers by human activity

Aliens

The performance assessment predicts how the repository will respond

Rate of inflow of groundwater into the repository

Hydrology current, long-term, fracturing, faulting, thermal stresses in host rock

Rate of corrosion of canister, other barriers, and primary waste form

Temperature, oxidation/reduction conditions, materials properties of waste package

Yucca mountain is an oxidizing environment, different than world standards

Radionuclide transport in groundwater, sorption on rock surfaces, actinide chemistry

Biosphere transport in potable water supplies, irrigation water, demography dose

How safe is safe enough?

Assumptions about future human activities and lifestyles

Who needs to know the repository is safe?

Implementer – to have confidence to present and defend a license application

Regulator – to be able to license the repository

Elected Representatives – to be justified in taking a decision to proceed

Public – to have confidence in and accept the whole process

The relative roles and participation is going to vary with location and people

Perception of safety is different for the public

ALARA v precautionary principle

Defining risk for specific context

Risk communication (resiliency) is extremely difficult

Public trust must be cultivated

The arguments they hear should be based upon what they can understand and relate to their experience

They need to see relevant information and that nothing is being hidden

Really public have different priorities and concerns than what experts think

Experts need to develop social science literacy

Consent-based siting should be social science led

Safety is dependent on an independent regulator

Assure that post-closure safety meets international accepted standards

Confirm that uncertainty has been properly considered

Are system models complete and valid?

Are appropriate parameter values and ranges identified?

Are analyses conducted to identify processes & parameters most important to safety?

Are ‘what if…’ scenarios analyzed?

Conduct independent review and assessment of safety

Require Implementer to present multiple lines of evidence

Public comment

What evidence could the operator provide?

There is no data cohort for a repository

Uncertainties in extrapolating conditions today into the future

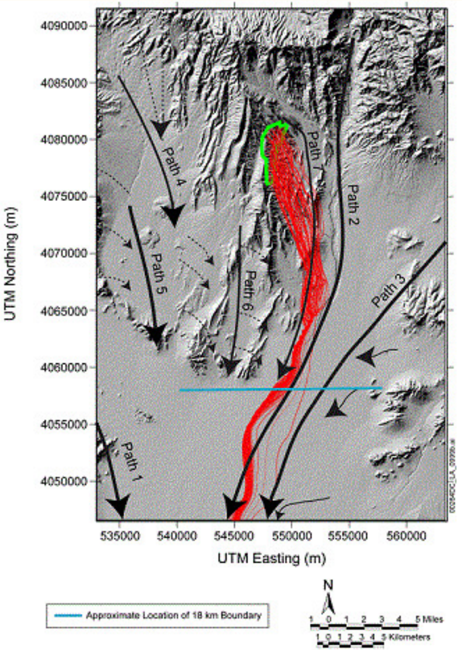
Increased confidence from defense-in-depth technical arguments

Matching of time scales between repository assessment and natural/archaeological analogues

Calculated doses can have relatively large uncertainties

Verification/validation of safety assessment codes

Analyze an analogue using the same component models



UTM Northing (m)
4090000
4085000
4080000
4075000
4070000
4065000
4060000
4055000
4050000
Path 1
Path 2
Path 3
Path 4
Path 5
Path 6
Path 7
535000
540000
545000
550000
555000
560000
UTM Easting (m)
Approximate Location of 18 km Boundary
N
1 0 1 2 3 4 5 Miles
1 0 1 2 3 4 5 Kilometers

Technical design of a repository

What is there to design for the repository?

Canister spacing – Why?

HLWis vitrified (not the used fuel) to a glass waste form

HLW producers required to have Waste Acceptance Program

Drift spacing – 81 m

Mid-pillar peak temperature of 96^oC

Waste package spacing – 0.1 m

Average emplacement drift line load – 1.45 kW/m

Maximum waste package thermal output – $18.0kW$

Ventilation flow rate – 15 m^3/s

23 years of waste emplacement followed by 50 years of forced ventilation following emplacement of the last waste package

Design of the process and the waste form determines thermal load

Specifications developed by DOE Environmental Management for the HLW form producers as their Waste Acceptance Program

Chemical

Waste form – 20–40 w/o waste oxides, 35–65 w/o silica, 5–10 w/o boron oxide, 10–20 w/o alkali oxides, and other oxide constituents

Chemical composition, crystalline phases expected to be present, and the amount of each crystalline phase should be identified

Oxide composition of waste form should be reported, including all elements (excluding oxygen) present in concentrations greater than 0.5% of the glass

Radionuclide inventory

Half-lives longer than 10 years that are or will be present in concentrations greater than 0.05% of total are, be, 0.05% radioactive inventory shall be reported (indexed to the year 2015 and 3115)

Total quantities of individual radionuclides to be shipped to the repository

Upper limit of these radionuclides for any canistered waste form calculated

Average calculated radionuclide inventory per canister

Quality control by comparing to Environmental Assessment Benchmark Glass

At time of shipment, temperature has to be below $400^o C$

Rules for nonradionuclide hazardous waste

For fissile content for all the isotopes

There are multi dimensional constraints on canister loading

Waste composition in canister

Number of canisters

Materials

Canister dimensions

Radiation

Repository conditions

Storage conditions

Linear programming

Linear programming is a good modeling approach

Waste composition –

$$\overline{N}_W = [x_{W_1}, \ldots, x_{W_i}, \ldots, x_{W_n}] \tag{1}$$

Glass composition –

$$\overline{N}_G = [x_{G_1}, \ldots, x_{G_i}, \ldots, x_{G_n}] \tag{2}$$

Total mass of solid waste –

$$M_S = M_W + M_G \tag{3}$$

Solid form composition –

$$\overline{N}_S = \theta \overline{N}_W + (1 - \theta)\overline{N}_G \tag{4}$$

$$\theta \equiv \frac{M_W}{M_S} \tag{5}$$

Linear programming is widely used for optimization

Optimize an objective function –

$$max(f) = \underline{c}\,\underline{x} \tag{6}$$

Where $\underline{x}$ is constrained –

$$\underline{\underline{A}}\,\underline{x} \leq \underline{b} \tag{7}$$

$c \equiv$ row vector of coefficients of objective function

$x \equiv$ column vector of independent variables

$A \equiv$ matrix of coefficients of constraint inequalities

$b \equiv$ column vector of RHS of constraint inequalities

A and b would be based on the regulations for the waste form/canister

Linear programming example

Maximize profit for a gas processing plant

A gas processing plant receives a fixed amount of raw gas each week

Either regular or premium quality is processed, but only one grade at a time

The plant is open 80 hours per week

Decision variables are quantity of regular gas and premium gas produced per week

Objective function is to maximize profit under given constraints

Table. Gas processing constraints

Constraint	Regular grade	Premium grade
Raw gas	7 m^3/ton	11 m^3/ton
Production time	10 h/ton	8 h/ton
Storage	9 tons	6 tons
Profit	\$150/ton	\$175/ton

Total gas volume – 77 m^3/week

Total production time – 80 h/week

Derive variables and constraints

Independent variables –

$$\underline{G} = \begin{bmatrix} r \\ p \end{bmatrix} \tag{8}$$

Coefficients of objective function –

$$\underline{c} = \begin{bmatrix} 150 & 175 \end{bmatrix} \tag{9}$$

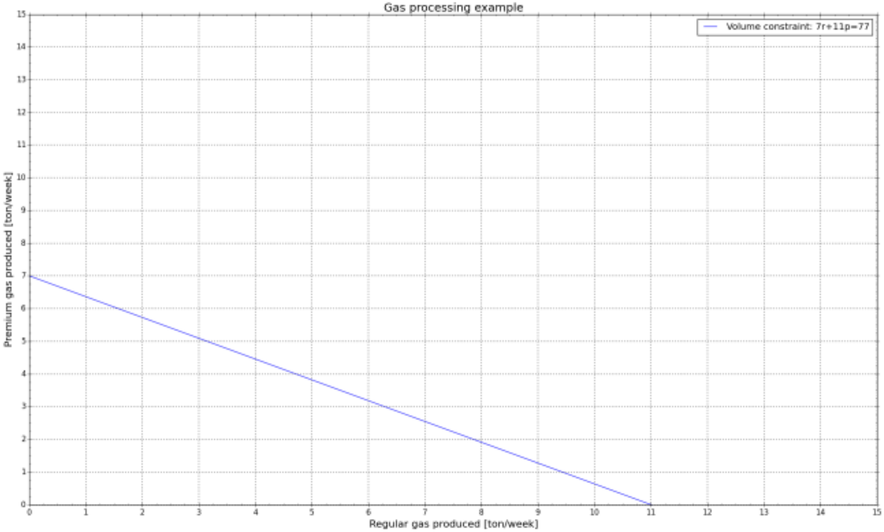
Constraints –

$$\begin{bmatrix} 7 & 11 \\ 10 & 8 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r \\ p \end{bmatrix} \leq \begin{bmatrix} 77 \\ 80 \\ 9 \\ 6 \end{bmatrix} \tag{10}$$

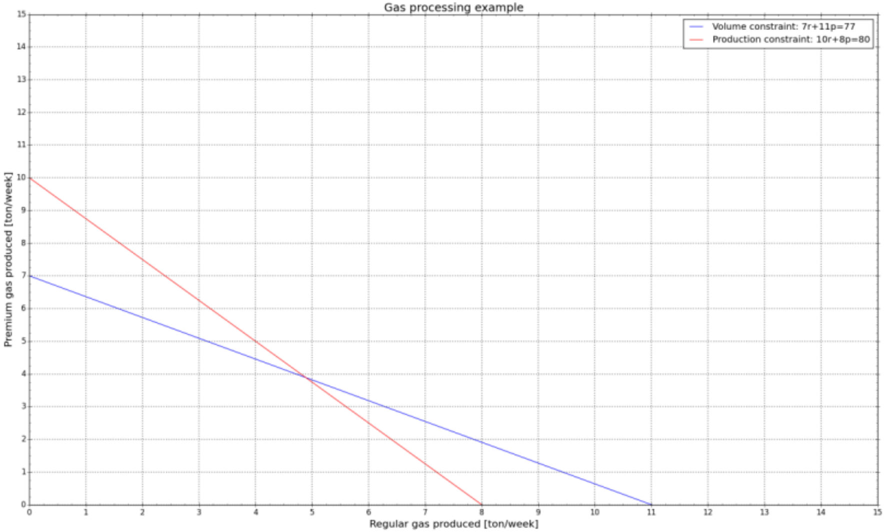
Derive objective function

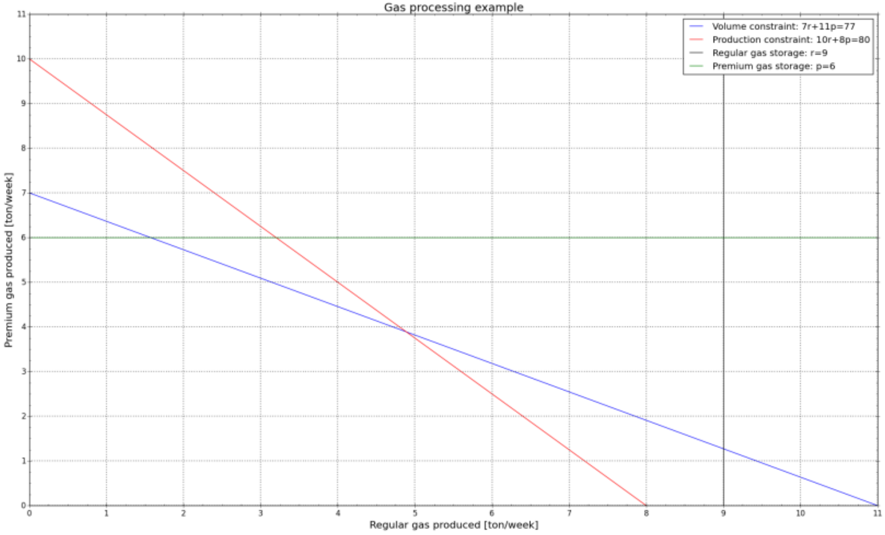
$$max(P) = [150 \;\; 175] \cdot \begin{bmatrix} r \\ p \end{bmatrix} \tag{11}$$

Linear programming results



Gas processing example
Volume constraint: 7r+11p=77
Premium gas produced [ton/week]
Regular gas produced [ton/week]
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15


Gas processing example
Volume constraint: 7r+11p=77
Production constraint: 10r+8p=80
Premium gas produced [ton/week]
Regular gas produced [ton/week]
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15



Gas processing example
Volume constraint: 7r+11p=77
Production constraint: 10r+8p=80
Regular gas storage: r=9
Premium gas storage: p=6
Premium gas produced [ton/week]
Regular gas produced [ton/week]
0
1
2
3
4
5
6
7
8
9
10
11

Then maximize profit

Typically, it's a good idea to plot the 'break even' function –

$$P = 150r + 175p = 0 \tag{12}$$

Not in feasible space

But we know the points of intersection bounding the feasible space $(0,0) - (0,6) - (1.7,6) - (4.9,3.9) - (0,8)$

Substitute into the objective function –

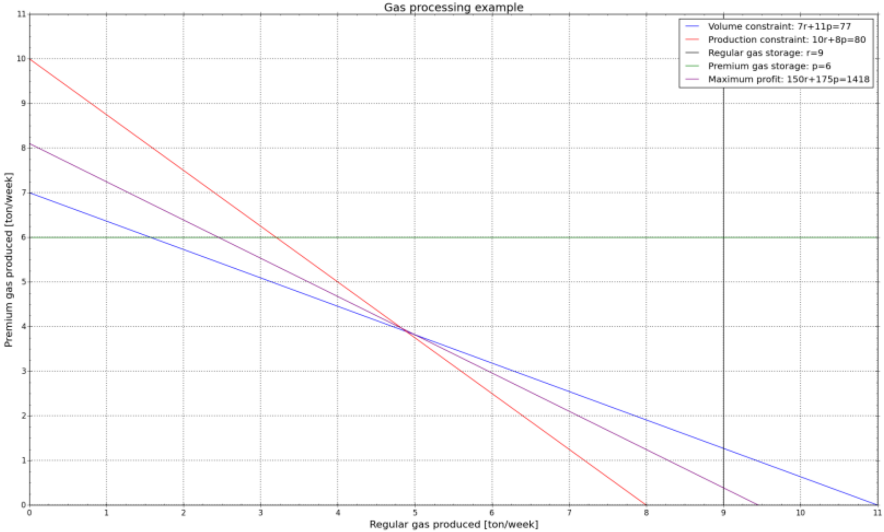
$P(0,0) = 0$

$P(0,6) = 1050$

$P(1.7,6) = 1305$

$P(4.9,3.9) = 1418$

$P(0,8) = 1400$



Gas processing example
Volume constraint: 7r+11p=77
Production constraint: 10r+8p=80
Regular gas storage: r=9
Premium gas storage: p=6
Maximum profit: 150r+175p=1418
Premium gas produced [ton/week]
Regular gas produced [ton/week]
0
1
2
3
4
5
6
7
8
9
10
11

Now for waste oxide loading

Apply to Japanese waste concept

Objective function

We want to maximize the amount of waste oxide that can be loaded into an HLW form of glass in a stainless steel canister

Decision variables

Waste Oxide Mass in the canister and Glass Oxide Mass in the canister

$M_W, \; M_G$

Constraints

(1) Total canister weight is 500 kg of waste mass + glass mass

Include 100 kg for mass of the canister

(2) Volume of waste and glass is 0.15 cubic meters

(3) Empirical formula for volume constraint –

$\rho = 1230r + 2419 \; \frac{kg}{m^3}$

r = mass fraction of waste in the waste glass

(4) Heat generation limit is 2300 Watts

$[CN_A \sum_i \frac{E_i \lambda_i x_i}{A_i}] \cdot M_W \leq 2300$

This is tedious to find or compute all the data and not particularly constructive

Just use 9.652