List 4 Report

Statistics and Linear Models

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Goal

This report aims to examine three different statistical tests

1. the Wilcoxon test based on the statistic $W = T^2_{\phi_1}$,
2. the Ansari-Bradley test based on the statistic $AB = T^2_{\phi_2}$,
3. the Lepage test based on the statistic $L = W + AB$,
4. the Kolmogorov-Smirnov test based on the statistic KS.

We generate data from the normal and the uniform distribution. Then we calculate the above statistics to investigate the behaviour of the critical values and the power functions.

Task 1

In the first task, we are asked to generate $n = 20$ observations from the $N(0,1)$ distribution. Then, we calculate the values of the statistics W, AB, L, and KS. We repeat the experiment 10 000 times. We compare them with the theoretical values.

Statistic	Value	Theoretical value
W	3.887	3.841
AB	3.888	3.841
L	6.012	5.991
KS	1.264	1.358

We can see that all statistics are close to the true asymptotic values despite the sample size being very low.

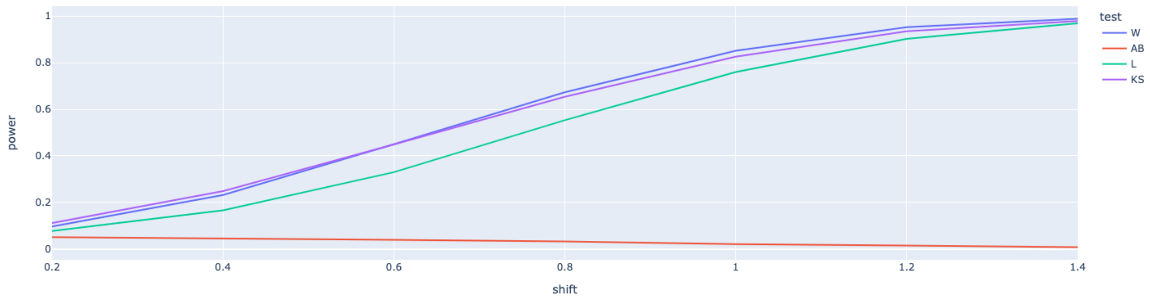
Tasks 2, 3, 4

In those tasks, we are asked to generate $n = 20$ observations from the normal, logistic and Cauchy distribution with different shift and scale parameters. As usual, we repeat the experiment 10 000 times, calculate the statistics' values and estimate the power functions' values.

First, consider graphs 2a), 3a), and 4a). They show tests performed on the normal distribution. We plot power functions relative to different parameters.

2a) normal distribution, where $\mu_1 = 0, \sigma_1 = 1$

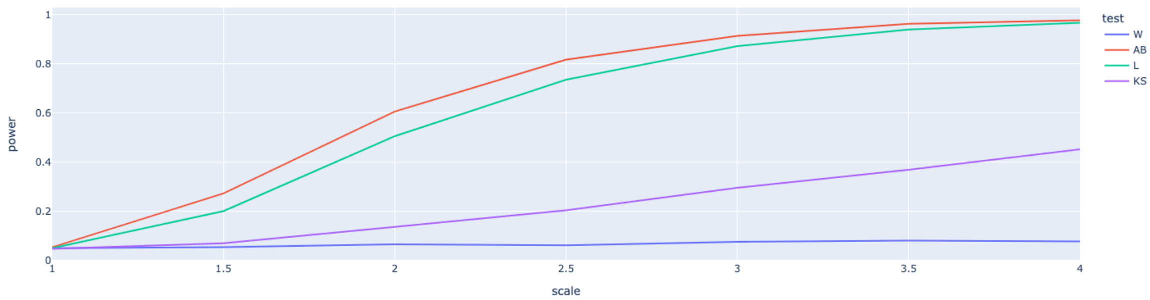
Here are power functions drawn as functions of the parameter μ_2.



The AB test constantly has low power and does not change with the shift parameter (even slightly decreases). Other tests perform here very well and converge to 1 when the shift parameter increases.

3a) normal distribution, where $\mu_1 = 0, \sigma_1 = 1$

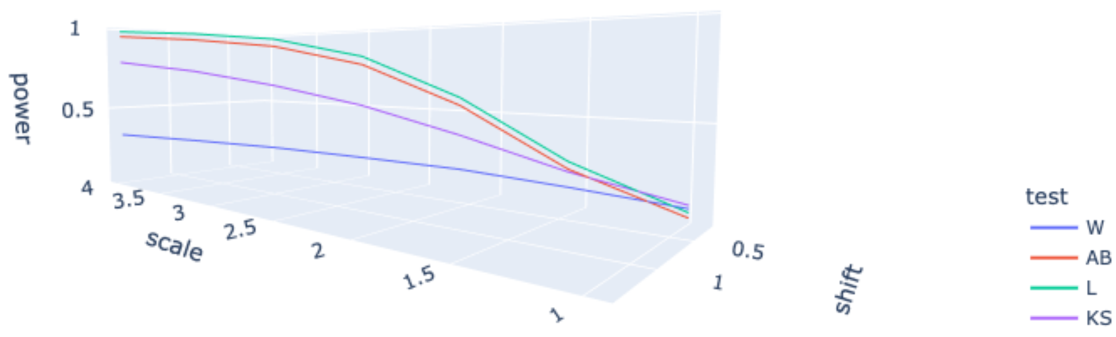
Here are power functions drawn as functions of the parameter σ_2.



The W test performs poorly as the AB test in the case above. KS test also doesn't yield high power. L and AB tests perform the best with power close to 1 when the scale parameter increases.

4a) normal distribution, where $\mu_1 = 0, \sigma_1 = 1$

Here are power functions drawn as functions of the vector of the parameters (μ_2, σ_2).

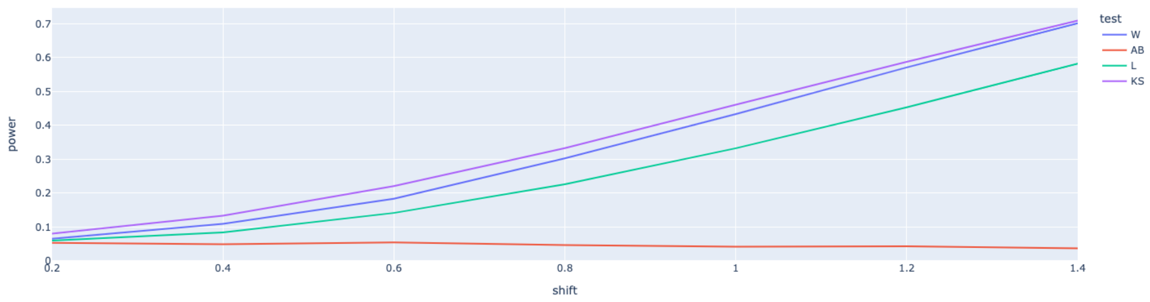


In this case, the clear winners are L and AB, like in the case above. Although the W tests performed better here, it's still the worst.

Now, let's consider graphs 2b), 3b), and 4b). They show tests performed on the logistic distribution. We plot power functions relative to different parameters.

2b) logistic distribution, where $\mu_1 = 0, \sigma_1 = 1$

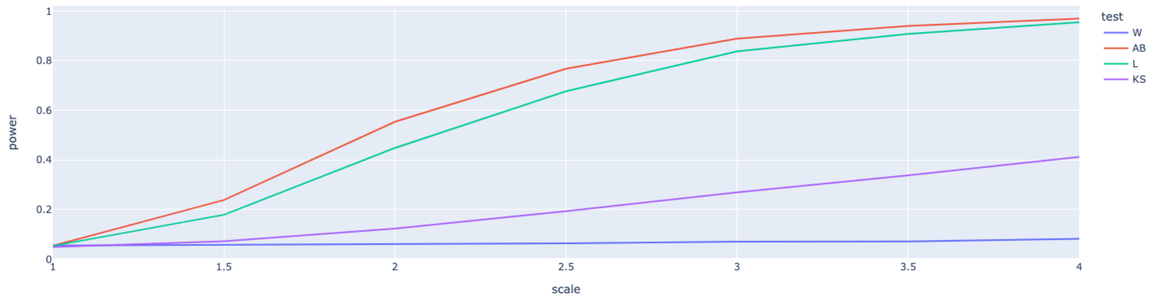
Here are power functions drawn as functions of the parameter μ_2.



Similar to case 2a) with the normal distribution, the AB test performs very poorly. The L test is almost as good as others.

3b) logistic distribution, where $\mu_1 = 0, \sigma_1 = 1$

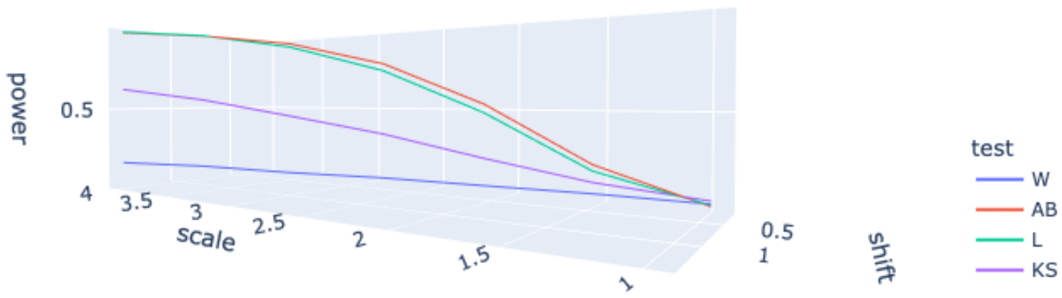
Here are power functions drawn as functions of the parameter σ_2.



Similar to case 3a) KS and W tests are the worst performers.

4b) logistic distribution, where $\mu_1 = 0, \sigma_1 = 1$

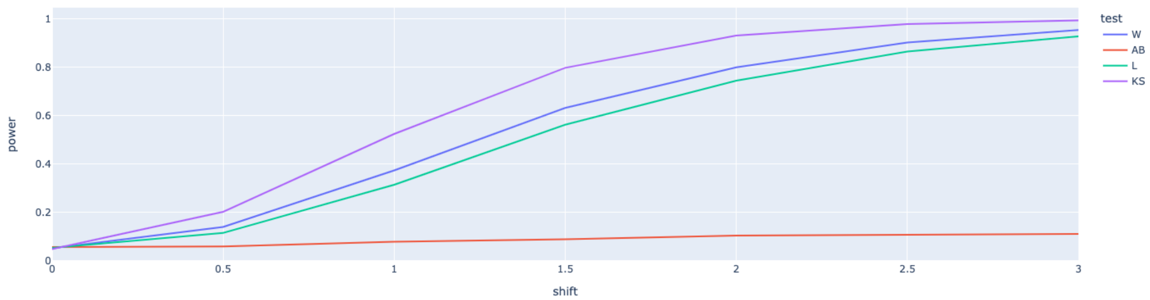
Here are power functions drawn as functions of the vector of the parameters (μ_2, σ_2).



The W and KS tests are still the worst performers, like in case 3b) above. Other tests nicely convert to the power equal to 1.

2c) Cauchy distribution, where $\mu_1 = 0, \sigma_1 = 1$

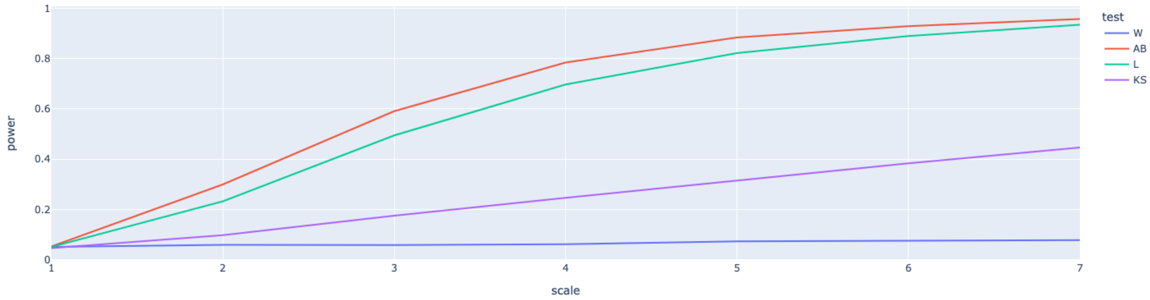
Here are power functions drawn as functions of the parameter μ_2.



Like in cases 2a) and 2b), the AB test also looks terrible.

3c) Cauchy distribution, where $\mu_1 = 0, \sigma_1 = 1$

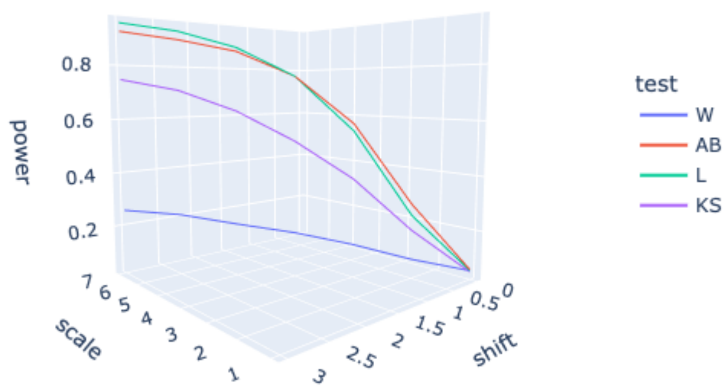
Here are power functions drawn as functions of the parameter σ_2.



Very similar situation here compared to 3a) and 3b). The KS and W tests generate the lowest powers.

4c) Cauchy distribution, where $\mu_1 = 0, \sigma_1 = 1$

Here are power functions drawn as functions of the vector of the parameters (μ_2, σ_2).



The W test is still the worst as in 2c) and 3c). But the second worst KS test does not look entirely amiss. It is partially correct as it gets close to 0.8 power.

Task 5

In the fifth task, we are asked to generate $n = 20$ observations from the $U(0,1)$ distribution. Then, we calculate the values of the statistics W, AB, L, and KS. We repeat the experiment 10 000 times. We compare them with the theoretical values.

Statistic	Value	Theoretical value
W	3.844	3.841
AB	3.817	3.841
L	5.828	5.991
KS	1.358	1.358

As in task 1, the estimated values are very close to the theoretical ones. Changing the distribution from normal to uniform did not change a lot.

Task 6

In the sixth task, we repeat calculations from tasks 2, 3, and 4, but we generate $n = 50$ observations.

We compare the power functions drawn as functions of the parameters μ_2, σ_2, and (μ_2, σ_2) with those generated in tasks 2, 3, and 4. In terms of the goodness of the statistics, it did not change that much. The ones that were performing poorly still performed poorly. The ones converging to power close to 1 still do, but they do it faster. Increasing the sample size made the charts steeper. On all graphs, but the shift one, the statistics convert more quickly. Additionally, one visible improvement is that the KS statistic always converges to the power close to 1.