Statistics

List 1

Exercise 1.

Generate n observations from a $N(\theta, \sigma^2)$ distribution.

(a) $n = 50, \theta = 1, \sigma = 1$,

(b) $n = 50, \theta = 4, \sigma = 1$,

(c) $n = 50, \theta = 1, \sigma = 2$.

Calculate a value of an estimator of the parameter θ of the form

(i) $\hat{\theta}_1 = \overline{X} = (1/n)\sum_{i=1}^{n} X_i$,

(ii) $\hat{\theta}_2 = Me\{X_1, \ldots, X_n\}$,

(iii) $\hat{\theta}_3 = \sum_{i=1}^{n} w_i X_i$, $\sum_{i=1}^{n} w_i = 1$, $0 \leq w_i \leq 1$, $i = 1, \ldots, n$, with an arbitrary weights' selection,

(iv) $\hat{\theta}_4 = \sum_{i=1}^{n} w_i X_{i:n}$, where $X_{1:n} \leq \cdots \leq X_{n:n}$ are the order statistics from the sample $X_1, \ldots, X_n$,

$$w_i = \varphi\Big(\Phi^{-1}(\frac{i-1}{n})\Big) - \varphi\Big(\Phi^{-1}(\frac{i}{n})\Big),$$

while φ is the density and Φ is the cumulative distribution function of the standard normal $N(0, 1)$ distribution.

Repeat the experiment 10 000 times. Estimate the variance, mean squared error (MSE), and bias of the estimators under consideration. Discuss the outcomes.

Exercise 2.

Discuss the command `set.seed(1)` and its potential applications.

Exercise 3.

Discuss the necessity of the numerical finding of the maximum likelihood estimator (MLE), on the basis of the shift parameter's estimation, in the logistic distribution (cf. Example 6.1.4, p. 315, Hogg et al. 2005).

Exercise 4.

Discuss a method of the numerical finding of the MLE (see, for instance, p. 329, Hogg et al., 2005).

Exercise 5.

Generate n observations from a logistic $L(\theta, \sigma)$ distribution with the shift parameter θ and the scale parameter σ.

(a) $n = 50, \theta = 1, \sigma = 1$,

(b) $n = 50, \theta = 4, \sigma = 1$,

(c) $n = 50, \theta = 1, \sigma = 2$.

Estimate a value of the MLE of the parameter θ. Discuss the influence of the selection of the starting point and the number of steps on the algorithm.

Repeat the experiment 10 000 times. Estimate the variance, MSE, and bias of the estimator. Discuss the outcomes.

Exercise 6.

Generate n observations form a Cauchy $C(\theta, \sigma)$ distribution with the shift parameter θ and the scale parameter σ.

(a) $n = 50, \theta = 1, \sigma = 1$,

(b) $n = 50, \theta = 4, \sigma = 1$,

(c) $n = 50, \theta = 1, \sigma = 2$.

Estimate a value of the MLE of the parameter θ. Discuss the influence of the selection of the starting point and the number of steps on the algorithm.

Repeat the experiment 10 000 times. Estimate the variance, MSE, and bias of the estimator. Discuss the outcomes.

Exercise 7.

Repeat the numerical experiment from Exercises 1, 5, and 6 for $n = 20$ and $n = 100$. Discuss the results in comparison to the previous outcomes.

References

Hogg, R. V., McKean, J. W., Craig, A. T. (2005). *Introduction to Mathematical Statistics.* Pearson Education International, London.