Course Title : Matrix Computations **Maximum Time : 1 Hour**
Course Code : 16B1NMA533 **Maximum Marks : 20**

After pursuing the above course, students will be able to:

C301-3.1	explain the basics of matrix algebra and inverse of a matrix by partitioning.
C301-3.2	solve the system of linear equations using direct and iterative methods
C301-3.3	explain the vector spaces and their dimensions, inner product space, norm of a vector and matrix.
C301-3.4	apply the Gram-schmidt process to construct orthonormal basis and Q-R decomposition of a matrix.
C301-3.5	construct Gerschgorin circles and solve eigenvalues problem using Jacobi, Givens, Householder, power and inverse power methods.
C301-3.6	analyze systems of differential and difference equations arising in dynamical systems using matrix calculus.

Note: All questions are compulsory.

1. [C301-3.1] (i) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \\ 2 & -1 & 4 & 3 \end{bmatrix}$ using partitioning method. **(4)**

(ii) Compute A^2, where $A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 8 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. **(2)**

2. [C301-3.2] Use Jacobi method to approximate the solution of the following system of linear equations:
$$3x + 4y - z = -2;\quad 5x + y - 2z = 2;\quad 2x - 3y + 5z = 10.$$
Perform only three iterations with initial approximation as (1,1,0). **(4)**

3. [C301-3.2] Consider the following system:
$$4x - y = 1;\quad -x + 4y - z = 0;\quad -y + 4z = 0.$$
(i) Check the positive definiteness of the coefficient matrix. **(1)**
(ii) Find the solution of above system using Cholesky method. **(3)**

4. [C301-3.3] (i) Let V be the set of all polynomials with real coefficients of degree n, where addition is defined by $a + b = ab$ and under usual scalar multiplication. Determine whether V is a vector space or not. **(1)**

(ii) Let M_{22} be the vector space of all matrices of order 2×2 with real entries with respect to usual vector addition of matrices and scalar multiplication of a matrix. Let W be the subset of M_{22} such that
$W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + b = c + d \right\}$. Determine whether W is a subspace or not. **(2)**

(iii) Let W be the set of all $(x_1, x_2, \ldots, x_5)$ in R^5 which satisfy
$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0;\quad x_1 + \frac{2}{3}x_3 - x_5 = 0;\quad 9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0.$$
Find a finite set of vectors which spans W.

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