

Jaypee Institute of Information Technology, Noida
End Term Examination, 2018
B.Tech III Semester

Course Title : Probability & Random Processes
Course Code : 15B11MA301

Maximum Time : 2 Hrs.
Maximum Marks : 35

Note: All the questions are compulsory.

Q1: Consider the probability density function $f(x) = ae^{-b|x|}$, $-\infty < x < \infty$. Find (i) the relationship between a and b (ii) the cumulative distribution function. [3M]

Q2: If the mean and variance of the binomial distribution are 6 and 1.5 respectively, find the value of $E\{X - P[X \leq 3]\}$. [4M]

Q3: If $X(t) = A + B \sin(\omega t + \phi)$, where A , B and ϕ are independent uniform random variables distributed over $(0,1)$, $(0,2)$ and $(0,2\pi)$ respectively and ω is a constant. Find autocorrelation function $R_{XX}(t, t + \tau)$ and also check whether $\{X(t)\}$ is mean ergodic? [4M]

Q4: Find the average power of the random process with following power spectrum density function [4M]

$$S_{XX}(\omega) = \frac{\omega^2 + 14}{(\omega^2 + 25)(\omega^2 + 36)}$$

Q5: (a) If $\{X(t)\}$ is a Poisson process, then prove that correlation coefficient between $X(t)$ and $X(t+2)$ is $\sqrt{\frac{t}{t+2}}$. (b) Show that interarrival time of the Poisson process with parameter λ obeys an exponential distribution. [4M]

Q6: The TPM of a Markov chain with three states 0, 1, 2 and initial probability vector are

$$P = \begin{bmatrix} 0 & 0.67 & 0.33 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } p^{(0)} = [0.7, 0.2, 0.1] \text{ respectively. Find (i) } P[X_2 = 2 / X_1 = 1, X_0 = 0]$$

(ii) $P[X_3 = 2, X_1 = 1]$ (iii) steady state solution, if possible. Is the chain irreducible? [4M]

Q7: There are two stables on a farm, one that houses 20 horses and 13 mules, the other with 25 horses and 8 mules. Without any pattern, animals occasionally leave their stables and then return to their stables. Suppose that during a period when all the animals are in their stables, a horse comes out of a stable and then returns. What is the probability that the next animal coming out of the same stable will also be a horse? [4M]

Q8: Man leaves for work between 8:00 am and 8:30 am and takes 40 to 50 minutes to get to the office. If departure time and time of travel are independent and uniformly distributed, find the probability that he arrives at office before 9:00 am. [4M]

Q9: The conditional probability density function of X given $Y=y$ and marginal density function of Y are $f_{X|Y}(x/y) = \begin{cases} C_1 \frac{x}{y^2}, & 0 < x < y, 0 < y < 1 \\ 0, & \text{elsewhere (if defined)} \end{cases}$ and $f_Y(y) = \begin{cases} C_2 y^4, & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$ respectively. Find (i) constants C_1 and C_2 , (ii) joint pdf of X and Y , (iii) $P[0.75 < X < 0.5 / Y = 0.625]$ [4M]