POSSESION OF MOBILES IN EXAM IS UFM PRACTICE.

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Jaypee Institute of Information Technology, Noida
End Semester Examination, 2023 (Even)
B.Tech. IV Semester

Course Title: Probability and Random Processes
Course Code: 15B11MA301

Maximum Time: 2 Hrs.
Maximum Marks: 35

After pursuing this course, students will be able to:

CO1: explain the basic concepts of probability, conditional probability and Bayes' theorem.

CO2: identify and explain one and two dimensional random variables along with their distributions and statistical averages.

CO3: apply some probability distributions to various discrete and continuous problems.

CO4: solve the problems related to the component and system reliabilities.

CO5: identify the random processes and compute their averages.

CO6: solve the problems on Ergodic process, Poisson process and Markov chain.

Note: All questions are compulsory. The use of non-programmable calculator is allowed.

1. In a coin tossing experiment, if the coin shows head, one die is thrown and the result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. Let A and B be the events that the recorded numbers are 2 and 5 respectively. Compare the probabilities of A and B. **[CO1, 3M]**

2. The joint probability mass function of X and Y is given as follows: **[CO2, 4M]**

$X\downarrow$ \ $Y\rightarrow$	0	1
-1	1/8	2/8
1	3/8	2/8

Make use of covariance of X and Y to find the correlation coefficient between X and Y.

3. The lifetime of a light bulb is exponentially distributed with mean of 300 hours. Apply the exponential distribution to find the probabilities that (a) a randomly selected light bulb will last over 400 hours, (b) 2 out of 6 randomly selected light bulbs will last over 400 hours, (c) a randomly selected light bulb will last between 200 hours and 600 hours. **[CO3, 3M]**

4. Let the hazard function for a machine be given by $\lambda(t) = \frac{3}{2\sqrt{30}} t^{\frac{1}{2}}$; t (in hours) ≥ 0. Apply the appropriate formula to compute the following:
(i) Reliability for a time of 30 hours, (ii) Reliability for a 30-hour run given that the wear in period is 10 hours, (iii) The design life for a reliability of e^{-30}. **[CO4, 4M]**

5. The autocorrelation function of a WSS process $\{X(t)\}$ is given by $R_{XX}(\tau) = 36 + \frac{5}{1+7\tau^2}$. Utilize the properties of autocorrelation function to find (i) mean and variance of the process $\{X(t)\}$, (ii) $E(Y^2)$, where $Y = X(3) - X(2)$. **[CO5, 4M]**

6. Let $\{X(t)\}$ be a random process given by $X(t) = A\cos\lambda t + B\sin\lambda t$, $\lambda \geq 0$, where A and B are independent normal variables with mean zero and variance 2. Identify whether $\{X(t)\}$ is WSS or not. Support your answer in each case. **[CO5, 4M]**

7. Make use of the properties of Poisson process to show that the sum of two independent Poisson processes is always Poisson but their difference is not. **[CO6, 4M]**

8. A communication source can generate 1 of 3 possible messages 1, 2 and 3. Assume that the generation can be described by a homogeneous Markov chain with the following transition probability matrix P.

	1	2	3
1	0.90	0.05	0.05
2	0.05	0.85	0.10
3	0.10	0.07	0.83

The initial state probability distribution is given by $p^{(0)} = (0.4, 0.4, 0.2)$. Utilize the properties of Markov chain to find (i) $p^{(1)}$ (ii) long-run probabilities. **[CO6, 3M]**

9. Let $\{X(t)\}$ be a wide sense stationary process given by $X(t) = 50\sin(20t + \lambda)$, where λ is a random variable uniformly distributed in the interval $(0, 2\pi)$. Identify whether the process $\{X(t)\}$ is correlation ergodic or not. Give reasons to support your answer. **[CO6, 3M]**

10. The autocorrelation function of a transmission process $\{X(t)\}$ is given by $R(\tau) = k\,e^{-5|\tau|}$, where k is a constant. Find the spectral density function $S(\omega)$ of $\{X(t)\}$. Utilize the condition given as $S(5) - S(10) = 12$ to find the average power of the process. **[CO6, 3M]**

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