Tutorial Sheet : 12

Probability & Random Processes (15B11MA301)

Q1: $Y(t) = \beta X(t)$

$E(Y(t)) = E(\beta)\, E(X(t))$

Here $E(\beta) = \frac{1}{4}\int_{-2}^{2} x\,dx = 0$ & $E(\beta^2) = 4/3$

$\Rightarrow E(Y(t)) = 0$

$R_{YY}(t_1, t_2) = E(Y(t_1)\, Y(t_2)) = E(\beta^2)\, E(X(t_1) . X(t_2))$

$= \frac{4}{3} e^{-2\lambda(t_1 - t_2)}$ $\left\{ \because X(t) \text{ is semi Random telegraph signal Process} \right\}$

$\Rightarrow Y(t)$ is WSS.

Q2: $E(X(t)) = \left[\lim_{\tau \to \infty} R(\tau)\right]^{1/2} = (45+4)^{\frac{1}{2}} = 7$

$var(X(t)) = R(0) - (E(X(t)))^2 = 49.5 - 49 = 0.5$

Q3: $X(t) = A\cos wt + B\sin wt$

$E(AB) = E(A) = E(B) = 0$

$E(X(t)) = \cos wt\, E(A) + \sin wt\, E(B) = 0$

$\lim_{T\to\infty} \bar{X}_T = \lim_{T\to\infty}\left(\frac{1}{2T}\int_{-T}^{T} X(t)\,dt\right)$

$= \lim_{T\to\infty} \frac{A}{2T}\int_{-T}^{T} \cos wt\,dt + \lim_{T\to\infty}\frac{B}{2T}\int_{-T}^{T}\sin(wt)\,dt$

$= 0$

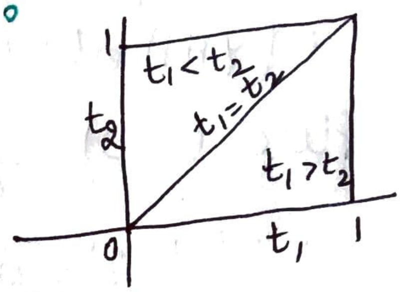
$\lim_{T\to\infty} \bar{X}_T = E(X(t)) = 0 \Rightarrow \{X(t)\}$ is Mean Ergodic Process

Q4: $E(X(t)) = 2, \quad R_{XX}(\tau) = 4 + e^{-|\tau|/10} \quad \{\tau = t_1 - t_2\}$

$\Rightarrow C_{XX}(\tau) = e^{-|\tau|/10}$

$E(S) = E(X(t)) = 2$ Here $S = \int_0^1 X(t)\,dt$

$S^2 = \int_0^1 X(t_1)\,dt_1 \int_0^1 X(t_2)\,dt_2$



$E(S^2) = \int_0^1\int_0^1 E(X(t_1)\,X(t_2))\,dt_1\,dt_2$

$= \int_0^1\int_0^1 R_{XX}(\tau)\,dt_1\,dt_2 = \int_0^1\int_0^1 4 + e^{-(|t_1-t_2|)/10}\,dt_1\,dt_2$

$= \int_0^1\int_0^{t_2} 4 + e^{(t_1-t_2)/10}\,dt_1\,dt_2 + \int_0^1\int_0^{t_1} 4 + e^{(t_2-t_1)/10}\,dt_2\,dt_1$

$(t_1 > t_2)$ $(t_2 < t_1)$

$= \int_0^1 \left(4t_1 + 10 - 10e^{-t_1/10}\right) dt_1 + \int_0^1 \left(4t_2 + 10 - e^{-t_2/10}\right) dt_2$

$= 24 + 200(e^{-1/10} - 1) = -176 + 200e^{-1/10}$

$\mathrm{var}(S) = E(S^2) - (E(S))^2 = -176 + 200e^{-1/10} - 4$

$= 200e^{-1/10} - 180$

Q5: $X(t) = 10\cos(100t + \theta)$; $\theta \sim U(-\pi, \pi)$

$E(X(t)) = 10\,E(\cos(100t + \theta))$

$= 10 \int_{-\pi}^{\pi} \frac{1}{2\pi} \cos(100t + \theta)\,d\theta$

$= 5/\pi \left[\sin(100t + \theta)\right]_{-\pi}^{\pi} = 0$

$\frac{1}{2T}\int_{-T}^{T} 10\cos(100t + \theta)\,dt = \frac{10}{2T} \left.\frac{\sin(100t+\theta)}{100}\right|_{-T}^{T}$

$= \frac{1}{10T}\sin(100T)$

lim $T \to \infty$ on both sides

$$\lim_{T\to\infty}\left(\frac{1}{2T}\int_{-T}^{T} 10\cos(100t+\theta)\,dt\right) = \lim_{T\to\infty}\left(\frac{1}{10T}\sin 100T\right)$$

$$= 0 = \mu = E(X(t))$$

$\Rightarrow$ It is mean ergodic Random Process.

$$R(\tau) = E(X(t)\,X(t+\tau))$$

$$= E\left((100\cos(100t+\theta))(\cos(100t+100\tau+\theta))\right)$$

$$= 50\,E\left[\cos(200t+100\tau+2\theta) + \cos(100\tau)\right]$$

$$= 50\cos(100\tau) + \frac{50}{2\pi}\int_{-\pi}^{\pi}\cos(200t+100\tau+2\theta)\,d\theta$$

$$= 50\cos(100\tau) + 0 = 50\cos(100\tau)$$

$$\overline{Y}_\tau = \frac{1}{2T}\int_{-T}^{T} X(t+\tau)\,X(t)\,dt$$

$$= \frac{1}{2T}\int_{-T}^{T} 100\cos(100t+100\tau+\theta)\cos(100t+\theta)\,dt$$

$$= \frac{100}{2T\times 2}\left[\cos(100\tau) + \frac{\sin(200t+100\tau+2\theta)}{200}\right]_{-T}^{T}$$

Taking limit $T \to \infty$

$$\lim_{T\to\infty}\overline{Y}_T = 50\cos(100\tau) = R(\tau)$$

$\Rightarrow \{X(t)\}$ is correlation ergodic.

Q6! $\overline{X}_T = \frac{1}{2T}\int_{-T}^{T} X(t)\,dt$

Given $R(\tau) = 1 - \frac{|\tau|}{T}$ & $E(X(t)) = E(\overline{X}_T) = 0$

$$C(\tau) = R(\tau) - (E(X(t)))^2 = 1 - \frac{|\tau|}{T}$$

$$\mathrm{var}(\overline{X}_T) = \frac{1}{T}\int_{-T}^{T}\left(1-\frac{|\tau|}{T}\right)^2 d\tau$$

$$= \frac{2}{T}\int_{0}^{T}\left(1-\frac{|\tau|}{T}\right)^2 d\tau$$

$$= \frac{2}{T}\int_{0}^{\tau}\left(1-\frac{\tau}{T}\right)^2 d\tau = 2/3$$

$$\lim_{T\to\infty}\{\mathrm{var}\,\overline{X}_T\} = 2/3 \neq 0$$

$\Rightarrow \{X(t)\}$ is not mean Ergodic