JAYPEE INSTITUTE OF INFORMATION TECHNOLOGY
Electronics and Communication Engineering
Digital Systems (18B11EC213)
Tutorial Sheet: 6

Solution 1 (a)

$$x(t) = 1 + \frac{1}{6}\cos(2\pi t) + \frac{1}{3}\cos(4\pi t) + \cos(6\pi t) =$$

$$1 + \frac{1}{12}e^{j2\pi t} + \frac{1}{12}e^{-j2\pi t} + \frac{1}{6}e^{j4\pi t} + \frac{1}{6}e^{-j4\pi t} + \frac{1}{2}e^{j6\pi t} + \frac{1}{2}e^{-j6\pi t}$$

The fundamental frequency is $\omega_0 = 2\pi$ and

$$a_0 = 1 \ (dc\ value)$$

$$a_1 = a_{-1} = \frac{1}{12}$$

$$a_2 = a_{-2} = \frac{1}{6}$$

$$a_3 = a_{-3} = \frac{1}{2}$$

$$a_k = 0 \ \ for \ \ k \neq 0, \pm 1, \pm 2, \pm 3$$

$$x(t) = \sum_{k=-3}^{3} a_k . e^{jk2\pi}$$

Solution 1 (b)

Sol 1 (b)

$$x(t) = \begin{cases} A & 0 \leq t \leq \pi \\ -A & \pi \leq t \leq 2\pi \end{cases}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{2\pi}\int_0^{\pi} A\,dt + \frac{1}{2\pi}\int_{\pi}^{2\pi}(-A)\,dt = 0$$

coefficients a_k for $k = 1, 2, 3 \ldots$

$$a_k = \frac{1}{T}\int_0^T x(t)\,e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{2\pi}\int_0^{\pi} A\,e^{-jkt}\,dt + \frac{1}{2\pi}\int_{\pi}^{2\pi}(-A)\,e^{-jkt}\,dt$$

$$= \frac{-A}{j2\pi k}\left[e^{-jk\pi} - 1 - e^{-jk2\pi} + e^{-jk\pi}\right] \qquad \{\because e^{-jk2\pi} = 1\}$$

$$= -\frac{A}{j2\pi k}\left[2e^{-jk\pi} - 2\right]$$

$$a_k = \frac{A}{j\pi k}\left[1 - e^{-jk\pi}\right]$$

if k is even

$$a_k = 0$$

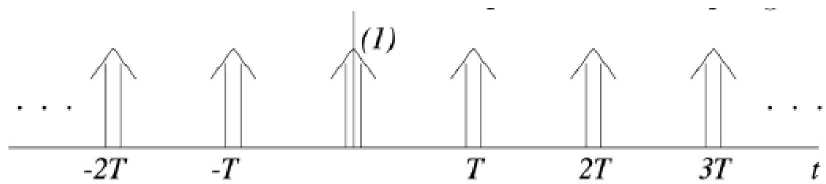
if k is odd

$$a_k = \frac{2A}{j\pi k}$$

$$a_k = \begin{cases} 0 & \text{for } k = 0, 2, 4 \ldots \text{ even} \\ \dfrac{2\pi}{j\pi k} & \text{for } k = 1, 3, 5, \ldots \text{ odd} \end{cases}$$

Solution 2

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



$$\begin{aligned} a_k &= \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt \\ &= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)e^{-jk\omega_0 t}dt \\ &= \frac{1}{T} \quad \text{for } all\ k\ \ ! \end{aligned}$$

ns-3 (a) $\quad F\{e^{-at}u(t)\} \Longleftrightarrow \dfrac{1}{a+j\omega}$

$$F\{e^{-a(t-t_0)}u(t-t_0)\} = \frac{e^{-j\omega t_0}}{a+j\omega}.$$

$$F\{e^{-2(t-1)}u(t-1)\} = \frac{e^{-j\omega}}{2+j\omega} \quad \underline{Ans}$$

3(b) $g(t) = e^{-2|t-1|}$

$$F\{e^{-a|t|}\} = \frac{2a}{a^2+\omega^2}$$

$$F\{e^{-2|t|}\} = \frac{4}{4+\omega^2}$$

By applying time shifting property

$$F\{e^{-2|t-1|}\} = \frac{4e^{-j\omega}}{4+\omega^2}$$ Ans.

Ans -4(a) $g_1(t) = e^{-t}\cos(w_c t)\, u(t)$.

$$= e^{-t}u(t)\left(\frac{e^{jw_c t} + e^{-jw_c t}}{2}\right)$$

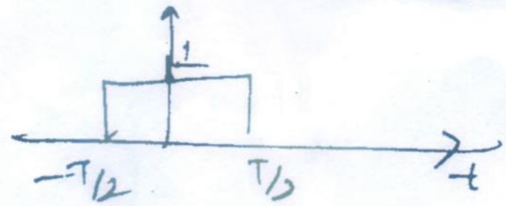
$$F\{e^{-t}u(t)\} = \frac{1}{1+jw}$$

$$F\left\{\frac{1}{2}e^{-t}e^{jw_c t}u(t)\right\} = \frac{1}{2}\left(\frac{1}{1+j(w-w_c)}\right)$$

$$F\left\{\frac{1}{2}e^{-t}e^{-jw_c t}u(t)\right\} = \frac{1}{2}\left[\frac{1}{1+j(w+w_c)}\right]$$

$$F\{e^{-t}\cos(w_c t)\,u(t)\} = \frac{1}{2}\left[\frac{1}{1+j(w-w_c)} + \frac{1}{1+j(w+w_c)}\right]$$

Ans



$$F\left\{ rect\left(\frac{t}{T}\right)\right\} = \int_{-T/2}^{T/2} 1 . e^{-j2\pi f t} \, dt$$

$$= \frac{e^{-j2\pi f}}{-j2\pi f} \Bigg|_{-T/2}^{T/2}$$

$$= \frac{-e^{-j\pi T f} + e^{j\pi T f}}{j2\pi f}$$

$$= \frac{\left[\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j}\right]}{\pi f}$$

$$= \frac{\sin(\pi f T)}{\pi f}$$

$$= T \frac{\sin(\pi f T)}{\pi f T}$$

$$= T . \operatorname{Sinc}(f T)$$

Solution 5 (a)

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$$

(b) Consider the Fourier transform of the unit step $x(t) = u(t)$

$$g(t) = \delta(t) \xleftrightarrow{F} 1$$

Also note that

$$x(t) = \int_{-\infty}^{t} g(\tau) d\tau$$

The Fourier transform of this function is

$$X(j\omega) = \frac{1}{j\omega} + \pi G(0)\delta(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)\,.$$