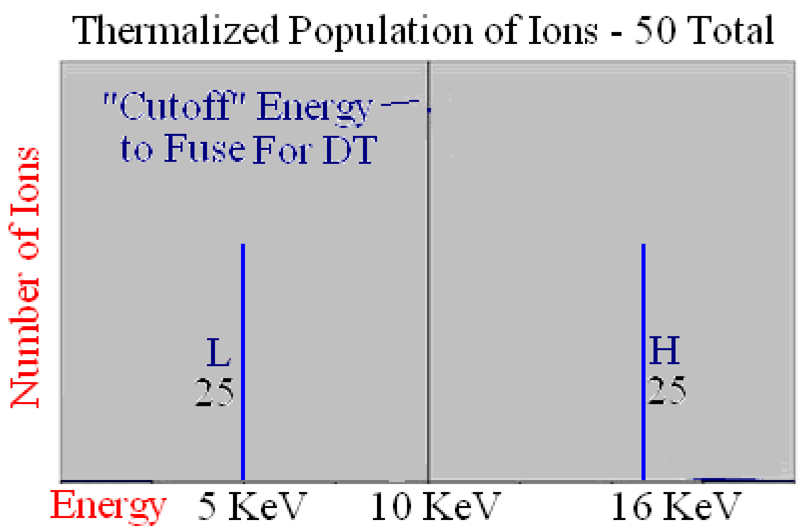
A very simple model for the polywell to spark discussion:

Imagine we had a population of 50 ions and 50 electrons inside the polywell. Now we spilt up the ions into two groups, half have enough energy to fuse and half do not. This is analogous to an ion population with a Boltzmann distribution illustrated below.



Now there are a couple of reactions that we can model. Fusion reactions are high energy ions hitting high energy ions. We will call these H ions; H is for high energy. When H and H hit they should fuse creating an F. F is for a fusion product. Assume that F's leave the system. We will call the other ions L; L stands for low energy. When an H meets an L, we would assume they would both leave with equal energy. We do not care what this energy is; suffice to say they both now lack enough energy to fuse. Bremsstrahlung reactions (or X-ray reactions) are electrons hitting either of these types. Anytime an electron whizzes past an ion, an X-ray can be made. Assume this reaction would drop H ions into L ions. Assume that the electron population remains constant; we feed in enough electrons to compensate for losses. If you move along this line of thinking, then you can setup six possible reactions. These are illustrated below.

1. $E + E \rightarrow E + E$
2. $E + H \rightarrow E + L$
3. $E + L \rightarrow E + L$
4. $H + H \rightarrow F$
5. $H + L \rightarrow L + L$
6. $L + L \rightarrow L + L$

Since we have a population of 100 electrons and ions, then we can figure out what the probability of each of the above reactions is. We can do this by applying basic statistics. For example the total number of reactions in a population of a 100 where it is just two things hitting one another, is 100 choose 2. Mathematically this is shown below.

$$NumberOfAllPossibleReactions = \binom{100}{2} = \frac{100!}{2!(100-2)!} = 4950$$

The same method is used anytime a reaction involves something hitting itself; for example H and H reactions. In other cases, you multiply the size of the two populations. Using this process we can calculate the likelihood of each reaction inside the polywell.

$$\text{Number of E + E Reactions} = \binom{50}{2} = 1225 \quad \text{or} \quad 25\%$$

$$\text{Number of H + H Reactions} = \binom{25}{2} = 300 \quad \text{or} \quad 6\%$$

$$\text{Number of L + L Reactions} = \binom{25}{2} = 300 \quad \text{or} \quad 6\%$$

$$\text{Number of H + L Reactions} = (25 * 25) = 625 \quad \text{or} \quad 13\%$$

$$\text{Number of H + E Reactions} = (50 * 25) = 1250 \quad \text{or} \quad 25\%$$

$$\text{Number of L + E Reactions} = (50 * 25) = 1250 \quad \text{or} \quad 25\%$$

Note that the fusion reactions are only 6% of all possible reactions. With all the possible reactions of the system enumerated and their probabilities laid out, the next step is to factor in the energy. If one looks at deuterium, deuterium reactions very quickly it becomes clear that H+H → F is a serious oversimplification. Here is a list of all the possible deuterium reactions from "Physics of Fusion Fuel Cycles".

Primary Reactions[a]

$D + T \rightarrow n + {}^{4}He + 17.586\ MeV\ (3.517\ MeV)$

$D + D \rightarrow p + T + 4.032\ MeV\ (4.032\ MeV)$

$D + D \rightarrow n + {}^{3}He + 3.267\ MeV\ (0.817\ MeV)$

$D + {}^{3}He \rightarrow p + {}^{4}He + 18.341\ MeV\ (18.341\ MeV)$

Fig 2: The two DD reactions, with two possible side reactions and the energy produced in millions of electron volts. Listed is the total energy released by reaction followed by the subset of that energy trapped in the hot neutron.

For our model, we want a simple amount of energy to assign to the H+H reaction. The number we will use will be positive 10.8 MeV. For a description of how this number was decided upon, please see the appendix. The average amount of energy an x-ray can have is between 12 eV and 12 KeV. We will assign negative 11 KeV to the E+H reaction, and negative 3 keV to the E+L reaction. This again is an oversimplification; the reader can refer to the NRL plasma foundry for a much better treatment of x-ray cooling. We can assume all the rest of the reactions are elastic. Based on this, below are the energies of each reaction.

1. $E + E \rightarrow E + E$
2. $E + H \rightarrow E + L \quad (-0.011\ \text{MeV})$
3. $E + L \rightarrow E + L \quad (-0.003\ \text{MeV})$
4. $H + H \rightarrow F \quad (+10.8\ \text{MeV})$
5. $H + L \rightarrow L + L$
6. $L + L \rightarrow L + L$

From this, one can see why fusion is so energetically desirable. Given all that has been outlined above it becomes possible to track material and energy into and out of our system. For a DT reaction, ions need about 10 keV to fuse together. The reaction we are treating is DD. Assume the H ions 16 keV and L ions 5 keV, so we can track the total energy across the system. Details on this calculation and the energy assignment can be found in the appendix. Therefore an energy balance across the system in MeV is:

$$\boxed{Energy = -LXRays * 0.003 - HXRay * 0.011 + F * 10.8 + HIon * 0.016 + LIon * 0.005}$$

Model:

Using this construct, one can create a little model in excel. This was done, using a random number generator, and the probabilities to choose which of the six reactions take place. Graphs of material and energy balance from a given run are presented below.

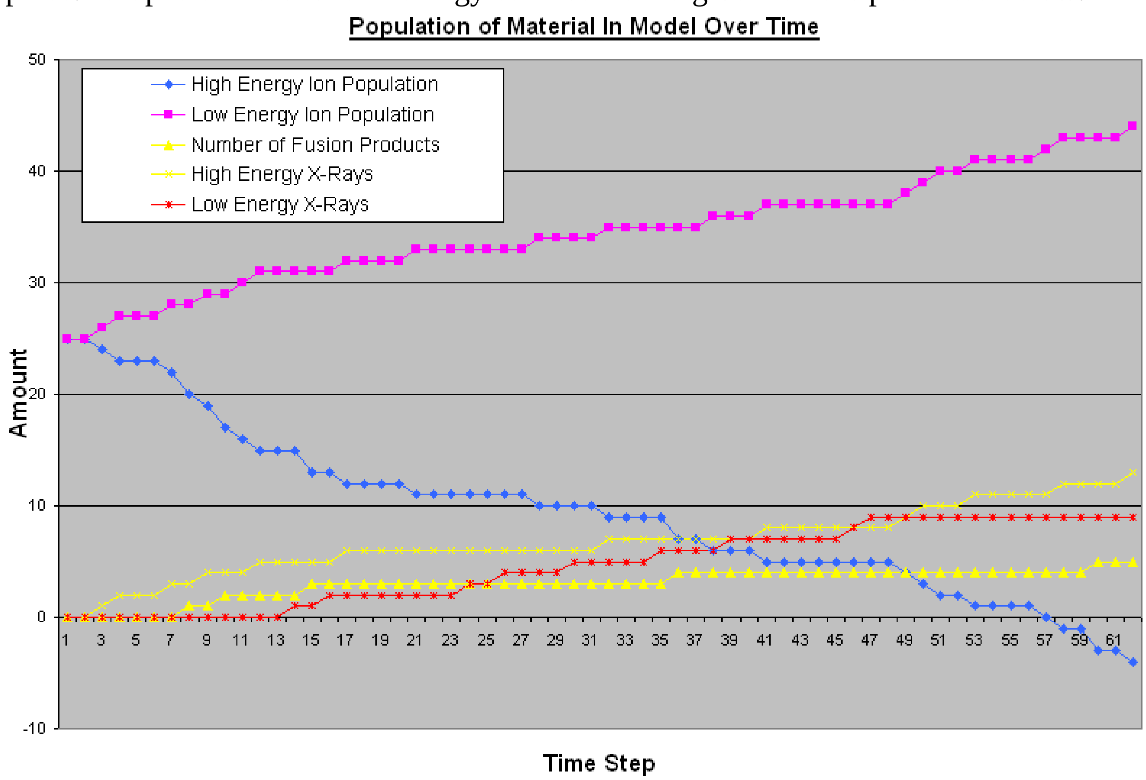


Fig 3: material overtime generated by model.

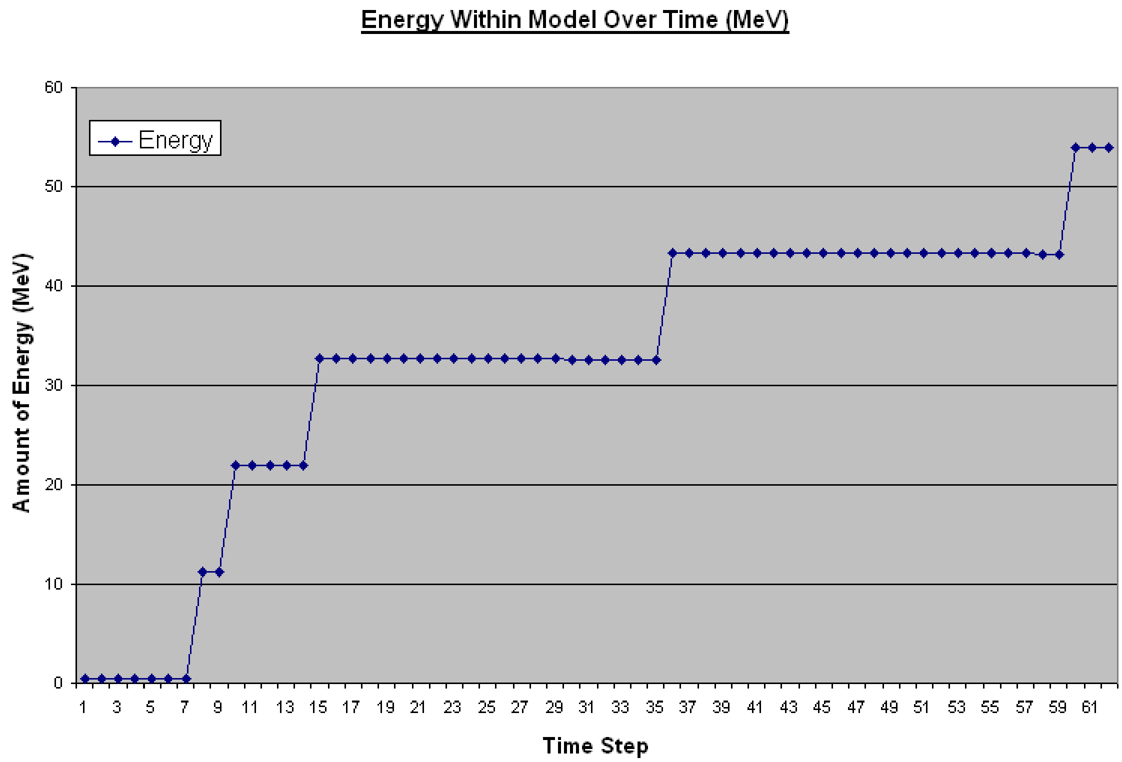


Fig 4: Energy within model over time.

Conclusion:

The point of this model is to make the reader think about the polywell from a rough engineering standpoint. It is by no means complete, nor does it claim to be. Many of the assumptions in it are weak. There are many effects that it does not take into account. Based on Bussard's work, a reasonable estimate is that there was about 1.4E12 electrons and about 1E6 fewer ions inside the WB-6 device (see appendix). One interesting conclusion from the model is only 6% of the reactions would be fusion reaction. This percentage does not change if one enters in the population of electrons, H and L ions estimated from Bussards' WB6 work. These would presumably be 1.4E12 electrons, 7E11 L and H ions and 2.8E12 total things in the system. Bussard also estimated that electrons have about 100,000 re-circulations before they hit a metal surface and leave the polywell[6]. This loss was not accounted for. The model also oversimplifies the energy of the ions, into two H and L ion groups. This assumption has been shown to be critical to the Polywell working. It has been shown that a population of ions with a bell curve inside the polywell will kill the device[14]. Bussard also claimed that his WB-6 experiment did deuterium-deuterium fusion with a 10 kilovolt potential well[6]. These claims now seem reasonable. If it takes about 10 KeV for DT fusion, Bussards reactor would be in the right ballpark for deuterium fusion.

Appendix:

Assigning Energy to the H+H reaction:

This is an estimate at best. One way to do this is to assume that all the reactions that can occur do occur. Then calculate the number of times each reaction occurs and the energy produced with each reaction. You can then get the weighted average energy. Assume that 16.66 deuterium ions go into the first round of fusion reactions; that means there are 8.33 pairs. You can calculate the probability of each of the first fusion reactions by the ratio of their energies.

1. $\text{Probabilty Of } D + D \rightarrow T \text{ is } \left(\frac{4.032}{7.299}\right) = 55\% \text{ or } 4.60 \text{ T generated}$
2. $\text{Probabilty Of } D + D \rightarrow He \text{ is } \left(\frac{3.267}{7.299}\right) = 45\% \text{ or } 3.73 \text{ He generated}$

Next, assume that all the T and He fuse with the all remaining deuterium. There is no probability here because T reactions do not compete with He reactions. Assuming this, 16.67 total reactions took place across the whole system. Then you can find the weighted average energy for our model for fusion reactions.

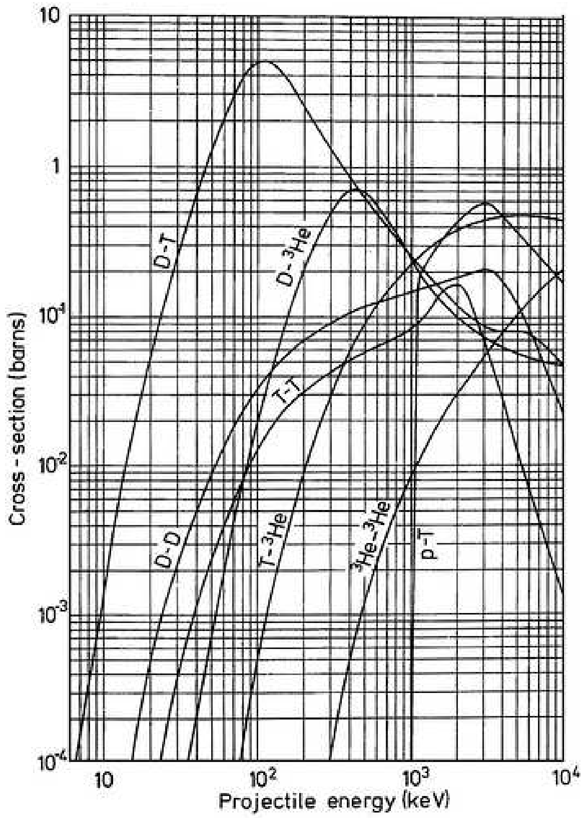
3. $T + D \rightarrow He, \text{ creates } 17.586 \text{ MeV of Energy, } 4.60 \text{ He generated}$
4. $He + D \rightarrow He, \text{ creates } 18.341 \text{ MeV of Energy, } 3.73 \text{ He generated}$

$$\text{Weighted Average Energy} = \left(\frac{4.6}{16.7}\right) * 4.032\text{Mev} + \left(\frac{3.7}{16.7}\right) * 3.26\text{Mev} + \left(\frac{4.6}{16.7}\right) * 17.58\text{Mev} + \left(\frac{4.6}{16.7}\right) * 18.34\text{Mev}$$

This means that the average energy for every H+H interaction is 10.80 MeV.

Assigning Energy to the H and L Ions:

The easiest type of fusion is deuterium tritium fusion. This is based off measurements where one ion was smashed into other ion. Researchers fired different ions at one another at various speeds and measured the amount of fusion reactions that occurred. This is quantified in a fusion reactions' cross section. Cross sections are area measurements in units of barns or 10^{-28} meters^2. Cross sections for various fusion reactions are plotted below[12].



This cross section is used to calculate the rate of fusion for large thermalized clouds of ions. The normal way to measure fusion within these clouds is some variation on the equation below.

$$F = N_1 * N_2 * v * \sigma$$

$$\left[\frac{Fusion}{Seconds * Volume}\right] = \left[\frac{Ions}{Volume}\right] * \left[\frac{Ions}{Volume}\right] * [Velocity] * \left[Meter^2\right]$$

Where, F is fusions per second per volume, n is ions per volume, v is the average velocity of the ions in the cloud and sigma the average cross section of the reaction.

One can also calculate the energy it would take to fuse just one deuterium and one tritium ion. The ions need to overcome the columbic repulsive forces, and come close enough together to where the nuclear force can take over. The nuclear force binds nucleouse together. This process is made easier by quantum tunneling. Quantum tunneling arises out of the wave-particle nature of protons. A proton, acting like a particle could not tunnel across a barrier; but if it acts like a wave it can. The effect lowers the energy needed to fuse DT. The energy is about 10 kiloelectronvolts[4]. Based on this, you can estimate what the temperature of a thermalized cloud of pure deuterium would need to fuse. You have to assume all the ions have an average temperature is 10 KeV.

$$T = \left(\frac{1}{\text{Cp}}\right) * \text{Average Energy Per Ion} * \text{Ions In Cloud} * \left(\frac{1}{\text{Mass Of Deterium In Cloud}}\right)$$

$$\text{Cp} = 5300 \frac{\text{Joules}}{\text{Kg} * \text{K}}, \text{For Gaseous Deterium}^{13}$$

$$\text{Average Energy Per Ion} = 1.602\text{E} - 15 \text{ Joules or } 10 \text{ KeV}$$

$$\text{Ions in Cloud} = 1\text{E}12, \text{Estaimated Ion Population In Bussards' WB6 Expirement}$$

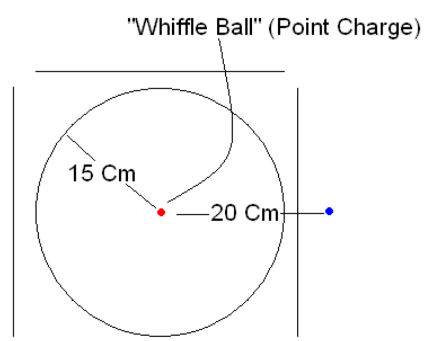
$$\text{Mass of 1 Deterium Ion} = 3.3444E - 27Kg^{13}$$

One reference cited a temperature of 120 million kelvins[4] for a DT cloud this is compared to our estimate of 92 million kelvins for a D cloud. For DT, one would imagine a cloud split between the ions. The temperature of this cloud could not be estimated because the heat capacity of tritium could not be found. In any case 10 kiloelectronvolts is a good estimate for what is needed for fusion for DD. It should be noted that Bussards' WB6 experiments

Calculating the amount of Ions & Electrons in the WB6 Test:

If you accept that one can form a Whiffle ball; then a number of conclusions naturally follow from that. For basic analysis, we can treat the Whiffle ball like a point charge. From a general physics textbook we can find the follow equation for a point charge:

$$FieldStrength = \left(\frac{q_2}{4\pi \varepsilon_0 r^2}\right)$$



For my analysis, I assume 20 cm there is no charge. Bussard claimed there was a 10 kilovolt well in the center, from a 12.5 kV drive voltage. Applying the above equation, this means there are about 1.4E12 electrons in the Whiffle ball. Bussard claimed that the electron density in the Whiffle ball was 1E13 electron/cm^3. This means the Whiffle ball would be 0.6 cm in diameter, about five one millionths of the total volume.

Calculating the amount of Electrons needed for a Diamagnetic Whiffle Ball:

The "Whiffle ball" is a theory. It is not necessarily happening. It also may not be needed by the polywell for it to produce power. The idea is if you jam enough electrons together and subject them to a strong magnetic field, they behave diamagnetically. They push the field back. Diamagnetism is when something that is not a magnet, acts like a magnet. The example used in the film is a frog which becomes magnetic. It took about 16 Teslas to make the frog fly. In Bussards Valencia paper we can find the quote describing conditions in WB-6. "If all the electrons were still at ca. 100 eV, … at B=1000 [Gauss]." Let's assume that the magnetic field was 1000 gauss or 0.1 teslas and the electrons have energy of 100 eV.

Electrons spin. Their spin creates a magnetic moment, like a tiny magnet. A quick and easy way to find out if the Whiffle ball is possible is to see if 1.4E12 electrons would have enough magnet strength to match the 0.1 teslas external field. This is an easy estimate to make.

$$\text{Magnetic Strength} = \left(\frac{1}{\text{Electron Moment}} \right) * \text{Energy Per Electron} * \text{Electrons In Cloud} \approx \text{1E17 Teslas}$$

$$\text{Electron Magnetic Moment} = 928.4\text{E - 26} \frac{\text{Joules}}{\text{Tesla}}$$

$$\text{Energy Per Electron} = 1.602\text{E} - 18 \text{ Joules Or } 100\text{eV}$$

$$\text{Electrons In Cloud} = 1.4\text{E12 Electrons}$$

It appears they would have enough magnetic strength.

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