1 Using inverse of CDF to generate different PDFs

All results are described in Figure 1.

1.1 Normal Distribution

The normal distribution's Probability Density Function (PDF) is given by

$$N(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{1}$$

The Cumulative Density Function (CDF) for the above equation is given by

$$C_N(t;\mu,\sigma^2) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{t-\mu}{\sqrt{2\sigma^2}}\right)\right) \tag{2}$$

Where erf is the *error function*, given by

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} \mathrm{d}x \tag{3}$$

The CDF equation $y = C_N(x;\mu,\sigma^2)$ can be inverted (get x for given y) as follows

$$\begin{aligned} y = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sqrt{2\sigma^2}}\right)\right) &\Rightarrow \operatorname{erf}\left(\frac{x-\mu}{\sqrt{2\sigma^2}}\right) = 2y-1 \\ \Rightarrow \frac{x-\mu}{\sqrt{2\sigma^2}} = \operatorname{erfinv}(2y-1) &\Rightarrow x = \mu + \sqrt{2\sigma^2}\operatorname{erfinv}(2y-1) \end{aligned} \tag{4}$$

Note that erfinv is the *inverse of the error function*.

Generating PDF through this is explored in Figure 1a with the corresponding code in **??**.

1.2 Rayleigh Distribution

The Rayleigh distribution's Probability Density Function is given by

$$R(x;\sigma^2) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right) \quad x \geq 0 \tag{5}$$

The Cumulative Density Function for the above equation is given by

$$C_R(t;\sigma^2) = 1 - \exp\left(\frac{-t^2}{2\sigma^2}\right) \quad t \geq 0 \tag{6}$$

The CDF equation $y = C_R(x;\sigma^2)$ can be inverted as follows

$$\begin{aligned} y = 1 - \exp\left(\frac{-x^2}{2\sigma^2}\right) &\Rightarrow (1-y) = \exp\left(\frac{-x^2}{2\sigma^2}\right) \\ \Rightarrow \frac{x^2}{2\sigma^2} = -\ln(1-y) &\Rightarrow x = \sigma\sqrt{-2\ln(1-y)} \\ \Rightarrow x = \sqrt{2\sigma^2 \ln\left(\frac{1}{1-y}\right)} & \end{aligned} \tag{7}$$

Generating PDF through this is explored in Figure 1b with the corresponding code in **??**.

1.3 Exponential Distribution

The Exponential distribution's Probability Density Function is given by

$$E(x;\lambda) = \lambda e^{-\lambda x} \quad x \geq 0 \tag{8}$$

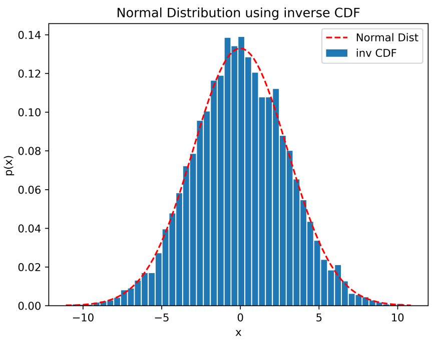
The Cumulative Density Function for the above equation is given by

$$C_E(t;\lambda) = 1 - e^{-\lambda t} \tag{9}$$

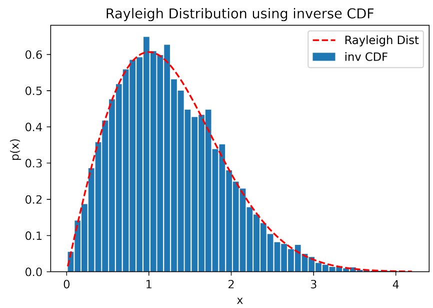
The CDF equation $y = C_E(x;\lambda)$ can be inverted as follows

$$\begin{aligned} y = 1 - e^{-\lambda x} \Rightarrow e^{-\lambda x} = 1 - y \Rightarrow x = \frac{-1}{\lambda} \ln(1-y) \\ \Rightarrow x = \frac{1}{\lambda} \ln\left(\frac{1}{1-y}\right) \end{aligned} \tag{10}$$

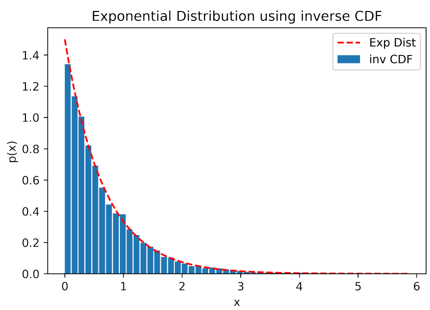
Generating PDF through this is explored in Figure 1c with the corresponding code in **??**.



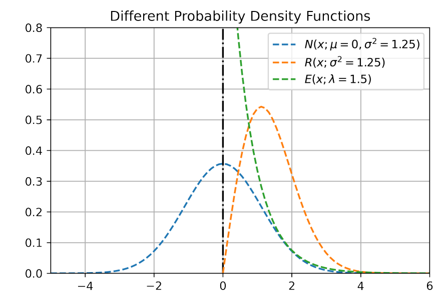
(a) $N(x; \mu = 0, \sigma^2 = 9)$



(b) $R(x; \sigma^2 = 1)$



(c) $E(x; \lambda = 1.5)$



(d) N, R and E PDFs

Figure 1: Using the inverse of CDFs to generate a PDF

Figure 1a consists of the *normal distribution* generated through inverse CDF in 50 bins (as blue bars) and the actual normal PDF through function (as red dotted line). Code for this is in **??**. Figure 1b consists of the *rayleigh distribution* generated through inverse of CDF (as blue bars) and the actual rayleigh PDF through function (as red dotted line). Code for this is in **??**. Figure 1c consists of the *exponential distribution* generated through inverse of CDF (as blue bars) and the actual exponential PDF through function (as red dotted line). Code for this is in **??**. Figure 1d consists of all three PDFs (with different parameters) in one plot, just for comparison. The *Normal PDF* shown in blue line has $\mu = 0$ and $\sigma^2 = 1.25$, the *Rayleigh PDF* shown in orange line has $\sigma^2 = 1.25$, and the *Exponential PDF* shown in green line has $\lambda = 1.5$. The code responsible for this is mentioned in Appendix **??**