1 Experimenting sum of samples from Uniform Distribution

The result is demonstrated in Figure 1 and code can be found in Appendix **??**.

The experiment is to generate multiple random numbers from a *Uniform Distribution* (0 to 1), sum them, store the result in a buffer and then do the same again. The resultant buffer has to be viewed as a normalized histogram.

The resultant distribution achieved is called an *Irwin-Hall distribution* which is defined as the sum of n independent and identically distributed $U(0,1)$ random variables.

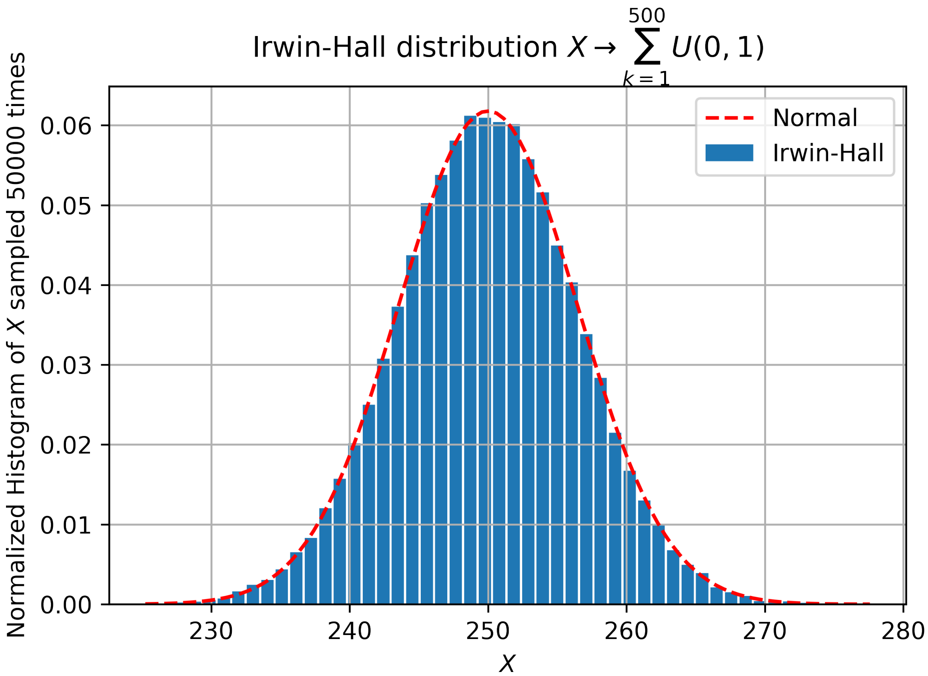

Figure 1: Irwin-Hall Distribution

This figure is obtained by visualizing the histogram of multiple experiments. In each experiment, 500 random numbers were generated from $U(0,1)$ (uniform distribution) and added, the result being returned. The histogram is in blue, the red line is a *normal distribution* with $\mu = \frac{N}{2} = 250$ and $\sigma^2 = \frac{N}{12} = 41.\bar{6}$. The code responsible for this figure can be found in Appendix A **??**.

Irwin-Hall Distribution

The Irwin-Hall distribution is defined as the distribution of a continuous random variable X, where

$$X = \sum_{k=1}^{n} U(k;0,1) \tag{1}$$

Basically, X is the sum of n independent and identically distributed uniform distributions (spanning 0 to 1). The probability density function is given by

$$f_X(x;n) = \frac{1}{2\,(n-1)!} \sum_{k=0}^{n} (-1)^k \binom{n}{k} (x-k)^{n-1} \operatorname{sgn}(x-k) \tag{2}$$

Where sgn is the sign function defined by

$$\operatorname{sgn}(x-k) = \begin{cases} -1 & x < k \\ 0 & x = k \\ +1 & x > k \end{cases} \tag{3}$$

This is **different** from the normal distribution, but it can be *approximated* to one by using $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{12}$ where n is the number of times the summation is done. Such an approximated fit is shown in Figure 1.